Challenges to Transport Theory for Heavy-Ion Collisions
ECT*, Trento, May 20-24th, 2019

Spin dynamics in heavy-ion collisions

Jun Xu (徐骏)

Shanghai Advanced Research Institute, Chinese Academy of Sciences

Shanghai Institute of Applied Physics, Chinese Academy of Sciences

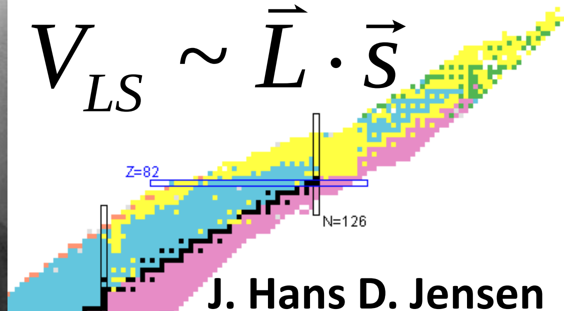
Content

1. Spin dynamics in intermediate-energy HIC
   - Probing nuclear spin-orbit interaction
2. Spin dynamics in relativistic HIC
   - Chiral anomalies

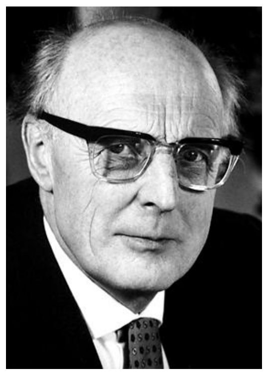
Spin-orbit coupling and spin dynamics

Maria Goeppert Mayer

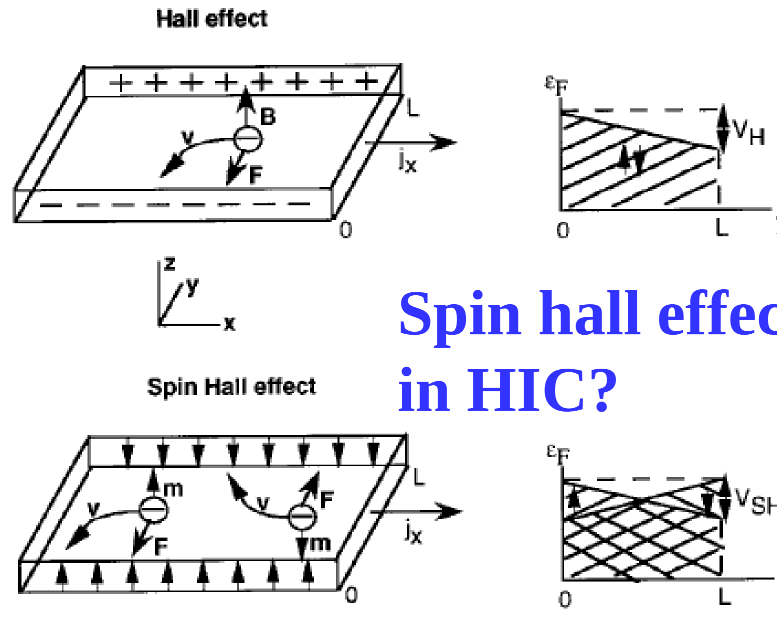
$$V_{LS} \sim \vec{L} \cdot \vec{s}$$



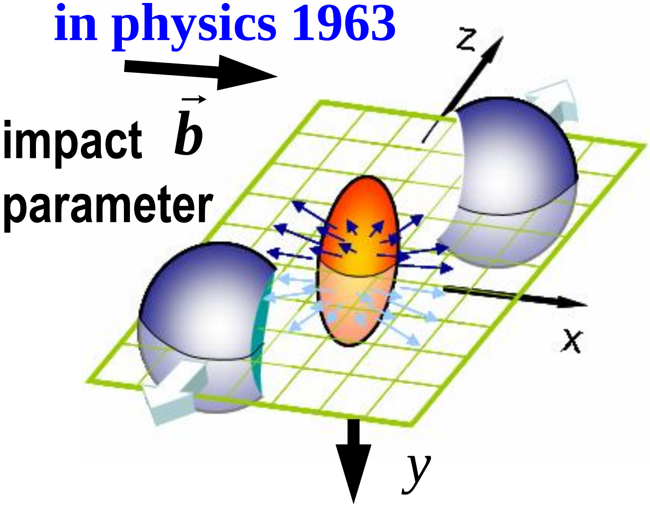
J. Hans D. Jensen



Nobel prize in physics 1963



Spin hall effect in HIC?



