Laboratoire Kastler Brossel
Physique quantique et applications

X-ray spectroscopy of antiprotonic atoms

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ECT*, Trento, June 16-21, 2019

EMMI · CNRS · Collège de France · ENS · Sorbonne Université

Outline

- QED contributions in normal and exotic atoms
- Status of comparison between theory and experiment for highly-charged ions
- The proton size puzzle
- The hyperfine structure of H-like and Li-like Bi
- Testing QED in an other framework
  - Muonic Atoms: measuring nuclear size or QED effects?
  - Antiprotonic atoms: nuclear size, strong interaction or QED?
- Conclusion

Why testing QED?

- Often asked question: the electromagnetic interaction is well understood, so what is left to test in QED?
  - the series of bound state QED corrections has a radius of convergence equal to zero
  - not obvious how orders beyond 2 behave… (should be convergent up to n=137 but…)
  - QED corrections to multi-electrons systems impossible to obtain with required accuracy
  - we can measure very accurately, and possibly be sensitive to effects beyond the standard model
  - need to understand better to make better predictions, e.g., for super-heavy elements
  - understand atoms in super-critical field (U^{92+} on U collisions for example $Z_1+Z_2>173$)
  - distinguish nuclear and QED contributions (either measure nuclear properties or to measure atomic properties)
  - low-energy tests of strong interaction or particle and anti-particle interaction
  - fundamental constant drifts…
  - better atomic clocks using highly-charge ions (insensitive to black-body radiation…)
  - …

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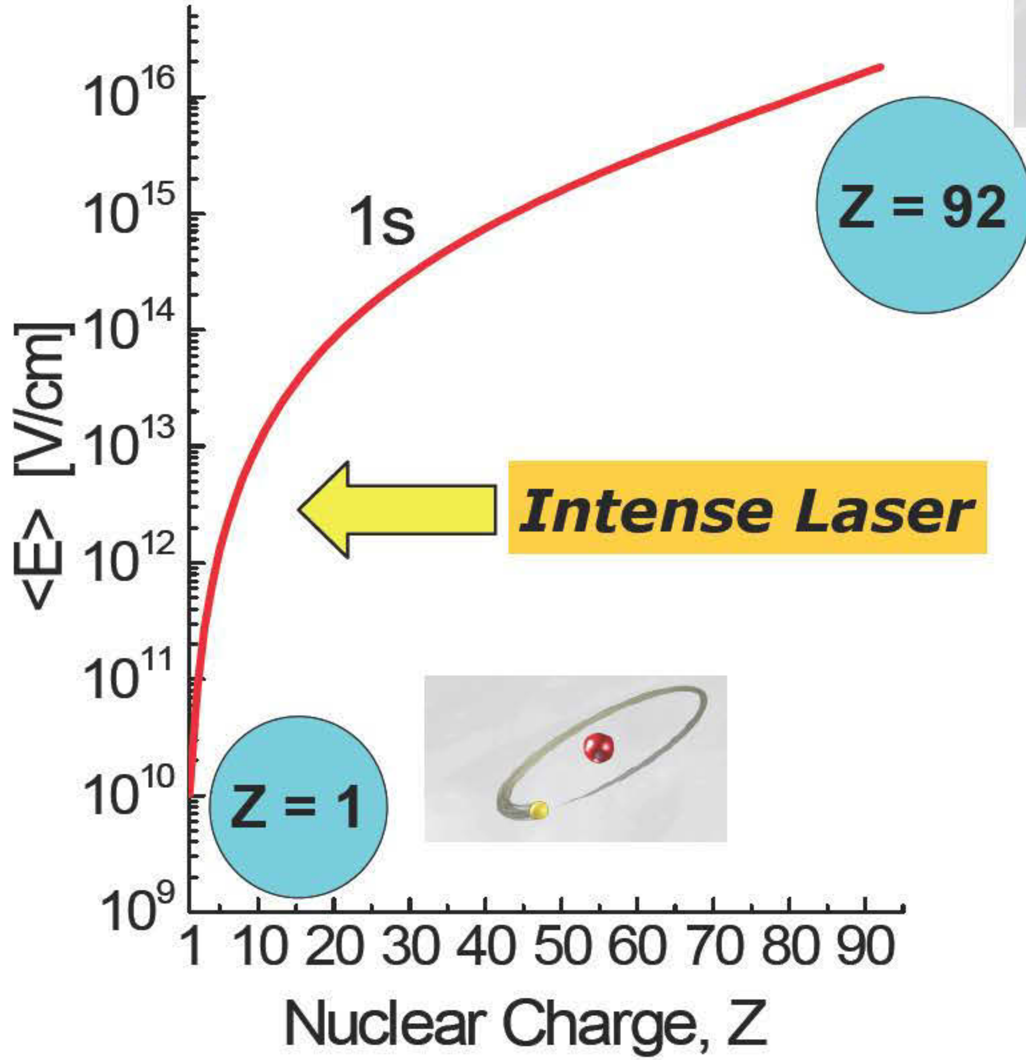
Atomic energies

All order Dirac+Vacuum polarization treatment+other QED

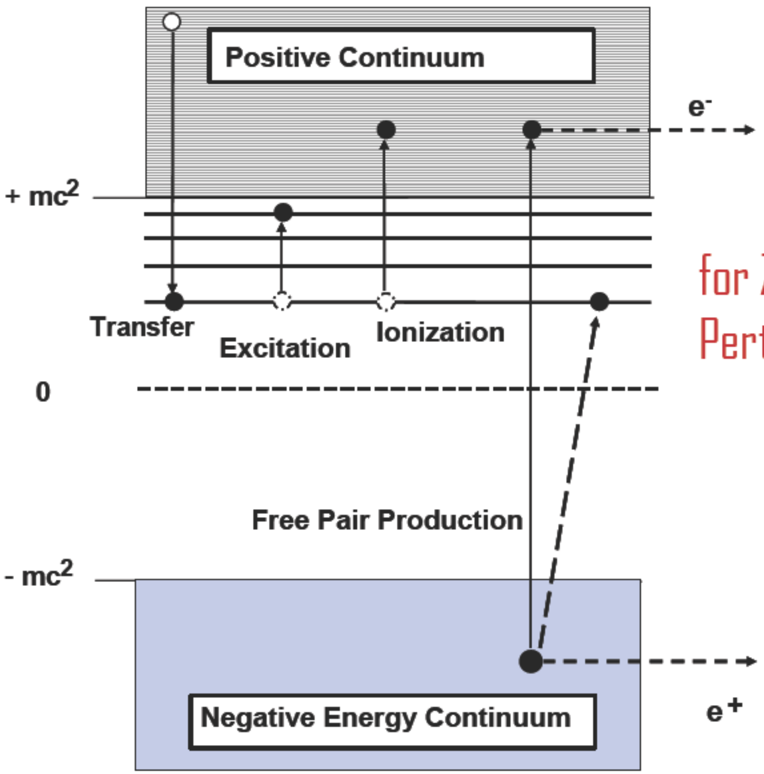
Atomic Physics in Strong Coulomb Fields



H-like Uranium
E_K = -132 • 10^3 eV
<E>= 1.8 • 10^{16} V/cm

Hydrogen
E_K = -13.6 eV
<E>= 1 • 10^{10} V/cm

Atomic Physics in Strong Coulomb Fields



for Z≥~173, 1s dives in the negative energy continuum
Perturbative QED no longer possible...

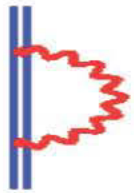
One electron BSQED corrections

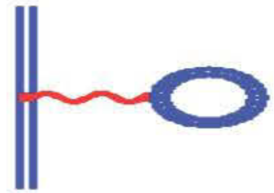
successive orders for QED

one electron QED at order α

Self Energy

$\frac{\alpha}{\pi}$





Vacuum Polarization

H-like "One Photon" order (α/π)

Many calculations, very accurate down to Z=1

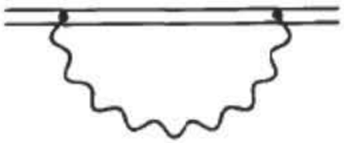
$$\Delta E_{SE} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}F(Z\alpha).$$

$$\frac{\alpha}{\pi} = 2.3 \times 10^{-3}$$

$$E_{NR} = \frac{(Z\alpha)^2}{n^2}$$

Content of a Feynman diagram: Self-energy

- Self-energy
  - A bound electron emit and and reabsorb a photon.



- Finite nuclear size contribution ditticult to evaluate for heavy particles (muon, $\bar{\mathrm{p}}$ in deep bound states)

No potential

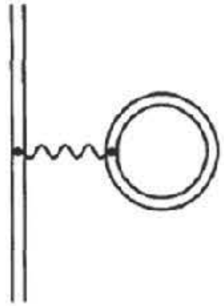
$$\Delta E_n = -i\pi\alpha \int d(t_2 - t_1)\, d^3\mathbf{x}_2\, d^3\mathbf{x}_1\, \bar{\psi}_n(x_2)\, \gamma_\mu S_F{}^e(x_2\,,\, x_1)\, \gamma^\mu \psi_n(x_1)$$
$$\times\, D_F(x_2 - x_1) - \delta m \int d^3\mathbf{x}\, \bar{\psi}_n(x)\, \psi_n(x),$$

Content of a Feynman diagram: vacuum polarization

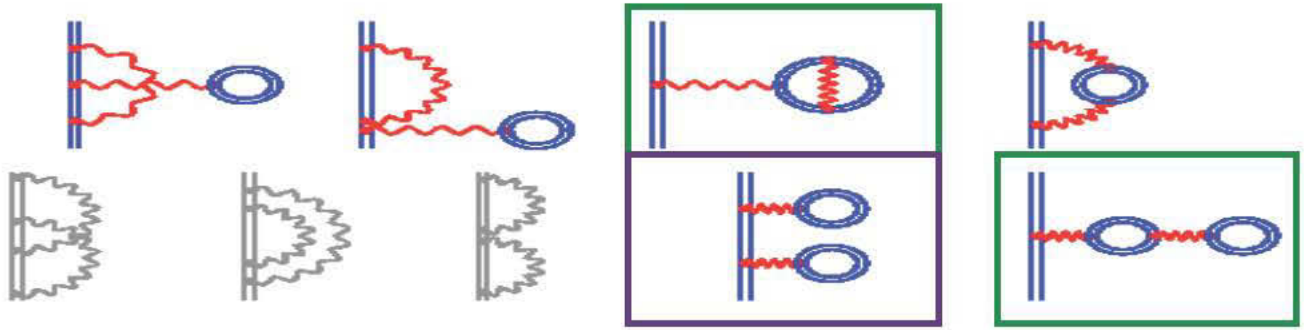
- **Vacuum polarization**
  - The electron interact with a virtual electron-positron (or muon-antimuon) pair in the field of the nucleus

$$E_{\mathrm{VP}}^{(2)} = 4\pi i\alpha \int d(t_2 - t_1) \int d\boldsymbol{x}_2 \int d\boldsymbol{x}_1\, D_{\mathrm{F}}(x_2 - x_1)$$
$$\times \mathrm{Tr}[\gamma_\mu S_{\mathrm{F}}(x_2, x_2)]\, \overline{\phi}_n(x_1)\gamma^\mu \phi_n(x_1)$$

→ Potential



one electron QED at order α^2



H-like "Two Photon" order $(\alpha/\pi)^2$

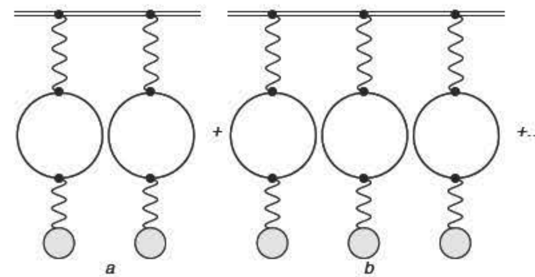
Loop after loop
VP in Dirac Eq.

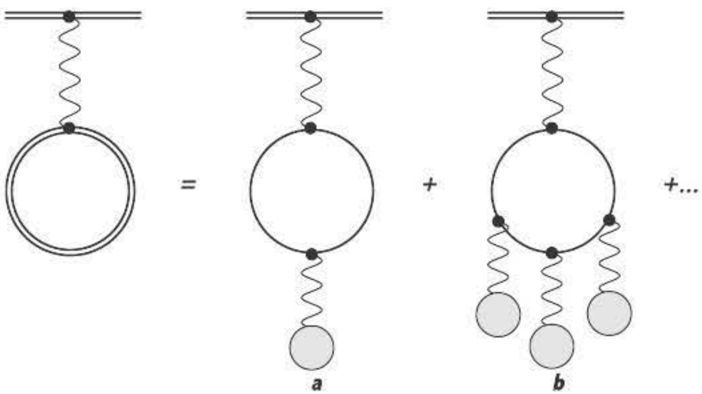
Källèn & Sabry
Potential

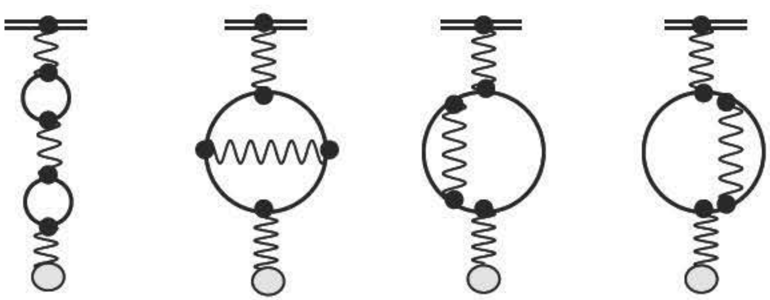
Others not precisely known for exotic atoms

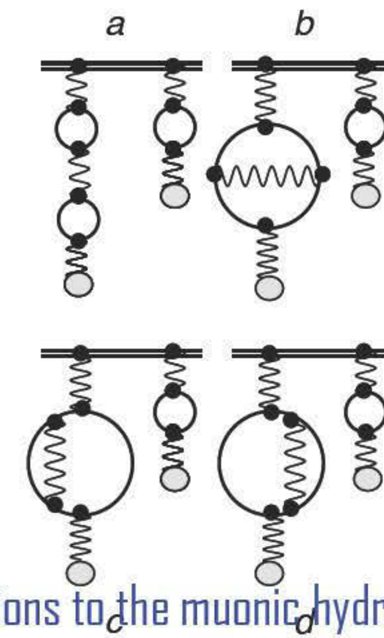
Contributions included to all-order

All-order: the charge distribution is included exactly in the wavefunction and in the operator, when relevant. Higher order Vacuum Polarization contribution included by numerical solution of the Dirac equation







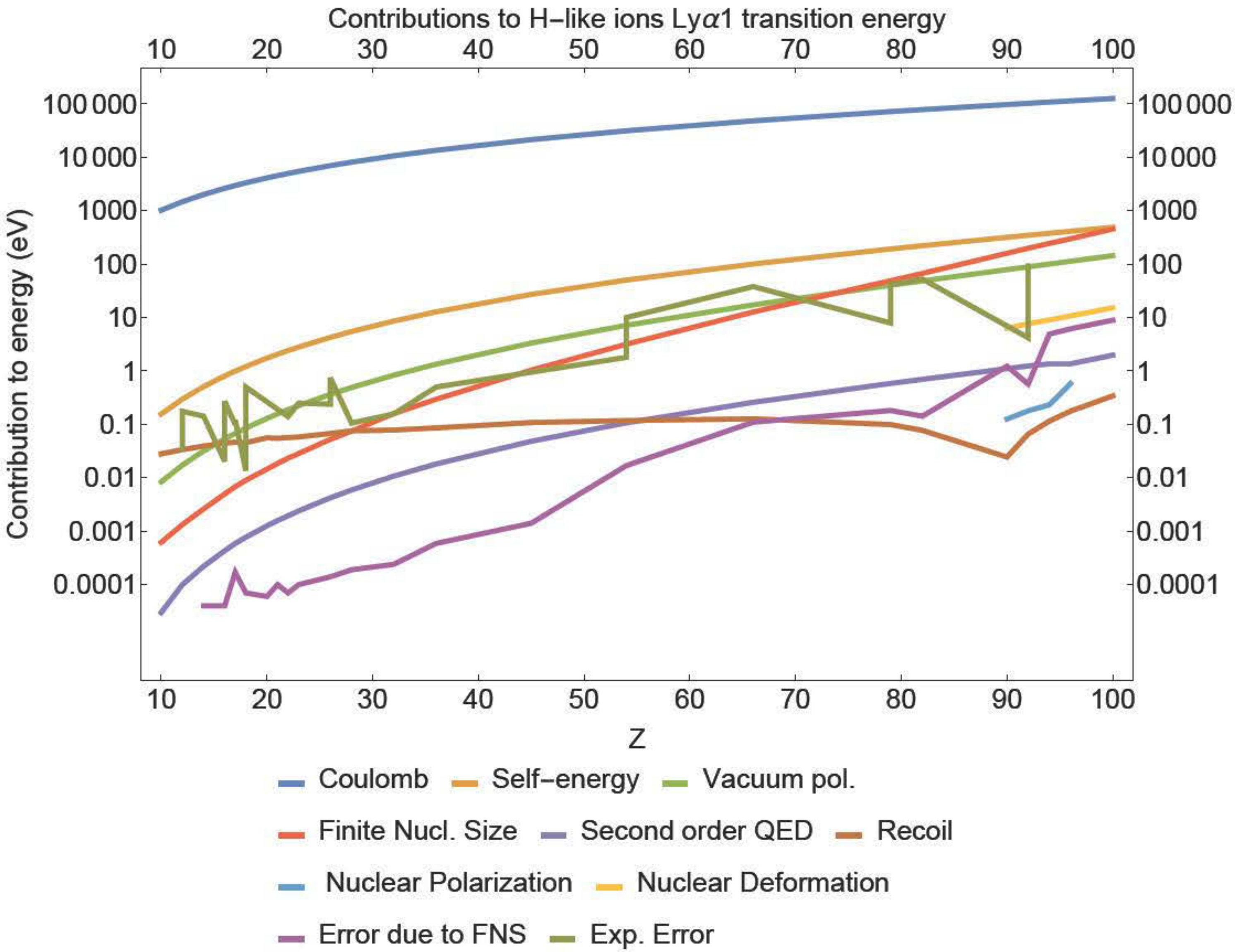


Nonperturbative evaluation of some QED contributions to the muonic hydrogen n=2 Lamb shift and hyperfine structure, P. Indelicato. Phys. Rev. A **87,** 022501 (2013).

The size of QED effects

in normal atoms and ions

What are we testing?



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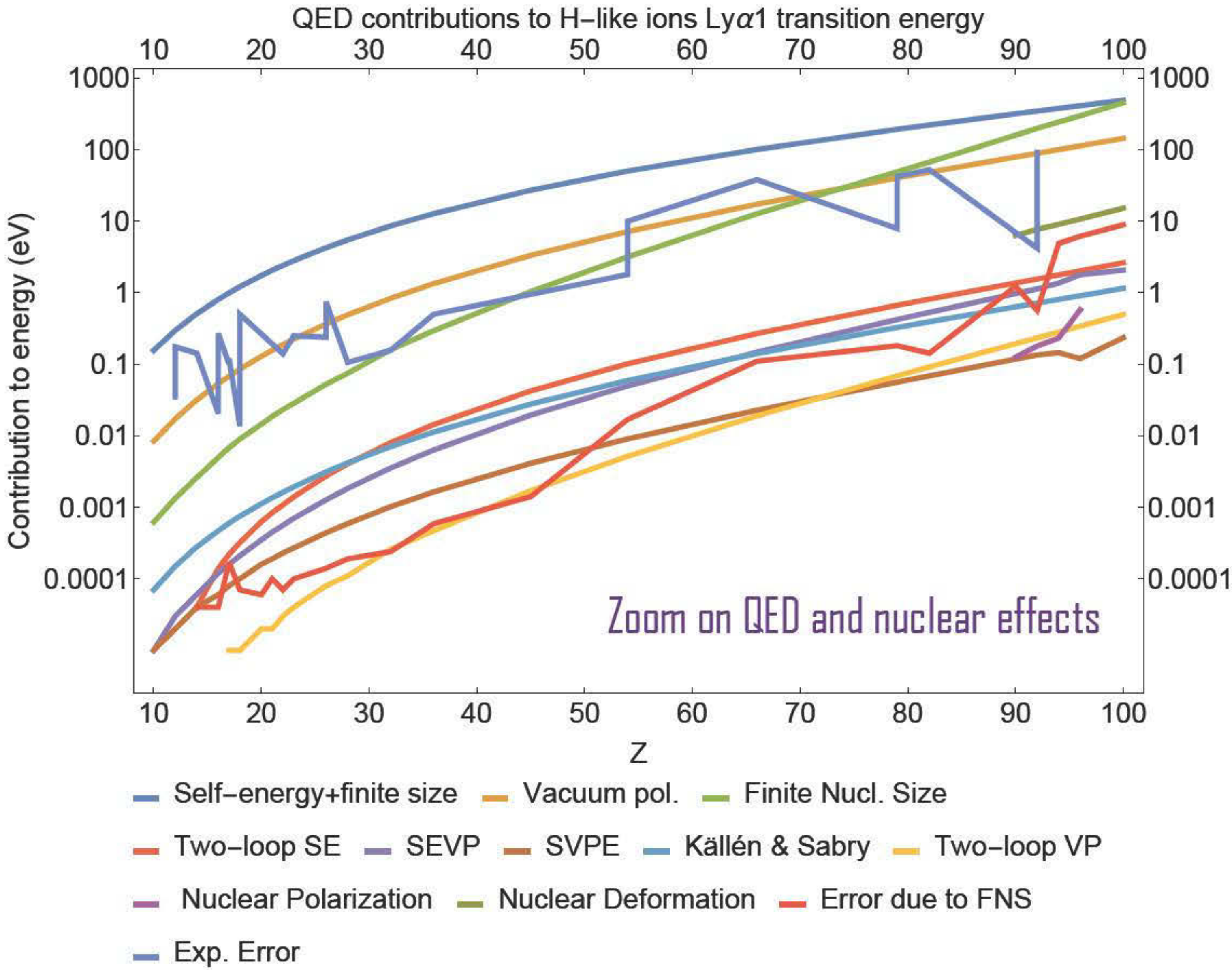

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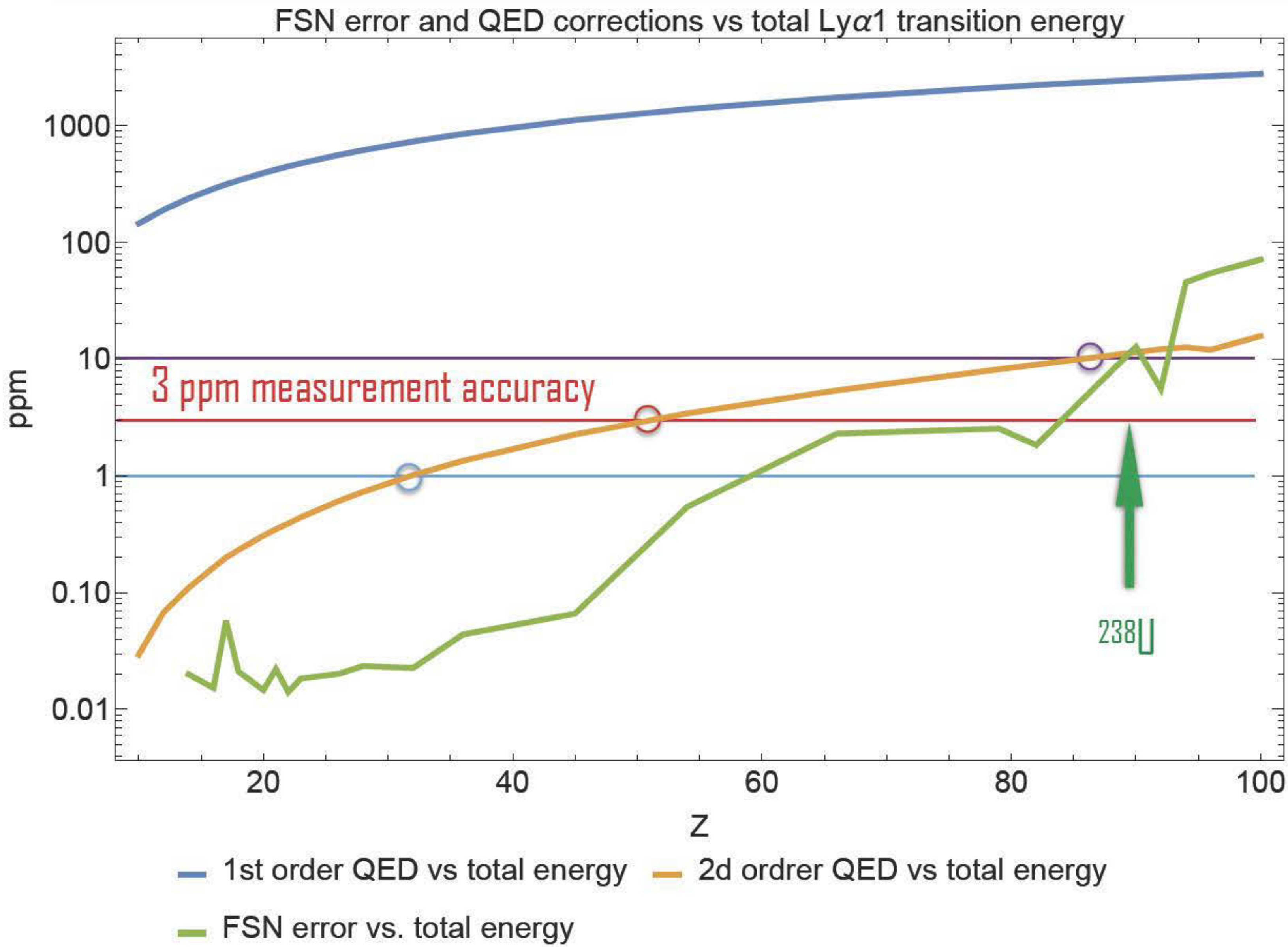
Where to do measurement to best test QED

Nuclear effects and QED tests on H-like ions

Is there an optimal Z to avoid nuclear uncertainties?



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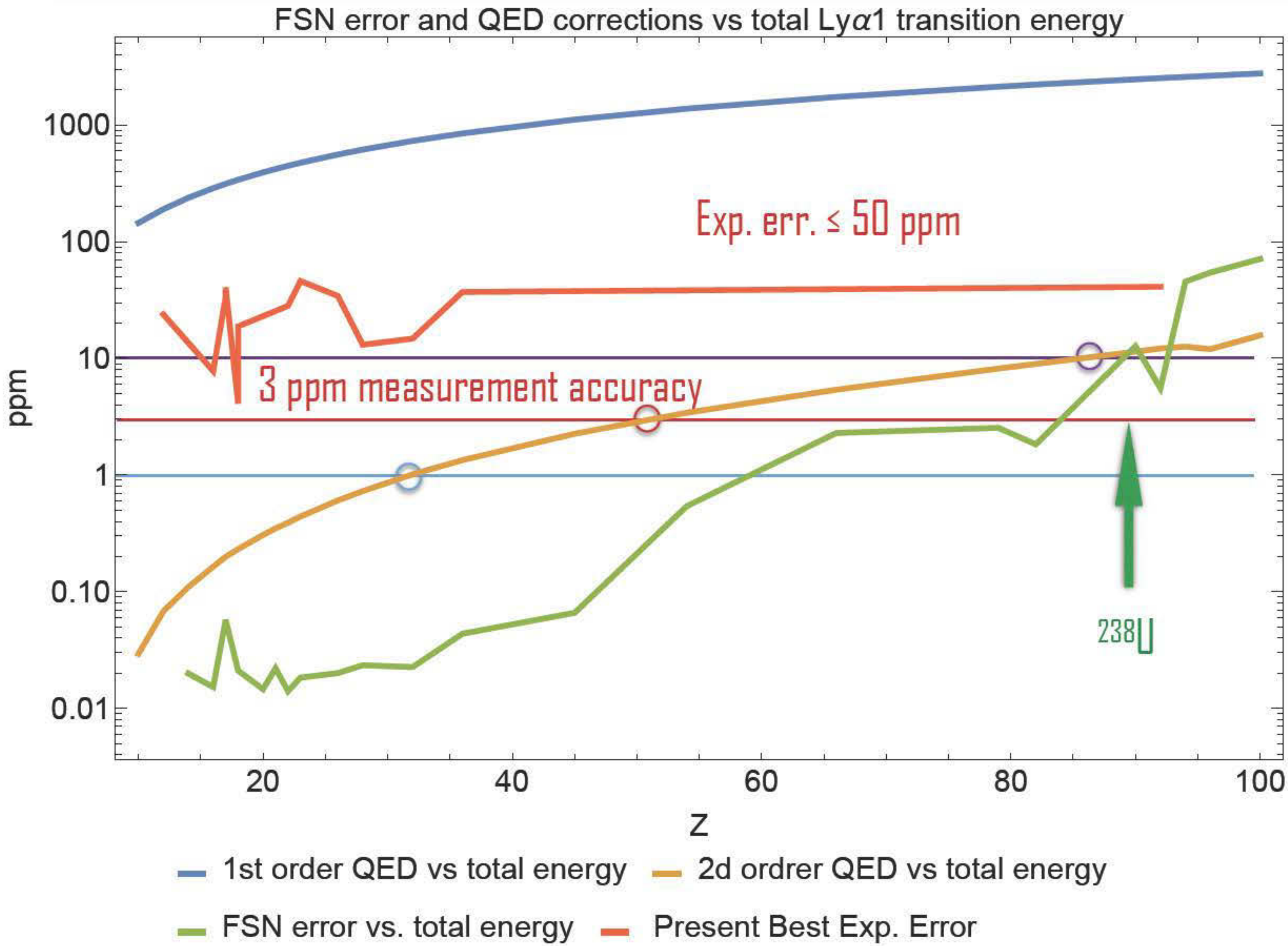

Other ways to test BSQED?

what about exotic atoms?

