Beta decay with the valence space IMSRG

What works, what doesn't, and what kind of precision can we expect?

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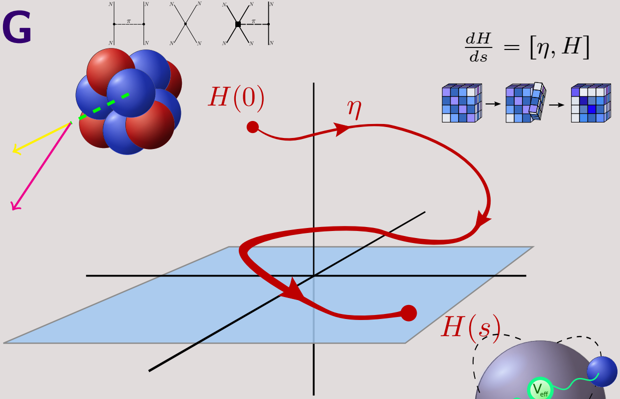
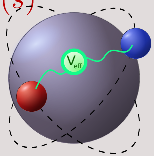
University of Washington

"Precise beta decay calculations for searches for new physics
ECT*, Trento
April 11, 2019



Based on work done with Matt Martin, Kyle Leach, and Jason Holt



The method

- Valence-space in-medium similarity renormalization group (VS-IMSRG)

Applications to β decay

- Superallowed Fermi decay
- Isobaric Mass Multiplet Equation
- Isospin breaking correction δ_C

Why is ab-initio shell model attractive for treating super-allowed $0^+ \to 0^+$?

- Relevent nuclei are mostly open-shell & medium mass
- We have lots of success/experience with phenomenologcial shell model
- Ab initio shell model interactions come with well-defined spatial wave functions
- More direct connection to the EFT degrees of freedom
- (Hopefully) more rigorous error estimation

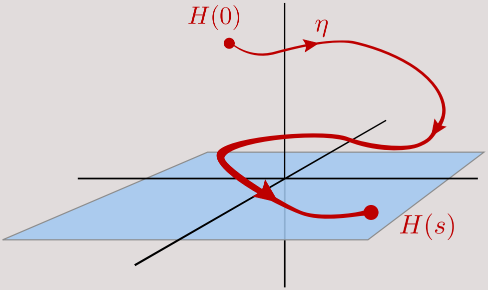
$$H = E_0 + \sum_{ij} f_{ij}\{a_i^\dagger a_j\} + \frac{1}{4}\sum_{ijkl}\Gamma_{ijkl}\{a_i^\dagger a_j^\dagger a_l a_k\} + \frac{1}{36}\sum_{\substack{ijk\\lmn}} W_{ijklmn}\{a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l\}$$

Flowing Hamiltonian: $H(s) = U(s)HU^\dagger(s)$

Generator: $\eta(s) = \frac{dU}{ds}U^\dagger \equiv \frac{H^{\text{od}}(s)}{\Delta}$

Flow equation: $\frac{dH}{ds} = [\eta(s), H(s)]$

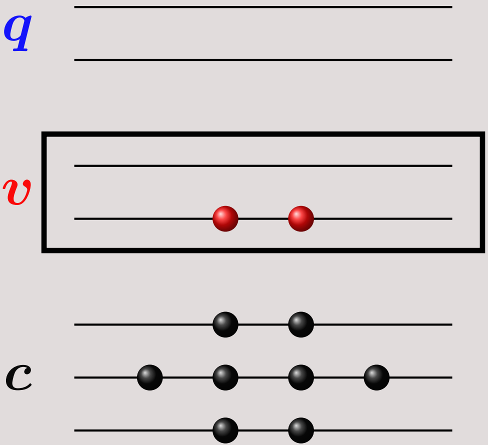
Fixed point: $H^{\text{od}}(s) = 0 \rightarrow \frac{dH}{ds} = 0$



IMSRG(2): Truncate operators at 2-body level

Wegner 1994; Głazek and Wilson 1993; Tsukiyama, Bogner, and Schwenk 2011; Hergert et al. 2016; Hergert et al. 2018

Valence Space In-Medium Similarity Renormalization Group



Define H^{od} as all terms connecting valence configurations to non-valence configurations.

$$
\begin{aligned}
H^{\mathrm{od}} \equiv\ & \langle \boldsymbol{v}|H|\boldsymbol{c}\rangle + \langle \boldsymbol{q}|H|\boldsymbol{c}\rangle + \langle \boldsymbol{q}|H|\boldsymbol{v}\rangle \\
& + \langle \boldsymbol{vv}|H|\boldsymbol{cc}\rangle + \langle \boldsymbol{qv}|H|\boldsymbol{cc}\rangle + \langle \boldsymbol{qq}|H|\boldsymbol{cc}\rangle \\
& + \langle \boldsymbol{vv}|H|\boldsymbol{vc}\rangle + \langle \boldsymbol{qv}|H|\boldsymbol{vc}\rangle + \langle \boldsymbol{qq}|H|\boldsymbol{vc}\rangle \\
& + \langle \boldsymbol{qv}|H|\boldsymbol{vv}\rangle + \langle \boldsymbol{qq}|H|\boldsymbol{vv}\rangle
\end{aligned}
$$

When $H^{\mathrm{od}}(s) \to 0$,

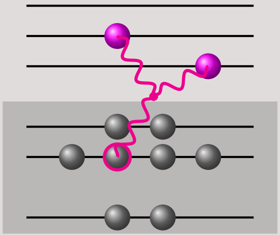
$$
\langle \mathbf{core}|H(s)|\mathbf{core}\rangle + \langle \boldsymbol{v}|H(s)|\boldsymbol{v}\rangle + \langle \boldsymbol{vv}|H(s)|\boldsymbol{vv}\rangle
$$

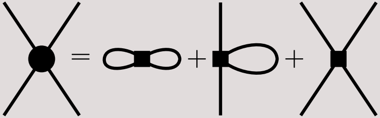
is a shell-model effective interaction.

