

Quantum Monte Carlo calculations of dark matter scattering off light nuclei

ECT*, Trento, Italy

Based on Phys. Rev. C, 99:025501, Feb 2019.

L. A., V. Cirigliano, S. Gandolfi, F. Pederiva

Lorenzo Andreoli

April 16, 2019



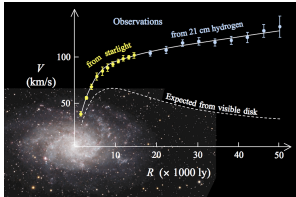
UNIVERSITY
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- Dark Matter and its direct detection
- Interaction: from DM-quarks to DM-nucleons
- Nuclear wave functions
- Results

Dark Matter

Dark Matter

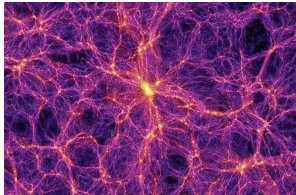
It's out there:



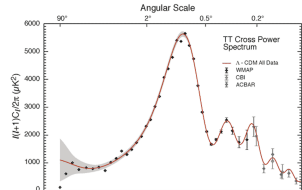
Rotation curves



Gravitational lensing



Structure formation

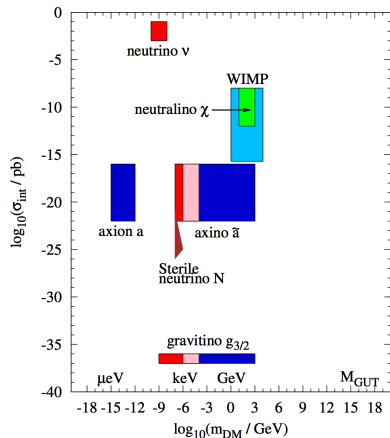


CMB anisotropy

Particle candidates

Requirements:

- Stable on cosmological time scales
- Very weakly interacting with EM radiation
- Correct density

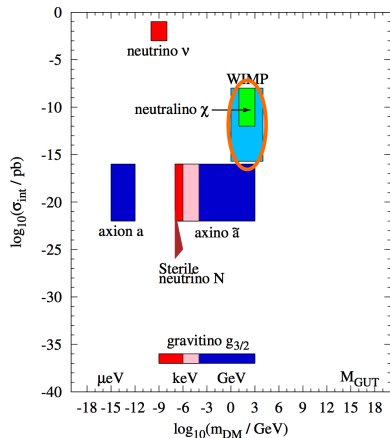


Roszkowski et al., arXiv:1707.06277

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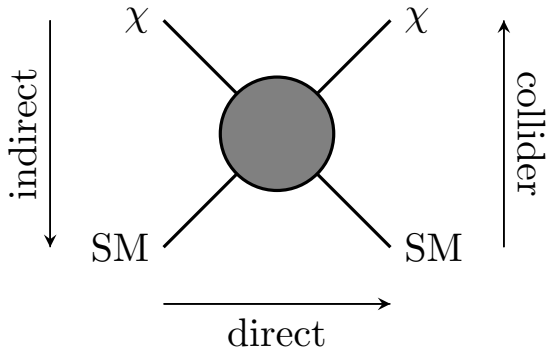
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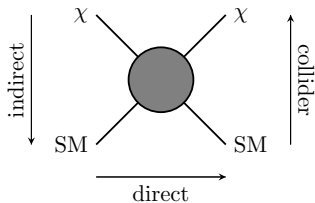
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Detection

Mainly three possible methods to detect WIMPs:

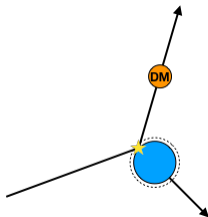


Detection

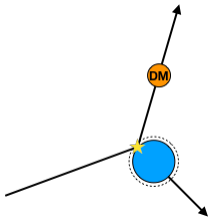
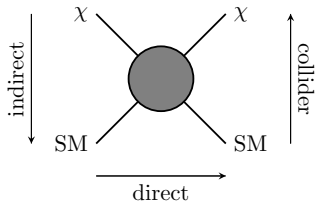


$$N(\mathbf{p}_i) + \chi(\mathbf{k}) \rightarrow N(\mathbf{p}_i') + \chi(\mathbf{k}')$$

