An EFT approach to the radiative corrections

Shung-Ichi Ando

Sunmoon University, Asan, Republic of Korea

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Outline

- Introduction:
  Radiative corrections for hadrons (and nuclei), *i.e.*, QED + QCD (+ Weak current as external fields)
- Radiative corrections of neutron beta decay in EFT
- Results and discussion

1. Introduction

- A photon, a massless particle, is coupled to every electric charge.
- The coupling is weak $\sim \alpha/(2\pi) \simeq 1.16 \times 10^{-3} \sim 10^{-3}$, and its effect is treated perturbately.
- UV and IR divergeces appear from photon loop diagrams.
- UV divergences are renormalized by mass and electric charge while IR divergences are exactly channeled with those from Bremsstrahlung diagrams.
- Those are well known as QED, and there are many good textbooks.

Photons in EFTs

- EFT for hardons; ChPT, a low energy EFT of QCD, $SU(2) \times SU(2)$ symmerty
- Isospin symmtery in SU(2) sector with a photon as an external vector field
- Explicite photon degrees of freedom in ChPT (with isospin symmetry breaking terms)
  - Muller and Meissner, NPB556, 265 (1999).
- Radiative corrections of neutron beta decay, $n \to p + e^- + \bar{\nu}_e$,
  $\alpha/(2\pi) \sim 10^{-3}$ while
  $\alpha/(2\pi)\ln(m_W/m_e) \simeq 0.0139 \sim 10^{-2}$
- Radiative corrections of nuclear beta decay may become larger, $\alpha/(2\pi) \to Z\alpha/(2\pi)$?

2. RCs for beta decay

- Many works about the radiative corrections (RCs) have already be done:
  - Hardy and Towner for $0^+ \rightarrow 0^+$ beta decay
  - Sirlin and Marciano for RCs
  - Garcia and Maya for RCs to the correlation coefficients of NBD
  - Kuryolv, Ramsey-Musolf, and Vogel for RCs for ν-d reactions
  - Kubota and Fukugita for RCs for ν-d reactions
  - and many others..

- Effective Field Theories (EFTs)
  - Model independent approach
  - Separation scale
  - Counting rules
  - Parameters should be fixed by experiments

EFT for neturon beta decay

- Neutron beta decay,

$$n \to p + e^- + \bar{\nu}_e \,,$$

- Typical momentum of the process

$$\bar{Q} = m_n - m_p - m_e \simeq 0.782 \text{ MeV},$$

- Separation (large) scales: nucleon mass, m_N, and pion mass, m_π,
- Expansion parameters

$$\frac{\alpha}{2\pi} \sim 10^{-3}, \quad \frac{\bar{Q}}{m_N} \simeq 0.8 \times 10^{-3} \sim 10^{-3}$$

Specifications

- Gauge symmetry,
- Contact weak interaction between lepton current and nucleon $V - A$ current,
- Heavy baryon formalizm with isospin symmtery breaking terms and without pions
- Dimentional regularization for UV and IR diverences using Lorentz gauge,
- Up to NLO, $\alpha/(2\pi)$ and $1/m_N$ corrections

