Imperfect fluid description of modified gravities

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Outline

1. Scalar-tensor gravity as an (imperfect, irrotational) effective fluid
2. An application: the limit of electrovacuum Brans-Dicke theory to GR
3. Conclusions & open problems

Motivation

Scalar-tensor gravity is the prototypical alternative to Einstein's GR. $f(R)$ gravity, a subclass of scalar-tensor theories, is extremely popular to explain the current acceleration of the universe without an *ad hoc* dark energy.
In many areas (especially cosmology, models of neutron star/white dwarf interiors), it is common to use fluids as the matter source of the Einstein equations. In alternative theories of gravity, the field equations are often recast as effective Einstein equations by moving geometric terms (other than G_{ab}) to the r.h.s., regarding them as an effective $T_{ab}^{(eff)}$. This approach has proven useful in reducing problems to known GR problems.

Can you interpret $T_{ab}^{(eff)}$ on r.h.s. as an (effective) fluid? Not obvious.

- For a scalar field in GR? Yes, and the effective fluid is a *perfect fluid*
- For a nonminimally coupled scalar field? Yes (Madsen '88)
- Partially done for scalar-tensor gravity (Pimentel '89)
- Well known for special spacetimes; in FLRW space, fluid reduces to a perfect fluid.
- Done in more general theories containing a scalar d.o.f. (k-essence, special cases of Horndeski, higher derivative mimetic dark matter scenario) for cosmological perturbations (Arroja & Sasaki '10; Diez-Tejedor '13; Avelino & Azevedo '18; Unnikrishnan & Sriramkumar '10; Christopherson & Malik '09; Lim, Sawicki, Vikman '10; Mirzagholi & Vikman '15).

Focus on simple scalar-tensor gravity but on *general* spacetimes. The (Jordan frame) action is

$$S_{ST} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}\nabla^c\phi\nabla_c\phi - V(\phi)\right] + S^{(m)}$$

$\phi \sim 1/G_{eff} > 0$ is the Brans-Dicke scalar, $\omega = \omega(\phi)$.
The field equations are

$$R_{ab} - \frac{1}{2}g_{ab}R = \frac{8\pi}{\phi}T^{(m)}_{ab} + \frac{\omega}{\phi^2}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla_c\phi\nabla^c\phi\right) + \frac{1}{\phi}\left(\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right) - \frac{V}{2\phi}g_{ab}\,,$$

$$\Box\phi = \frac{1}{2\omega+3}\left(\frac{8\pi T^{(m)}}{\phi} + \phi\frac{dV}{d\phi} - 2V - \frac{d\omega}{d\phi}\nabla^c\phi\nabla_c\phi\right)$$

Effective fluid description?

Kinematics of the scalar field fluid

The correspondence is possible if gradient $\nabla^a\phi$ is *timelike*; fluid 4-velocity is

$$u^a = \frac{\nabla^a\phi}{\sqrt{-\nabla^e\phi\nabla_e\phi}} \, ,$$

(normalized, $u^c u_c = -1$)
3-D space "seen" by the comoving observers of the fluid with time direction u^a has 3-metric

$$h_{ab} \equiv g_{ab} + u_a u_b \, ,$$

while $h_a{}^b$ is the usual projection operator on this 3-space,

$$\begin{aligned} h_{ab}u^a &= h_{ab}u^b = 0 \, , \\ h^a{}_b \, h^b{}_c &= h^a{}_c \, , \qquad h^a{}_a = 3 \, . \end{aligned}$$

Fluid 4-acceleration is

$$\dot{u}^a \equiv u^b \nabla_b u^a$$

(of course, orthogonal to 4-velocity, $\dot{u}^c u_c = 0$).

