Towards analog reheating of the universe in the laboratory

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Synthetic Quantum Systems SynQS

Reheating of the universe after inflation

Cosmic inflation

The very early universe underwent a phase of accelerated expansion.

?

Hot big bang cosmology

The early universe was a hot thermal plasma of matter and radiation

Reheating of the universe

The inflaton decays into particles

Kofman et al., PRL 73 1994

What aspects of the reheating dynamics can be simulated with trapped atomic gases?

- Cosmic expansion
- Particle production after inflation
- Thermalization

Cosmic expansion in the lab

- Cosmology:
  - Expanding universe
    - Flat **FRW metrics**: characterized by a scale factor $a(t)$

$$\Delta x_{\mathrm{phys}} = a(t)\Delta x$$

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- Cosmology:
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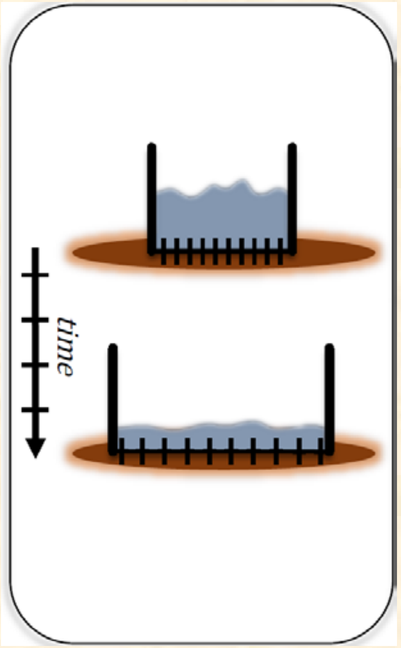
- Expansion in a trapped Bose gas:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\Delta}{2m} + U_{\rm ext}\right)\psi + g\psi^{+}\psi\psi,$$

$$+$$

physically expand the trapping potential

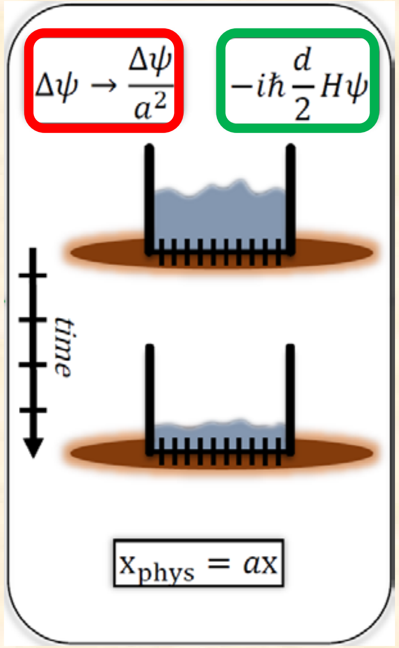
$$U_{\rm ext} \rightarrow U_{\rm ext}[a(t)]$$



M. Uhlmann et al. New J. Phys. 7, 2005, Fedichev and Fischer PRL 91, 2003
S. Eckel et al. PRX 8 2018

Cosmic expansion in the lab

Alternative approach: identify the (lab) coordinates with co-moving ones:



$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\Delta}{a^2} + U_{\text{ext}}\right)\psi + g\psi^+\psi\psi - i\hbar\frac{d}{2}H\psi,$$

Red-shift

Hubble friction: $H = \frac{\dot{a}}{a}$

Non-relativistic limit of KG equation

$$\frac{1}{c^2}\ddot{\varphi} + \frac{1}{c^2}dH\dot{\varphi} - \frac{\Delta\varphi}{a^2} + \frac{\delta U}{\delta\varphi} = 0,$$

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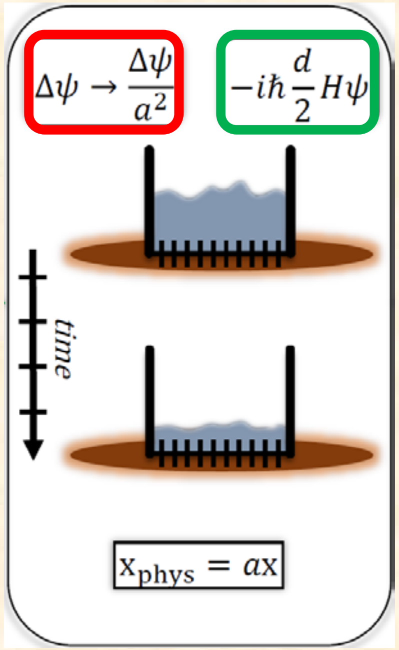