Bogner et al. 2014; SRS et al. arXiv:1902.06154

(Ensemble) Normal Ordering

Minimize impact of neglected 3+ body terms by *normal ordering* all operators w.r.t finite reference $|\Phi\rangle$.

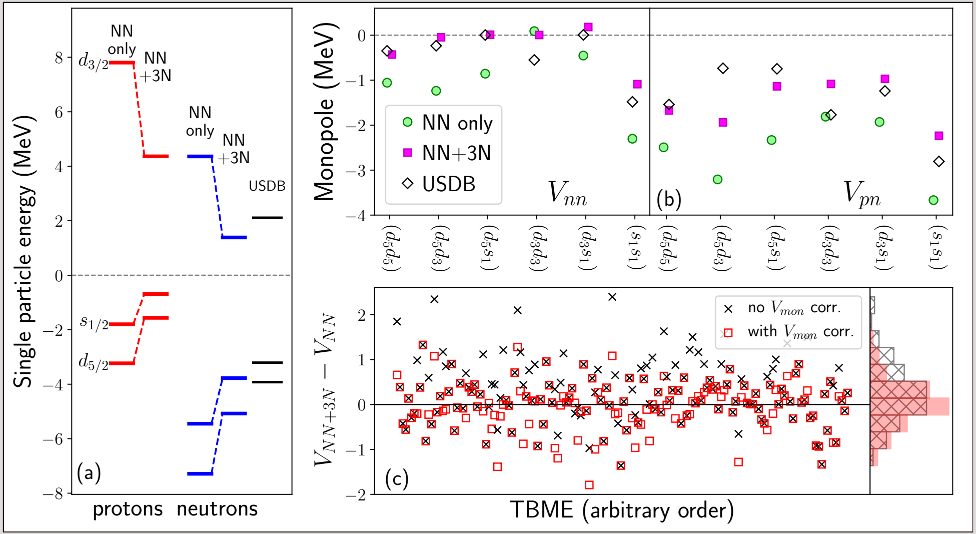




$$\langle\Phi|\{a^\dagger a^\dagger \ldots aa\}|\Phi\rangle = 0$$

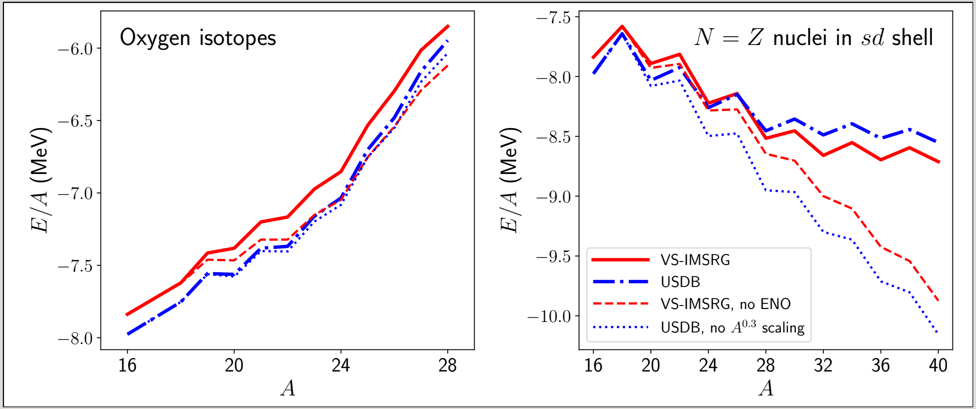
$$|\Psi\rangle \approx |\Phi\rangle \quad \Rightarrow \quad \langle\Psi|\{a^\dagger a^\dagger a^\dagger aaa\}|\Psi\rangle \approx 0$$

For open-shell systems, use equal filling approx (ensemble normal ordering) for reference.
N.B. This rewriting is exact. Any choice yields exact answer when A-body terms are kept.

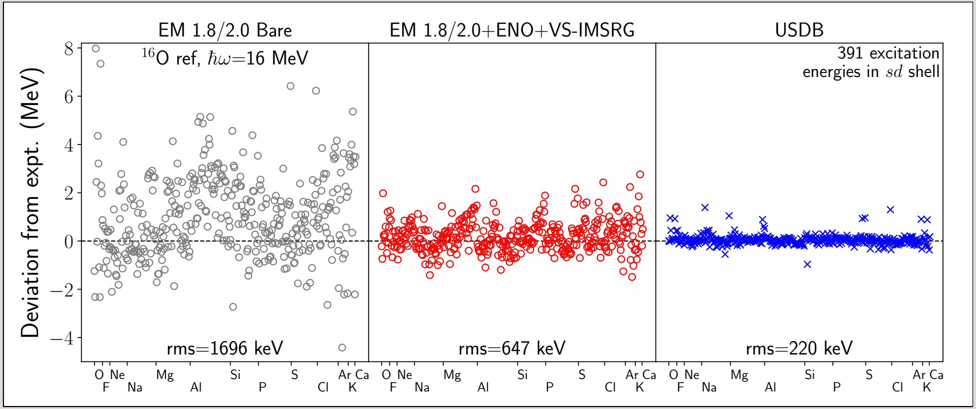


SRS, Hergert, Bogner, and Holt arXiv:1902.06154

Phenomenological 2-body matrix elements are scaled as $A^{1./3}$



SRS, Hergert, Bogner, and Holt arXiv:1902.06154



SRS, Hergert, Bogner, and Holt arXiv:1902.06154

Consistent transformation of observables

$$H(s) = U(s)HU^\dagger(s) \quad \Rightarrow \quad \mathcal{O}(s) = U(s)\mathcal{O}U^\dagger(s)$$

$$\frac{dH}{ds} = [\eta(s), H(s)] \quad \Rightarrow \quad \frac{d\mathcal{O}}{ds} = [\eta(s), \mathcal{O}(s)]$$

