EW Radiative Corrections: Context

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Beta Decay Workshop
ECT* April 2019

Goals For This Talk

- *Provide a brief BSM context & basic formalism*
- *Introduce the dispersion relation framework*
- *Set the stage for following talks & discussion*

Outline

I. Context & Formalism

II. $W\gamma$ Box: Dispersion Relations

III. Questions for the Day

I. Context

Weak Decays: CKM Unitarity

$$d \to u\, e^-\, \bar{\nu}_e$$

$$s \to u\, e^-\, \bar{\nu}_e$$

$$b \to u\, e^-\, \bar{\nu}_e$$

$$\begin{pmatrix} u & c & t \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\Delta_{\rm CKM} = (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)_{\rm exp} - 1$$

0.94906 ± 0.00041 $\quad$ 0.05031 ± 0.00022 $\quad$ 0.00002

$$\Delta_{\rm CKM} = -0.0006 \pm 0.0005$$

Precision ~ BSM Mass Scale

Precision Goal:

$$\delta\,\Delta_{CKM} \sim O(10^{-4})$$

Heavy BSM Physics:

$$\Delta_{CKM} \sim C\ \ (\,v/\Lambda\,)^2$$

$\Lambda \sim 10$ TeV (tree)

$\Lambda < 1$ TeV (loop)

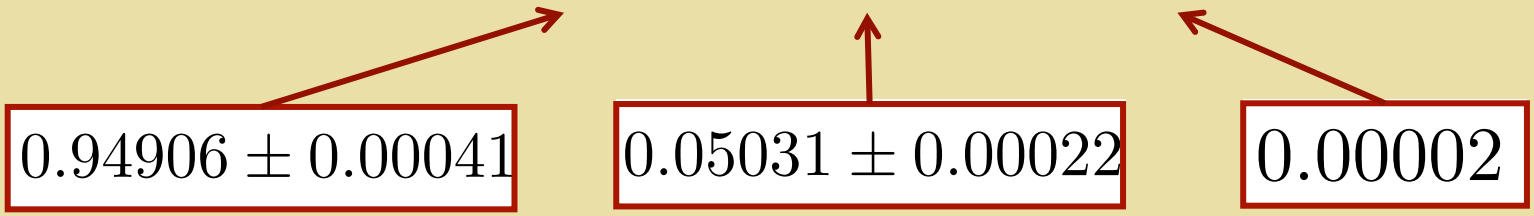
Ultralight BSM Physics:

$$\Delta_{CKM} \sim \varepsilon^2\ \ (\,\alpha/4\pi\,)$$

$\varepsilon < 1$ (loop)

Error Budget

$$\Delta_{\mathrm{CKM}} = (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)_{\mathrm{exp}} - 1$$



