β decay with the nuclear shell model: capabilities, limitations, uncertainties

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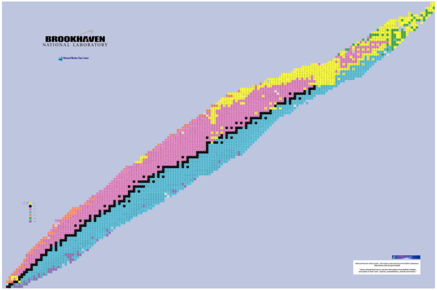
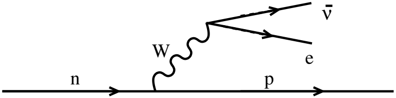
東京大学
THE UNIVERSITY OF TOKYO

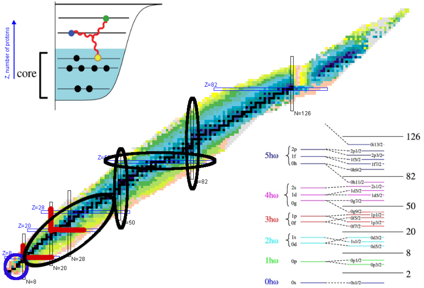
科研費
KAKENHI

Nuclear β decay

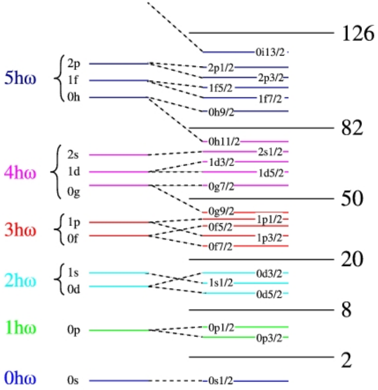
β decays (e^- capture) main decay model along nuclear chart
In principle can be described by nuclear structure theory: shell model...







Nuclear shell model



Solve many-body problem
by direct diagonalization
in limited configuration space

- Excluded orbitals: always empty
- Valence space:
  configuration space where to solve
  the many-body problem
- Inner core: always filled

Diagonalize valence space, rest in H_{eff}:

Dimension $\sim$

$$\binom{(p+1)(p+2)_\nu}{N}\binom{(p+1)(p+2)_\pi}{Z}$$

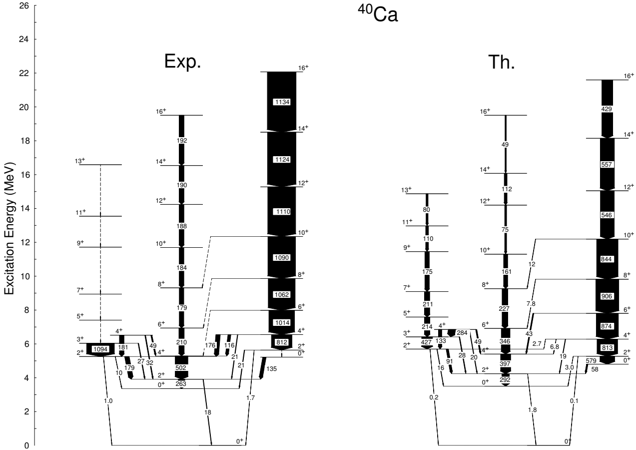
$$H\left|\Psi\right\rangle = E\left|\Psi\right\rangle \;\rightarrow H_{eff}\left|\Psi\right\rangle_{eff} = E\left|\Psi\right\rangle_{eff}$$

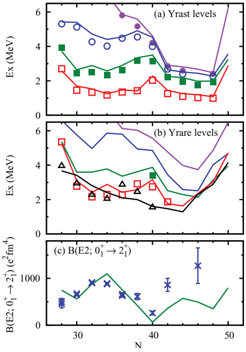
$$\left|\Psi\right\rangle_{eff} = \sum_\alpha c_\alpha \left|\phi_\alpha\right\rangle, \quad \left|\phi_\alpha\right\rangle = a^+_{i1} a^+_{i2} ... a^+_{iA} \left|0\right\rangle$$

Exact diagonalization: 10^{11} dimension Caurier et al. RMP77 427 (2005)
Monte Carlo shell model: 10^{24} dimension Togashi et al. PRL121 062501 (2018)

Excitation energies, EM transitions

The shell model is the method of choice for medium-mass nuclei: energies, deformation, electromagnetic transition rates...