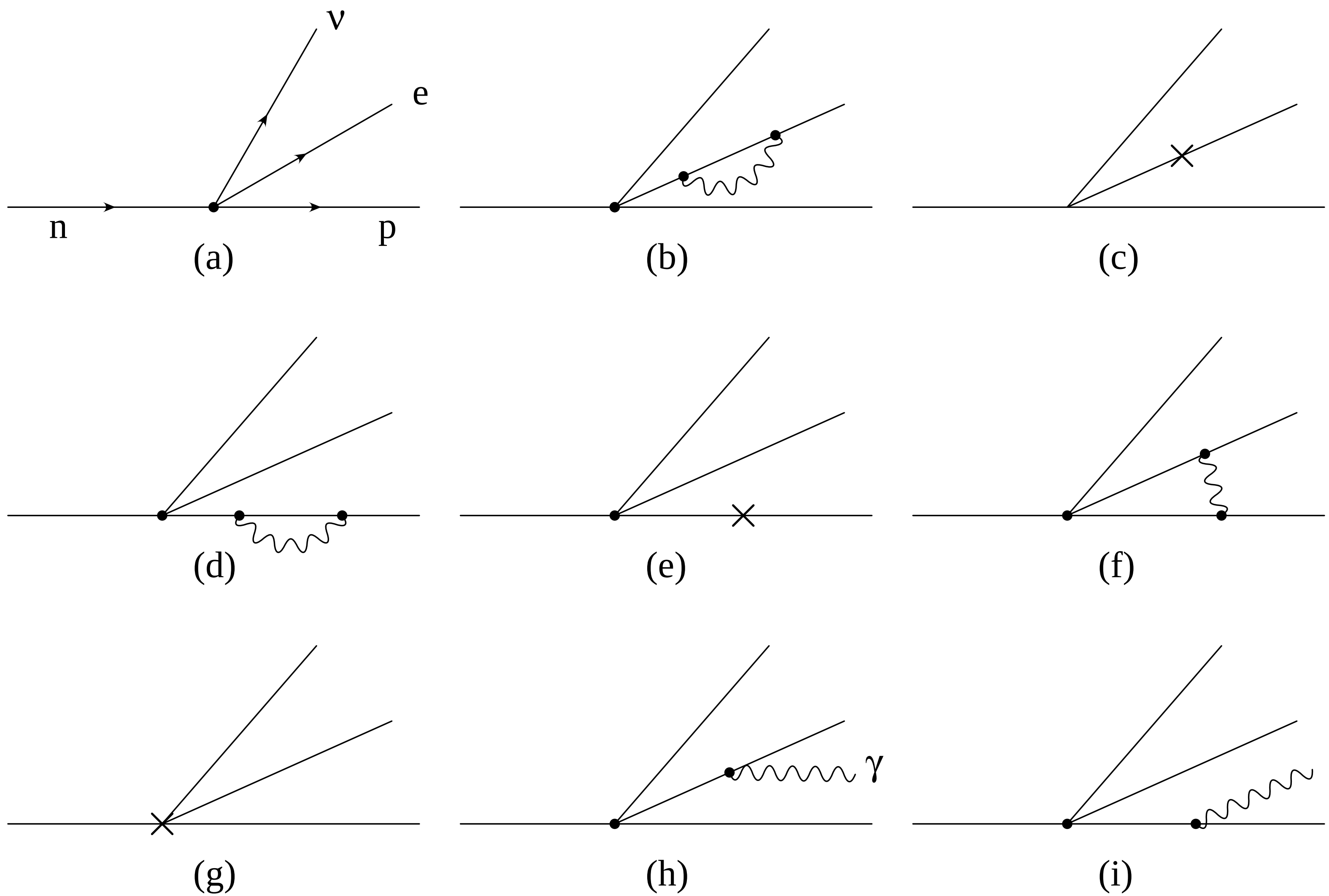
- Effective Lagrangian

$$\mathcal{L}_{\beta} = \mathcal{L}_{e\nu\gamma} + \mathcal{L}_{NN\gamma} + \mathcal{L}_{e\nu NN},$$

with

$$\begin{aligned}
\mathcal{L}_{e\nu\gamma} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi_A}(\partial\cdot A)^2 \\
&+ \left(1+\frac{\alpha}{4\pi}e_1\right)\bar{\psi}_e\,(i\gamma\cdot D)\psi_e - m_e\bar{\psi}_e\psi_e + \bar{\psi}_\nu i\gamma\cdot\partial\psi_\nu, \\
\mathcal{L}_{NN\gamma} &= \bar{N}\left[1+\frac{\alpha}{8\pi}e_2(1+\tau_3)\right] iv\cdot DN, \\
\mathcal{L}_{e\nu NN} &= -\frac{(\mathring{G}_F\,V_{ud})}{\sqrt{2}}\,\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_\nu\left\{\bar{N}\tau^+\left[\left(1+\frac{\alpha}{4\pi}e_V\right)v^\mu - 2\,\mathring{g}_A\left(1+\frac{\alpha}{4\pi}e_A\right)S^\mu\right]\right. \\
&+ \frac{1}{2m_N}\bar{N}\tau^+\left[i(v^\mu v^\nu - g^{\mu\nu})(\overleftarrow{\partial}-\overrightarrow{\partial})_\nu - 2i\,\mathring{\mu}_V\,[S^\mu, S\cdot(\overleftarrow{\partial}+\overrightarrow{\partial})]\right. \\
&\left.\left. -2i\,\mathring{g}_A\,v^\mu S\cdot(\overleftarrow{\partial}-\overrightarrow{\partial})\right] N\right\},
\end{aligned}$$