The (double) projection of the velocity gradient onto the 3-space orthogonal to u^c is the purely spatial tensor

$$V_{ab} \equiv h_a{}^c \, h_b{}^d \, \nabla_d u_c$$

which decomposes as

$$V_{ab} = \theta_{ab} + \omega_{ab} = \sigma_{ab} + \frac{\theta}{3} h_{ab} + \omega_{ab}$$

where $\theta \equiv \theta^c{}_c = \nabla^c u_c =$ expansion scalar, and these tensors are purely spatial,

$$\theta_{ab} u^a = \theta_{ab} u^b = \omega_{ab} \, u^a = \omega_{ab} \, u^b = \sigma_{ab} u^a = \sigma_{ab} u^b = 0 \, ,$$

The shear and vorticity scalars are

$$\sigma^2 \equiv \frac{1}{2}\,\sigma_{ab}\sigma^{ab}$$
$$\omega^2 \equiv \frac{1}{2}\,\omega_{ab}\omega^{ab}\,,$$

In general (Ellis '71)

$$\nabla_b u_a = \sigma_{ab} + \frac{\theta}{3}\,h_{ab} + \omega_{ab} - \dot{u}_a u_b = V_{ab} - \dot{u}_a u_b\,.$$

Let's specialize these general definitions to our particular case.
Kinematic quantities (not given in Pimentel '89):

$$h_{ab} = g_{ab} - \frac{\nabla_a\phi\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}$$

$$\nabla_b u_a = \frac{1}{\sqrt{-\nabla^e\phi\nabla_e\phi}}\left(\nabla_a\nabla_b\phi - \frac{\nabla_a\phi\nabla^c\phi\nabla_b\nabla_c\phi}{\nabla^e\phi\nabla_e\phi}\right).$$

4-acceleration, its norm, and its divergence are:

$$\dot{u}_a = \left(-\nabla^e\phi\nabla_e\phi\right)^{-2}\nabla^b\phi\Big[(-\nabla^e\phi\nabla_e\phi)\nabla_a\nabla_b\phi + \nabla^c\phi\nabla_b\nabla_c\phi\nabla_a\phi\Big],$$

$$\dot{u}^a\dot{u}_a = \frac{\left[-\nabla^e\phi\nabla_e\phi\nabla_b\phi\nabla^d\phi\nabla^b\nabla^a\phi\nabla_d\nabla_a\phi + \left(\nabla^a\phi\nabla^b\phi\nabla_a\nabla_b\phi\right)^2\right]}{(-\nabla^e\phi\nabla_e\phi)^3},$$

$$\nabla_a\dot{u}^a = \left(-\nabla^e\phi\nabla_e\phi\right)^{-2}\left[-\nabla^e\phi\nabla_e\phi\nabla^b\phi\,\Box\,(\nabla_b\phi) + \nabla^c\phi\nabla^a\phi\nabla^b\phi\nabla_b\nabla_a\nabla_c\phi\right]$$

$$+(-\nabla^e\phi\nabla_e\phi)^{-3}\Big[(\nabla^e\phi\nabla_e\phi)^2\nabla^a\nabla^b\phi\nabla_a\nabla_b\phi - \nabla^e\phi\nabla_e\phi\nabla^b\phi\nabla^c\phi\nabla_b\nabla_c\phi\,\Box\,\phi$$

$$-4\left(\nabla^e\phi\nabla_e\phi\right)\nabla^c\phi\nabla_b\phi\nabla_a\nabla_c\phi\nabla^b\nabla^a\phi + 4\left(\nabla^a\phi\nabla^b\phi\nabla_b\nabla_a\phi\right)^2\Big]$$

$$
\begin{aligned}
V_{ab} &= \frac{\nabla_a \nabla_b \phi}{\left(-\nabla^e \phi \nabla_e \phi\right)^{1/2}} \\
&+ \frac{\left(\nabla_a \phi \nabla_b \nabla_c \phi + \nabla_b \phi \nabla_a \nabla_c \phi\right) \nabla^c \phi}{\left(-\nabla^e \phi \nabla_e \phi\right)^{3/2}} \\
&+ \frac{\nabla_d \nabla_c \phi \nabla^c \phi \nabla^d \phi}{\left(-\nabla^e \phi \nabla_e \phi\right)^{5/2}} \nabla_a \phi \nabla_b \phi .
\end{aligned}
$$

Vorticity $\omega_{ab} \equiv V_{[ab]}$ vanishes identically, because 4-velocity originates from a gradient, so

$$
V_{ab} = \theta_{ab} , \qquad \nabla_b u_a = \theta_{ab} - \dot{u}_a u_b ,
$$

u^a is hypersurface-orthogonal, line element can be diagonalized.