Perpendicular to the reaction plane:

1） vorticity $\vec{L} \cdot \vec{s}$

2） magnetic field $\vec{\mu} \cdot \vec{B}$

Different types of spin-orbit couplings

$$H^{SO} = A(\vec{p})\sigma_x - B(\vec{p})\sigma_y + C(\vec{p})\sigma_z = \vec{b}\cdot\vec{\sigma}$$

2D system	A(p)	B(p)	C(p)
Rashba	$\beta_R p_y$	$\beta_R p_x$	
Dresselhaus [001]	$\beta_D p_x$	$\beta_D p_y$	
Dresselhaus [110]	βp_x	$-\beta p_x$	
Rashba - Dresselhaus	$\beta_R p_y - \beta_D p_x$	$\beta_R p_x - \beta_D p_y$	
Cubic Rashba (hole)	$i\frac{\beta_R}{2}(p_-^3 - p_+^3)$	$\frac{\beta_R}{2}(p_-^3 + p_+^3)$	
Cubic Dresselhaus	$\beta_D p_x p_y^2$	$\beta_D p_y p_x^2$	
Wurtzite type	$(\alpha + \beta p^2)p_y$	$(\alpha + \beta p^2)p_x$	
Single-layer graphene	$v p_x$	$-v p_y$	
Bilayer graphene	$\frac{p_-^2 + p_+^2}{4m_e}$	$\frac{p_-^2 - p_+^2}{4m_e i}$	
3D system	A(p)	B(p)	C(p)
Bulk Dresselhaus	$p_x(p_y^2 - p_z^2)$	$p_y(p_x^2 - p_z^2)$	$p_z(p_x^2 - p_y^2)$
Cooper pairs	Δ	0	$\frac{p^2}{2m} - \epsilon_F$
Extrinsic			
$\beta = \frac{i}{\hbar}\lambda^2 V(p)$	$q_y p_z - q_z p_y$	$q_z p_x - q_x p_z$	$q_x p_y - q_y p_x$
Neutrons in nuclei			
$\beta = i W_0(n_n + \frac{n_p}{2})$	$q_z p_y - q_y p_z$	$q_x p_z - q_z p_x$	$q_y p_x - q_x p_y$

K. Morawetz, Phys. Rev. B, 2015

$$\vec{\sigma}\cdot(\nabla\rho\times\vec{p})$$

Single-particle energy

$$\hat{\varepsilon}(\vec{r},\vec{p}) = \varepsilon(\vec{r},\vec{p})\hat{I} + \vec{h}(\vec{r},\vec{p})\cdot\vec{\sigma}$$

Spin-independent   Spin-dependent

Distribution function

$$\hat{f}(\vec{r},\vec{p}) = f_0(\vec{r},\vec{p})\hat{I} + \vec{g}(\vec{r},\vec{p})\cdot\vec{\sigma}$$

Spin-averaged

$$f(\vec{r}\vec{p},t,0) = f_{1,1}(\vec{r}\vec{p},t) + f_{-1,-1}(\vec{r}\vec{p},t)$$

Polarization in z direction

$$\tau(\vec{r}\vec{p},t,z) = f_{1,1}(\vec{r}\vec{p},t) - f_{-1,-1}(\vec{r}\vec{p},t)$$

Polarization in x direction

$$\tau(\vec{r}\vec{p},t,x) = f_{-1,1}(\vec{r}\vec{p},t) + f_{1,-1}(\vec{r}\vec{p},t)$$

Polarization in y direction

$$\tau(\vec{r}\vec{p},t,y) = -i[f_{-1,1}(\vec{r}\vec{p},t) - f_{1,-1}(\vec{r}\vec{p},t)]$$

Single-particle Hamiltonian from Skyrme interaction

Skyrme spin-orbit interaction:

$$V_{so} = iW_0(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2)\vec{k}'$$

Single-particle Hamiltonian:

$$h_q^{so}(\vec{r}, \vec{p}) = h_1 + h_4 + (\vec{h}_2 + \vec{h}_3) \cdot \vec{\sigma}.$$

Y.M. Engel et al., NPA (1975)

$$h_1 = -\frac{W_0}{2}\nabla_{\vec{r}} \cdot [\vec{J}(\vec{r}) + \vec{J}_q(\vec{r})]$$ **time-even** $$\vec{J}(\vec{r}) = \int d^3p \frac{\vec{p}}{\hbar} \times \vec{\tau}(\vec{r}, \vec{p}).$$

$$\vec{h}_2 = -\frac{W_0}{2}\nabla_{\vec{r}} \times [\vec{j}(\vec{r}) + \vec{j}_q(\vec{r})]$$ **time-odd** $$\vec{j}(\vec{r}) = \int d^3p \frac{\vec{p}}{\hbar} f(\vec{r}, \vec{p}),$$

$$\vec{h}_3 = \frac{W_0}{2}\nabla_{\vec{r}}[\rho(\vec{r}) + \rho_q(\vec{r})] \times \vec{p}$$ **time-even** $$\rho(\vec{r}) = \int d^3p f(\vec{r}, \vec{p}),$$

$$h_4 = -\frac{W_0}{2}\nabla_{\vec{r}} \times [\vec{s}(\vec{r}) + \vec{s}_q(\vec{r})] \cdot \vec{p}$$ **time-odd** $$\vec{s}(\vec{r}) = \int d^3p \vec{\tau}(\vec{r}, \vec{p}),$$