Morris, Parzuchowski, and Bogner 2015; Parzuchowski et al. 2017

$$H(s) = U(s)HU^{\dagger}(s) \quad \Rightarrow \quad \mathcal{O}(s) = U(s)\mathcal{O}U^{\dagger}(s)$$

$$\frac{dH}{ds} = [\eta(s), H(s)] \quad \Rightarrow \quad \frac{d\mathcal{O}}{ds} = [\eta(s), \mathcal{O}(s)]$$

Magnus formulation:

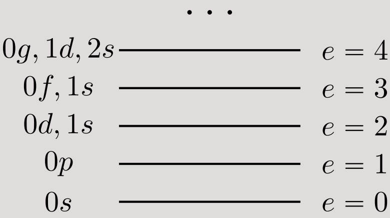
$$U(s) \equiv e^{\Omega(s)} \quad \Rightarrow \quad \mathcal{O}(s) = e^{\Omega(s)}\mathcal{O}e^{-\Omega(s)}$$
$$= \mathcal{O} + [\Omega(s), \mathcal{O}] + \tfrac{1}{2}[\Omega(s), [\Omega(s), \mathcal{O}]] + \dots$$

Flow equation for $\Omega(s)$: $$\frac{d\Omega}{ds} = \eta(s) + \tfrac{1}{2}[\eta(s), \Omega(s)] + \dots$$

Morris, Parzuchowski, and Bogner 2015; Parzuchowski et al. 2017

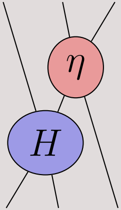
Two approximations:

1. Truncation of single particle basis: $2n + \ell \leqslant e_{max} \sim 14$

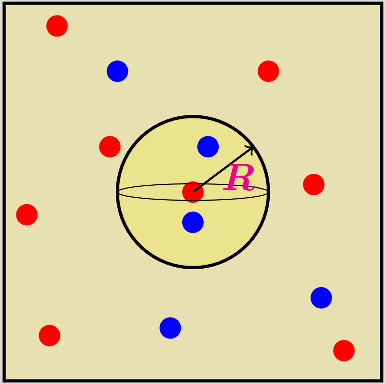


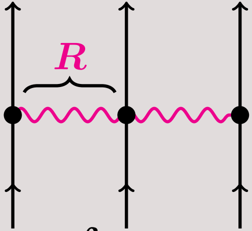
2. Truncation of 3+ -body operators

$$[\eta_{2b}, H_{2b}] \sim$$



Cluster hierarchy $H_{2b} > H_{3b} > H_{4b} \ldots$ justified if $\rho R^3 \ll 1$





$$\rho \sim 0.16\ \mathrm{fm}^{-3}$$

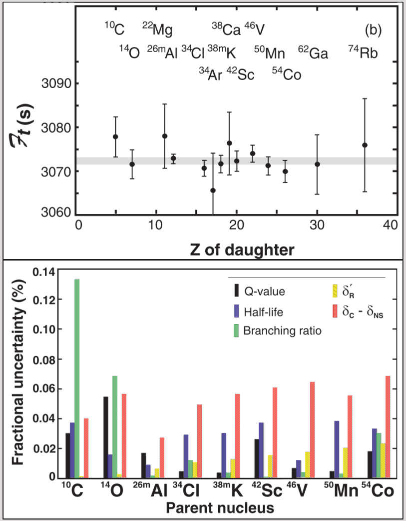
$$\Rightarrow R \ll 1.1\ \mathrm{fm}$$

$$\langle H_{3b} \rangle \sim \int d^3r_1 d^3r_2 d^3r_3\ |\psi_1|^2 |\psi_2|^2 \underbrace{|\psi_3|^2}_{\rho} \underbrace{V(r_{12}, r_{13})}_{\sim V_0 \theta(R-r)}$$

$$\sim \left(\tfrac{4\pi}{3} R^3 \rho\right)^2 V_0 \int d^3r_1 |\psi_1|^2$$

Rules of thumb:

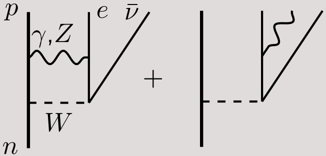
- So long as the IMSRG is integrating out **short-distance** interactions, the cluster hierarchy is maintained.
- Elimination of **long-distance** physics (collective rotational/vibrational modes) will induce unsuppressed many-body forces.
- Mean-field type effects will typically be captured, as will correlations in the valence space. (More on this later).



$$\mathcal{F}t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C)$$

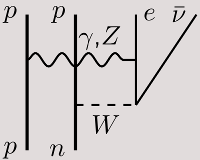
$$|M_F|^2 = |M_F^0|^2(1 - \delta_C)$$ (isospin breaking)