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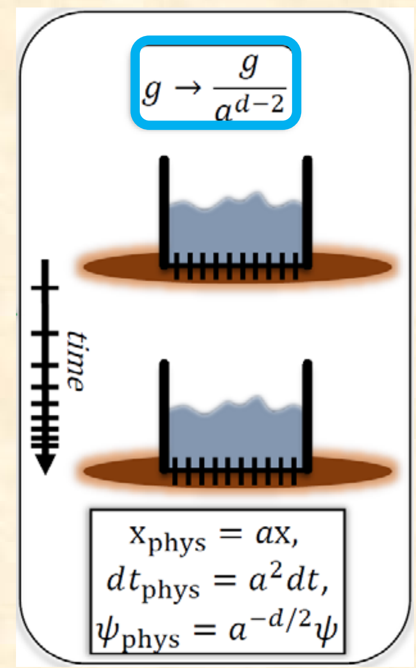
- Transform to new variables

$$\psi = \tilde{\psi}a^{-d/2}, \qquad dt = a^2 d\tau,$$

$$i\hbar\frac{\partial\tilde{\psi}}{\partial\tau} = \left(-\frac{\hbar^2}{2m}\Delta + \tilde{U}_{\rm ext}\right)\tilde{\psi} + \left(\frac{g}{a^{d-2}}\right)\tilde{\psi}^+\tilde{\psi}\tilde{\psi}.$$

Time-dependent coupling



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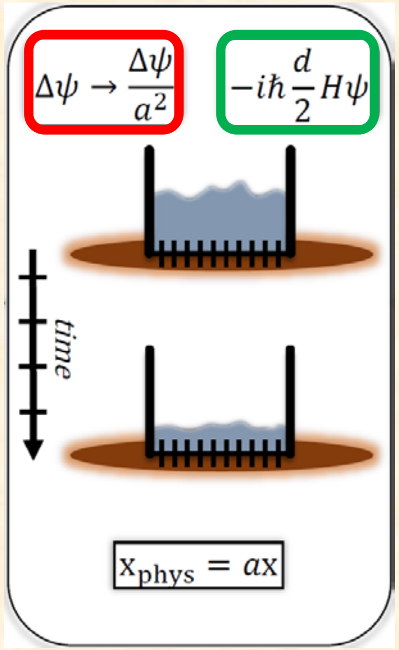

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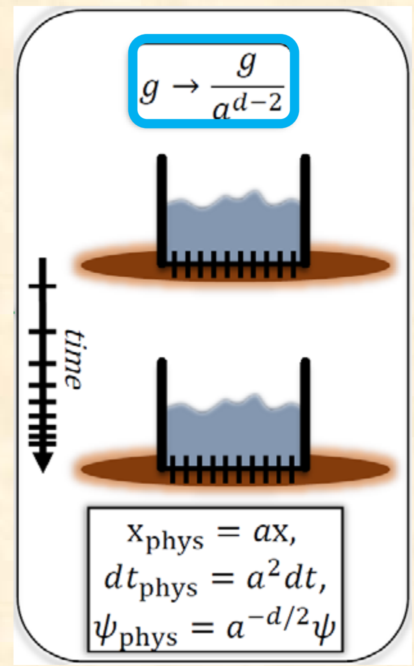


- $d = 2$: scale-invariance:

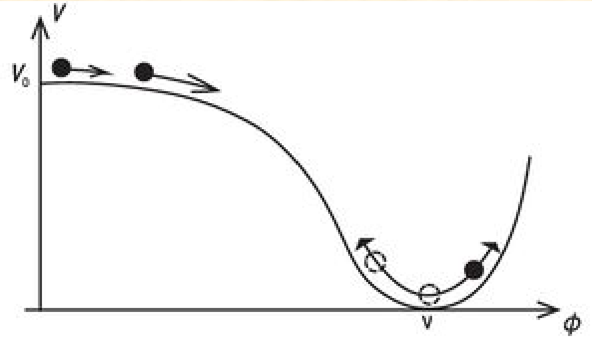
Pitaevskii and Rosch PRA 55, 1997,
Saint-Jalm et al, PRX9, 2019,

expansion enters only via back-transformation of the variables:

Particle production after inflation

The universe after inflation:

- The inflaton oscillates around the minimum of its potential
- Decays into particles



Ultracold atomic gas:

The total number of atoms is conserved

⇩

Prevents the decay of the condensate

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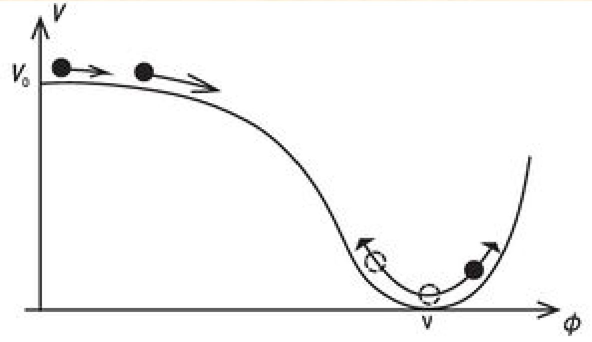

Early stages (preheating):

- Particles/fluctuations live on an effective $\phi(t)$-dependent potential

$$\delta\ddot{\varphi}_{\mathbf{p}} + M^2_{\mathbf{p}}(\phi(t))\delta\varphi_{\mathbf{p}} = 0$$

⇩

Parametric resonance
(explosive particle production)

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Prevents the decay of the condensate

Sound waves on top of the BEC:

$$\ddot{\Phi}_{\mathbf{p}} + \frac{g n_{\text{cond}}}{m} \mathrm{p}^2 \Phi_{\mathbf{p}} = 0$$

