Seeded vacuum decay in cold atoms

Florent Michel
CPT Group, Durham University

Based on ***[T. Billam, R. Gregory, FM, I. Moss, arXiv:1811.09169]***

BEC with a false vacuum *[O. Fialko* **et al** *'16]* (see Ian Moss' talk)

Setup:

- Two-components BEC (atoms with two different spin states)
- Coupling through time-dependent radio-frequency field

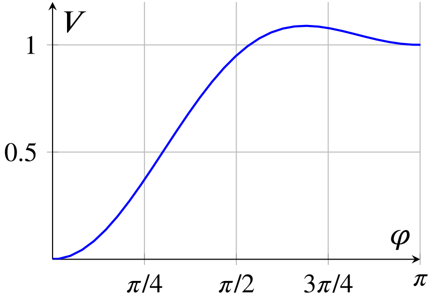
Hamiltonian:

$$\hat{H} = \sum_{\sigma=\pm 1} \left[\int \hat{\psi}_\sigma^\dagger \left(-\frac{\hbar^2 \nabla^2}{2m} - \mu \right) \hat{\psi}_\sigma \, \mathrm{d}^d x + \frac{g}{2} \int \hat{\psi}_\sigma^\dagger \hat{\psi}_\sigma^\dagger \hat{\psi}_\sigma \hat{\psi}_\sigma \, \mathrm{d}^d x \right.$$
$$\left. - \nu(t) \int \hat{\psi}_\sigma^\dagger \hat{\psi}_{1-\sigma} \mathrm{d}^d x \right]$$

Canonical commutation relations

$$\left[\hat{\psi}_\sigma^\dagger(\vec{x}), \hat{\psi}_{\bar{\sigma}}(\vec{y}) \right] = \delta_{\sigma,\bar{\sigma}} \delta(\vec{x} - \vec{y})$$

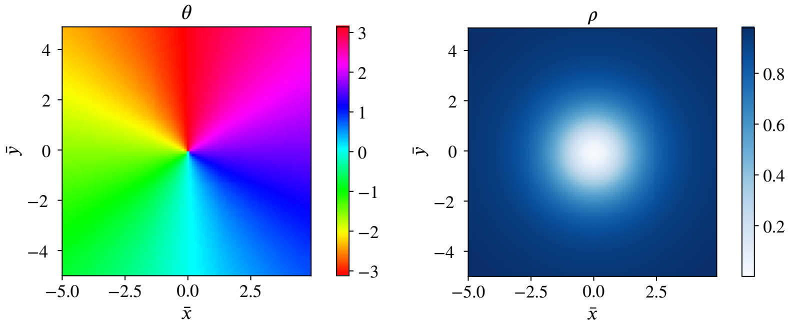
Testing vacuum decay with a nonrelativistic BEC (see Ian Moss' talk)



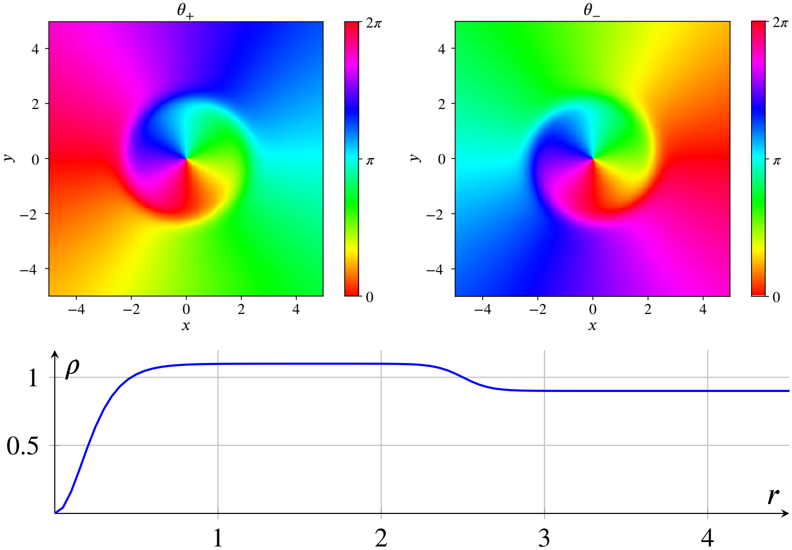
- System: two-component BEC with a time-dependent radio-frequency coupling
- Parameters chosen to have two minima → true and false vacua
- Relativistic instanton approach to compute the decay rate