$$\mathbf{q} = \mathbf{k}' - \mathbf{k}$$



Detection



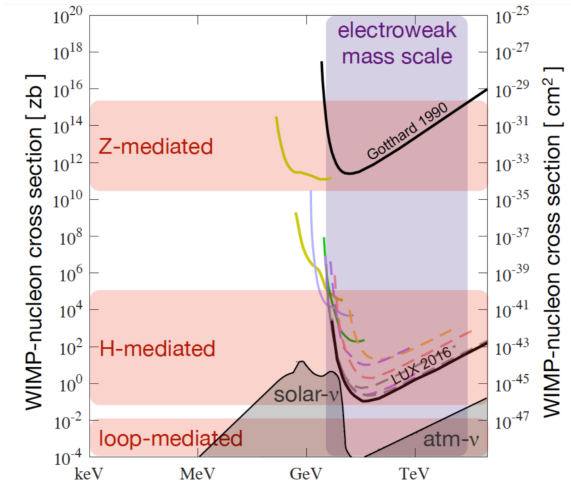
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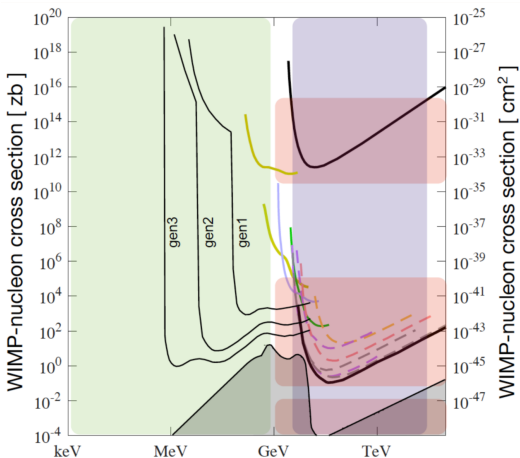
$$|\mathbf{q}| \lesssim 2\mu v_{\chi}^{(\text{esc})} \lesssim A \times 2.5 \text{ MeV}$$

$$E_R < \frac{|\mathbf{q}|^2}{2m_A}$$

Great experimental progress

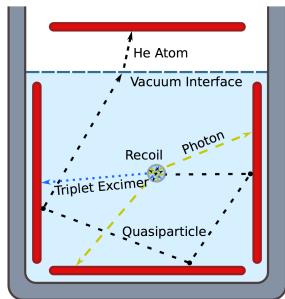


From nuclear recoil on ^4He



S. A. Hertel, A. Biekert, J. Lin, V. Velan, and D. N. McKinsey,
arXiv:1810.06283

Nuclear recoil in liquid He

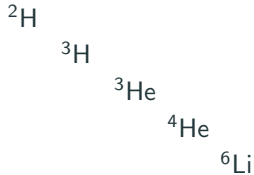


Superfluid He:

- better kinematic matching with light DM
- high impedance to external vibration noise
- mechanism for observing phonon-like modes via liberation atoms into a vacuum

S. A. Hertel, A. Biekert, J. Lin, V. Velan, and D. N. McKinsey,
arXiv:1810.06283

- Benchmark: accurate nuclear calculations
- Potential experimental targets: He
- More sensitive to relatively light dark matter



Interaction

A solid theoretical control of nuclear effects is necessary to interpret direct detection experiments

¹ J. Fan, M. Reece, and L.-T. Wang, JCAP 1011, 042 (2010), arXiv:1008.1591

² A. L. Fitzpatrick, W. Haxton, E. Katz, N. Lubbers, and Y. Xu, JCAP 1302, 004 (2013), arXiv:1203.3542

³ M. Hoferichter, P. Klos, J. Menendez, and A. Schwenk, Phys. Rev. D 94, 063505 (2016).

⁴ E. Chang et al., Phys. Rev. Lett. 120, 152002 (2018)

A solid theoretical control of nuclear effects is necessary to interpret direct detection experiments

A variety of approaches based on effective field theory have been proposed:

- non-relativistic DM-nucleus interactions¹
- non-relativistic DM-nucleon interactions²
- DM-nucleon interactions derived from DM-quark and DM-gluon interactions³
- First principle, lattice-QCD calculations⁴

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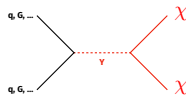
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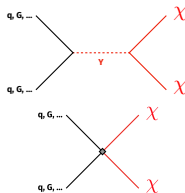
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E $\mathcal{L}_{BSM}$ 