Modulate the scattering length: $g(t)$

Robertson et al., PRD 98, 2018

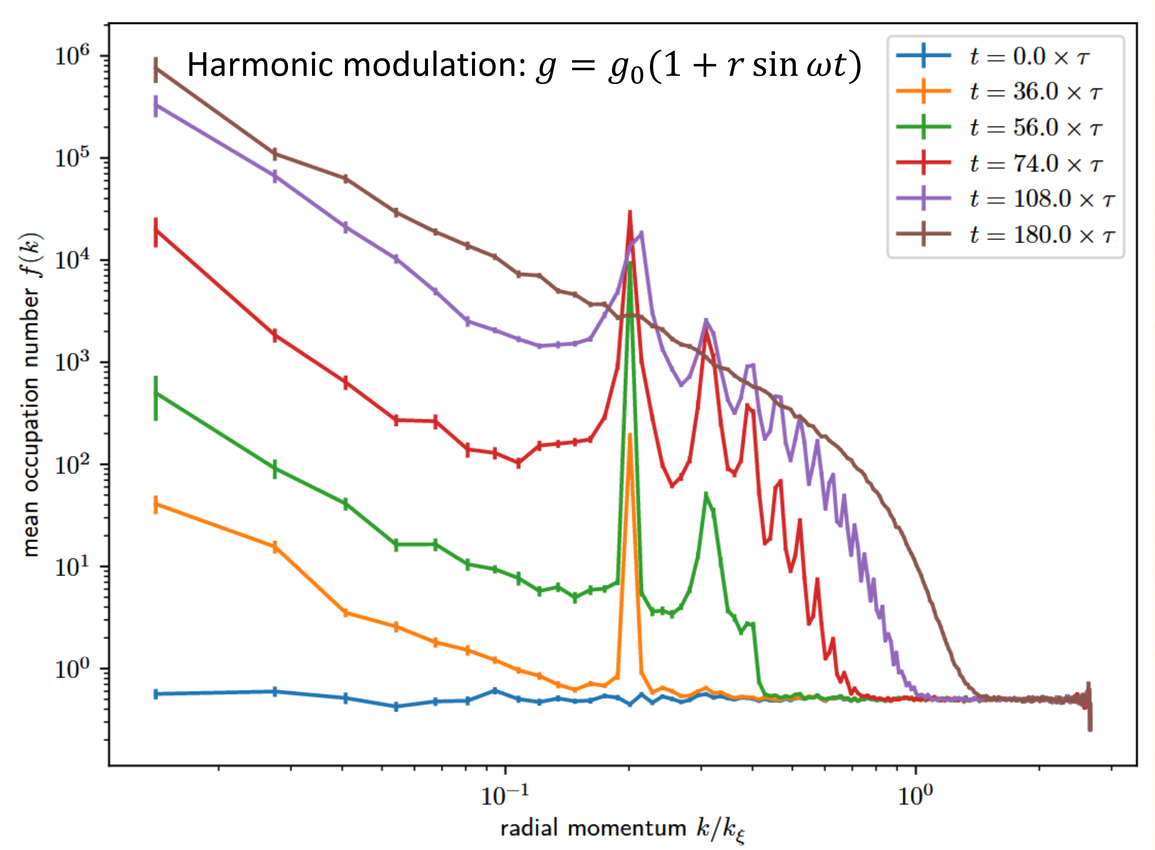
Pollack et al,. PRA 81, 2010

Vidanović et al, PRA 84, 2011

Nguyen et al., PRX 9, 2019

Particle production after inflation

Lattice simulations ($2D$, $N \sim 10^8$ atoms) based on TWA



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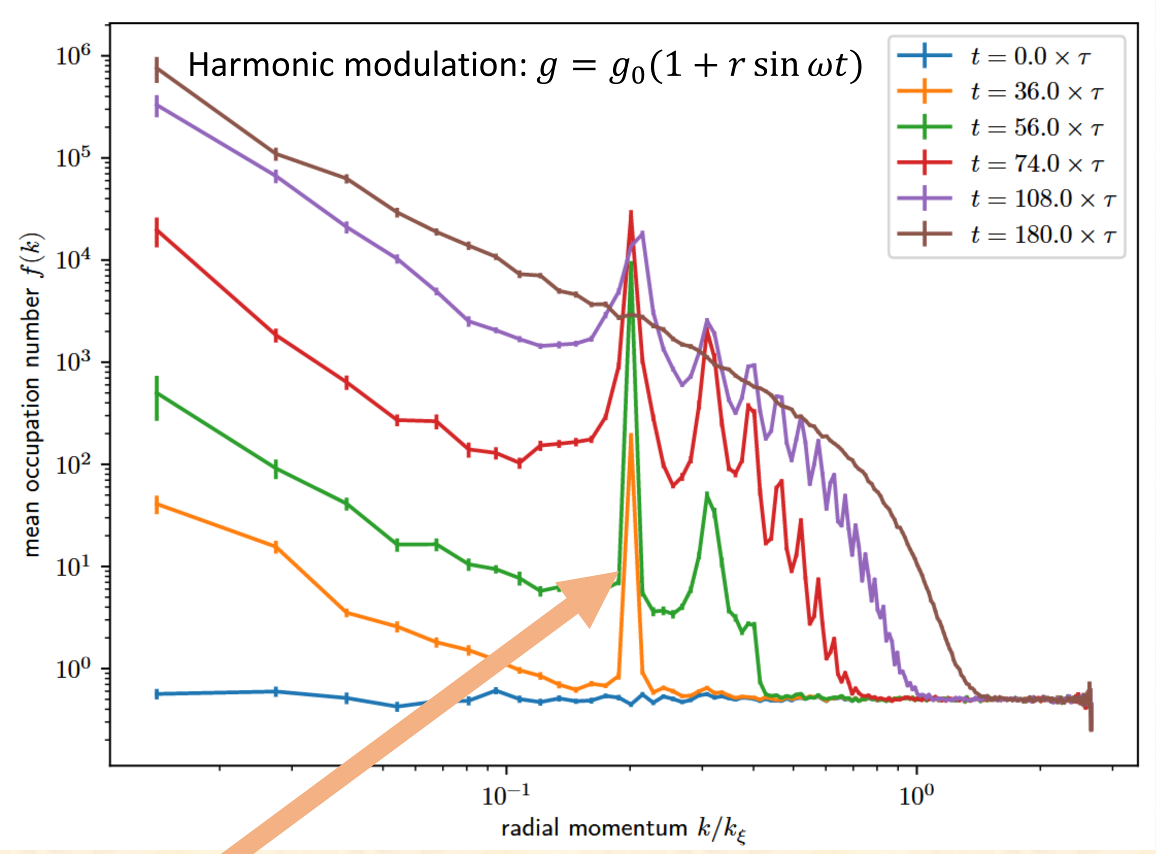

Linear regime

Parametric resonance peak at $\omega_1 = \omega/2$.