The expansion scalar reduces to

$$\theta = \nabla_a u^a = \frac{\Box\phi}{\left(-\nabla^e\phi\nabla_e\phi\right)^{1/2}} + \frac{\nabla_a\nabla_b\phi\nabla^a\phi\nabla^b\phi}{\left(-\nabla^e\phi\nabla_e\phi\right)^{3/2}} \, ,$$

shear tensor is

$$\sigma_{ab} = \left(-\nabla^e\phi\nabla_e\phi\right)^{-3/2}\Big[-\left(\nabla^e\phi\nabla_e\phi\right)\nabla_a\nabla_b\phi$$

$$-\frac{1}{3}\left(\nabla_a\phi\nabla_b\phi - g_{ab}\nabla^c\phi\nabla_c\phi\right)\Box\,\phi$$

$$-\frac{1}{3}\left(g_{ab} + \frac{2\nabla_a\phi\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}\right)\nabla_c\nabla_d\phi\nabla^d\phi\nabla^c\phi$$

$$+\left(\nabla_a\phi\nabla_c\nabla_b\phi + \nabla_b\phi\nabla_c\nabla_a\phi\right)\nabla^c\phi\Big] \, ,$$

shear scalar reads

$$\sigma = \left(\frac{1}{2} \sigma^{ab} \sigma_{ab} \right)^{1/2} = \left(-\nabla^e \phi \nabla_e \phi \right)^{-3/2} \left\{ \frac{1}{2} \left(\nabla^e \phi \nabla_e \phi \right)^2 \left[\nabla^a \nabla^b \phi \nabla_a \nabla_b \phi \right. \right.$$

$$\left. - \frac{1}{3} \left(\Box \phi \right)^2 \right] + \frac{1}{3} \left(\nabla_a \nabla_b \phi \nabla^a \phi \nabla^b \phi \right)^2$$

$$\left. - \left(\nabla^e \phi \nabla_e \phi \right) \left(\nabla_a \nabla_b \phi \nabla^b \nabla_c \phi - \frac{1}{3} \Box \phi \nabla_a \nabla_c \phi \right) \nabla^a \phi \nabla^c \phi \right\}^{1/2} ,$$

Writing the vacuum field eqs as the effective Einstein eqs.

$$8\pi T_{ab}^{(\phi)} = \frac{\omega}{\phi^2}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla^c\phi\nabla_c\phi\right) + \frac{1}{\phi}\left(\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right) - \frac{V}{2\phi}g_{ab}\,.$$

the r.h.s. can be decomposed as

$$\boxed{T_{ab} = \rho u_a u_b + q_a u_b + q_b u_a + \Pi_{ab}\,,}$$

where

$$
\begin{aligned}
\rho &= T_{ab} u^a u^b \,, \\
q_a &= -T_{cd}\, u^c h_a{}^d \,, \\
\Pi_{ab} &\equiv P h_{ab} + \pi_{ab} = T_{cd}\, h_a{}^c\, h_b{}^d \,, \\
P &= \frac{1}{3}\, g^{ab} \Pi_{ab} = \frac{1}{3}\, h^{ab} T_{ab} \,, \\
\pi_{ab} &= \Pi_{ab} - P h_{ab} \,,
\end{aligned}
$$

with

$$
q_c u^c = 0 \,,
$$

$$
\Pi_{ab} u^b = \pi_{ab} u^b = \Pi_{ab} u^a = \pi_{ab} u^a = 0 \,, \qquad \pi^a{}_a = 0 \,.
$$