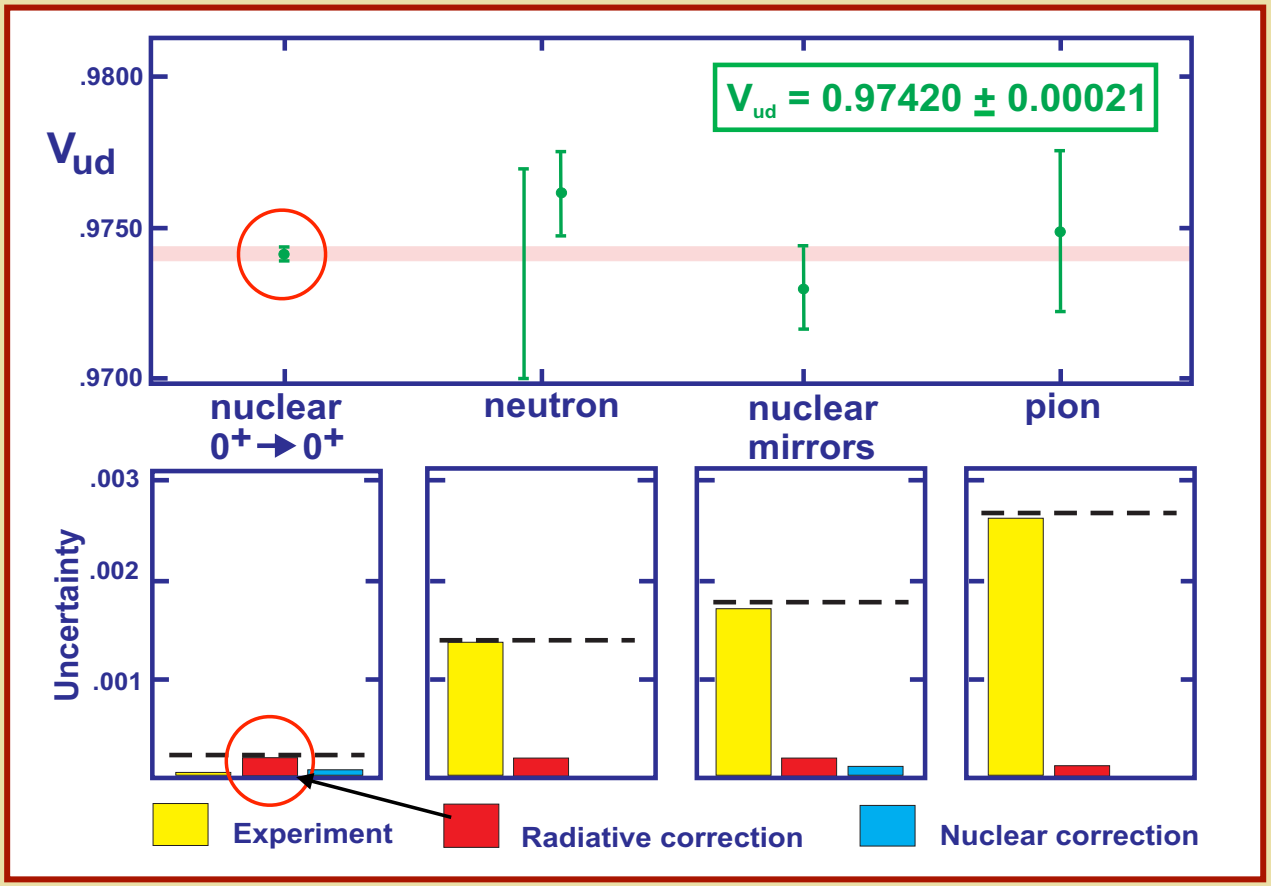
Error Budget

$$\Delta_{\mathrm{CKM}} = (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)_{\mathrm{exp}} - 1$$

0.94906 ± 0.00041

Radiative Correction

Factor of 2 reduction using disp relations



Thanks: J. Hardy

Error Budget

$$\Delta_{\mathrm{CKM}} = (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)_{\mathrm{exp}} - 1$$