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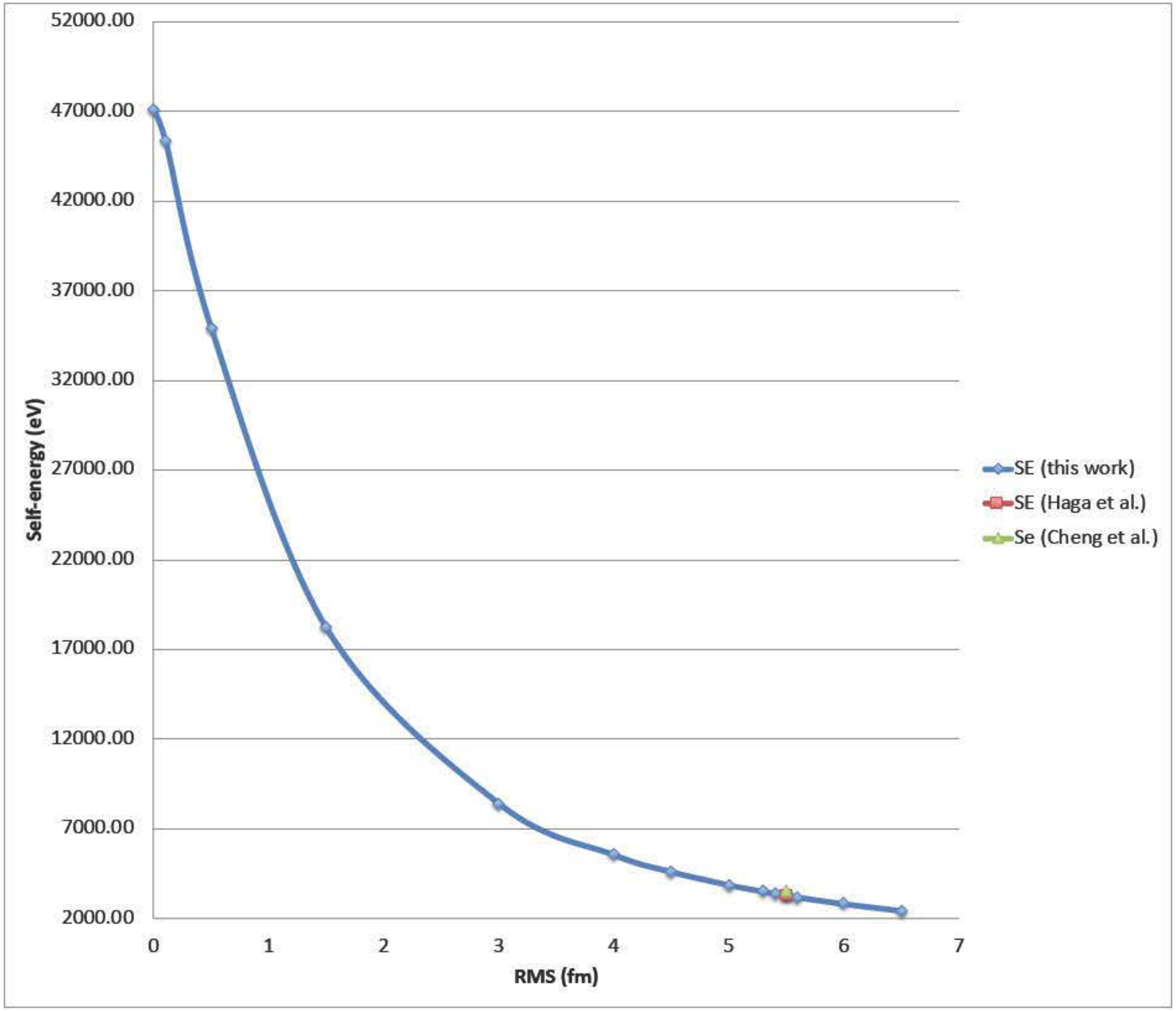
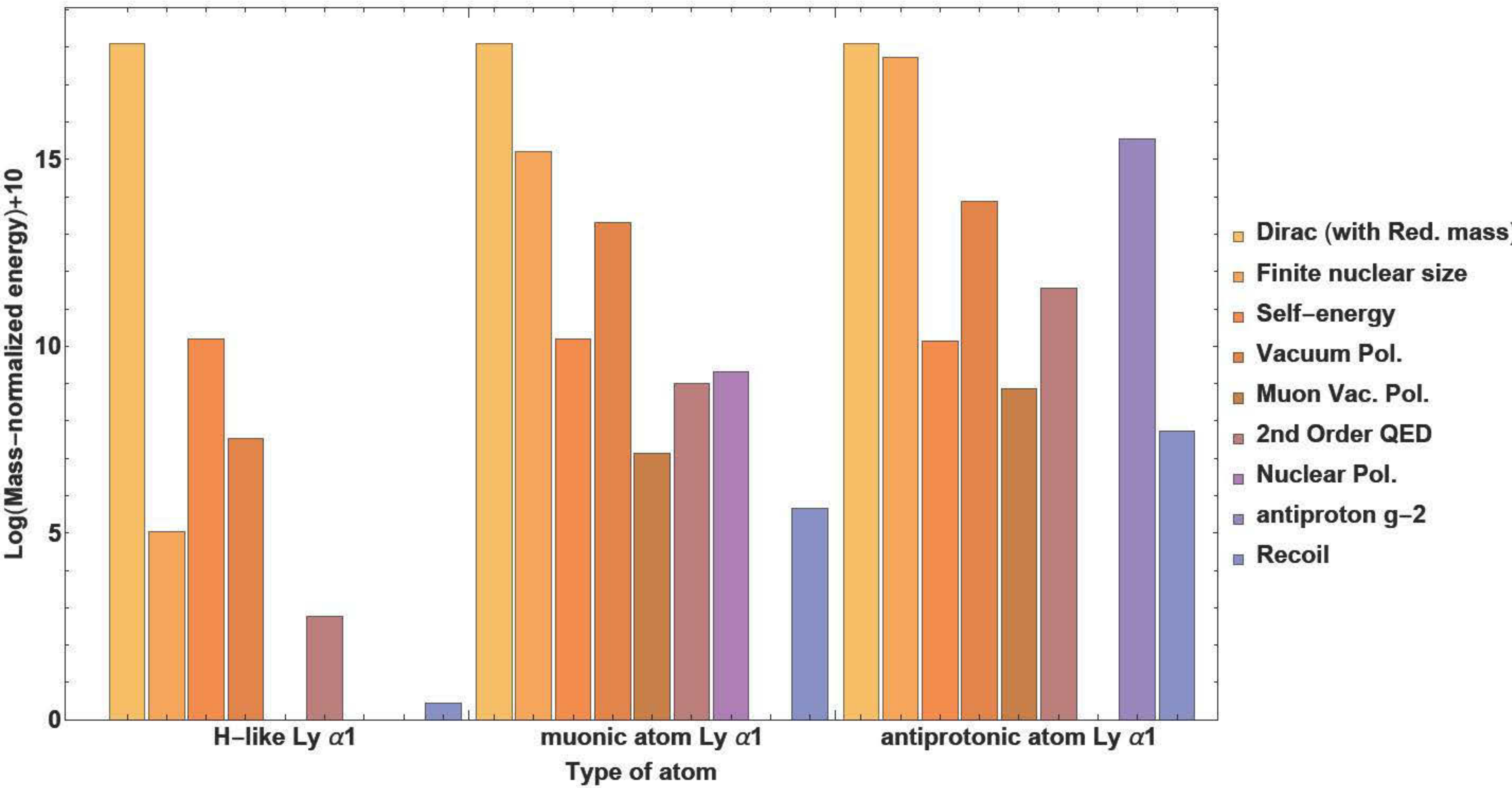
Finite size on Self-energy: exemple for Muonic lead



With Peter Mohr (NIST)

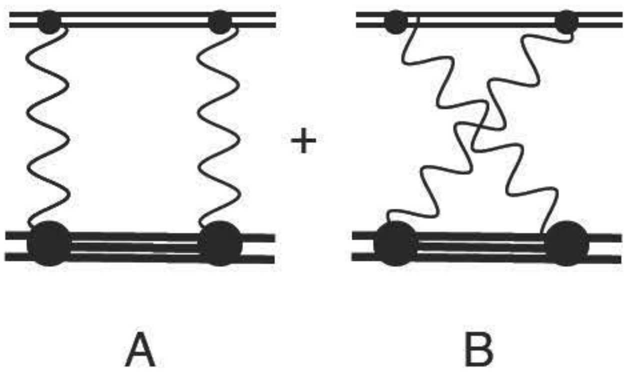
Some orders of magnitude



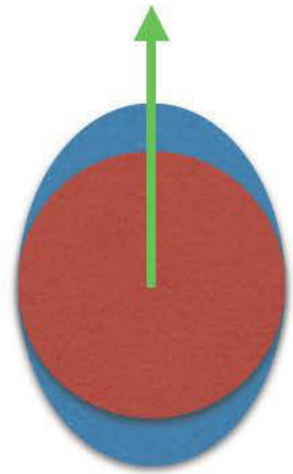
Nuclear contributions

beyond finite size

Internal nucleus structure



Nuclear polarization



Nuclear deformation

$$c = R_0\left[1 + \beta_2 Y_{20}(\theta,\phi) + \beta_4 Y_{40}(\theta,\phi)\right]$$

$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{(r-c)}{a}}}$$

The proton and deuteron size puzzle

An example of the different effects for the lightest element

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8 juillet 2010

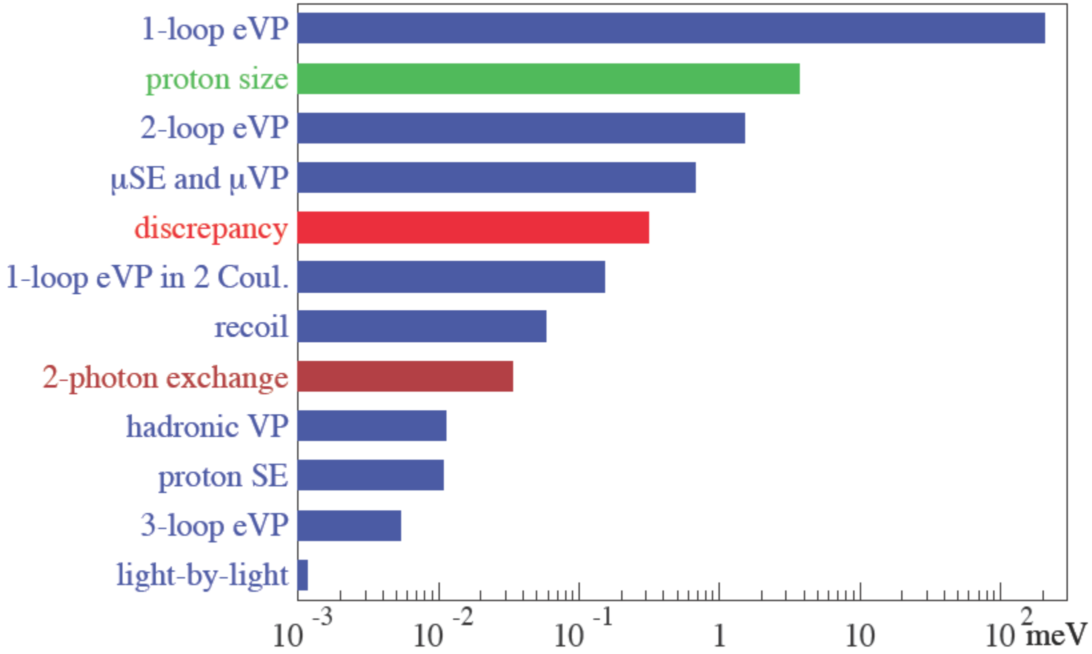
The size of the proton, R. Pohl, A. Antognini, F. Nez *et al.* Nature **466,** 213-216 (2010).

Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen, A. Antognini, F. Nez, K. Schuhmann *et al.* Science **339,** 417-420 (2013).

µP theory

Discrepancy	= 0.31 meV
Theory uncertainty	= 0.0025 meV

$$\Delta E^{\mathrm{th}} = 206.0668(25) - 5.2275(10)\, r_p^2 \quad [\mathrm{meV}]$$



Pachucki, PRA 60, 3593 (1999)

Borie, arXiv: 1103.1772-v7

Jentschura, Ann. Phys. 326, 500 (2011)

Karshenboim, J. Phys. Chem. Ref. Data 44, 031202 (2015)

Indelicato, Martynenko, Miller, Eides, Pineda....

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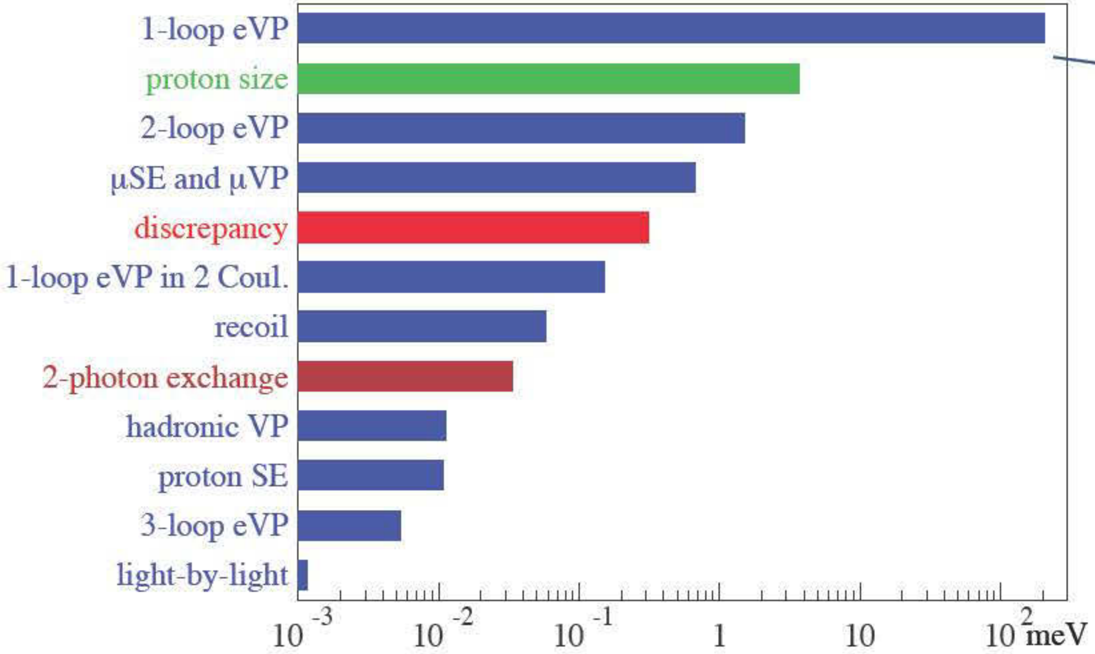

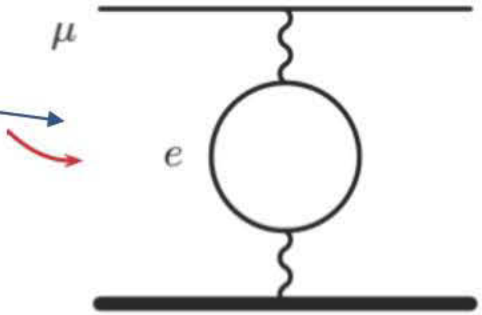

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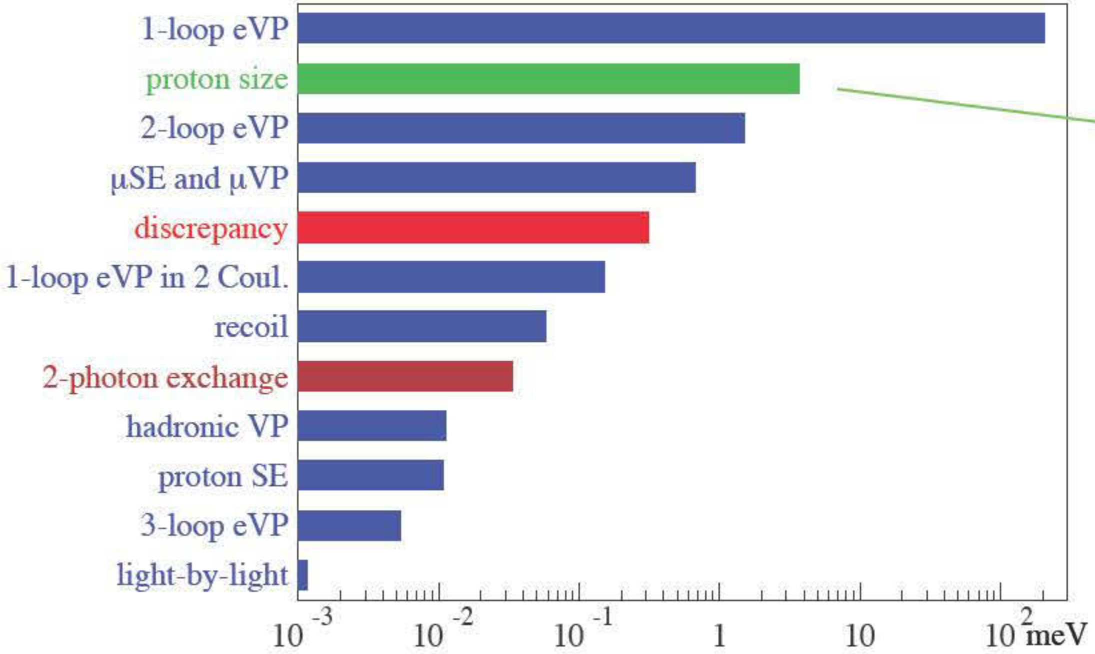

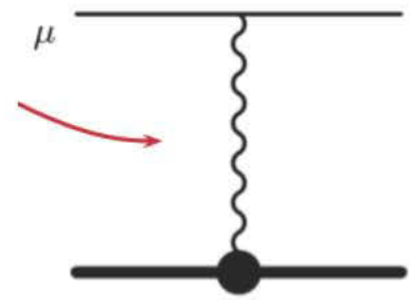

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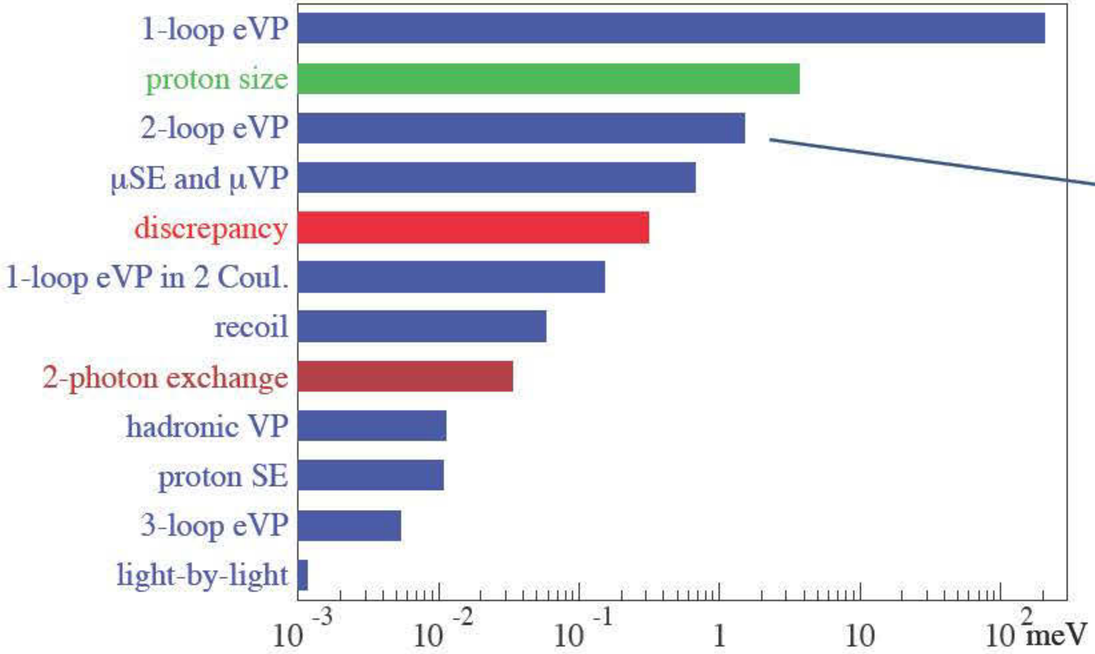

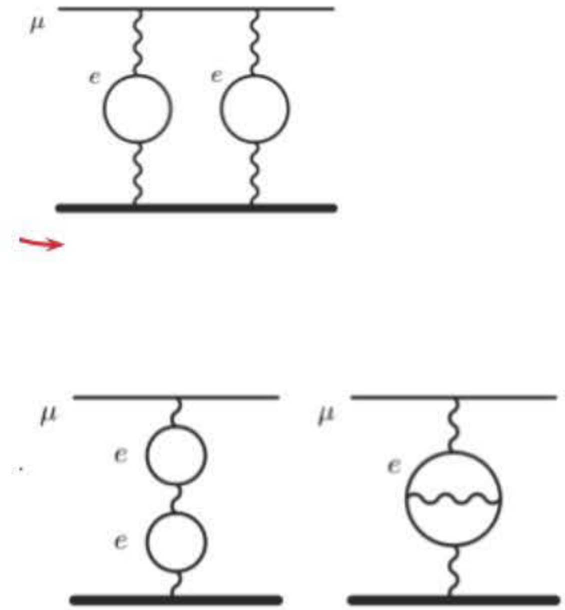

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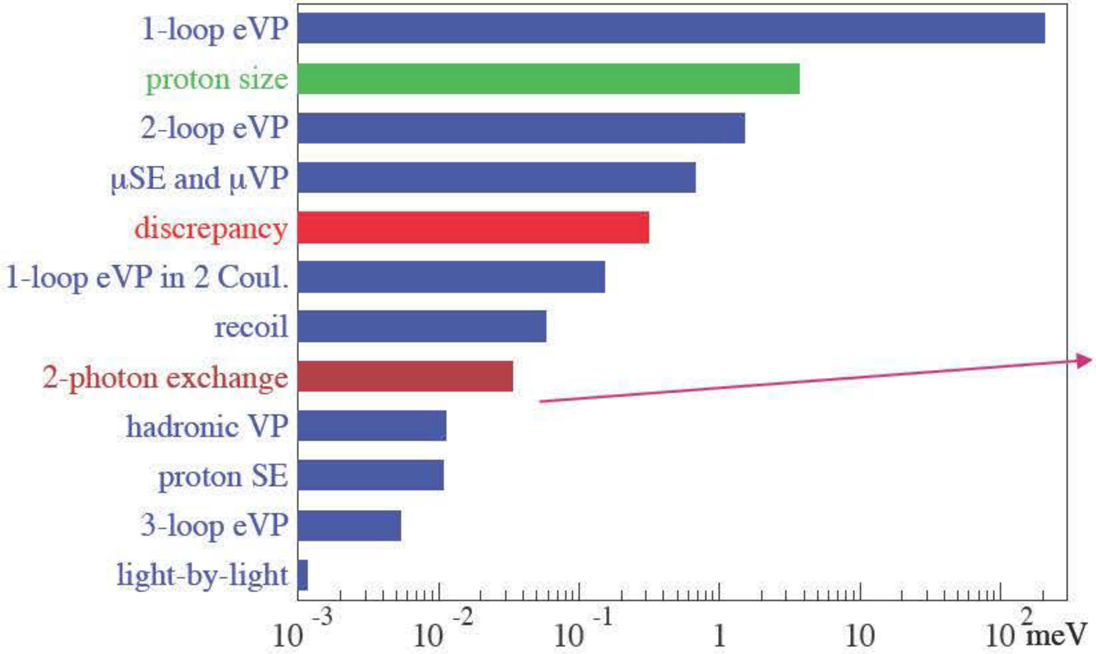

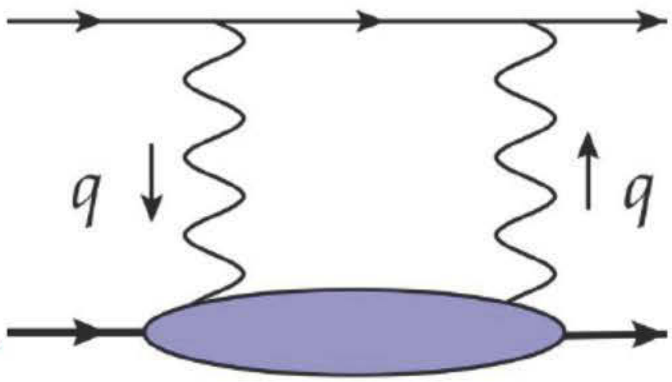

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Indelicato, Martynenko, Miller, Eides, Pineda....