Calculating these quantities explicitly,

$$
\begin{aligned}
8\pi\rho^{(\phi)} &= -\frac{\omega}{2\phi^2}\nabla^e\phi\nabla_e\phi + \frac{V}{2\phi} + \frac{1}{\phi}\left(\Box\phi - \frac{\nabla^a\phi\nabla^b\phi\nabla_a\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}\right), \\
8\pi q_a^{(\phi)} &= -\frac{\nabla^c\phi\nabla_a\nabla_c\phi}{\phi\left(-\nabla^e\phi\nabla_e\phi\right)^{1/2}} - \frac{\nabla^c\phi\nabla^d\phi\nabla_c\nabla_d\phi}{\phi\left(-\nabla^e\phi\nabla_e\phi\right)^{3/2}}\nabla_a\phi\,, \\
8\pi\Pi_{ab}^{(\phi)} &= \left(-\frac{\omega}{2\phi^2}\nabla^c\phi\nabla_c\phi - \frac{\Box\phi}{\phi} - \frac{V}{2\phi}\right)h_{ab} + \frac{1}{\phi}h_a{}^c h_b{}^d\nabla_c\nabla_d\phi\,, \\
8\pi P^{(\phi)} &= -\frac{\omega}{2\phi^2}\nabla^e\phi\nabla_e\phi - \frac{V}{2\phi} - \frac{1}{3\phi}\left(2\Box\phi + \frac{\nabla^a\phi\nabla^b\phi\nabla_b\nabla_a\phi}{\nabla^e\phi\nabla_e\phi}\right),
\end{aligned}
$$

$$8\pi \pi_{ab}^{(\phi)} = \frac{1}{\phi \nabla^e\phi\nabla_e\phi}\left[\frac{1}{3}\left(\nabla_a\phi\nabla_b\phi - g_{ab}\nabla^c\phi\nabla_c\phi\right)\left(\Box\phi - \frac{\nabla^c\phi\nabla^d\phi\nabla_d\nabla_c\phi}{\nabla^e\phi\nabla_e\phi}\right)\right.$$

$$+\nabla^d\phi\left(\nabla_d\phi\nabla_a\nabla_b\phi - \nabla_b\phi\nabla_a\nabla_d\phi - \nabla_a\phi\nabla_d\nabla_b\phi\right.$$

$$\left.\left.+\frac{\nabla_a\phi\nabla_b\phi\nabla^c\phi\nabla_c\nabla_d\phi}{\nabla^e\phi\nabla_e\phi}\right)\right],$$

The heat flux density

$$\boxed{q_a^{(\phi)} = -\frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi}\,\dot{u}_a\,,}$$

does not vanish: *imperfect* fluid.

Thermodynamics of spacetime?

Previous equation has a physical consequence in Eckart's first order thermodynamics (Eckart '40, notoriously plagued by non-causality and instability but still widely used as an approximation)

The heat flux density is related to the temperature $\mathcal{T}$ by the generalized Fourier law

$$q_a = -K \left(h_{ab} \nabla^b \mathcal{T} + \mathcal{T} \dot{u}_a \right) \,,$$

where $K =$ thermal conductivity. This gives

$$K\mathcal{T} = \frac{\sqrt{-\nabla^c \phi \nabla_c \phi}}{8\pi\phi} \,,$$

for a ST spacetime.

Alternative approach: trade temperature with chemical potential, assign zero temperature and entropy but nonzero chemical potential to the effective fluid (as in Vikman).

All this needs to be developed.

Fluid symmetry for electrovacuum Brans-Dicke gravity

Now restrict to (electro-)vacuum Brans-Dicke theory with $\omega =$ const. The Brans-Dicke action is invariant in form under the 1-parameter group of symmetries (VF '99)

$$g_{ab} \rightarrow \tilde{g}_{ab} = \phi^{2\alpha} g_{ab} \,,$$

$$\phi \rightarrow \tilde{\phi} = \phi^{1-2\alpha} \,, \qquad \alpha \neq 1/2 \,,$$

provided that

$$\omega \rightarrow \tilde{\omega}(\omega, \alpha) = \frac{\omega + 6\alpha(1-\alpha)}{(1-2\alpha)^2} \,,$$

$$V \rightarrow \tilde{V}(\tilde{\phi}) = \tilde{\phi}^{\frac{-4\alpha}{1-2\alpha}} V\left(\tilde{\phi}^{\frac{1}{1-2\alpha}}\right) .$$