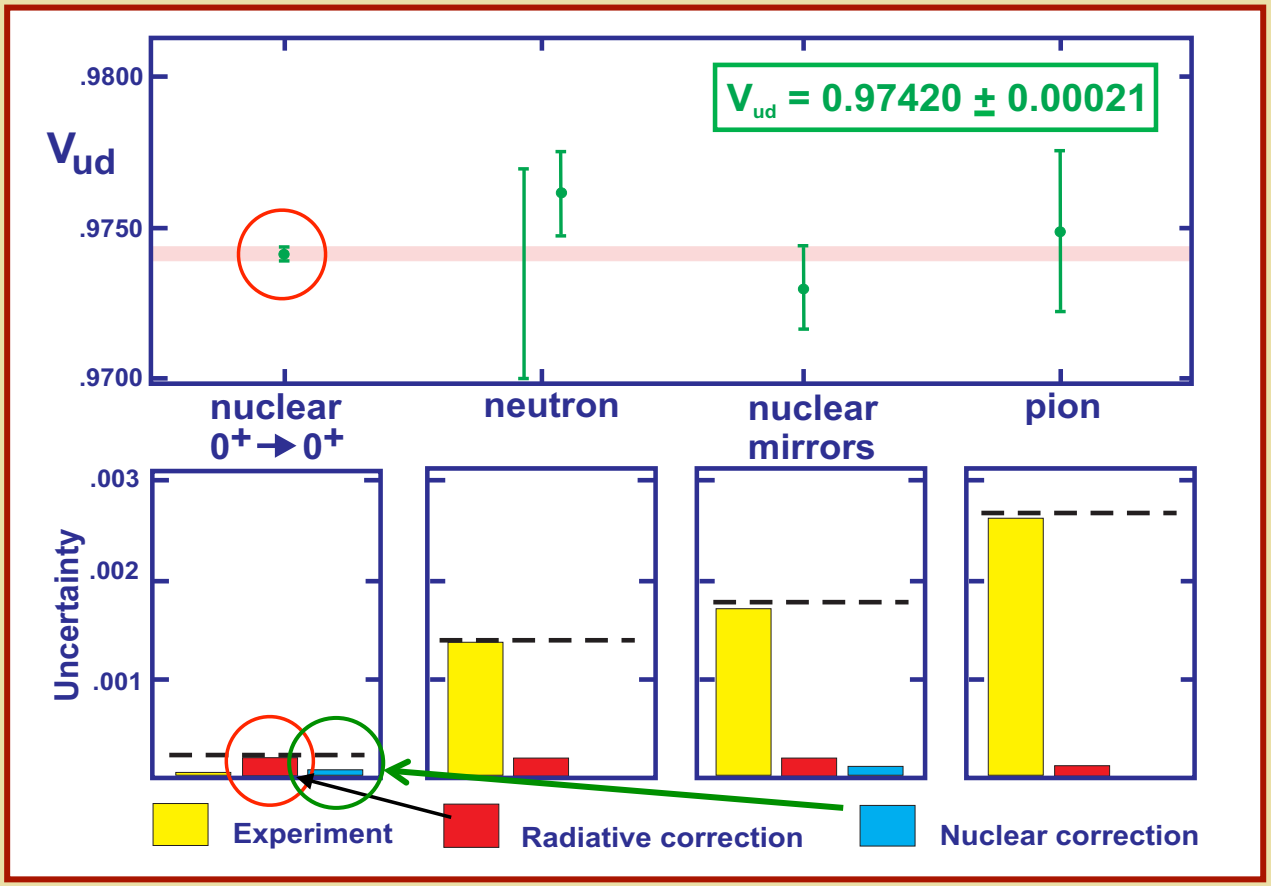
0.94906 ± 0.00041

Radiative Correction

Factor of 2 reduction using disp relations

Nuclear Correction

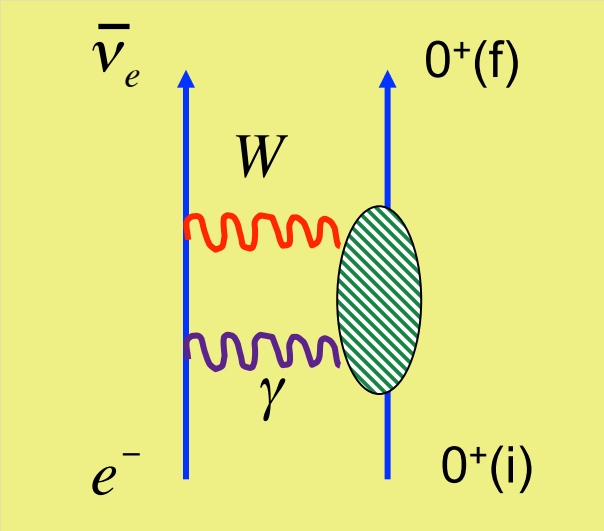
Increase due to previously omitted contributions



Thanks: J. Hardy

Radiative Corrections

Dominant source of uncertainty:



$$M_{\gamma W} = \frac{G_F V_{ud}}{\sqrt{2}} \frac{\alpha}{8\pi} \left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right) + C_{\gamma W}(\Lambda) \right]$$

Short distance ***Long distance***

Long distance

Sensitive to hadronic & nuclear dynamics

Radiative Corrections & Ft Values

Corrected ft values:

$$\mathcal{F}t = ft\left(1+\delta'_R\right)\left(1+\delta_{NS}-\delta_C\right)\left(1+\Delta_E^{NS}\right)$$