$\delta'_R, \Delta_R \sim$



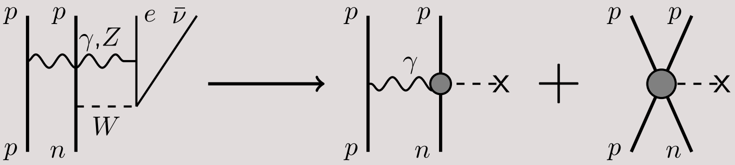
(structure-independent radiative correction)

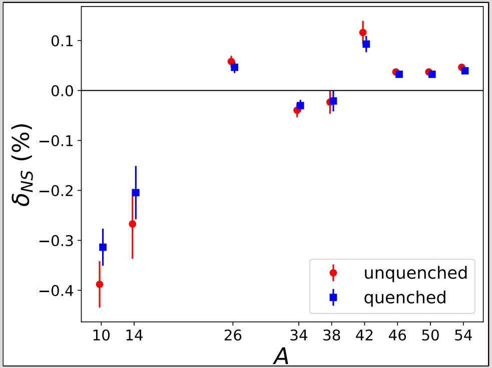
$\delta_{NS} \sim$



(structure-dependent radiative correction)

Towner and Hardy 2010

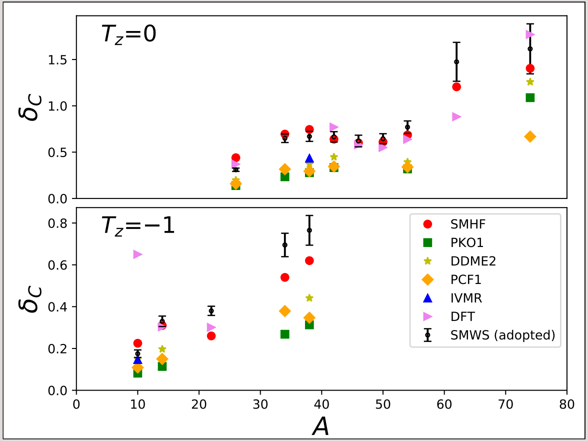




- Weak axial current contributes to $\Delta J = 0$ decay.
- Issues with g_A quenching?

Towner 1994

Towner and Hardy split it up: $\delta_C = \underbrace{\delta_{C1}}_{\text{configuration mixing}} + \underbrace{\delta_{C2}}_{\text{wave function mismatch}}$

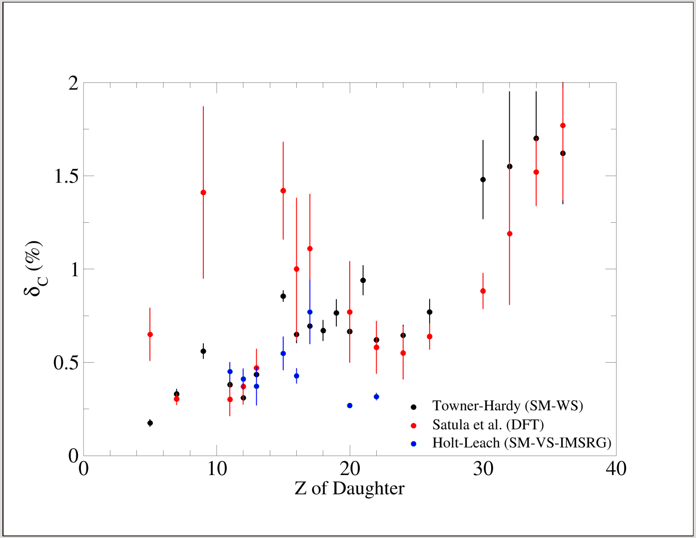


Hardy and Towner 2015

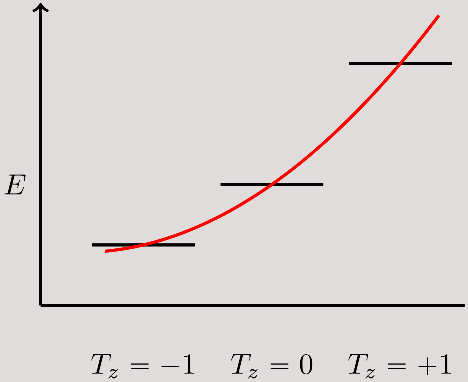
VS-IMSRG:

$$\delta_C = \left\{ H_{pp}(s) \neq H_{nn}(s) \neq H_{pn}(s) \right\} + \left\{ \tau(s) = U(s)\tau U^\dagger(s) \right\} + \left\{ \langle \phi_p^{HF} | \tau | \phi_n^{HF} \rangle \neq 1 \right\}$$

(The separation is not totally clean.)



K. Leach, CIPANP 2018 conference



$$E(T_z) = a + bT_z + cT_z^2$$

Connection to the δ_C correction:

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}\tau_\pm|\Psi_0\rangle + |\delta\Psi_\pm\rangle$$