Assuming $\nabla^c\phi$ is timelike, then the “new” fluid quantities are

$$u_c \rightarrow \tilde{u}_c = \phi^{\alpha} u_c \,,$$

$$u^c \rightarrow \tilde{u}^c = \phi^{-\alpha} u^c \,.$$

(still normalized) and

$$\tilde{T}_{ab}^{(\tilde{\phi})} = T_{ab}^{(\phi)} + \frac{\alpha}{4\pi\phi}\left[\frac{(1+\alpha)}{\phi}\nabla_a\phi\nabla_b\phi \right.$$
$$\left. + \frac{(\alpha-2)}{2\phi}\nabla^e\phi\nabla_e\phi\, g_{ab} - \left(\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right)\right] ,$$

effective energy density:

$$\tilde{\rho}^{(\tilde{\phi})} = \phi^{-2\alpha}\left[(1-2\alpha)\rho^{(\phi)} - \frac{\alpha(3\alpha+\omega)}{8\pi\phi^2}\nabla^e\phi\nabla_e\phi + \frac{\alpha V}{8\pi\phi}\right].$$

effective heat flux density:

$$\tilde{q}_a^{(\tilde{\phi})} = (1-2\alpha)\,\phi^{-\alpha}\,q_a^{(\phi)}\,,$$

effective pressure:

$$\tilde{P}^{(\tilde{\phi})} = \phi^{-2\alpha}\left[(1-2\alpha)\,P^{(\phi)} + \frac{\alpha\,(\alpha-\omega-2)}{8\pi\phi^2}\nabla^e\phi\nabla_e\phi - \frac{\alpha V}{8\pi\phi}\right],$$

effective spatial stress tensor:

$$\tilde{\Pi}_{ab}^{(\tilde{\phi})} = (1-2\alpha)\,\Pi_{ab}^{(\phi)} + \frac{\alpha}{8\pi\phi}\left[\frac{(\alpha-\omega-2)}{\phi}\,\nabla^e\phi\nabla_e\phi - V\right]h_{ab}\,,$$

effective anisotropic stresses

$$\tilde{\pi}_{ab}^{(\tilde{\phi})} = (1-2\alpha)\,\pi_{ab}^{(\phi)}$$

Now apply all this to the GR limit of Brans-Dicke theory.

Limit of Brans-Dicke to GR

- Related to the weak-field limit and the PPN formalism (the basis for constraining ω with Solar System experiments) (Bertotti, Iess & Tortora '03, Will '06).
- There are attractor mechanisms forcing ST gravity toward GR in the early universe (Damour & K. Nordtvedt '93; Mimoso & Wands '95; Gérard & Mahara '95; Santiago *et al.* '98; Serna *et al.* '02; ...).
- ST gravity could be an "excitation" of GR in the thermodynamics of spacetime–Einstein eqs. derived as a macroscopic equation of state (Jacobson '95). GR as a "state of equilibrium", ST deviations from it could be non-equilibrium states (Eling, Guedens, Jacobson '06; Chirco, Liberati, Eling '10-'11).

$\omega \to \infty$: the standard lore

GR is reproduced from Jordan frame Brans-Dicke gravity when ϕ becomes constant: then $G_{eff} \simeq \phi^{-1} \to$ const.
The contentious issue is how fast ϕ approaches a constant.
Common lore (*e.g.*, Weinberg '72) is that BD gravity reduces to GR as $\omega \to \infty$, with the BD scalar asymptotics

$$\phi = \phi_\infty + \mathcal{O}\left(\frac{1}{\omega}\right),$$

where $\phi_\infty > 0$ is a constant.

An anomaly in the $\omega \to \infty$ limit

A number of analytic solutions of electrovacuum BD have been reported which fail to reduce to the corresponding solutions of GR as $\omega \to \infty$ (Matsuda '72; Romero & Barros '92-'93; Paiva & Romero '93; Banerjee & Sen '97; Anchordoqui *et al.* '98; Bhadra & Nandi '01; Järv *et al.* '07, '15; Kirezli & Delice '15)

Apparently, also in matter different from (electro)vacuum (Chauvineau '03, 07; Brando *et al.* '18).