Tsunoda et al. PRC89 031301 (2014)

Caurier et al. PRC75 054317 (2007)

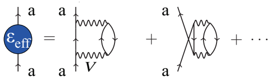
Shell model interaction

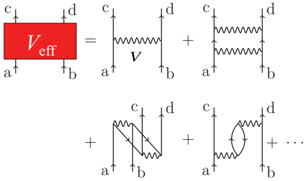
Effective interaction, with effects beyond valence space

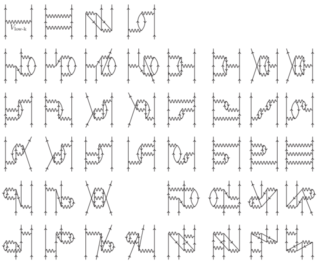
Hjorth-Jensen, Coraggio, Holt, Tsunoda...

$$H\,|\Psi\rangle = E\,|\Psi\rangle \rightarrow H_{eff}\,|\Psi\rangle_{eff} = E\,|\Psi\rangle_{eff}\,, \qquad H_{eff} = \varepsilon_{eff} + V_{eff}$$

Start with nucleon-nucleon interaction (Argonne, CD-Bonn...),
obtain H_eff based on many-body perturbation theory (to third order)







Problem: Nuclear properties given by H_{eff} do not reproduce data well

Phenomenological modifications: KB family

Agreement with data demands to modify “monopole" matrix elements which give information on single-particle degrees of freedom   Zuker, Poves...

$$\frac{\sum_J \langle ab|(2J+1)V^{JT}|ab\rangle}{\sum_J 2J+1}$$

Correlations (pairing, quadrupole) well described by *G*-matrix interaction

Philosophy: make minimal changes,
due to absence of 3N forces in initial Hamiltonian?

$$V^0_{\rm ff}({\rm KB1}) = V^0_{\rm ff}({\rm KB}) - 350\ {\rm keV}$$

$$V^1_{\rm ff}({\rm KB1}) = V^1_{\rm ff}({\rm KB}) - 110\ {\rm keV}$$

$$V^T_{\rm fr}({\rm KB1}) = V^T_{\rm fr}({\rm KB}) - (-)^T\ 300\ {\rm keV}$$

KB2: $W^{\Gamma}_{\rm ffff}$(KB1) – 300 keV for $\Gamma = 10, 30$, KB1 centroïds

KB3: $W^{21}_{\rm ffff}$(KB2) – 200 keV, KB1 centroïds kept.

$$V_{fp}^{T=1}({\rm KB3G}) = V_{fp}^{T=1}({\rm KB3}) - 50\ {\rm keV},$$

$$V_{fp}^{T=0}({\rm KB3G}) = V_{fp}^{T=0}({\rm KB3}) - 100\ {\rm keV},$$

$$V_{ff_{5/2}}^{T=1}({\rm KB3G}) = V_{ff_{5/2}}^{T=1}({\rm KB3}) - 100\ {\rm keV},$$

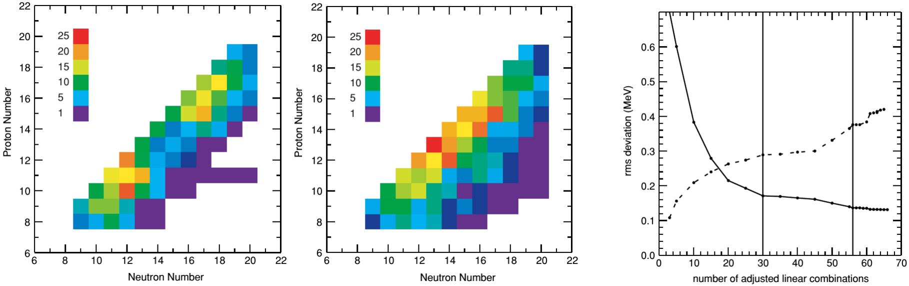
$$V_{ff_{5/2}}^{T=0}({\rm KB3G}) = V_{ff_{5/2}}^{T=0}({\rm KB3}) - 150\ {\rm keV},$$

$$V_{pp}^{T}({\rm KB3G}) = V_{pp}^{T}({\rm KB3}) + 400\ {\rm keV},$$

Alternatively, “improve" original interaction
by fitting to data all single-particle energies and two-body matrix elements

$$\langle a|\epsilon_a|a\rangle, \qquad \langle ab|V^{JT}|cd\rangle$$

Example:
USD (Wildenthal, 1984), USDA, and USDB (Brown, Richter, 2006) interactions