Pachucki, PRA 60, 3593 (1999)
Nevado, Pineda, PRC 77, 035202 (2008)
Peset, Pineda, EPJA 51, 32 (2015)
Peset, Pineda, NPB 887, 69 (2014)
Carlson, Vanderhaeghen, PRA 84, 020102 (2011)
Hill, Paz, PRL 107, 160402 (2011)
Miller, arXiv:1209.4667 (2012)
Birse, McGovern, EPJA 48, 120 (2012)
Miller, PLB 718, 1078 (2013)
Gorchtein et al., PRA 87, 052501 (2013)]
Alarcon, Lensky, Pascalutsa,EPJC 74, 2852 (2014)
Tomalak, Vanderhaeghen, PRD 90, 013006 (2014)
Tomalak, Vanderhaeghen, EPJC 76, 125 (2016)
Hill, Paz, PRD 95, 094017 (2017)
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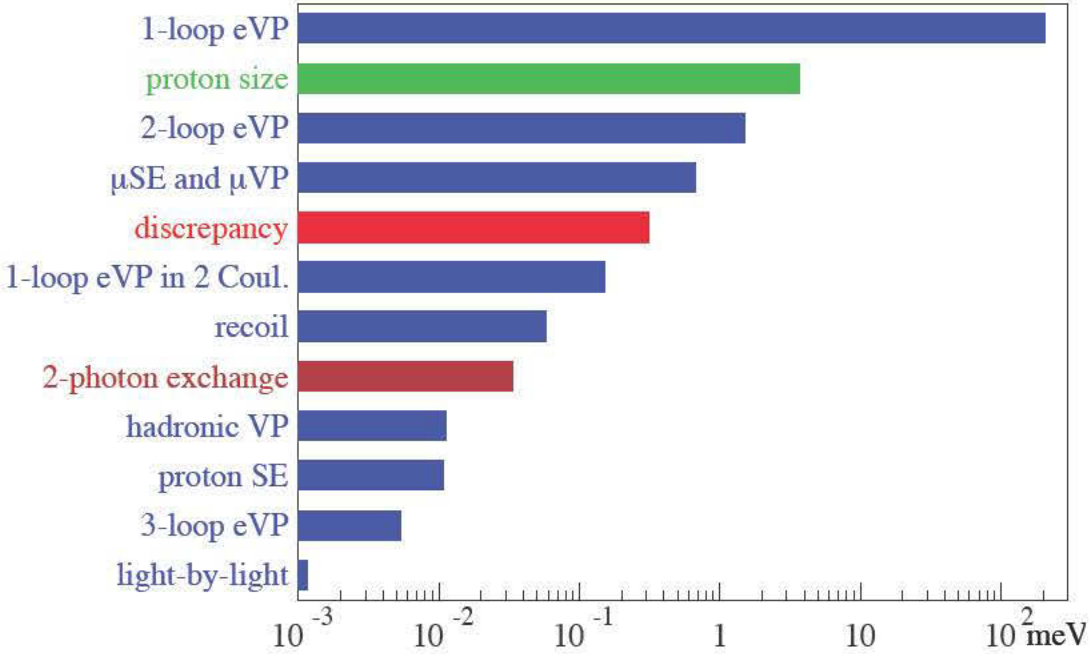

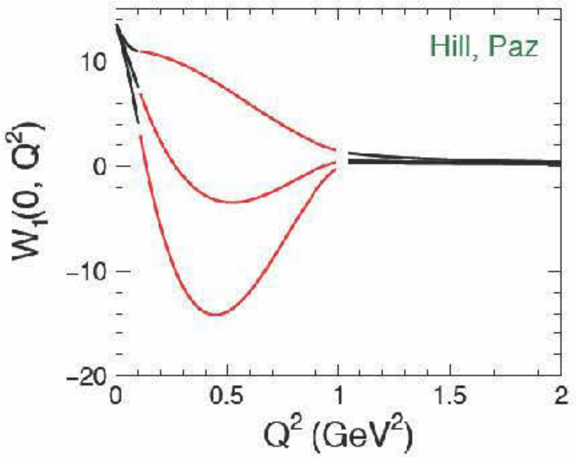

Subtraction term:
- low Q^2: NRQED + LEC
- medium Q^2: unknown
- high Q^2: OPE expansion

Discrepancy: 0.3 meV
TPE uncertainty: ± 0.01 meV
(**Conservatively**: Hill & Paz, 2017)

Pachucki, PRA 60, 3593 (1999)

Borie, arXiv: 1103.1772-v7

Jentschura, Ann. Phys. 326, 500 (2011)

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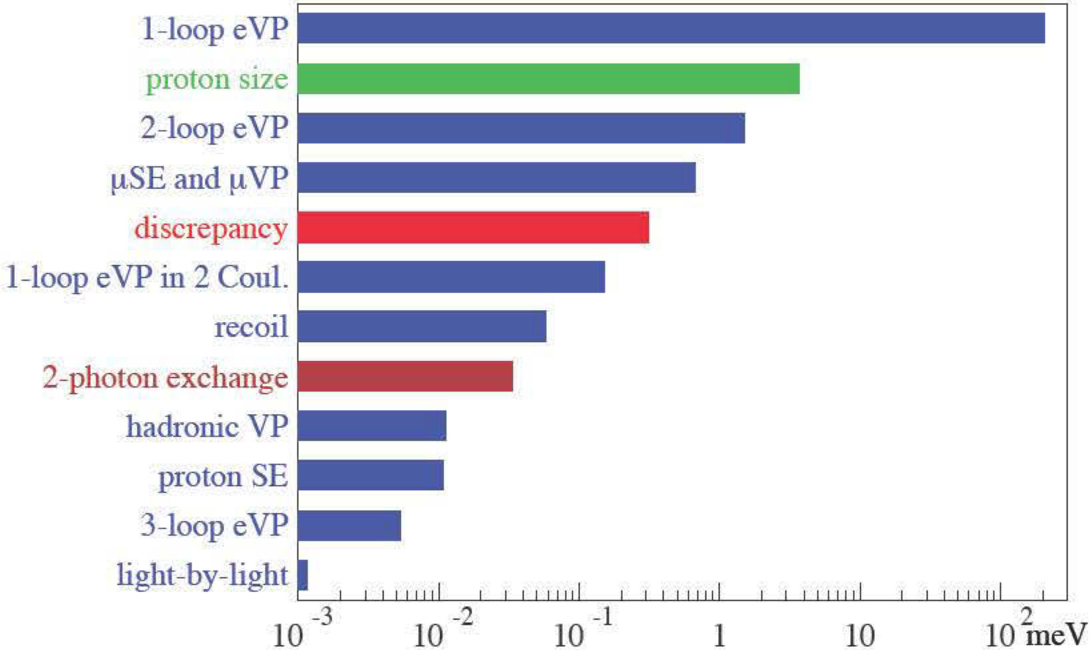

Δ (1232)-Resonance in the Hydrogen Spectrum

arXiv:1801.09790 Hagelstein 0.95μeV

Pachucki, PRA 60, 3593 (1999)

Borie, arXiv: 1103.1772-v7

Jentschura, Ann. Phys. 326, 500 (2011)

Karshenboim, J. Phys. Chem. Ref. Data 44, 031202 (2015)

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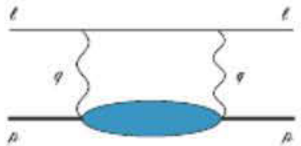


Fig. 1 Two-photon-exchange diagram in forward kinematics: the horizontal lines correspond to the lepton and the proton (bold). The "blob" represents all possible excitations in the non-Born diagrams.

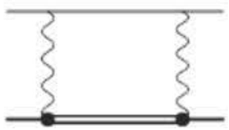
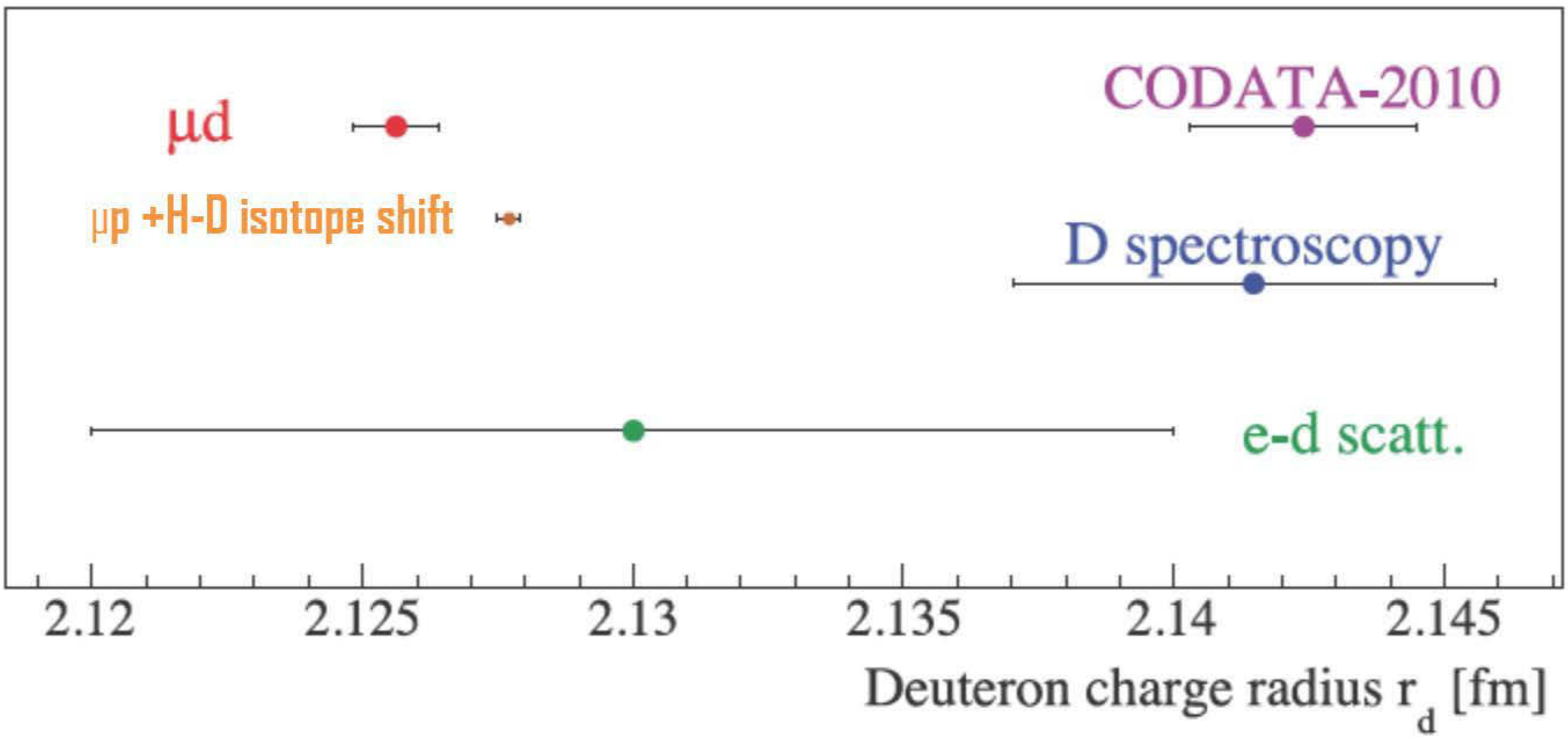
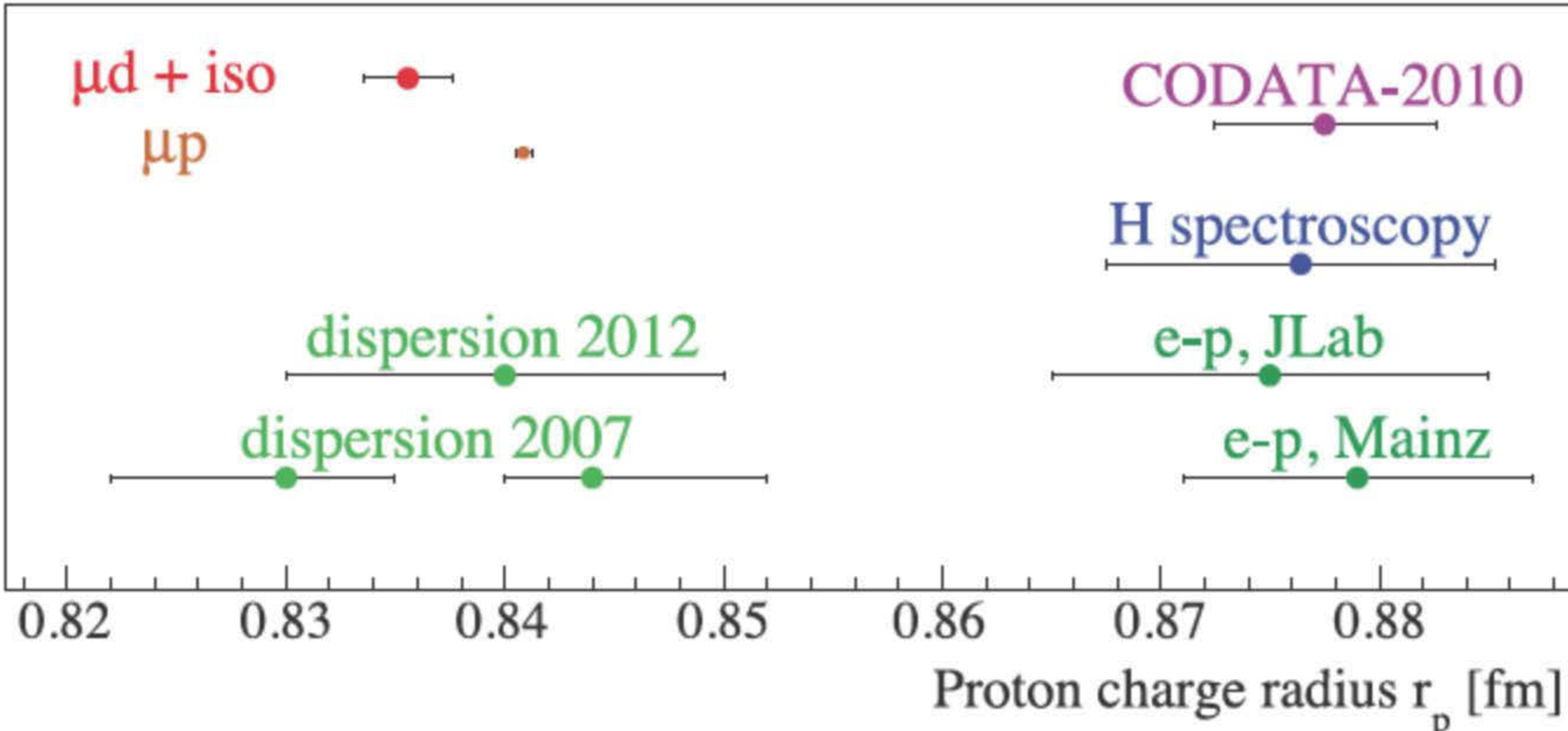


Fig. 2 Two-photon-exchange diagram with intermediate $\Delta(1232)$-excitation. The crossed diagram is not drawn.

deuteron charge radius



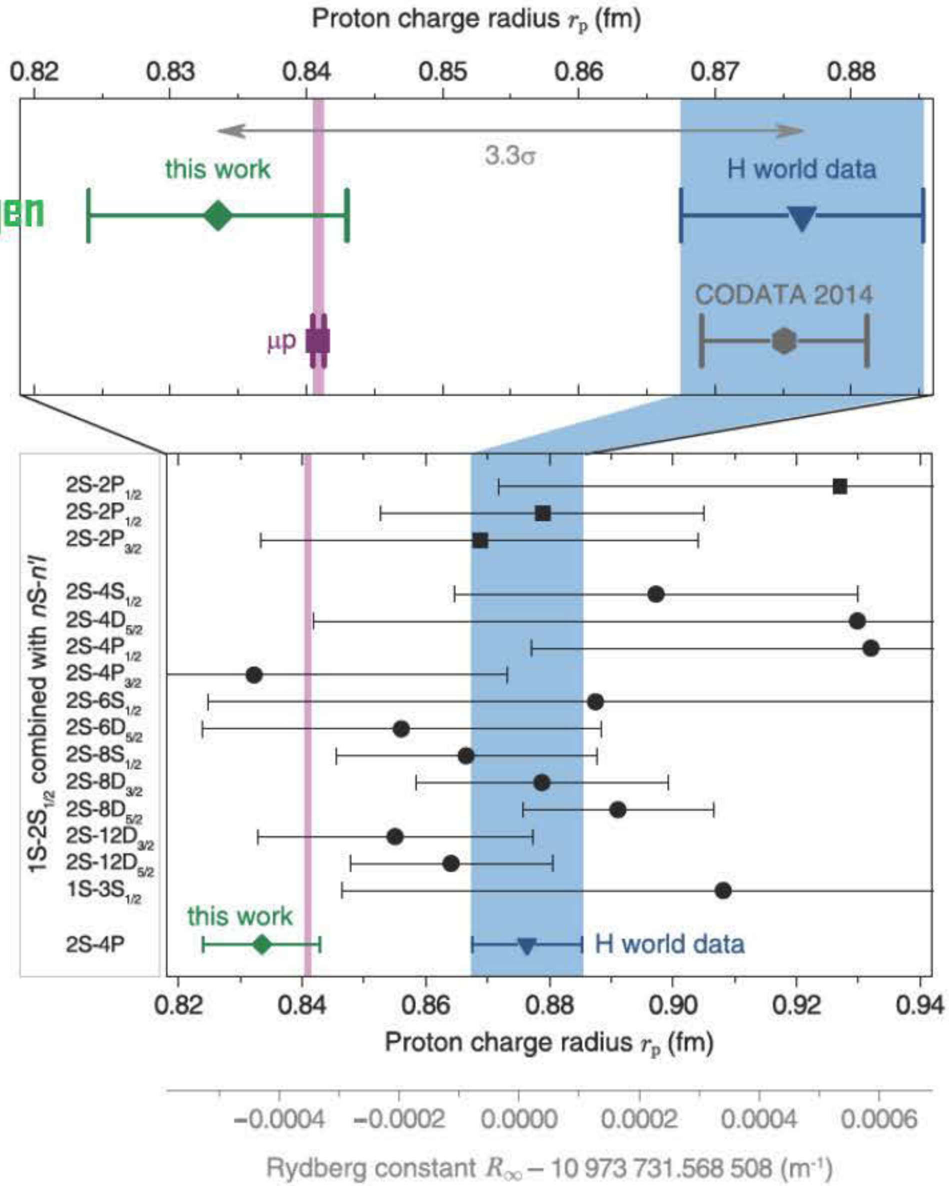
proton charge radius from deuteron



Deuton and proton from muonic atoms in agreement

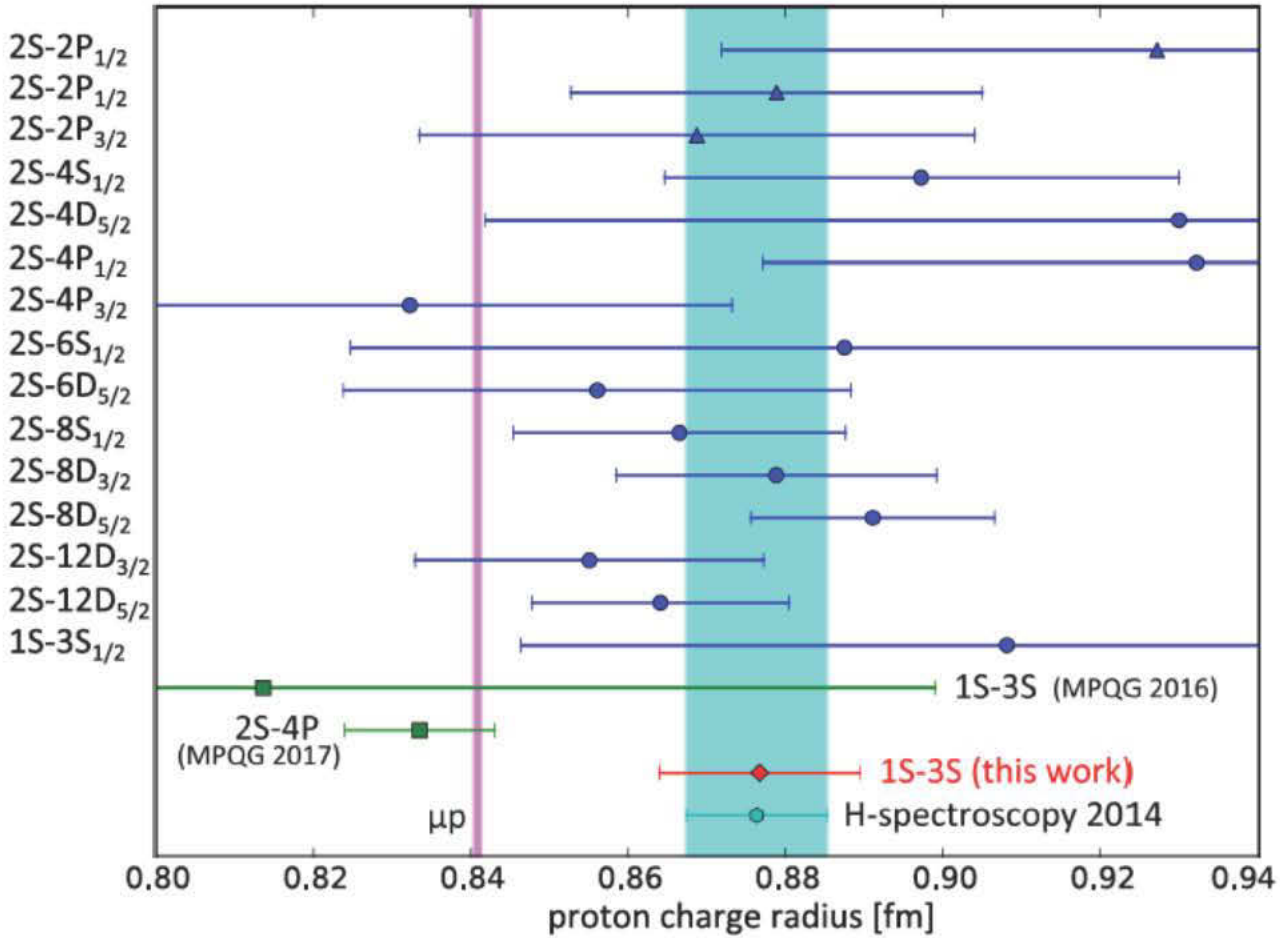
Status at the end of 2017

2S-4P transition in hydrogen



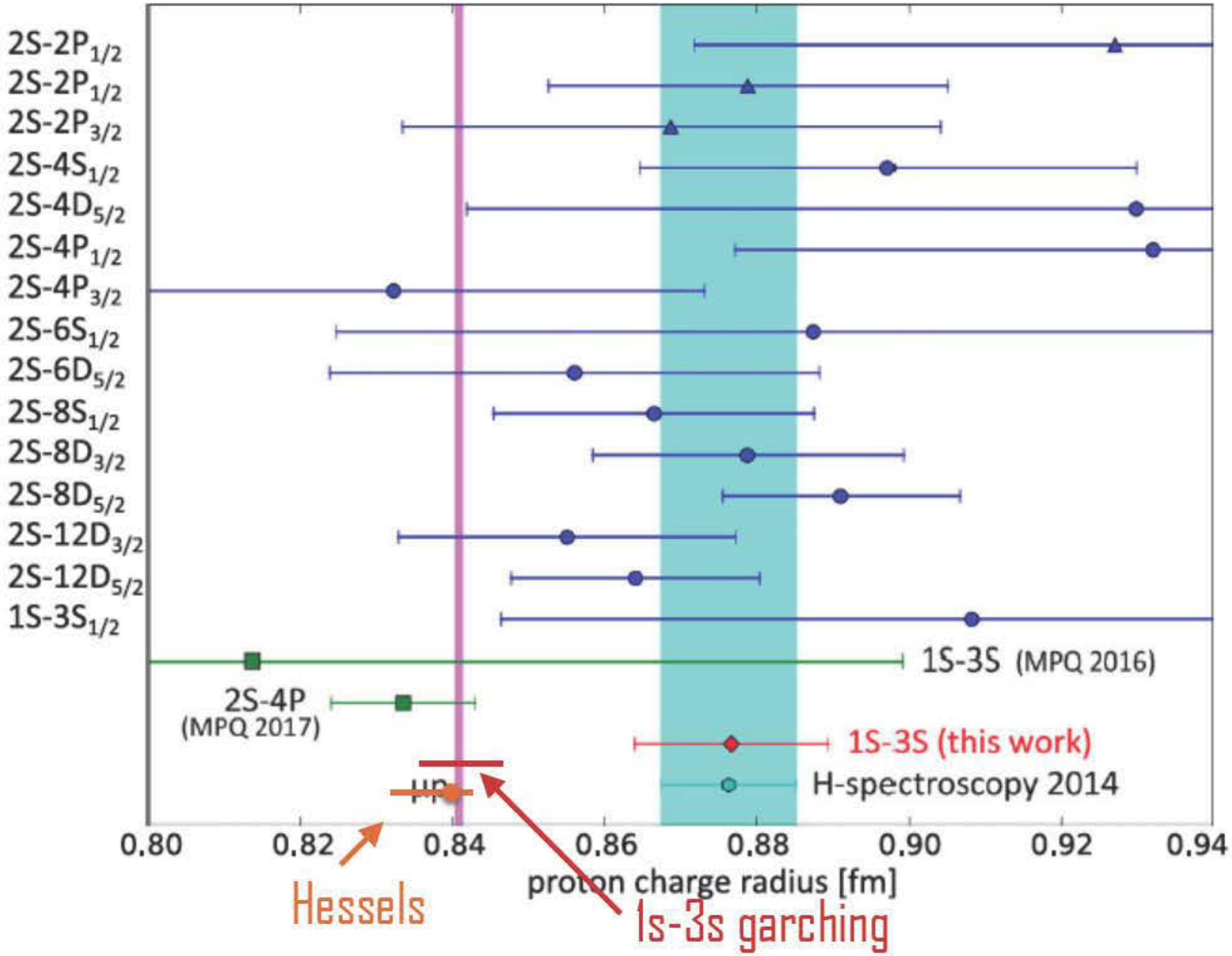
Beyer, A., et al. <u>Science</u> **358**(6359): 79-85.

Status at the beginning of 2018



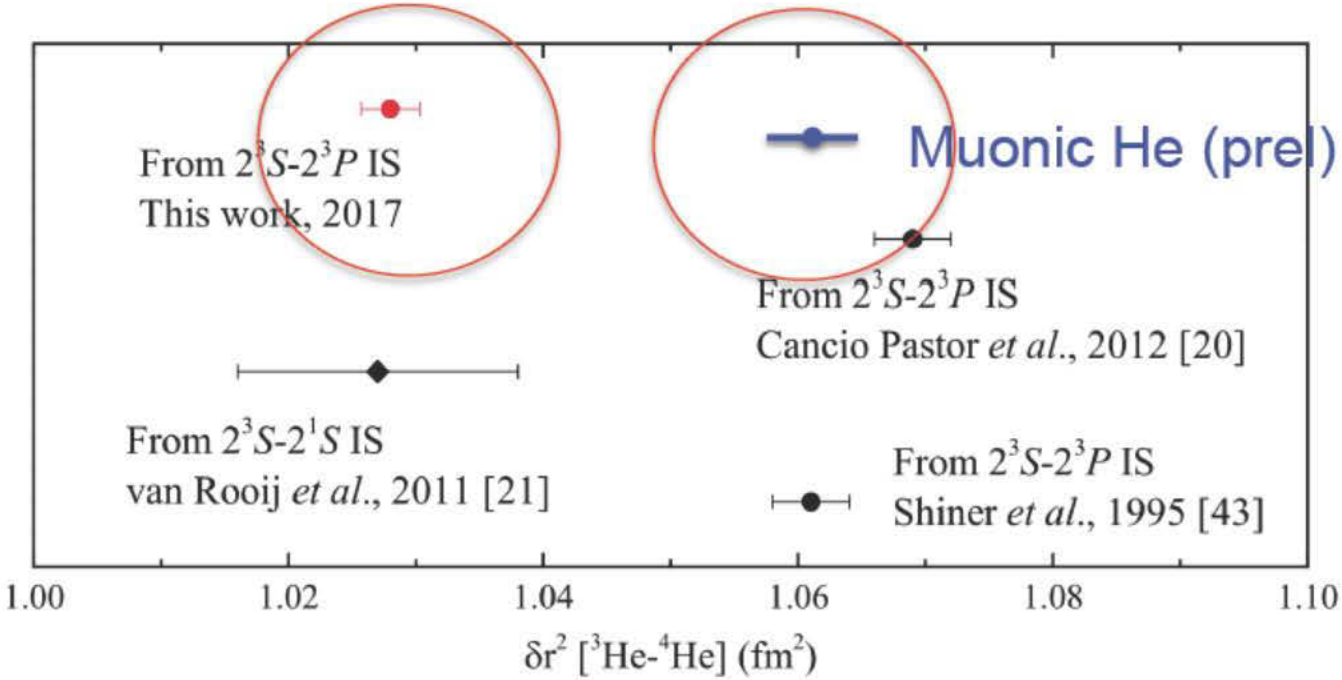
H. Fleurbaey, S. Galtier, S. Thomas, et al., Phys. Rev. Lett. **120**, 183001 (2018).

Proton charge radius (summer 2018)



H. Fleurbaey, S. Galtier, S. Thomas, et al., Phys. Rev. Lett. **120**, 183001 (2018).

The ^{3}He-^{4}He isotopic shift puzzle



[1] X. Zheng, Y. R. Sun, J. J. Chen, et al., Phys. Rev. Lett. **119**, 263002 (2017).

