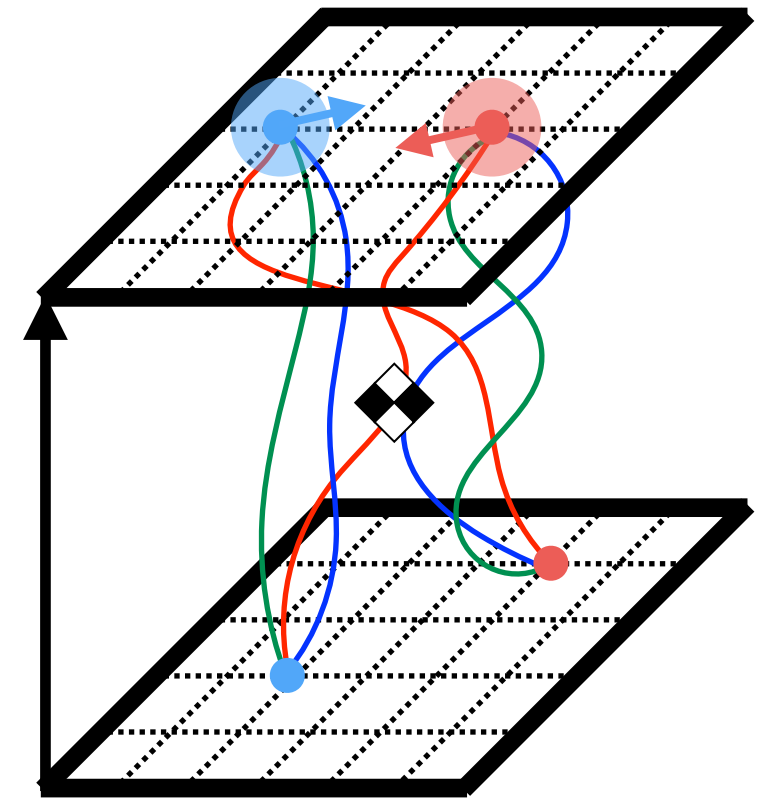
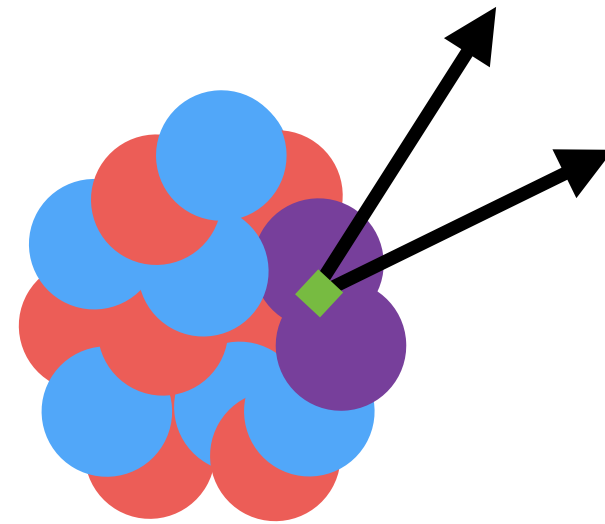


Lattice QCD and nuclear probes of BSM Physics



Evan Berkowitz

Institut für Kernphysik

Institute for Advanced Simulation

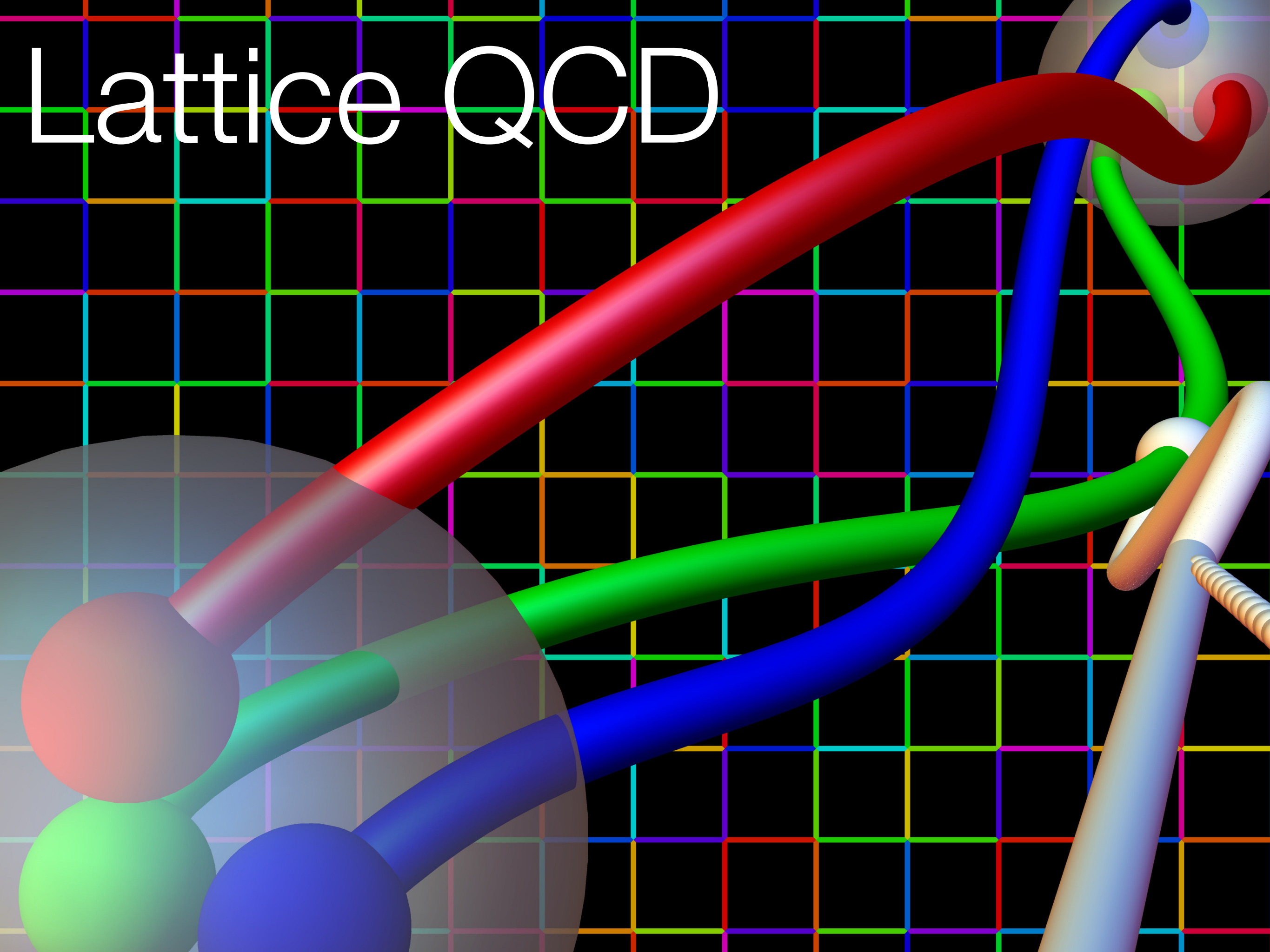
Forschungszentrum Jülich

Atomic Nuclei as Laboratories for BSM Physics

ECT*

17 April 2019

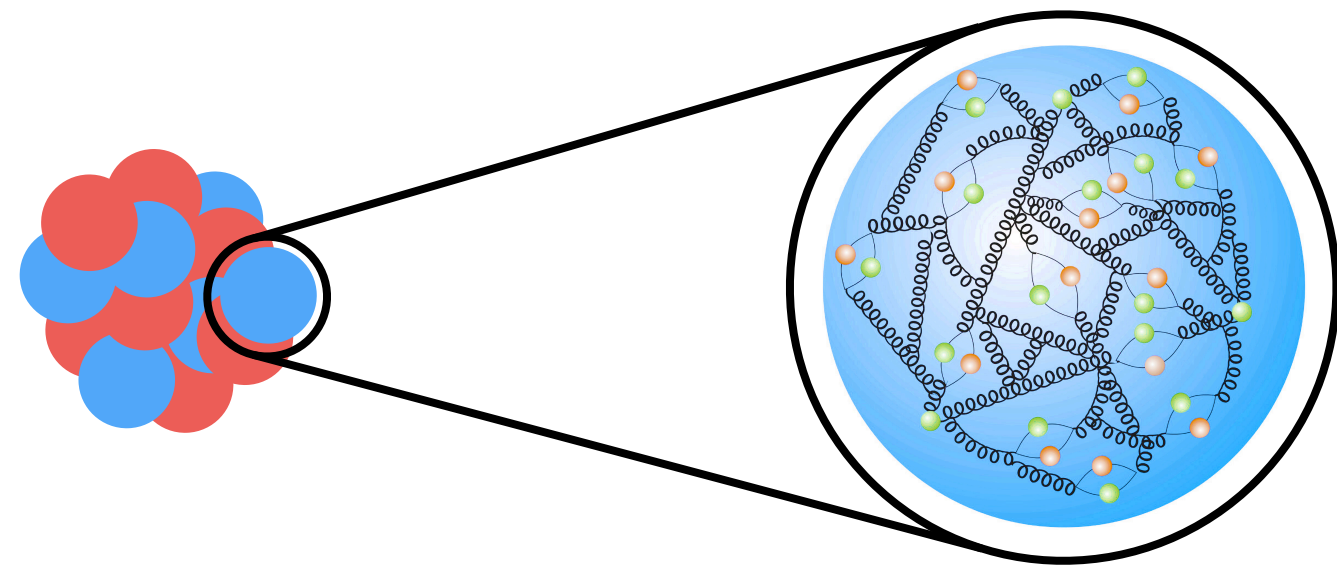
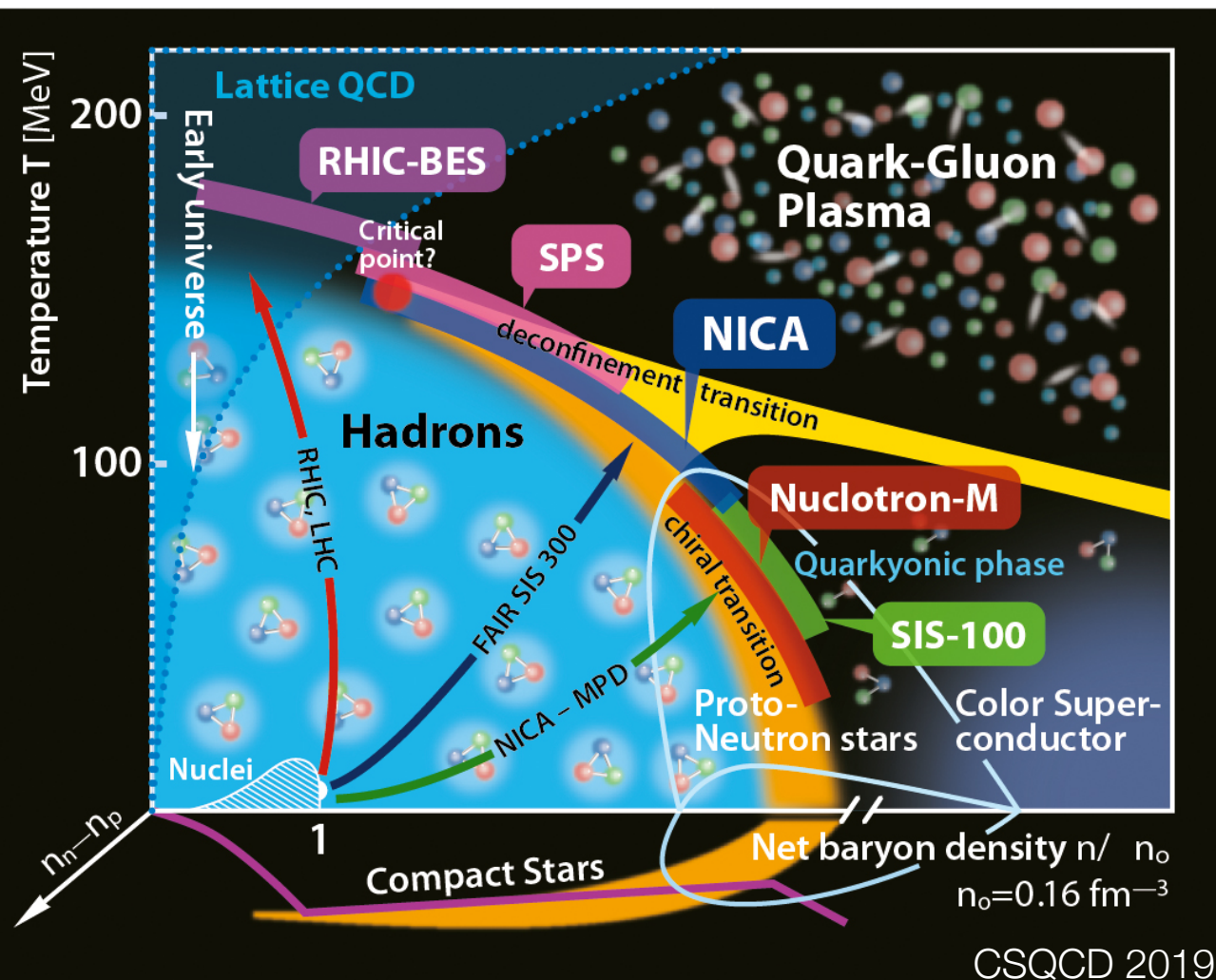




Lattice QCD

Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4} \overset{\text{Glue}}{F^2} + \overset{\text{Quarks}}{\bar{\psi}} \underset{\text{Gauge theory}}{(i \not{D} + m)} \underset{\text{masses}}{\psi}$$



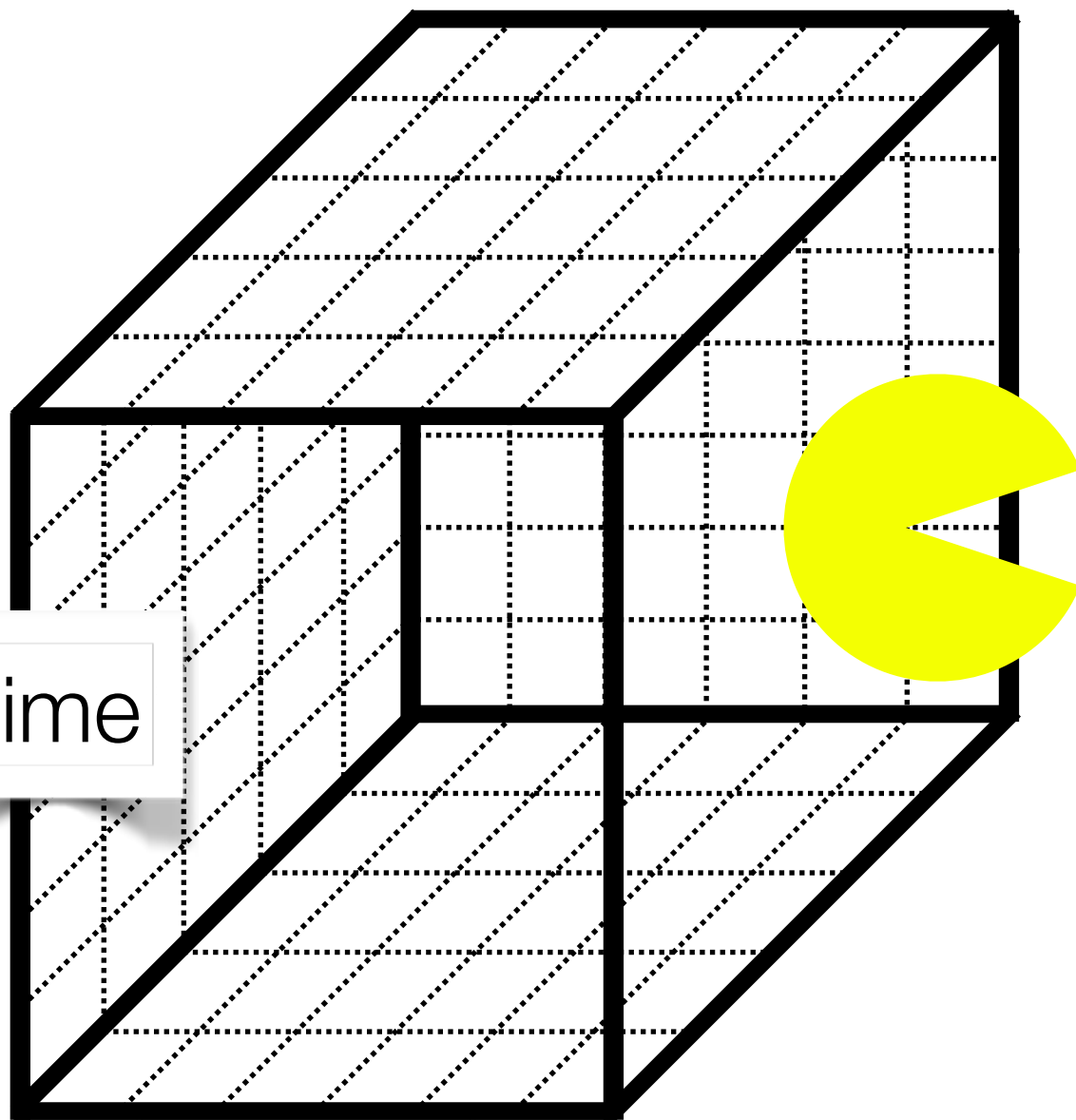
Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t) \mathcal{O}^\dagger(0) e^{-S[\bar{\psi}, \psi, U]}$$

$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \det(\not{D} + M) e^{-S[U]} \mathcal{O}(t) \mathcal{O}^\dagger(0)$$

lattice
finite volume

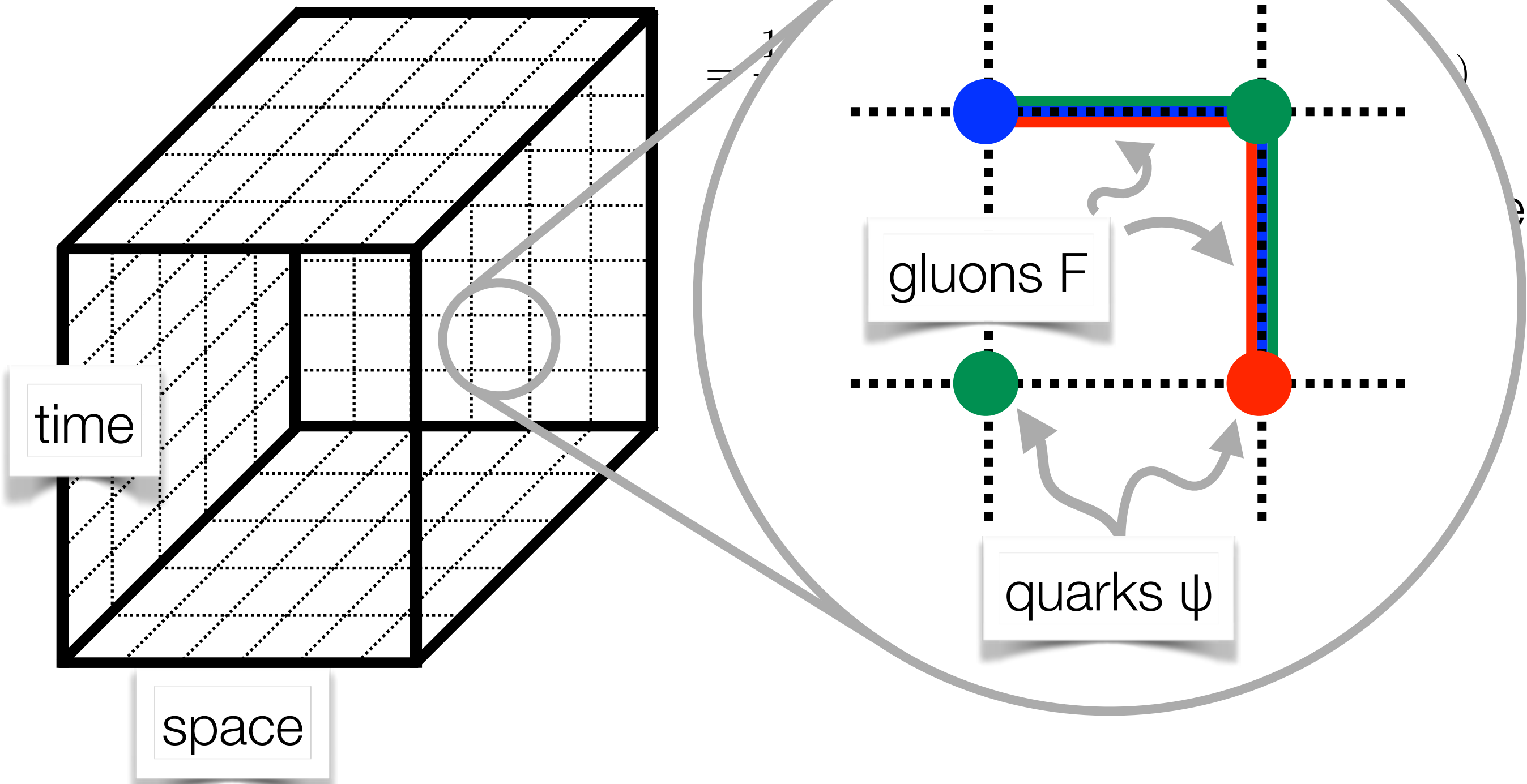


space

Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle = \frac{1}{Z} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t) \mathcal{O}^\dagger(0) e^{iS[\bar{\psi}, \psi, U]}$$



Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

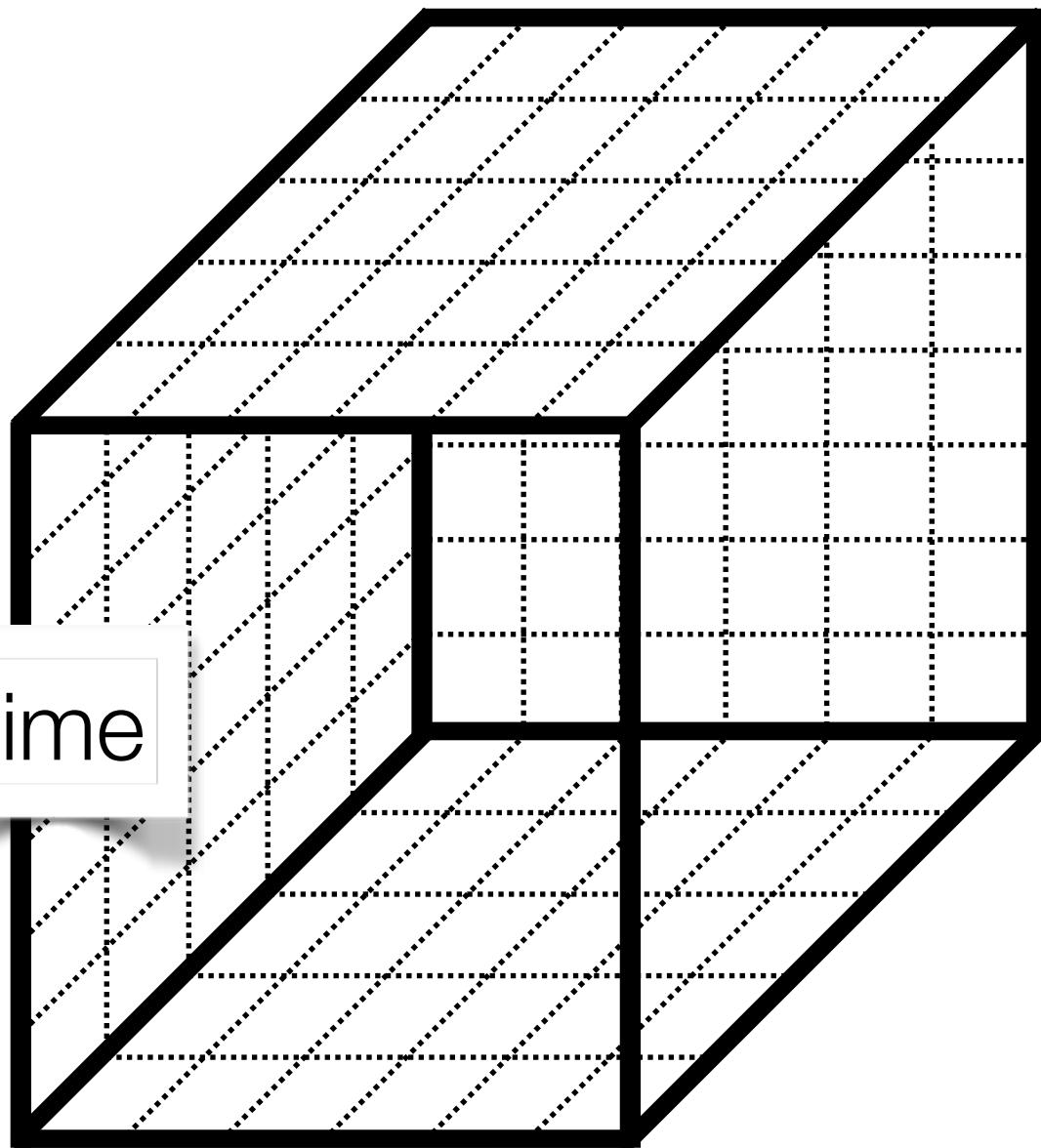
$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t)\mathcal{O}^\dagger(0) e^{-S[\bar{\psi},\psi,U]}$$
$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \underbrace{\det(\not{D} + M) e^{-S[U]}}_{\text{Probability}} \mathcal{O}(t)\mathcal{O}^\dagger(0)$$

Probability

$$\{U_1, U_2, U_3, \dots, U_N\}$$

Markov Chain Monte Carlo

$$\approx \frac{1}{N} \sum_{i=1}^N \mathcal{O}(t)\mathcal{O}^\dagger(0)[U_i] + O\left(\frac{1}{\sqrt{N}}\right)$$



time

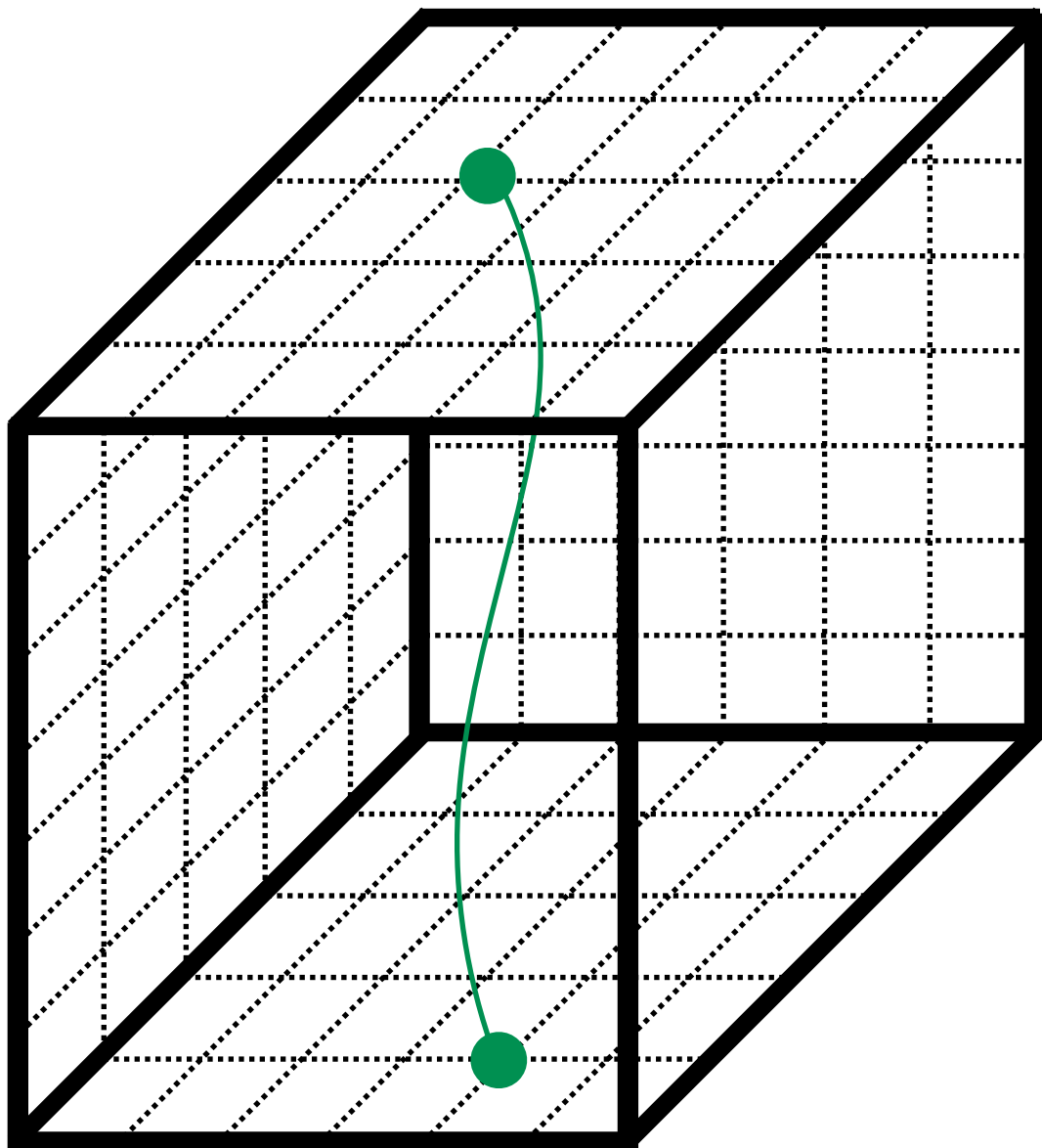
space

Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t) \mathcal{O}^\dagger(0) e^{-S[\bar{\psi}, \psi, U]}$$