Outer correction

Nuclear struct part of $M_{\gamma W}$

E-independent

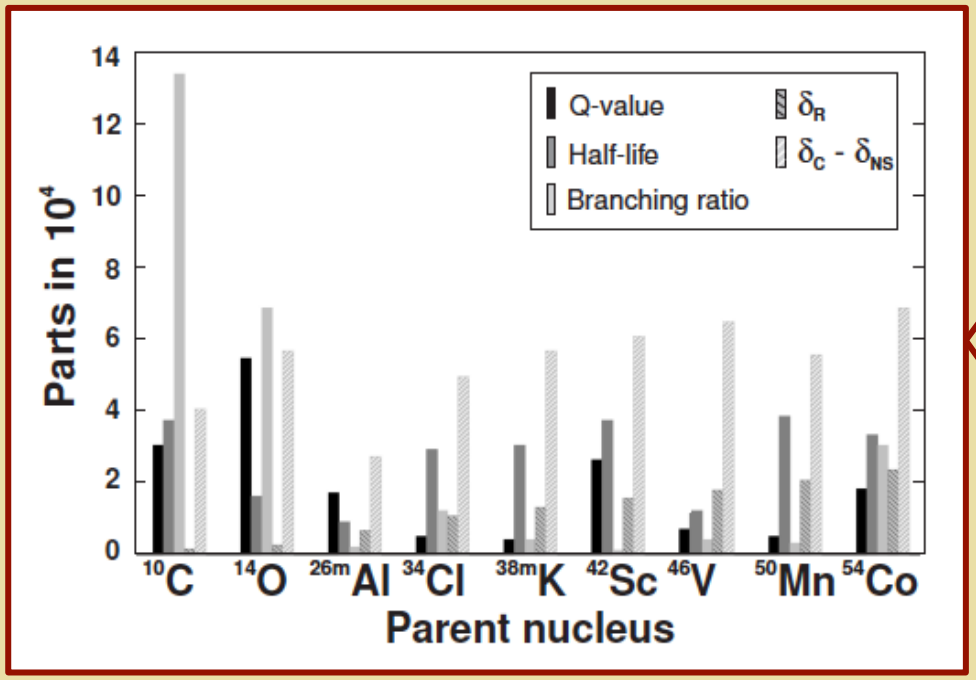
Nucl wavef'n

Not a RC

Nuclear struct part of $M_{\gamma W}$

E-dependent

$0^+ \rightarrow 0^+$ Dispersion Corrections: δ_{NS}



b_F : scalar currents

Input for V_{ud} & CKM unitarity test

Towner & Hardy, PRC 91 (2015) 2, 025501

Radiative Corrections & V_{ud}

Superallowed

$$|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1+\Delta_R^V)}$$

Hadronic & short distance part of $M_{\gamma W}$

Neutron

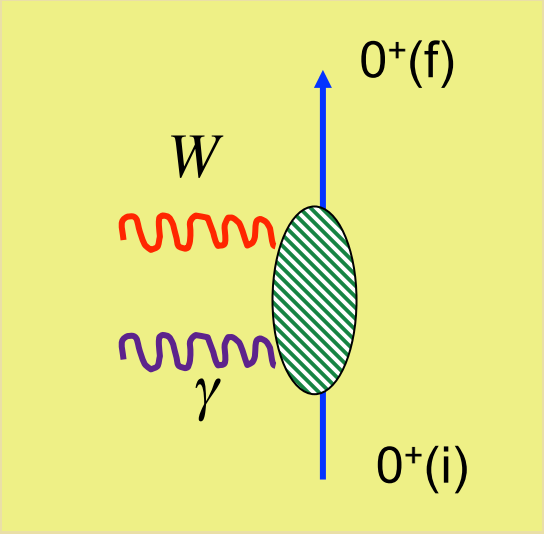
$$|V_{ud}|^2 = \frac{5099.34s}{\tau_n(1+3\lambda^2)(1+\Delta_R)}$$

$(g_A / g_v)^2$

Contains Δ_R^V

II. Wγ Box: Dispersion Relations

Dispersion Relations



Electroweak virtual Compton amplitude:

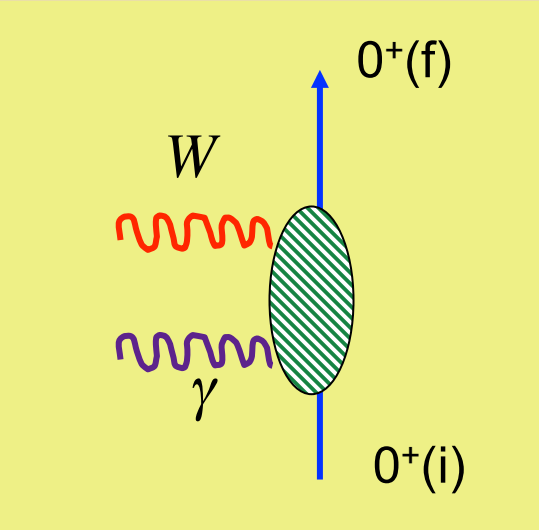
$$T^{\mu\nu}_{\gamma W} = \left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right] T_1 + \frac{\hat{p}^\mu \hat{p}^\nu}{(p\cdot q)} T_2 + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p\cdot q)} T_3$$

Radiative correction:

$$\Box^{VA}_{\gamma W} = 4\pi\alpha \mathrm{Re} \int \frac{d^4q}{(2\pi)^4} \frac{M_W^2}{M_W^2 + Q^2} \frac{Q^2+\nu^2}{Q^4} \frac{T_3(\nu, Q^2)}{M\nu}$$

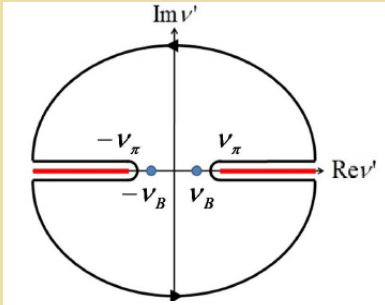
$$\Box^{VA}_{\gamma W} = \frac{1}{2} \left(\Delta_R^V\right)^{VA}_{\gamma W}$$

Dispersion Relations



Dispersion relation:

Write T_3 as integral over discontinuity along cut



$$T_3^{(I)}(\nu, Q^2) = \frac{2}{i}\int_0^\infty d\nu' \left[\frac{1}{\nu' - \nu} + \frac{\xi^I}{\nu' + \nu}\right] F_3^{(I)}(\nu', Q^2)$$