- Diagrams for neutron beta decay up to NLO



- Result at LO

$$\frac{d\Gamma}{dE_e d\Omega_{\hat{p}_e} d\Omega_{\hat{p}_\nu}} \simeq \frac{(G_F V_{ud})^2}{(2\pi)^5}(1+3g_A^2)|\vec{p}_e|E_e E_\nu^2 \left[1 + a\,(\vec{\beta}\cdot\hat{p}_\nu) + b\left(\frac{m_e}{E_e}\right) + \hat{n}\cdot\left(A\,\vec{\beta} + B\,\hat{p}_\nu + D\,\frac{\vec{p}_e\times\vec{p}_\nu}{E_e E_\nu}\right)\right].$$

with

$$a = \frac{1-g_A^2}{1+3g_A^2}, \quad A = \frac{-2g_A^2+2g_A}{1+3g_A^2}, \quad B = \frac{2g_A^2+2g_A}{1+3g_A^2}, \quad b = D = 0\,,$$

- Result up to NLO

$$e^R_{V,A}(\mu) \quad = \quad e_{V,A} - \frac{1}{2}(e_1 + e_2) + \frac{3}{2}\left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + 1\right] + 3\,\ln\left(\frac{\mu}{m_N}\right)\,,$$

in $d = 4 - 2\epsilon$ space-time dimensions.

$$\frac{d\Gamma}{dE_e d\Omega_{\hat{p}_e} d\Omega_{\hat{p}_\nu}} = \frac{(G_F V_{ud})^2}{(2\pi)^5}\,\frac{F(Z,E_e)\;|\vec{p}_e|\;E_\nu}{m_n\;[E_p + E_\nu + E_e\;(\vec{\beta}\cdot\hat{p}_\nu)]}|M|^2,$$

where

$$|M|^2 = m_n m_p E_e E_\nu \left(1 + \frac{\alpha}{2\pi}\;e^R_V\right)\left(1 + \frac{\alpha}{2\pi}\;\delta^{(1)}_\alpha\right)$$
$$\times C_0(E_e)(1 + 3\tilde{g}^2_A)\left\{1 + \left(1 + \frac{\alpha}{2\pi}\;\delta^{(2)}_\alpha\right)C_1(E_e)\vec{\beta}\cdot\hat{p}_\nu\right.$$
$$\left. + \left(1 + \frac{\alpha}{2\pi}\;\delta^{(2)}_\alpha\right)[C_2(E_e) + C_3(E_e)\vec{\beta}\cdot\hat{p}_\nu]\hat{n}\cdot\vec{\beta} + [C_4(E_e) + C_5(E_e)\vec{\beta}\cdot\hat{p}_\nu]\hat{n}\cdot\hat{p}_\nu\right\}.$$

- Renomalization of g_A:

$$\tilde{g}_A = g_A \left[1 + \frac{\alpha}{4\pi}(e_A^R - e_V^R)\right] ,$$

- Fermi function,

$$F(Z, E_e) \simeq 1 + (\alpha/2\pi)\delta_\alpha^{(Coul)} = 1 + \alpha\pi/\beta,$$

for $Z = 1$.

- $\alpha/(2\pi)$ corrections

$$\begin{aligned}\delta_\alpha^{(1)} &= 3\ln\left(\frac{m_N}{m_e}\right) + \frac{1}{2} + \frac{1+\beta^2}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - \frac{1}{\beta}\ln^2\left(\frac{1+\beta}{1-\beta}\right) + \frac{4}{\beta}L\left(\frac{2\beta}{1+\beta}\right) \\ &+4\left[\frac{1}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right]\left[\ln\left(\frac{2(E_e^{max} - E_e)}{m_e}\right) + \frac{1}{3}\left(\frac{E_e^{max} - E_e}{E_e}\right) - \frac{3}{2}\right] \\ &+ \left(\frac{E_e^{max} - E_e}{E_e}\right)^2 \frac{1}{12\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) ,\end{aligned}$$

- $\alpha/(2\pi)$ corrections

$$\delta_\alpha^{(2)} = \frac{1-\beta^2}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) + \left(\frac{E_e^{max}-E_e}{E_e}\right)\frac{4(1-\beta^2)}{3\beta^2}\left[\frac{1}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)-1\right] + \left(\frac{E_e^{max}-E_e}{E_e}\right)^2\frac{1}{6\beta^2}\left[\frac{1-\beta^2}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)-1\right] ,$$

where $E_e^{max} = (m_n^2 - m_p^2 + m_e^2)/2m_n$, and $L(z)$ is the Spence function.

