Seeded vacuum decay in cold atoms

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BEC with a false vacuum \cite{Fialko16} (see Ian Moss’ talk)

Setup:
- Two-components BEC (atoms with two different spin states)
- Coupling through time-dependent radio-frequency field

Hamiltonian:
\[
\hat{H} = \sum_{\sigma=\pm 1} \left[ \int \hat{\psi}_\sigma^\dagger \left( -\frac{\hbar^2 \nabla^2}{2m} - \mu \right) \hat{\psi}_\sigma \, d^d x + \frac{g}{2} \int \hat{\psi}_\sigma^\dagger \hat{\psi}_\sigma^\dagger \hat{\psi}_\sigma \hat{\psi}_\sigma \, d^d x \\
- \nu(t) \int \hat{\psi}_\sigma^\dagger \hat{\psi}_{1-\sigma} \, d^d x \right]
\]

Canonical commutation relations
\[
\left[ \hat{\psi}_\sigma^\dagger(\vec{x}), \hat{\psi}_{\bar{\sigma}}(\vec{y}) \right] = \delta_{\sigma,\bar{\sigma}} \delta(\vec{x} - \vec{y})
\]
Testing vacuum decay with a nonrelativistic BEC (see Ian Moss’ talk)

- System: two-component BEC with a time-dependent radio-frequency coupling
- Parameters chosen to have two minima → true and false vacua
- Relativistic instanton approach to compute the decay rate
Seeded vacuum decay: motivation

- \( \Gamma \propto e^{-S_E/\hbar} \Rightarrow \) very small decay rate in general

- relativistic case: a defect can efficiently catalyze the decay [R. Gregory, I. Moss, B. Withers '13], [N. Oshita, M. Yamada, M. Yamaguchi '19] → similar effect in the analogue model?

- test of this idea: compute the decay rate with a vortex
Instanton with vortex: qualitative picture

\[ \theta_+ \]

\[ \theta_- \]

\[ \rho \]

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Adding a vortex

Two new difficulties:

- \( \rho_+, \rho_- \to 0 \) in the vortex core \( \Rightarrow \) Need to include density fluctuations
- Largest effects expected for small bubbles \( \Rightarrow \) Need to include higher-order terms in \( \partial_r \phi \)

Back to the full field equations

\[
\begin{align*}
\frac{\hbar^2}{2m} \nabla^2 \psi_i + i \hbar \partial_t \psi_i - \frac{\partial V}{\partial \bar{\psi}_i} &= 0 \\
\frac{\hbar^2}{2m} \nabla^2 \bar{\psi}_i + i \hbar \partial_t \bar{\psi}_i - \frac{\partial V}{\partial \psi_i} &= 0
\end{align*}
\]

+ Wick rotation \( \Rightarrow \) look for instanton solutions?
Let us try the usual Wick rotation \( t \rightarrow \tau = it \):

Field equations in real time:

\[
\begin{align*}
\left\{ \begin{array}{l}
\imath \hbar \partial_t \psi_i &= -\frac{\hbar^2}{2m} \nabla^2 \psi_i + \frac{\partial V}{\partial \psi_i} \\
\imath \hbar \partial_t \bar{\psi}_i &= -\frac{\hbar^2}{2m} \nabla^2 \bar{\psi}_i + \frac{\partial V}{\partial \bar{\psi}_i}
\end{array} \right.
\]

Field equations in Euclidean time:

\[
\begin{align*}
\left\{ \begin{array}{l}
-\hbar \partial_\tau \psi_i &= -\frac{\hbar^2}{2m} \nabla^2 \psi_i + \frac{\partial V}{\partial \psi_i} \\
\hbar \partial_\tau \bar{\psi}_i &= -\frac{\hbar^2}{2m} \nabla^2 \bar{\psi}_i + \frac{\partial V}{\partial \psi_i}
\end{array} \right.
\]

→ Equations in Euclidean time consistent only if \( \partial_\tau \psi_i = 0 \) → no nontrivial instanton
**Idea:** deform the integration contour into a wider region of complex function space where $\bar{\psi}_i$ is not the complex-conjugate of $\psi_i$

2D two-components BEC with a vortex:

$$\psi_i = \rho_m^{1/2} (1 \pm \epsilon \sigma) e^{\pm i \varphi/2 \pm i n \theta}$$

$$\bar{\psi}_i = \rho_m^{1/2} (1 \pm \epsilon \sigma) e^{\mp i \varphi/2 \mp i n \theta}$$

$\epsilon$: parameter describing the coupling between the fields  
$n$: winding number

Leading order in $\epsilon$: $\sigma \in i \mathbb{R} \rightarrow \bar{\psi}_i \neq \psi_i^*$
Two examples of solutions

\[ \Delta \tilde{S}_E \approx 19.3 \]

\[ \Delta \tilde{S}_E \approx 13.8 \]
Results for the decay rate

![Graph showing decay rates for homogeneous and vortex cases](image-url)
Truncated Wigner approach

- Idea: Perform many classical simulations with random initial conditions mimicking the statistics of the false vacuum

- Similar to the approach of [Braden et al. ’18], [Braden et al. ’19]

- Two set-ups: periodic vortex array and circular trap
Numerical results: periodic array of vortices

- with vortices $\rightarrow$ bubbles form (nearly) always around one vortex
- without vortices $\rightarrow$ nucleation happens significantly later

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Numerical results: circular trap

- no vortex $\rightarrow$ bubbles form (nearly) exclusively on the walls
- with vortex $\rightarrow$ the vortex can nucleate a bubble

$\mathbf{R}_0$

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Seeded vacuum decay in cold atoms
1D case, comparison of the Euclidean actions in the homogeneous case, with a potential defect, and with a soliton
Outlook

How exactly is this related to *relativistic* vacuum decay?

Experimentally realistic model?
Thank you for your attention!