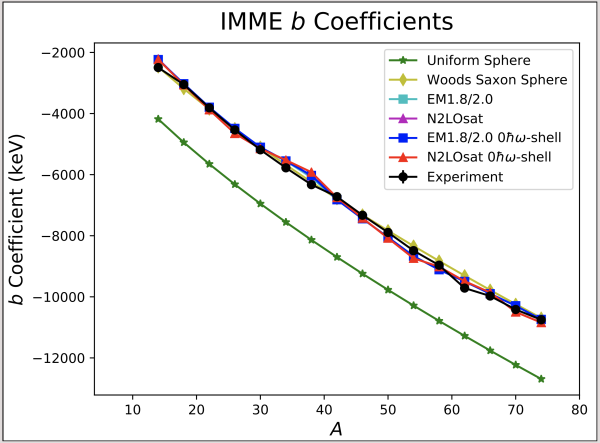
$$\delta_C =\sim \langle\delta\Psi_\pm|\tau_\pm|\Psi_0\rangle$$

$$b \sim \langle\Psi_0|\tau_- H\tau_+ - \tau_+ H\tau_-]|\Psi_0\rangle + \langle\delta\Psi_+|\tau_+ H|\Psi_0\rangle$$

$$c \sim \langle\Psi_0|\tau_- H\tau_+ + \tau_+ H\tau_-|\Psi_0\rangle + \langle\delta\Psi_+|\tau_+ H|\delta\Psi_0\rangle$$

$$E(T_z) = a + bT_z + +cT_z^2$$

$J^{\pi}{=}0^{+}$, $T{=}1$ multiplets with $14{\leqslant}A{\leqslant}74$



M. Martin M.S. Thesis, Colorado School of Mines (2019)

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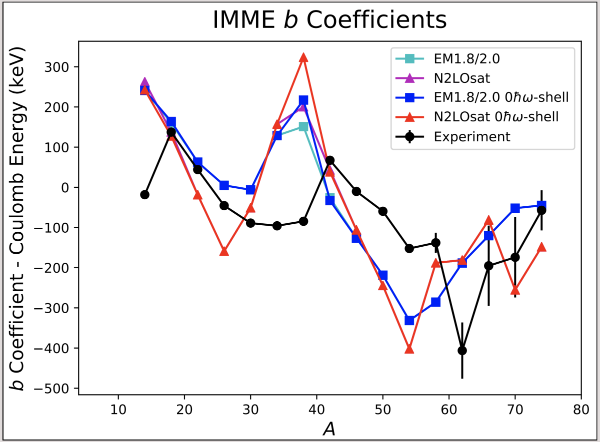

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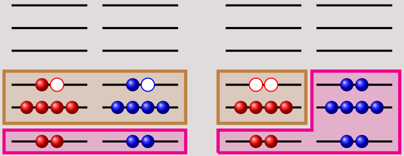
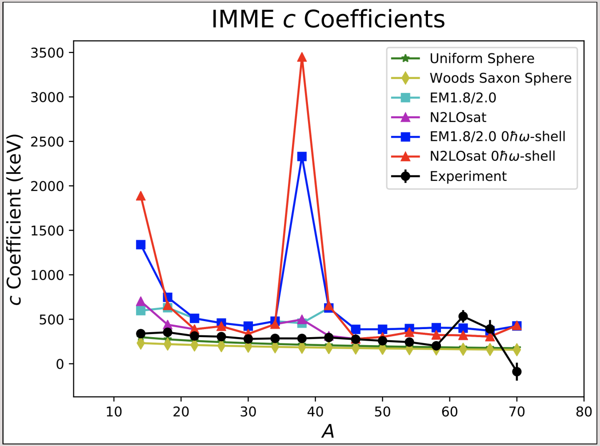
valence space definition:



^{14}N ^{14}O



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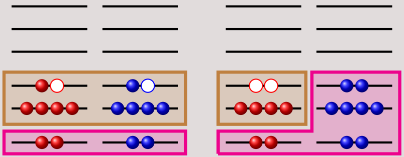

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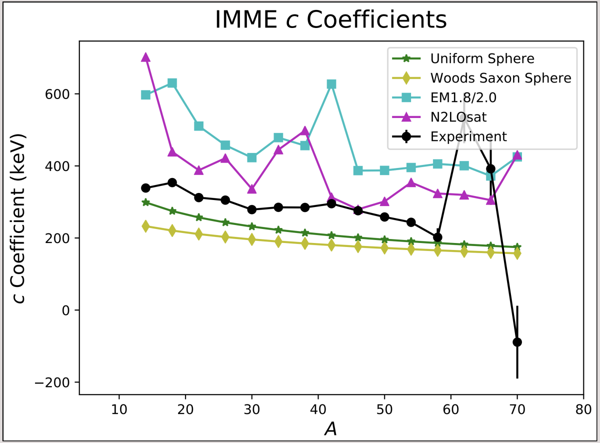
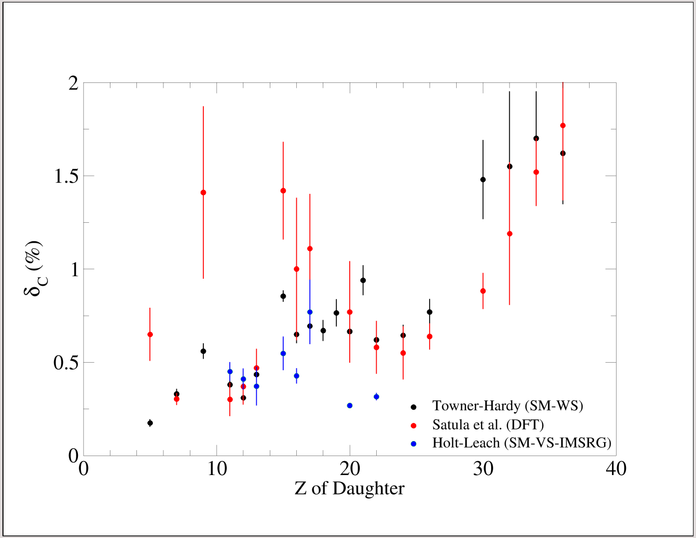


M. Martin M.S. Thesis, Colorado School of Mines (2019)

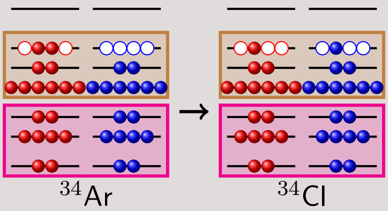
Isospin mixing correction δ_C



K. Leach, CIPANP 2018 conference

Some possible issues:

- Is it converged in the basis truncation?
- Ambiguity in choice of normal-ordering ref. $|\Phi\rangle$: Should we take the initial or final nucleus? Something else? Does it matter??