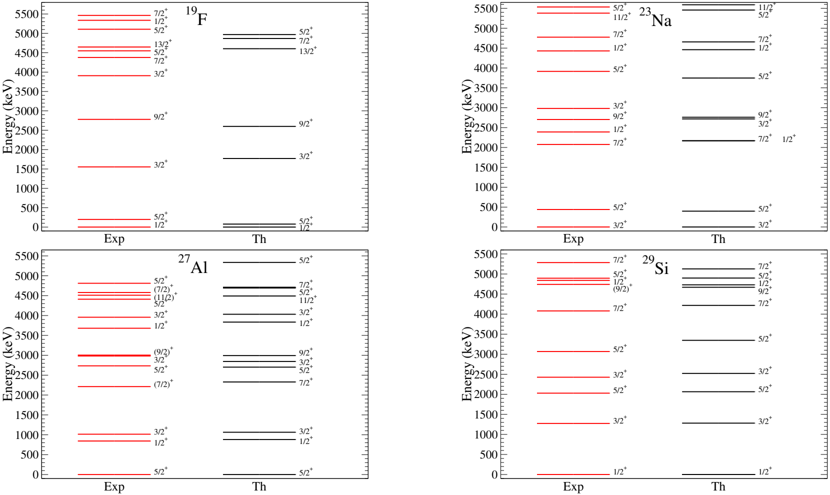


Brown, Richter PRC74 034315 (2006)

Fit linear combination of TBME's to minimize deviation with respect to data
Better constrained linear combinations (USDA) vs full fit (USDB)

Excitation spectra in *sd*-shell

Good agreement in *sd*-shell, $17 \leq A \leq 40$, universal interaction "USDB"



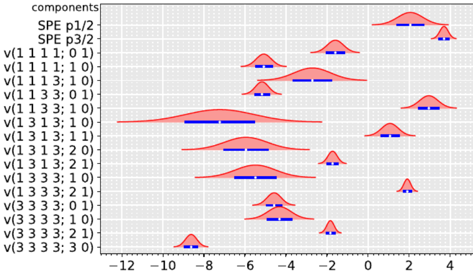
Klos, Menéndez, Gazit, Schwenk PRD88 083516 (2013)

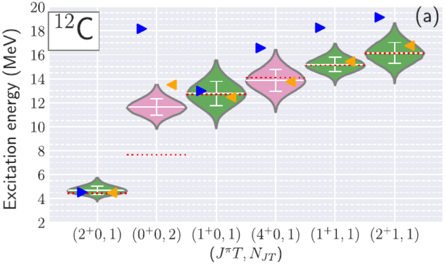
Theoretical uncertanties?

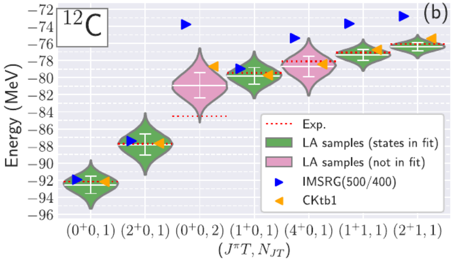
Fit all single-particle energies and two-body matrix elements, considering each as a probability distribution instead of a point

Example: p-shell

Yoshida et al. PRC98 061301 (2018)







No previous information on nucleon-nucleon interaction

Ab initio shell model interaction

Coupled Cluster method: operators (correlations) acting on reference impose no particle-hole excitations present in the reference state

Hagen, Papenbrock, Hjorth-Jensen...

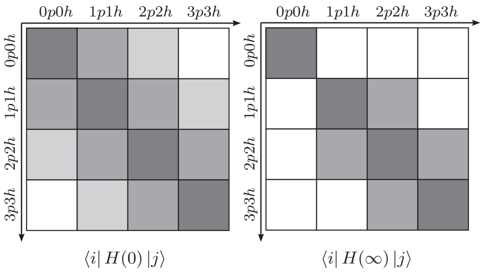
$$|\Psi\rangle = e^{-(T_1+T_2+T_3\cdots)}\,|\Phi\rangle$$