E

 $\mathcal{L}_{BSM}$

$$\mathcal{L}_{SM} + \sum_{k,i} \frac{c_i^{(k)}}{\tilde{\Lambda}^k} \mathcal{O}_i^{(k)}[\chi^\dagger, \chi; q, G, \dots]$$

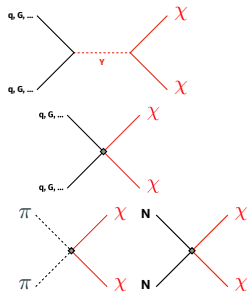


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$$\mathcal{L}_{ChPT}[\chi; N, \pi, \dots]$$



E

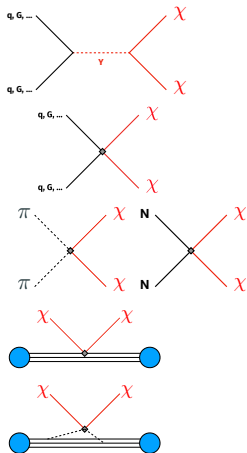
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$$\mathcal{L}_{ChPT}[\chi; N, \pi, \dots]$$

$$H_I = V_{\chi N} + V_{\chi NN} + \dots$$

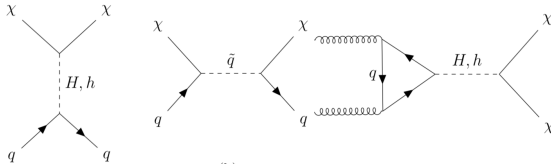
$$\langle \psi'_{(A,Z)}, X' | H_I | \psi_{(A,Z)}, X \rangle$$



Effective Lagrangian

Scalar case:

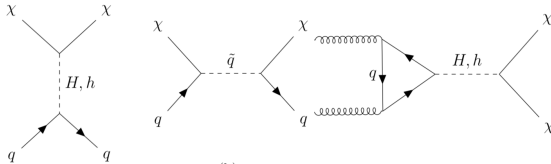
- Well motivated, e.g. Higgs and squark exchange mediate scalar interactions



Effective Lagrangian

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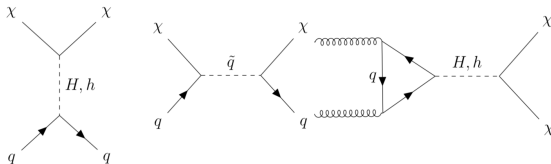


- interesting two body currents at NLO

Effective Lagrangian

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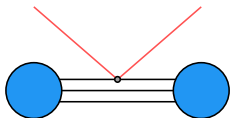
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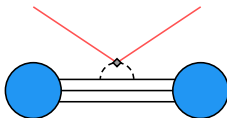
- interesting two body currents at NLO

$$\mathcal{L}_{\text{eff}} = \frac{1}{\tilde{\Lambda}^3} \left(\sum_{q=u,d,s} c_q \bar{\chi} \chi m_q \bar{q} q + c_G \bar{\chi} \chi \alpha_s G_{\mu\nu}^a G_a^{\mu\nu} \right)$$

Effective Lagrangian



LO

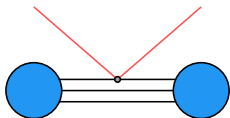


NLO

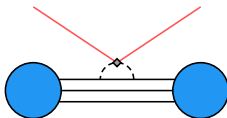
$$J^{(1)}(\mathbf{q}_i) = \frac{c_{is}}{\tilde{\Lambda}^3} \left[\sigma_{\pi N} - \frac{9g_A^2 \pi m_\pi^3}{4(4\pi f_\pi)^2} F\left(\frac{|\mathbf{q}_i|}{2m_\pi}\right) \right] - \frac{c_{iv}}{\tilde{\Lambda}^3} \frac{\delta m_N}{4} \tau_i^z$$

$$+ \frac{c_s}{\tilde{\Lambda}^3} (\sigma_s - \dot{\sigma}_s \mathbf{q}^2) - \frac{c_G}{\tilde{\Lambda}^3} \frac{8\pi m_N^G}{9}$$

Effective Lagrangian



LO



NLO

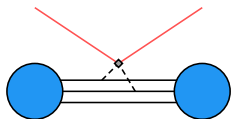
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$$F(x) = \frac{-x + (1 + 2x^2) \arctan x}{3x}$$

$$c_{is} = \frac{c_u m_u + c_d m_d}{m_u + m_d}, \quad c_{iv} = 2 \frac{c_d m_d - c_u m_u}{m_d - m_u}$$