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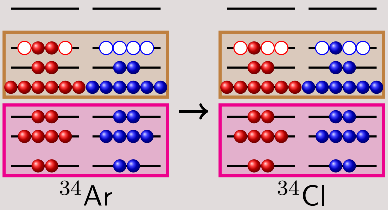
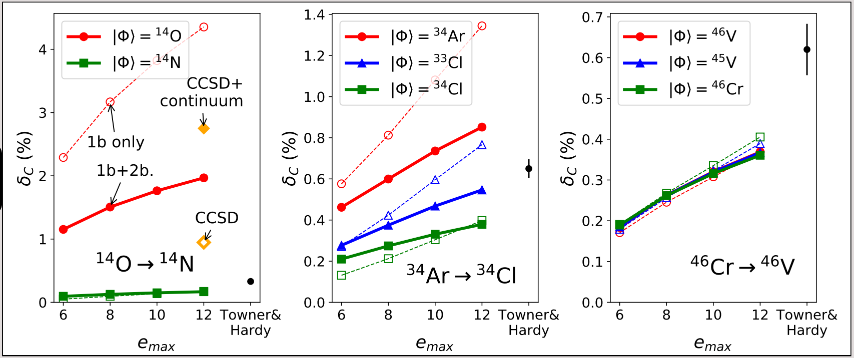
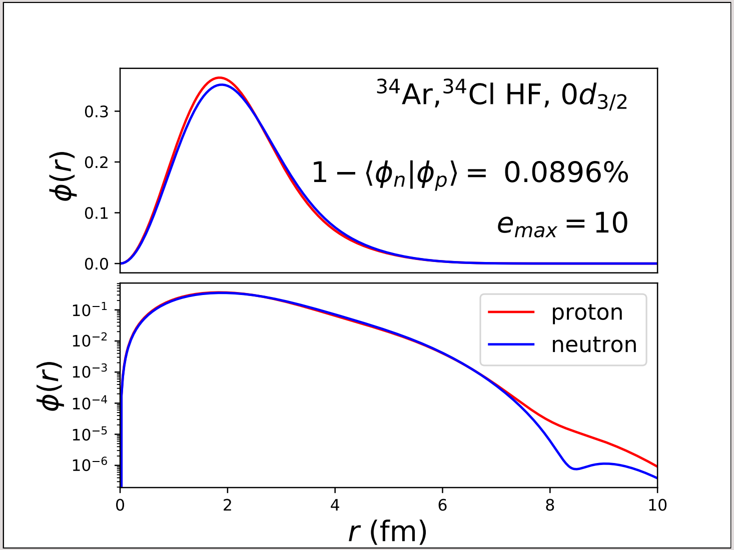


CC results from Gaute Hagen




^{34}Ar,^{34}Cl HF, $0d_{3/2}$
$1-\langle\phi_n|\phi_p\rangle =$ 0.0896%
$e_{max} = 10$
$\phi(r)$
0.0
0.1
0.2
0.3
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
proton
neutron
0
2
4
6
8
10
r (fm)

Summary

- IMSRG allows a non-perturbative derivation of shell model parameters
- Historical phenomenological adjustments can be understood as correcting for missing 3N physics
- Structure correction δ_C can be calculated
- Dependence of δ_C on interaction, basis size, reference needs careful study
- Also in the (nearish) future: δ_{NS}

Collaborators:

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NSCL/FRIB/MSU S. Bogner, H. Hergert

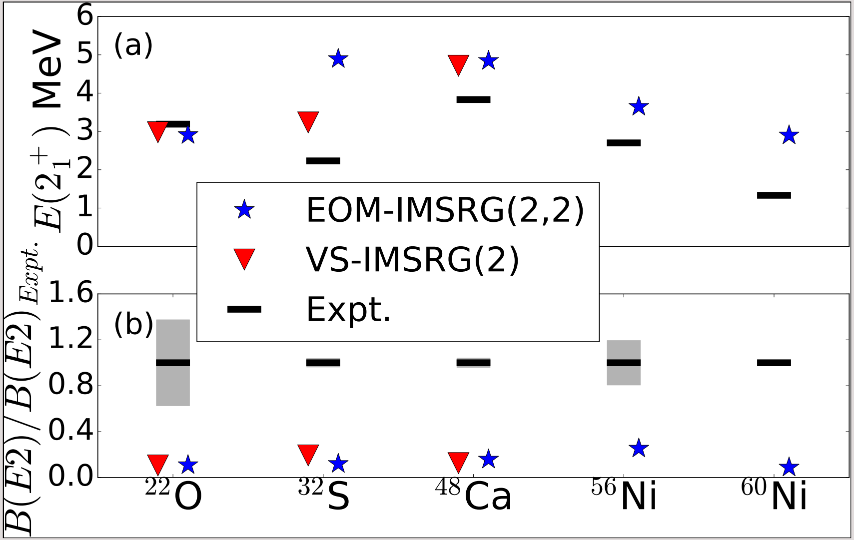
Colorado School of Mines K. Leach, M. Martin