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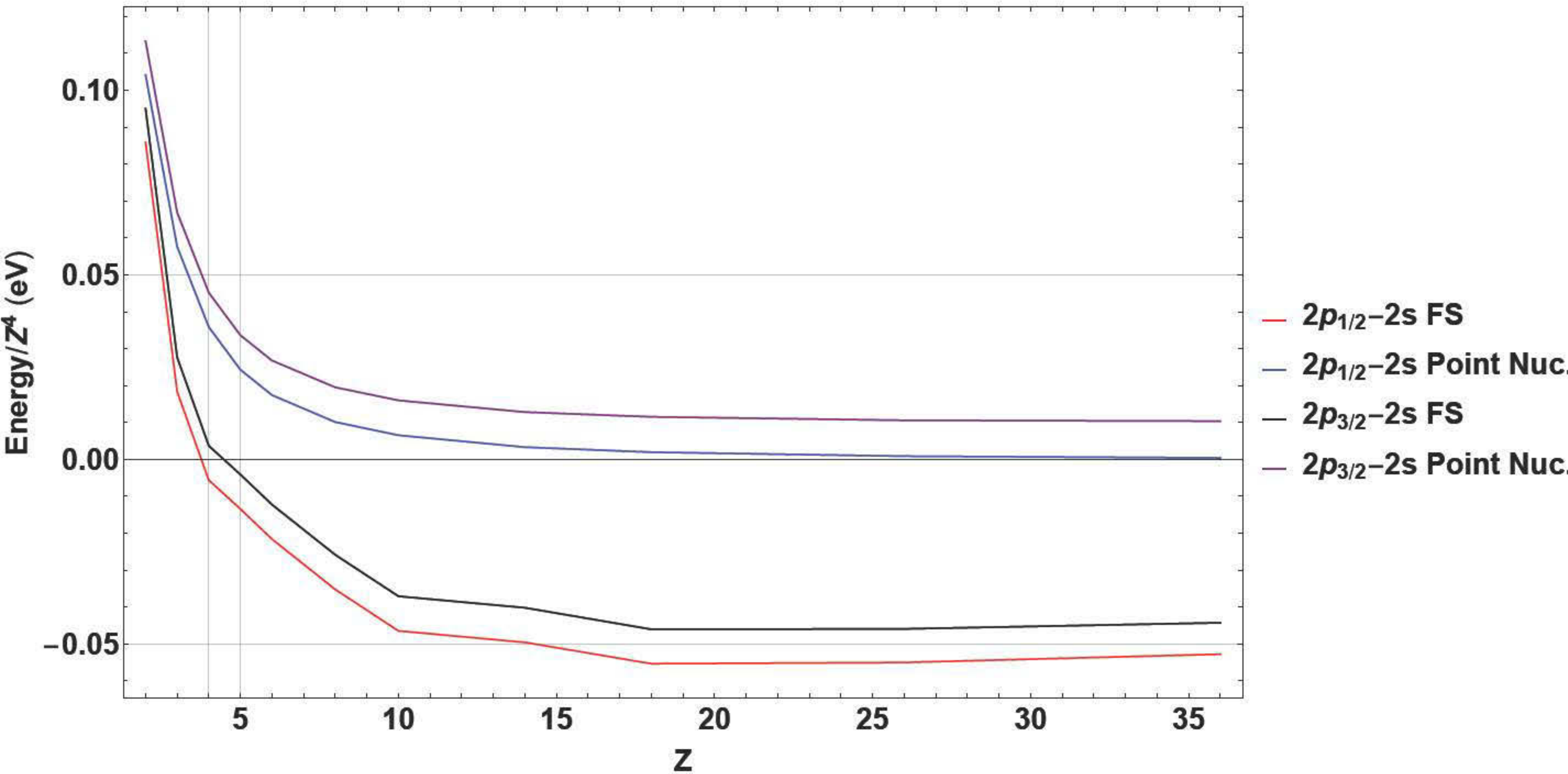
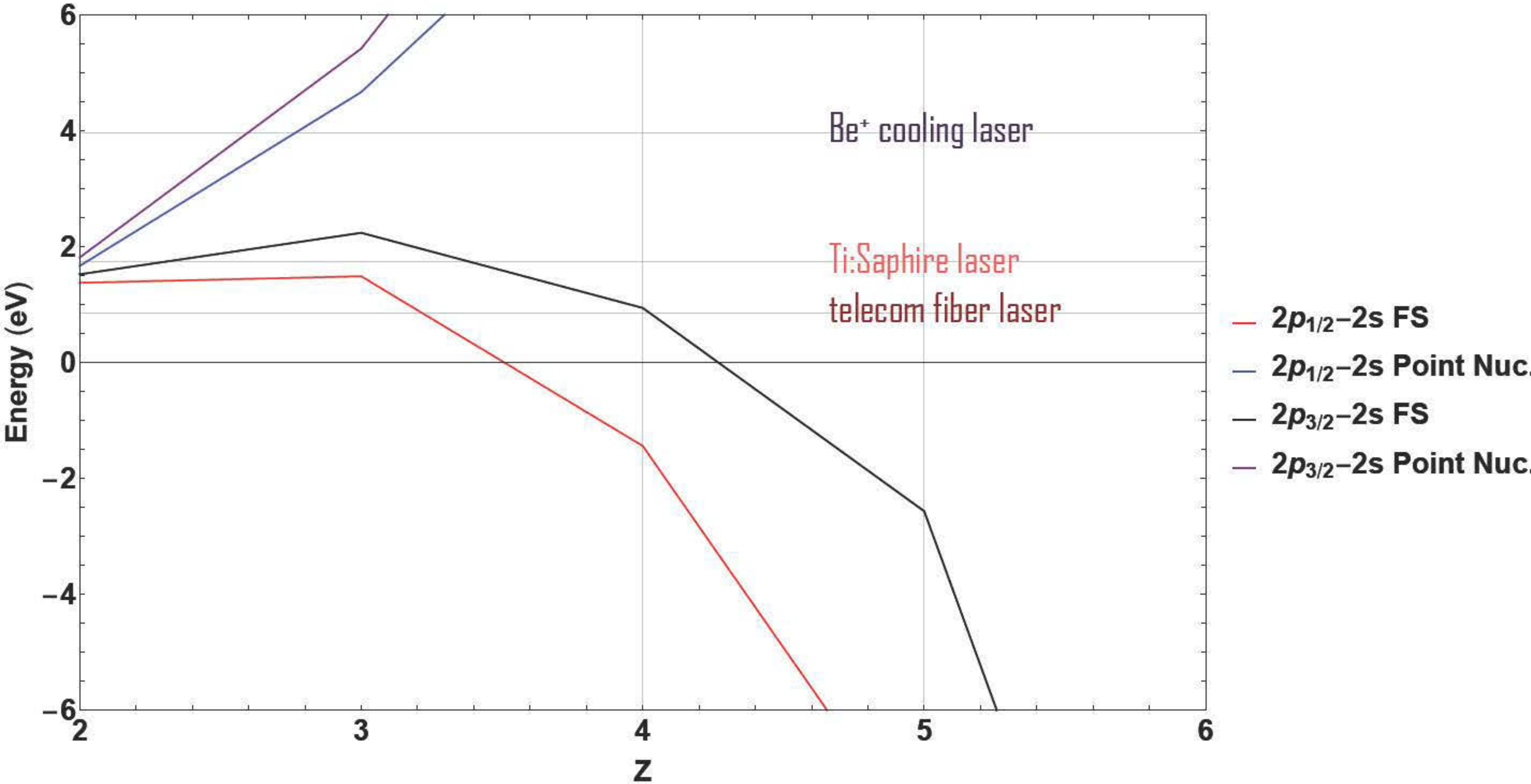
Can it be done for higher Z

Interplay between QED and finite nuclear size

Exemple: Lamb-shift on muonic atoms

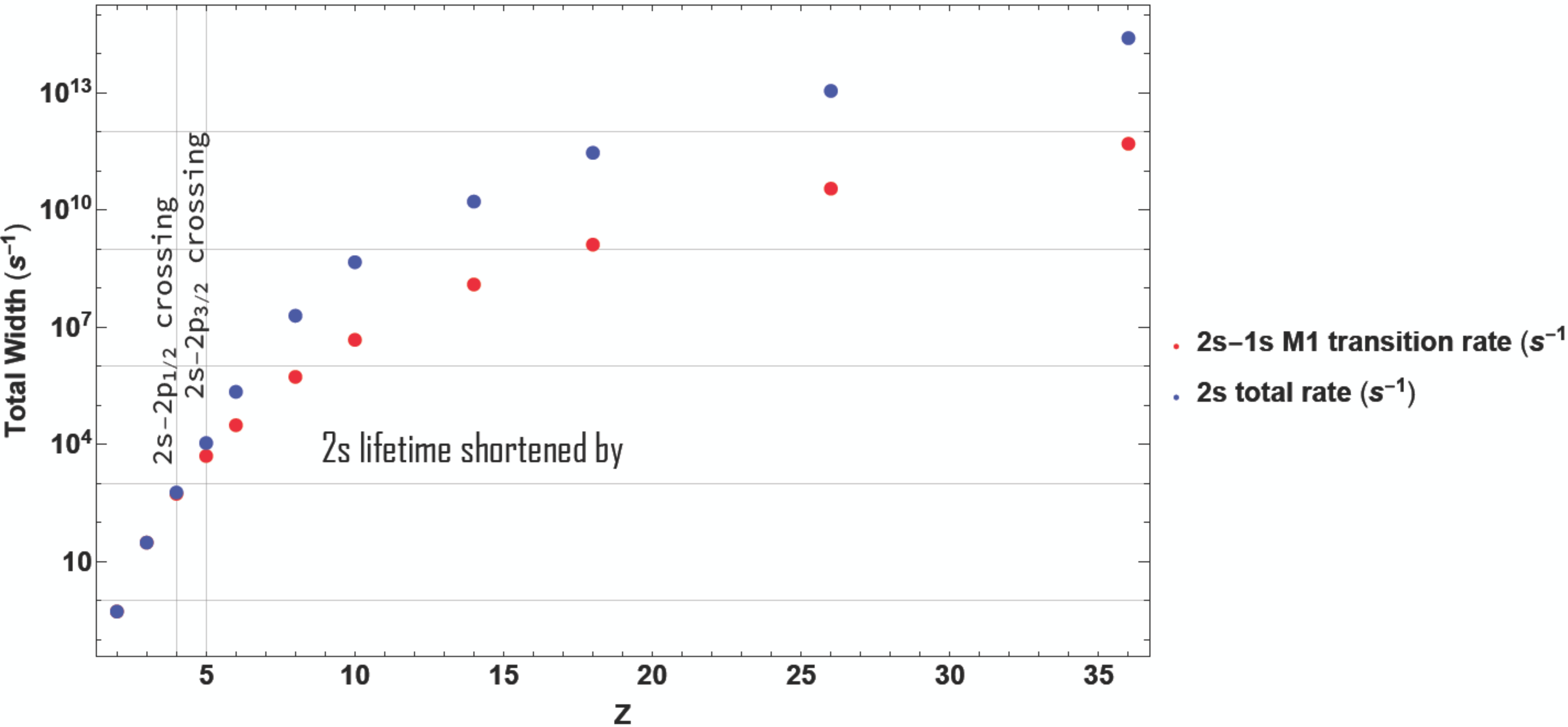


2s-2p splitting



look at the HFS structure before concluding...

2s lifetime changes due to crossing



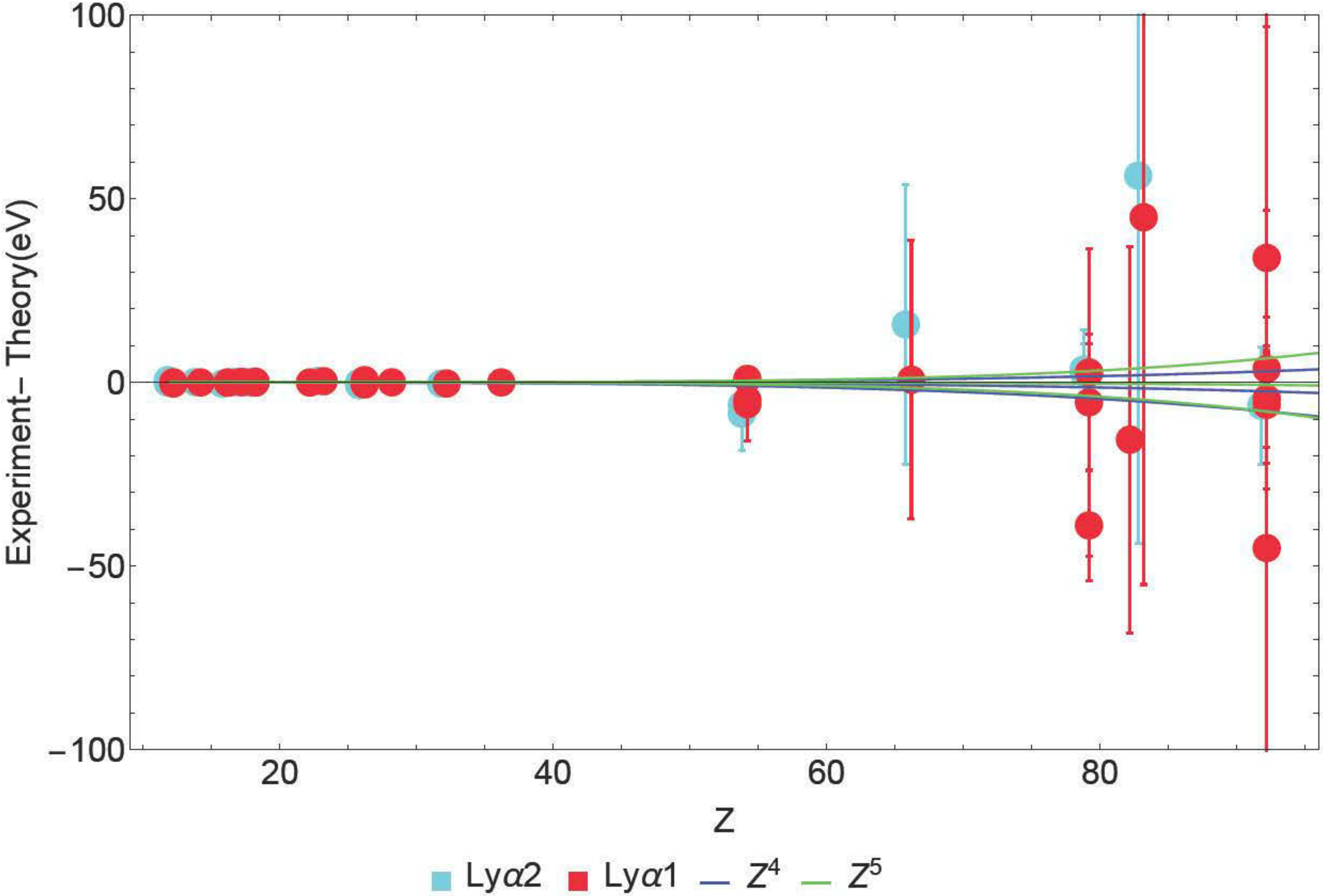
Comparison between theory and experiment

Few electron ions

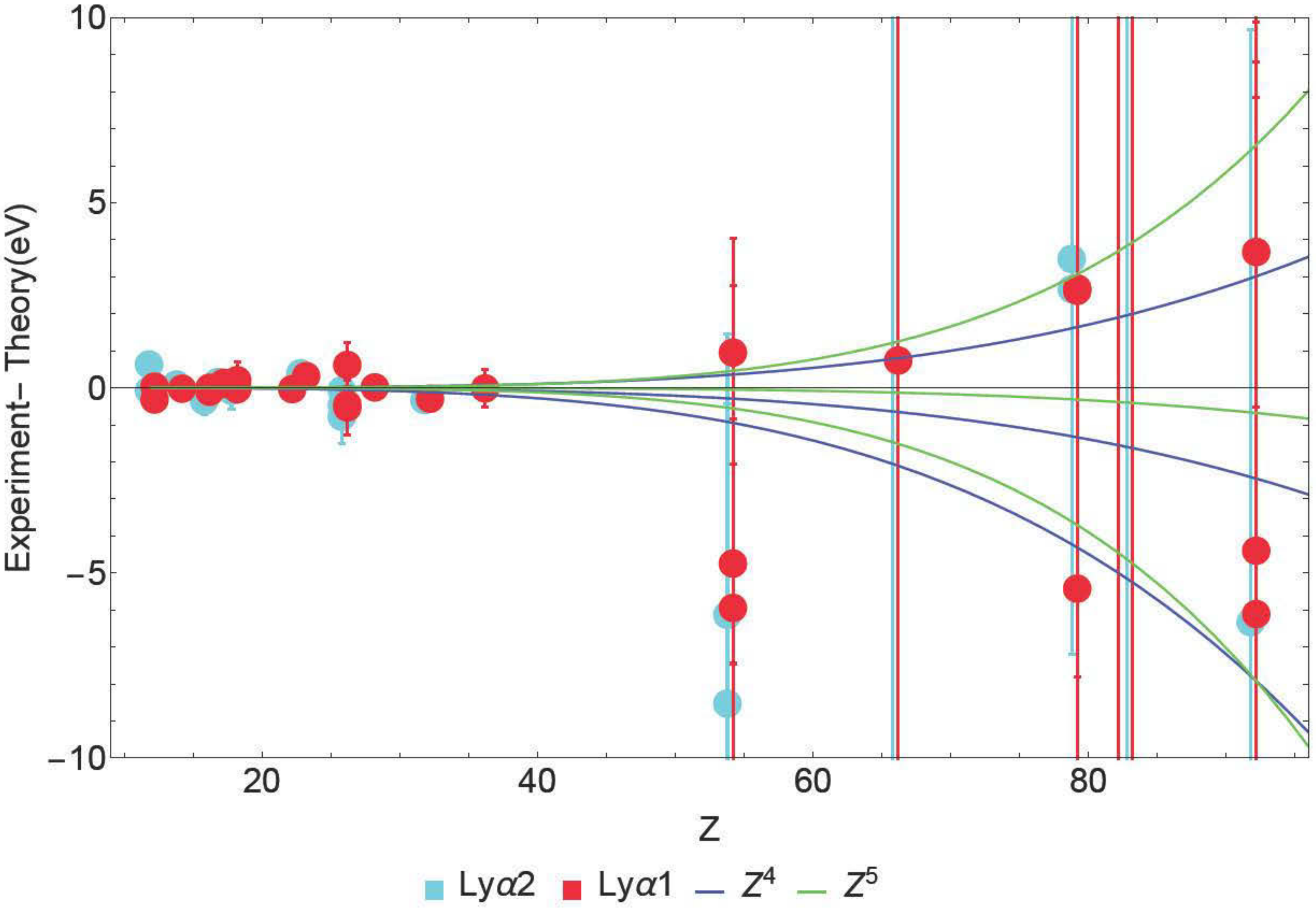
Analysis for one-electron ions

So far so good…

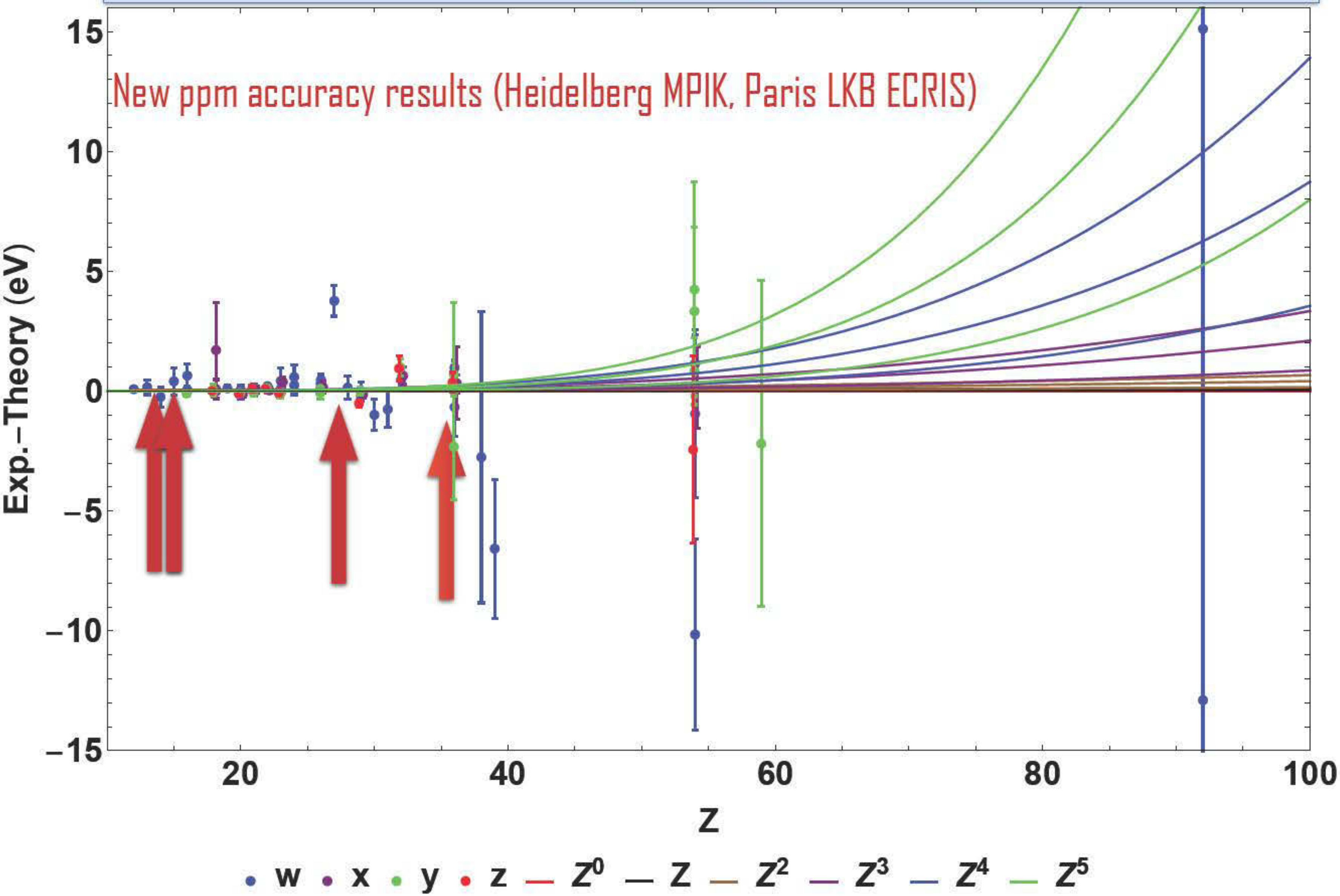
Comparison Theory-Experiment for H-like ions



Comparison Theory-Experiment for H-like ions



He-like ions: improving the comparison



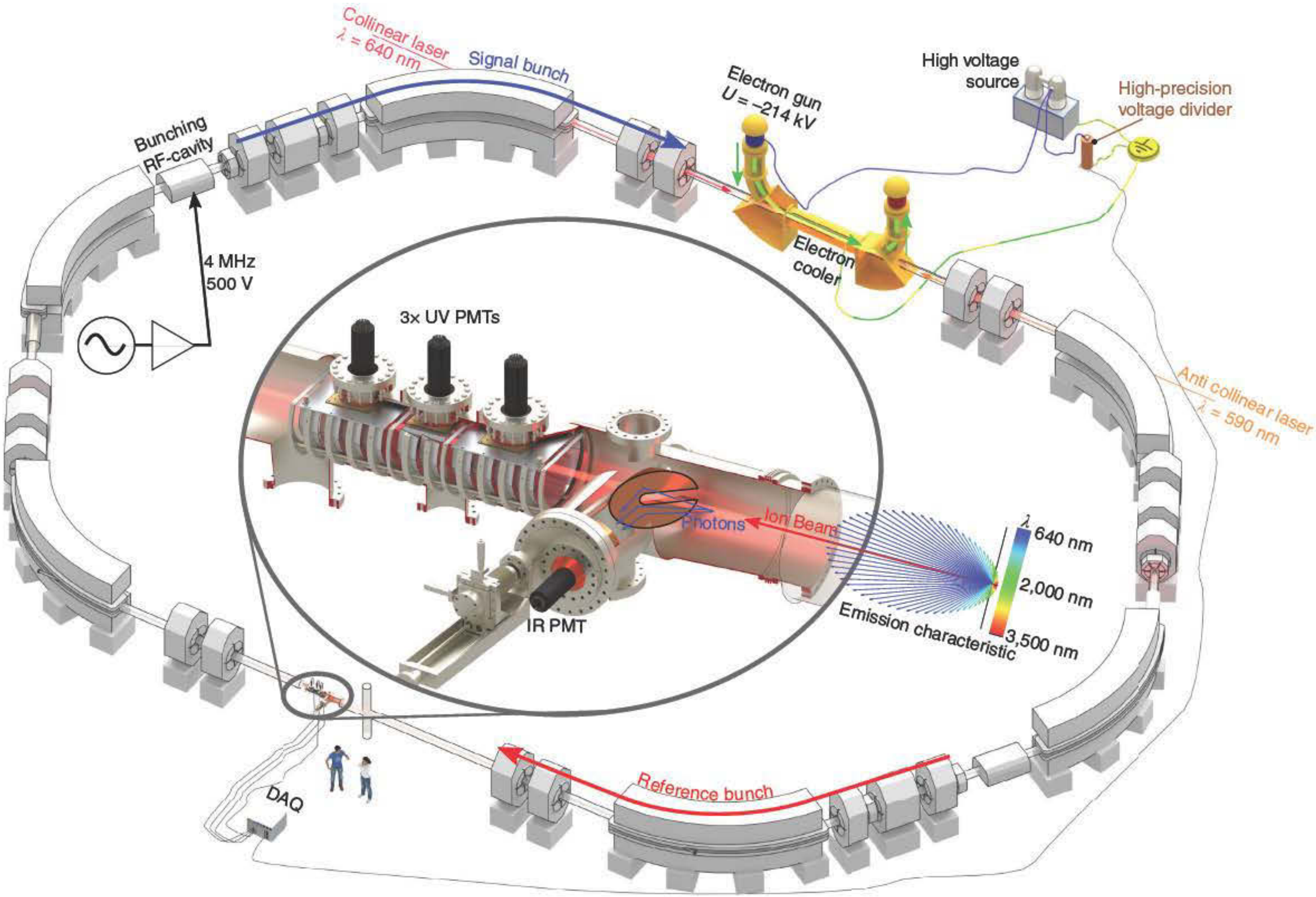
w: $1s2p\ ^1P_1 \rightarrow 1\ s^2$; z: $1s2p\ ^3S_1 \rightarrow 1\ s^2$; x: $1s2p\ ^3P_1 \rightarrow 1\ s^2$; x: $1s2p\ ^3P_2 \rightarrow 1\ s^2$

LKB

Hyperfine structure of H-like ions

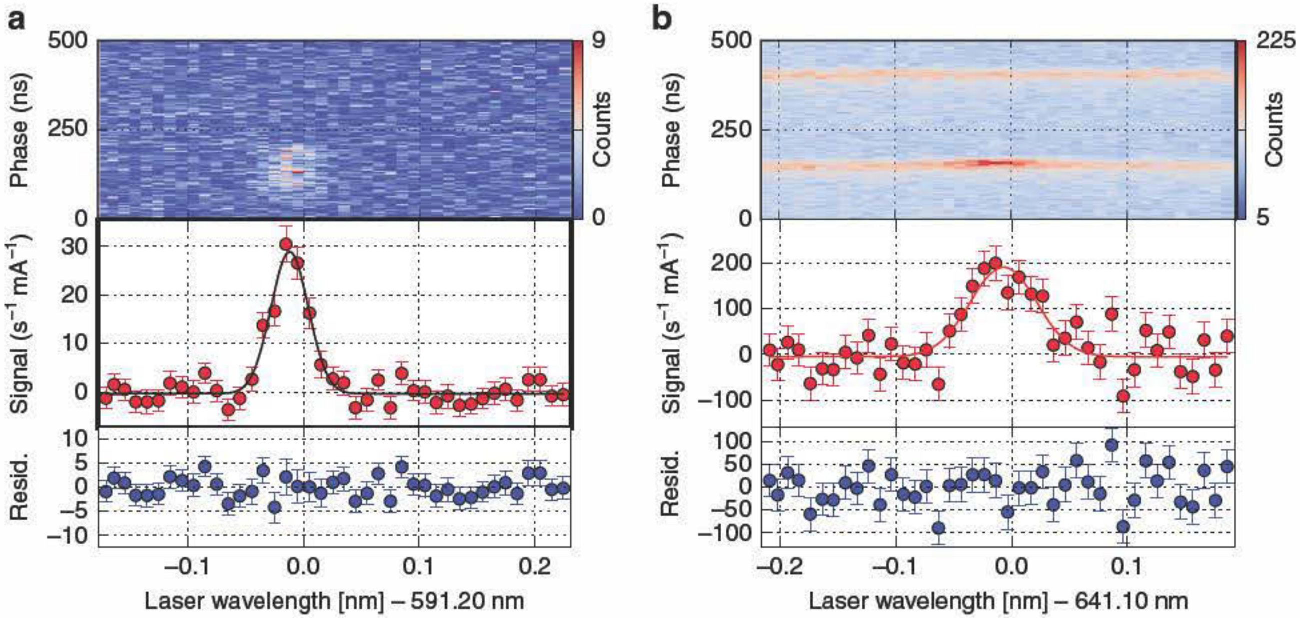
Bi^{82+} and Bi^{80+}

Hydrogen-like Bi Hyperfine structure



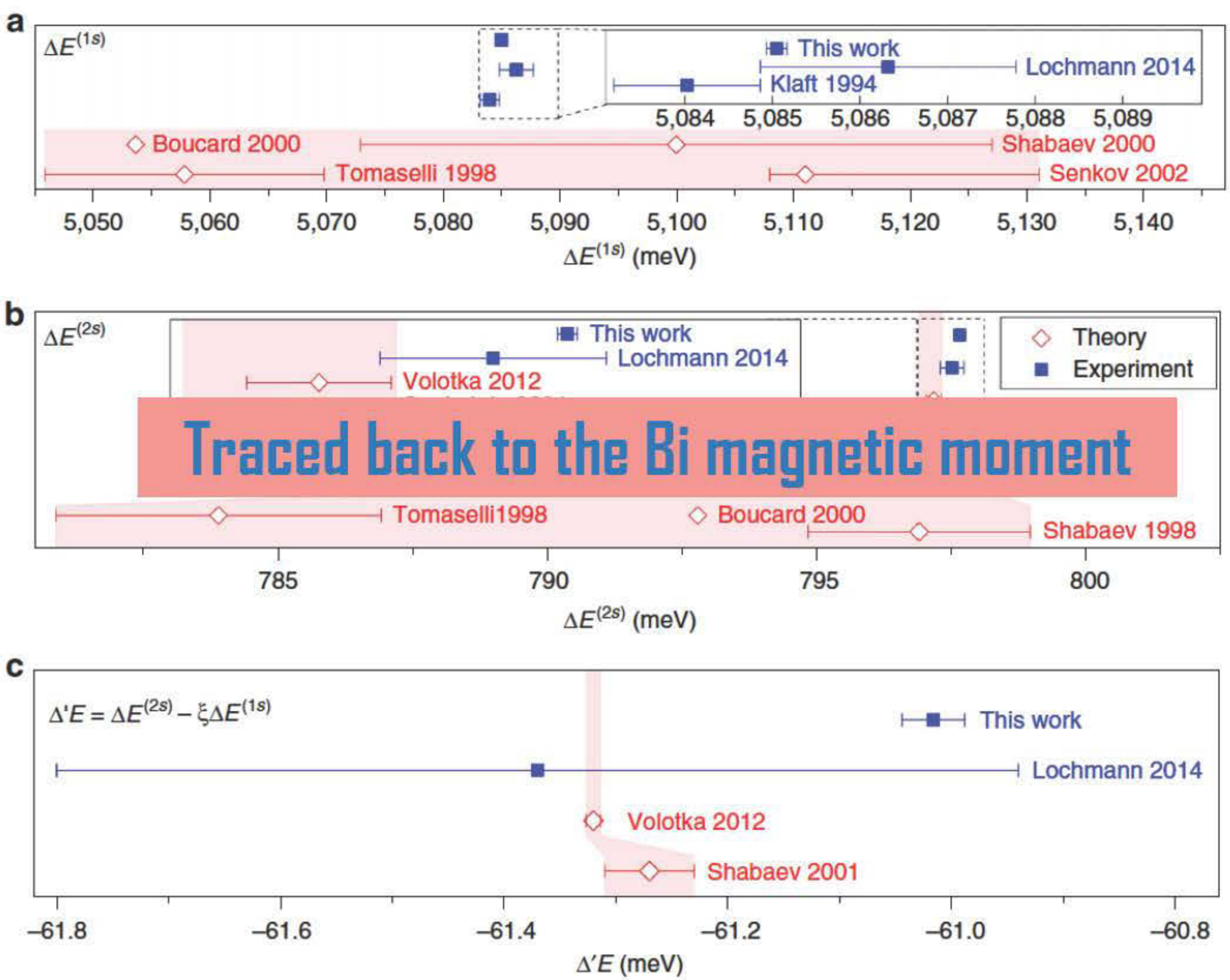
J. Ullmann, Z. Andelkovic, C. Brandau, et al., Nature Communications 8, 15484 (2017).