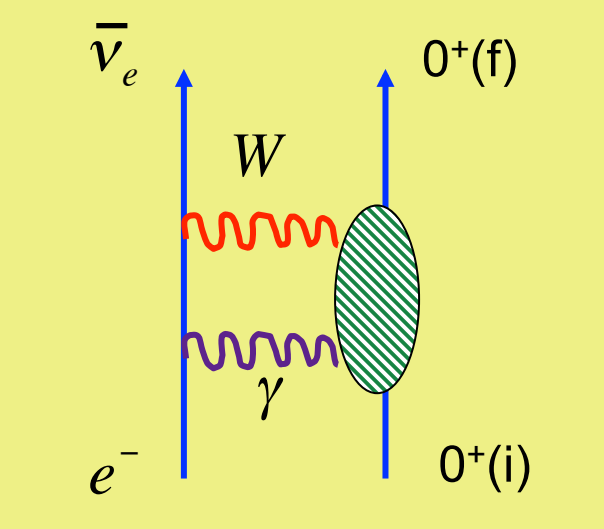
Electroproduction structure functions:

$$W_{\gamma W}^{(I)\mu\nu} = \frac{1}{8\pi}\sum_X (2\pi)^4 \delta^4(p + q - p_X) \langle p| J_{em}^{(I)\mu} |X\rangle \langle X| J_W^\nu |n\rangle$$

$$= \left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right] F_1^{(I)} + \frac{\hat{p}^\mu \hat{p}^\nu}{(p\cdot q)} F_2^{(I)} + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p\cdot q)} F_3^{(I)}$$

$$d\sigma \propto L_{\mu\nu} W^{\mu\nu}$$

Dispersion Relations



Radiative Correction:

$$\Box_{\gamma W}^{VA\,(0)} = \frac{\alpha}{\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu \frac{(\nu+2q)}{\nu(\nu+q)^2}F_3^{(0)}(\nu,Q^2)$$

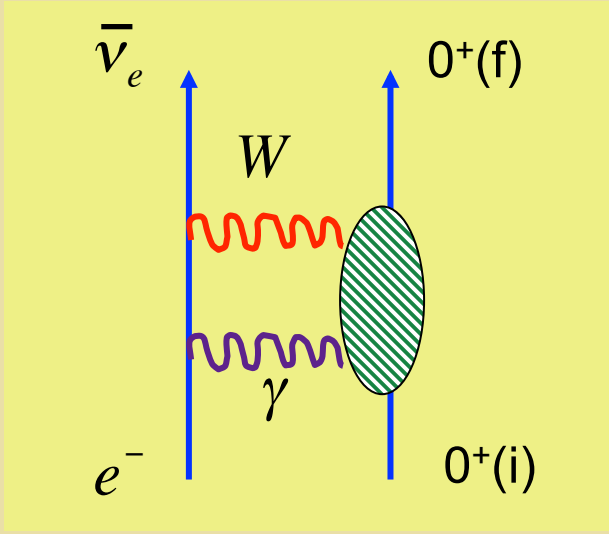
$$= \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2 M_W^2}{Q^2[M_W^2+Q^2]}M_3^{(0)}(1,Q^2)$$

Nachtmann Moments:

$$M_3^{(0)}(N,Q^2) = \frac{N+1}{N+2}\int_0^1 \frac{dx\xi^N}{x^2}\left[2x - \frac{N\xi}{N+1}\right]F_3^{(0)}$$

$$\xi = 2x\left(1+\frac{4M^2x^2}{Q^2}\right)^{-1}$$

Dispersion Relations



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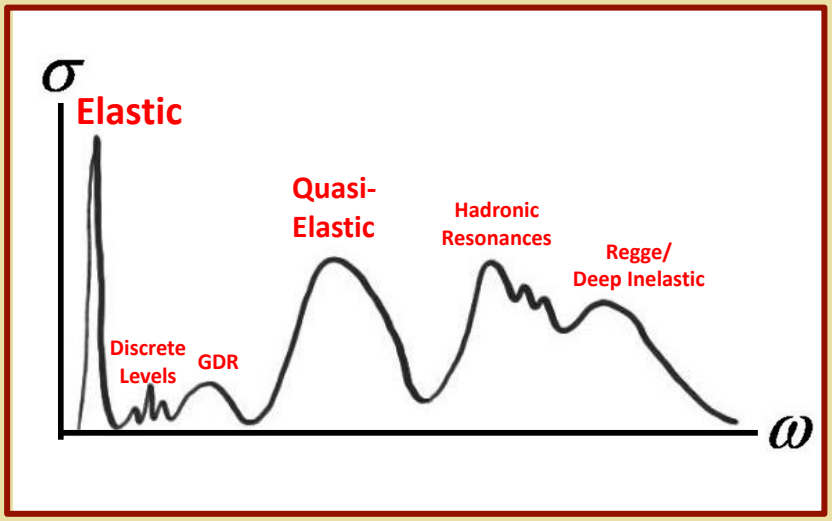
$$= \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2 M_W^2}{Q^2[M_W^2+Q^2]}M_3^{(0)}(1,Q^2)$$