TU Darmstadt A. Schwenk

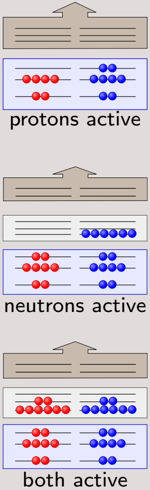
Mainz J. Simonis

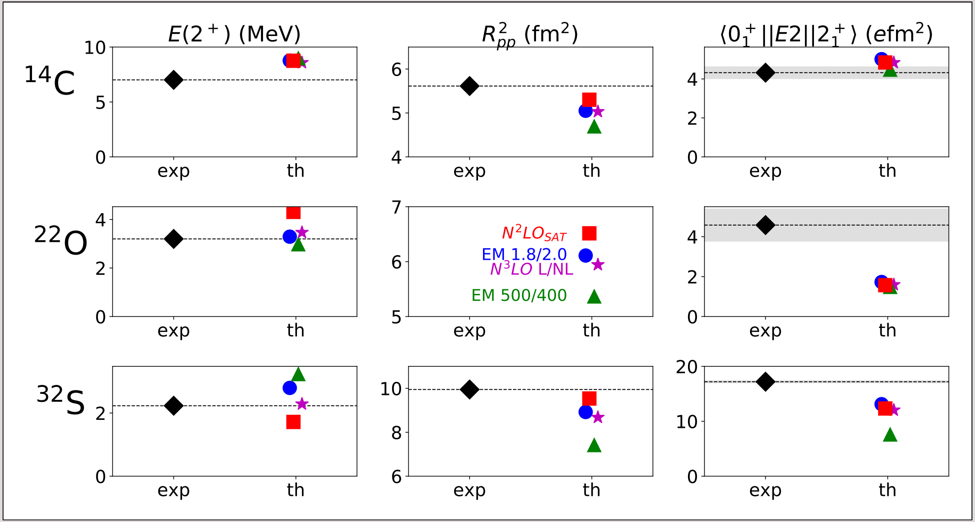
Additional figures

$E2$ transitions—severe underprediction of strength

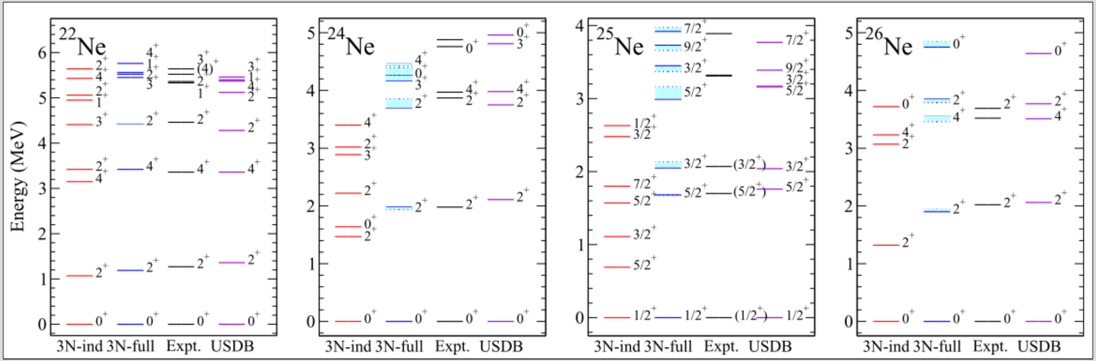


Parzuchowski et al. 2017



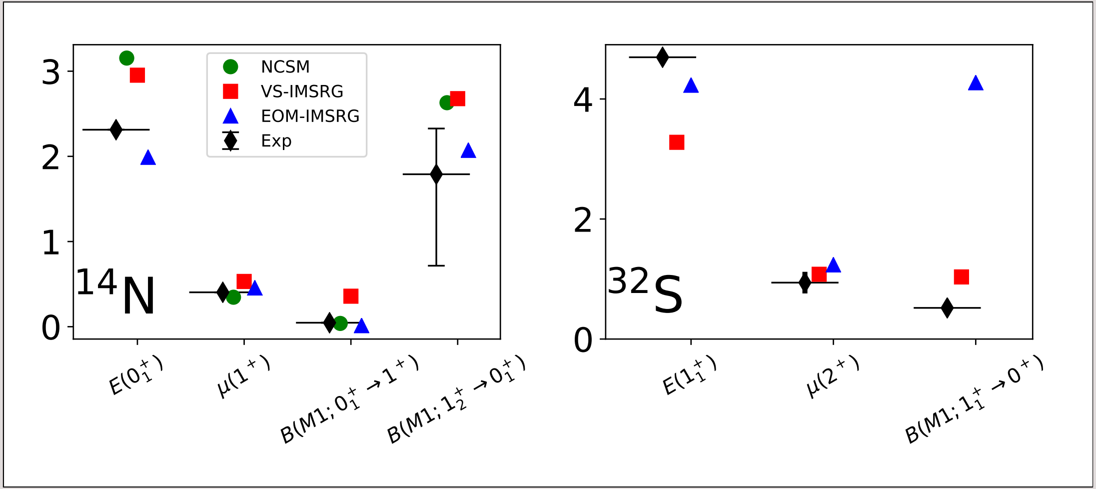


Entem and Machleidt 2003; Navrátil 2007; Gazit, Quaglioni, and Navrátil 2009; Ekström et al. 2015; Simonis et al. 2017



Stroberg et al. 2016

Magnetic dipole observables



Parzuchowski, Morris, and Bogner 2017

Bogner, S. K. et al. (2014). "Nonperturbative shell-model interactions from the in-medium similarity renormalization group". In: *Phys. Rev. Lett.* 113.14, p. 142501. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.113.142501. URL: http://link.aps.org/doi/10.1103/PhysRevLett.113.142501.