$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \det(\not{D} + M) e^{-S[U]} \mathcal{O}(t) \mathcal{O}^\dagger(0)$$



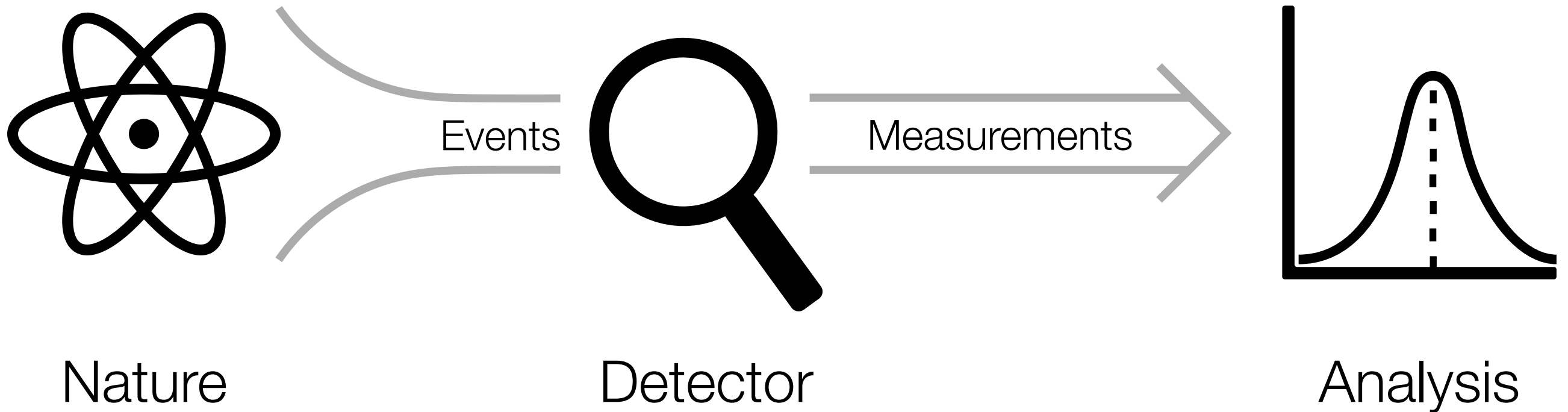
$\mathbf{D}$ is a sparse matrix
U-dependent

$(N_x N_t N_c N_s)$ on a side

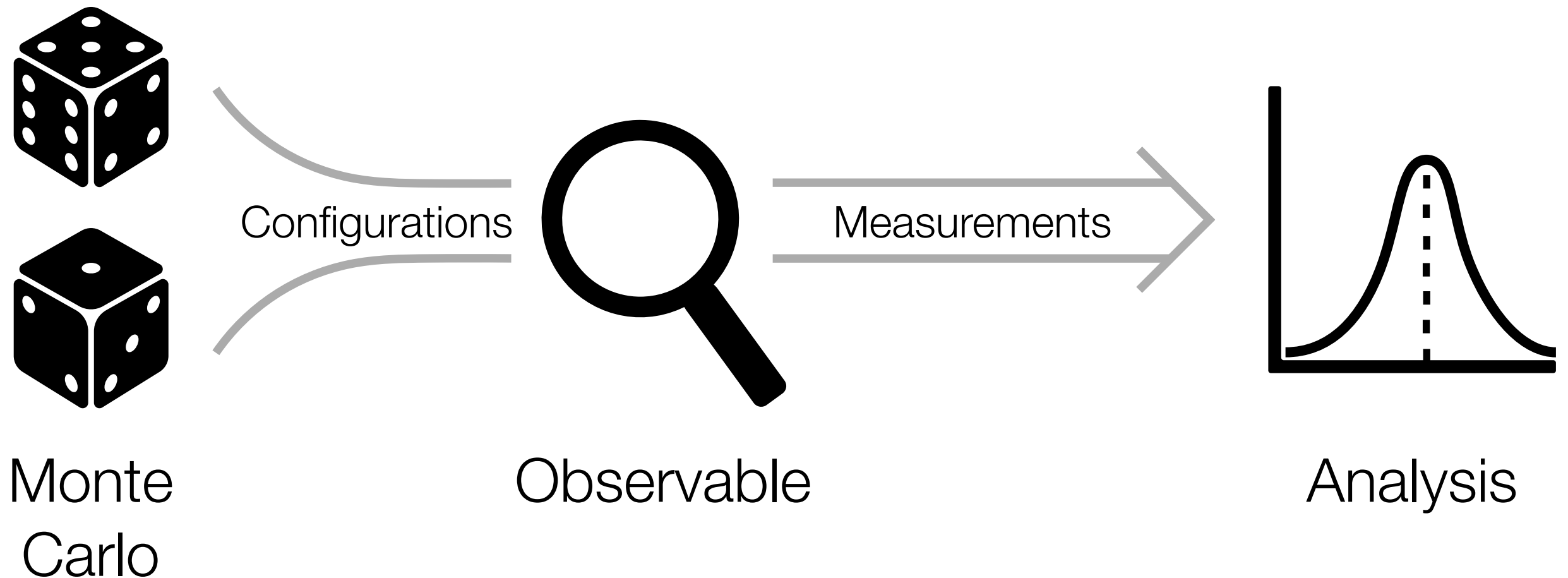
$$(48^3 \times 64 \times 3 \times 4) = 127\,401\,984$$

$$(\not{D}[U] + M)_{xf} S_{fi} = i\delta_{xi}$$

Analogy with Experiment

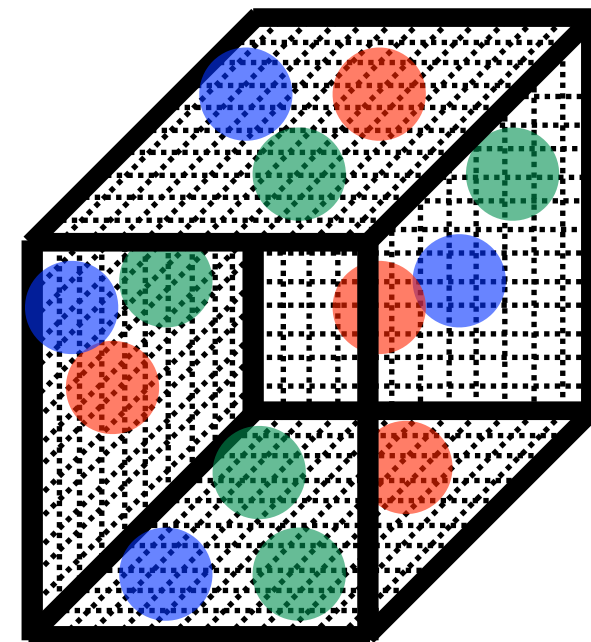
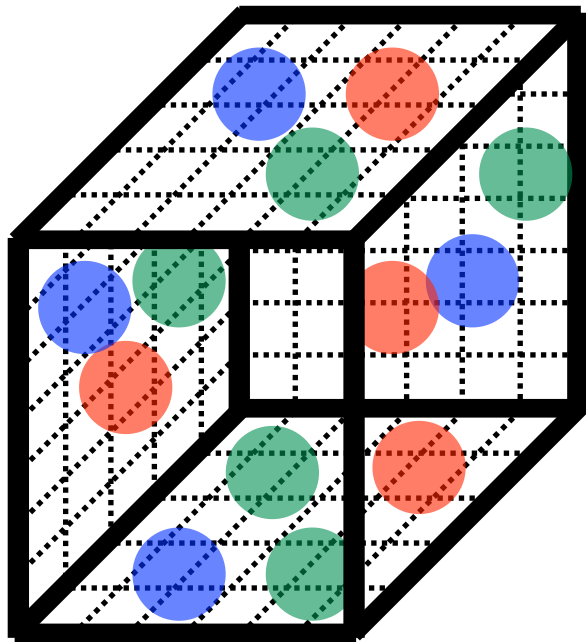


Analogy with Experiment

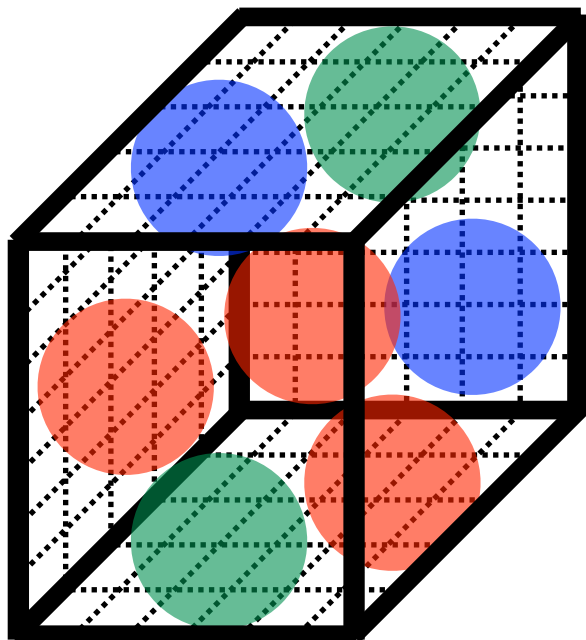


- ✓ Statistical errors are improved with run time (budget)
- ✓ Systematic errors are understood + controlled

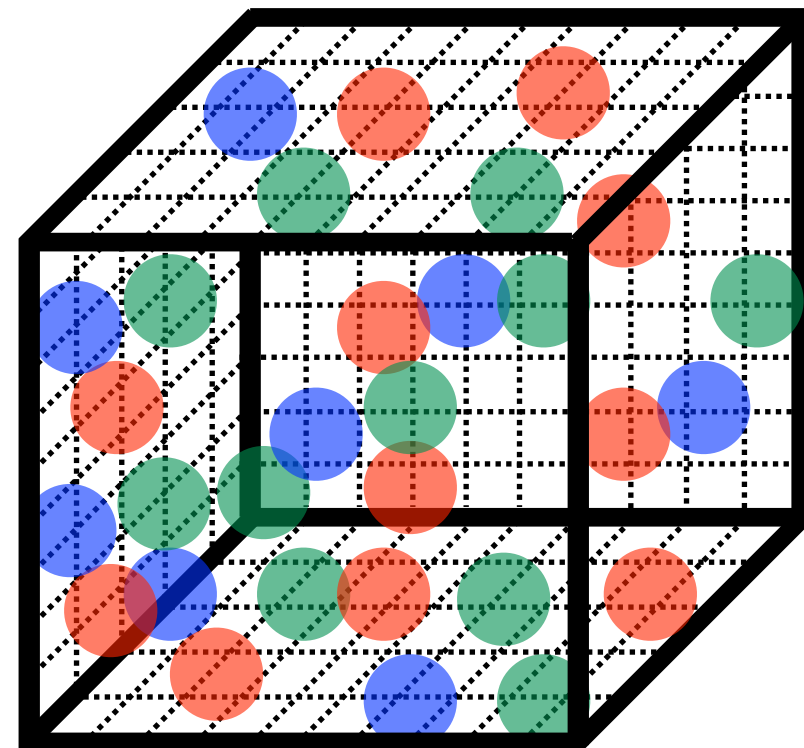
LQCD Systematics



continuum limit



physical quark masses



infinite volume limit

What can Lattice QCD do for BSM Physics?

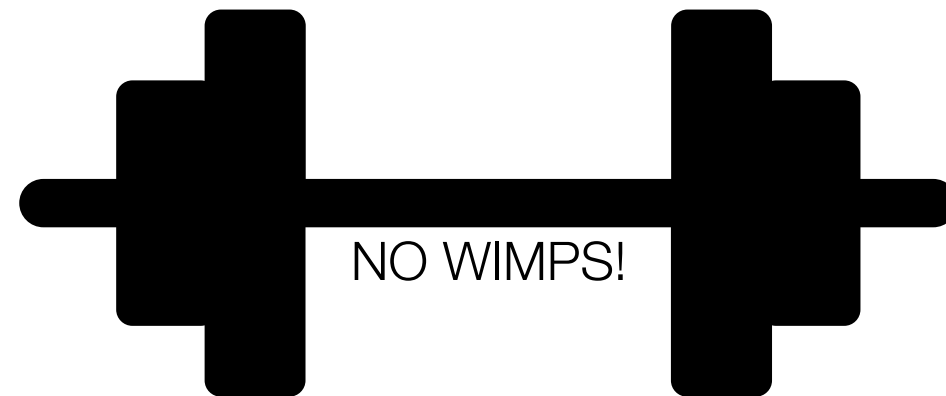
Axion Cosmology



Colgate-Palmolive

1505.07455 EB, M. Buchoff
1506.00370 Kitano & Yamada
1508.06917 Borsanyi et al.
1512.06746 Bonati et al.
1606.07494 Borsanyi et al.
Nature **539** 69–71 Borsanyi et al.

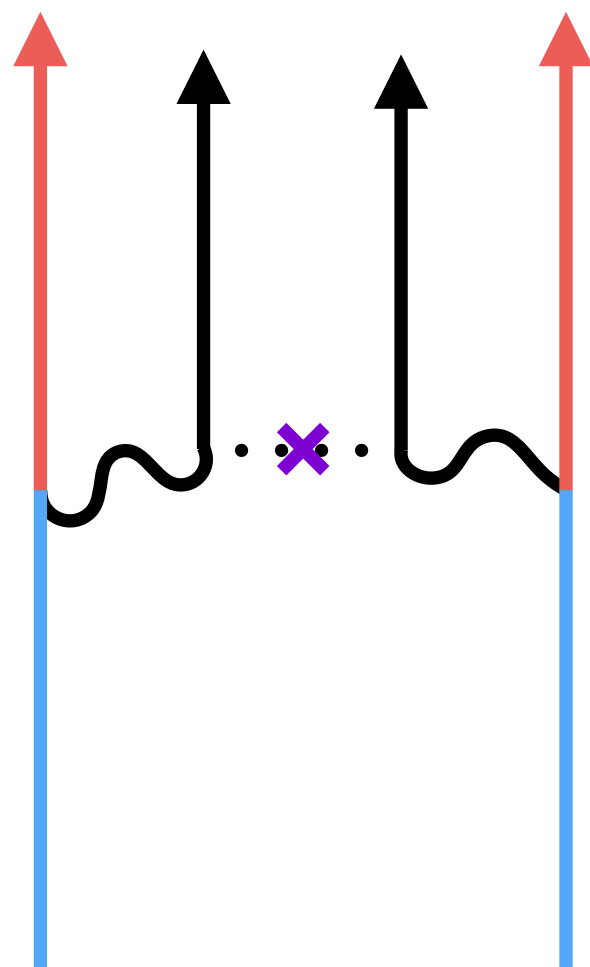
Non-abelian Dark Sectors



Reviews:
1604.04627 Kribs & Neil
1707.04591 US Cosmic Visions 2017

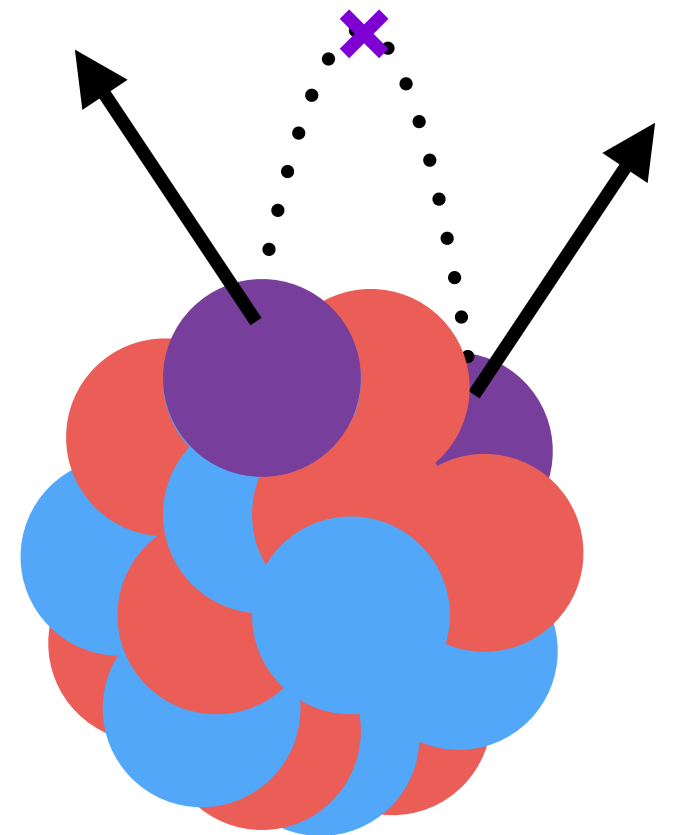
One proposal:
Stealth Dark Matter, LSD collaboration
1402.6656, 1503.04203, 1503.04205 ...

The Standard Model as an EFT: Model-Independence

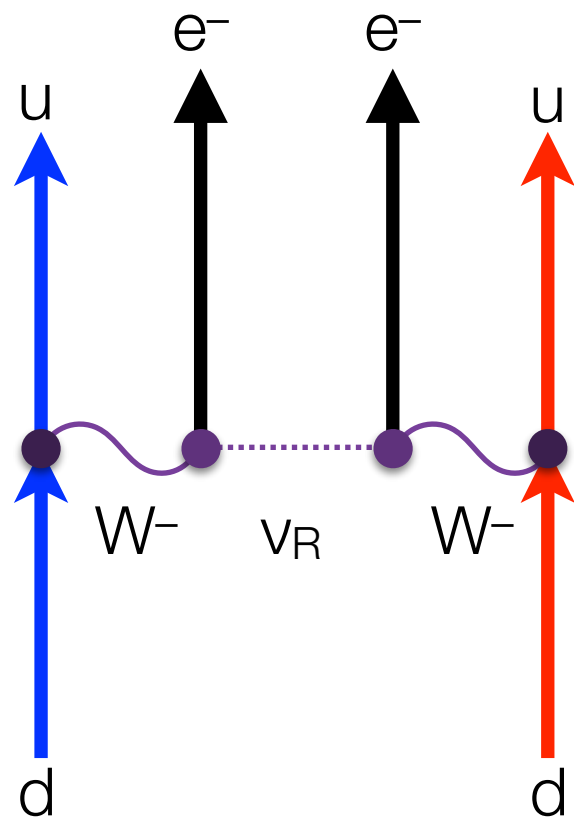


Your favorite
theory of new
physics

Calculate!



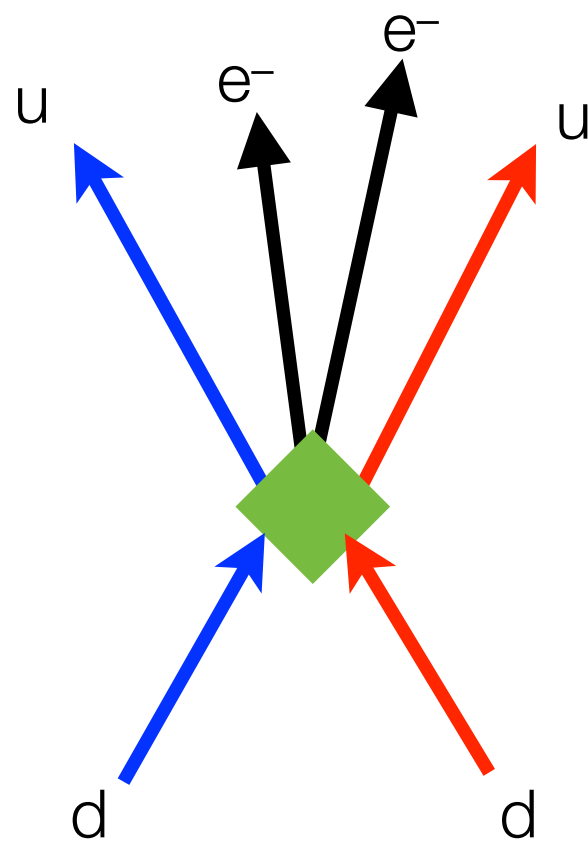
decay rates
in big nuclei



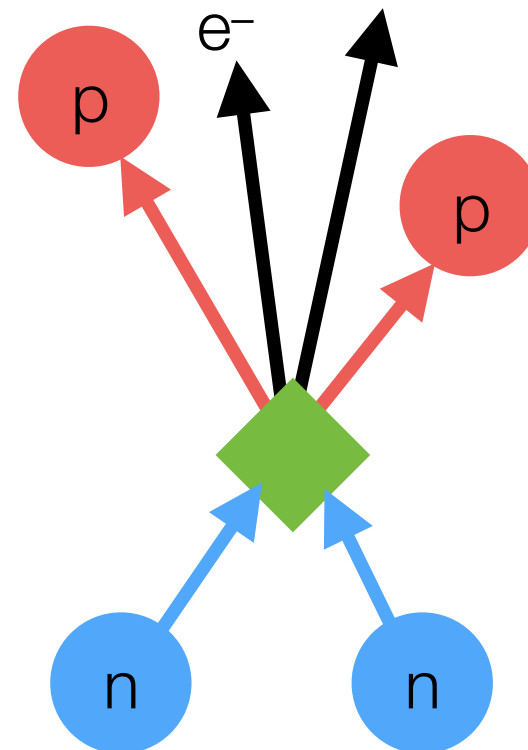
Your favorite
theory of new
physics

Solve QCD

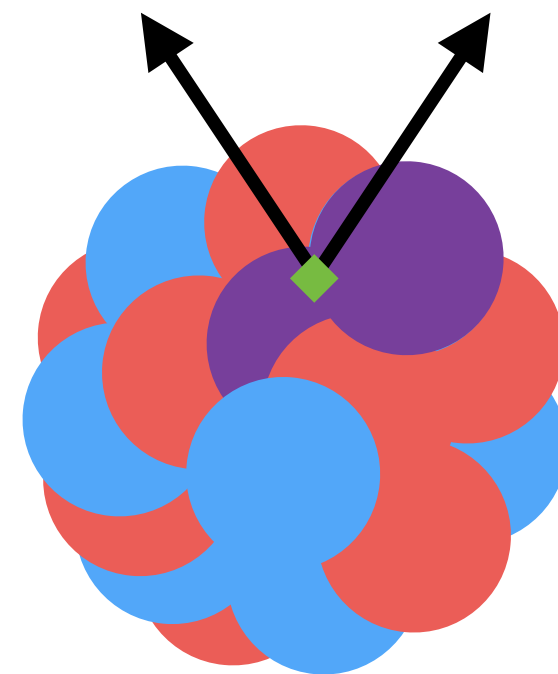
Solve NP



effective 4-quark
operators in QCD



two nucleon
amplitudes



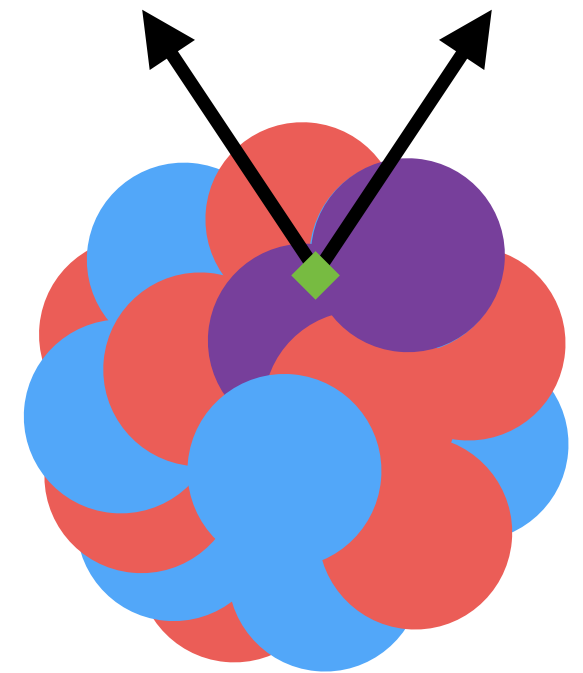
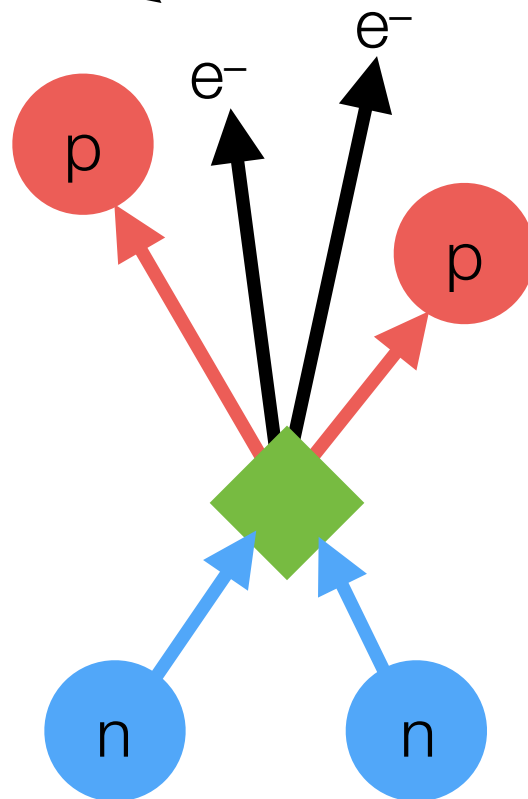
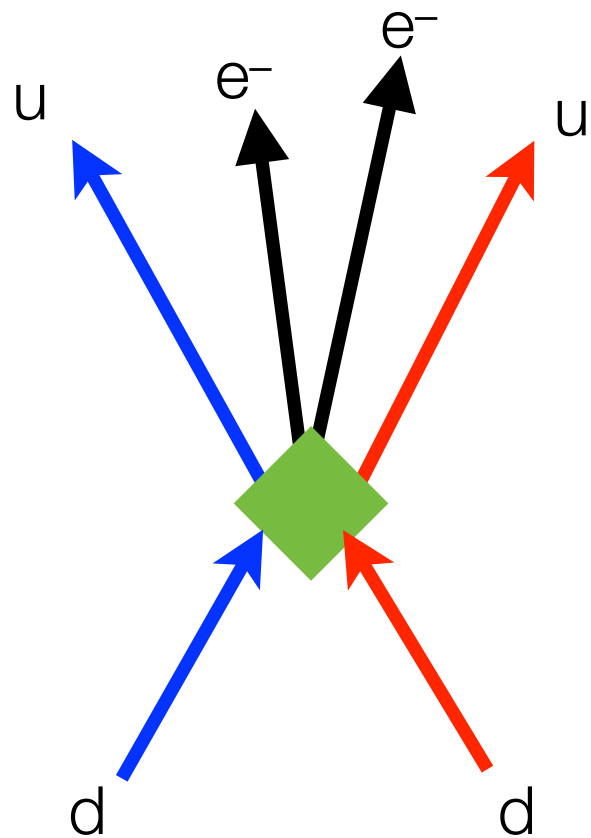
decay rates
in big nuclei

Pen and paper?

Numerical calculation?