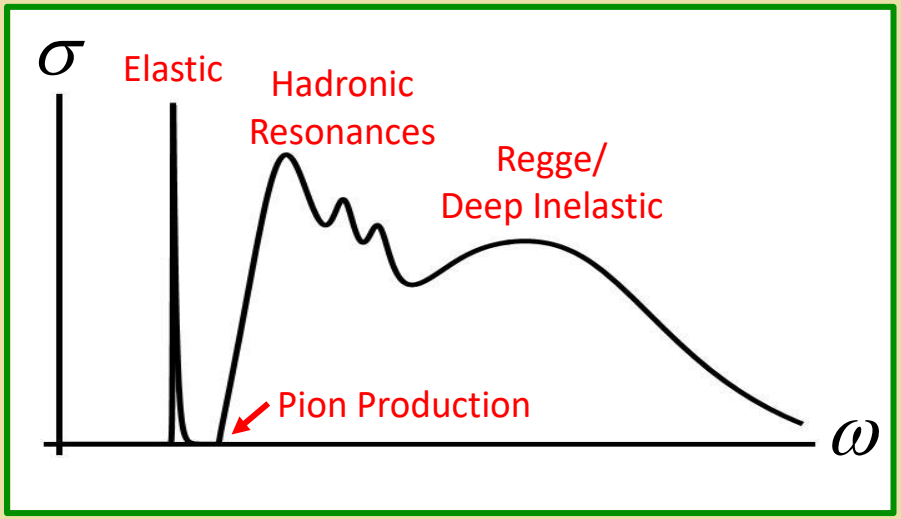
- ***Relate $F_3^{(0)}$ and $M_3^{(0)}$ to data and/or***
- ***Compute $F_3^{(0)}$ and $M_3^{(0)}$ using same methods used to describe semi-leptonic scattering processes with nucleon & nuclear targets***