Ekström, A. et al. (2015). "Accurate nuclear radii and binding energies from a chiral interaction". In: *Phys. Rev. C* 91.5, p. 051301. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.91.051301. URL: http://link.aps.org/doi/10.1103/PhysRevC.91.051301.

Entem, D. R. and R. Machleidt (2003). "Accurate charge-dependent nucleon-nucleon potential at fourth order of chiral perturbation theory". In: *Phys. Rev. C* 68.4, p. 041001. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.68.041001. URL: http://link.aps.org/doi/10.1103/PhysRevC.68.041001.

Gazit, Doron, Sofia Quaglioni, and Petr Navrátil (2009). "Three-Nucleon Low-Energy Constants from the Consistency of Interactions and Currents in Chiral Effective Field Theory". In: *Phys. Rev. Lett.* 103.10, p. 102502. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.103.102502. URL: http://link.aps.org/doi/10.1103/PhysRevLett.103.102502.

Głazek, Stanisław D. and Kenneth G. Wilson (1993). "Renormalization of Hamiltonians". In: *Phys. Rev. D* 48.12, pp. 5863–5872. ISSN: 0556-2821. DOI: 10.1103/PhysRevD.48.5863. URL: http://link.aps.org/doi/10.1103/PhysRevD.48.5863.

Hardy, J. C. and I. S. Towner (2015). "Superallowed $0+$ $0+$ nuclear β decays: 2014 critical survey, with precise results for V u d and CKM unitarity". In: *Phys. Rev. C* 91.2, p. 025501. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.91.025501. URL: http://link.aps.org/doi/10.1103/PhysRevC.91.025501.

Hergert, H. et al. (2016). "The In-Medium Similarity Renormalization Group: A Novel Ab Initio Method for Nuclei". In: *Phys. Rep.* 621, pp. 165–222. ISSN: 03701573. DOI: 10.1016/j.physrep.2015.12.007. URL: http://arxiv.org/abs/1512.06956.

Hergert, Heiko et al. (2018). "Nuclear Structure from the In-Medium Similarity Renormalization Group". In: arXiv: 1805.09221. URL: http://arxiv.org/abs/1805.09221.

Morris, T. D., N. M. Parzuchowski, and S. K. Bogner (2015). “Magnus expansion and in-medium similarity renormalization group”. In: *Phys. Rev. C* 92.3, p. 034331. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.92.034331. URL: http://journals.aps.org.ezproxy.library.ubc.ca/prc/abstract/10.1103/PhysRevC.92.034331.

Navrátil, P. (2007). “Local three-nucleon interaction from chiral effective field theory”. In: *Few-Body Syst.* 41.3-4, pp. 117–140. ISSN: 0177-7963. DOI: 10.1007/s00601-007-0193-3. URL: http://link.springer.com/10.1007/s00601-007-0193-3.

Parzuchowski, N. M., T. D. Morris, and S. K. Bogner (2017). “Ab initio excited states from the in-medium similarity renormalization group”. In: *Phys. Rev. C* 95.4, p. 044304. ISSN: 2469-9985. DOI: 10.1103/PhysRevC.95.044304. URL: http://link.aps.org/doi/10.1103/PhysRevC.95.044304.

Parzuchowski, N. M. et al. (2017). “Ab initio electromagnetic observables with the in-medium similarity renormalization group”. In: *Phys. Rev. C* 96, p. 034324. ISSN: 2469-9985. DOI: 10.1103/PhysRevC.95.044304. arXiv: 1705.05511. URL: http://arxiv.org/abs/1705.05511.

Simonis, J. et al. (2017). “Saturation with chiral interactions and consequences for finite nuclei”. In: *Phys. Rev. C* 96.1, p. 014303. ISSN: 2469-9985. DOI: 10.1103/PhysRevC.96.014303. arXiv: 1704.02915. URL: http://arxiv.org/abs/1704.02915http://link.aps.org/doi/10.1103/PhysRevC.96.014303.

Stroberg, S. R. et al. (2016). “Ground and excited states of doubly open-shell nuclei from ab initio valence-space Hamiltonians”. In: *Phys. Rev. C* 93, 051301(R). DOI: 10.1103/PhysRevC.93.051301. URL: http://journals.aps.org/prc/abstract/10.1103/PhysRevC.93.051301.

Towner, I. S. (1994). “Quenching of spin operators in the calculation of radiative corrections for nuclear beta decay”. In: *Phys. Lett. B* 333.1-2, pp. 13–16. ISSN: 03702693. DOI: 10.1016/0370-2693(94)91000-6.

Towner, I S and J C Hardy (2010). “The evaluation of Vud and its impact on the unitarity of the CabibboKobayashiMaskawa quark-mixing matrix”. en. In: *Reports Prog. Phys.* 73.4, p. 046301. ISSN: 0034-4885. DOI: 10.1088/0034-4885/73/4/046301. URL: http://iopscience.iop.org/article/10.1088/0034-4885/73/4/046301.

Tsukiyama, K., S. K. Bogner, and A. Schwenk (2011). “In-Medium Similarity Renormalization Group For Nuclei”. In: *Phys. Rev. Lett.* 106.22, p. 222502. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.106.222502. URL: http://link.aps.org/doi/10.1103/PhysRevLett.106.222502.

Wegner, Franz (1994). “Flow-equations for Hamiltonians”. In: *Ann. der Physic* 3, pp. 77–91.