Department of Physics at Wake Forest University

Correlation patterns in a BEC Black Hole Analog with Massive Phonons

Trento Workshop 2019

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In collaboration with
Paul R. Anderson, Alessandro Fabbri and Roberto Balbinot

September 16, 2019

- Most work done theoretically and experimentally for BEC analog black holes in 1D systems is with no excitations of the transverse modes for the phonons (A. Coutant, A. Fabbri, R. Parentani, R. Balbinot, and P.R. Anderson, PRD **86** (2012) 064022 and G. Jannes, P. Maı, T.G. Philbin, and G. Rousseaux, PRD **83** (2011) 104028 are examples of exceptions)

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- In the analog spacetime this results in a massive minimally coupled scalar field with a potential

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Introduction

Overview

- Previous studies of massive fields in black hole spacetimes show the energy density and pressure are smaller than the massless case
- One might expect therefore that structure in the density density correlation function might be similar to the massless case but on a smaller scale
- Instead we find **fundamental differences** between the massless and massive cases

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- Introduce the simple model we've been working with

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- Introduce the simple model we've been working with
- Review results for the two point and density density correlation functions for the case of massless phonons
- Show the very different results for the massive case

The Model
The Gross-Pitaevskii Equation

- $i\hbar\partial_T \Psi_0 = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V_{ext} + g\left|\Psi_0\right|^2\right)\Psi_0.$
- Let $\Psi_0 = \sqrt{n e^{i\theta}}$

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- Choose a solution with $n =$ constant.
- The BEC moves with a constant velocity in the negative x direction.
  $\vec{v} = \frac{\hbar}{m}\vec{\nabla}\theta = -v_0\hat{x}$

The Model
The Horizon

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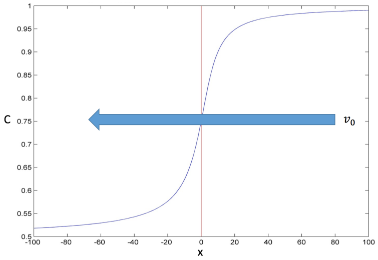

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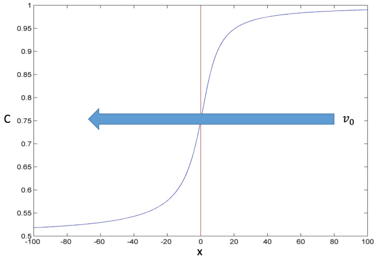

- A sonic horizon occurs at $x = 0$
  For $x > 0, \quad v_0 > c$
  For $x < 0, \quad v_0 < c$

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- Considering scales much larger than the healing length(hydrodynamic approximation)

$$\left[-\frac{c}{c^2 - v_0^2}\partial_T^2 + c\partial_x\left(\frac{c^2 - v_0^2}{c^2}\partial_x\right) + c\left(\partial_y^2 + \partial_z^2\right)\right]\hat{\theta}_1 = 0 \qquad (1)$$

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- Excitation of a mode in the transverse direction yields an effective mass term.

$$e^{i(k_y y + k_z z)} \rightarrow m^2 = k_y^2 + k_z^2$$

(F. Chevy, V. Bretin, P. Rosenbusch, K.W. Madison, and J. Dalibard, PRL **88** (2002) 250402)

- It is possible to show this is equivalent to a massive minimally coupled scalar field propagating in the 2D spacetime

$$ds^2 = \frac{c^2 - v_0^2}{c} dt^2 + \frac{c}{c^2 - v_0^2} dx^2 = -(c^2 - v_0^2)(dt^2 - dx^{*2})$$

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$$V_{\text{eff}} \equiv \frac{c^2 - v_0^2}{c} \left[\frac{1}{2} \frac{d^2 c}{dx^2} \left(1 + \frac{v_0^2}{c^2}\right) - \frac{1}{4c} \left(\frac{dc}{dx}\right)^2 + \frac{5 v_0^2}{4c^3} \left(\frac{dc}{dx}\right)^2 \right].$$

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- V_{eff} vanishes at the horizon and at $x = \pm\infty$ if $c \to$ constant.