Hydrogen-like Bi Hyperfine structure



J. Ullmann, Z. Andelkovic, C. Brandau, et al., Nature Communications 8, 15484 (2017).

Hyperfine structure in few-electron Bi



ξ is evaluated to cancels out Bohr-Weisskopf effect

Effect on atoms

Is there an effect of the nuclear charge distribution shape beyond RMS radii on atomic energies

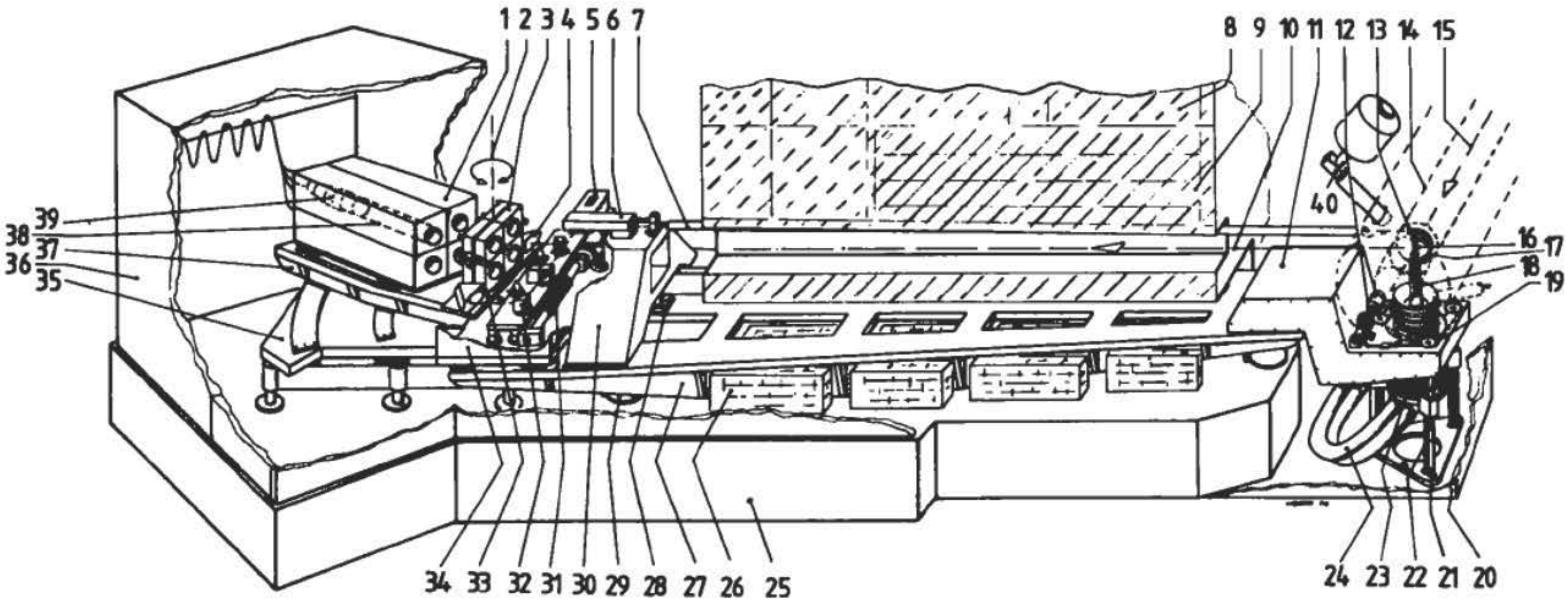
Beyond mean square radius?

- For electronic atoms, it is usually claimed that the atomic energy depends only on the mean square radius
  - are there higher order effects?
  - how can we compare distributions beyond RMS radii?
- Is it true for muonic atoms?
- Method: compare the effect of different models by high precision numerical calculations on normal and muonic atoms
  - mcdfgme 2019 Dirac-Fock code
  - many points inside the nucleus (a few thousand)
  - Use higher-order $\langle r^n \rangle$ moments to compare distributions

Results

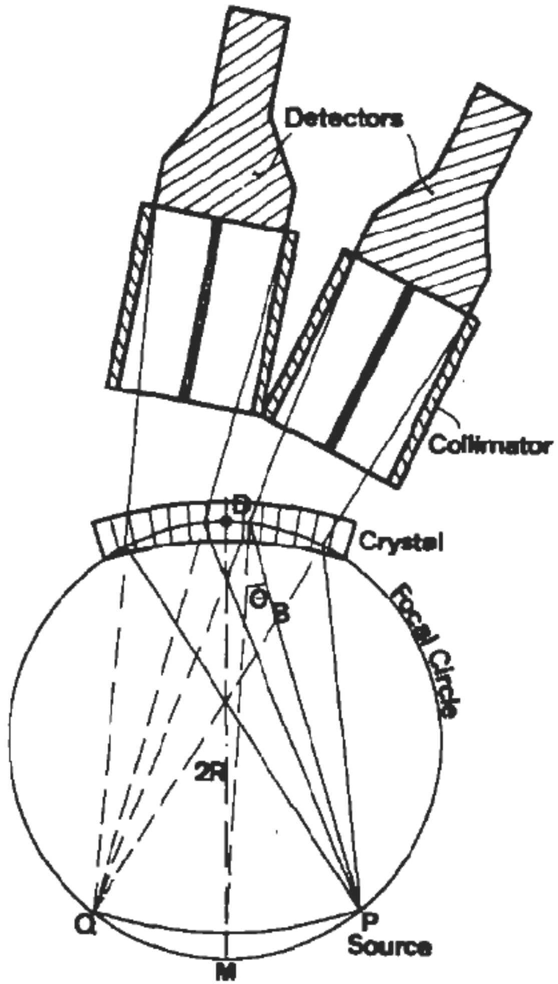
muonic ^{12}C

High-energy transmission spectrometer



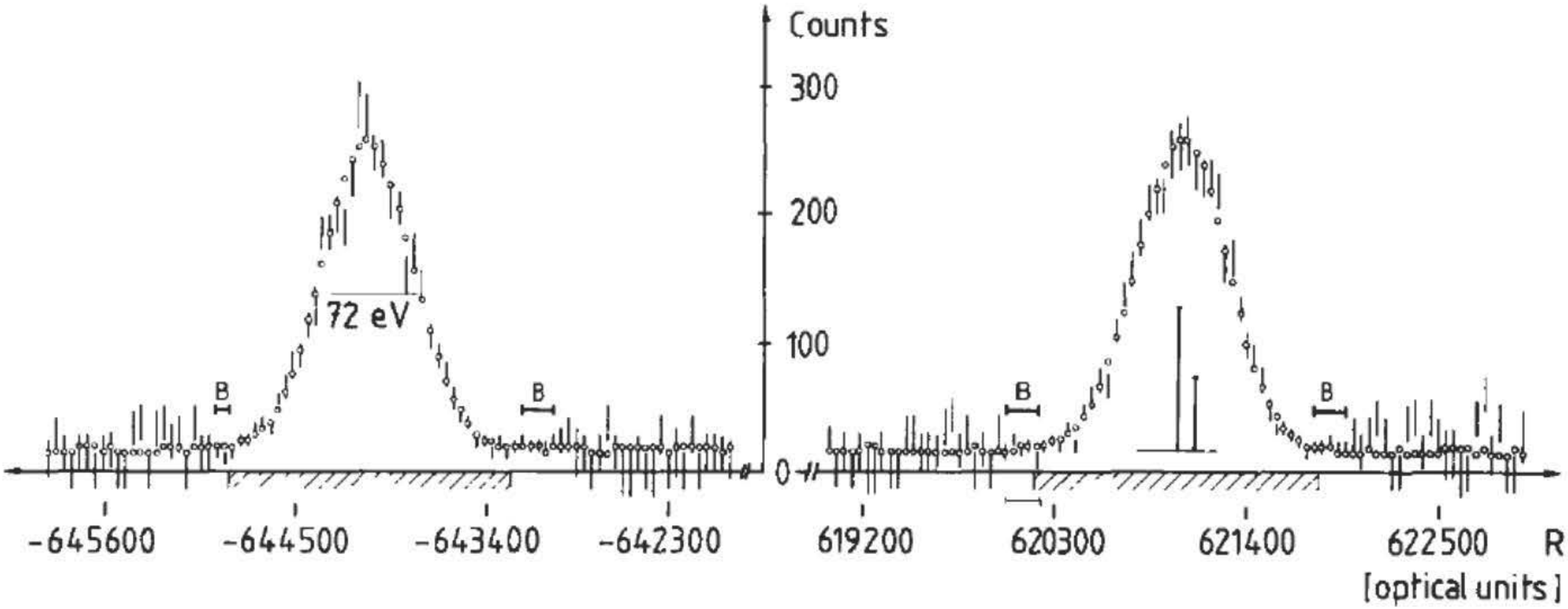
Aas, B., R. Eichler and H. J. Leisi (1982). "Transition wavelengths of muonic X-rays of magnesium, silicon and phosphorus." Nuclear Physics A **375**(3): 439-452.

High-energy transmission spectrometer



Aas, B., R. Eichler and H. J. Leisi (1982). "Transition wavelengths of muonic X-rays of magnesium, silicon and phosphorus." Nuclear Physics A **375**(3): 439-452.

muonic Carbon 1984



(a)

new all order calculations, recalibration of the reference used (^{170}Tm g-ray) for 2010 Si lattice spacing and wavelength to energy conversion

W. Ruckstuhl, B. Aas, W. Beer, et al., Nuclear Physics A **430**, 685 (1984).

New muonic ^{12}C evaluation

- All-order Dirac equation solution with charge distribution and Vacuum polarization
- All contributions known to date
- CODATA fundamental constants to re-evaluate experimental energy
  - silicon lattice spacing
  - conversion from wavelength to energy
- Check ^{170}Tm γ-ray energy with current tables

- Result: 75261.69±0.40 eV

"Model independent" charge distribution descriptions

- Ingo Sick's Sum of Gaussians model

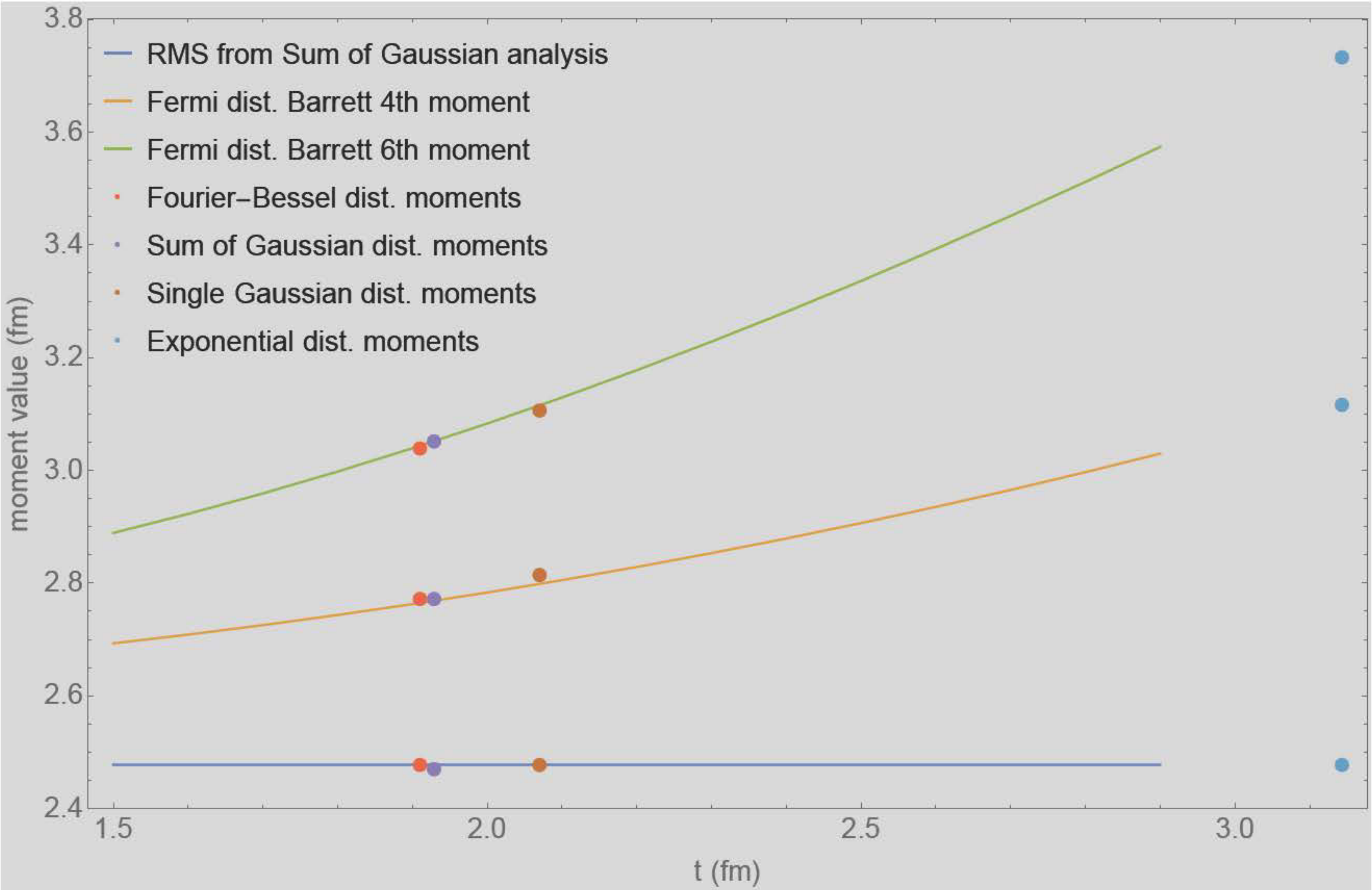
$$\rho_{\mathrm{SOG}}(r) = \sum_i \frac{3\sqrt{\frac{3}{2}}Q_i\left(e^{-\frac{3(r-R_i)^2}{2R_{\mathrm{rms}}^2}}+e^{-\frac{3(r+R_i)^2}{2R_{\mathrm{rms}}^2}}\right)}{4\pi^{3/2}R_{\mathrm{rms}}^3\left(\frac{3R_i^2}{R_{\mathrm{rms}}^2}+1\right)}$$

- Fourier-Bessel model

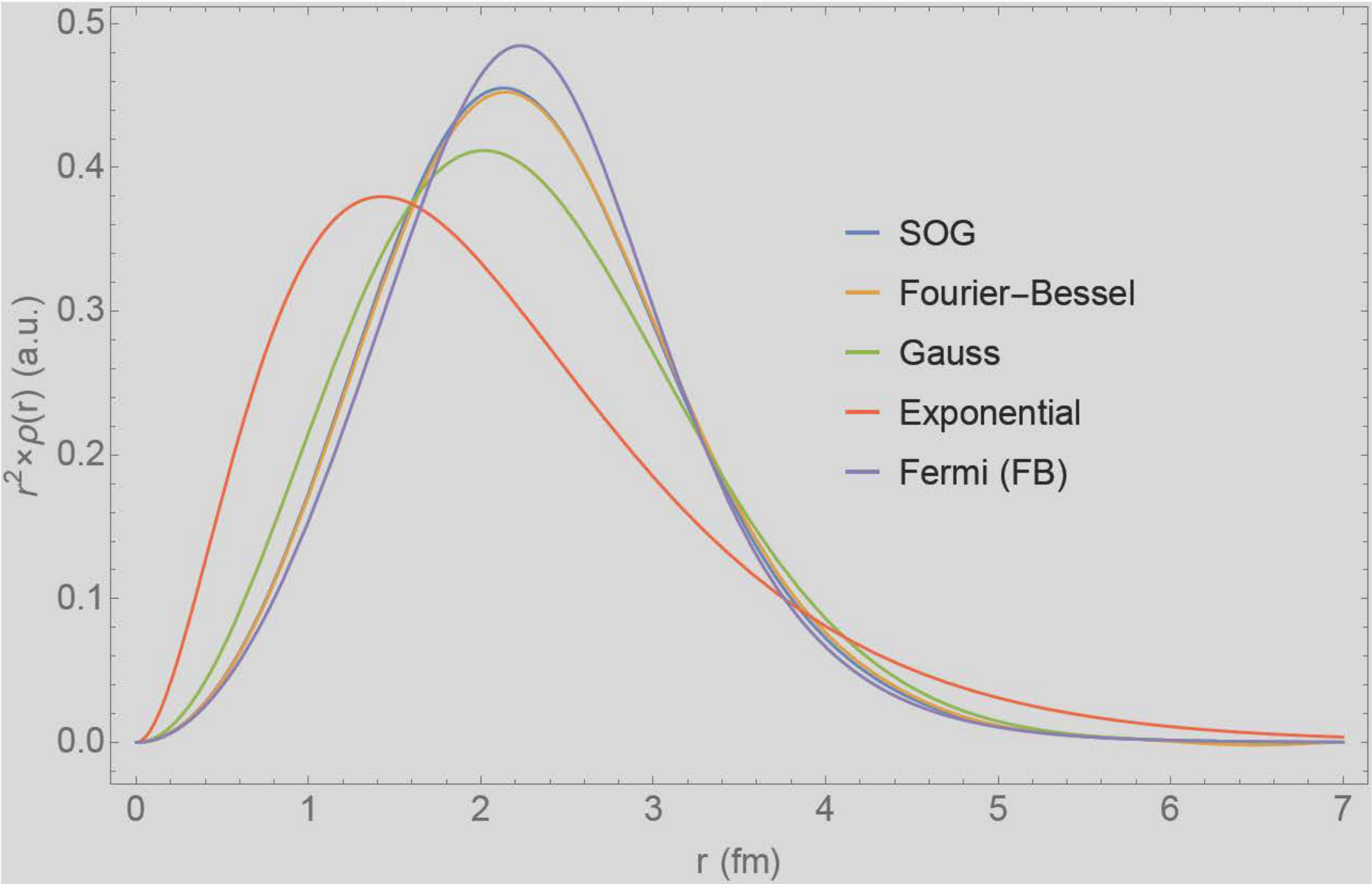
$$\rho(r) = \sum_{\upsilon} a_{\upsilon} j_0(\upsilon\pi r/R) \quad \text{for} \quad r \leqslant R,$$

$$\rho(r) = 0 \quad \text{for} \quad r > R.$$

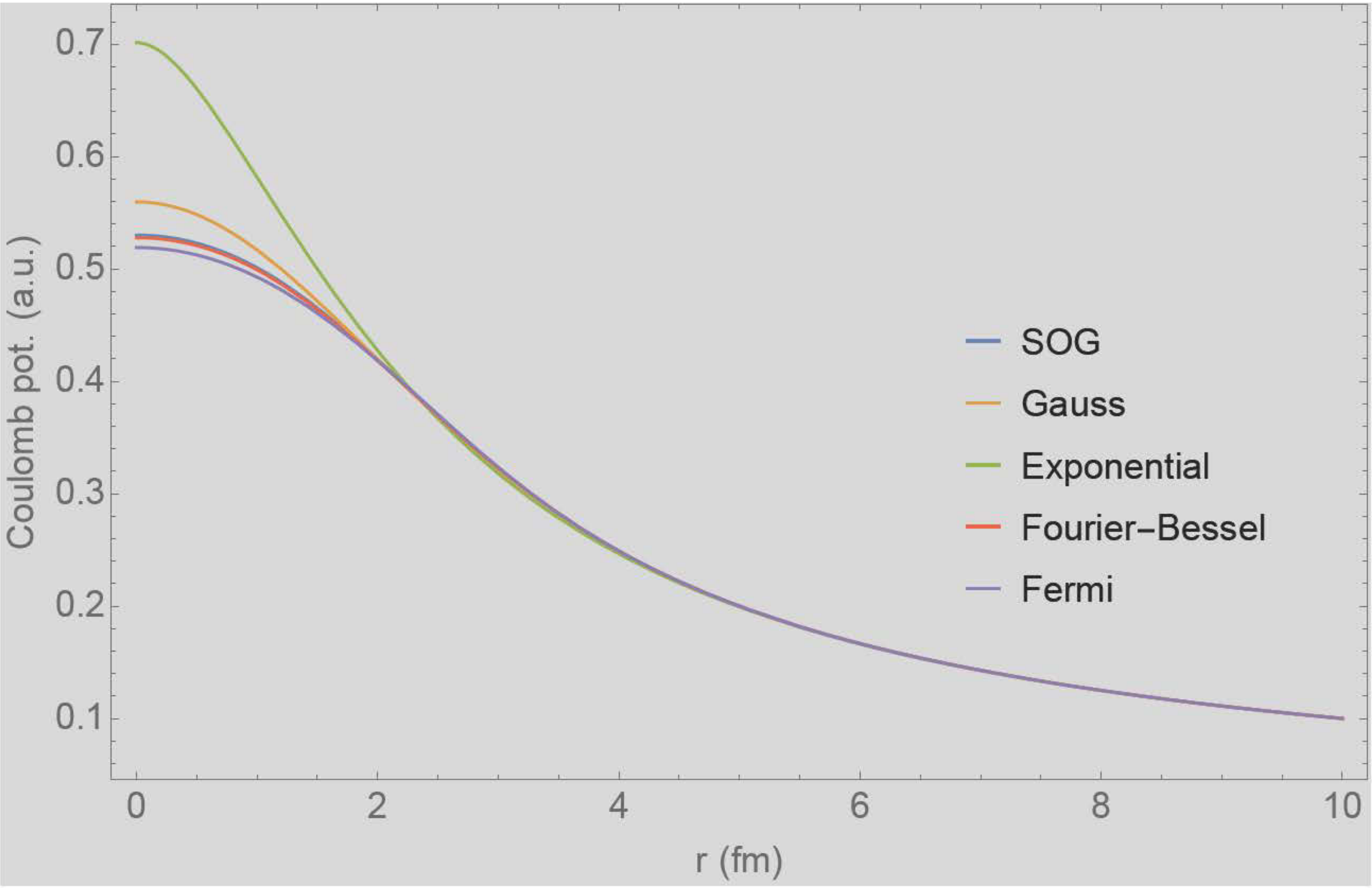
^{12}C <r^{2n}>moments model dependence



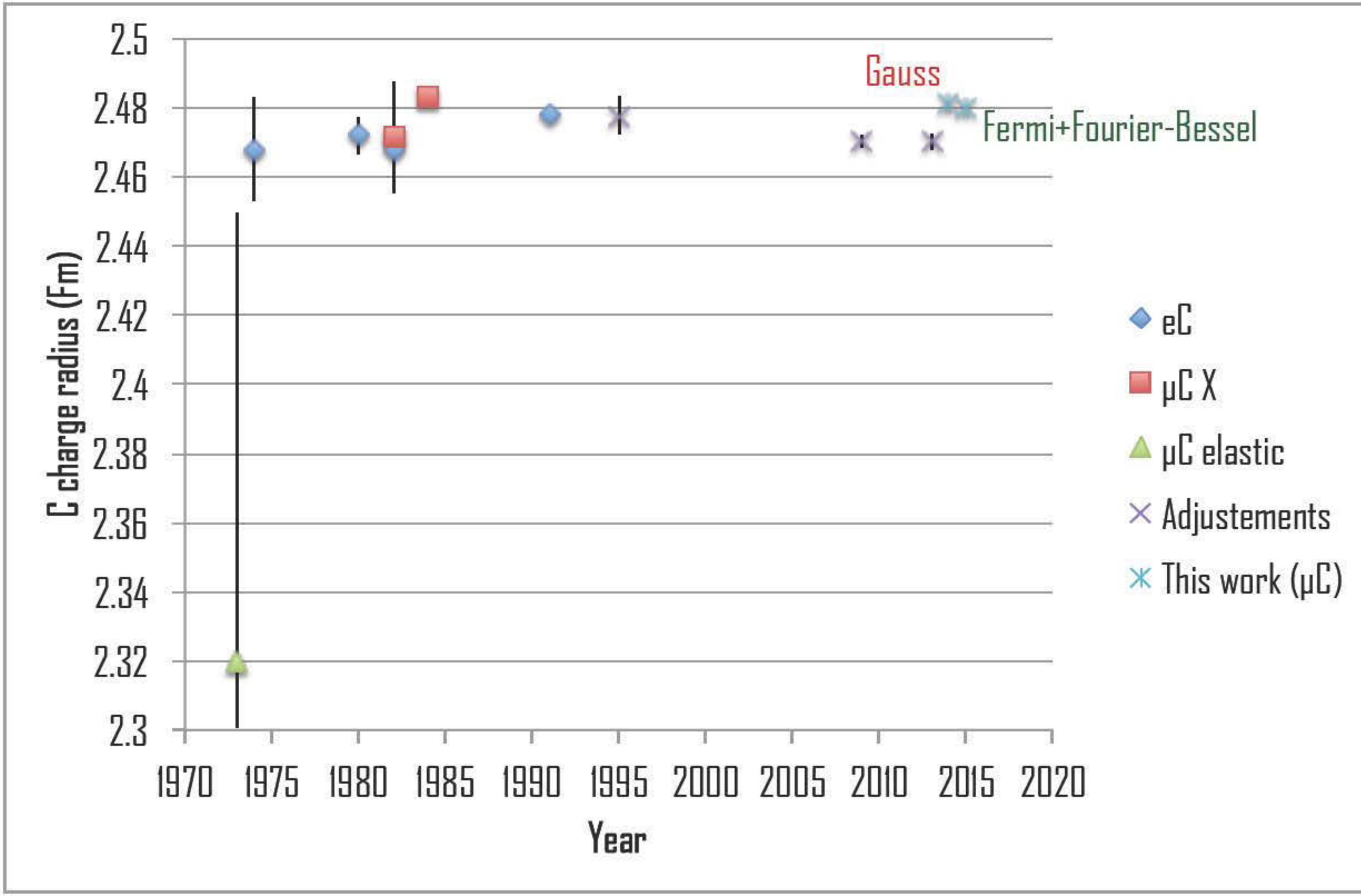
Charge distribution for ^{12}C



Potential for different models ^{12}C



^{12}C charge radius 1974-2015

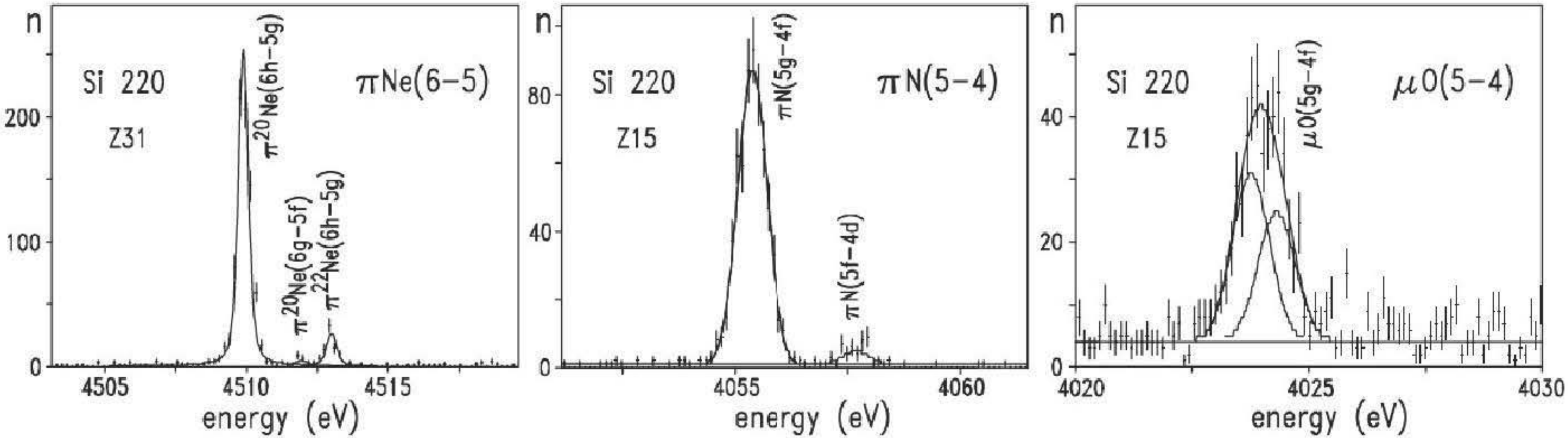


LKB

QED without nuclear effects?