$$\text{with}\quad T_1 = \sum_{\alpha,\bar{\alpha}} t_{\alpha}^{\bar{\alpha}} \left\{ a^{\dagger}_{\bar{\alpha}}, a_{\alpha} \right\}\ ,\ T_2 = \sum_{\alpha\beta,\bar{\alpha}\bar{\beta}} t_{\alpha\beta}^{\bar{\alpha}\bar{\beta}} \left\{ a^{\dagger}_{\bar{\alpha}} a^{\dagger}_{\bar{\beta}}, a_{\alpha} a_{\beta} \right\}\ ,\cdots$$

$$\text{solve}\quad \left\langle \Phi_{\alpha}^{\bar{\alpha}} \right| e^{\sum T_i} H e^{-\sum T_i} \left|\Phi\right\rangle = 0\ ,\ \left\langle \Phi_{\alpha\beta}^{\bar{\alpha}\bar{\beta}} \right| e^{\sum T_i} H e^{-\sum T_i} \left|\Phi\right\rangle = 0$$

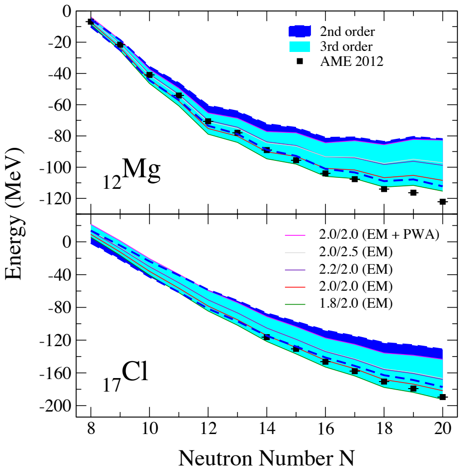
In-medium similarity renormalization group method: apply a similarity (unitary) transformation to decouple reference state from particle-hole excitations

Bogner, Schwenk, Hergert, Stroberg...



sd-shell nuclei: theoretical uncertainties

Valence-space shell model in *sd*-shell nuclei,
based on many-body perturbation theory



Simonis et al. PRC93 011302 (2016)

Explore the theoretical sensitivity:

Initial chiral Hamiltonian

RG evolution of NN, 3N forces

Convergence in MBPT

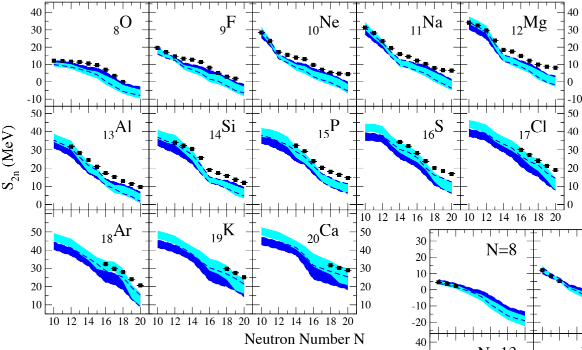
Use Hamiltonians with good nuclear saturation properties

Hebeler et al. PRC 83 031301 (2011)

Magnesium ground-state energies underbound, Chlorine good agreement to experiment

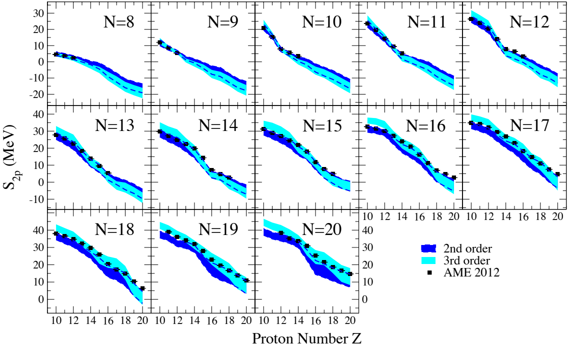
Uncertainties dominated by initial nuclear Hamiltonian