Solve QCD

Solve NP



effective 4-quark
operators in QCD

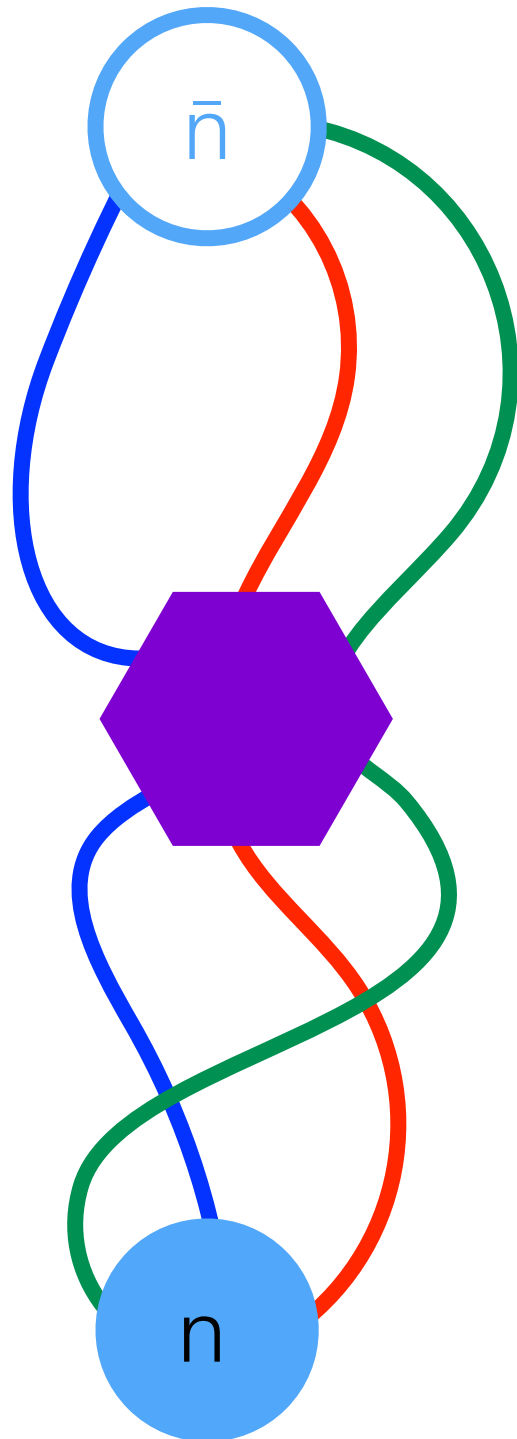
two nucleon
amplitudes

decay rates
in big nuclei

Single Nucleon Matrix Elements

$\bar{n}n$ Oscillations

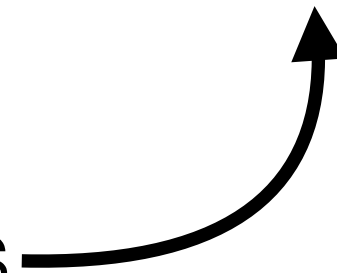
Rinaldi, Syritsyn, Wagman, Buchoff, Schroeder, and Wasem 1809.00246



✓ Physical point
✗ continuum, infinite volume

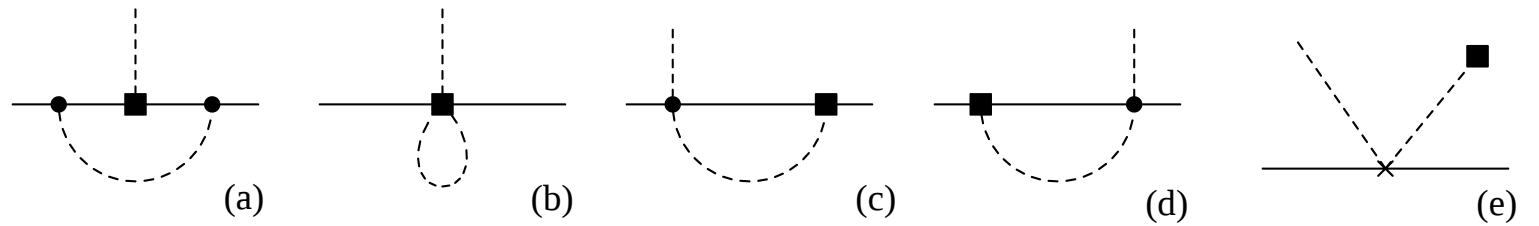
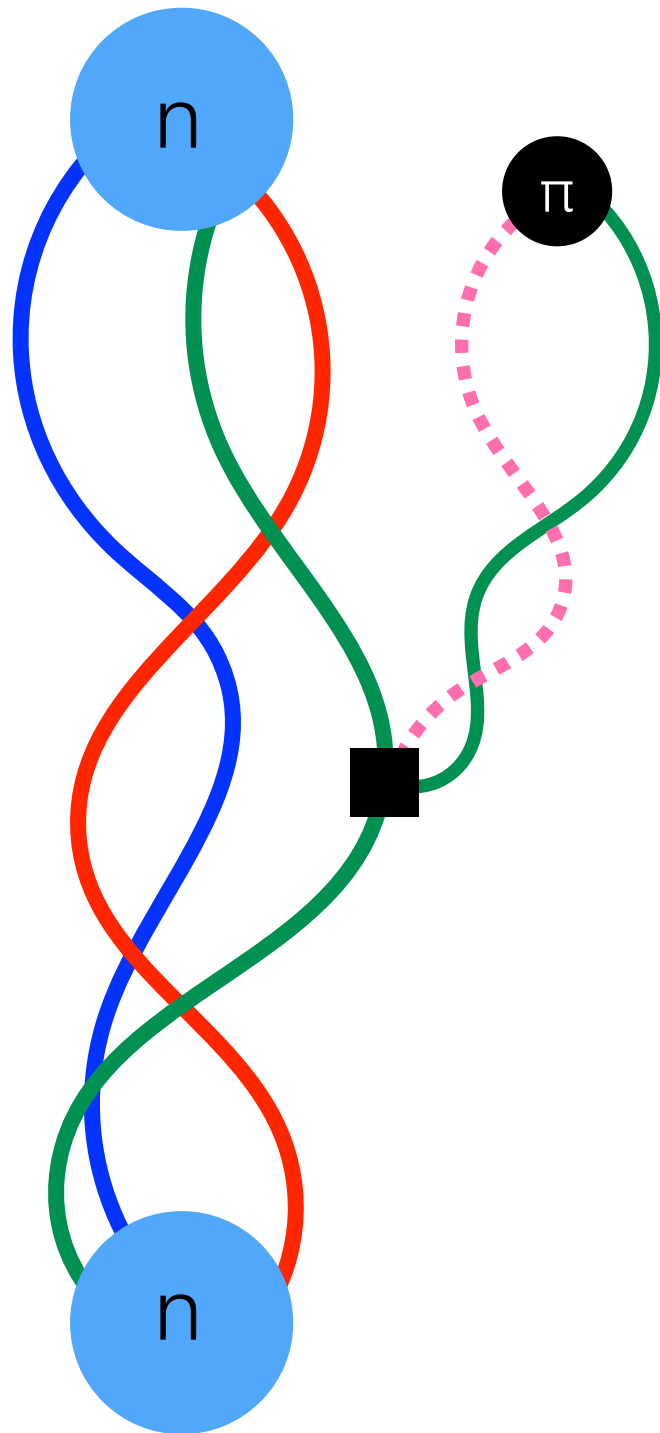
Operator	$\overline{\text{MS}}(2 \text{ GeV}),$ 10^{-5} GeV^6	$\frac{\overline{\text{MS}}(2 \text{ GeV})}{\text{MIT bag B}}$	Bare, 10^{-5} l.u.	χ^2/dof
Q_1	-44(19)	5.0	-3.7(1.6)	0.75
Q_2	140(40)	12.8	11.8(3.2)	0.69
Q_3	-79(23)	9.7	-6.6(1.9)	0.72
Q_5	-1.43(64)	2.1	-0.096(42)	0.73

enhancement
means experiments
have greater reach

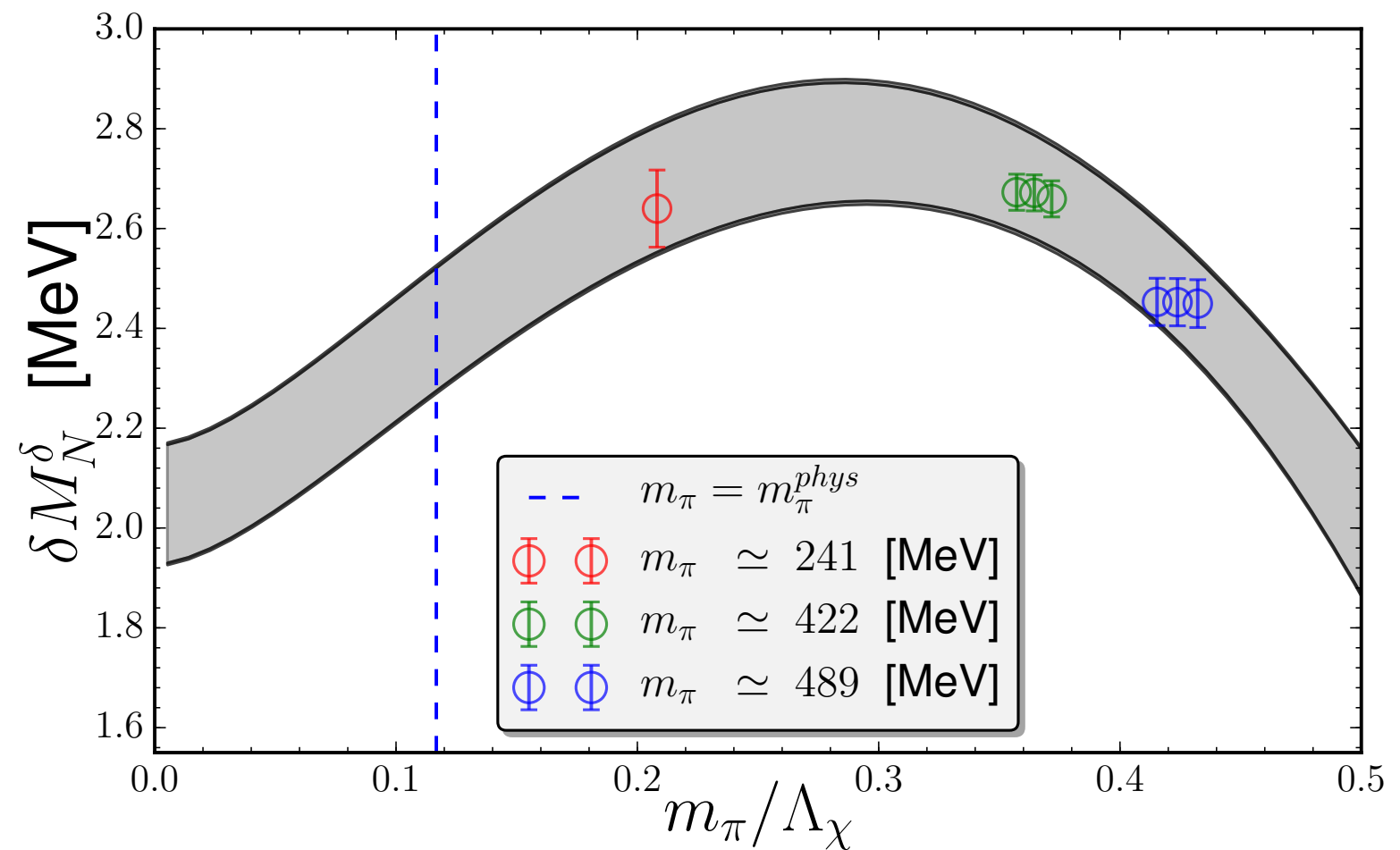


CP Violating πN Coupling

Brantley, Joo, Mastropas, Mereghetti, Monge-Camacho, Tiburzi, and Walker-Loud arXiv:1612.07733 [See also PLB 766 (2017) 254-262]



$$\bar{g}_0 = 14.7(1.8)^{\text{stat}}(1.4)^{\text{sys}} 10^{-3} \bar{\Theta} f_\pi \sqrt{2}$$



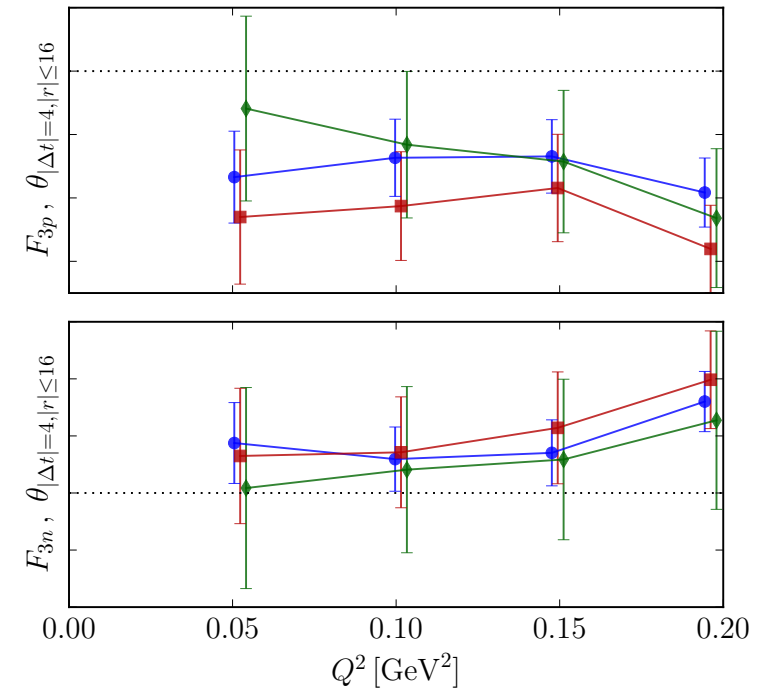
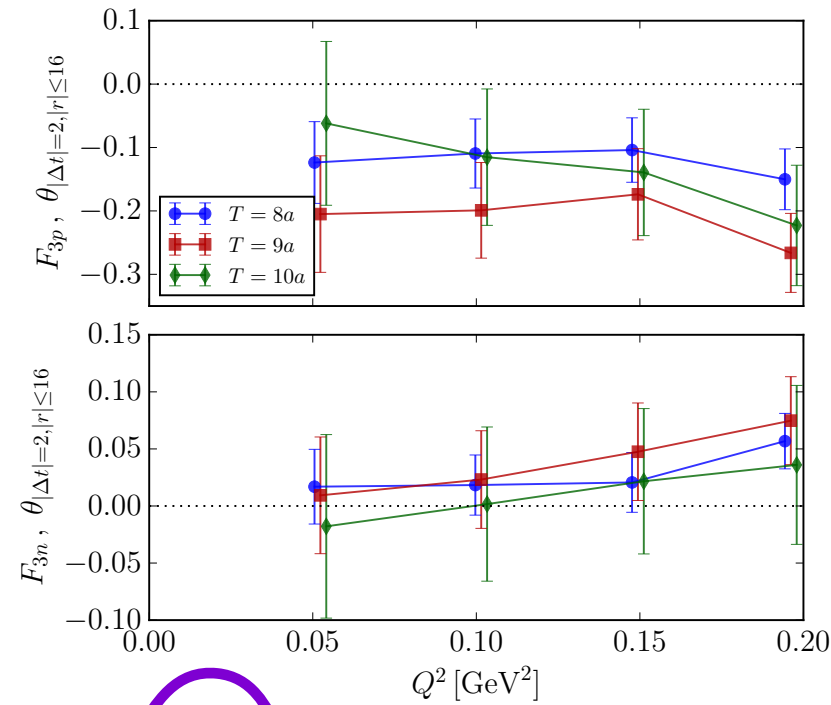
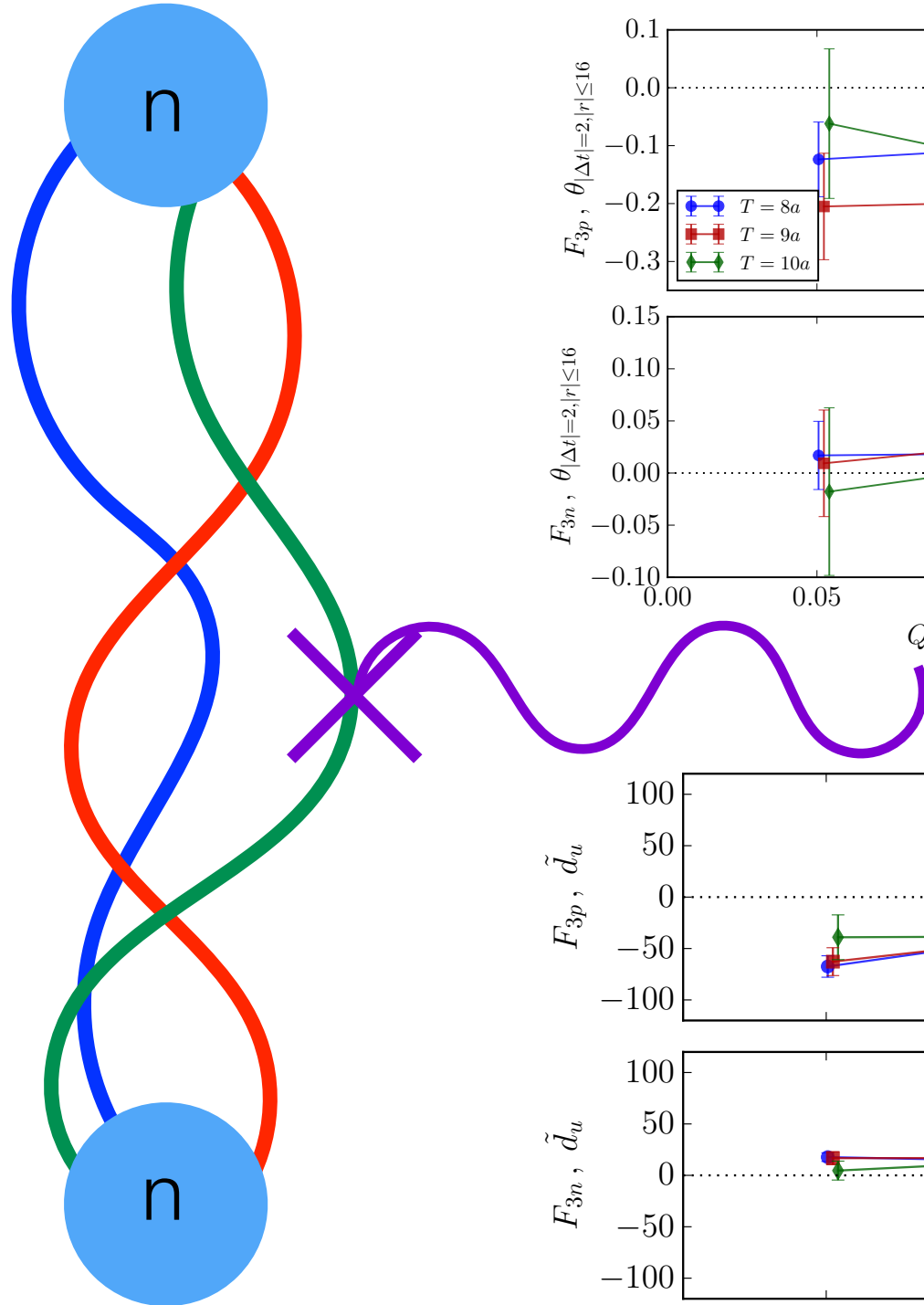
n EDM: Quark Bilinears

Syritsyn, Ohki, Izubuchi CIPANP 2018 1810.03721

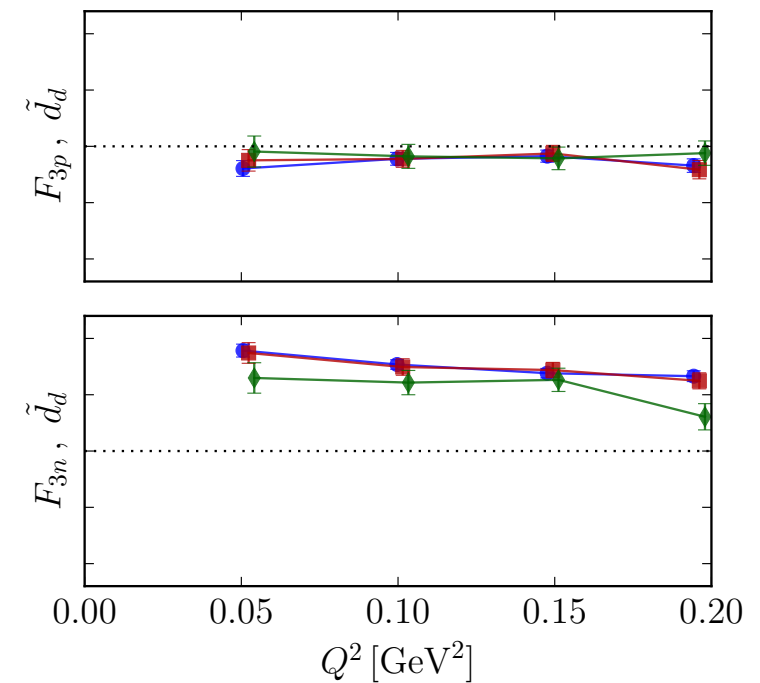
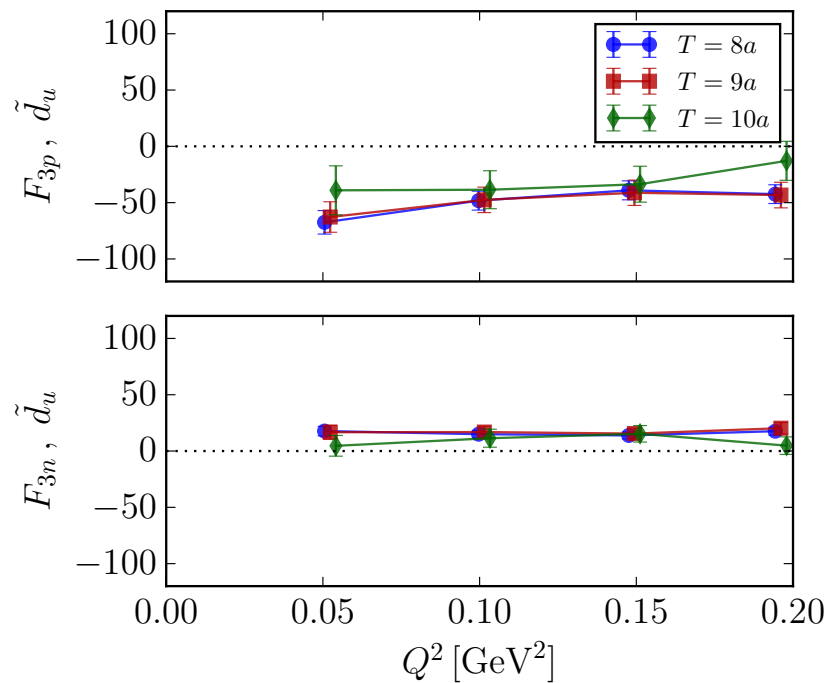
$$V = 48^3 \times 96 (\times 24 \text{ DWF})$$

$$a = 0.114 \text{ fm}$$

$$m_\pi = 139.2 \text{ MeV}$$



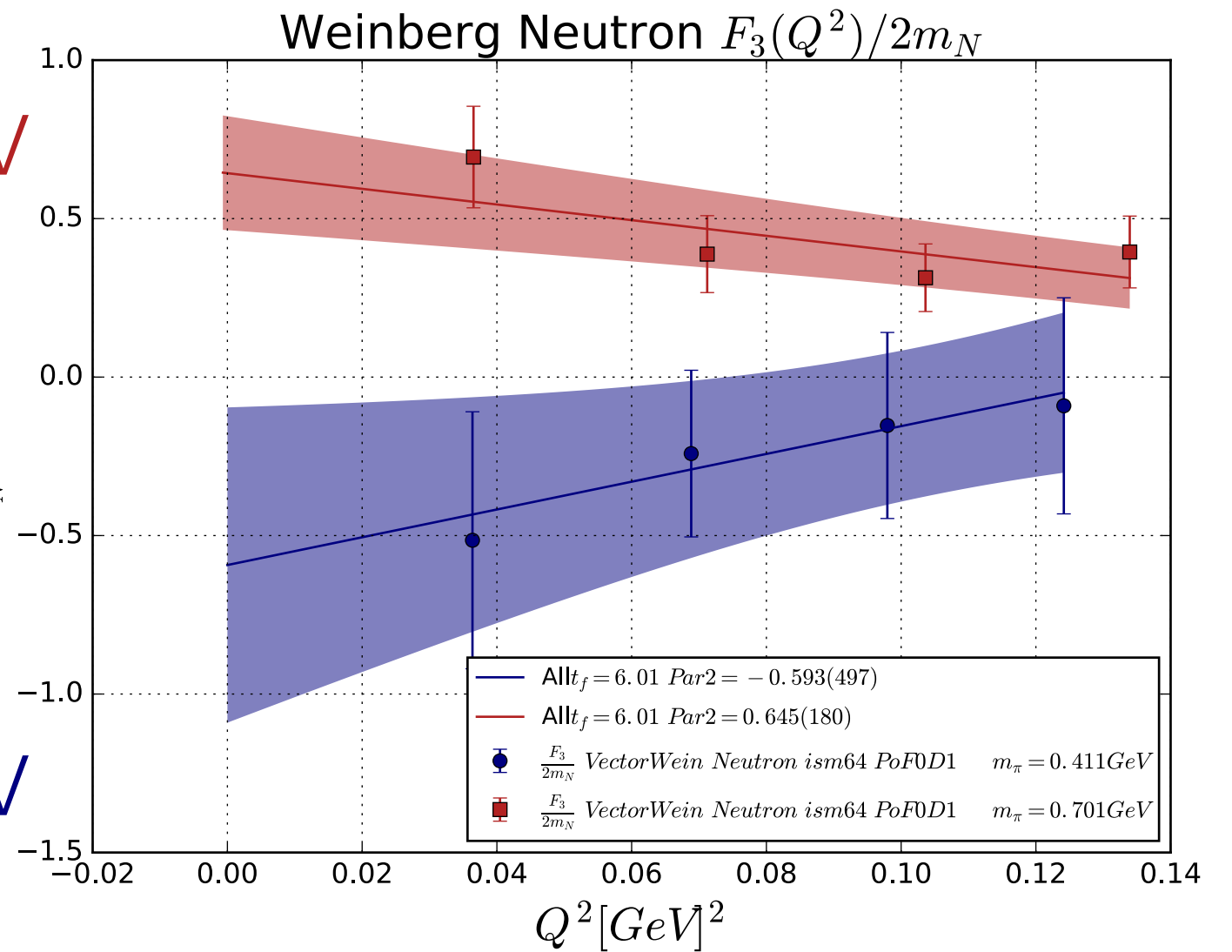
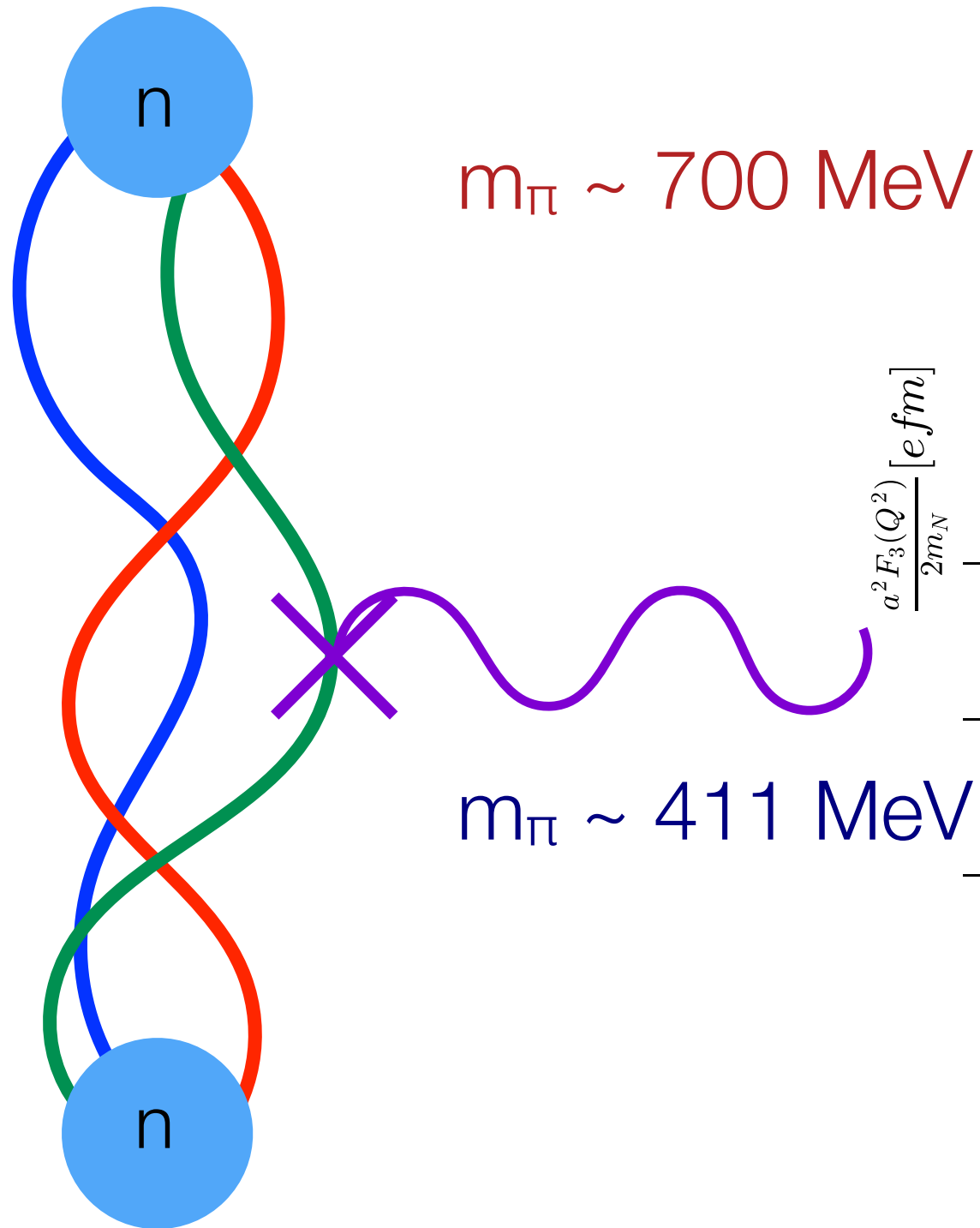
θ_{QCD}



cEDM

n EDM: Weinberg Operator

Dragos, Luu, Shindler, de Vries LATTICE 2017 EPJ Web Conf 175 (2018) 06018 arXiv:1711.04730

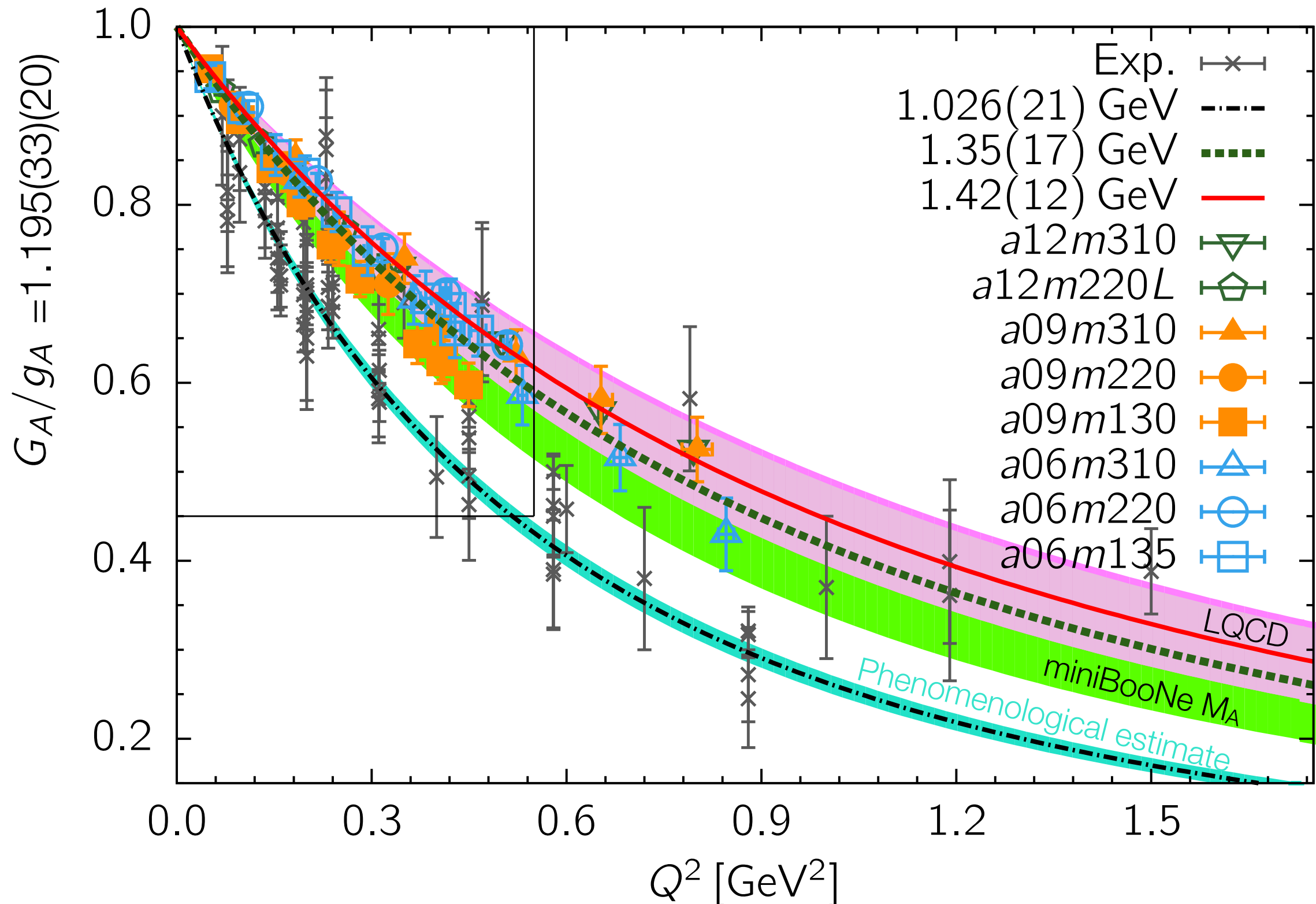


$$-i \frac{\alpha_{\tilde{G}}}{\Lambda^2} \frac{1}{3} f^{ABC} \tilde{G}_{\mu\nu}^A G_{\mu\rho}^B G_{\nu\rho}^C$$

see also community whitepaper
1711.07916 (PDFs, charges...)