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- Set $V_{\text{eff}} = 0$

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$$
\begin{aligned}
c^2 - v_0^2 &\rightarrow c_R^2 - v_0^2 \Theta(x^* -_R x_0^*), \quad x > 0 \\
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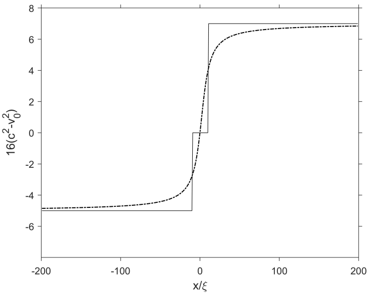

Correlation functions

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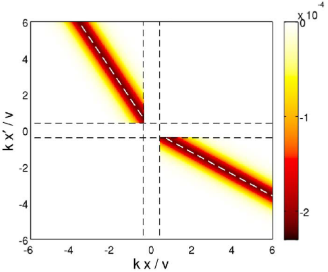
- The two point function $\left\langle\{\hat{\theta}_2(T,x)\hat{\theta}_2(T'x')\}\right\rangle$
- The D-D correlation function can be written in terms of the two point function as follows:

$$G_2(T,x;T',x') = \frac{\hbar n}{2ml_\perp c(x)^2 c(x')^2}\lim_{T\to T'} D\sqrt{c(x)c(x')}\left\langle\{\hat{\theta}_2(T,x)\hat{\theta}_2(T'x')\}\right\rangle \tag{2}$$

$$D = \partial_T\partial_{T'} - v_0\partial_T\partial_{x'} - v_0\partial_x\partial_{T'} + v_0^2\partial_x\partial_{x'}. \tag{3}$$

- DD correlation function from R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati, and I. Carusotto, PRA **78** (2008) 021603



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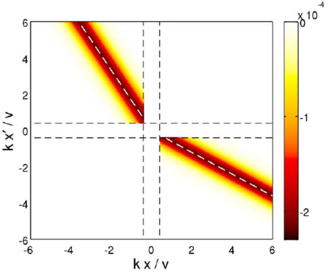

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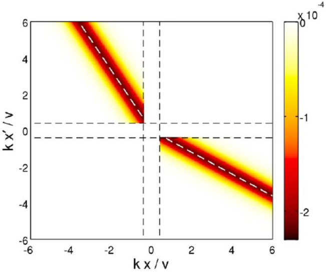
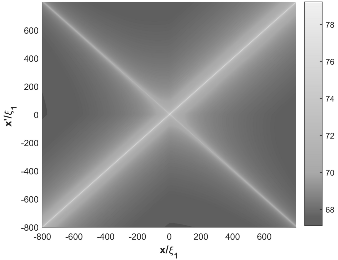
- Two -Point function for current speed of sound profile





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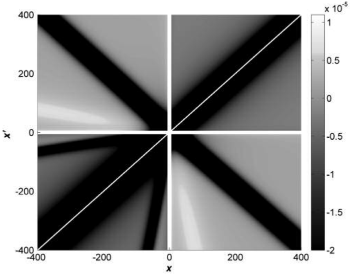

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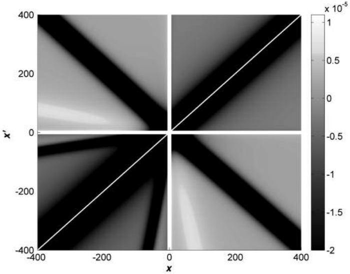

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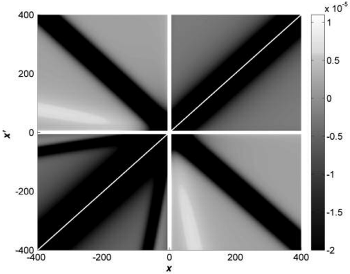

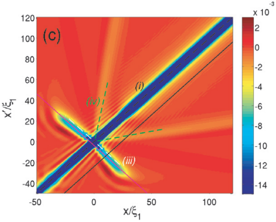

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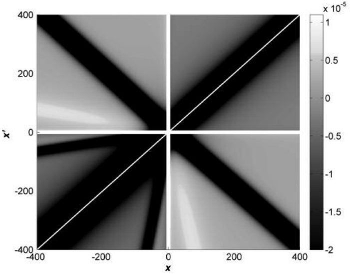