$f(\vec{r}, \vec{p})$ **and** $\tau(\vec{r}, \vec{p})$ **are calculated from the test-particle method.**

- **Skyrme-Hartree-Fock model**

Strength W_0 **= 80 ~ 150 MeVfm**^{5}

Hartree-Fock method →

$$\vec{W}_q = \frac{W_0}{2}\left(\nabla\rho + \nabla\rho_q\right)$$

q = n, p

- **Relativistic mean field model**

Dirac equation

Non-relativistic expansion →

$$\vec{W}_q = \frac{C}{(2m - C\rho)^2}\nabla\rho, C = \frac{g_\sigma^2}{m_\sigma^2} + \frac{g_\omega^2}{m_\omega^2}$$

P. G. Reinhard and H. Flocard, NPA, 1995

the isospin dependence of the SO potential

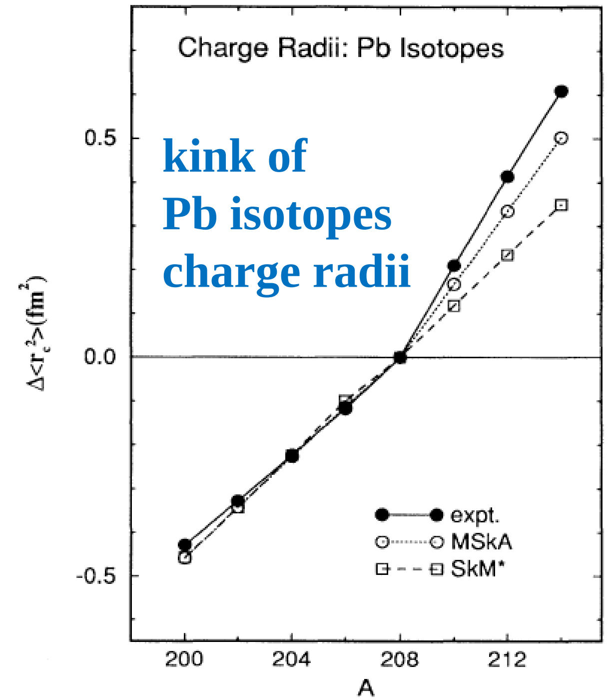
$$\vec{W}_q = \frac{W_0}{2}(1+\chi_w)\nabla\rho_q + \frac{W_0}{2}\nabla\rho_{q'}.(q \neq q')$$

M. M. Sharma *et al.*, Phys. Rev. Lett., 1995

the density dependence of the SO potential

$$v_{ij} = v_{ij}^0 + (i/\hbar^2)W_1(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\cdot\mathbf{p}_{ij} \times (\rho_{q_i} + \rho_{q_j})^\gamma \delta(\mathbf{r}_{ij})\mathbf{p}_{ij}.$$

$$\vec{W}_q = \frac{W_0}{2}\nabla(\rho+\rho_q) + \frac{W_1}{2}[(\rho)^\gamma\nabla(\rho-\rho_q) + (2+\gamma)(2\rho_q)^\gamma\nabla\rho_q] + \frac{W_1}{4}\gamma\rho^{\gamma-1}(\rho-\rho_q)\nabla\rho$$



J. M. Pearson and M. Farine, Phys. Rev. C 50, 185 (1994).

spin effects on low-energy nuclear reactions

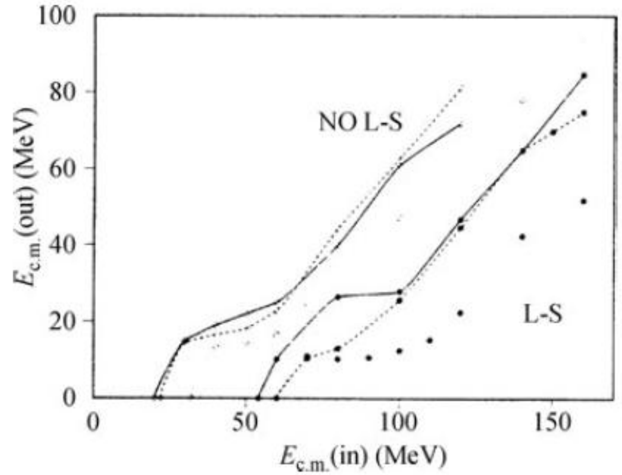
Effects of spin-orbit coupling illustrated by $O^{16}+O^{16}$ from TDHF:

Fusion threshold

TABLE I. Thresholds for the inelastic scattering of $^{16}O + ^{16}O$ system.