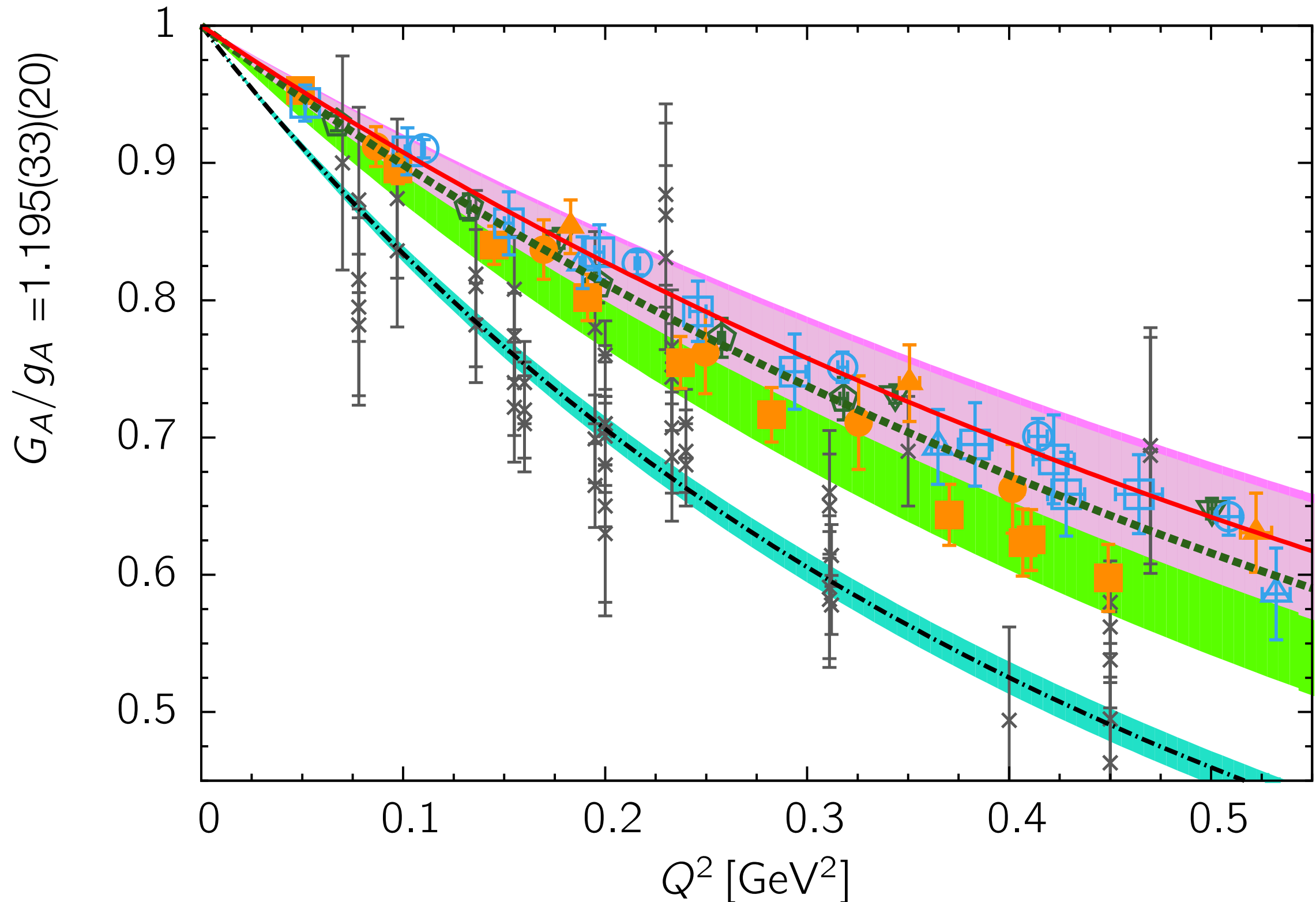
Charges + Form Factors

Gupta, Jang, Lin, Yoon, and Bhattacharya (PNDME) $q=0$: 1506.06411, **1606.07049** $q \neq 0$: **1705.06834**, 1801.01635 1901.00060



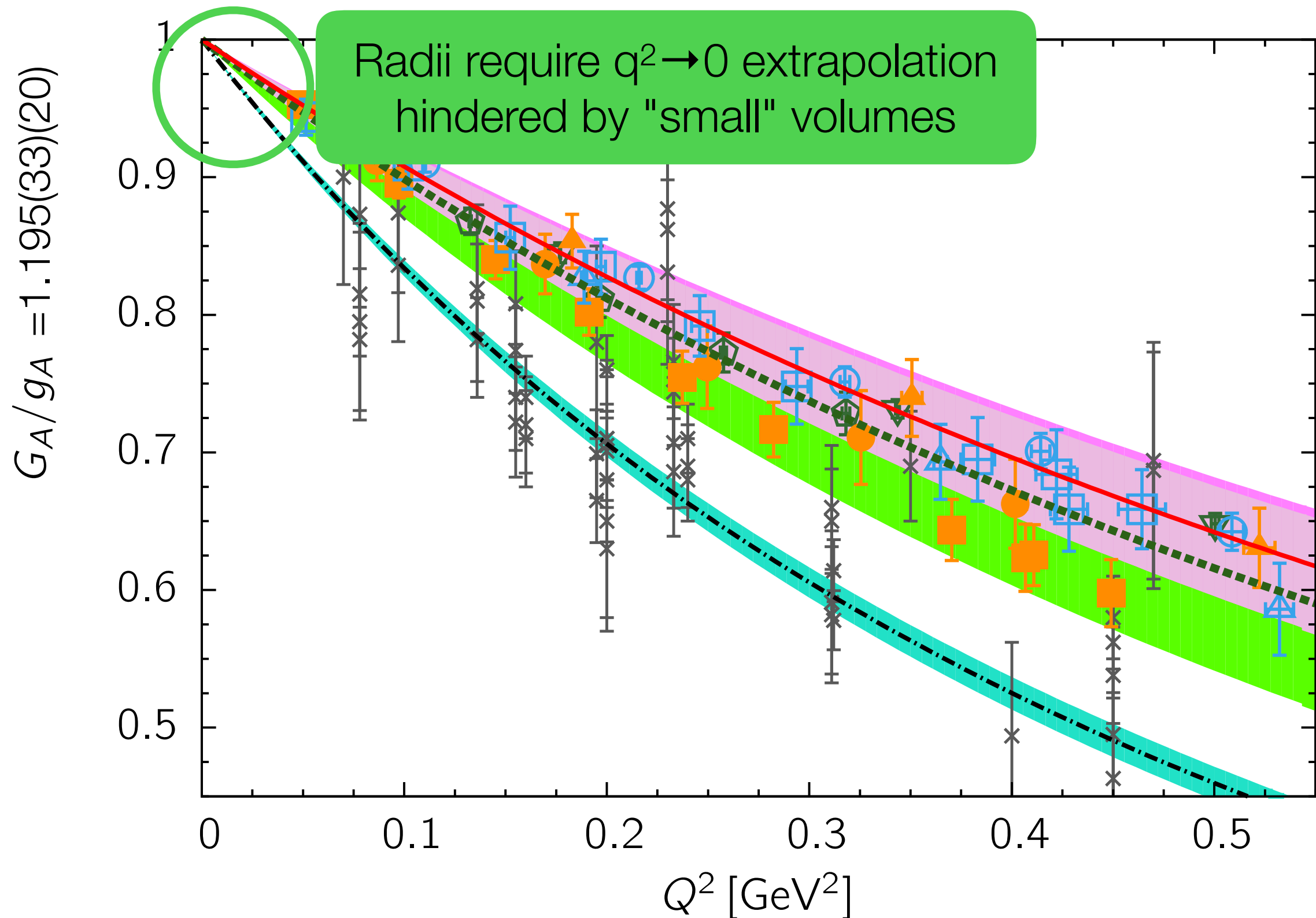
Charges + Form Factors

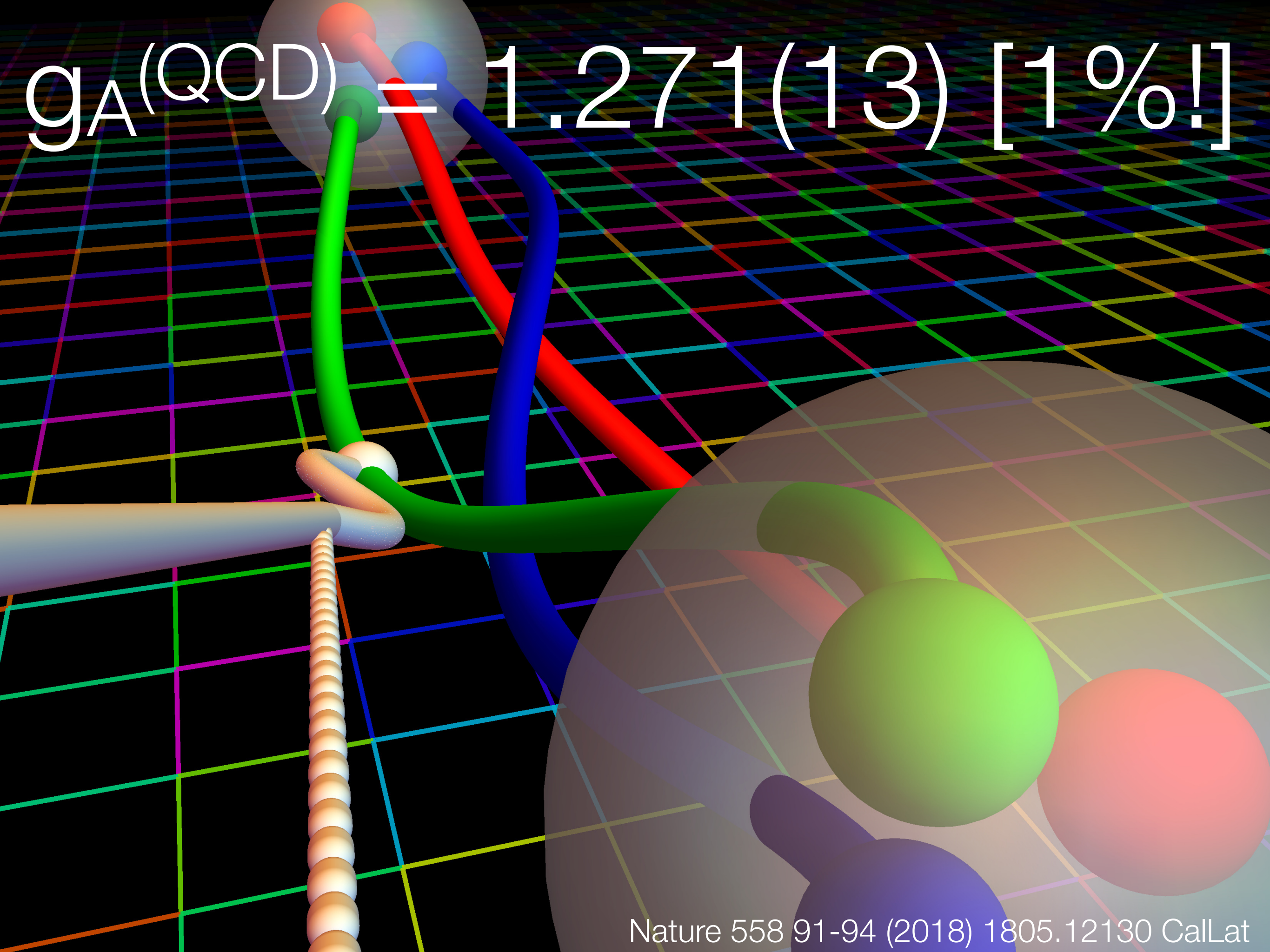
Gupta, Jang, Lin, Yoon, and Bhattacharya (PNDME) $q^2=0$: 1506.06411, **1606.07049** $q^2 \neq 0$: **1705.06834**, 1801.01635 1901.00060



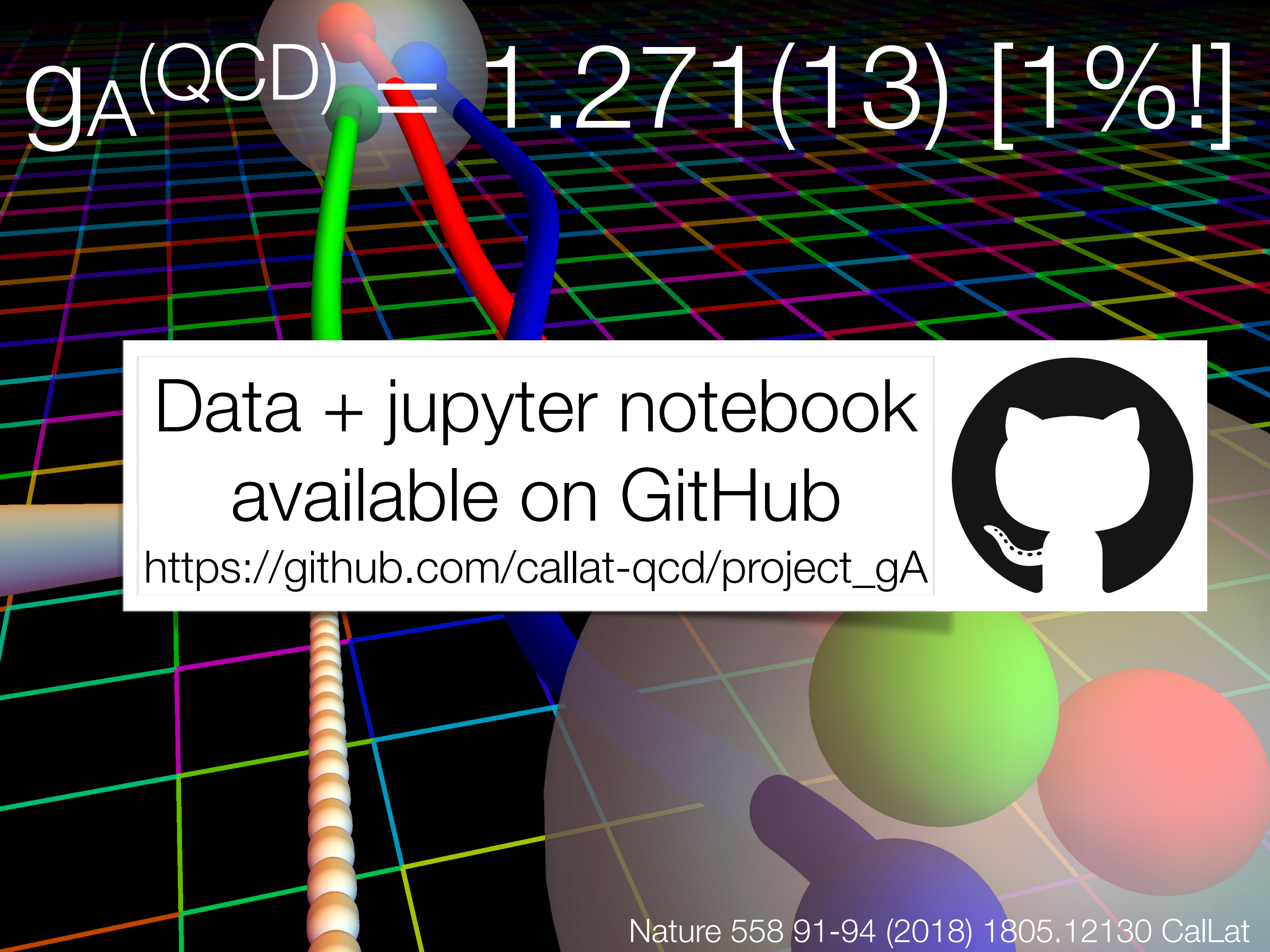
Charges + Form Factors

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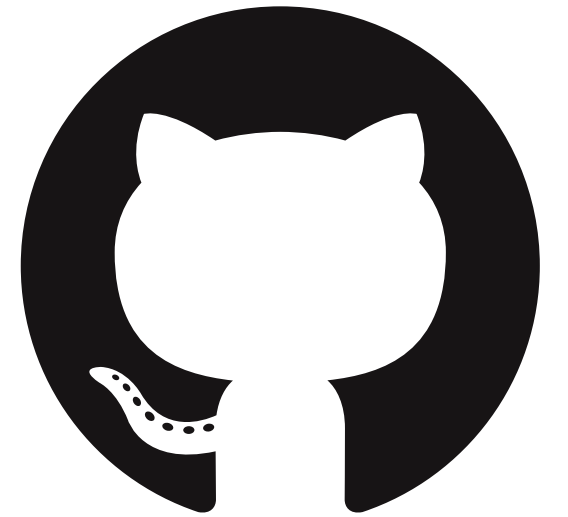
$$g_A(\text{QCD}) = 1.271(13) [1\%!]$$



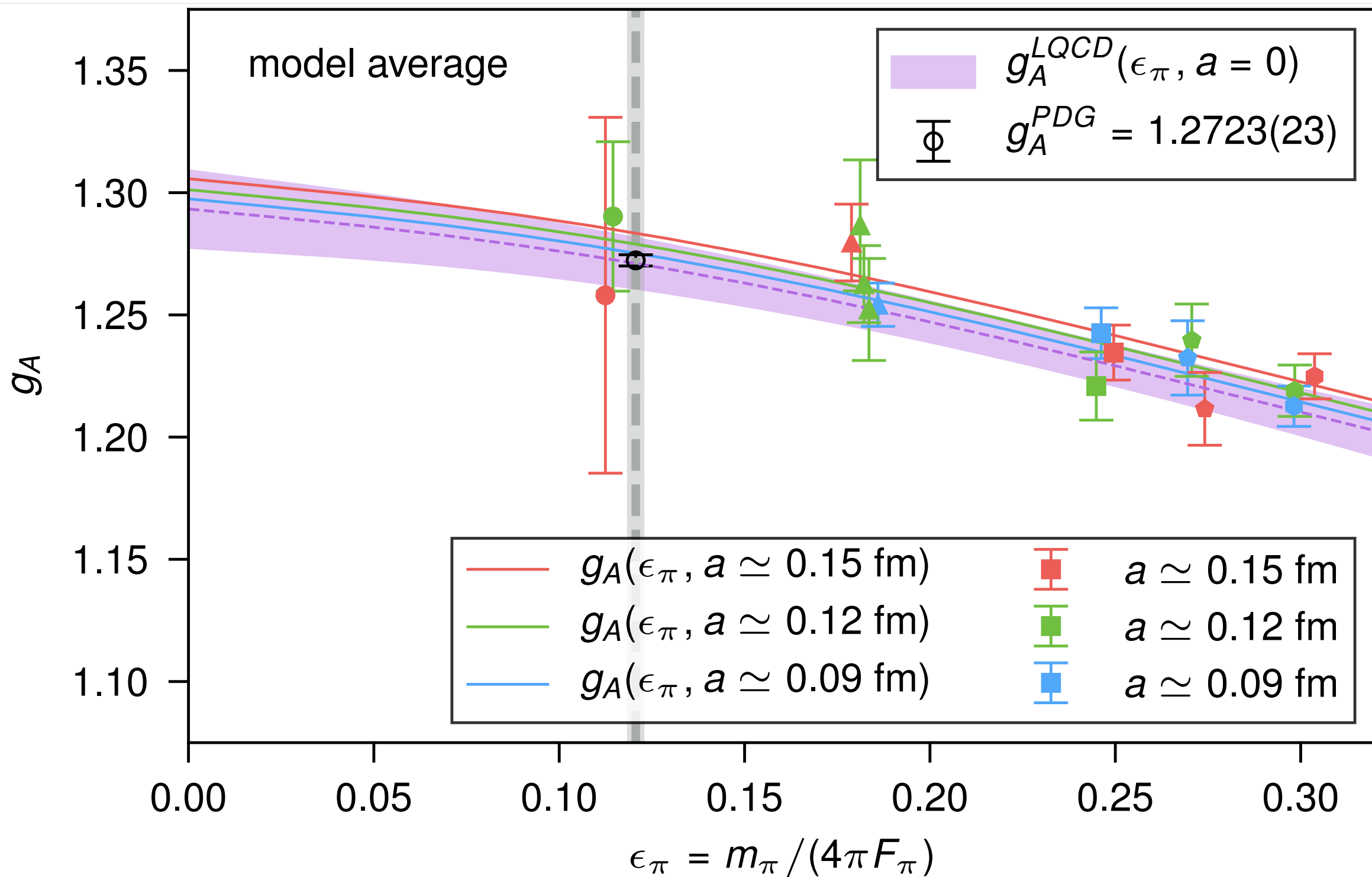
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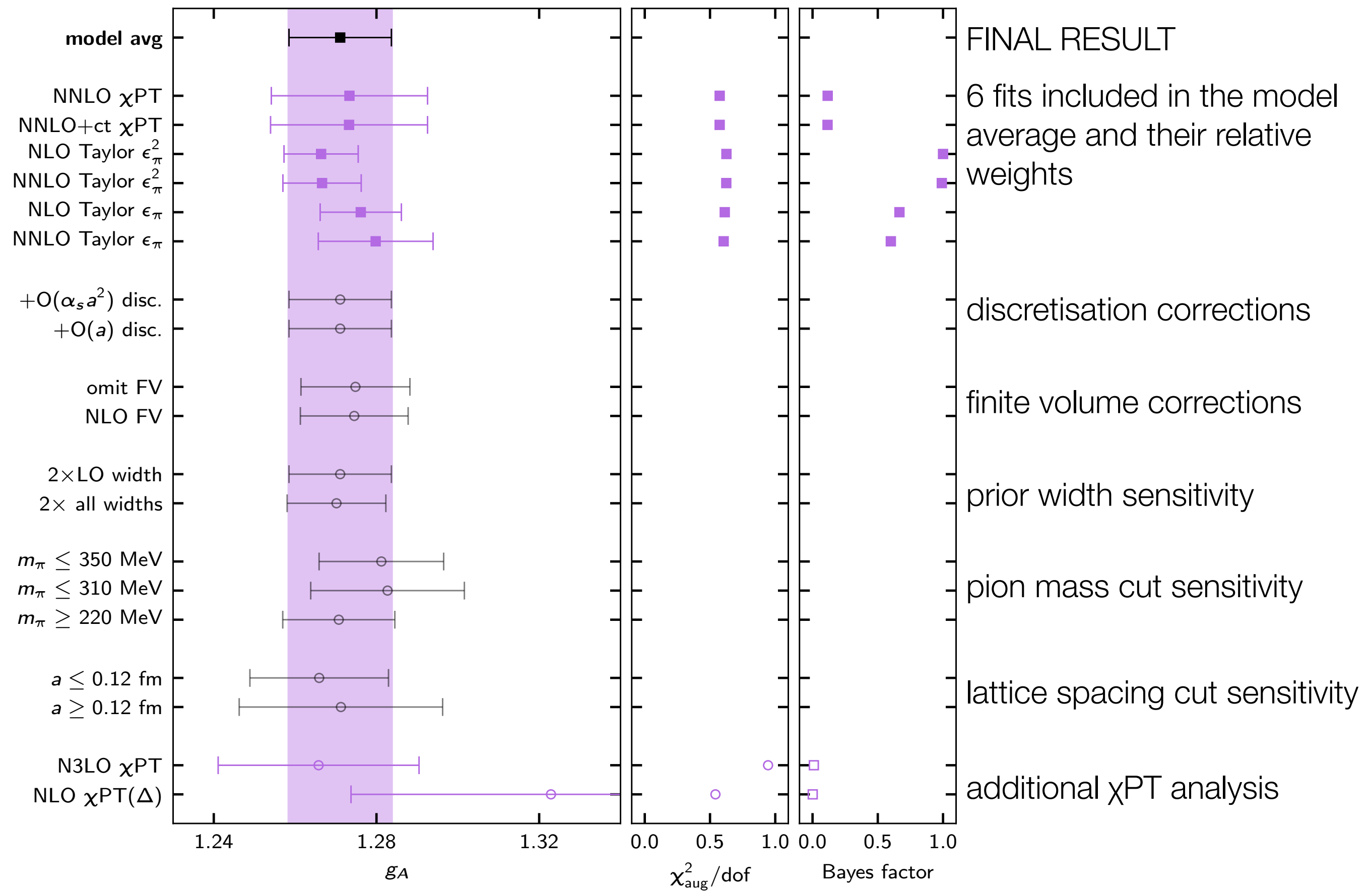
Data + jupyter notebook
available on GitHub