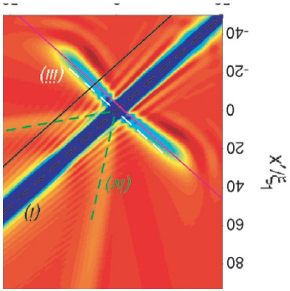

Results

Massive Two-Point Function

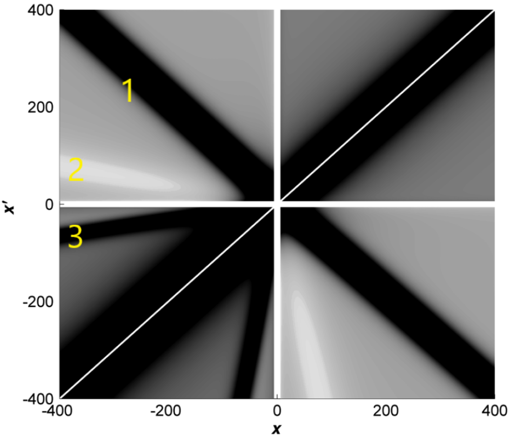

Figure: PRD 2013 Density Density results.

The Massless Case

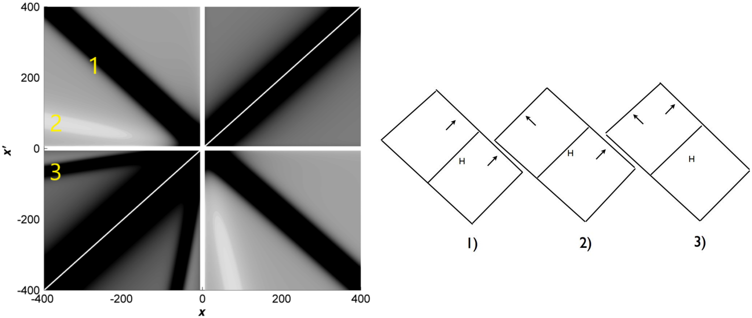

Figure: Origin of peaks in massless case.

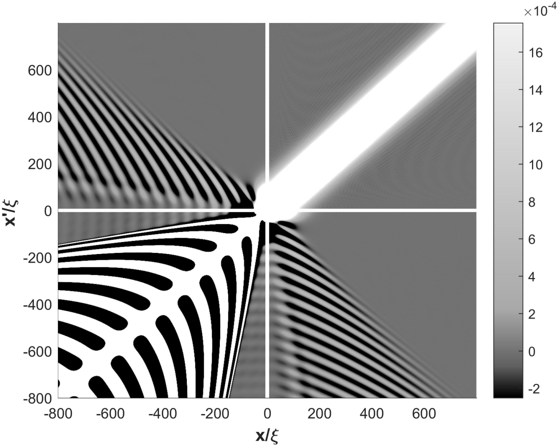

Figure: Massive two-point function correlation function for 1+1D BEC BH analog with masslike term, where $m = 4 \times 10^{-2} m_a$.

Results

Massive Two-Point Function with massless peaks superimposed

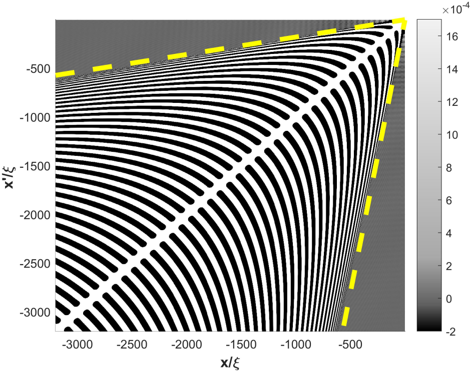

Figure: massive two-point function correlation function for 1+1D BEC BH analog with masslike term where $m = 4 \times 10^{-2} m_a$ with both points in the interior of the analog BH. The yellow dashed lines are the locations of the peaks found in the massless case.