Seeded vacuum decay: motivation

- $\Gamma \propto e^{-S_E/\hbar} \Rightarrow$ very small decay rate in general
- relativistic case: a defect can efficiently catalyze the decay ***[R. Gregory, I. Moss, B. Withers '13], [N. Oshita, M. Yamada, M. Yamaguchi '19]*** $\rightarrow$ similar effect in the analogue model?
- test of this idea: compute the decay rate with a vortex



Instanton with vortex: qualitative picture



Adding a vortex

Two new difficulties:

- $\rho_+, \rho_- \to 0$ in the vortex core $\Rightarrow$ Need to include density fluctuations
- Largest effects expected for small bubbles $\Rightarrow$ Need to include higher-order terms in $\partial_r \varphi$

Back to the full field equations

$$\begin{cases} \dfrac{\hbar^2}{2m} \nabla^2 \psi_i + \mathrm{i}\hbar\, \partial_t \psi_i - \dfrac{\partial V}{\partial \bar{\psi}_i} = 0 \\ \dfrac{\hbar^2}{2m} \nabla^2 \bar{\psi}_i + \mathrm{i}\hbar\, \partial_t \bar{\psi}_i - \dfrac{\partial V}{\partial \psi_i} = 0 \end{cases}$$

+ Wick rotation $\to$ look for instanton solutions?

Nonrelativistic case: naive Wick rotation

Let us try the usual Wick rotation $t \to \tau = \mathrm{i}\,t$:

Field equations in real time:

$$\begin{cases} \mathrm{i}\,\hbar\,\partial_t \psi_i = -\dfrac{\hbar^2}{2\,m}\,\nabla^2 \psi_i + \dfrac{\partial V}{\partial \bar{\psi}_i} \\ -\mathrm{i}\,\hbar\,\partial_t \bar{\psi}_i = -\dfrac{\hbar^2}{2\,m}\,\nabla^2 \bar{\psi}_i + \dfrac{\partial V}{\partial \psi_i} \end{cases}$$

Field equations in Euclidean time:

$$\begin{cases} -\hbar\,\partial_\tau \psi_i = -\dfrac{\hbar^2}{2\,m}\,\nabla^2 \psi_i + \dfrac{\partial V}{\partial \bar{\psi}_i} \\ \hbar\,\partial_\tau \bar{\psi}_i = -\dfrac{\hbar^2}{2\,m}\,\nabla^2 \bar{\psi}_i + \dfrac{\partial V}{\partial \psi_i} \end{cases}$$

$\to$ Equations in Euclidean time consistent only if $\partial_\tau \psi_i = 0$ $\to$ no nontrivial instanton

Generalized Wick rotation

Idea: deform the integration contour into a wider region of complex function space where $\bar{\psi}_i$ is not the complex-conjugate of ψ_i

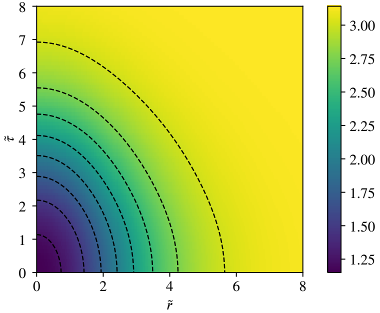
$2D$ two-components BEC with a vortex:

$$\psi_i = \rho_m^{1/2}\,(1 \pm \epsilon\,\sigma)\,\mathrm{e}^{\pm \mathrm{i}\,\varphi/2 + \mathrm{i}\,n\,\theta}$$
$$\bar{\psi}_i = \rho_m^{1/2}\,(1 \pm \epsilon\,\sigma)\,\mathrm{e}^{\mp \mathrm{i}\,\varphi/2 - \mathrm{i}\,n\,\theta}$$

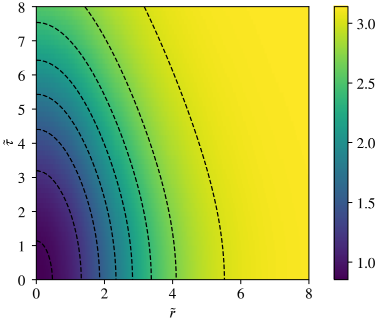
ϵ: parameter describing the coupling between the fields
n: winding number

Leading order in ϵ: $\sigma \in \mathrm{i}\,\mathbb{R} \rightarrow \bar{\psi}_i \neq \psi_i^*$