https://github.com/callat-qcd/project_gA



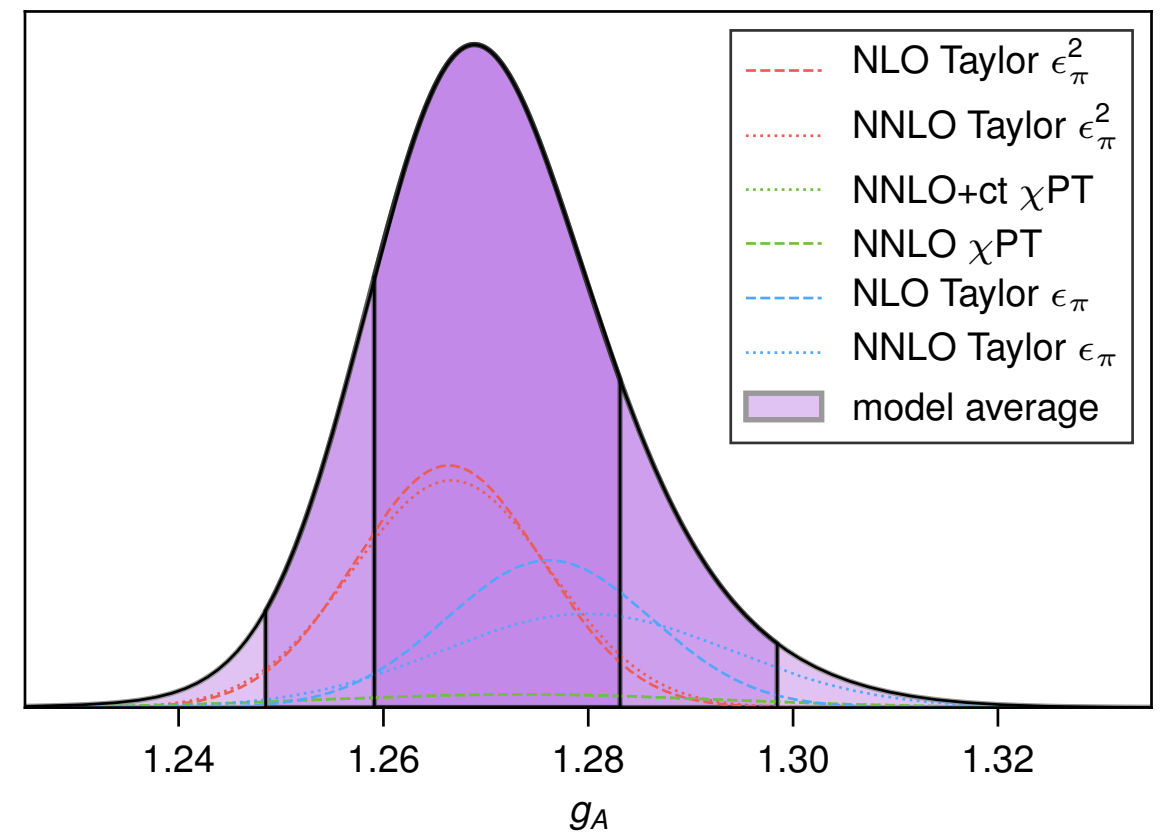
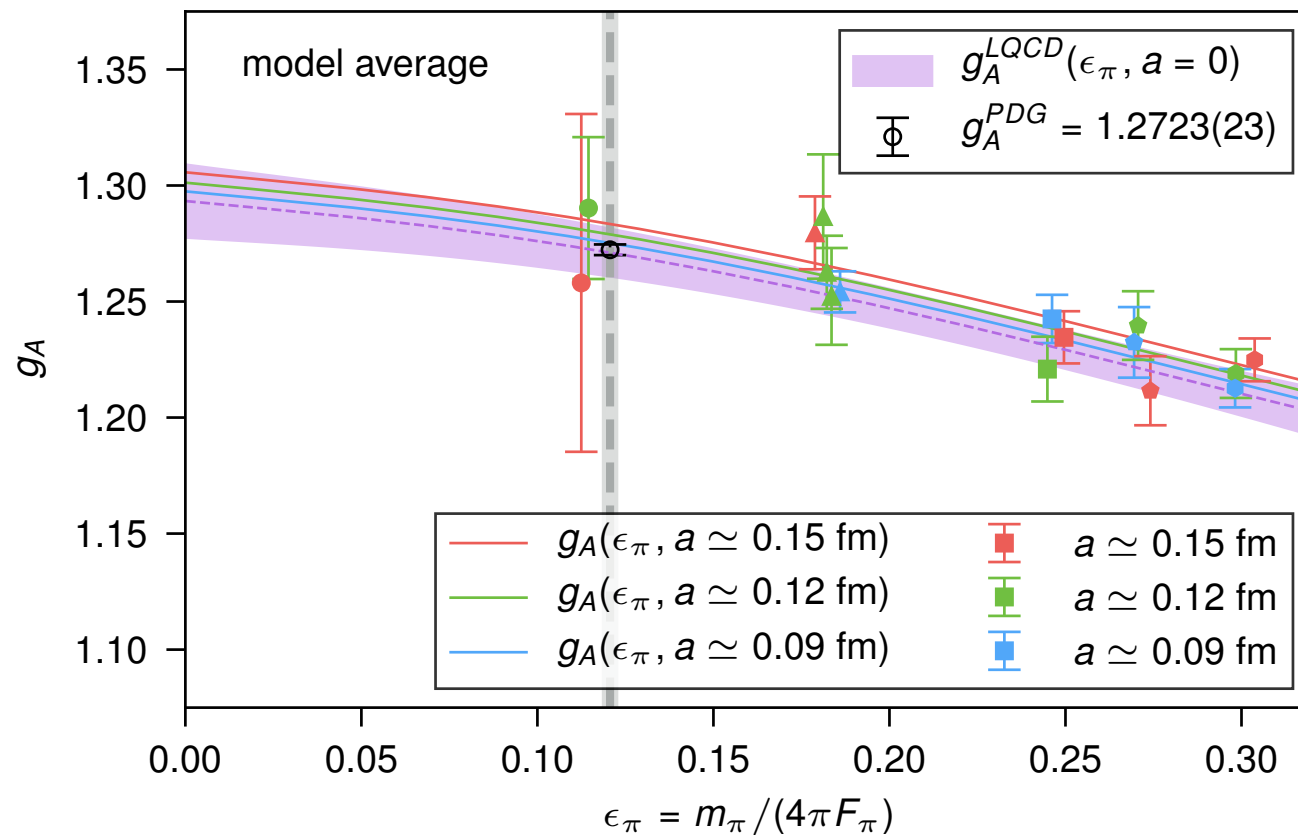
$g_A(\text{QCD}) = 1.271(13) [1\%!]$





Model Average $g_A = 1.2711(103)^s(39)^x(15)^a(19)^v(04)^I(55)^M$

Nature 558 (2018) 91



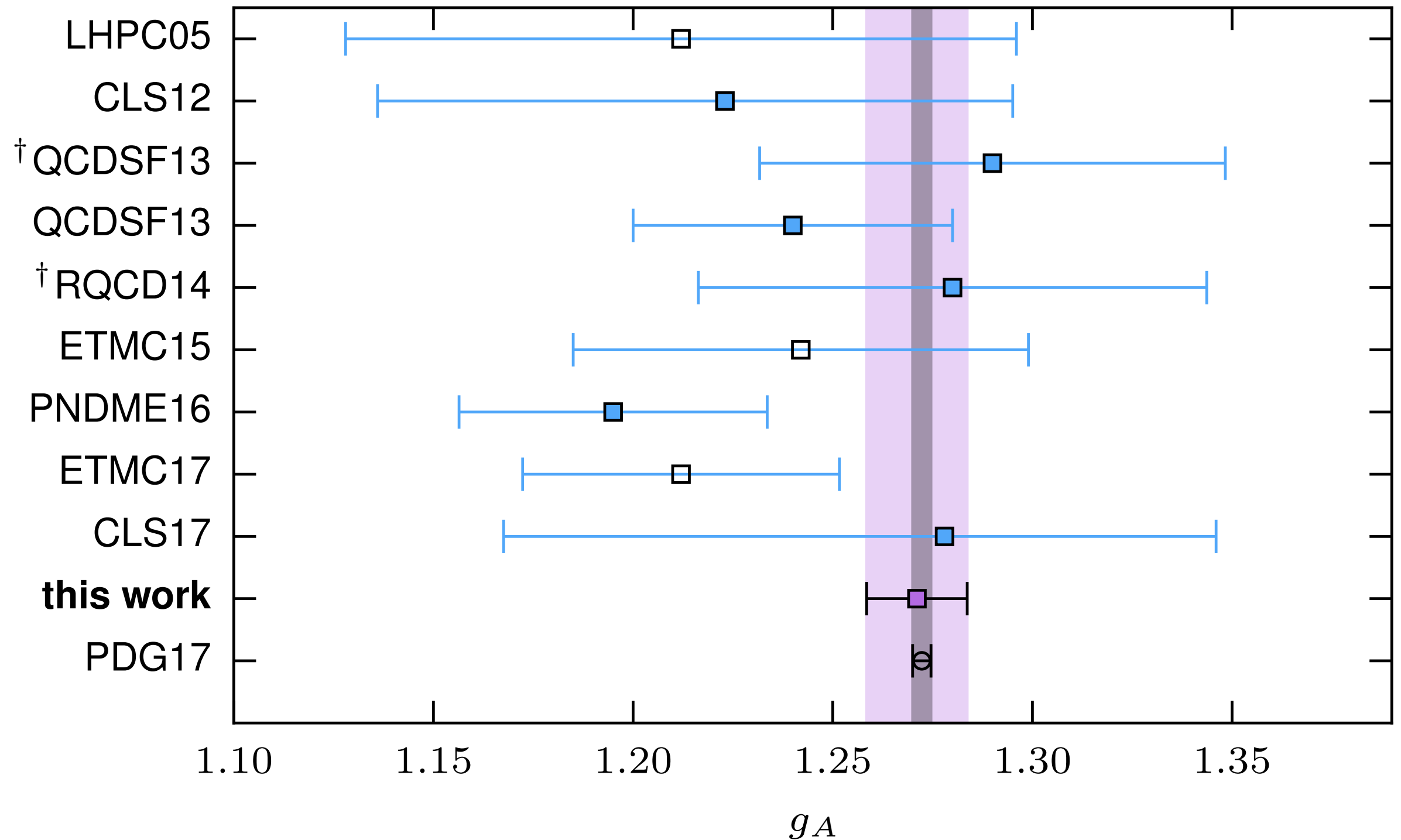
statistical	0.81%
chiral	0.30%
continuum	0.12%
infinite volume	0.15%
isospin breaking	0.03%
model selection	0.43%
total (added in quadrature)	0.98%

Final uncertainty is dominated by statistics, model selection, and chiral extrapolation.

Better control over the $m_\pi \sim 130$ MeV points will improve all three.

Comparison with previous LQCD results

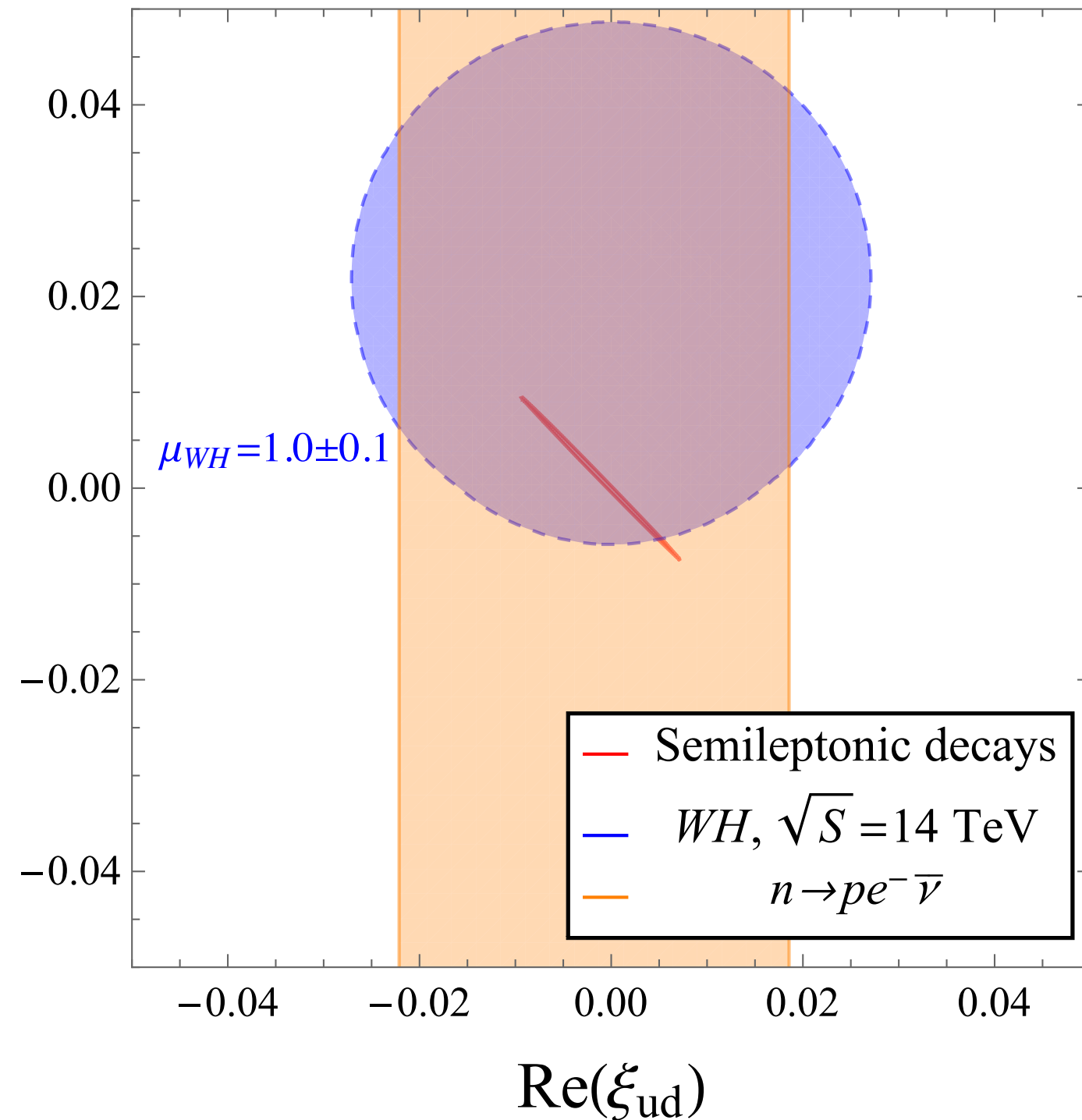
Nature 558 (2018) 91



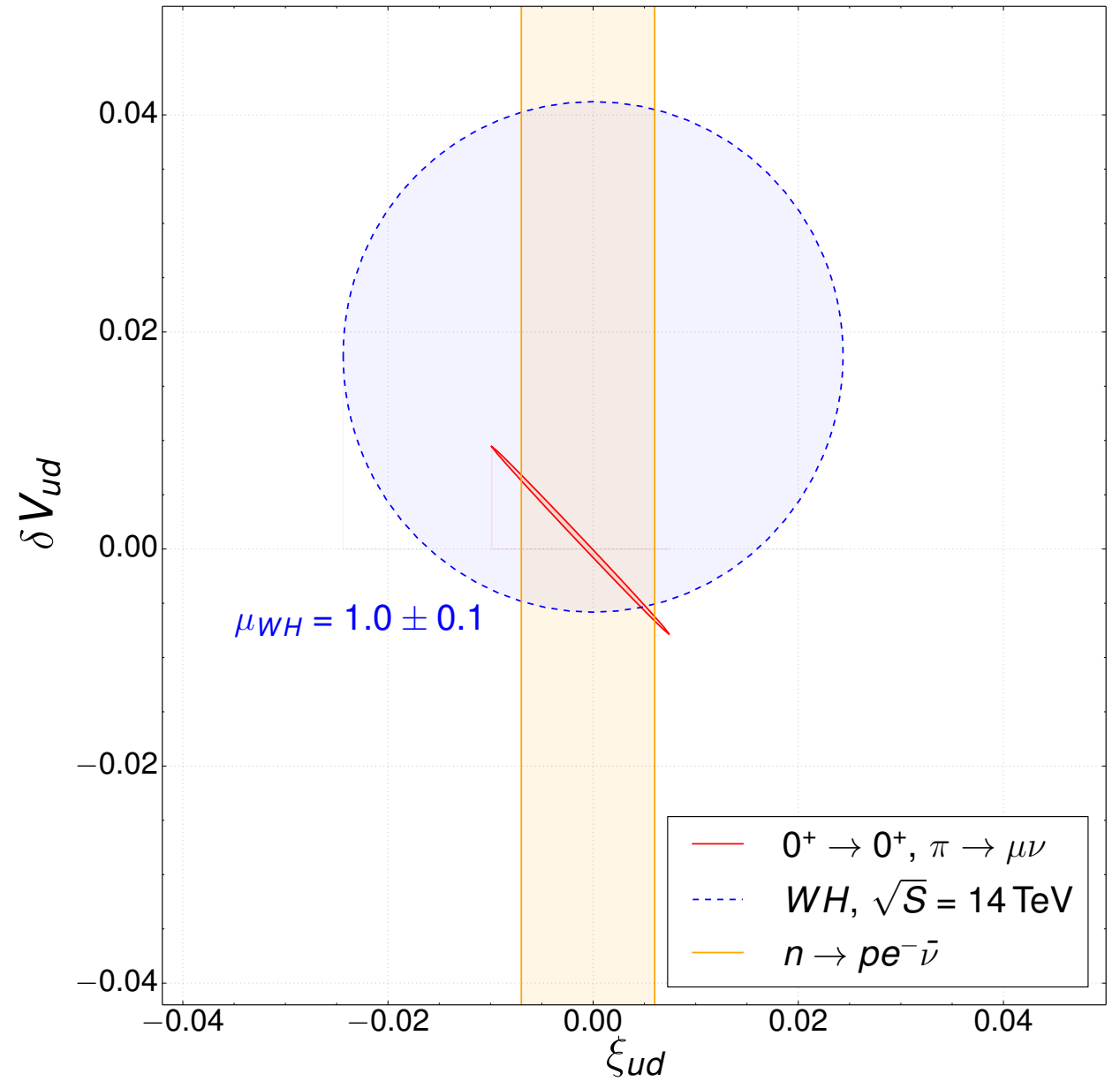
Searches for violations of V–A

Alioli, Cirigliano, Dekens, de Vries, Mereghetti JHEP 1705 (2017) 086 arXiv:1703.04751

$$g_A^{\text{QCD}} = 1.27 \pm 0.05$$



$$g_A^{\text{QCD}} = 1.271(13)$$

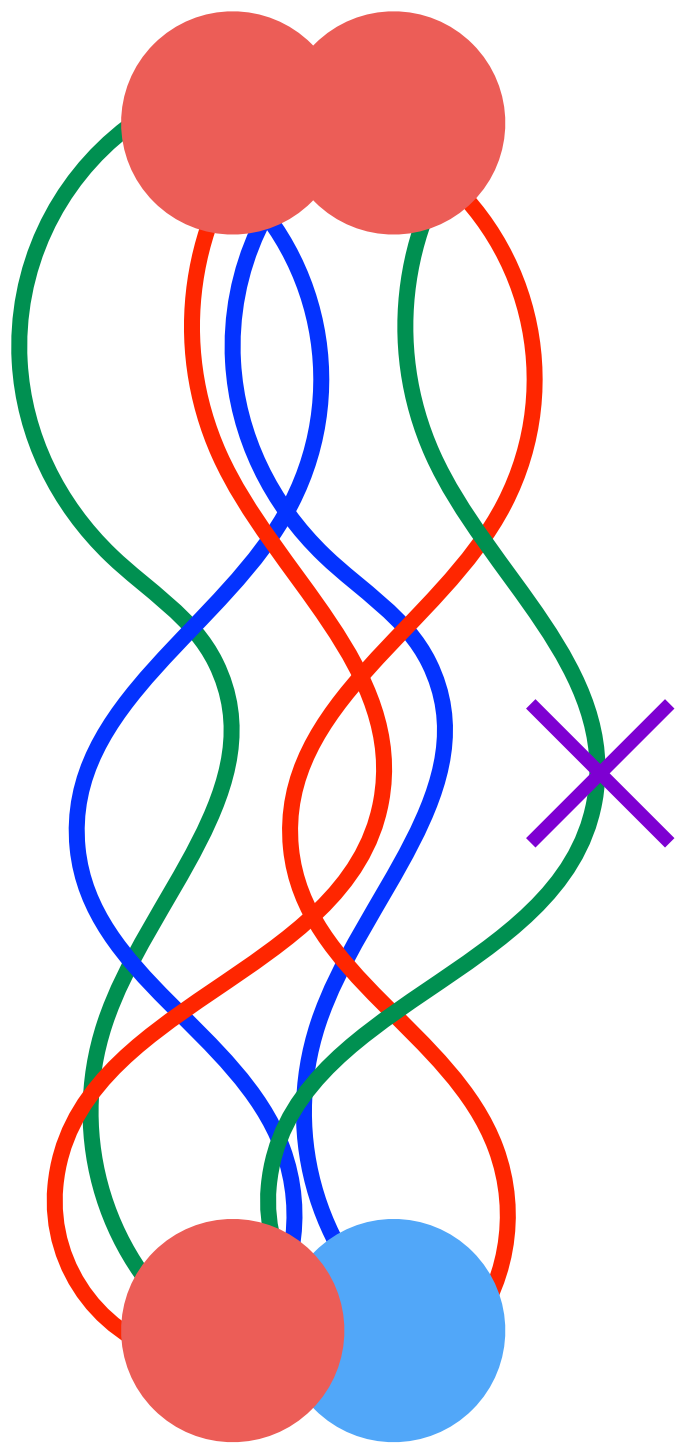


Two Nucleon Matrix Elements

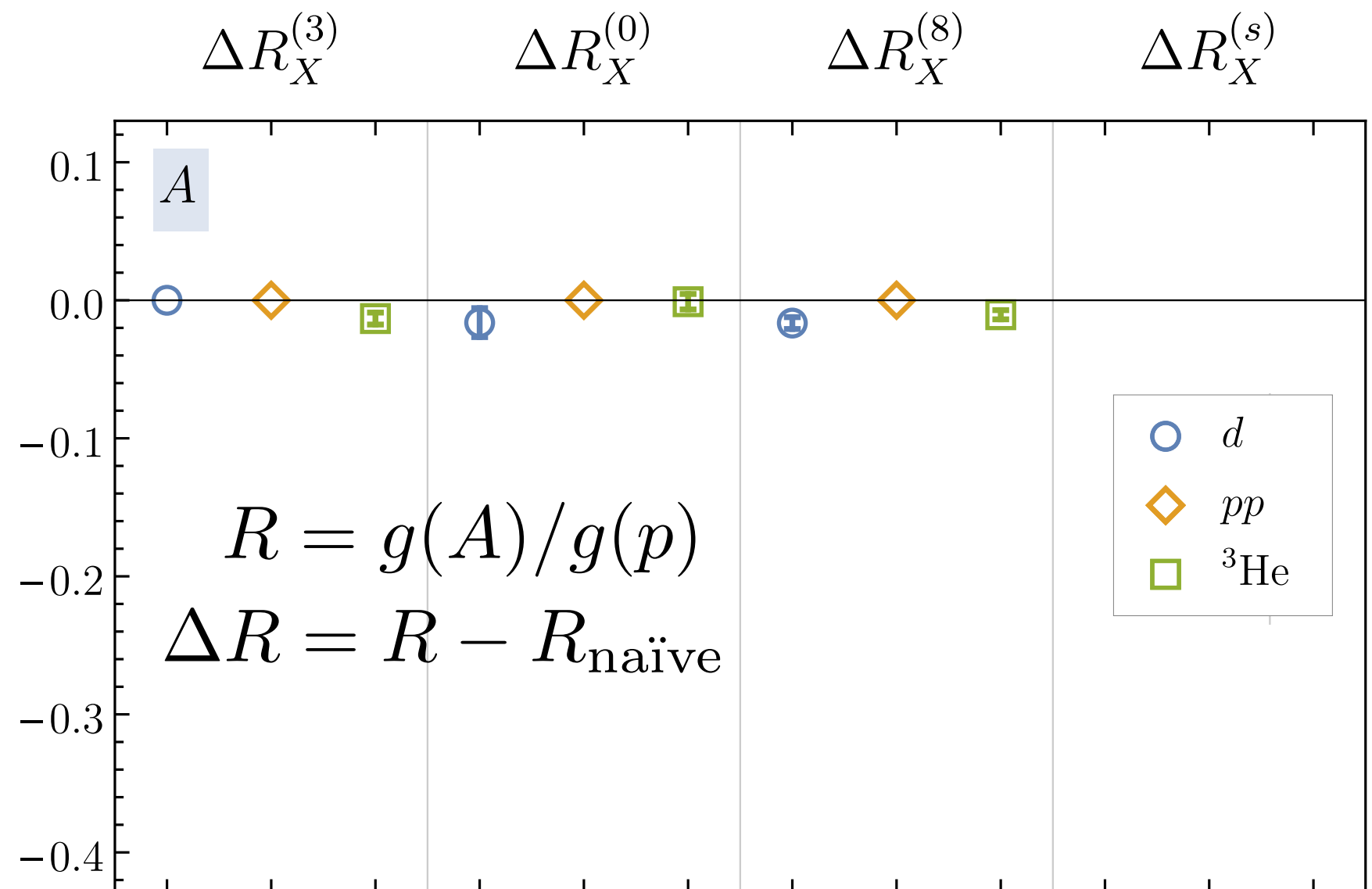
g_A Quenching?

NPLQCD PRL 120 (2018) 15 152002 arXiv:1712.03221

$m_\pi \sim 800$ MeV; $a \sim 0.145$ fm



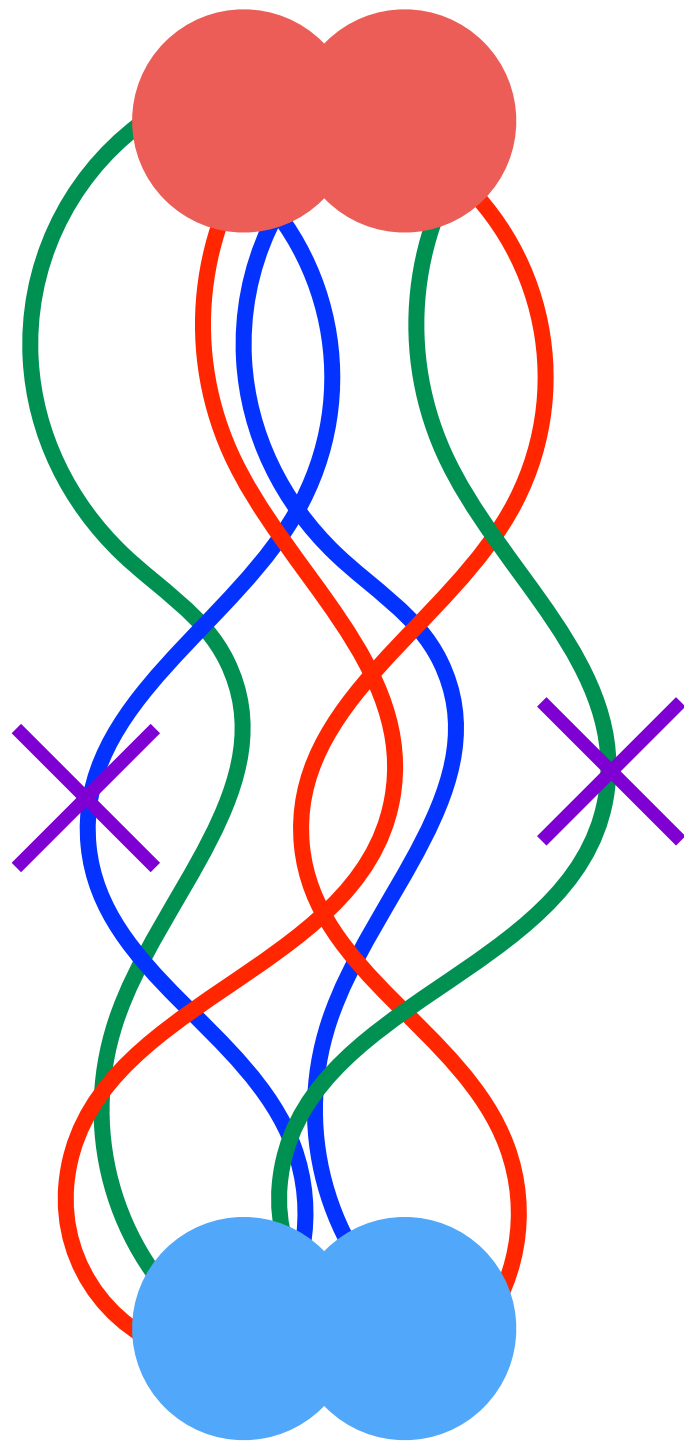
different flavor structures



Isotensor Polarizability ($2\beta_{vv}$)

NPLQCD PRL 119 (2017) 06 062003 arXiv:1701.03456

$m_\pi \sim 800 \text{ MeV}; a \sim 0.145 \text{ fm}$



Effect can be described by 'quenching' g_A

$$i\mathcal{C}_{nn \rightarrow pp} = \text{diagram with } g_A + \text{diagram with } L_{1,A} + \text{diagram with } H_{2,s} + \mathcal{O}(\lambda^4)$$

Figure from P. Shanahan 6 July 2017 INT-17-67W

previously-neglected short-range contribution

$$H_{2,s} = 4.7(1.3)^{\text{stat}}(1.8)^{\text{sys}}$$

- Pionless EFT
- Dineutron bound at 800 MeV
- Binding energies, matrix element from the lattice