- $1/m_N$ corrections

$$C_0(E_e) = 1 + \frac{1}{m_N(1+3\tilde{g}_A^2)}\left\{(\tilde{g}_A^2 - 2\mu_V\tilde{g}_A + 1)E_e^{max} - \frac{m_e^2}{E_e}(1+\tilde{g}_A^2) + 2\mu_V\tilde{g}_A(\beta^2+1)E_e\right\} ,$$

$$
\begin{aligned}
C_1(E_e) &= \tilde{a}\left\{1+\frac{1}{m_N}\left[\frac{(\tilde{g}_A^2+2\mu_V\tilde{g}_A+1)}{1+3\tilde{g}_A^2}\frac{m_e^2}{E_e}\right.\right. \\
&\left.\left. +\frac{(\tilde{g}_A^2+1)[8\mu_V\tilde{g}_A E_e-4E_e^{max}\tilde{g}_A(\tilde{g}_A+\mu_V)]}{(\tilde{g}_A^2-1)(1+3\tilde{g}_A^2)}\right]\right\}, \\
C_2(E_e) &= \tilde{A}\left\{1+\frac{1}{m_N}\left[\frac{(\tilde{g}_A^2-1)(\tilde{g}_A+\mu_V)}{2\tilde{g}_A(1+3\tilde{g}_A^2)}(E_e^{max}-E_e)+\frac{E_e(\mu_V-1)}{\tilde{g}_A-1}\right.\right. \\
&\left.\left. -\beta^2 E_e\frac{\tilde{g}_A^2+2\tilde{g}_A\mu_V+1}{1+3\tilde{g}_A^2}\right]\right\}, \\
C_3(E_e) &= \tilde{A}\frac{E_e(\tilde{g}_A-\mu_V)}{2m_N\tilde{g}_A}, \\
C_4(E_e) &= \tilde{B}\left\{1+\frac{1}{m_N}\left[\frac{E_e\beta^2(\tilde{g}_A^2-1)(\tilde{g}_A-\mu_V)}{2\tilde{g}_A(1+3\tilde{g}_A^2)}+\frac{(\tilde{g}_A+\mu_V)(\tilde{g}_A-1)^2}{(\tilde{g}_A+1)(1+3\tilde{g}_A^2)}(E_e-E_e^{max})\right]\right\} \\
C_5(E_e) &= \tilde{B}\frac{(\tilde{g}_A+\mu_V)}{2m_N\tilde{g}_A}(E_e^{max}-E_e),
\end{aligned}
$$

- Corrections from the phase space

$$\frac{m_p E_\nu^2}{(E_p + E_\nu + E_e \vec{\beta} \cdot \hat{p}_\nu)} \simeq (E_e^{max} - E_e)^2 \left[1 + \frac{1}{m_N}(3E_e - E_e^{max} - 3E_e \vec{\beta} \cdot \hat{p}_\nu)\right],$$

where we have used

$$E_\nu \simeq (E_e^{max} - E_e)\left[1 + \frac{E_e}{m_N}(1 - \vec{\beta} \cdot \hat{p}_\nu)\right].$$

Result and discussion:

- We reproduce all RCs and $1/m$ corrections which have been reported in the previous studies with one unknown LEC e_V^R, which is fixed by using the Sarlin's result as

$$e_V^R \simeq -\frac{5}{4} - 4\ln\left(\frac{m_W}{m_Z}\right) + 3\ln\left(\frac{m_W}{m_N}\right) + \ln\left(\frac{m_W}{m_A}\right) + 2C + A_g\,.$$

- What else ?
  - Neutron-neutron fusion including the RCs
  - A correction to the D term from heavy pions

$$D = D_{CT} + D_\pi, \quad D_{CT} \propto \alpha\mu_V \bar{Q}/m_N \sim 10^{-5}, \quad D_\pi \propto \alpha(\bar{Q}/m_\pi)^2 \sim 10^{-8}.$$

- What could be next ?
  - ^{16}N beta decay and β delayed α distribution from ^{16}N in cluster EFT (Z=8) ?