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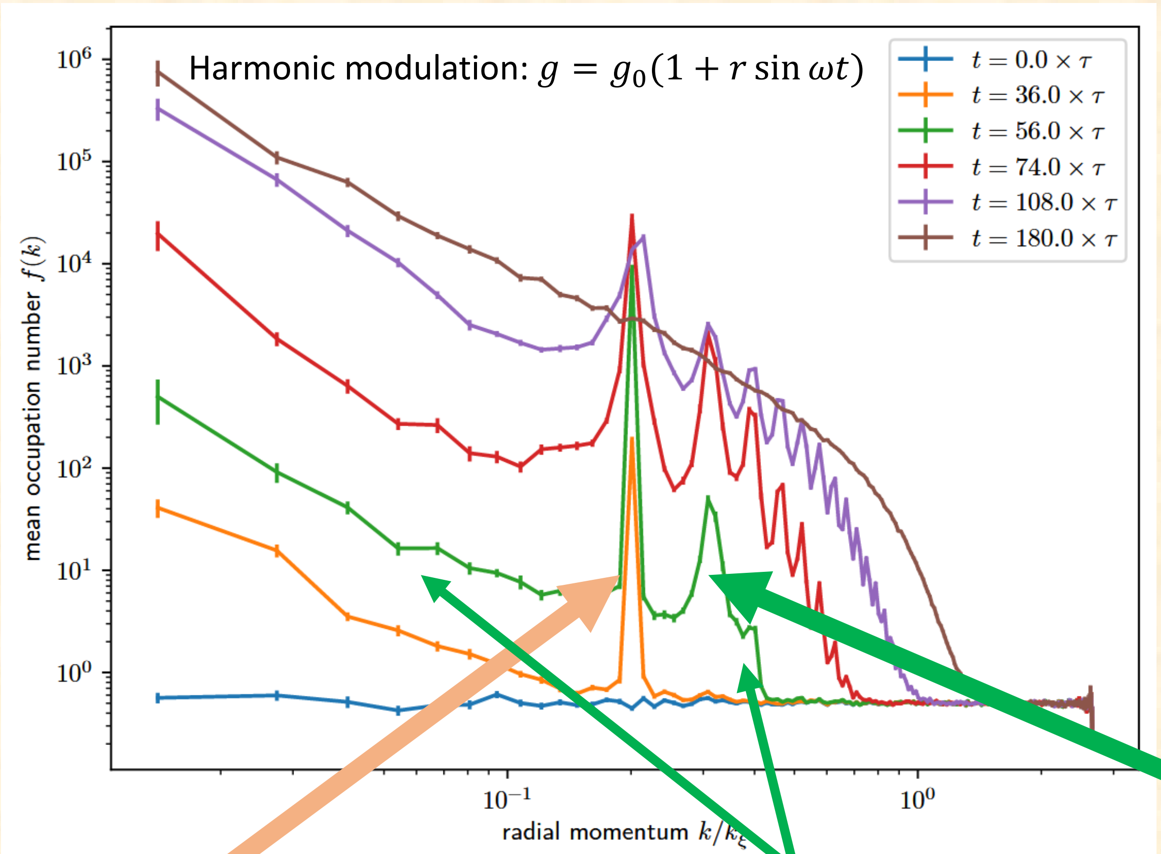

Linear regime

First non-linear corrections set in when $N_{exc}(t) \sim \sqrt{N}$.

Parametric resonance peak at $\omega_1 = \omega/2$.

Broad band from off-shell re-scattering at $0 < \mathrm{p} < 2\mathrm{p}_1$.

Narrow peak from on-shell re-scattering at $\omega_2 = 2\omega_1$.