Two examples of solutions

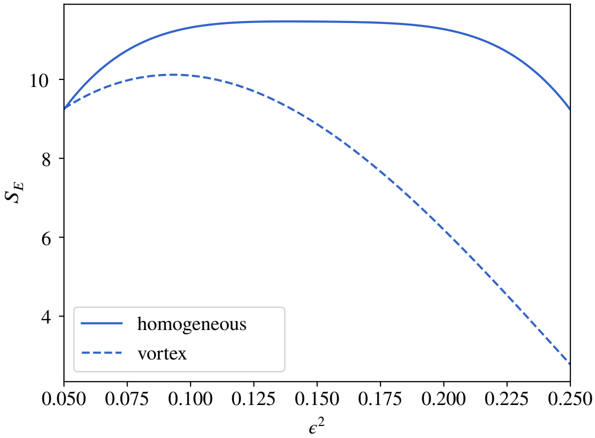


$\Delta \tilde{S}_E \approx 19.3$



$\Delta \tilde{S}_E \approx 13.8$

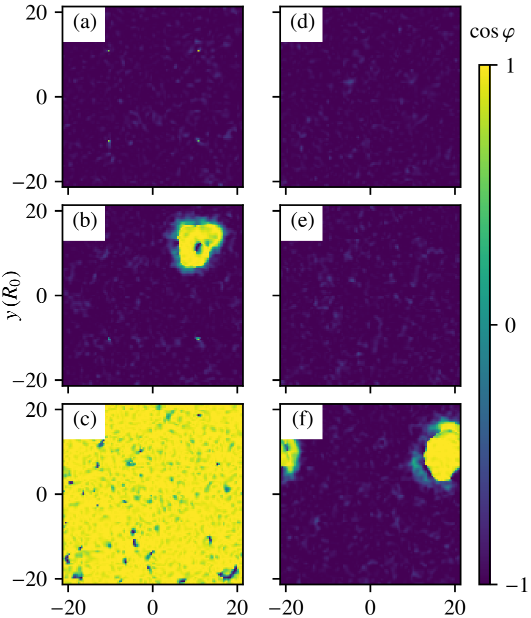
Results for the decay rate



Truncated Wigner approach

- Idea: Perform many classical simulations with random initial conditions mimicking the statistics of the false vacuum
- Similar to the appoach of ***[Braden et al. ’18]***, ***[Braden et al. ’19]***
- Two set-ups: periodic vortex array and circular trap

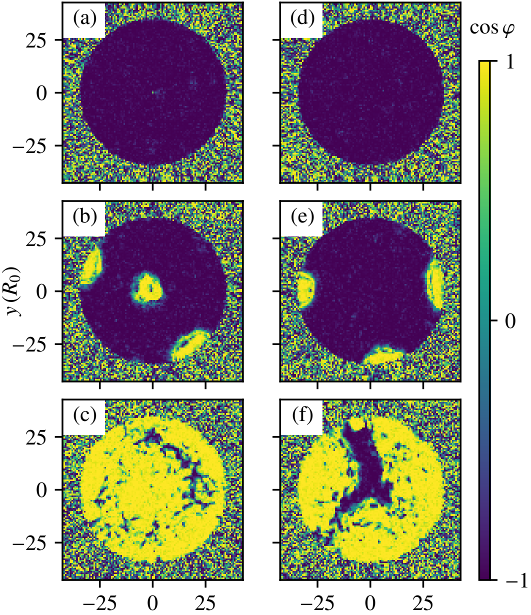
Numerical results: periodic array of vortices



- with vortices → bubbles form (nearly) always around one vortex
- without vortices → nucleation happens significantly later

run

Numerical results: circular trap

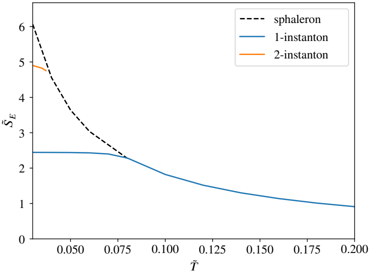


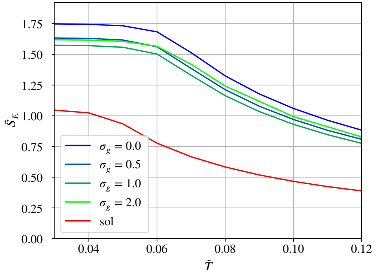
- no vortex → bubbles form (nearly) exclusively on the walls
- with vortex → the vortex can nucleate a bubble

run

Thermal case (work in progress)

1D case, comparison of the Euclidean actions in the homogeneous case, with a potential defect, and with a soliton





Outlook

How exactly is this related to *relativistic* vacuum decay?

Experimentally realistic model?

Thank you for your attention!