sd-shell: separation energies



Full sd-shell calculation with chiral NN+3N forces

Fit only to two, three and four-body systems

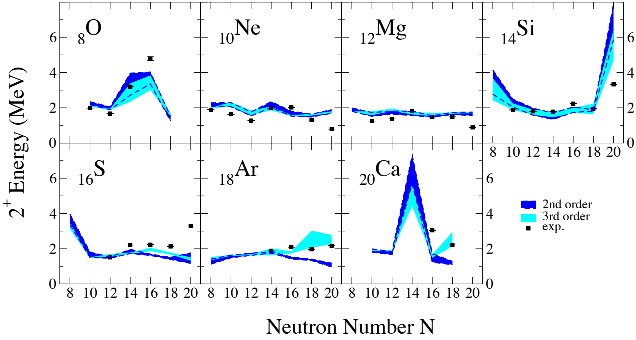


Uncertainly band includes SRG resolution scales and low-energy couplings

Typical uncertainties 5MeV, more in neutron-rich nuclei

2^+ energies in *sd*-shell nuclei

Energies of lowest 2^+ states less sensitive to the initial Hamiltonian



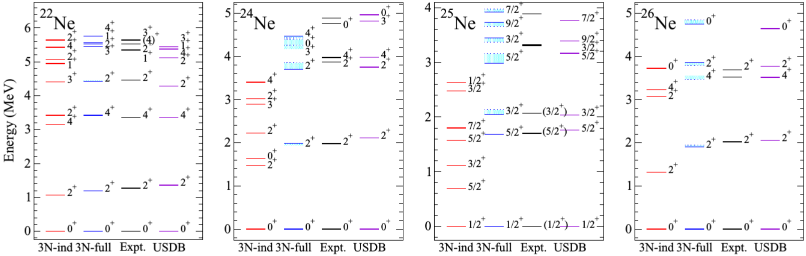
Simonis et al. PRC93 011302 (2016)

2^+ energies in *sd* shell reasonably well reproduced

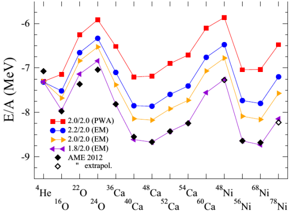
Typical uncertainties $\sim$ 500 keV, convergence important in magic nuclei

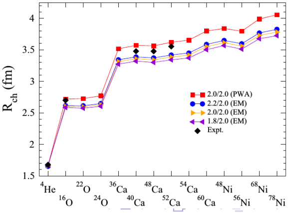
Island-of-inversion states in Ne, Mg, not reproduced in sd shell

Valence-space in-medium similarity renormalization group (VS-IMSRG) quality comparable to USD interactions Stroberg et al. PRC93 051301 (2016)



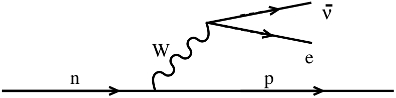
Sensitivity to nuclear Hamiltonian Simonis et al. PRC96 014303 (2017)





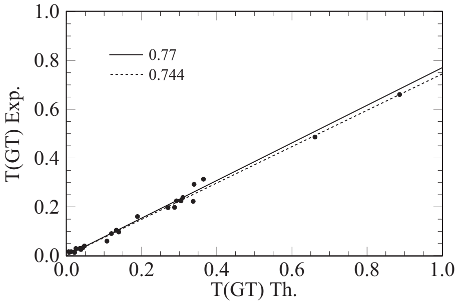
β Gamow-Teller transitions: quenching

Single β decays well described by nuclear structure (shell model)



$$\langle F | \sum_i g_A^{\text{eff}} \, \boldsymbol{\sigma}_i \tau_i^- \, | I \rangle$$

$$g_A^{\text{eff}} = q g_A, \quad q \sim 0.7 - 0.8.$$



Martínez-Pinedo et al. PRC53 2602 (1996)

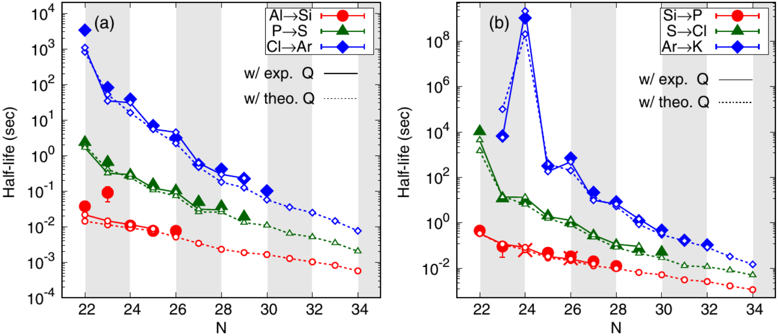
Theory needs to "quench" Gamow-Teller operator to reproduce Gamow-Teller lifetimes: problem in nuclear many-body wf or operator?

This puzzle has been the target of many theoretical efforts:

Arima, Rho, Towner, Bertsch and Hamamoto, Wildenthal and Brown...