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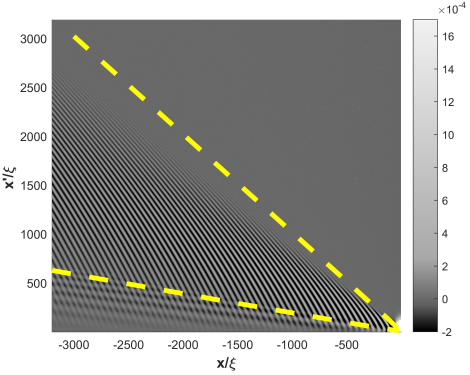

Figure: massive two-point function correlation function for 1+1D BEC BH analog with masslike term where $m = 4 \times 10^{-2} m_a$ with one point in the interior and 1 point in the exterior of the analog BH. The yellow dashed lines are the locations of the peaks found in the massless case.

Results

Massive Two-Point Function Decreasing Mass

Wake Forest University
Department of Physics

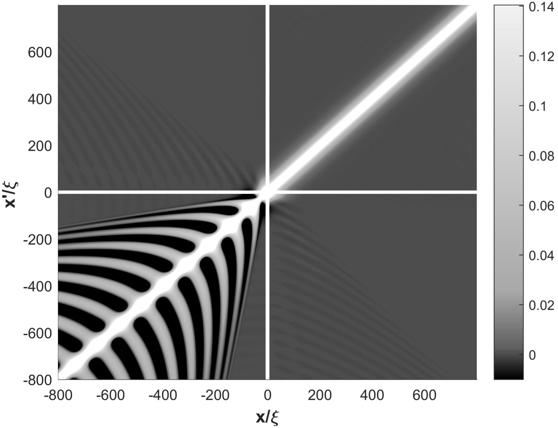

Figure: $m = 4 \times 10^{-2} m_a$

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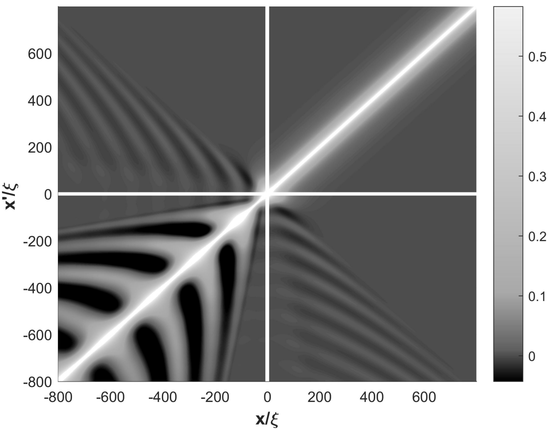