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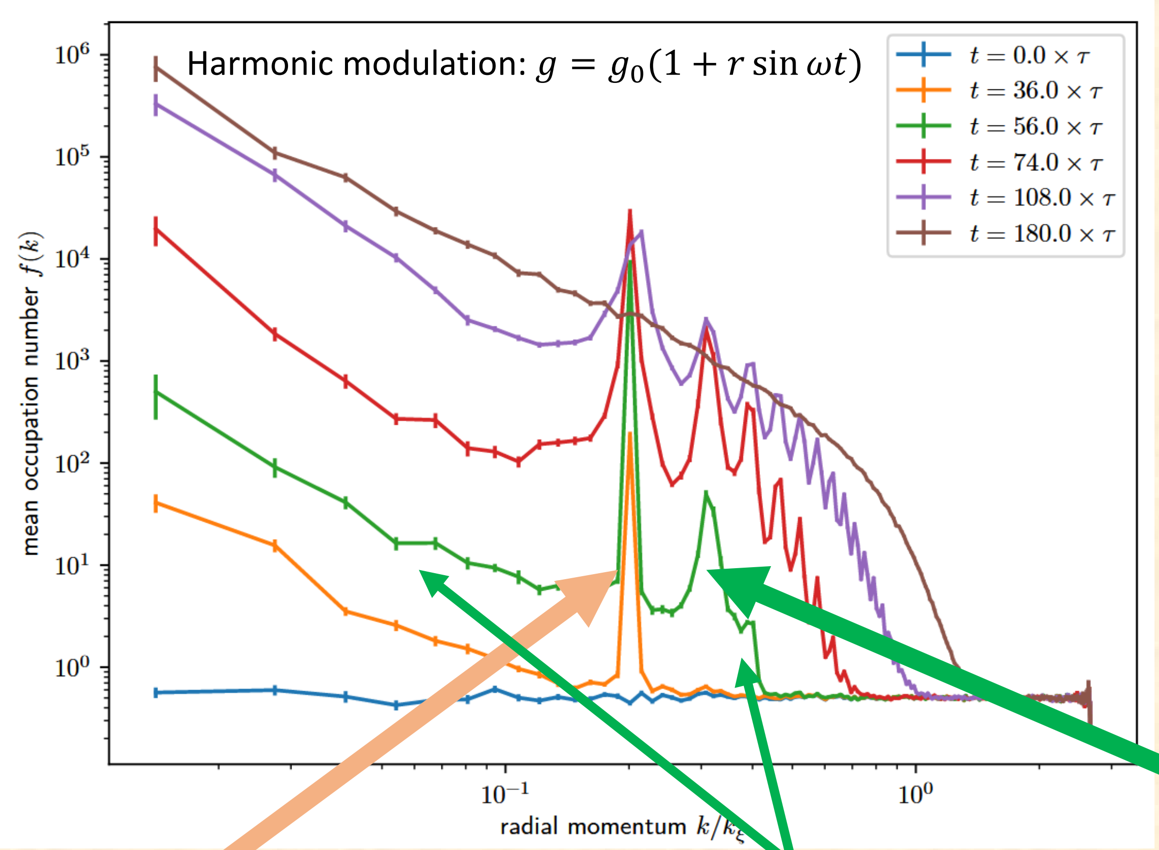

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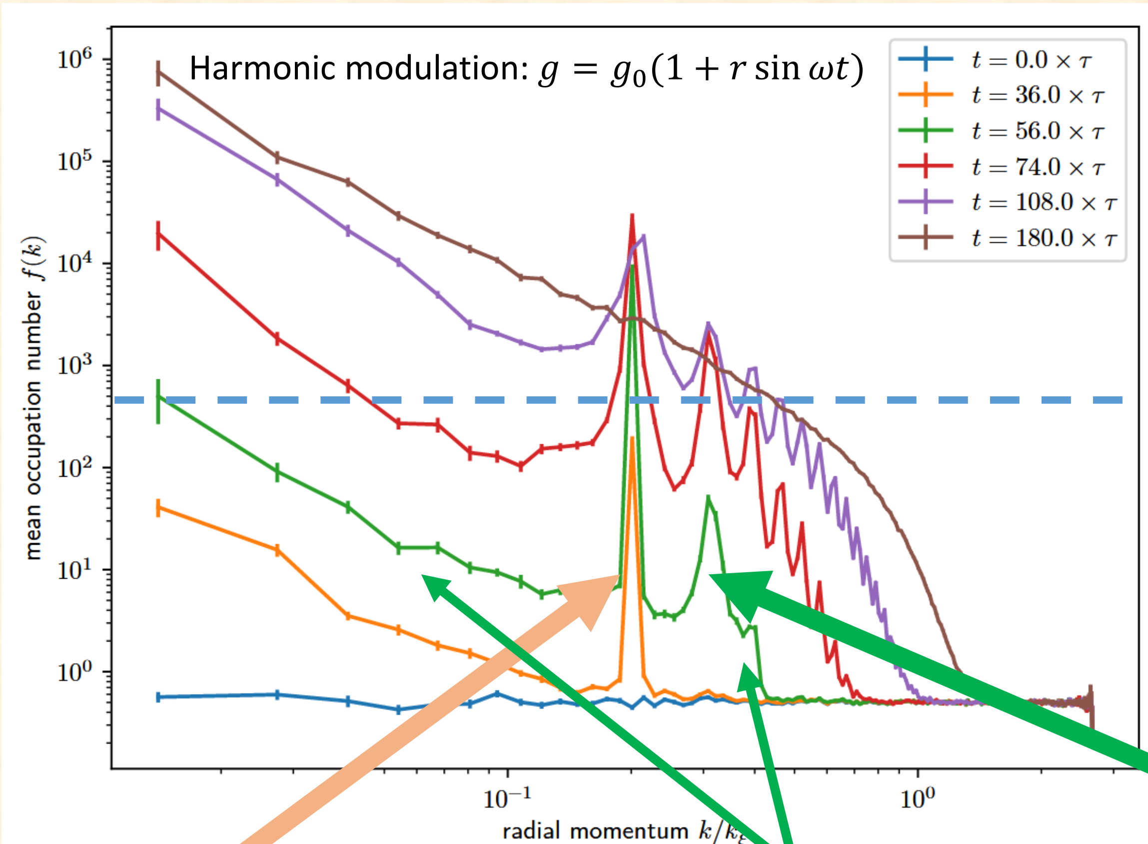