Restrict to *electrovacuum* $T^{(m)} = 0$: the asymptotic behavior of ϕ is not the one expected but

$$\phi = \phi_\infty + \mathcal{O}\left(\frac{1}{\sqrt{\omega}}\right)$$

This is related to the restricted conformal symmetry of BD theory when $T^{(m)} = 0$: the relation between $\bar{\omega}$ and α can be inverted →

$$\alpha = \frac{2\bar{\omega} + 3 \pm \sqrt{(2\bar{\omega}+3)(2\omega+3)}}{2(2\bar{\omega}+3)} \,,$$

and $\alpha \to 1/2$ as $\bar{\omega} \to \infty$. Trade these two limits and obtain larger and larger $\bar{\omega}$ by means of consecutive symmetry transformations. In (electro)vacuo, this transformation connects theories within an equivalence class and the change $\omega \to \bar{\omega}$ simply moves a BD theory within this equivalence class (VF '99). The $\bar{\omega} \to \infty$ limit is a transformation to larger and larger $\bar{\omega}$; it cannot break this restricted conformal invariance and move the theory outside of the equivalence class. GR, which is not conformally invariant, does not belong to this class and cannot be reproduced by this $\omega \to \infty$ limit (one needs to first break the conformal invariance).

Apply the effective fluid formalism

(restricted to timelike $\nabla_a \phi$ — if not, another method applies)

$$\omega \to \infty \leftrightarrow \alpha \to 1/2$$

- in this limit the imperfect fluid quantities $\bar{q}_a^{(\bar{\phi})}, \bar{\pi}_{ab}^{(\bar{\phi})} \propto (1 - 2\alpha)$ vanish identically;
- there remain non-vanishing contributions to the effective energy density and pressure

$$\bar{T}_{ab}^{(\bar{\phi})} \to \bar{T}_{ab}^{(\infty)} = \frac{1}{8\pi} \left[\frac{(2\omega + 3)}{2\phi^2} \left(\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi \right) - \frac{V(\phi)}{2\phi} g_{ab} \right],$$

$$\bar{\rho}^{(\bar{\phi})} \to \bar{\rho}_{(\infty)} = \frac{1}{\phi} \left[-\frac{(2\omega + 3)}{32\pi\phi^2} \nabla^c \phi \nabla_c \phi + \frac{V}{16\pi\phi} \right],$$

$$\bar{P}^{(\bar{\phi})} \to \bar{P}_{(\infty)} = \frac{1}{\phi} \left[-\frac{(2\omega + 3)}{32\pi\phi^2} \nabla^c \phi \nabla_c \phi - \frac{V}{16\pi\phi} \right].$$

Introducing the Einstein frame quantities

$$\tilde{g}_{ab} \equiv \phi g_{ab} \,,$$

$$\Phi = \sqrt{\frac{|2\omega + 3|}{16\pi}} \ln\left(\frac{\phi}{\phi_0}\right) \,,$$

we have

$$\bar{T}^{(\infty)}_{ab} = \text{sign}\,(2\omega + 3)\left[\tilde{\nabla}_a\Phi\tilde{\nabla}_b\Phi - \frac{1}{2}\,\tilde{g}_{ab}\left(\tilde{g}^{cd}\tilde{\nabla}_c\Phi\tilde{\nabla}_d\Phi\right)\right] - U(\Phi)\tilde{g}_{ab}$$

The stress-energy tensor reduces to that of a minimally coupled scalar field, which has the perfect fluid form. If $2\omega + 3 > 0$, the "standard" field Φ left in the limit is nothing but the Einstein frame scalar corresponding to the Jordan frame ϕ. However, the Einstein frame was not used in any way!

CONCLUSIONS/OPEN PROBLEMS

- ST gravity can be presented as effective Einstein eqs. with an *imperfect* (irrotational) fluid in the r.h.s.. Is this useful for analogue gravity?
- The anomaly in the $\omega \to \infty$ limit of electrovacuum BD theory is explained in a simple way using this fluid formalism: a minimally coupled scaler field survives in the limit (*i.e.*, ϕ does not become constant as it should). It is the Einstein frame scalar associated with ϕ.
- Open problems:
  - How relevant is this for the constraints on ST gravity?
  - Other anomalies in the GR limit for non-conformally invariant matter?
  - Other applications of the effective fluid formalism? Thermodynamics of spacetime?

THANK YOU