β decay including forbidden matrix elements

Forbidden nuclear matrix elements in *sd* – *pf* shell



Yoshida et al. PRC97 054321 (2018)

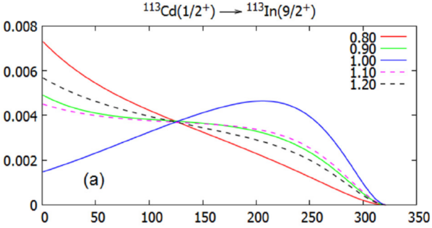
Shell model deficiencies
corrected with different factor
(different "quenching")
for each type of forbidden operator

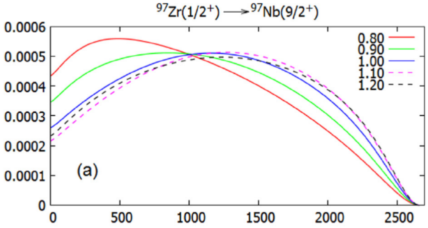
	GT	FF				
		M_0^S	M_0^T	x	u	z
present	0.74	1	1.7	1	1	0.55
Ref. [41]		1.1	1.5	1	1	0.51

β decay spectrum

Forbidden β decays sometimes assume common “correction"
for all axial (g_A) nuclear matrix elements

$$C(w_e) = g_V^2 C_V(w_e) + g_V\, g_A C_{AV}(w_e) + g_A^2 C_V(w_e)$$





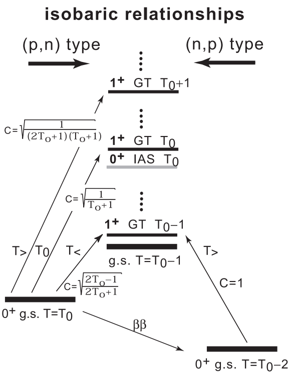
Kostensalo, Haraanen, Suhonen PRC95 044313 (2017)

May not be sufficient when two “axial" nuclear matrix elements compete

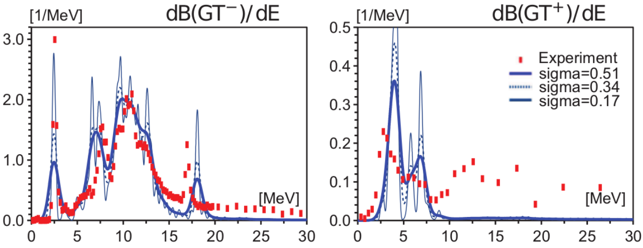
Important to simultaneously describe lifetime and β shape

Gamow-Teller strength distributions, $\beta\beta$ decay

Gamow-Teller (GT) distributions well described by theory (quenched)



Freckers et al.
NPA916 219 (2013)



Iwata et al. JPSCP 6 03057 (2015)

$$\langle 1_f^+ | \sum_i [\boldsymbol{\sigma}_i \tau_i^{\pm}]^{\rm eff} | 0_{\rm gs}^+ \rangle , \quad [\boldsymbol{\sigma}_i \tau^{\pm}]^{\rm eff} \approx 0.7 \boldsymbol{\sigma}_i \tau^{\pm}$$

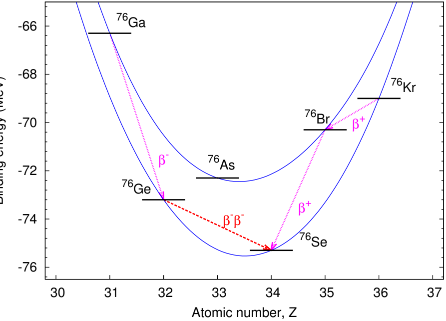
$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \boldsymbol{\sigma}_n \tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \boldsymbol{\sigma}_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

GT strengths combined related to $2\nu\beta\beta$ decay, but relative phase unknown

Nuclear $\beta\beta$ decay

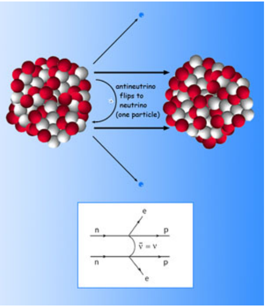
$\beta\beta$ decay second order process observable if β-decay is energetically forbidden or hindered by large ΔJ

Neutrinoless double-beta decay ($0\nu\beta\beta$):
Lepton-number violation, Majorana nature of neutrinos