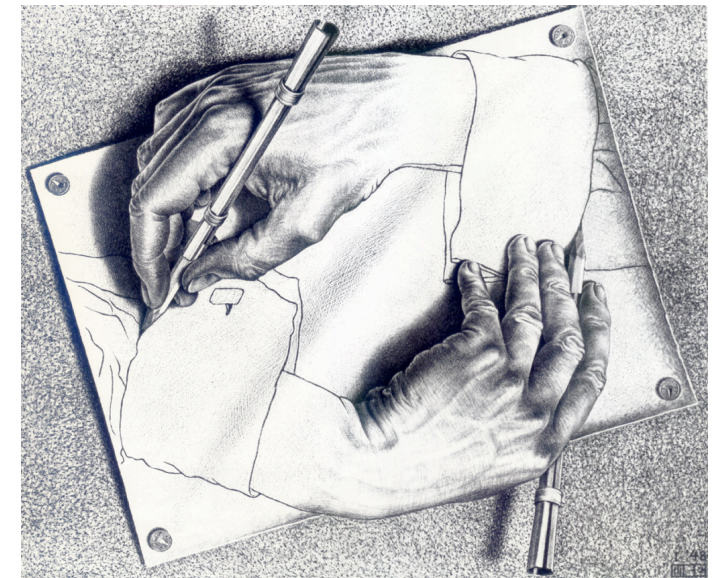
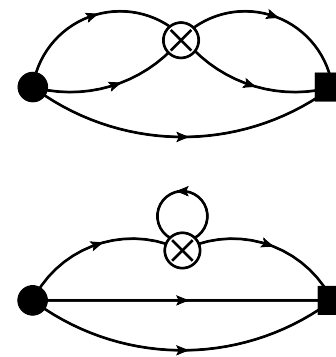
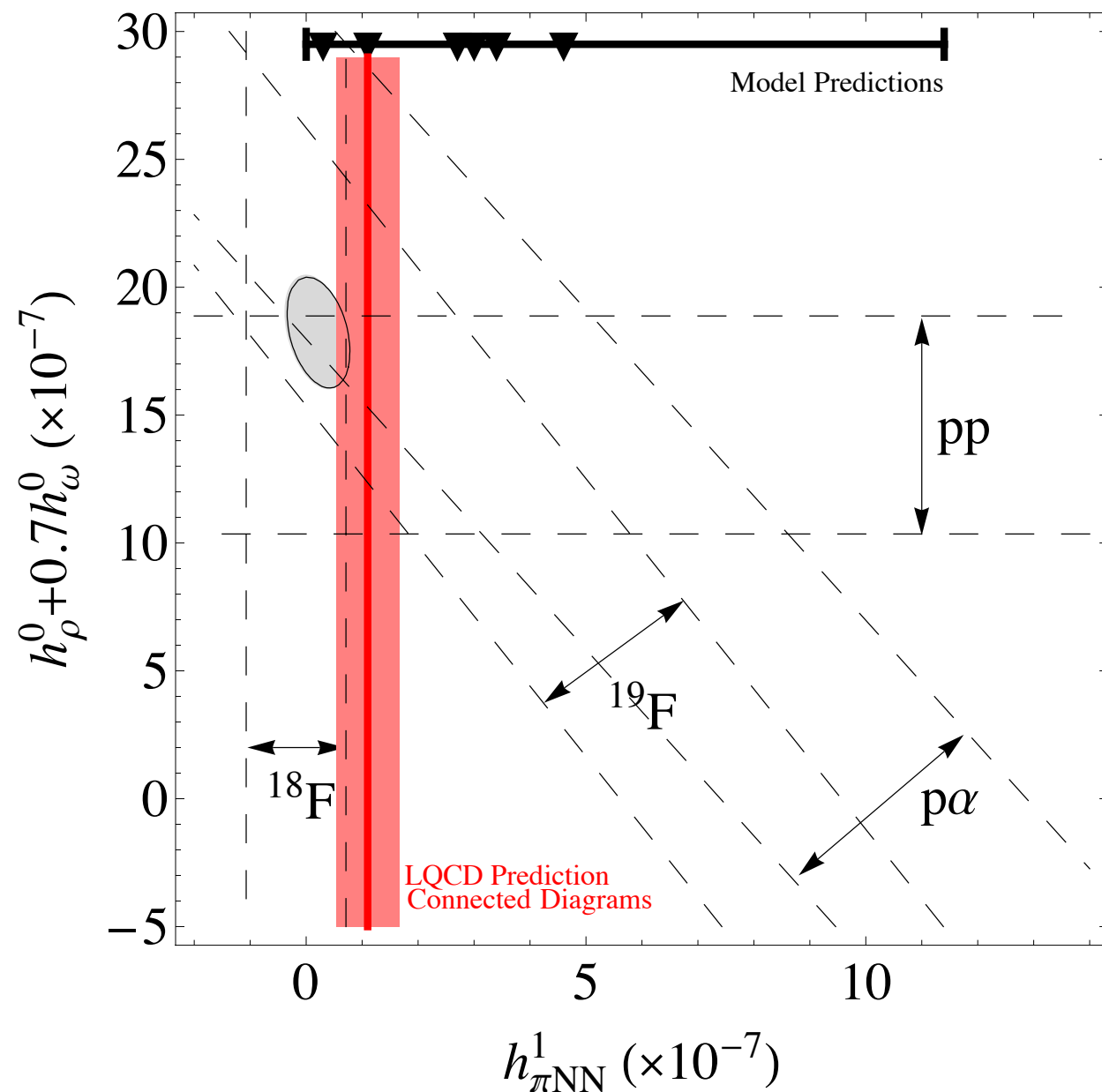
Hadronic Parity Violation

Wasem PRC85 (2012) 022501 arXiv:1108.1151

$m_\pi \sim 389 \text{ MeV}; a \sim 0.123 \text{ fm}$

$$\mathcal{L}_{PV}^{\pi NN} = h_{\pi NN}^1 (\bar{p}\pi^+n - \bar{n}\pi^-p)$$

$$h_{\pi NN}^{1,\text{con}} = (1.099 \pm 0.505^{+0.058}_{-0.064}) 10^{-7}$$



$PV \neq VP$



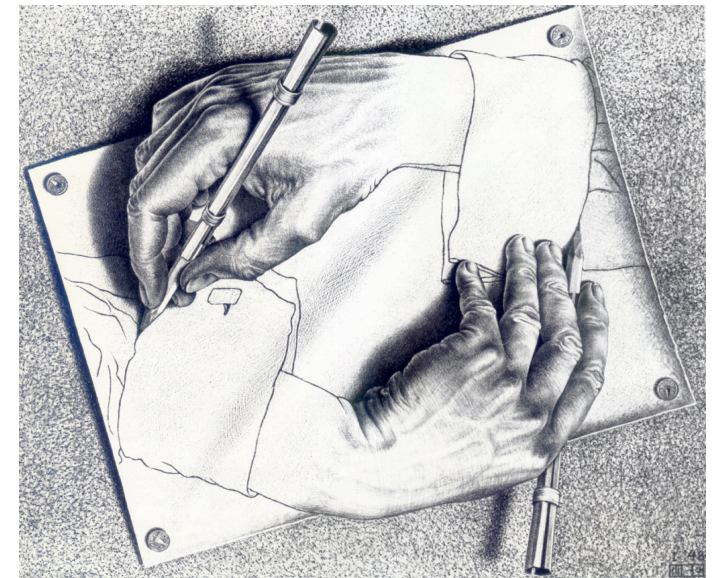
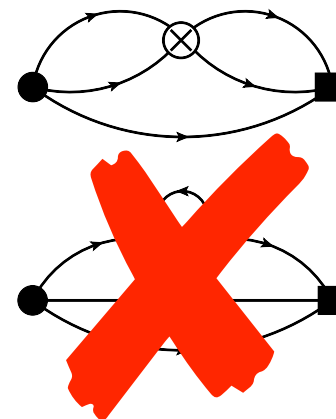
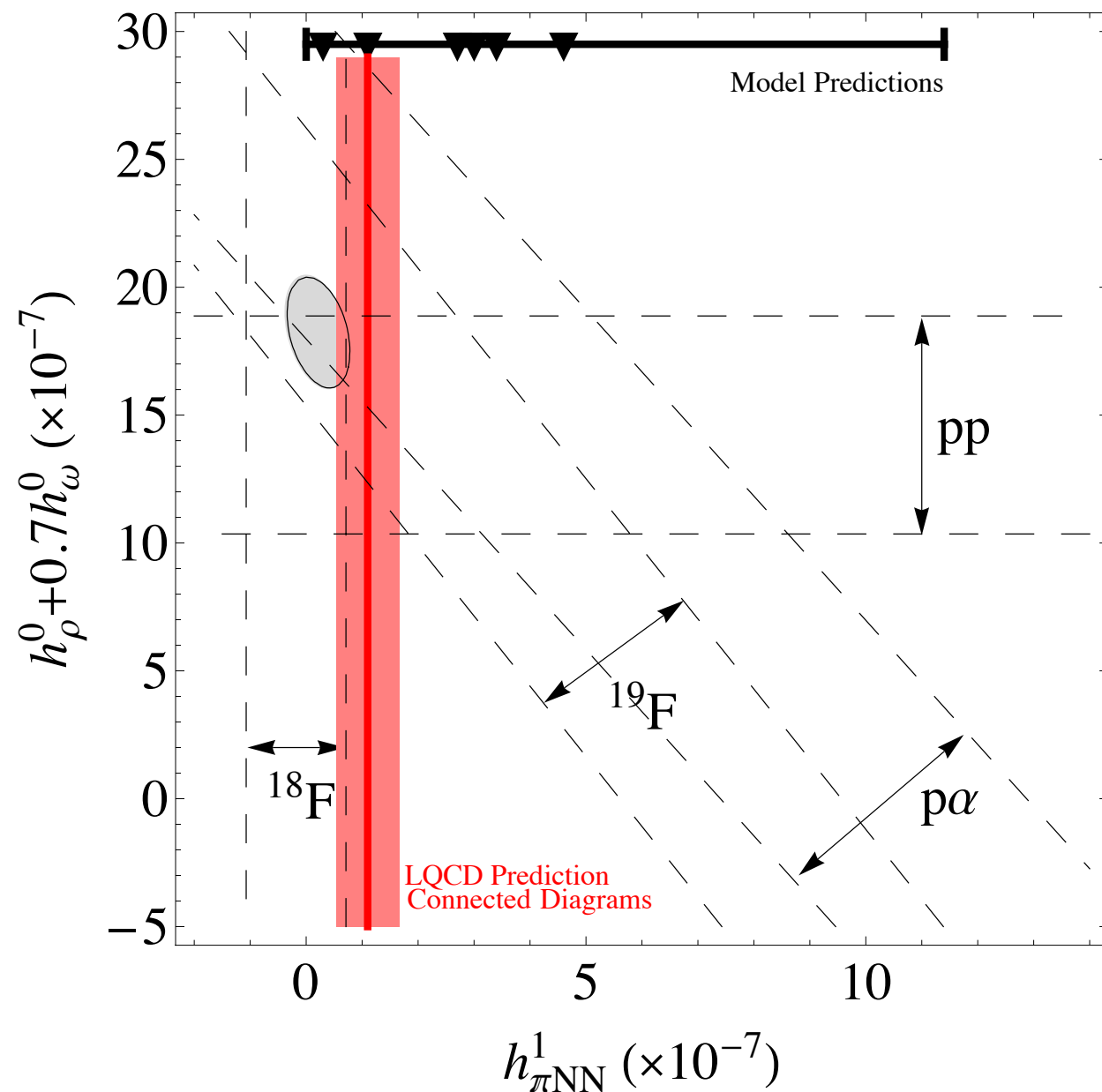
Hadronic Parity Violation

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$$h_{\pi NN}^{1,\text{con}} = (1.099 \pm 0.505^{+0.058}_{-0.064}) 10^{-7}$$



$PV \neq VP$



Short-Range $0\nu\beta\beta$ Matrix Elements



Berkeley
LBL

David Brantley, Henry Monge Camacho, Chia
Cheng (Jason) Chang, Ken McElvain, André
Walker-Loud



RBRC

Enrico Rinaldi

FZJ

EB



JLab

Bálint Joó



Liverpool
Plymouth

Nicolas Garron



LLNL

Pavlos Vranas



NERSC

Thorsten Kurth



NVIDIA

UNC

Amy Nicholson

nVidia

Kate Clark



Glasgow

Chris Bouchard



Rutgers

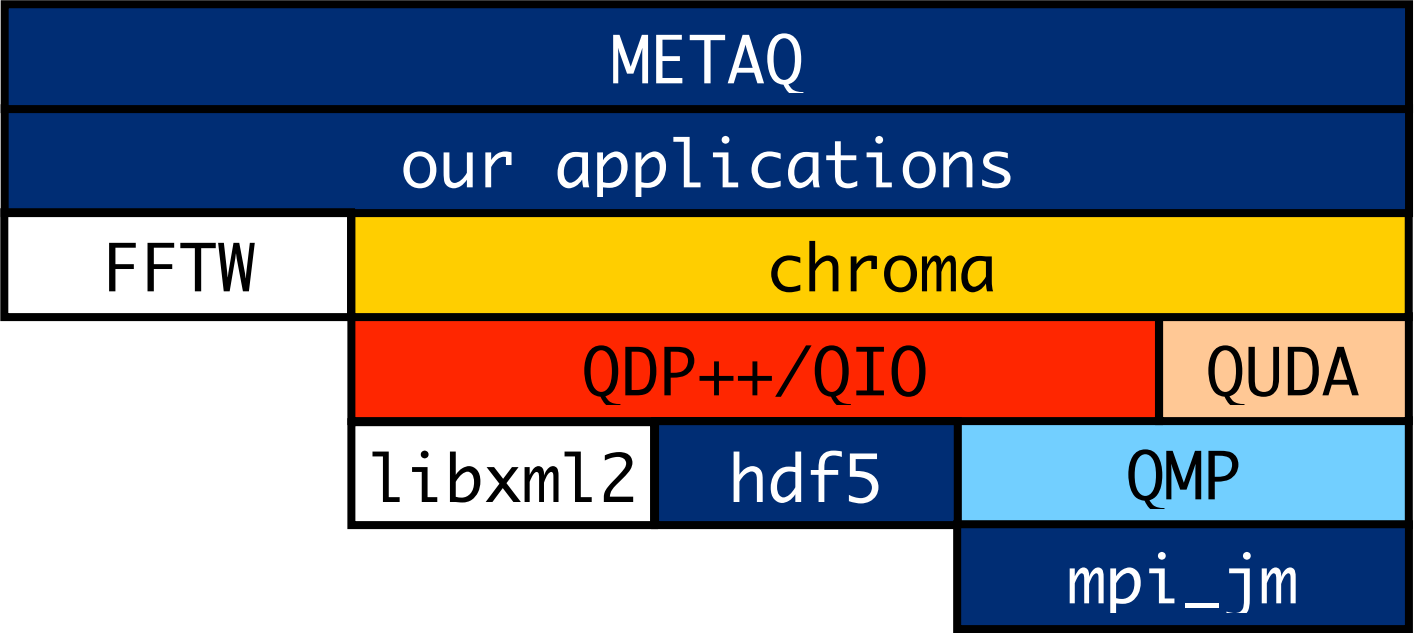
Chris Monahan



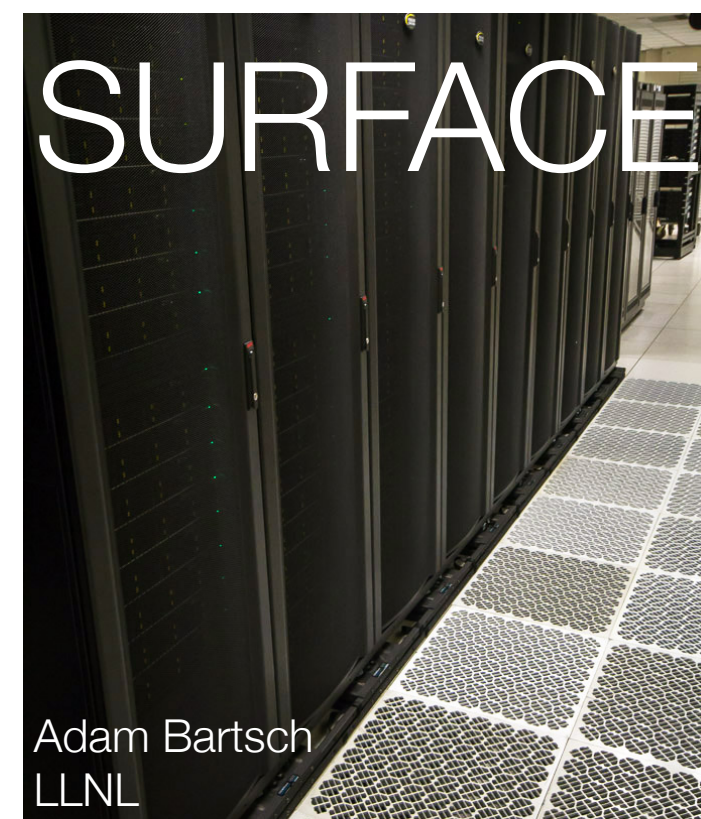
William &
Mary

Kostas Orginos





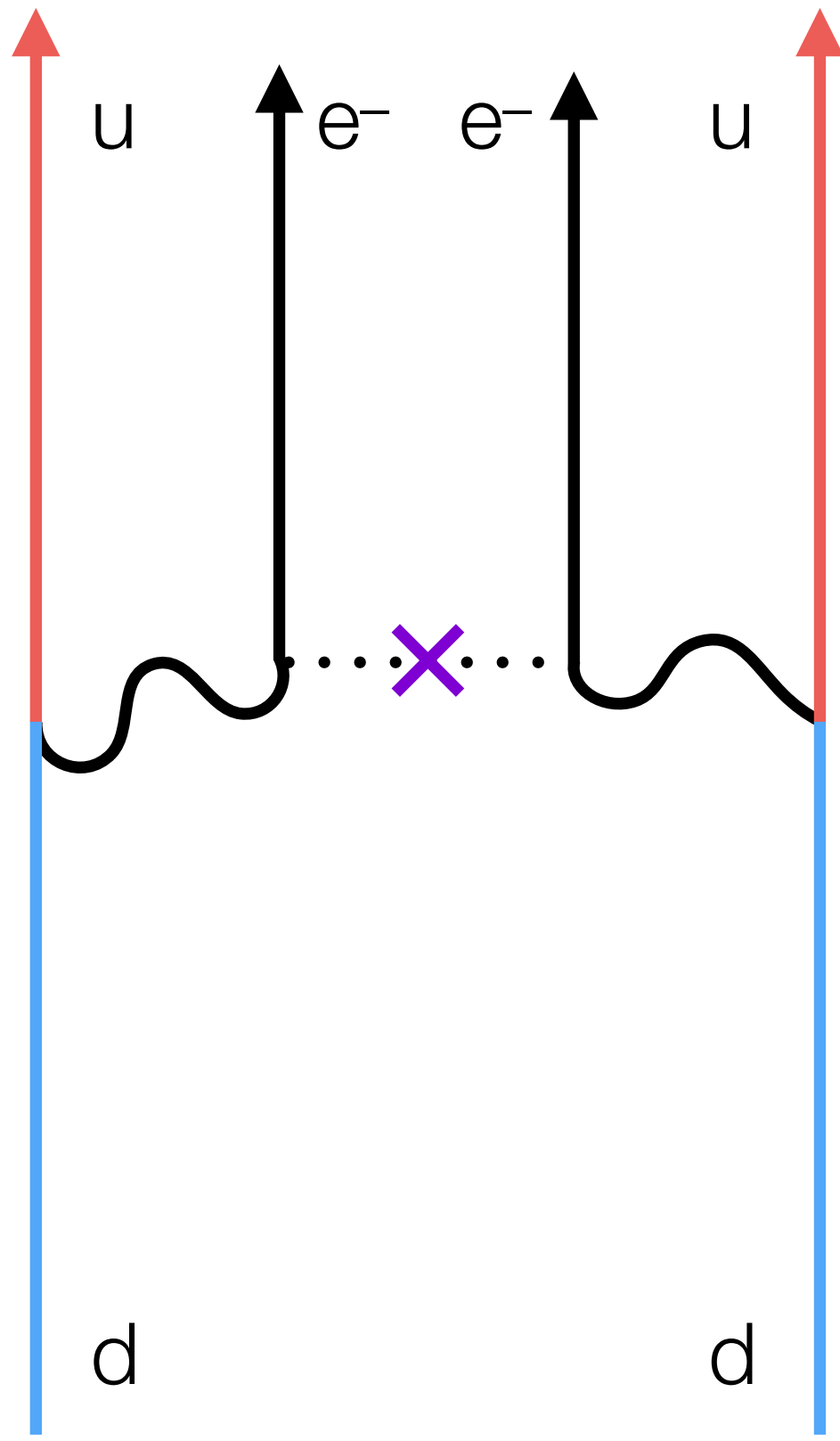
Software	References
METAQ	Berkowitz arXiv:1702.06122 github.com/evanberkowitz/metaq Berkowitz et al. EPJ (LATTICE2017) 175 09007 (2018)
chroma	Edwards and Joo (SciDAC, LHPC and UKQCD Collaborations) Nucl. Phys. Proc. Suppl 140, 832 (2005)
QDP++	
QUDA	Clark et al. Comput. Phys. Commun. 181 1517 (2010) Babich et al. Supercomputing 11, 70
hdf5 in QDP++	Kurth et al PoS LATTICE2014 045 (2015)
qmp	Chen, Edwards, and Watson et al. https://github.com/usqcd-software/qmp
mpi_jm	Berkowitz et al. EPJ (LATTICE2017) 175 09007 (2018) McElvain et al. https://github.com/kenmcelvain/mpi_jm/



Data + jupyter notebook available on GitHub https://github.com/callat-qcd/project_0vbb

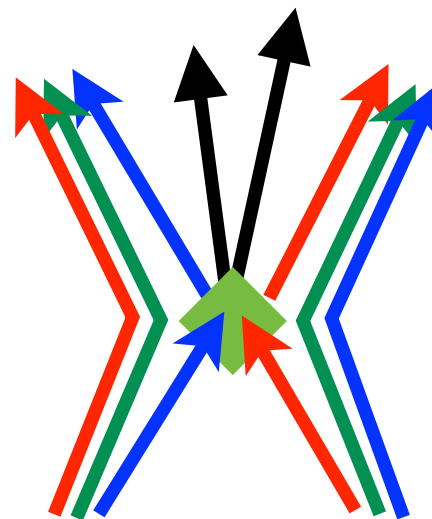
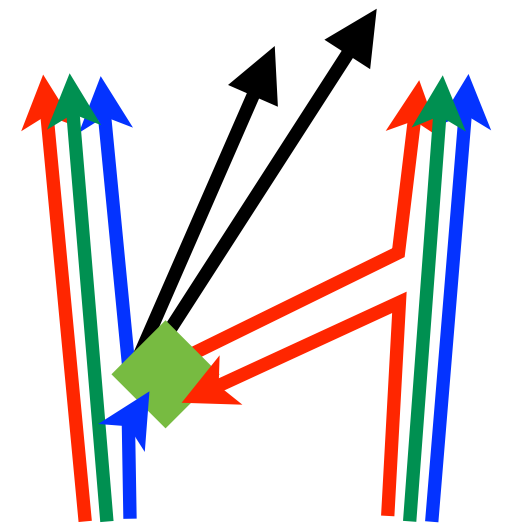
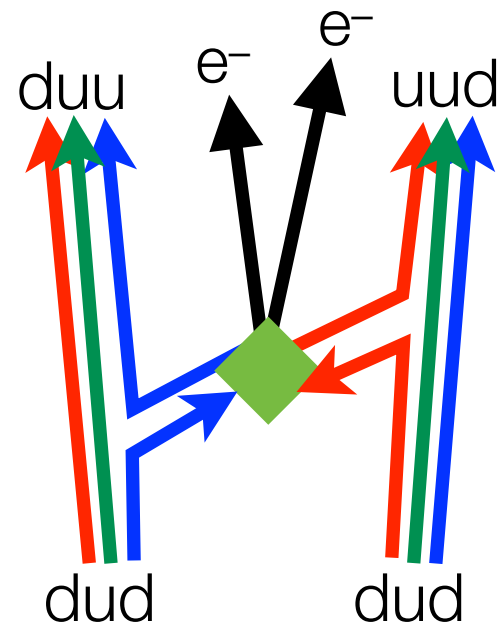
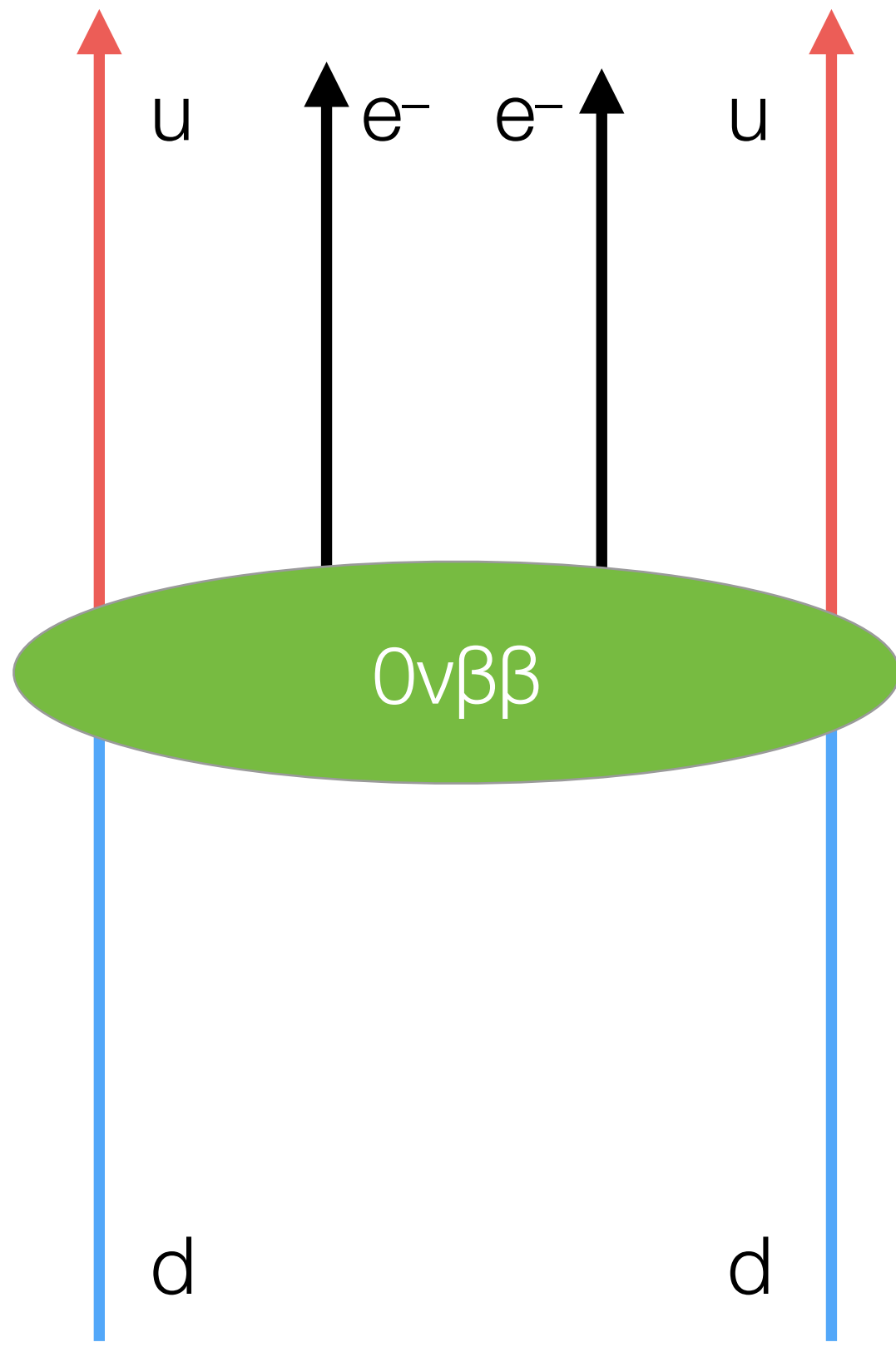
Short Range $0\nu\beta\beta$

CalLat PRL 121 (2018) 17 172501 arXiv:1805.02634

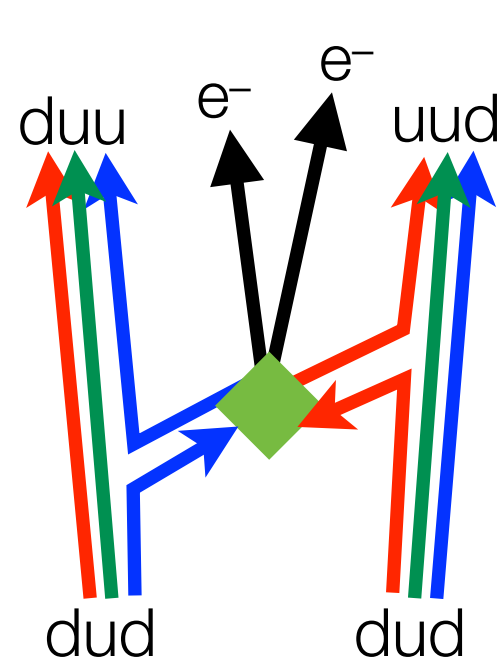


Short Range $0\nu\beta\beta$

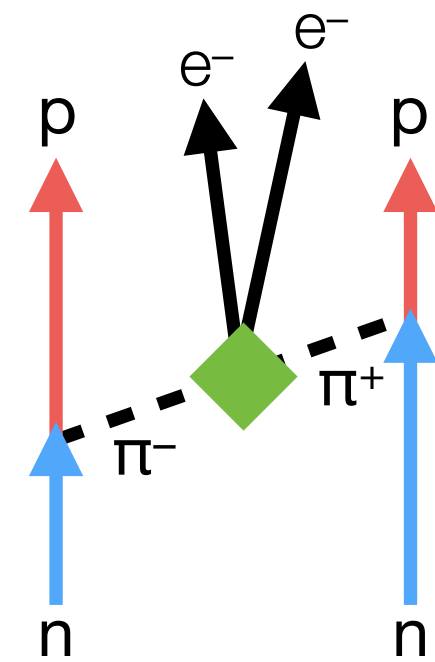
CallLat PRL 121 (2018) 17 172501 arXiv:1805.02634