Can one find muonic atoms and states to measure with high-resolution detectors

Measurements in a gaz with crystal spectrometer

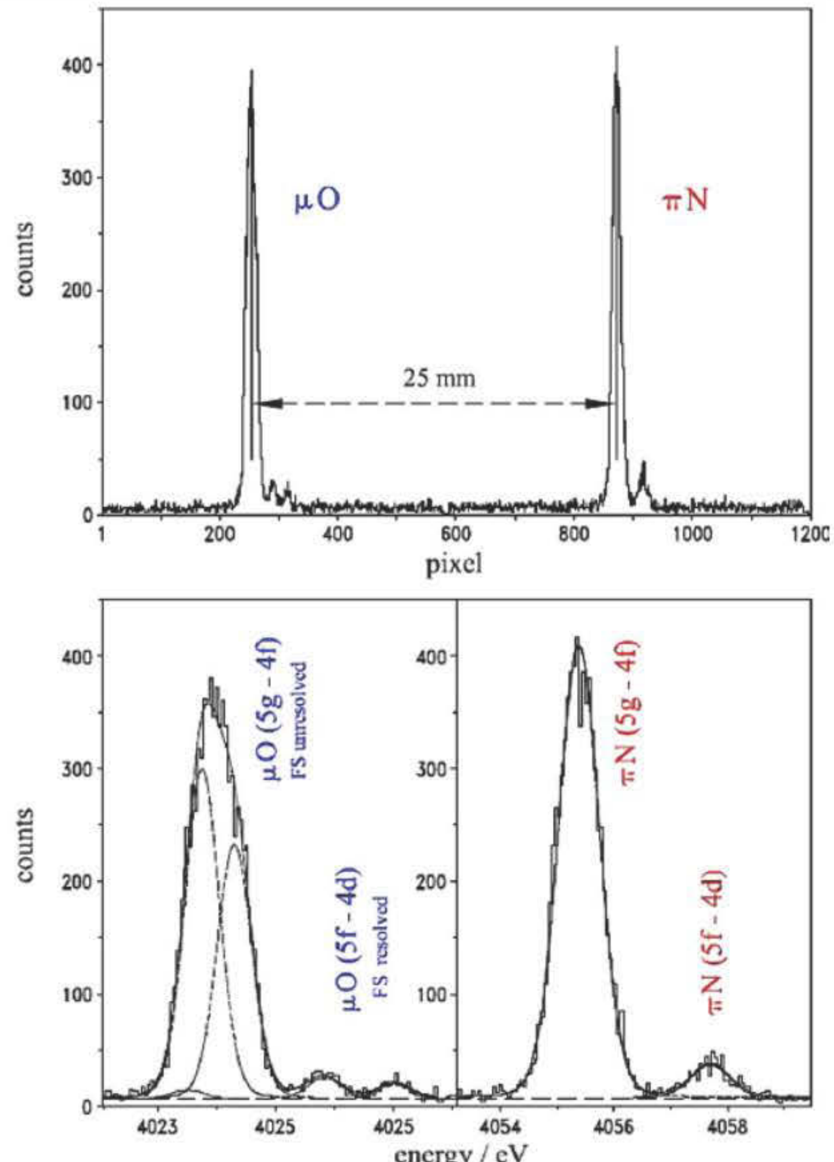


Siems, T., D. F. Anagnostopoulos, G. Borchert, D. Gotta, P. Hauser, K. Kirch, L. M. Simons, P. El-Khoury, P. Indelicato, M. Augsburger, D. Chatellard and J.-P. Egger (2000). "First direct observation of coulomb explosion during the formation of exotic atoms." Physical Review Letters **84**(20): 4573-4576.

used the cyclotron trap at PSI

Transition	transition energy (with QED, uniform nucl, eV)	QED contribution (eV)	QED as a Fraction of trans. Ener. (%)
5f5/2→4d3/2	4026.9926	2.3963	0.06%
5f5/2→4d5/2	4025.3962	2.3875	0.06%
5f7/2→4d5/2	4025.8036	2.3886	0.06%
5g7/2→4f5/2	4024.2987	0.8838	0.02%
5g7/2→4f7/2	4023.5082	0.8868	0.02%
5g9/2→4f7/2	4023.7505	0.8854	0.02%

Measurements in a gaz with crystal spectrometer



Trassinelli, M., D. F. Anagnostopoulos, G. Borchert, A. Dax, J. P. Egger, D. Gotta, M. Hennebach, P. Indelicato, Y. W. Liu, B. Manil, N. Nelms, L. M. Simons and A. Wells (2016). "Measurement of the charged pion mass using X-ray spectroscopy of exotic atoms." Physics Letters B 759: 583.

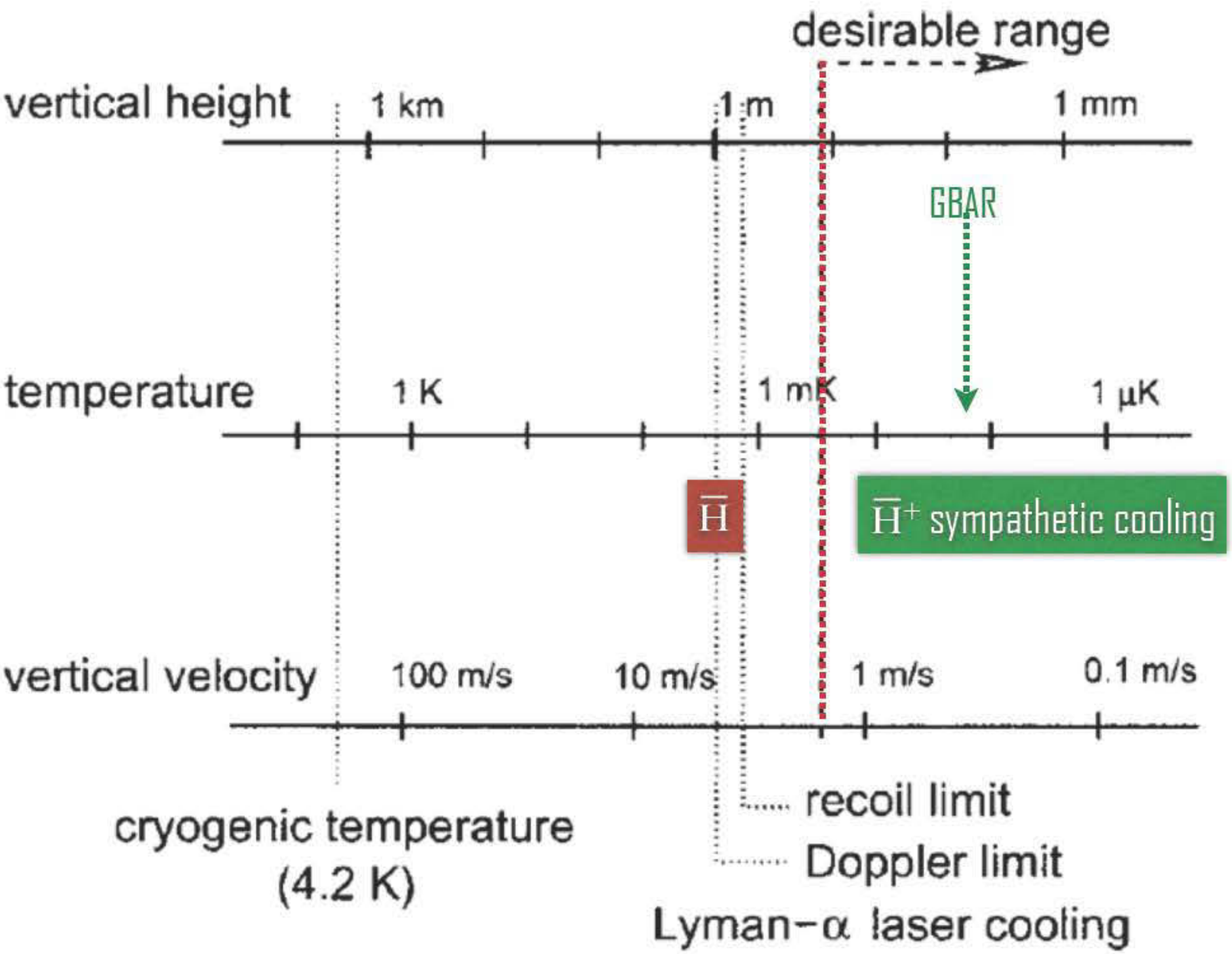
used the cyclotron trap at PSI

Context: the GBAR experiment

Would antiprotonic atoms be possible candidate for QED tests?

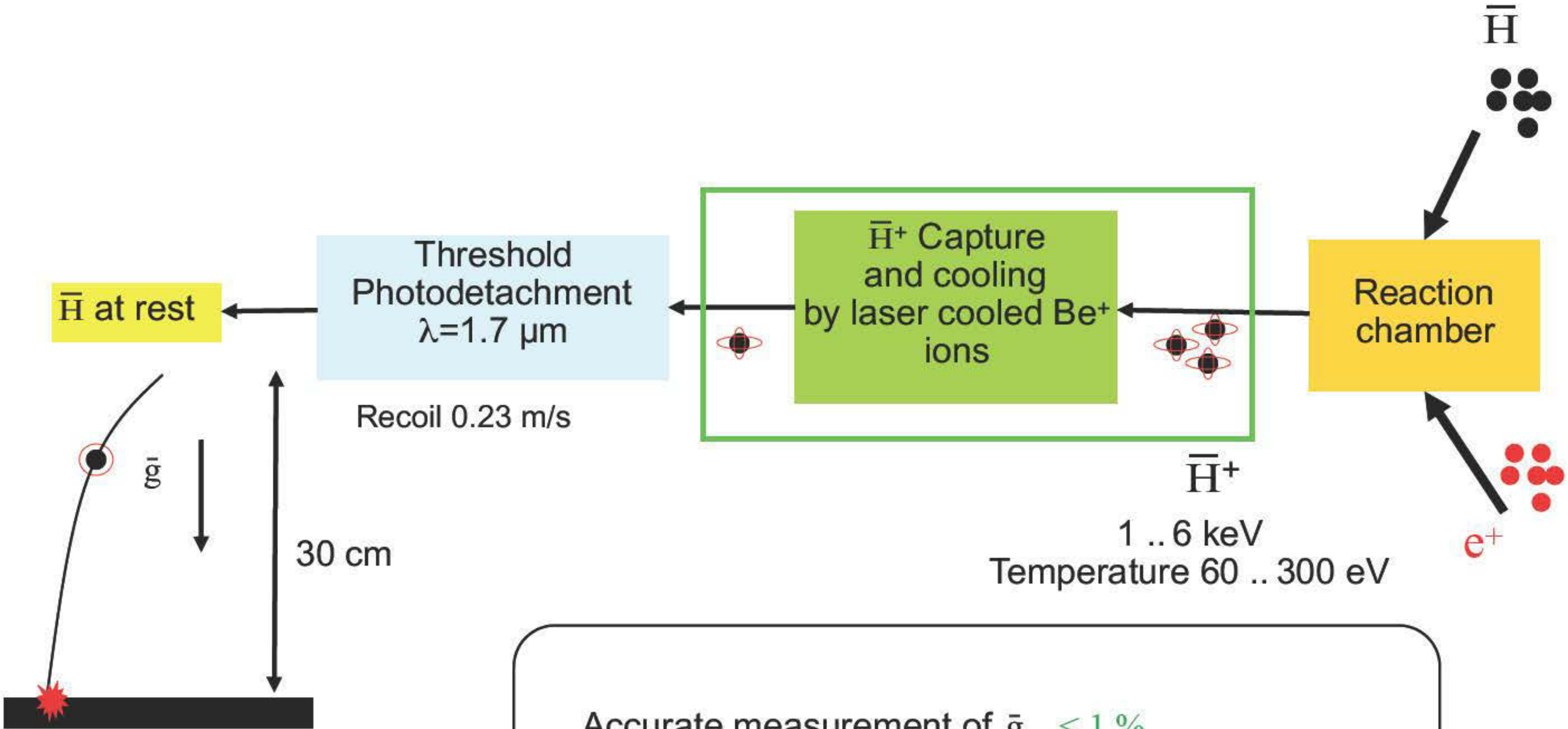
GBAR
Gravitational Behavior of Antihydrogen at Rest

How to make a direct measurement of $\bar{g}$?



A Proposal to Measure Antimatter Gravity Using Ultracold Antihydrogen Atoms. J. Walz et T.W. Hänsch. General Relativity and Gravitation. **36** 561-570, (2004).

Last GBAR steps

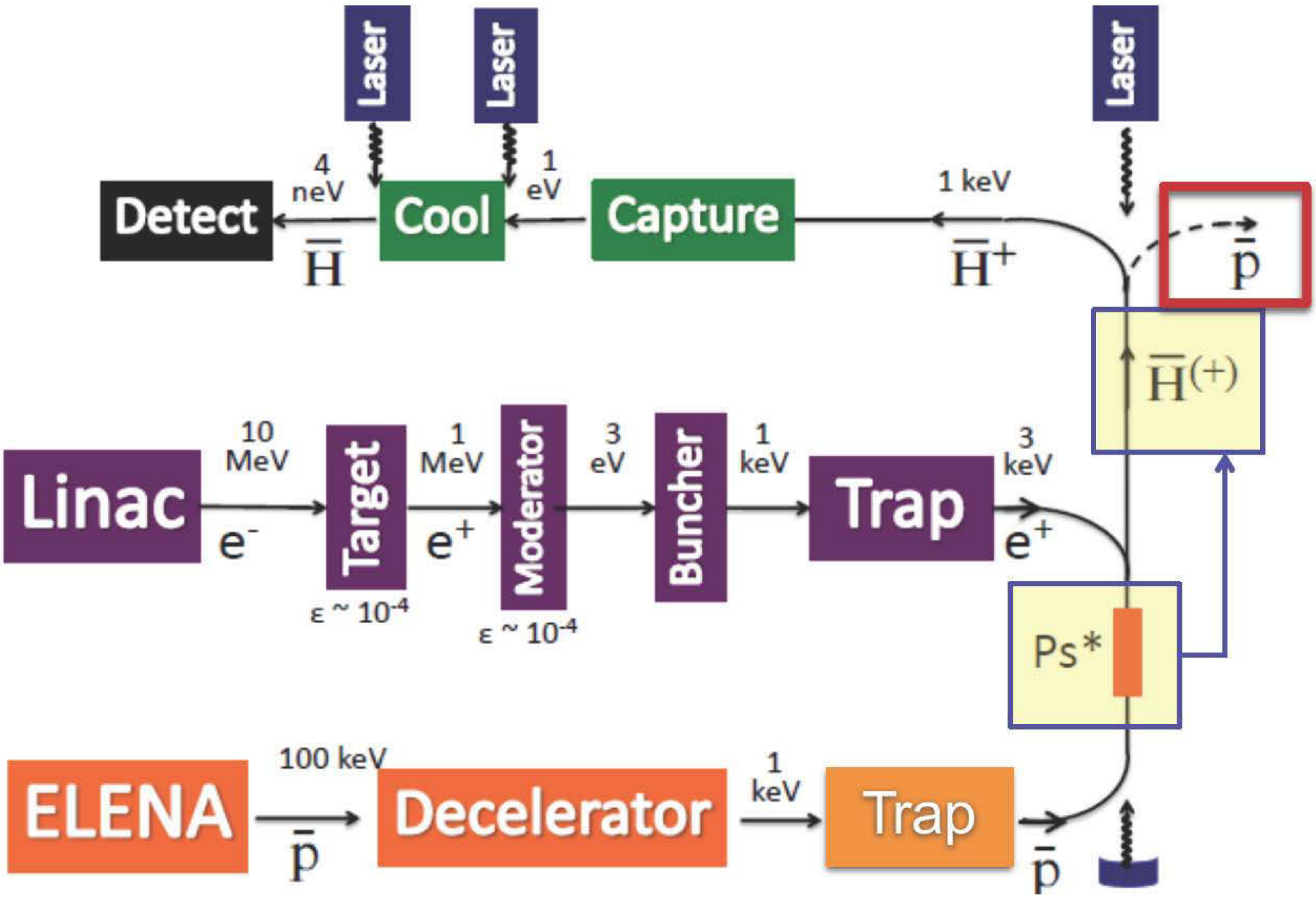


Accurate measurement of $\bar{g}$ $< 1\ \%$

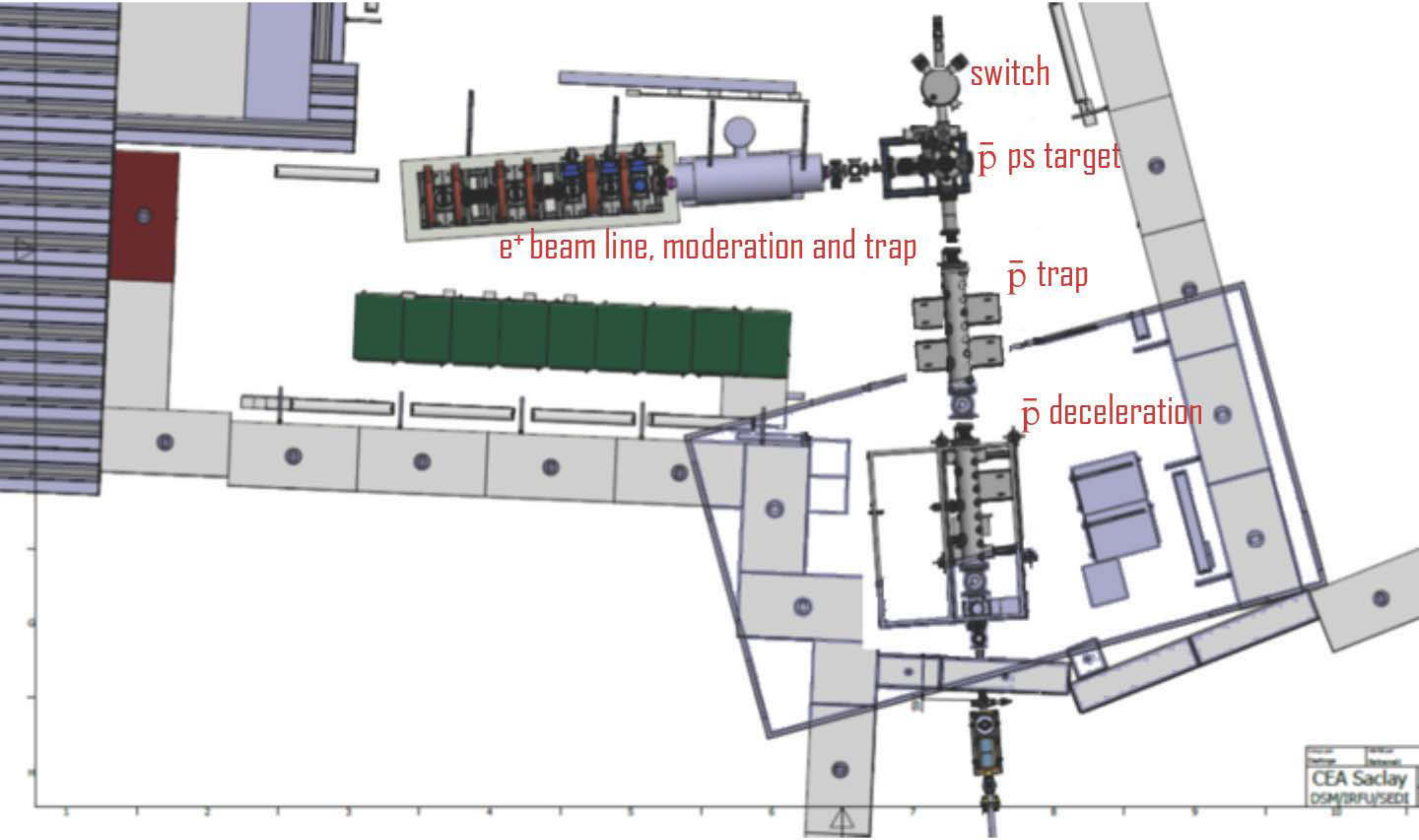
$\bar{H}$, $\bar{H}^+$ } initial velocity $\Delta v_0 < 0.8$ m/s