Leptoproduction: Had & Nuc Response

Nuclei

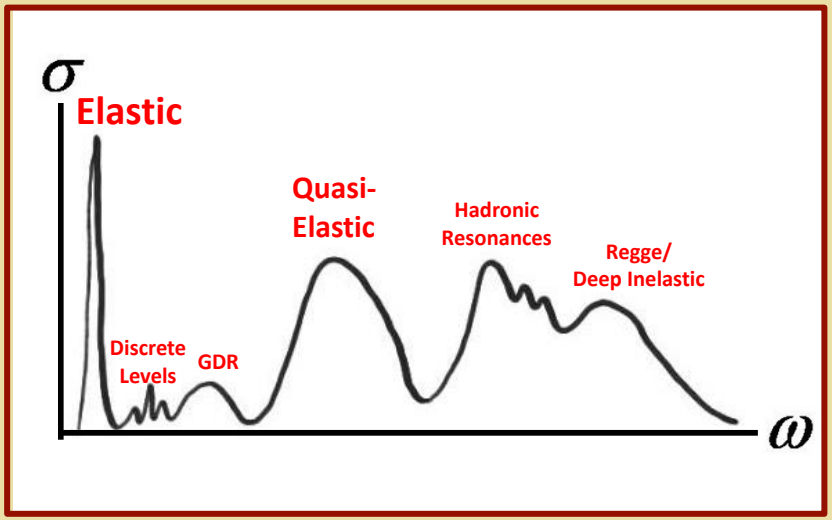


Free nucleons

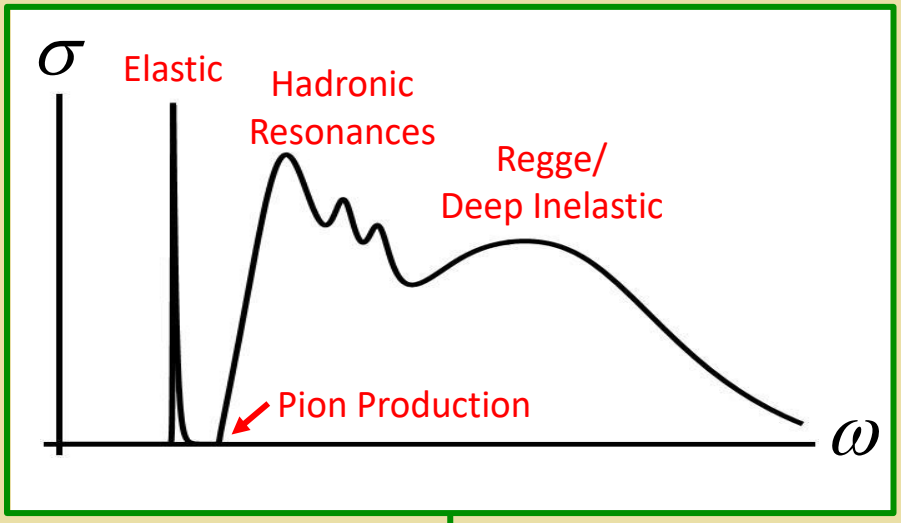


Leptoproduction: Had & Nuc Response

Nuclei



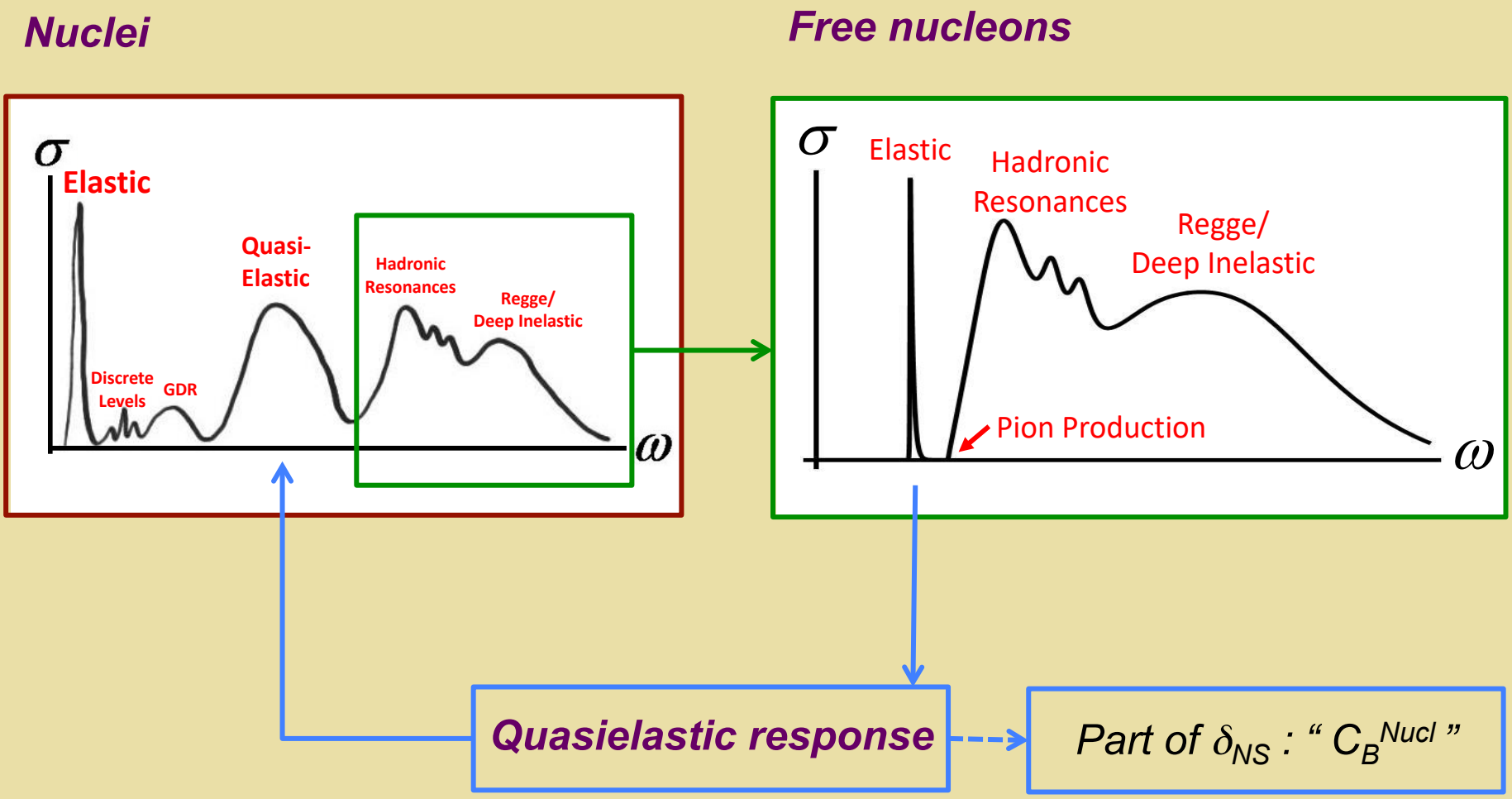
Free nucleons



Single nucleon: PRL 121 (2008) 241804

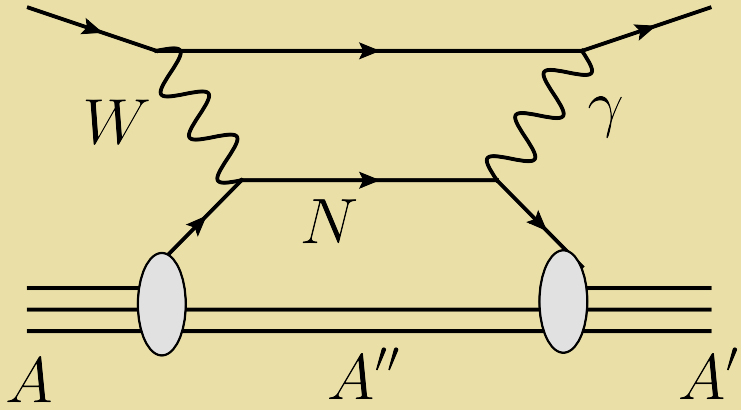
Δ_R^V = 0.02361(38) → 0.02467 (22)

Leptoproduction: Had & Nuc Response



New work

Impact on δ_{NS}



$$\Delta\,\delta_{NS} = \frac{\alpha}{\pi}\left(C_{QE} - q_S^{(0)} q_A C_B\right) = -(4.6 \pm 0.9) \times 10^{-4}$$

Our new work: QE response