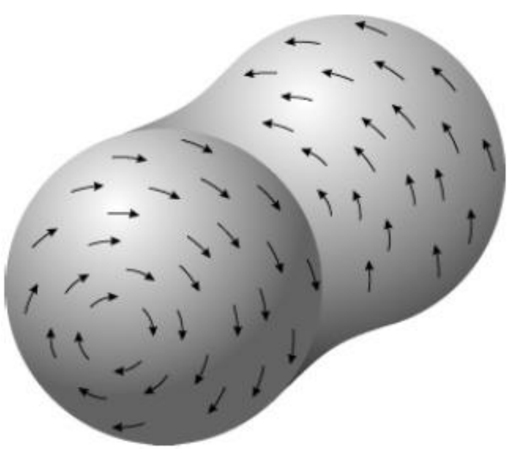
Force	Skyrme II (MeV)	Skyrme M* (MeV)
Spin orbit	68	70
No spin orbit	31	27

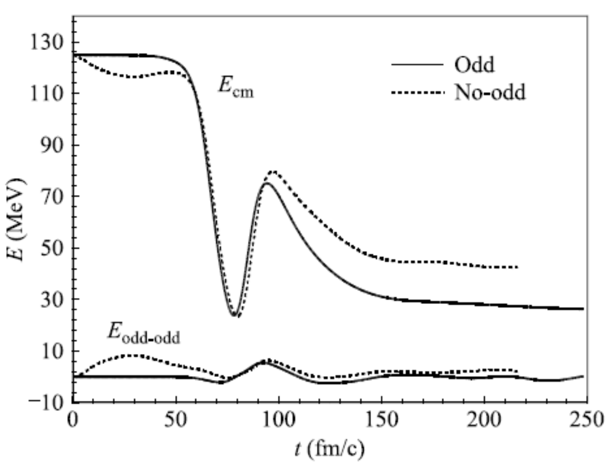
A.S. Umar *et al.*, Phys. Rev. Lett., 1986



P.G. Reinhard *et al.*, Phys. Rev. C, 1988

Spin twist & time-odd terms





J. A. Maruhn *et al.*, Phys. Rev. C, 2006

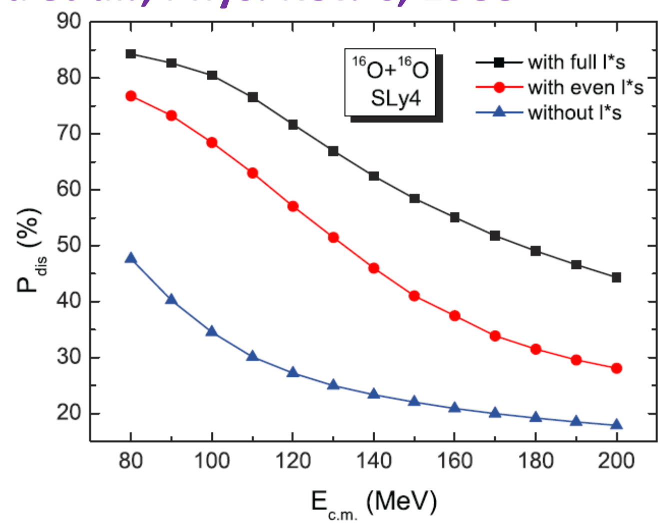


FIG. 2. (Color online) Percentage of energy dissipation as a function of center-of-mass energy for head-on collisions of $^{16}O + ^{16}O$ with parametrization SLy4. The black, red, and blue lines represent the TDHF calculations involving full l*s, time-even l*s, and no l*s force.

G.F. Dai, L. Guo, E.G. Zhao, and S.G. Zhou, Phys. Rev. C, 2014

Spin dynamics in intermediate- and low-energy heavy-ion collisions

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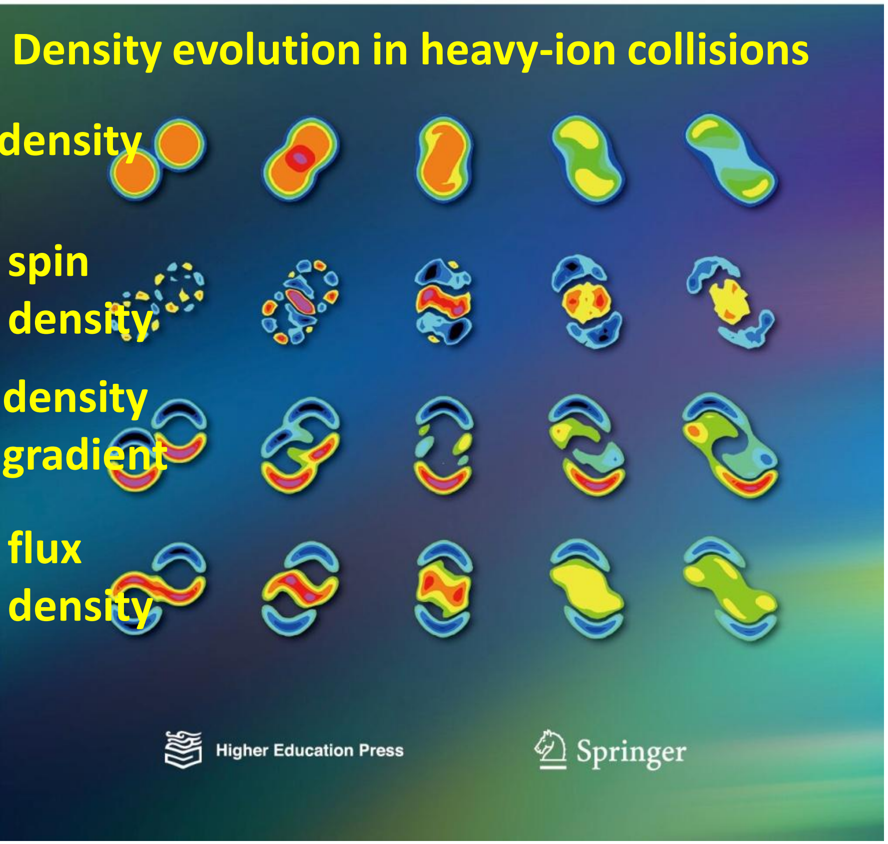