Effective Lagrangian



NLO

$$J_{\pi\pi}^{(2)}(\mathbf{q}_i, \mathbf{q}_j) = -\frac{c_{\text{is}}}{\tilde{\Lambda}^3} \left(\frac{g_A}{2F_\pi} \right)^2 m_\pi^2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \frac{\boldsymbol{\sigma}_i \cdot \mathbf{q}_i \boldsymbol{\sigma}_j \cdot \mathbf{q}_j}{(\mathbf{q}_i^2 + m_\pi^2)(\mathbf{q}_j^2 + m_\pi^2)}$$

Nuclear Wave Functions

End goal: calculate matrix elements for elastic scattering

$$J(\mathbf{q}) = \sum_i e^{i\mathbf{q}\cdot\mathbf{r}_i} J^{(1)}(\mathbf{q}) + \sum_{i<j} J_{\pi\pi}^{(2)}(\mathbf{q}; \mathbf{r}_i, \mathbf{r}_j)$$

$J(\mathbf{q})$ is the Fourier transform of the regularized one- and two-body currents

$$\frac{d\sigma}{d\mathbf{q}^2} = \frac{1}{4\pi v_\chi^2} \frac{1}{2j+1} \sum_{m_j, m'_j=-j}^j \left| \langle \psi_{jm'_j} | J(\mathbf{q}) | \psi_{jm_j} \rangle \right|^2$$

Nuclear Wave Functions

Nuclear Hamiltonian: Argonne v_{18} ⁵ + Urbana IX⁶

$$H = \sum_i T_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} v_{ijk}$$

⁵R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C 51, 38 (1995)

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Use nuclear wave functions that minimize the expectation value of E

$$E_V = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} \geq E_0$$

The evaluation is performed using Metropolis sampling

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Nuclear Wave Functions

Variational wave function for nucleus in J state:

$$|\psi\rangle = \mathcal{S} \prod_{i < j}^A \left[1 + U_{ij} + \sum_{k \neq i,j}^A U_{ijk} \right] \left[\prod_{i < j} f_c(r_{ij}) \right] |\Phi(JMTT_3)\rangle .$$

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Two-body spin- and isospin-dependent correlations

$$U_{ij} = \sum_p f^p(r_{ij}) O_{ij}^p$$

$$O_{ij}^p = [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}] \otimes [1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j] .$$

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$$U_{ijk} = \epsilon v_{ijk}(\bar{r}_{ij}, \bar{r}_{jk}, \bar{r}_{ki}).$$

Results

Event rate:

$$\frac{dN}{dE_R} \propto \Phi \otimes \frac{d\sigma}{dE_R}$$

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Flux factor: DM local density and velocity distribution

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Wimp-nucleus cross section: particle physics $\times$ hadronic & nuclear physics

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Focus on the cross section

Nuclear response function

It is convenient to expand the total cross section in terms of nuclear response functions:

$$\frac{d\sigma}{d\mathbf{q}^2} = \frac{c_{\text{is}}^2}{\tilde{\Lambda}^6} \frac{\sigma_{\pi N}^2 A^2}{4\pi v_\chi^2} \times$$
$$\left| \mathcal{F}_{\text{is}}^{(0)}(\mathbf{q}^2) + \mathcal{F}_{\text{is,r}}^{(1)}(\mathbf{q}^2) + \mathcal{F}_{\text{is,2b}}^{(1)}(\mathbf{q}^2) - \frac{c_{\text{iv}}}{c_{\text{is}}} \frac{\delta m_N}{4\sigma_{\pi N}} \mathcal{F}_{\text{iv}}^{(0)}(\mathbf{q}^2) \right|^2$$

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We concentrate on the case $c_{\text{is}} \neq 0$, setting $c_{\text{iv},s,G}/c_{\text{is}} = 0$

Additional couplings do not introduce independent nuclear responses

$$\mathcal{F}_{\text{iv}}^{(0)}(\mathbf{q}^2) = \frac{2Z - A}{A} \times \mathcal{F}_{\text{is}}^{(0)}(\mathbf{q}^2)$$