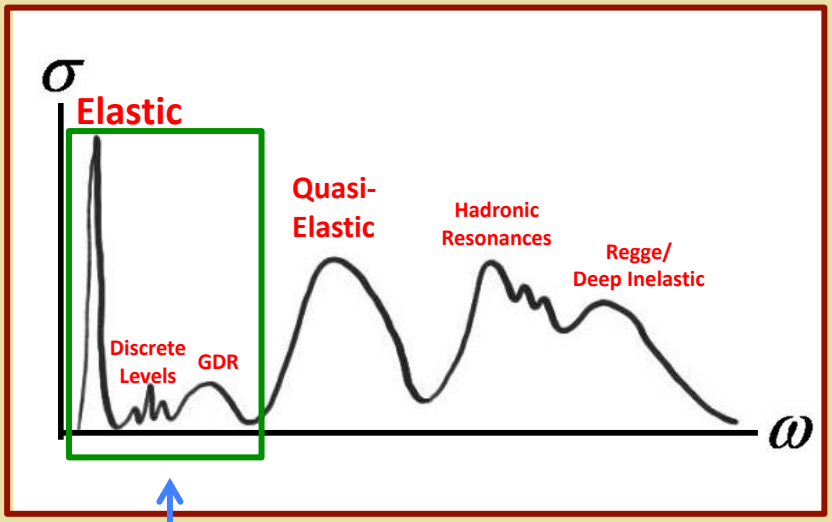
Towner & Hardy

Features:

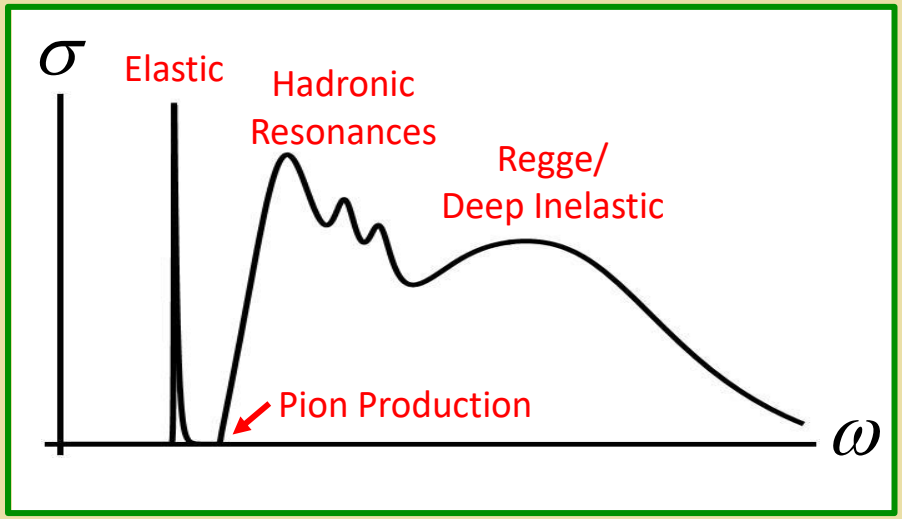
- *Few x 10^-4*
- *Error bar ?*
- *Refinements ?*

Other Nuclear Corrections

Nuclei



Free nucleons



Low-lying transitions → *Part of* δ_{NS}

Questions for the Day

- ***How robust is the quoted uncertainty on the new value of Δ_R^V ?***
- ***What additional tests (theory, experiment) are available ?***
- ***What is the roadmap to refined computation of δ_{NS} (QE) ?***
- ***How important are contributions from other region of the low-E nuclear response ? How to compute & how to test computations ?***