Figure: $m = 2 \times 10^{-2} m_a$

Results

Massive Two-Point Function Decreasing Mass

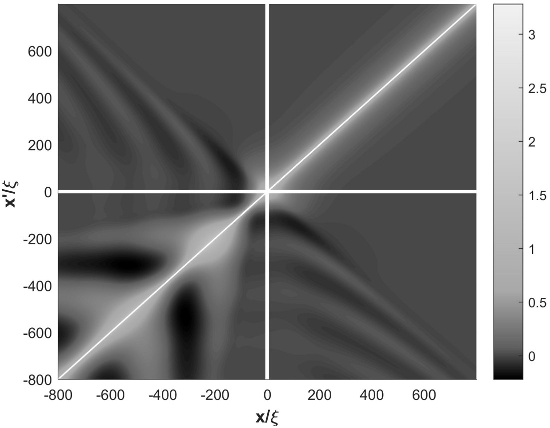


Figure: $m = 1 \times 10^{-2} m_a$

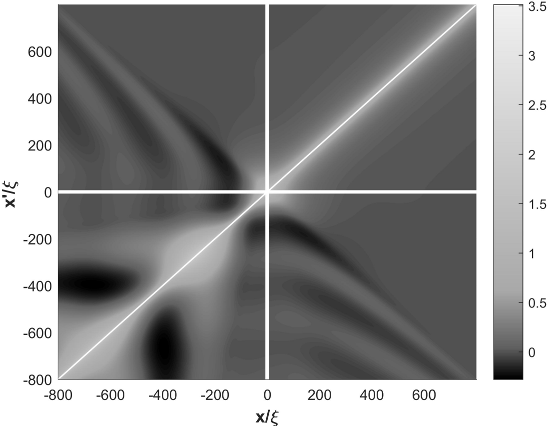


Figure: $m = 8 \times 10^{-3} m_a$

Massive Two-Point Function Decreasing Mass

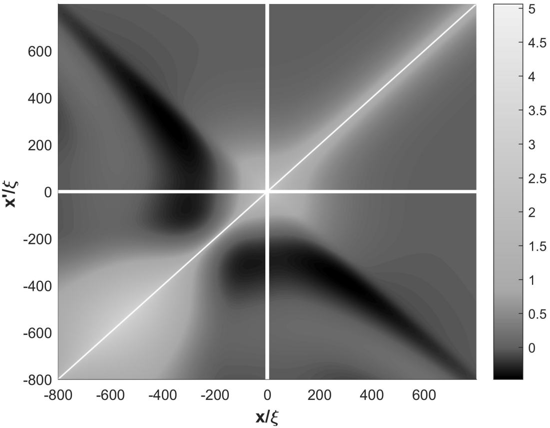


Figure: $m = 6 \times 10^{-3} m_a$

Results

Massive Two-Point Function Decreasing Mass

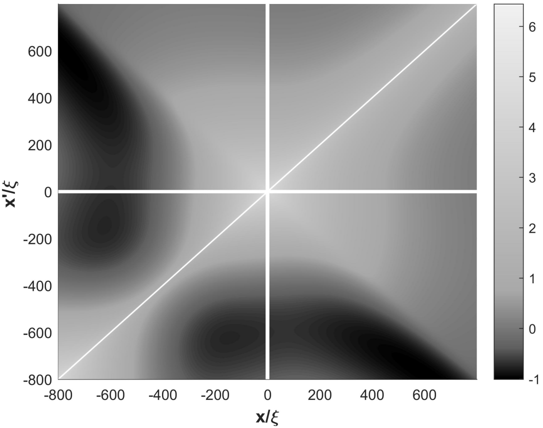


Figure: $m = 4 \times 10^{-3} m_a$

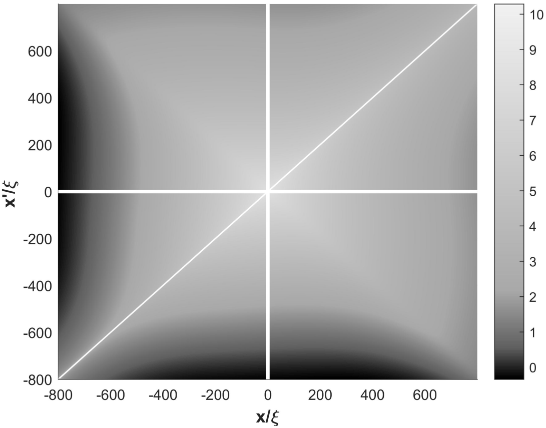


Figure: $m = 2 \times 10^{-3} m_a$

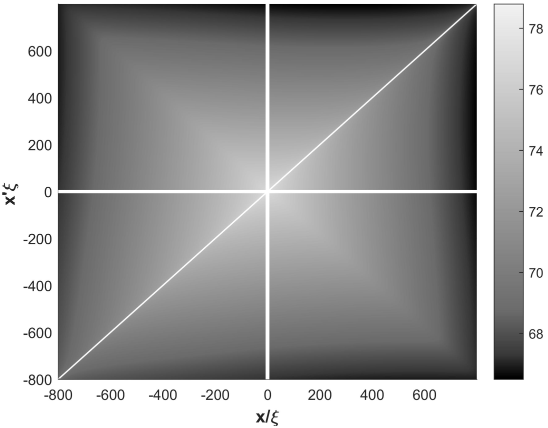


Figure: $m = 1 \times 10^{-4} m_a$

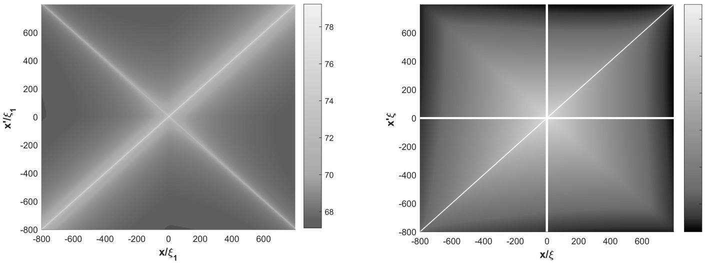


Figure: Left: Analytic solution for the massless two point function. Right: Massive two-point function correlation function for 1+1D BEC BH analog with masslike term where $m = \times 10^{-4} m_a$.