$Q_{\beta\beta} > 2$MeV

^{48}Ca $\rightarrow$ ^{48}Ti
^{76}Ge $\rightarrow$ ^{76}Se
^{82}Se $\rightarrow$ ^{82}Kr
^{96}Zr $\rightarrow$ ^{96}Mo
^{100}Mo $\rightarrow$ ^{100}Ru
^{110}Pd $\rightarrow$ ^{110}Cd
^{116}Cd $\rightarrow$ ^{116}Sn
^{124}Sn $\rightarrow$ ^{124}Te
^{130}Te $\rightarrow$ ^{130}Xe
^{136}Xe $\rightarrow$ ^{136}Ba
^{150}Nd $\rightarrow$ ^{150}Sm



Comparison of predicted $2\nu\beta\beta$ decay vs data

Shell model reproduce $2\nu\beta\beta$ data including “quenching" but matrix elements vary with nuclear interaction

Shell model prediction previous to ^{48}Ca measurement!

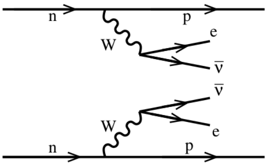
Caurier, Poves Zuker

PLB252 13(1990)

Table 2
The ISM predictions for the matrix element of several 2ν double beta decays (in MeV^{-1}). See text for the definitions of the valence spaces and interactions.

	$M^{2\nu}$(exp)	q	$M^{2\nu}$(th)	INT
^{48}Ca → ^{48}Ti	0.047 ± 0.003	0.74	0.047	kb3
^{48}Ca → ^{48}Ti	0.047 ± 0.003	0.74	0.048	kb3g
^{48}Ca → ^{48}Ti	0.047 ± 0.003	0.74	0.065	gxpf1
^{76}Ge → ^{76}Se	0.140 ± 0.005	0.60	0.116	gcn28:50
^{76}Ge → ^{76}Se	0.140 ± 0.005	0.60	0.120	jun45
^{82}Se → ^{82}Kr	0.098 ± 0.004	0.60	0.126	gcn28:50
^{82}Se → ^{82}Kr	0.098 ± 0.004	0.60	0.124	jun45
^{128}Te → ^{128}Xe	0.049 ± 0.006	0.57	0.059	gcn50:82
^{130}Te → ^{130}Xe	0.034 ± 0.003	0.57	0.043	gcn50:82
^{136}Xe → ^{136}Ba	0.019 ± 0.002	0.45	0.025	gcn50:82

Caurier, Nowacki, Poves PLB711 62(2012)

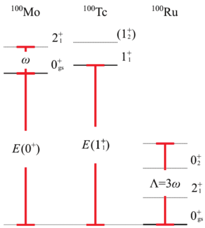


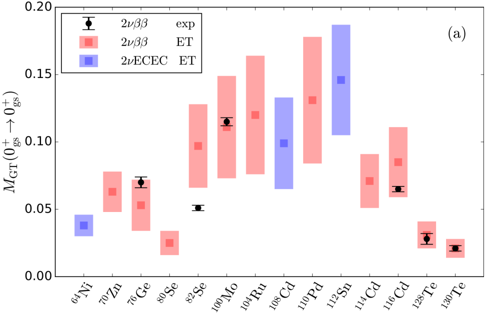
$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

Effective theory of β decay of heavy nuclei

Effective theory (ET) for $\beta\beta$ decay:
spherical core coupled to one nucleon

Couplings adjusted to experimental data,
uncertainty given by effective theory
(breakdown scale, systematic expansion)



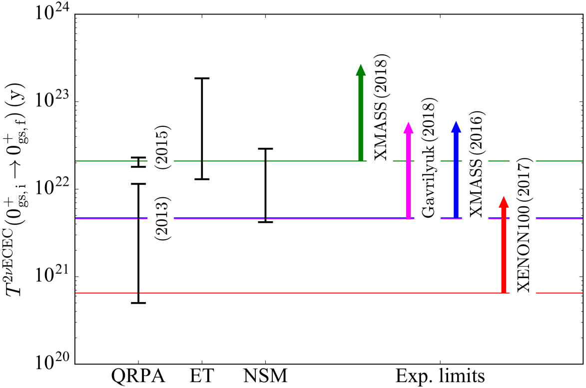


Use β decay data
to predict $2\nu\beta\beta$ decay
Good agreement, large error
(leading-order in ET)