Also, general case can be obtained rescaling $\mathcal{F}_{\text{is}}^{(0)}(\mathbf{q}^2)$ by the factor

$$1 + \left(\frac{c_s}{c_{\text{is}}} \right) \frac{\sigma_s - \dot{\sigma}_s \mathbf{q}^2}{\sigma_{\pi N}} - \left(\frac{c_G}{c_{\text{is}}} \right) \frac{8\pi m_N^G}{9\sigma_{\pi N}}$$

Radius correction

We define a “radius correction” as the percentual contributio of the NLO one-body current to the total cross section:

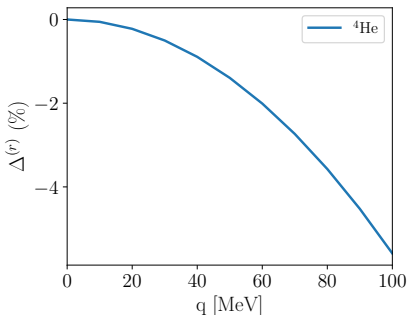
$$\Delta^{(r)} = \frac{|\mathcal{F}_{\text{is}}^{(0+1)}(\mathbf{q}^2)|^2 - |\mathcal{F}_{\text{is}}^{(0)}(\mathbf{q}^2) + \mathcal{F}_{\text{is},2\text{b}}^{(1)}(\mathbf{q}^2)|^2}{|\mathcal{F}_{\text{is}}^{(0+1)}(\mathbf{q}^2)|^2}$$

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$$\begin{aligned}\Delta(r) &\sim \frac{2\mathcal{F}_{\text{is},r}^{(1)}(\mathbf{q}^2)}{\mathcal{F}_{\text{is}}^{(0+1)}(\mathbf{q}^2)} \\ &\sim -\frac{2}{\sigma_{\pi N}} \frac{9g_A^2 \pi m_\pi^3}{4(4\pi f_\pi)^2} F\left(\frac{|\mathbf{q}|}{2m_\pi}\right)\end{aligned}$$



Two-body correction

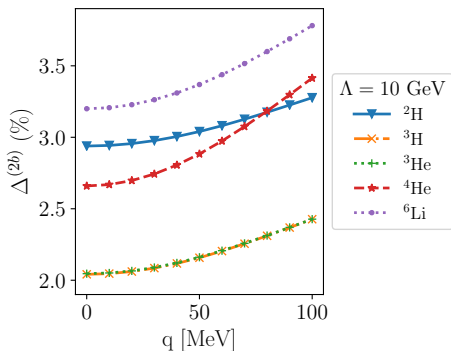
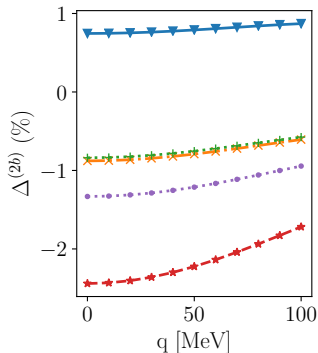
Similarly, for the two-body current:

$$\Delta^{(2b)} = \frac{|\mathcal{F}_{\text{is}}^{(0+1)}(\mathbf{q}^2)|^2 - |\mathcal{F}_{\text{is}}^{(0)}(\mathbf{q}^2) + \mathcal{F}_{\text{is},r}^{(1)}(\mathbf{q}^2)|^2}{|\mathcal{F}_{\text{is}}^{(0+1)}(\mathbf{q}^2)|^2}.$$

Two-body correction

Similarly, for the two-body current:

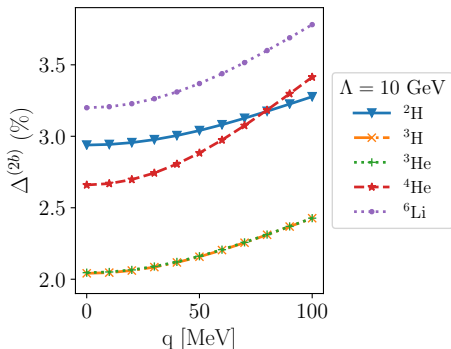
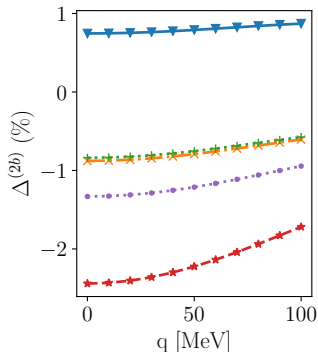
$$\Delta^{(2b)} = \frac{|\mathcal{F}_{\text{is}}^{(0+1)}(\mathbf{q}^2)|^2 - |\mathcal{F}_{\text{is}}^{(0)}(\mathbf{q}^2) + \mathcal{F}_{\text{is},r}^{(1)}(\mathbf{q}^2)|^2}{|\mathcal{F}_{\text{is}}^{(0+1)}(\mathbf{q}^2)|^2}.$$