3 neV, 20 μK

GBAR Synoptic Scheme

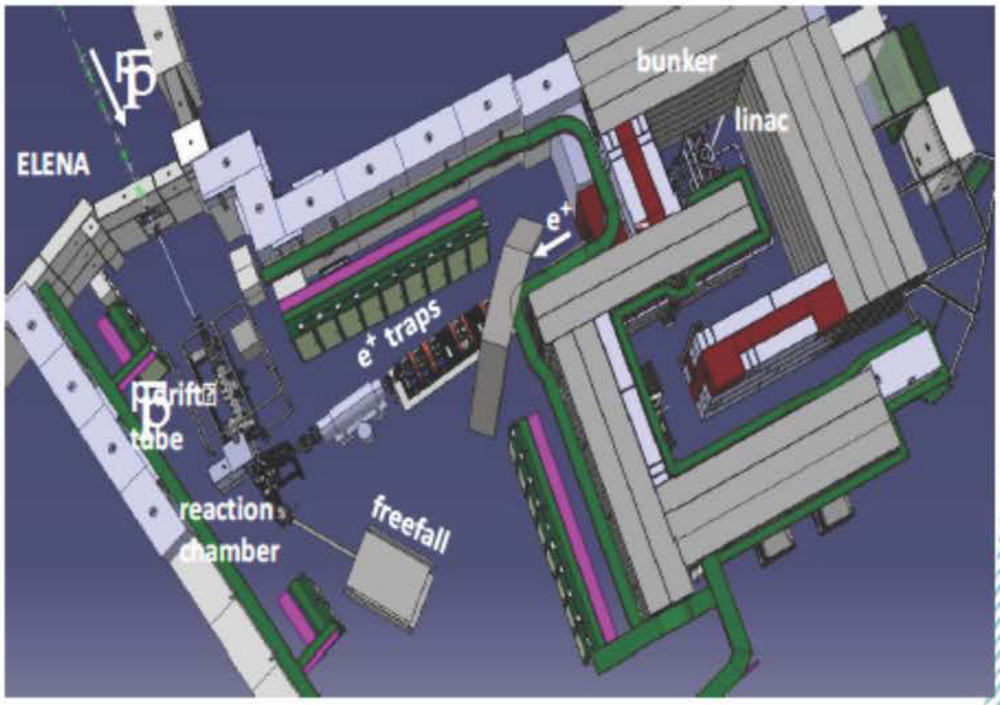


GBAR setup

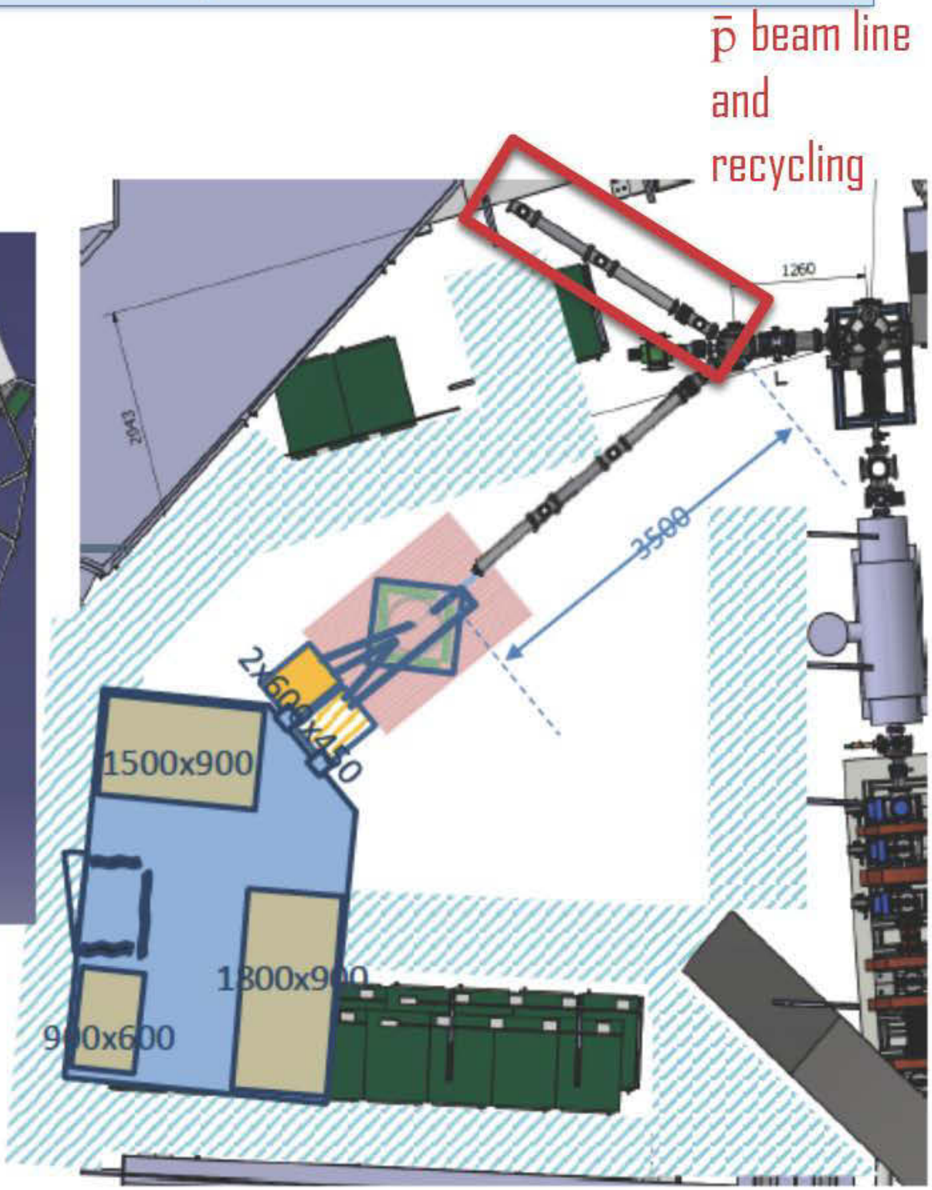


1:40
2,5 cm = 1 m

GBAR setup

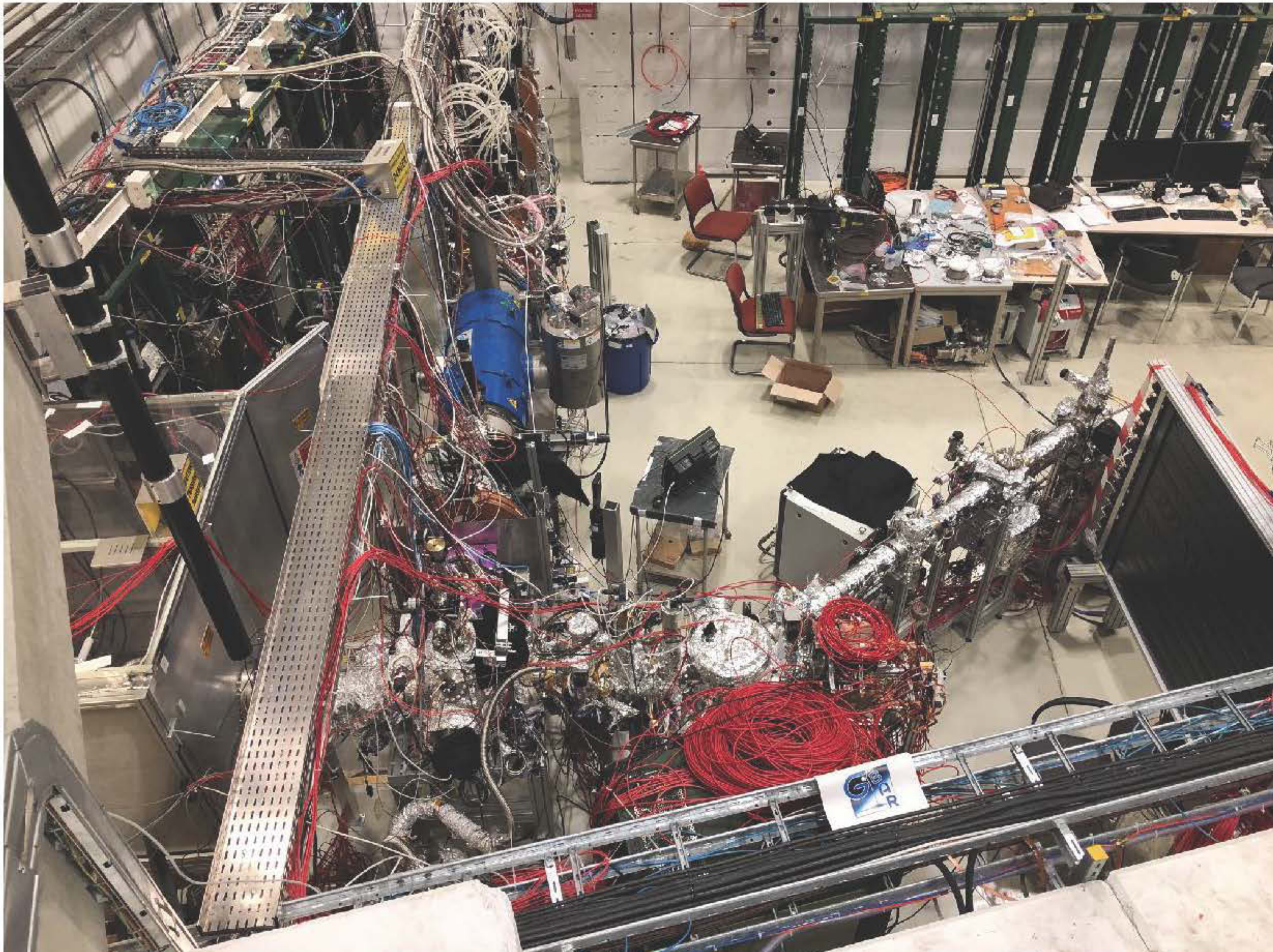


p̄
ELENA
bunker
linac
e+
e+ traps
p̄ drift tube
reaction chamber
freefall



p̄ beam line and recycling
1260
2043
3500
2x600x450
1500x900
1800x900
900x600

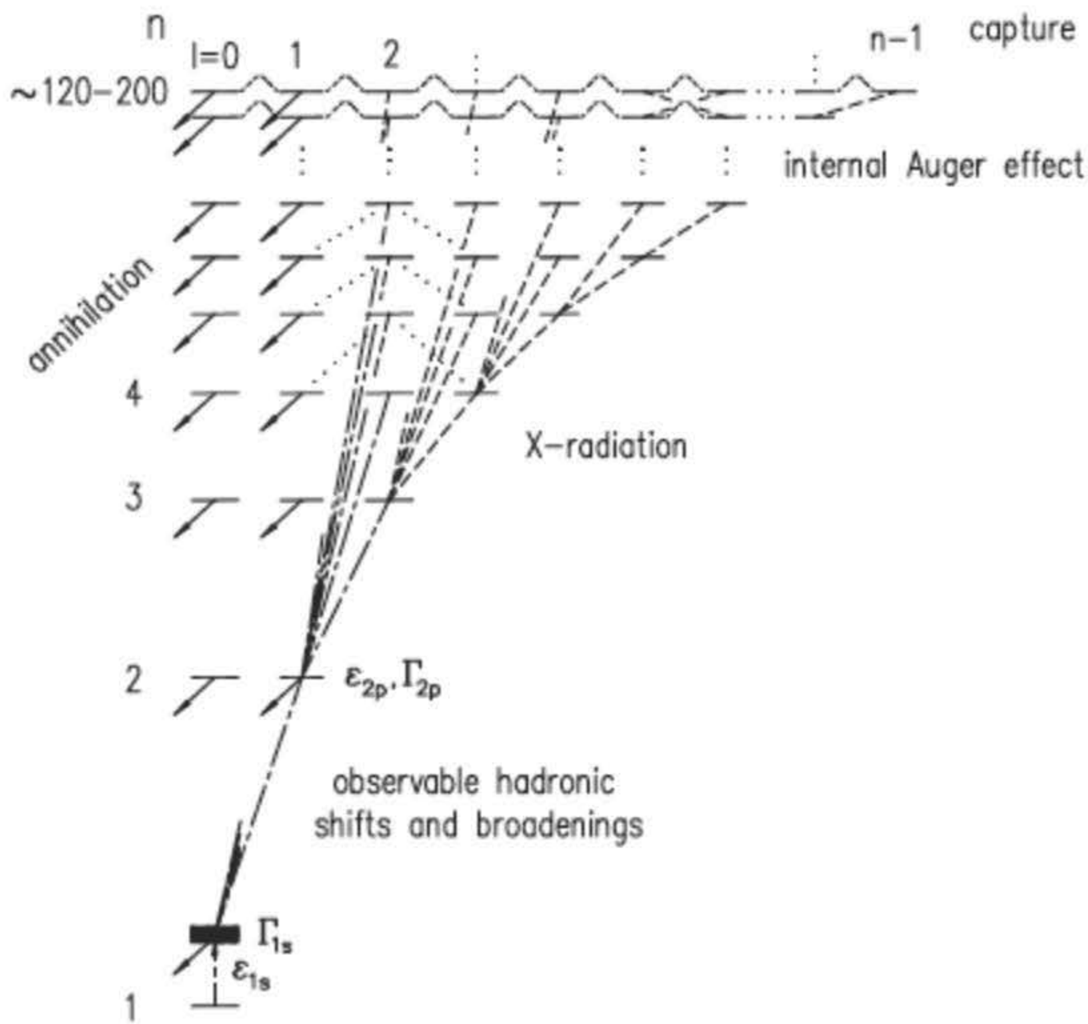
Status December 2018



QED in antiprotonic atoms

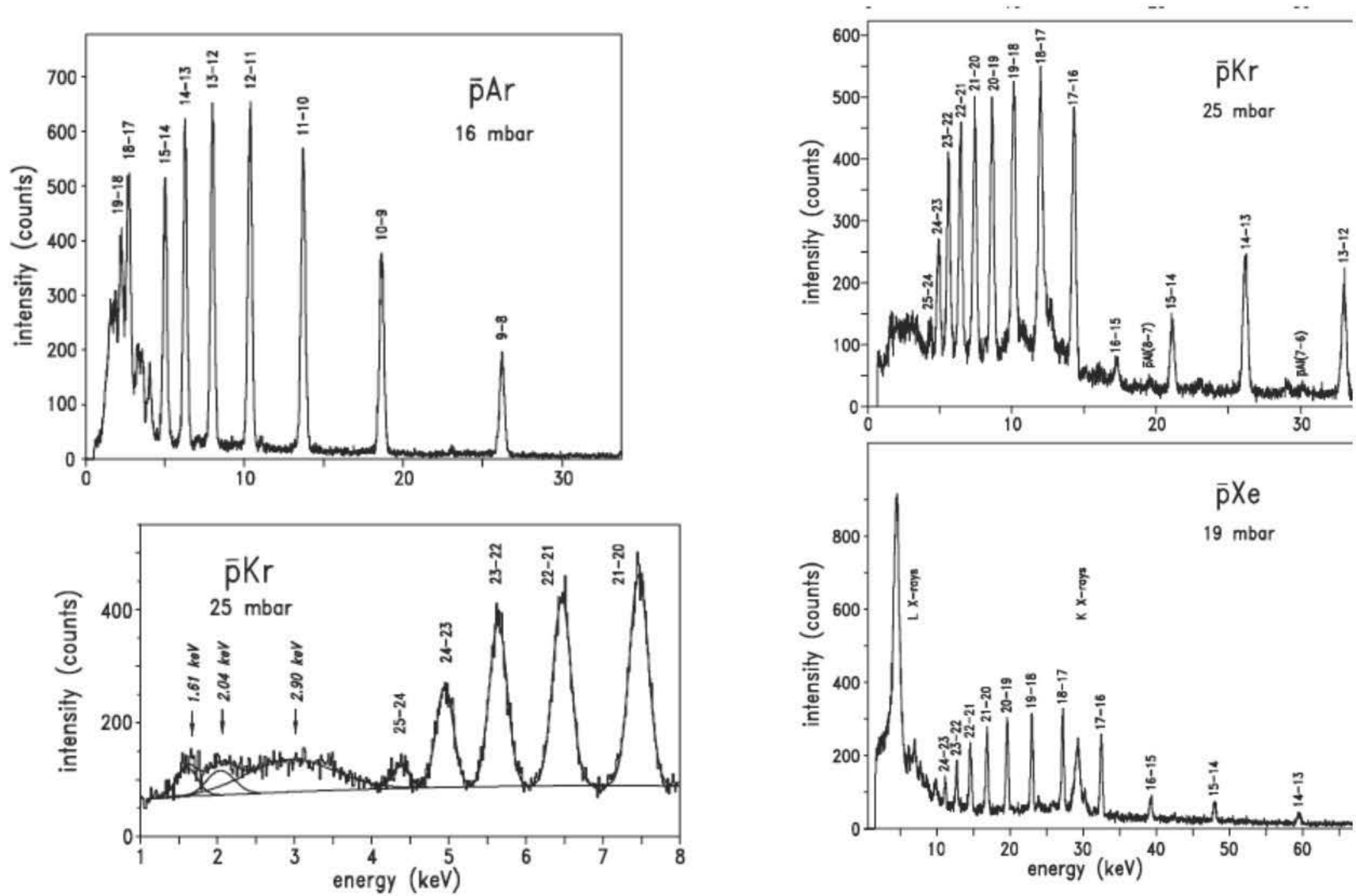
Would antiprotonic atoms be possible candidate for QED tests?

Exemple: rare gaz antiprotonic atoms x rays



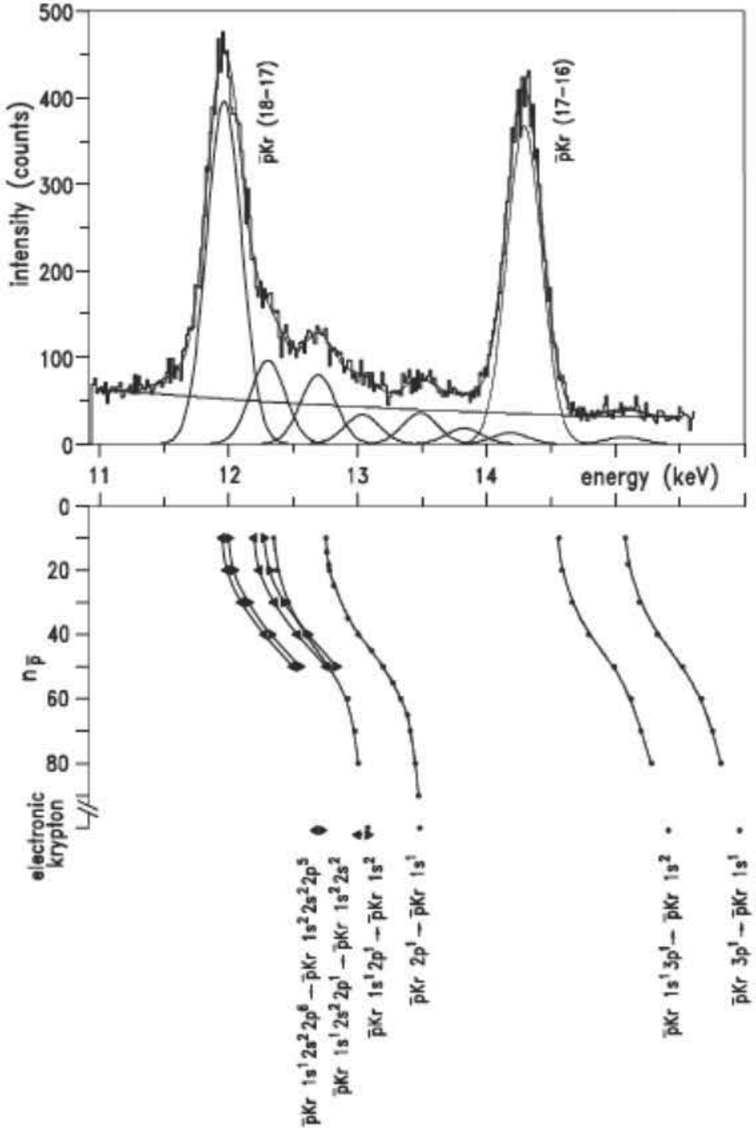
Gotta, D., K. Rashid, B. Fricke, P. Indelicato and L. M. Simons (2008). "X-ray transitions from antiprotonic noble gases." The European Physical Journal D - Atomic, Molecular, Optical and Plasma Physics **47**(1): 11-26.

Exemple: rare gaz antiprotonic atom x rays



Gotta, D., K. Rashid, B. Fricke, P. Indelicato and L. M. Simons (2008). "X-ray transitions from antiprotonic noble gases." The European Physical Journal D - Atomic, Molecular, Optical and Plasma Physics **47**(1): 11-26.

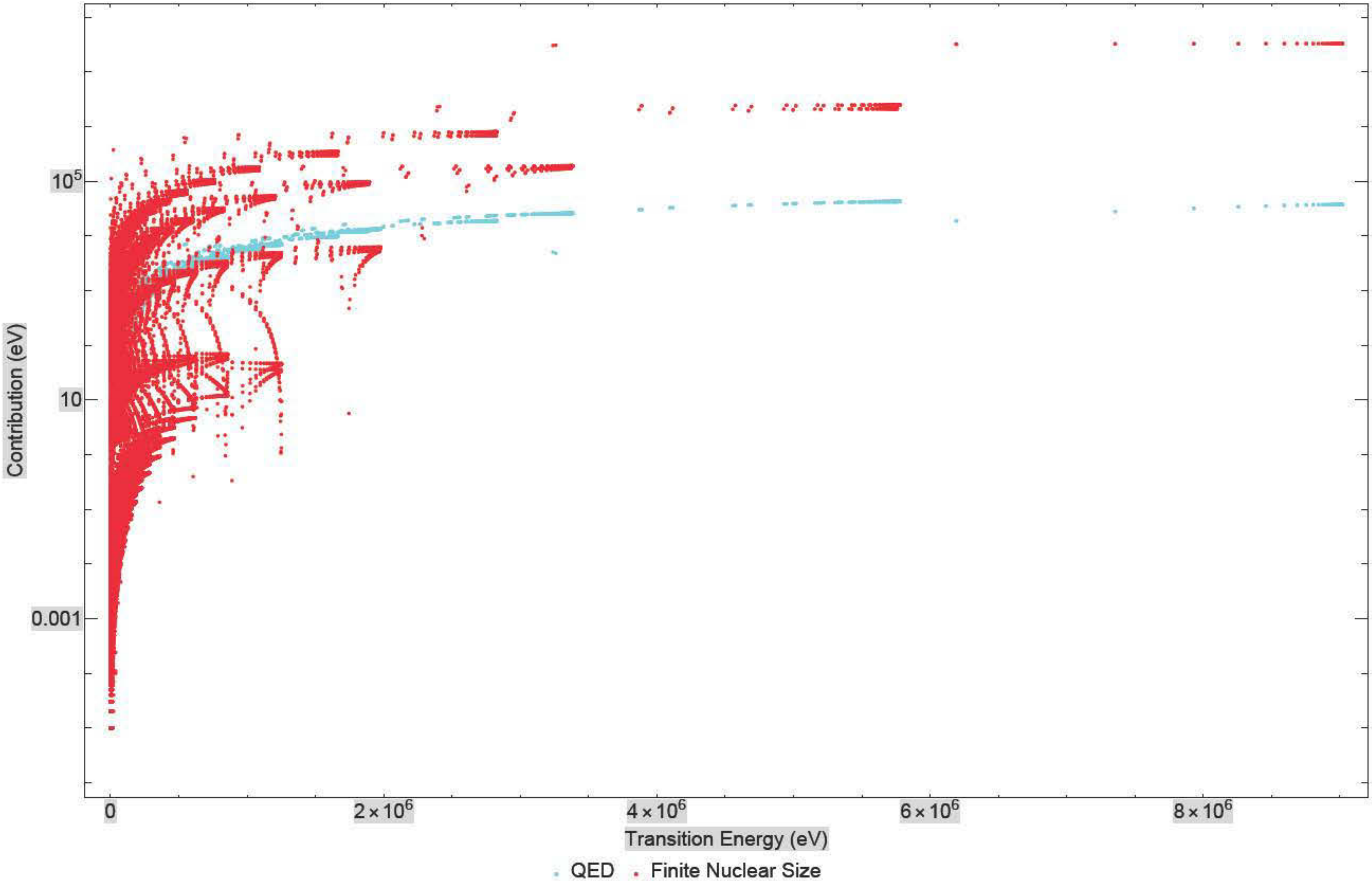
Exemple: rare gaz antiprotonic atom x rays



Calculation

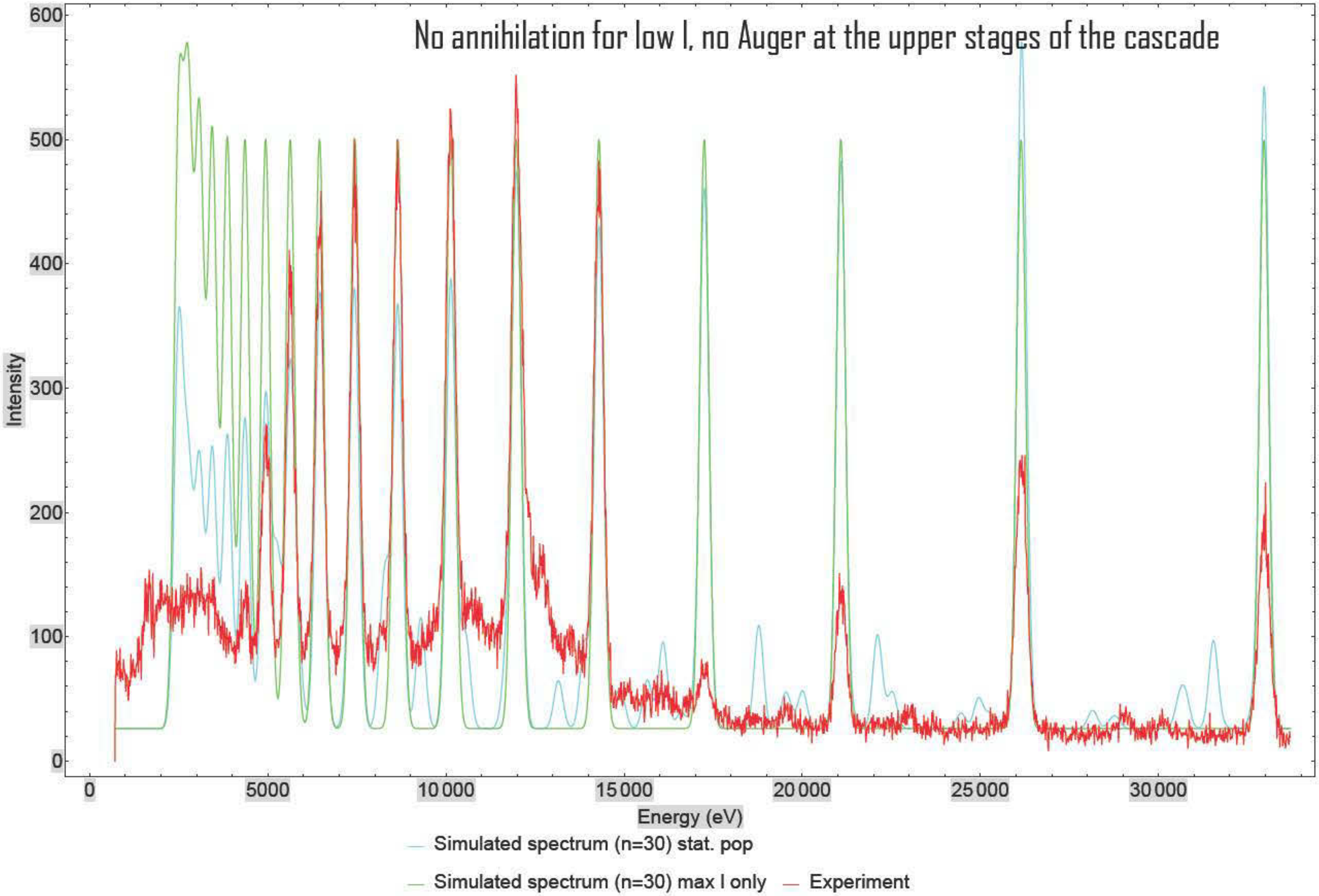
- Generate all possible E1, E2, M1, M2 transitions input data between levels, starting at n=30 as an example (60 initial levels, 900 in total, 41000 transitions)
- Use general MCDF code with exotic atoms capabilities to evaluate all transitions energies and probabilities
  - Vacuum polarization included in the Dirac Equation (re-summing all loop-after-loop diagrams)
  - Vacuum polarization in perturbation
  - point nucleus
- Calculate the QED and finite nuclear size (FNS) contribution to alll transition energies
- extract results with large QED and small FNS correction
- Simulate « cascade » (no Auger for now) and compare calculation to experiment
  - statistical distribution for n = 30
  - distribution peaked in the two n = 30, l = 29 levels…
  - evaluate levels width and branching ratios

Results



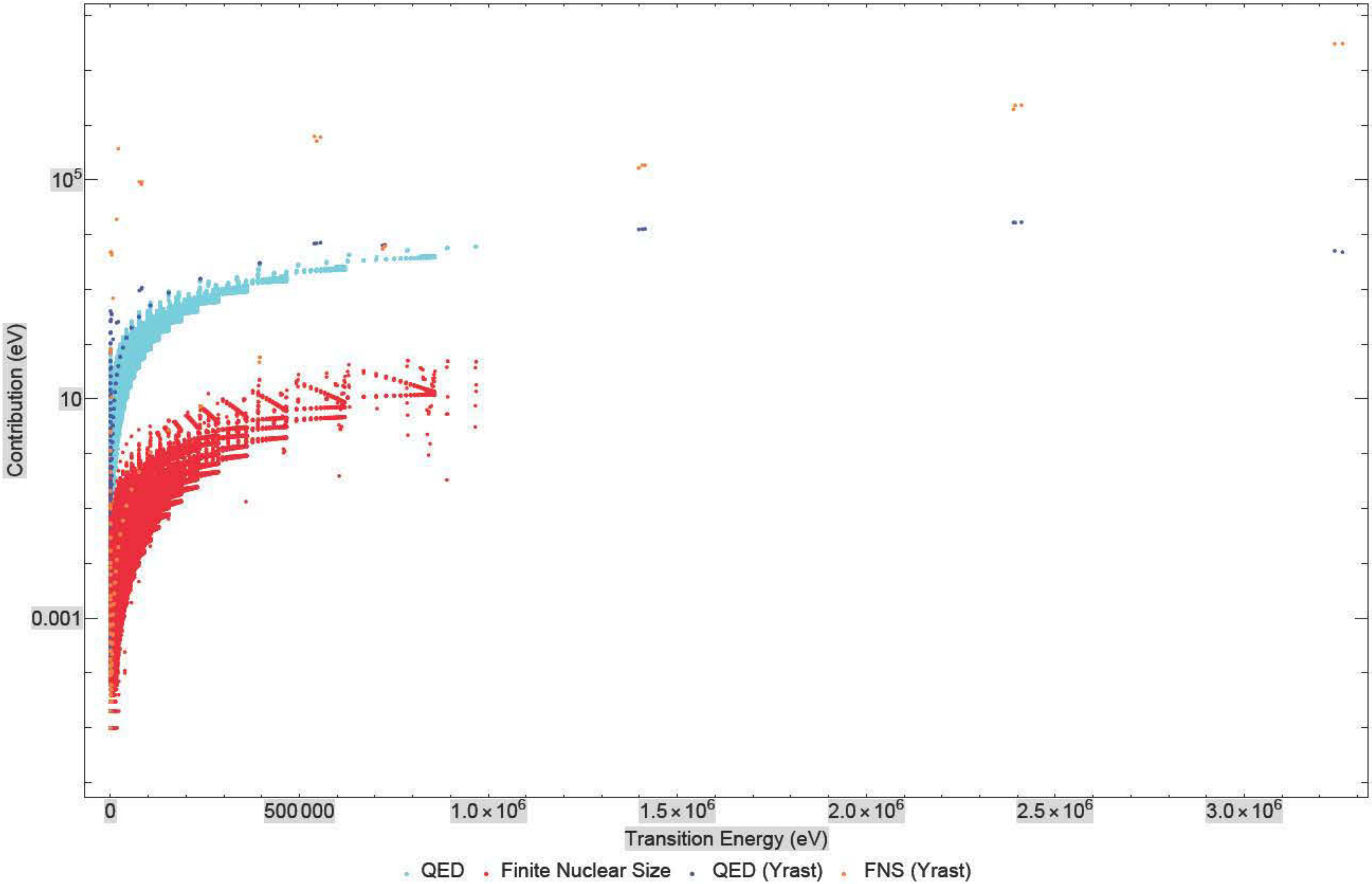
Contribution (eV)
10^5
10
0.001
0
2×10^6
4×10^6
6×10^6
8×10^6
Transition Energy (eV)
QED
Finite Nuclear Size