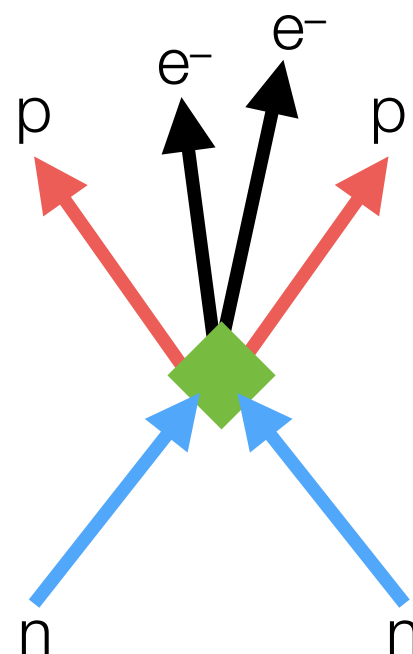
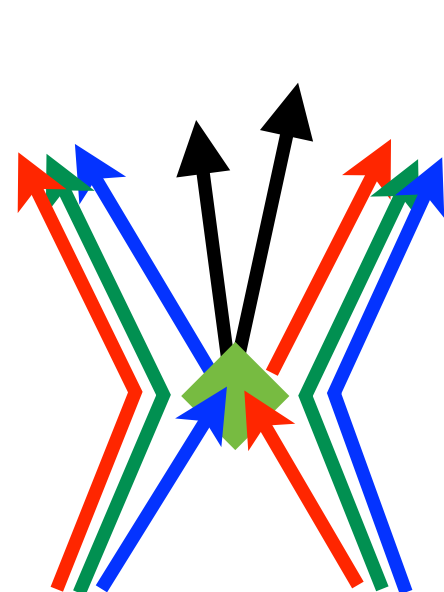
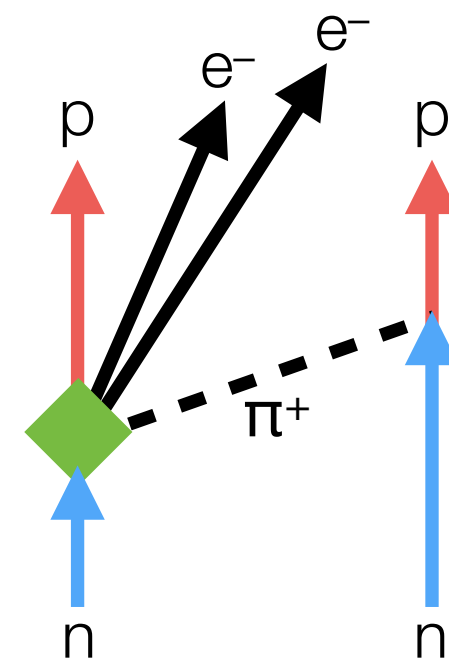
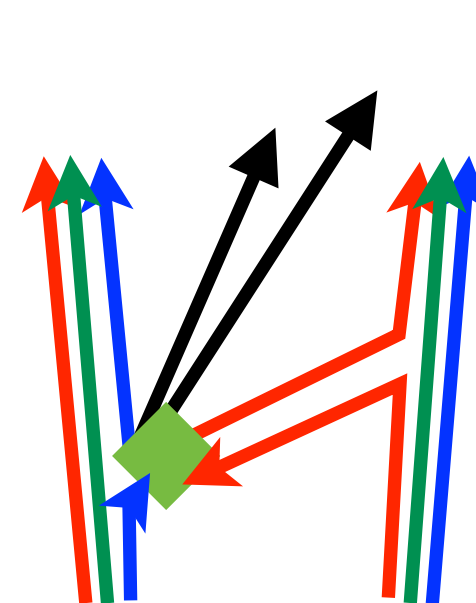
In χ PT



$\mathcal{O}(p^{-1})$ new πN vertex



$\mathcal{O}(p^{-2})$ long-range π exchange

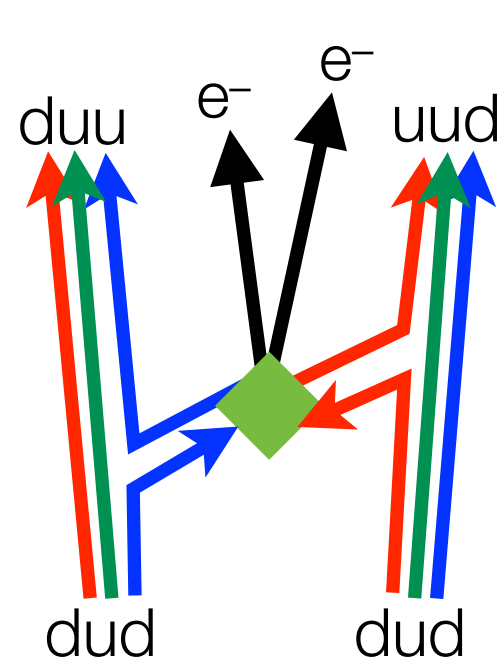


$\mathcal{O}(p^0)$ NN contact operator

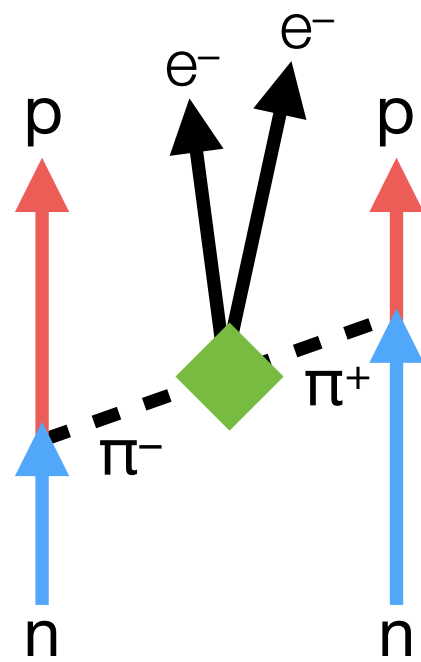
Promoted, as in Cirigliano, Dekens, Mereghetti and Walker-Loud PRC 97 (2018) 06 065501 arXiv:1710.01729
Cirigliano, Dekens, de Vries, Graesser, Mereghetti, Pastor and van Kolck, PRL 120 (2018) 202001 arXiv:1802.10097

Prézeau, Ramsey-Musolf, Vogel PRD68 034016 hep-ph/0303205

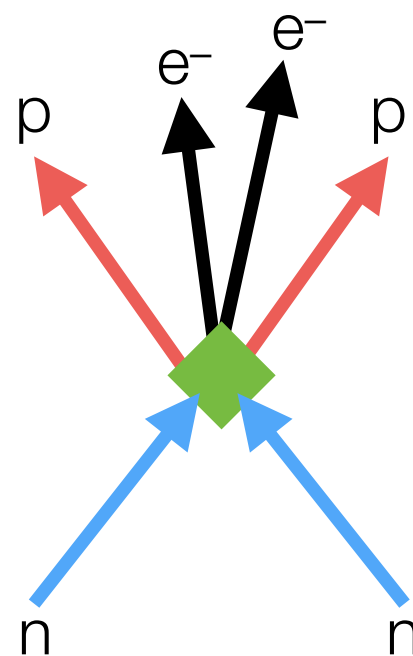
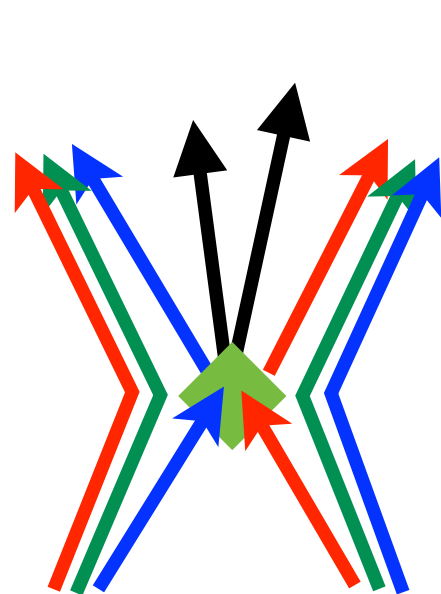
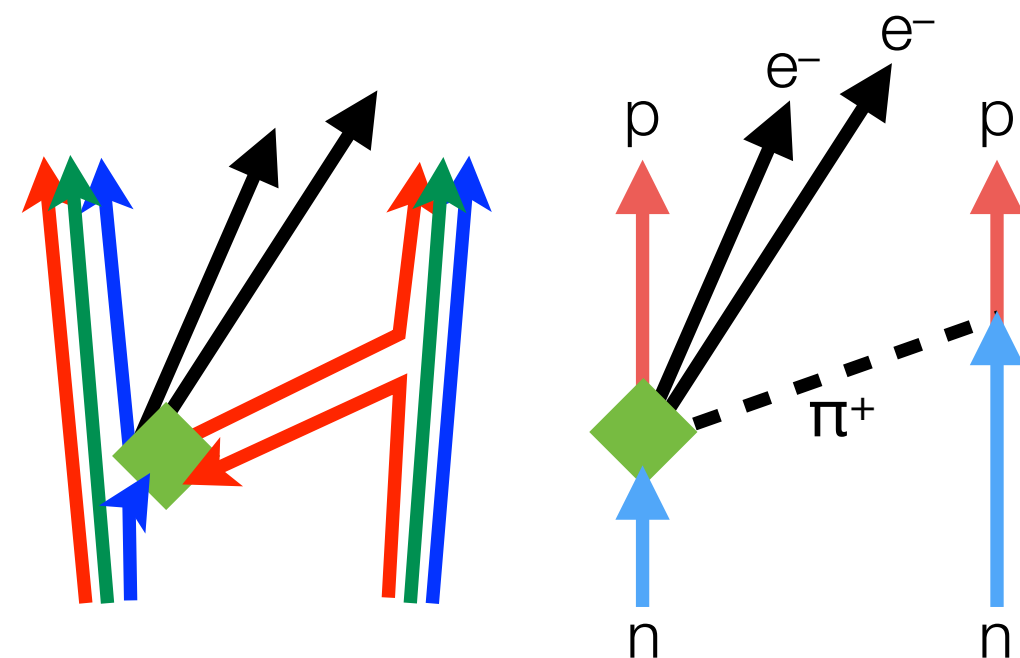
In χ PT



$\mathcal{O}(p^{-1})$ new πN vertex



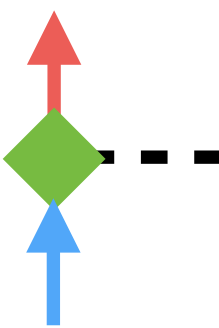
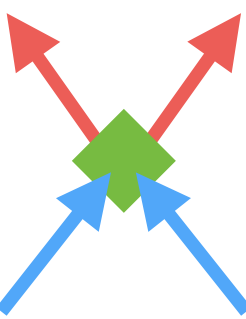
$\mathcal{O}(p^{-2})$ long-range π exchange



$\mathcal{O}(p^{-2})$ NN contact operator

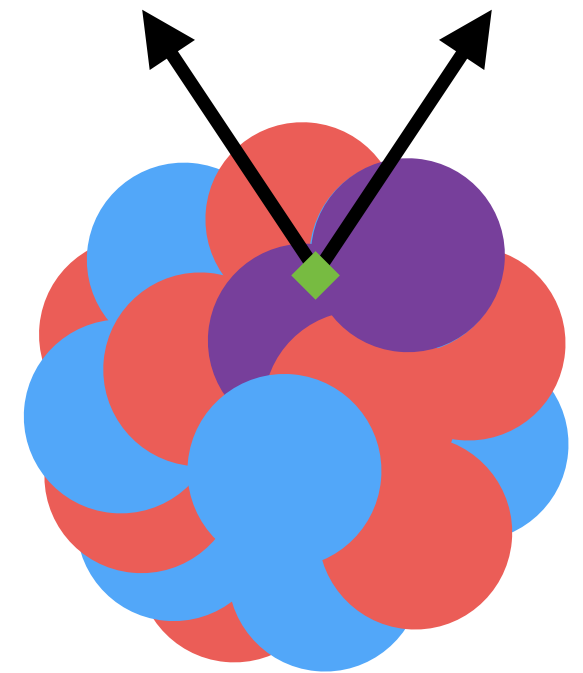
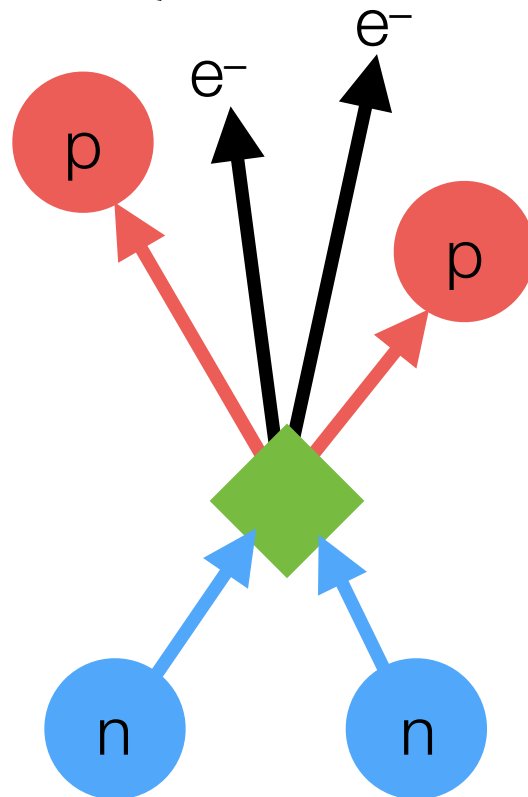
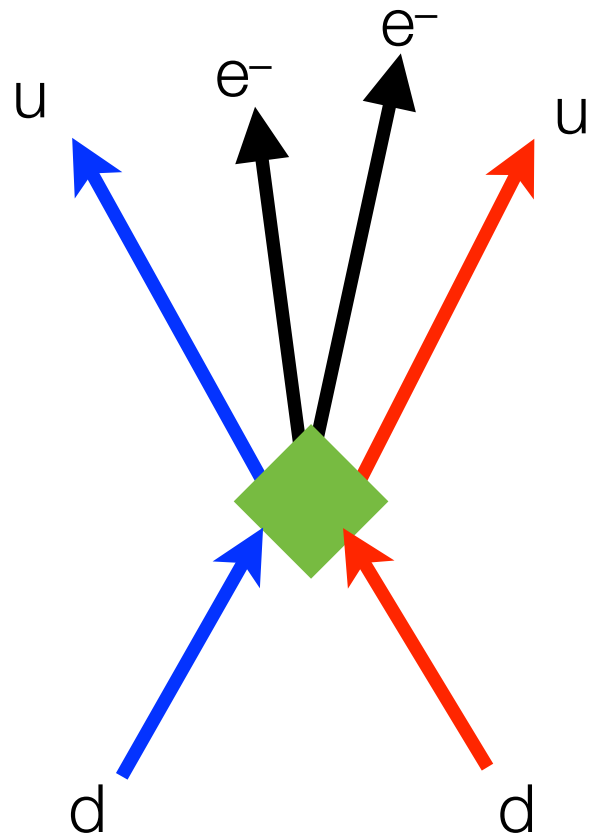
Promoted, as in Cirigliano, Dekens, Mereghetti and Walker-Loud PRC 97 (2018) 06 065501 arXiv:1710.01729
Cirigliano, Dekens, de Vries, Graesser, Mereghetti, Pastor and van Kolck, PRL 120 (2018) 202001 arXiv:1802.10097

Prézeau, Ramsey-Musolf, Vogel PRD68 034016 hep-ph/0303205

	Order	No Disconnected Diagrams	Signal to Noise	Leading Order Operators Induce 0^+ to 0^+ transition
	p^{-2}	✓	✓	✓
	p^{-1}	✗	✗	✗
	p^{-1} ⁻²	✓	✗✗	✓

Solve QCD

Solve NP



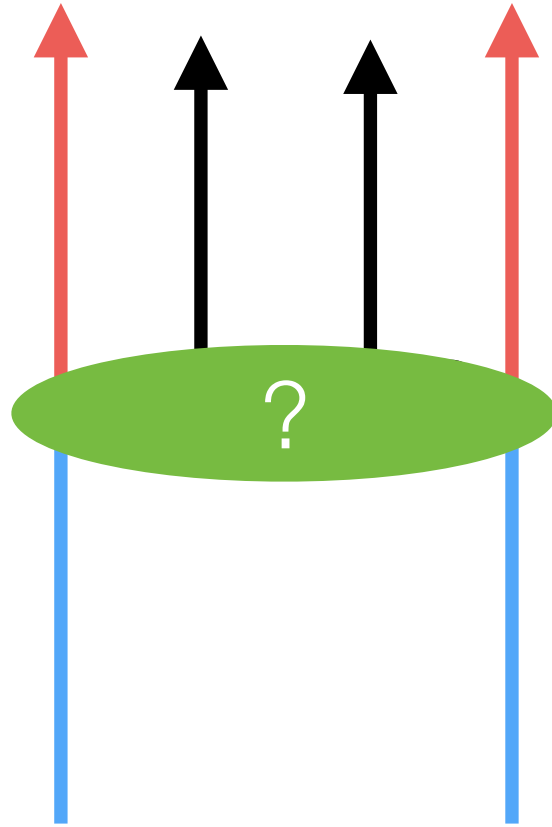
effective 4-quark
operators in QCD

two nucleon
amplitudes

decay rates
in big nuclei

Operators

Prézeau, Ramsey-Musolf, Vogel PRD68 (2003) 034016 hep-ph/0303205



$$\mathcal{O}_{1+}^{ab} = (\bar{q}_L \tau^a \gamma^\mu q_L)(\bar{q}_R \tau^b \gamma_\mu q_R),$$

$$\mathcal{O}_{2\pm}^{ab} = (\bar{q}_R \tau^a q_L)(\bar{q}_R \tau^b q_L) \pm (\bar{q}_L \tau^a q_R)(\bar{q}_L \tau^b q_R),$$

$$\mathcal{O}_{3\pm}^{ab} = (\bar{q}_L \tau^a \gamma^\mu q_L)(\bar{q}_L \tau^b \gamma_\mu q_L) \pm (\bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_R \tau^b \gamma_\mu q_R),$$

$$\mathcal{O}_{4\pm}^{ab,\mu} = (\bar{q}_L \tau^a \gamma^\mu q_L \mp \bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_L \tau^b q_R - \bar{q}_R \tau^b q_L),$$

$$\mathcal{O}_{5\pm}^{ab,\mu} = (\bar{q}_L \tau^a \gamma^\mu q_L \pm \bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_L \tau^b q_R + \bar{q}_R \tau^b q_L).$$

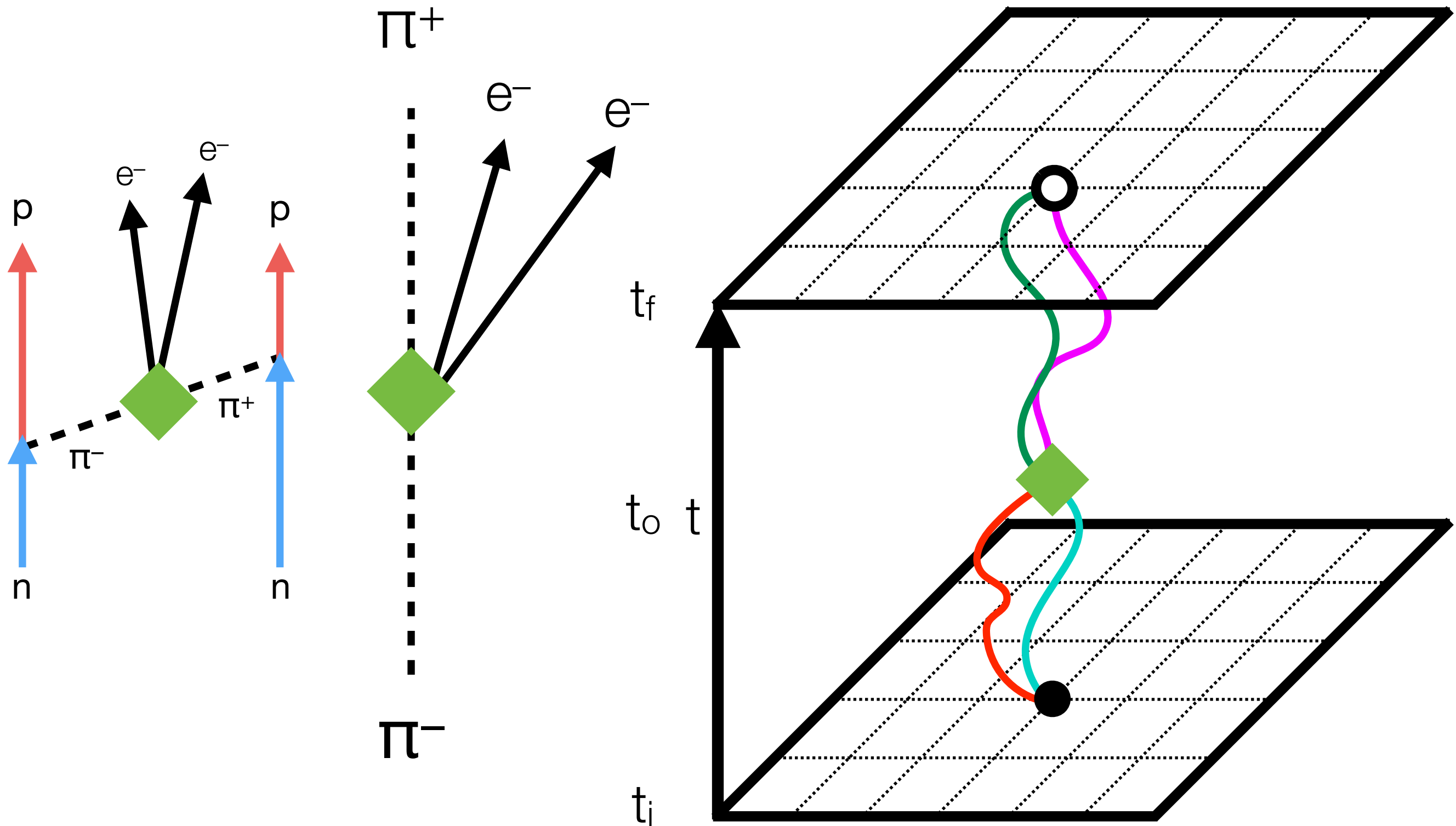
+ $\mathcal{O}'$ color-mixed operators:

$$(\bar{q}\Gamma_1 q) [\bar{q}\Gamma_2 q] = \bar{q}_a \Gamma_1 q_b \quad \bar{q}_b \Gamma_2 q_a$$

$0\nu\beta\beta$ -decay ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}_{5+}^{\pm\pm,\mu}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
$\pi\pi ee$ LO	✓	✓	X	X	X	X	X	X	X
$\pi\pi ee$ NNLO	✓	✓	X	✓	X	X	X	X	X
$NN\pi ee$ LO	X	X	✓	X	X	✓	✓	✓	✓
$NN\pi ee$ NLO	X	✓	X	✓	X	✓	✓	✓	✓
$NNN\pi ee$ LO	✓	✓	X	✓	X	✓	✓	✓	✓

Short Distance $0\nu\beta\beta$ $\pi^+ \pi^-$ Transition

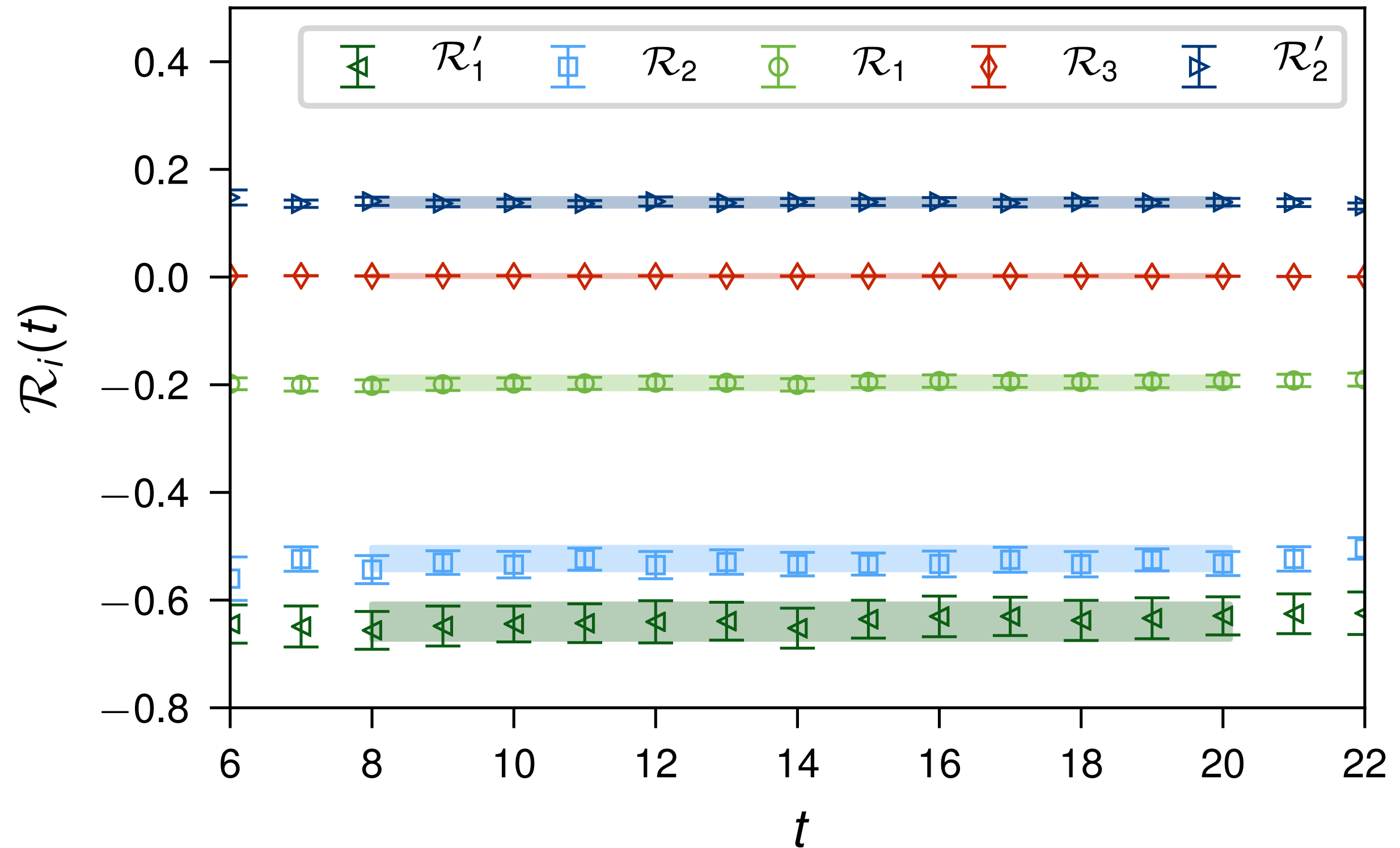
CalLat PRL 121 (2018) 17 172501 arXiv:1805.02634