invited review, cover story

JX, B.A. Li, W.Q. Shen, and Y. Xia, Front. Phys. (2015)

SIBUU12

Boltzmann-Uehling-Uhlenbeck equation

$$\frac{\partial f}{\partial t} + v\cdot\nabla_r f - \nabla U\cdot\nabla_p f = -\int \frac{d^3p_2\, d^3p_1'\, d^3p_2'}{(2\pi)^9}\sigma v_{12}[f f_2(1-f_{1'})(1-f_{2'}) - f_{1'}f_{2'}(1-f)(1-f_2)](2\pi)^3\delta^3(p+p_2-p_1'-p_2')$$

test-particle method

C.Y. Wong, PRC 25, 1460 (1982)

equations of motion

$$\frac{d\vec r}{dt} = \frac{\vec p}{m} \qquad \frac{d\vec p}{dt} = -\nabla U$$

Spin-dependent Boltzmann-Uehling-Uhlenbeck eq

2×2 **matrix equation**

$$\frac{\partial\hat f}{\partial t} + \frac{i}{\hbar}\left[\hat\varepsilon,\hat f\right] + \frac{1}{2}\left(\frac{\partial\hat\varepsilon}{\partial\vec p}\cdot\frac{\partial\hat f}{\partial\vec r}+\frac{\partial\hat f}{\partial\vec r}\cdot\frac{\partial\hat\varepsilon}{\partial\vec p}\right) - \frac{1}{2}\left(\frac{\partial\hat\varepsilon}{\partial\vec r}\cdot\frac{\partial\hat f}{\partial\vec p}+\frac{\partial\hat f}{\partial\vec p}\cdot\frac{\partial\hat\varepsilon}{\partial\vec r}\right) = I_c$$

test-particle method

Y. Xia, JX, B.A. Li, and W.Q. Shen, Phys. Lett. B (2016)

spin-dependent equations of motion

$$\frac{d\vec r}{dt} = \frac{\vec p}{m} + \nabla_p(\varepsilon + \vec h\cdot\vec n) \qquad \frac{d\vec p}{dt} = -\nabla(\varepsilon + \vec h\cdot\vec n)$$

$$\frac{d\vec n}{dt} = 2\vec h\times\vec n$$

$\vec n \sim \vec g$ or $\vec\tau$

spin expectation direction

Transverse flow $\langle p_x \rangle \sim y$

sensitive to nuclear interaction

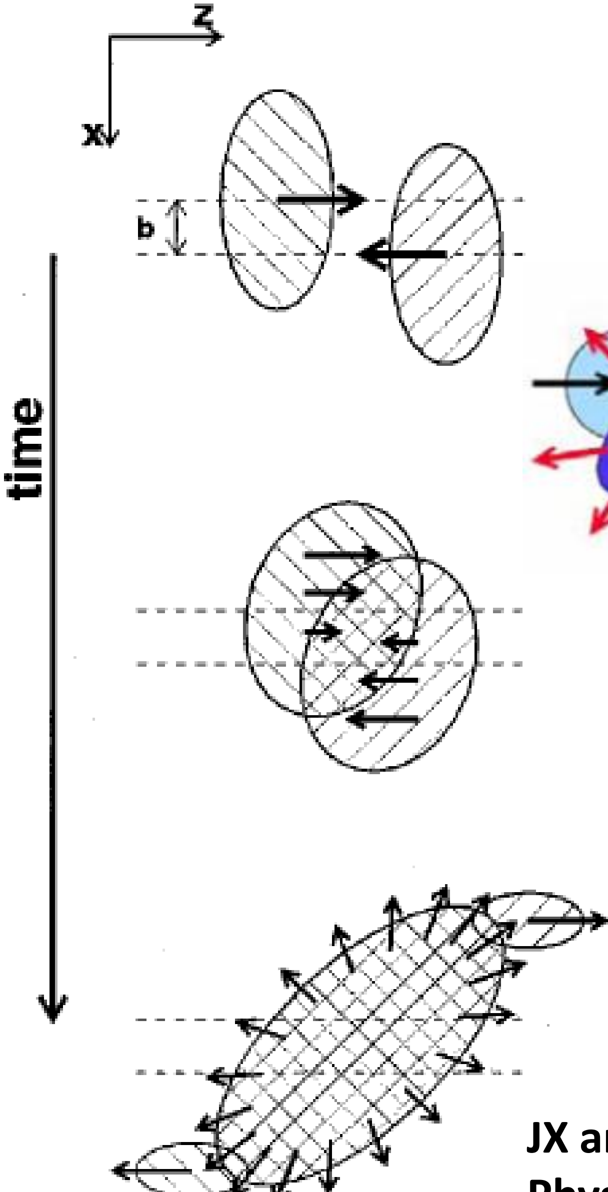
Spin up-down differential transverse flow