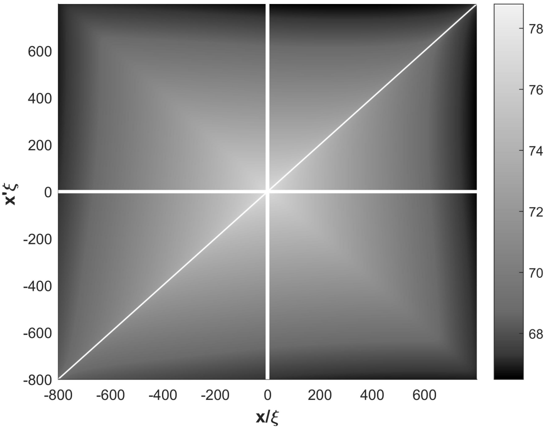


Figure: $m = 1 \times 10^{-4} m_a$

Results

Massive Two-Point Function Low Mass Result

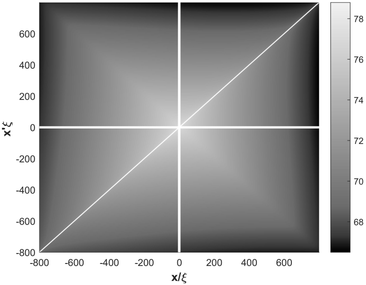


Figure: $m = 1 \times 10^{-4} m_a$

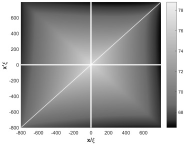


Figure: $m = 1 \times 10^{-4} m_a$

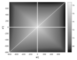
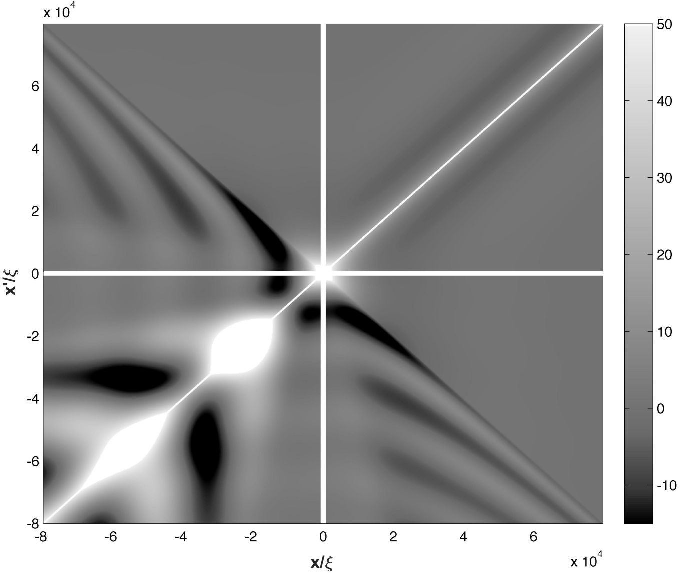


Figure: $m = 1 \times 10^{-4} m_a$

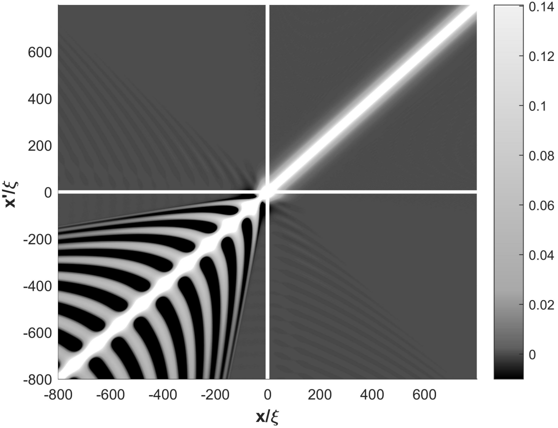
Results

Massive Two-Point Function Low Mass Result



Undulations exist in the massive case. (A. Coutant, A. Fabbri, R. Parentani, R. Balbinot, and P.R. Anderson, PRD **86** (2012) 064022)