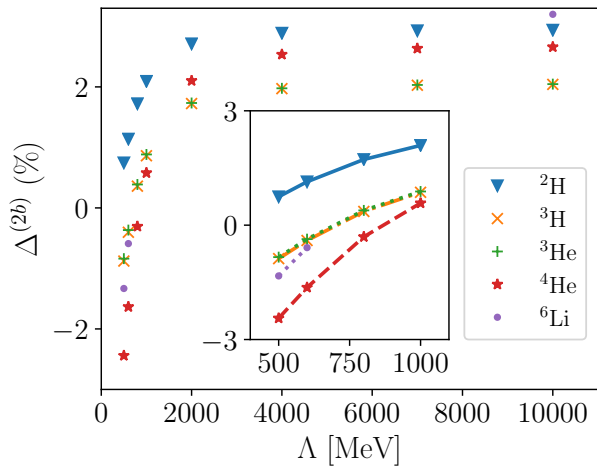
Two-body correction

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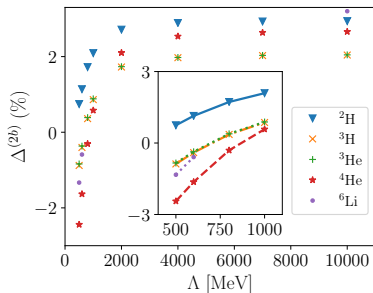


Two-body correction



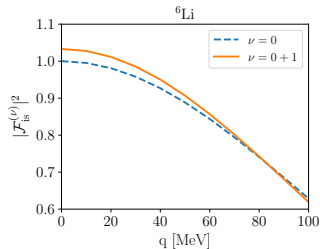
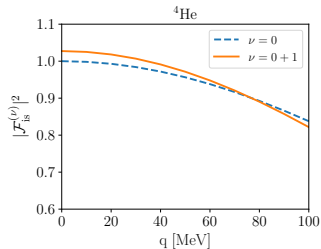
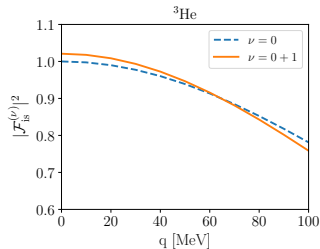
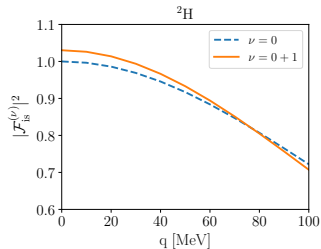
Two-body correction

Sources of uncertainty:

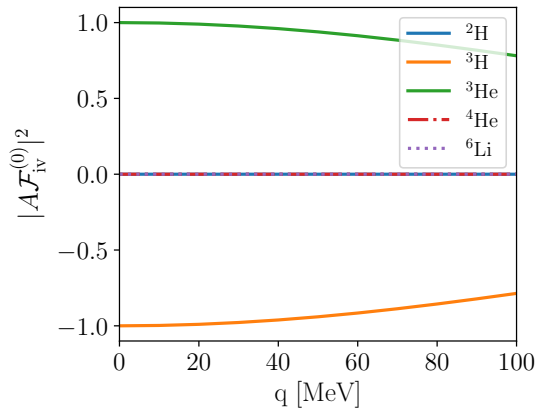


- Cutoff dependence
- Sigma term, tension between dispersion relation and lattice-QCD calculation

Total cross section



Total cross section



DM-nucleus scattering calculations for light nuclei, using QMC

- Light nuclei, first calculation for two-body currents in $A = 4, 6$
- Overall size of NLO correction of the order of few %, even assigning a conservative estimate on the two-body matrix elements
- Our results provide nuclear structure input needed to assess the sensitivity of future experimental searches of light dark matter using ^3He and ^4He targets.

Future studies

- Other mediators - Vector and Axial-Vector
- Moving beyond the hybrid approach: chiral interactions + QMC
- Exploring consistency of Weinberg power counting

Thank you!