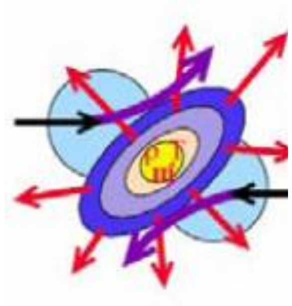
$$U = U_0 + \sigma U_{spin}$$

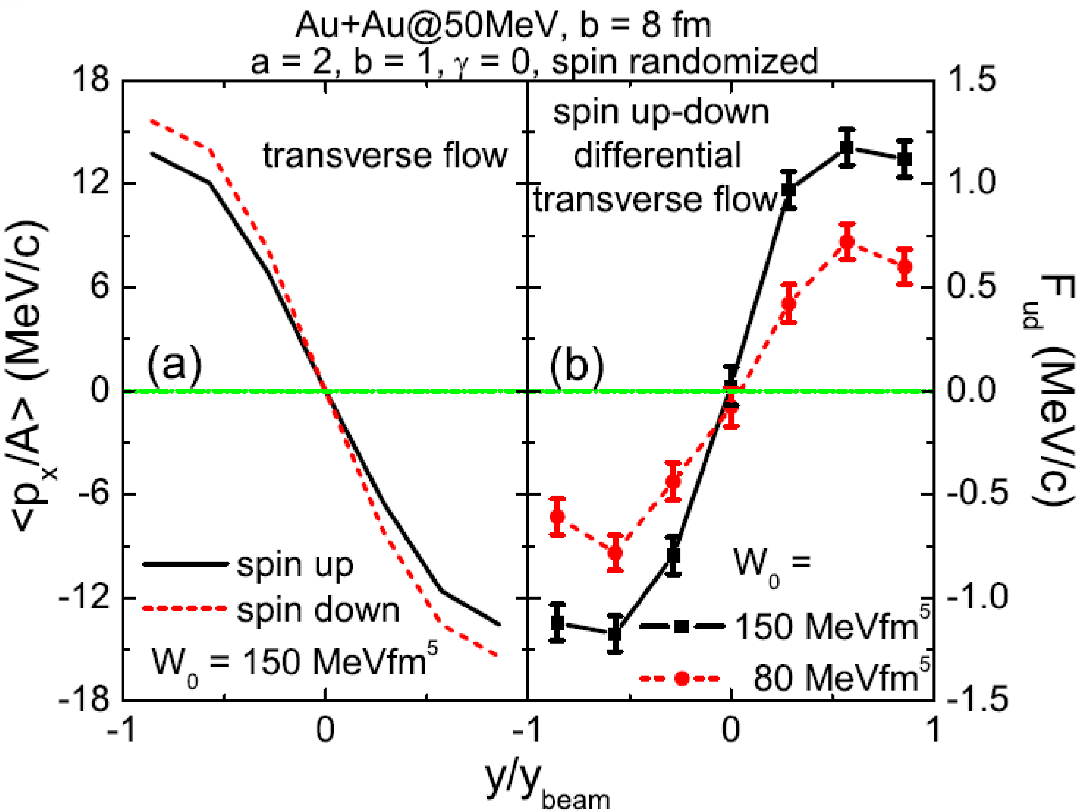
$$\sigma = 1(\uparrow) or -1(\downarrow)$$

$$F_{ud}(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} \sigma_i (p_x)_i$$