$$\cos(p_U^m x)\cos(p_U^m x') \quad \text{where } p_U \propto m$$



Results

Massive Two-Point Function Undulations

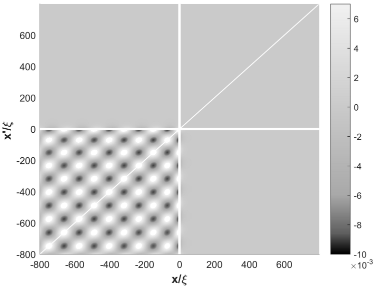


Figure: Massive two-point function correlation function for 1+1D BEC BH analog with masslike term, where $m = 4 \times 10^{-2} m_a$. The plot only includes the frequency range $0 < \omega < m/100$.

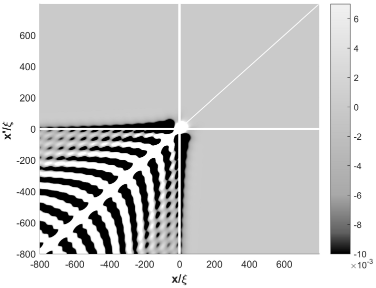


Figure: Massive two-point function correlation function for 1+1D BEC BH analog with masslike term, where $m = 4 \times 10^{-2} m_a$. The plot only includes the frequency range $0 < \omega < m/10$.

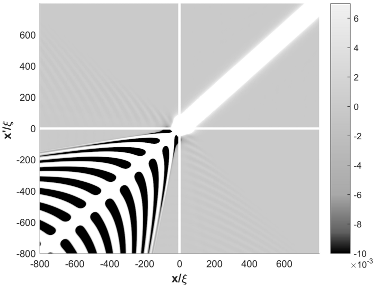
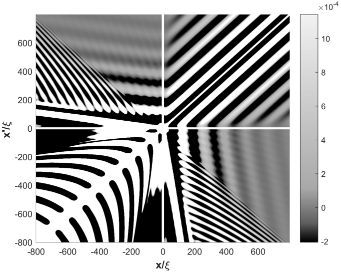
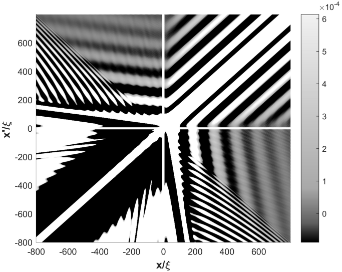


Figure: Massive two-point function correlation function for 1+1D BEC BH analog with masslike term, where $m = 4 \times 10^{-2} m_a$.

Thank you for your time!

- Left: H_{-} contribution. Right: i_{-} contribution.

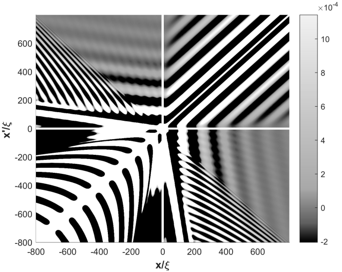


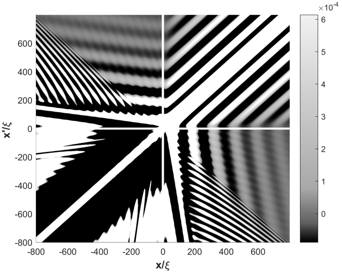


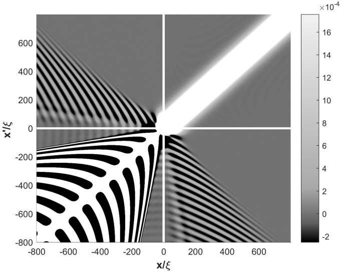
Results

Massive Two-Point Function Cancellations

- Left: H_- contribution. Right: i_- contribution.







- Left: H_{-} contribution. Right: i_{-} contribution.