Result

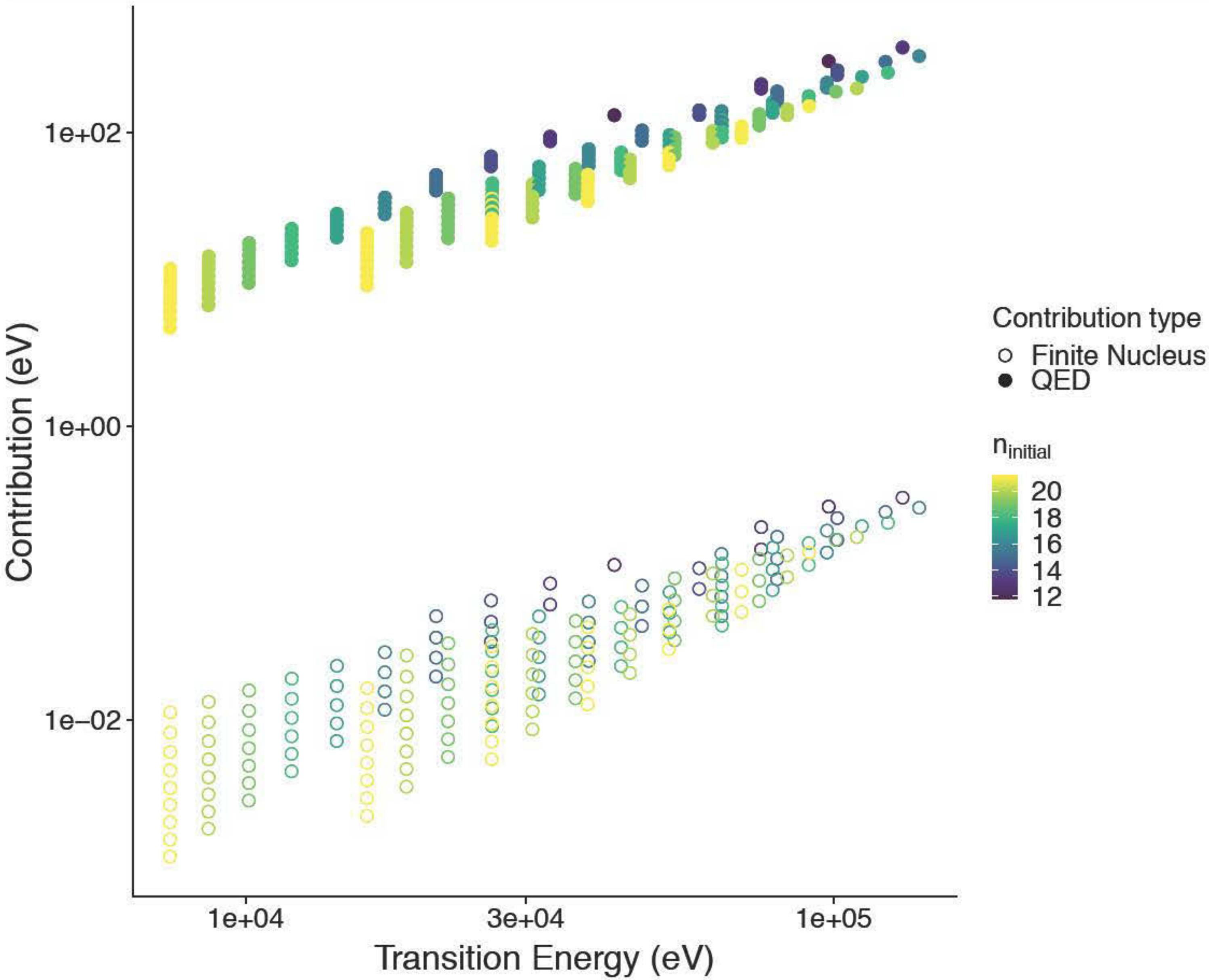


No annihilation for low l, no Auger at the upper stages of the cascade
Intensity
Energy (eV)
0
100
200
300
400
500
600
0
5000
10 000
15 000
20 000
25 000
30 000
Simulated spectrum (n=30) stat. pop
Simulated spectrum (n=30) max l only
Experiment

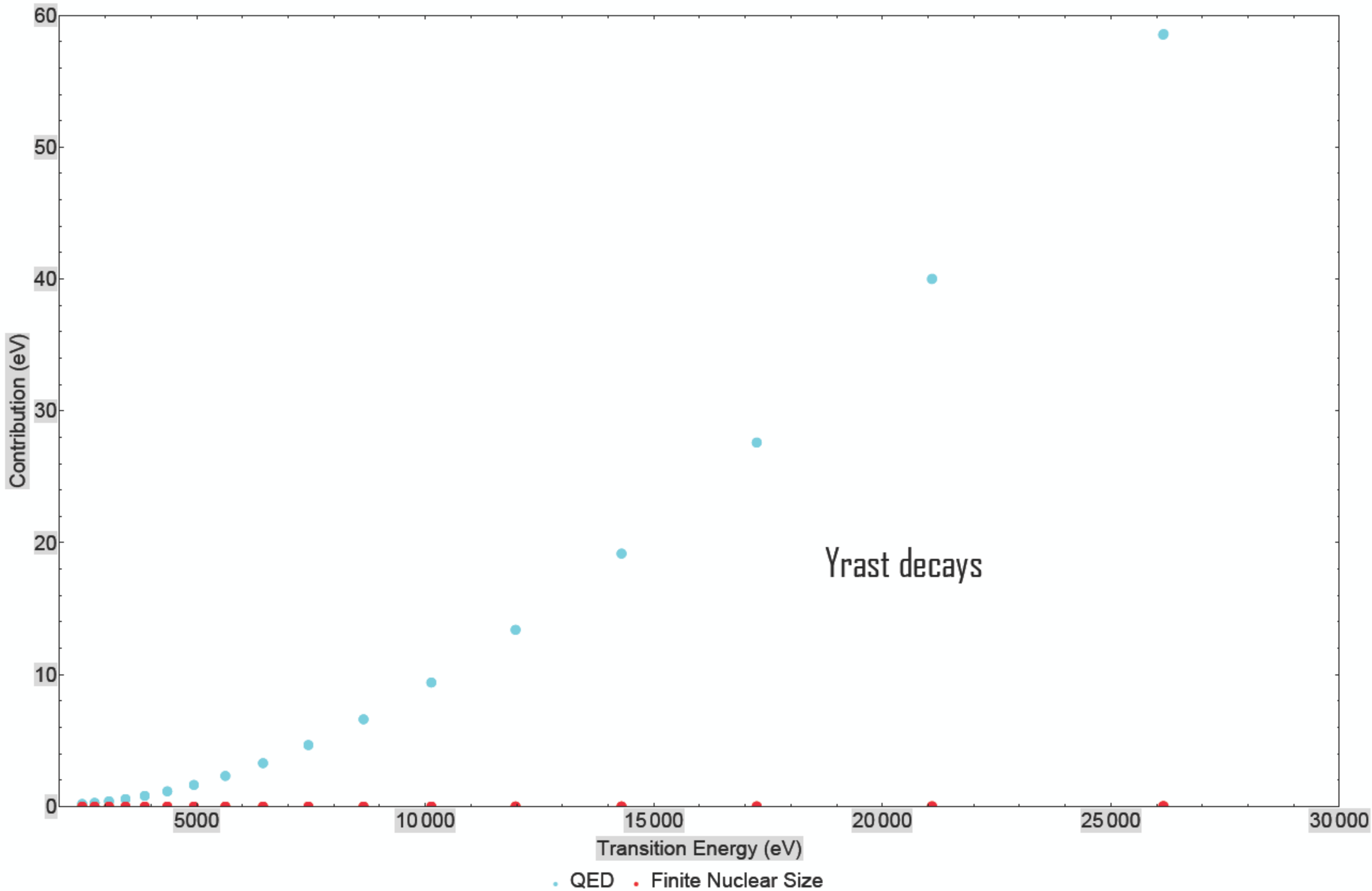
Lines with large QED and small FNS



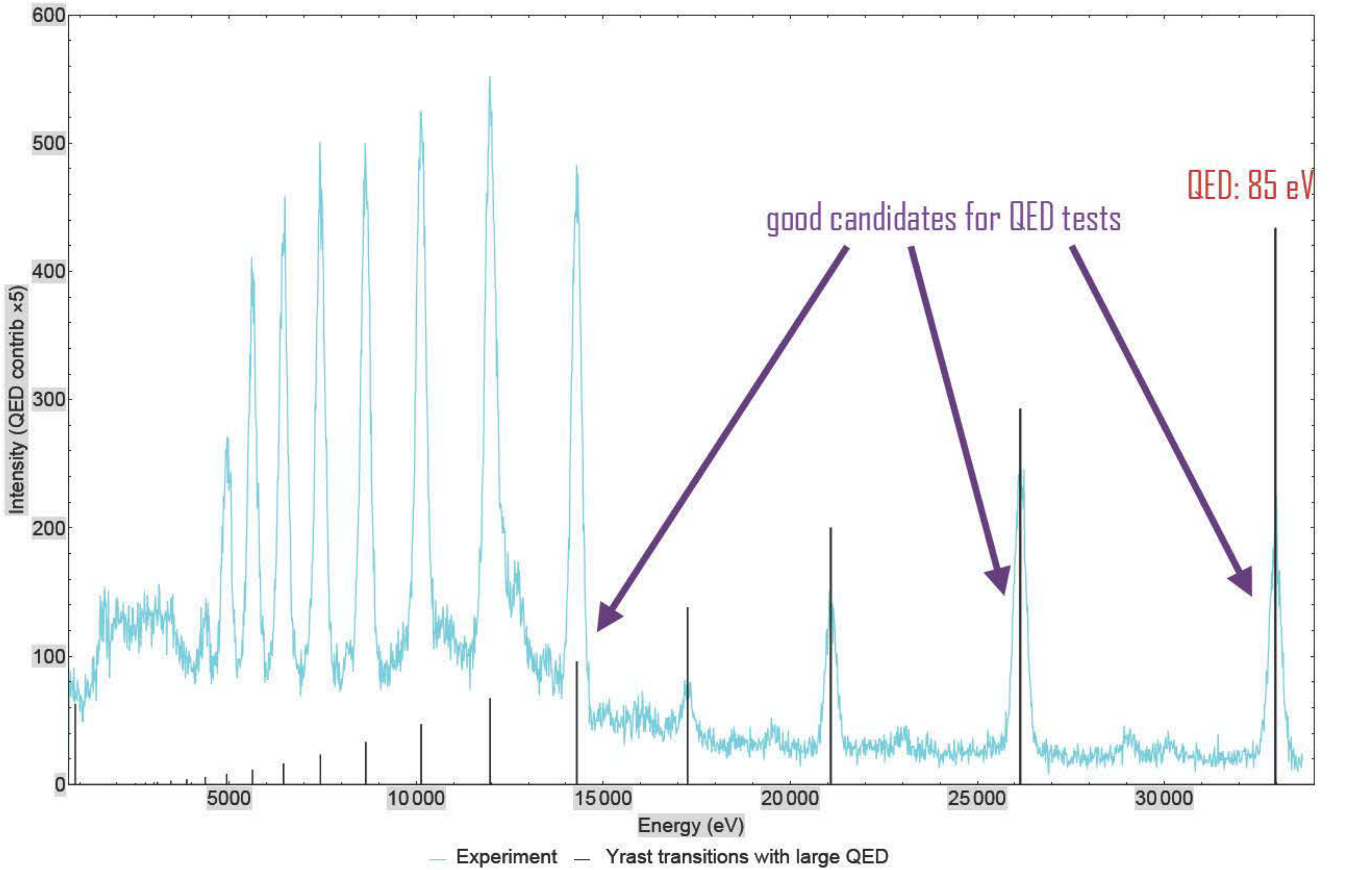
Lines with large QED and small FNS



Lines with large QED and small FNS



Line with large QED and no FNS



H-like Ar vs antiprotonic Kr

Ar^{17+}	Ly-α_1	$2p_{3/2} \rightarrow 1s_{1/2}$	abs.	**3322.993(4)(13)**
LS Ar^{17+}				1.201(4)(13)

K. Kubiček, P. H. Mokler, V. Mäckel, et al., Phys. Rev. A **90**, 032508 (2014).

Antiprotonic transition 17v-16u

Transition	Transition energy (eV)	QED	Nucl. Size corr.
27xx51/2→ 26zz49/2	3441.45617	-0.56596	-0.00011
27xx53/2→ 26zz51/2	3441.54138	-0.56604	-0.00010
27xx51/2→ 26uu51/2	3441.94795	-0.56616	-0.00010

H-like Kr vs antiprotonic Kr

	Experiment	Theory (Mohr 1983)
Ly-α_1 (eV)	13 508.95 ± 0.5	13 509.046
(1s) Lamb shift (eV)	11.95 ± 0.5	11.856

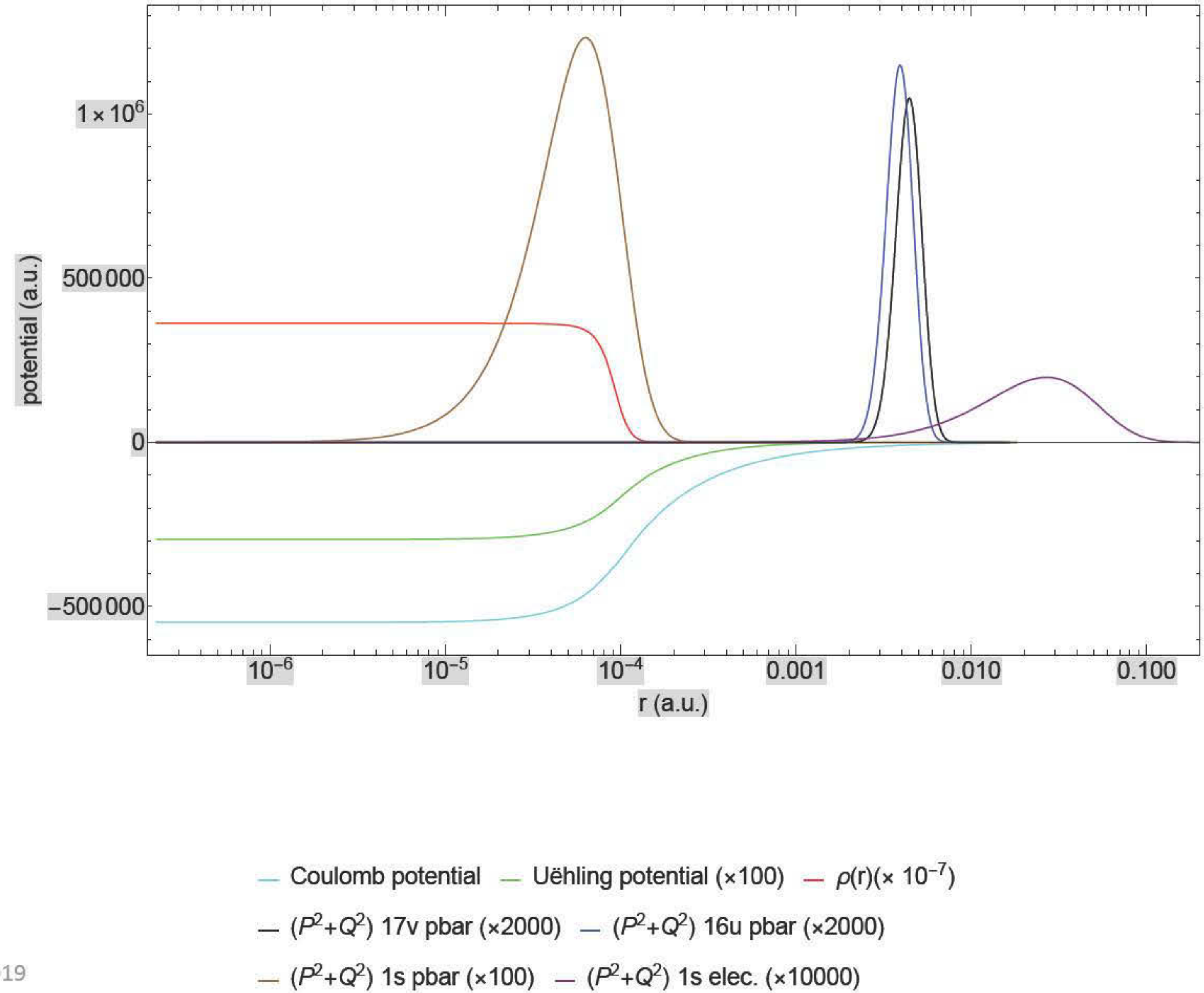
M. Tavernier, J. P. Briand, P. Indelicato, et al., Journal of Physics B: Atomic, Molecular and Optical Physics **18**, L327 (1985).

Antiprotonic transition 17v-16u

Transition	Transition energy (eV)	QED	Nucl. Size corr.
17v31/2→16u29/2	14290.6080	-19.1675	-0.0072
17v33/2→16u31/2	14292.1255	-19.1741	-0.0072
17v31/2→16u31/2	14296.3395	-19.1847	-0.0072

14 288 ± 2	95.5 ± 2.3	$\bar{p}$Kr(17–16)1s^2	14 286

Wavefunctions and potentials



Sizes…

- RMS radius ^{84}Kr: 7.91×10^{-5} a.u. (4.19 fm)
- Nuclear radius (50% density): 9.05×10^{-5} a.u.
- $\bar{p}\,Kr^{36+}$
  - n = 30, l=29 orbital: 1.40×10^{-2} a.u.
  - 17v orbital: 4.55×10^{-3} a.u.
  - 16u orbital: 4.04×10^{-3} a.u.
  - 1s orbital: 7.48×10^{-5} a.u.
- Kr^{35+}
  - 1s orbital: 4.07×10^{-2} a.u

H-like Xe vs antiprotonic Kr

J. P. Briand, P. Indelicato, A. Simionovici, et al., Europhysics Letters **9**, 225 (1989).

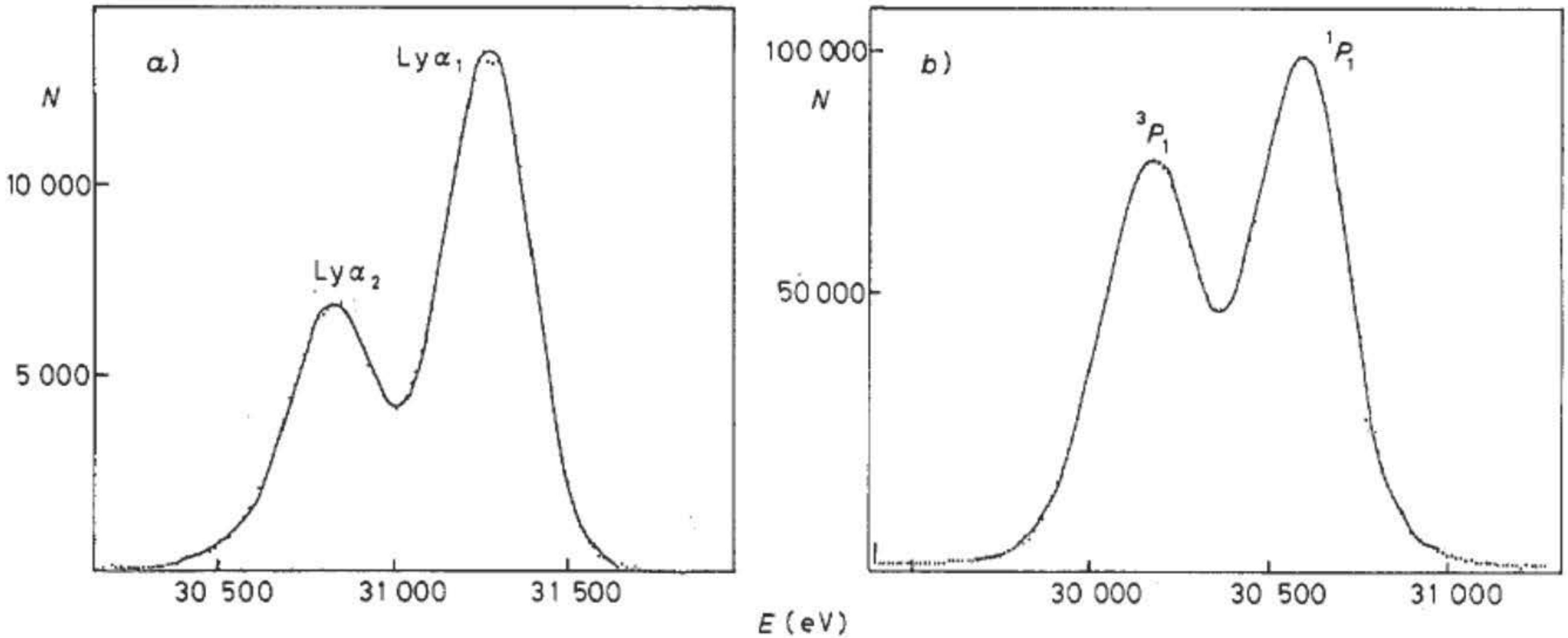


Fig. 5. – Lyman α spectra of hydrogenlike (a)), and heliumlike (b)) xenon after cleaning from double capture or excitation.

Cleaning done by using bare beam for H-like and H-like beam for He-like

H-like Xe vs antiprotonic Kr

	Experimental	Theory
Lyman α_1	31 278 ± 10 eV	31 284.0 eV
Lyman α_2	30 848 ± 10 eV	30 856.6 eV

(1s) *Lamb Shift*

	Experimental	Theory
mean value from Ly α_1 and Ly α_2	54 ± 10 eV	46.7 eV

J. P. Briand, P. Indelicato, A. Simionovici, et al., Europhysics Letters **9**, 225 (1989).

Antiprotonic transition 13q-12o

Transition	Transition energy (eV)	QED	Nucl. Size corr.
13q23/2→12o21/2	32956.86131	-86.65357	-0.06067
13q25/2→12o23/2	32965.16478	-86.71557	-0.06065
13q23/2→12o23/2	32981.63957	-86.79718	-0.06063

32 962±3 80.5±2.2 $\bar{p}$Kr(13–12) 32 965

What do we learn?

Part.	Level	Dirac	FNS	Self-Energy	Vac. Pol. (e^+e^-)	VP ($\mu^+\mu^-$)	g-2 $\bar{p}$	2nd order QED	Recoil	Total
$\bar{p}$	17v	-110709.7	0.0	0.0	-44.7	0.0	3.0	-0.4	-0.1	-110751.8
$\bar{p}$	16u	-124982.2	0.0	0.0	-63.7	0.0	4.1	-0.6	-0.1	-125042.4
$\bar{p}$	1s	-32578167.8	23565617.4	?	-58348.8	-1393.7	-17042.4	-549.4	12616.2	-9077268.4
e^-	1s	-17956.7	8.9	12.9	-1.3	0.0		0.0	0.0	-17936.2

identifying other lines?

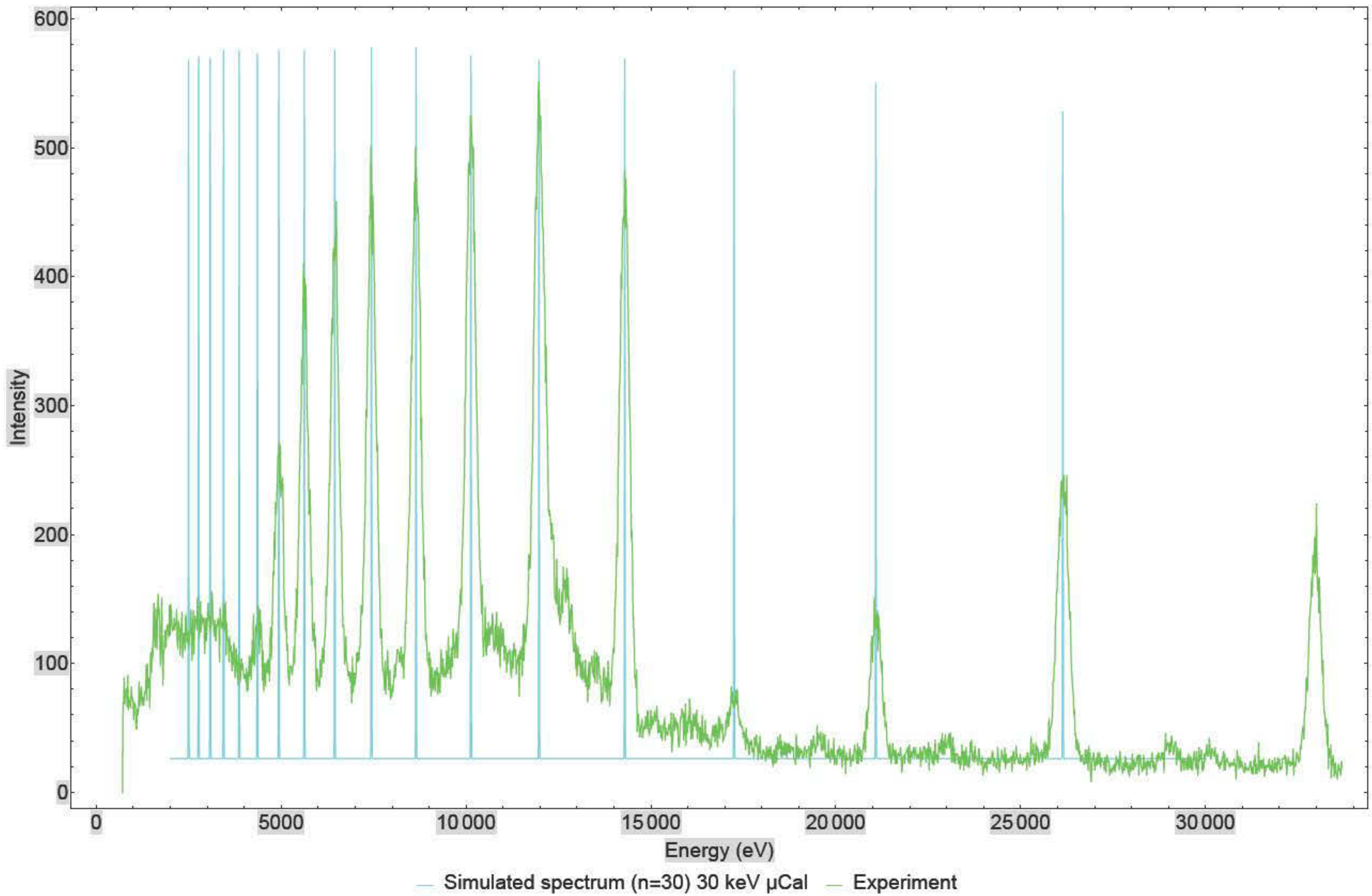
Experimental energy (eV)	Relative intensity (%)	Explanation	Theoretical energy (keV)
4369±13	17.6±2.4	$\bar{p}$Kr(25–24)$1s^2$	4341
4939±3	64.2±4.2	$\bar{p}$Kr(24–23)$1s^2$	4923
5627±2	88.9±2.8	$\bar{p}$Kr(23–22)$1s^2$	5613
6447±2	87.1±2.6	$\bar{p}$Kr(22–21)$1s^2$	6438
7445±5	89.7±3.8	$\bar{p}$Kr(21–20)$1s^2$	7431
8645±2	85.0±2.4	$\bar{p}$Kr(20–19)$1s^2$	8639
10 133±2	96.2±2.6	$\bar{p}$Kr(19–18)$1s^2$	10 123
10 642±32	6.1±1.1	*not identified*	
11 968±7	100	$\bar{p}$Kr(18–17)$1s^2$+"Br" Kα?	11 967
12 287±33	24.3±1.5	$\bar{p}$Kr $n \approx 20$ $1s2p \to 1s^2$ el.	12 290
12 696±24	20.4±1.4	$\bar{p}$Kr $n \approx 20$ $2p \to 1s$ el.	12 760
		Kα *fluorescence?*	12 630
12 954±32	8.6±1.2	*not identified*	
13 487±20	9.0±1.4	"Br" Kβ?	
13 814±39	4.6±1.0	$\bar{p}$Kr(22–20) with $1s^2$ el.	13 860
14 288±2	95.5±2.3	$\bar{p}$Kr(17–16)$1s^2$	14 286
15 090±43	2.4±0.7	$\bar{p}$Kr $n \leq 14$ $3p \to 1s$ el.	15 080
17 261±12	11.6±1.0	$\bar{p}$Kr(16–15)	17 250
19 537±27	3.2±0.8	$\bar{p}$Al(8–7)	19 523
21 090±4	33.6±1.2	$\bar{p}$Kr(15–14)	21 084
26 147±3	89.4±2.3	$\bar{p}$Kr(14–13)	26 145
29 052±19	4.4±0.6	*not identified*	
30 146±25	3.4±0.5	$\bar{p}$Al(7–6)	30 107
32 962±3	80.5±2.2	$\bar{p}$Kr(13–12)	32 965

28d5/2→25f 5/2	10639.970	-15.7999:	-282.74559

17f3/2→16f5 /2	12933.098	-35.31058	1626.81227
18d3/2→17f 5/2	12973.982	-30.77978	-984.06331

23p1/2→19d 3/2	29049.709	-58.8919	-754.07286

Simulation for $\bar{p}$Kr with a μCalorimeter



Conclusion

- The « proton size puzzle » is still alive
- There are apparently no problem with QED in normal ions
- For ^{12}C, muonic atoms and electronic atoms gives very close nuclear size, contrary to H and D
- Nuclear size effects for lower levels strongly dominates the transition energy values
- One can find, in both muonic and $\bar{p}$ atoms, transitions with lamb-shift contributions as large or larger than in electronic highly-charged ions, that can be measured precisely using modern techniques like μ-Calorimeters
- QED corrections are dominated by vacuum polarization while they are dominated by self-energy in normal atoms
- More work:
  - 1st-order self-energy with realistic potentials
  - 2nd order QED diagrams with mixed SE-VP loops
  - add Auger to transitions/level width code (!?)