reflects different transverse flows of spin-up and spin-down nucleons





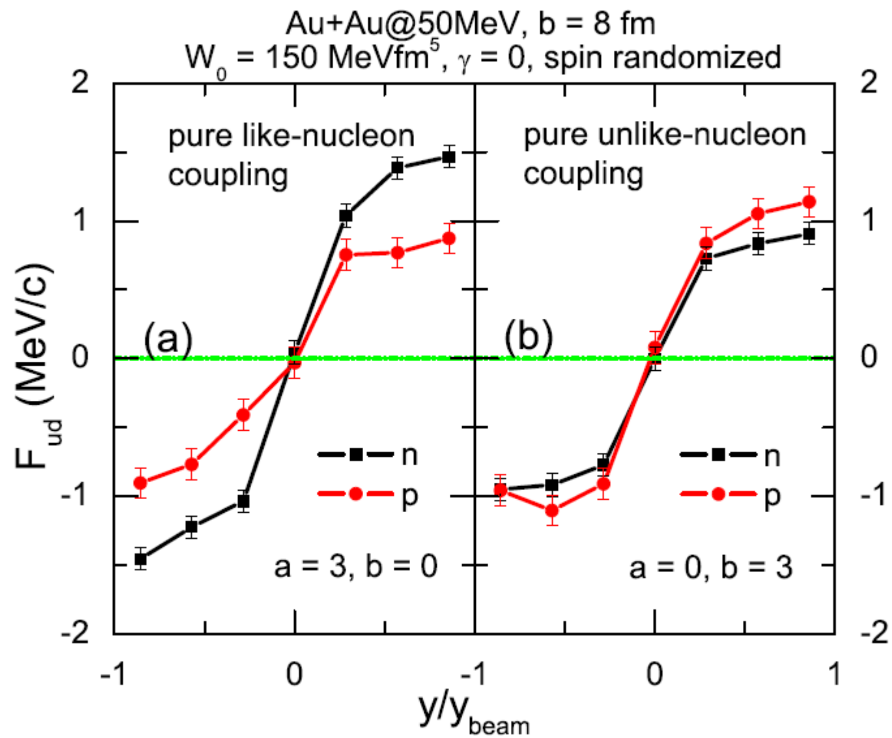


JX and B.A. Li
Phys. Lett. B (2013)

F_{ud} is sensitive to W_0, the strength of the spin-orbit interaction.

Measuring spin differential flows for neutrons and protons

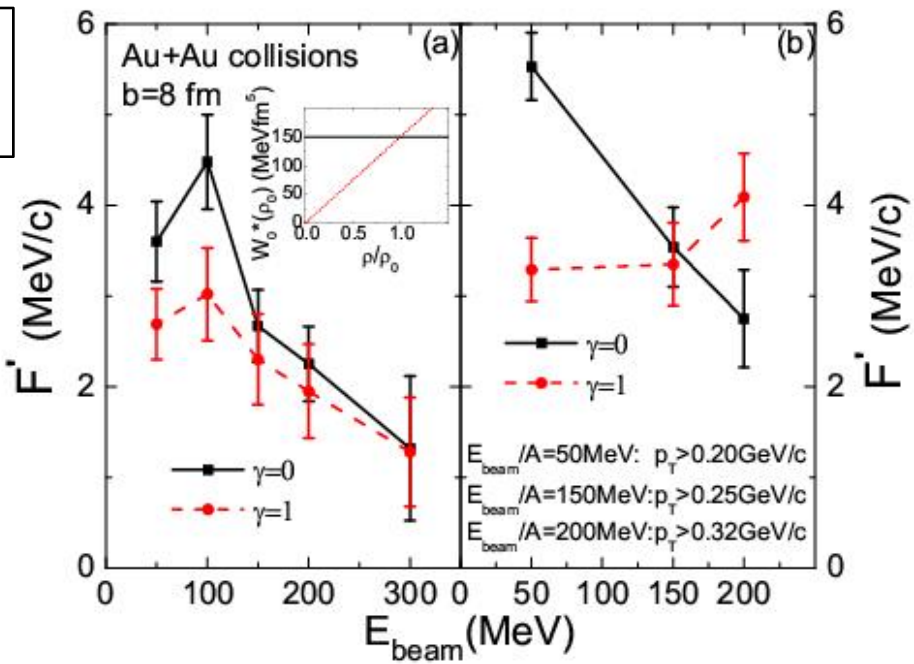
Probing the isospin dependence of the spin-orbit coupling



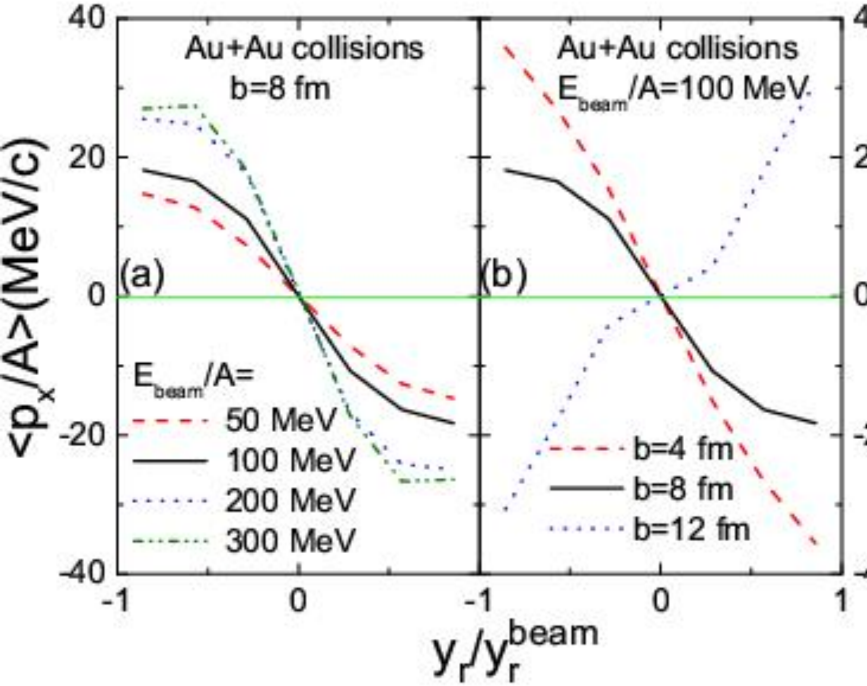
Measuring slope of spin differential flow for energetic nucleons at different E_{beam}