Coello-Pérez, JM, Schwenk
PRC 98, 045501 (2018)

Two-neutrino $\beta\beta$ decay and ECEC

Uncertainties in predicted 2νECEC:
largely dominated by “quenching" uncertainty (taken from similar nuclei)



Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk
arXiv:1809.04443

^{124}Xe 2νECEC pursued by XMASS, XENON collaborations
Shell model, QRPA and Effective theory (ET) predictions
suggest experimental detection possible in near future

Subleading matrix elements in $\beta\beta$ decay

Precise $2\nu\beta\beta$ half-life:

$$\left(T^{2\nu}_{1/2}\right)^{-1} \simeq \left(g_A^{\text{eff}}\right)^4 \left| \left(M^{2\nu}_{GT}\right)^2 G^{2\nu}_0 + \left(M^{2\nu}_{GT-3}\right)^2 G^{2\nu}_2 + ... \right|$$

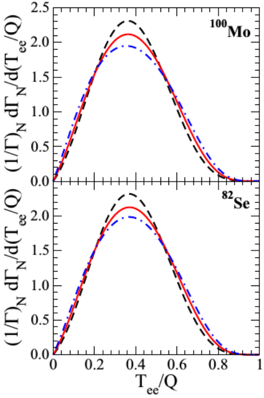
with

$$M^{2\nu}_{GT} = \sum_j \frac{\langle 0^+_f | \sum_l \boldsymbol{\sigma}_l \tau^-_l | 1^+_j \rangle \langle 1^+_j | \sum_l \boldsymbol{\sigma}_l \tau^-_l | 0^+_i \rangle}{\Delta}, \quad M^{2\nu}_{GT3} = \sum_j \frac{4 \langle 0^+_f | \sum_l \boldsymbol{\sigma}_l \tau^-_l | 1^+_j \rangle \langle 1^+_j | \sum_l \boldsymbol{\sigma}_l \tau^-_l | 0^+_i \rangle}{\Delta^3}$$

Summed electron differential decay rate:

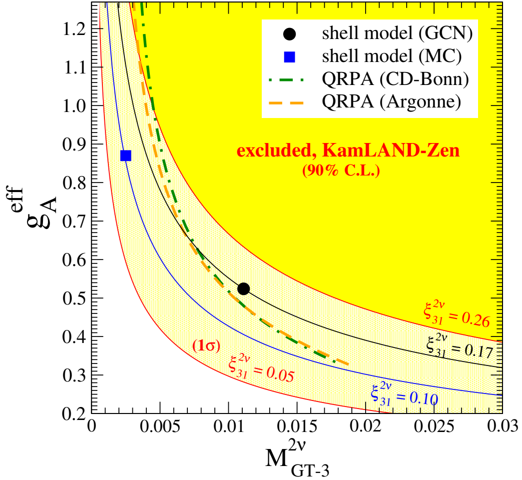
$$\frac{d\Gamma^{\beta\beta}}{dT_{ee}} \sim \frac{dG_0}{dT_{ee}} + \frac{M^{2\nu}_{GT}}{M^{2\nu}_{GT-3}} \frac{dG_2}{dT_{ee}}$$

Experimental sensitivity
to the ratio of nuclear matrix elements
$\xi^{2\nu}_{31} = M^{2\nu}_{GT-3} / M^{2\nu}_{GT}$



Ratio of leading/subleading $\beta\beta$ matrix elements

Shape of $\beta\beta$ spectrum constrains matrix element ratio $\xi_{31}^{2\nu} = M_{GT-3}^{2\nu}/M_{GT}^{2\nu}$



Theory deficiencies in $M_{GT}^{2\nu}$ fixed adjusting g_A (“queching")

$\xi_{31}^{2\nu}$ measurement test theoretical models

Theory-experiment work with KamLAND-Zen collaboration

Theory: JM, Šimkovic, Dvornicky

Shell model $\xi_{31}^{2\nu}$ predictions cosistent with 90% C.L. limit

Gando et al. arXiv:1901.03871

Summary

Nuclear shell model traditionally used with phenomenological interactions, recently ab initio shell model Hamiltonians available

- Phenomenological interactions:
  good description of nuclear structure,
  prevents uncertainty quantification
- Predictive power in β, $\beta\beta$ decays
  if the deficiencies ("quenching") are known
- Theoretical uncertainties
  can only be addressed with
  ab initio shell model Hamiltonians