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Non-linearities: analytics

- Real-time (2PI) evolution equations for two-point functions:

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  - Statistical propagator:

$$\widetilde{n}(x,y) = \frac{1}{2}\langle\{\hat{\psi}(x)\hat{\psi}^{+}(y)\}\rangle - \Psi(x)\Psi^{*}(y), \qquad \widetilde{n}(t,t,\mathbf{p}) = f(t,\mathbf{p})$$

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$$\Psi(x) = \langle\hat{\psi}(x)\rangle. \qquad \widetilde{m}(x,y) = \frac{1}{2}\langle\{\hat{\psi}(x)\hat{\psi}(y)\}\rangle - \Psi(x)\Psi(y).$$

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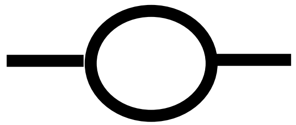
Linear regime

$$+icg(t)g(t')\int_{\mathbf{q}}\Big[4\Psi(t)\Psi^{*}(t')\Big(\tilde{n}(t,t',\mathbf{q})\tilde{n}^{*}(t,t',\mathbf{p}-\mathbf{q}) + \text{similar terms}\Big)$$

$$+\ldots$$

First non-linear correction



Zache et al., PRA 95, 2017
Berges et al., PRL 91 2003

Thermalization

Once a sufficient amount of atoms is produced, switch off the modulation

How does the universe approach thermal equilibrium?