Results for a Single Ensemble

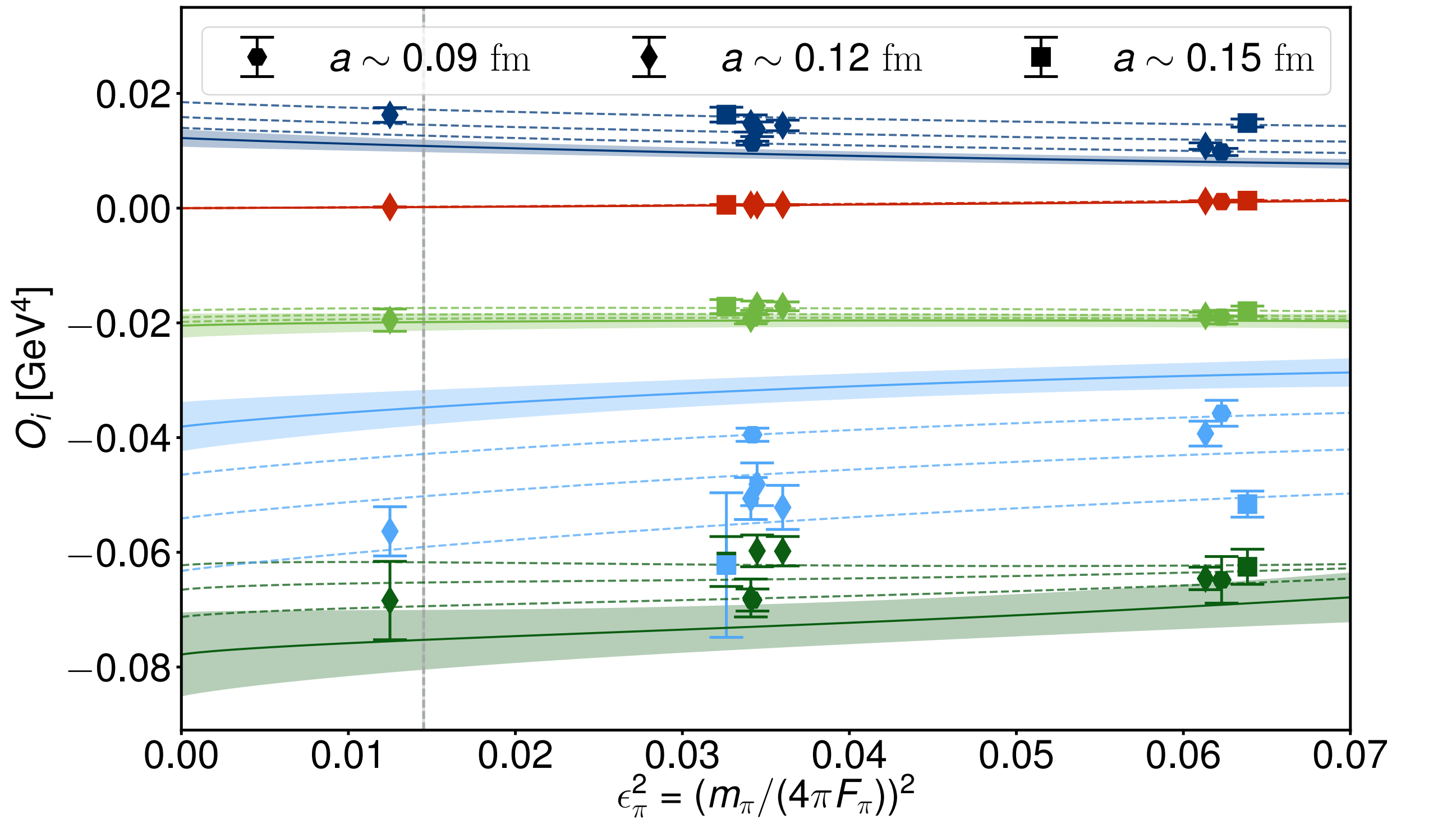
PRL 121 (2018) 172501 arXiv:1805.02634

$a \sim 0.12$ fm
 $m_\pi \sim 130$ MeV

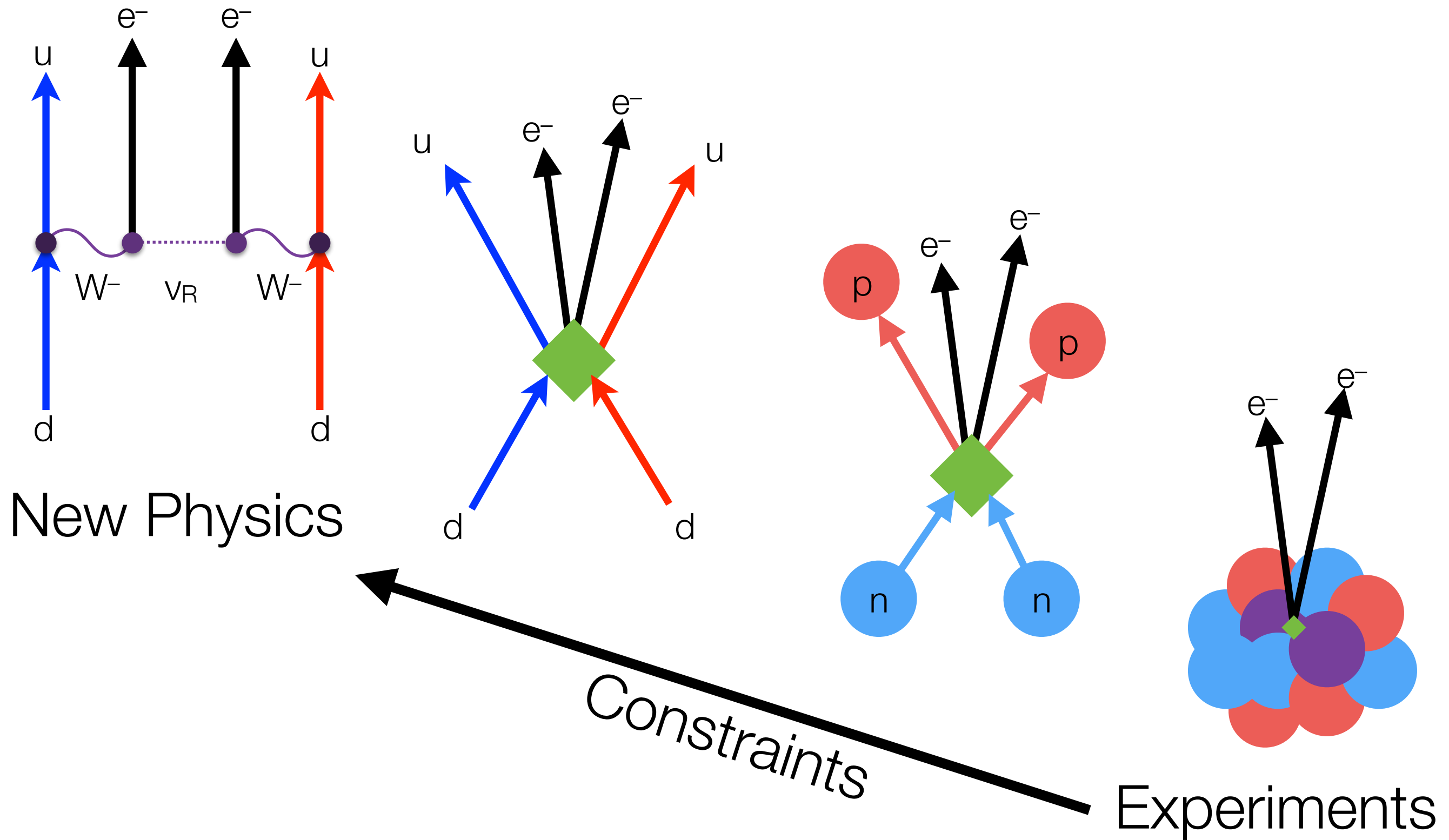


Short-distance $0\nu\beta\beta$

Callat PRL 121 (2018) 17 172501 arXiv:1805.02634



Nuclear Laboratory





SIERRA

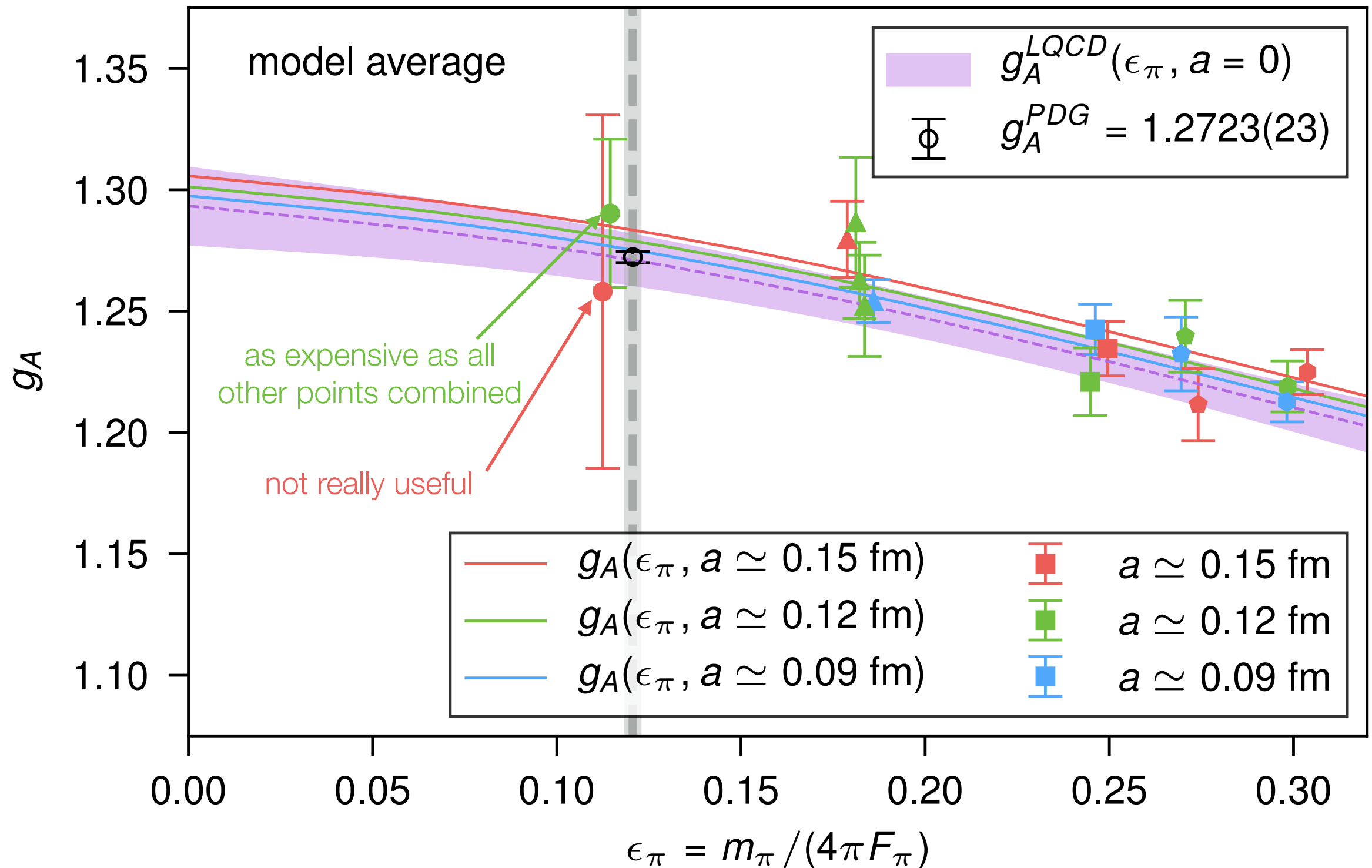
LLNL



LASSEN

LLNL

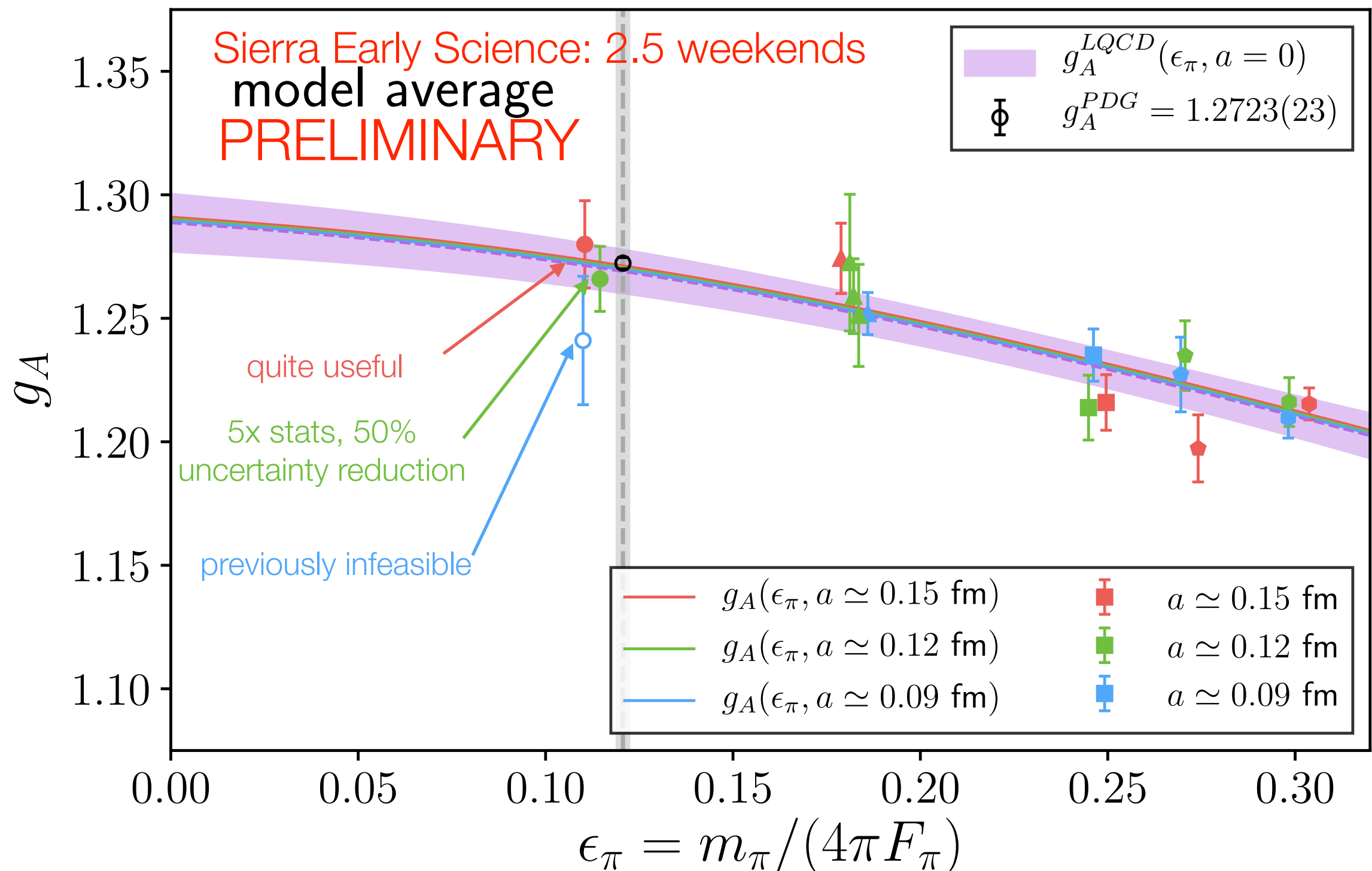
Published $g_A^{\text{QCD}} = 1.271(13)$



Published $g_A^{\text{QCD}} = 1.271(13)$

UPDATE $g_A^{\text{QCD}} = 1.2690(98)$ [0.77%] PRELIMINARY

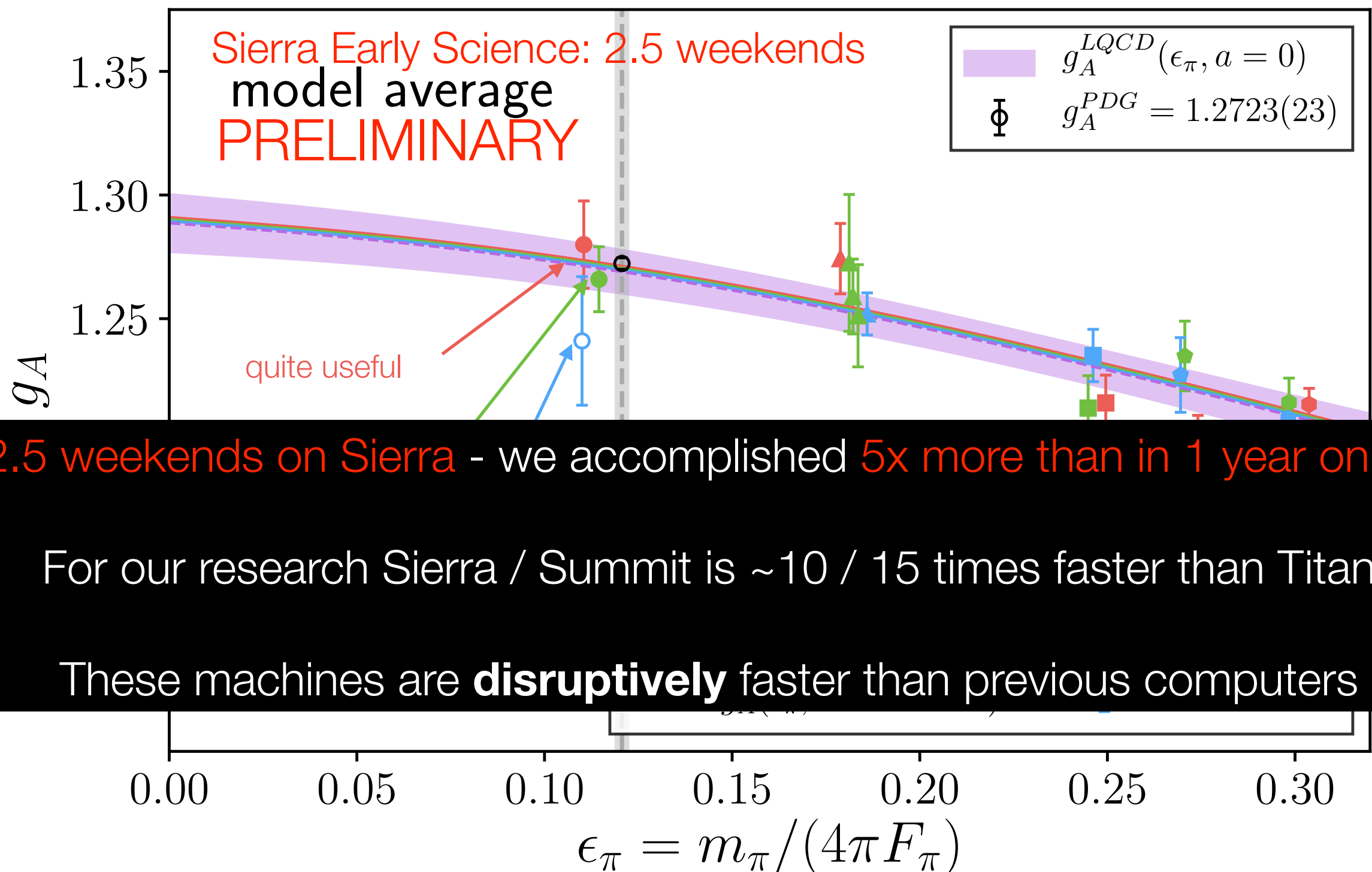
PRELIMINARY!!!!



Published $g_A^{\text{QCD}} = 1.271(13)$

UPDATE $g_A^{\text{QCD}} = 1.2690(98)$ [0.77%] PRELIMINARY

PRELIMINARY!!!!



In 2.5 weekends on Sierra - we accomplished 5x more than in 1 year on Titan

For our research Sierra / Summit is ~10 / 15 times faster than Titan

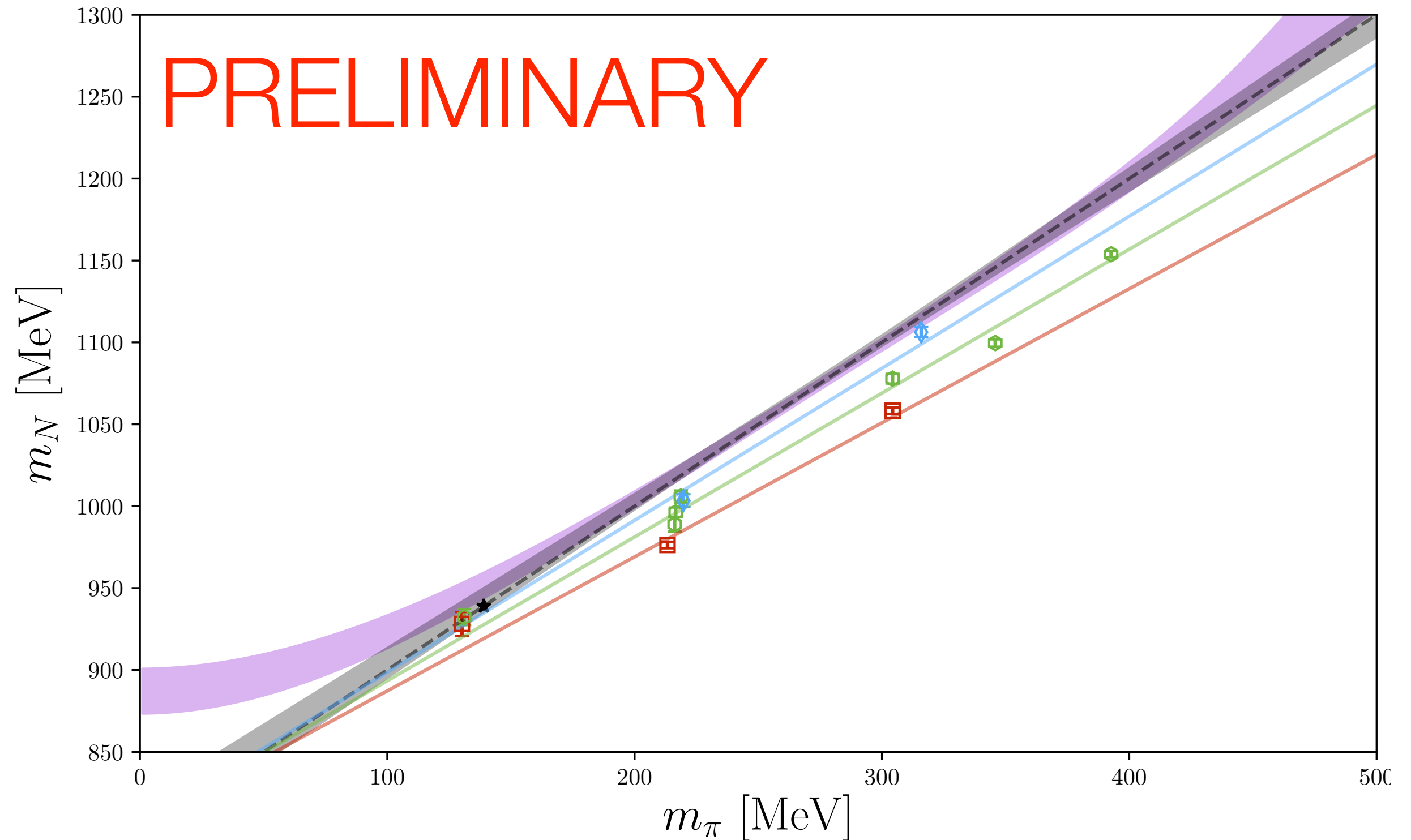
These machines are **disruptively** faster than previous computers

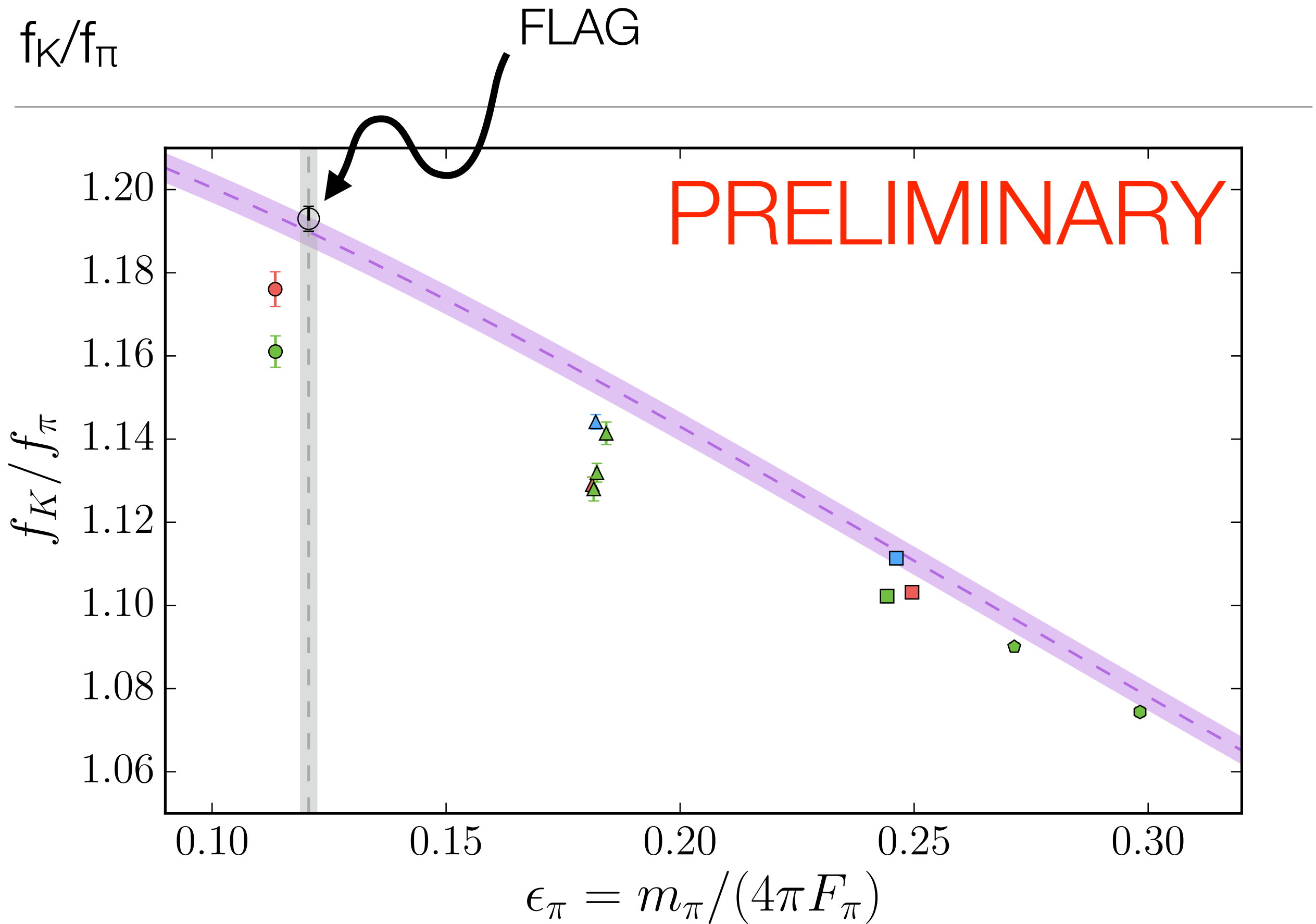
Summary

- g_A , $n\bar{n}$: Looking good!
- θEDM + $c\text{EDM}$: progress, but indistinguishable from zero;
Weinberg operator: not as much progress
- CP violating πN coupling
- FFs: M_A doesn't work; new techniques + faster machines will unlock $q^2 \rightarrow 0$
Chang, Bouchard, Orginos, Richards 1610.02354
- NN: g_A quenching, pp fusion, isotensor polarizability ($2\nu\beta\beta$), $\Delta I=2$ Hadronic PV.
1 lattice spacing, very heavy m_π .
- Complete $\pi^+\pi^-$ transition ($0\nu\beta\beta$) calculation!

Backup Slides

The Ruler still rules: $m_N(m_\pi) \approx 800 \text{ MeV} + m_\pi$





MILC Ensembles

MILC Collaboration Phys. Rev. D87 (2013) 054505
PRD 96 (2017) 054513 arXiv:1701.07559

+ additional HISQ ensembles generated at LLNL

coarser

middle

finer

HISQ gauge configuration parameters							valence parameters							
abbr.	N_{cfg}	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$	N_{src}	L_5/a	aM_5	b_5	c_5	$am_l^{\text{val.}}$	σ_{smr}	N_{smr}
a15m400	1000	$16^3 \times 48$	0.15	0.334	400	4.8	8	12	1.3	1.5	0.5	0.0278	3.0	30
a15m350	1000	$16^3 \times 48$	0.15	0.255	350	4.2	16	12	1.3	1.5	0.5	0.0206	3.0	30
a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2	5	24	1.3	2.25	1.25	0.00216	4.5	60
a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8	8	8	1.2	1.25	0.25	0.02190	3.0	30
a12m350	1000	$24^3 \times 64$	0.12	0.255	350	5.1	8	8	1.2	1.25	0.25	0.01660	3.0	30
a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5	8	8	1.2	1.25	0.25	0.01260	3.0	30
a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m130	1000	$48^3 \times 64$	0.12	0.036	130	3.9	3	20	1.2	2.0	1.0	0.00195	7.0	150
a09m400	1201	$32^3 \times 64$	0.09	0.335	400	5.8	8	6	1.1	1.25	0.25	0.0160	3.5	45
a09m350	1201	$32^3 \times 64$	0.09	0.255	350	5.1	8	6	1.1	1.25	0.25	0.0121	3.5	45
a09m310	784	$32^3 \times 96$	0.09	0.2	310	4.5	8	6	1.1	1.25	0.25	0.00951	7.5	167
a09m220	1001	$48^3 \times 96$	0.09	0.1	220	4.7	6	8	1.1	1.25	0.25	0.00449	8.0	150

MDWF pion mass tuned to taste-5 HISQ
pion mass within 1-2% - ensuring the
unitary limit is recovered in the continuum

Free to use; large statistics available
Capable of controlling all systematics
We use domain wall valence on the HISQ sea,
 $\mathcal{O}(a^2)$ errors [1701.07559].

$0\nu\beta\beta$ Lattices

PRL 121 (2018) 172501 arXiv:1805.02634

$a \setminus m_\pi$	310 MeV	220 MeV	135 MeV
0.15 fm	$16^3 \times 48$ $m_\pi L \sim 3.78$	$24^3 \times 48$ $m_\pi L \sim 3.99$	$32^3 \times 48$ $m_\pi L \sim 3.25$
0.12 fm		$24^3 \times 64$ $m_\pi L \sim 3.22$	
0.12 fm	$24^3 \times 64$ $m_\pi L \sim 4.54$	$32^3 \times 64$ $m_\pi L \sim 4.29$	$48^3 \times 64$ $m_\pi L \sim 3.91$
0.12 fm		$40^3 \times 64$ $m_\pi L \sim 5.36$	
0.09 fm	$32^3 \times 96$ $m_\pi L \sim 4.50$	$48^3 \times 96$ $m_\pi L \sim 4.73$	

- Möbius DWF on HISQ sea MILC Collaboration Phys. Rev. D87 (2013) 054505
- Gradient flow with very small m_{res} for reasonable L_5
Narayanan, Neuberger (2006), Luscher (2010)
- ~1000 configurations, 1 wall and 1 point per config

Black Box Theorem: $0\nu\beta\beta$ implies Majorana Mass

Schechter and Valle PRD 25 (1982) 774