Micha and Tkachev
PRD 70 2004

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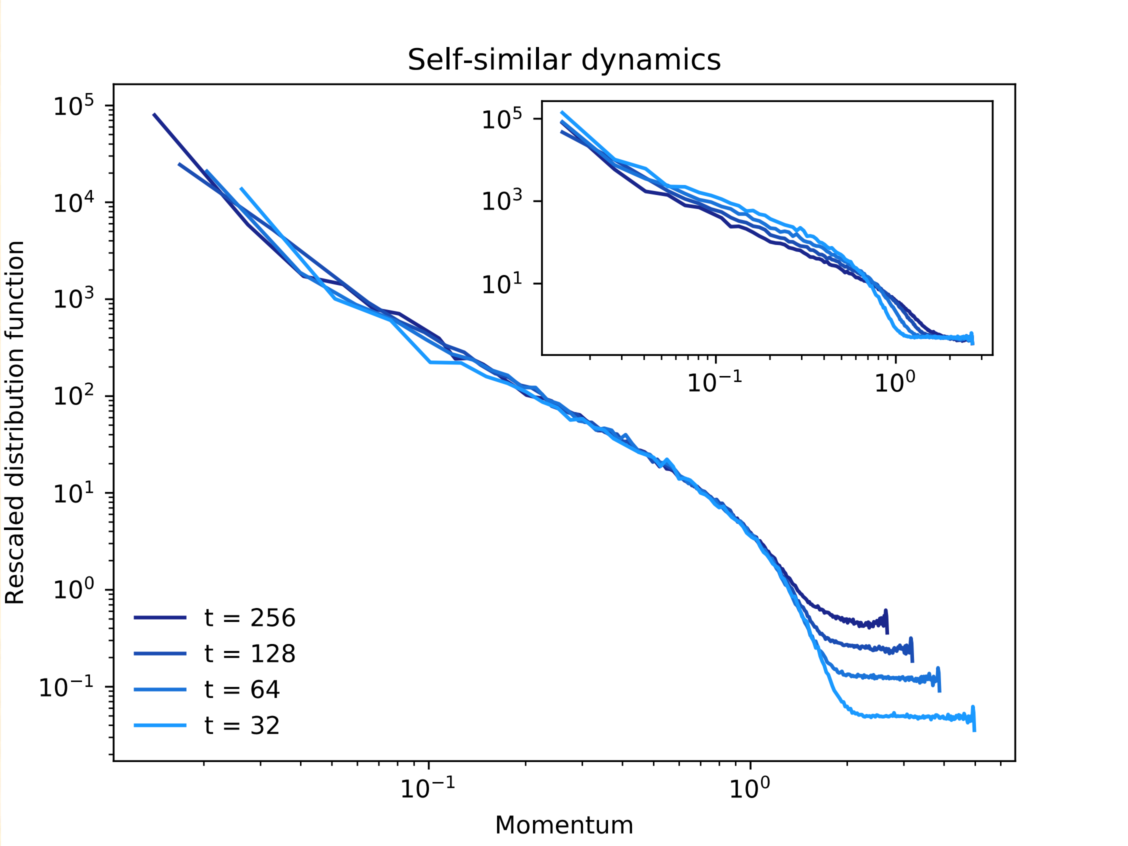

Energy transport with self-similar scaling behavior:

$$f(t, \mathrm{p}) = s^{\alpha} f_S(s^{\beta} \mathrm{p})$$

where $s = t/t_S$

⇩

Non-thermal fixed point

Numerically extracted exponents:

$$\alpha \approx 1,\ \beta \approx 0.3$$

Thermalization vs expansion

- When $f_{\mathrm{char}} \sim 1$, TWA becomes inapplicable

  ⮱ Relaxation to a Bose-Einstein distribution

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- Interaction rates decrease with expansion
  - Can the particles achieve thermal equilibrium?

Kolb Turner,
Front.Phys. 69 (1990)

$\Gamma \gg H$: thermalization possible

$\Gamma \ll H$: freeze-out

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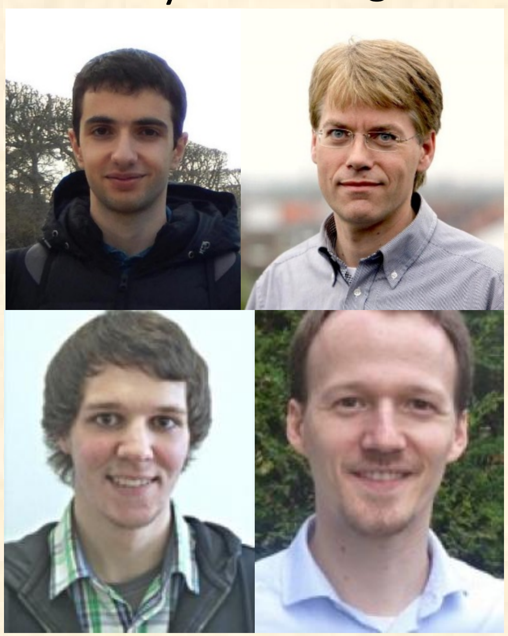
- These effects are present in the Bose gas
  - $d = 2$ ⇨ $g = \text{const}$ ⇨ Freeze-out happens at $\tau(t_{\text{phys}} = \infty)$, if $\tau_{\text{therm}} > \tau(t_{\text{phys}} = \infty)$
  - $d = 3$ ⇨ $g \propto a^{-1}$ ⇨ Freeze-out can happen before $\tau(t_{\text{phys}} = \infty)$

My collaborators

Paper: in preparation

Theory + Future implementation in “Oberthaler lab”

Aleksandr Chatrchyan | Jürgen Berges



Kevin Geier | Philipp Hauke

Markus Oberthaler | Helmut Strobel

Celia Viermann | Maurus Hans

Thank you for your attention!