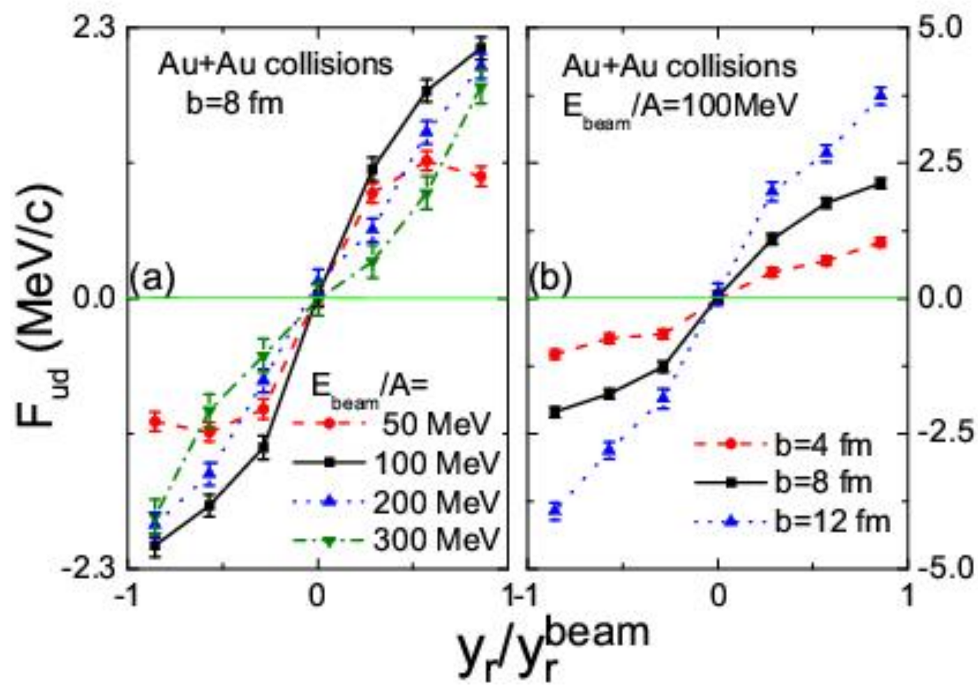
$$F' = \left[\frac{dF_{ud}}{d(y_r/y_r^{beam})}\right]_{y_r=0}$$

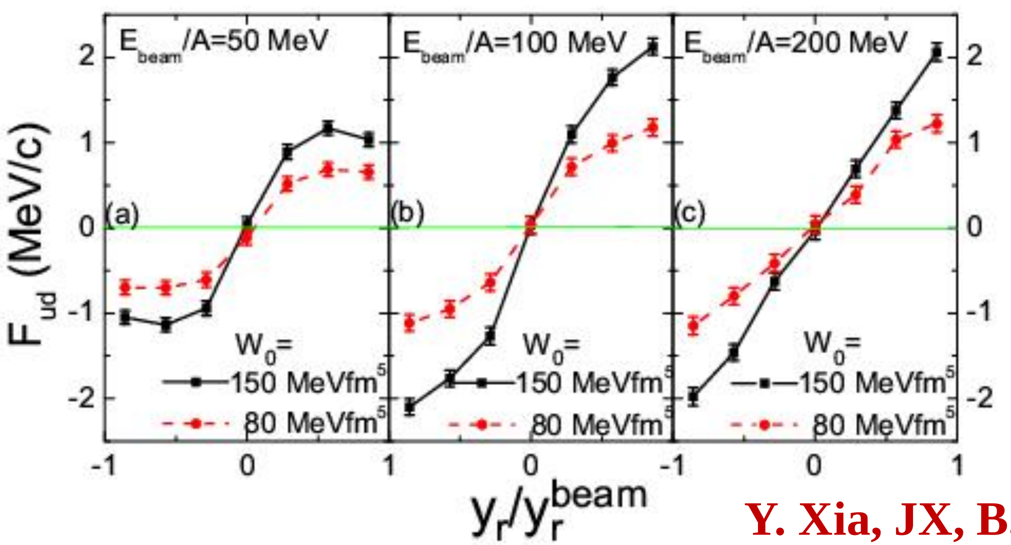
Probing the density dependence of the spin-orbit coupling



Energy and impact parameter dependence







The transverse flow

repulsive NN scatterings

attractive mean-field potential

Spin up-down differential transverse flow

density gradient (surface)

angular momentum (current)

violent NN scatterings

Y. Xia, JX, B.A. Li, and W.Q. Shen, Phys. Rev. C (2014)

System size dependence

Directed flow:

$$v_1 = \langle \cos(\phi) \rangle = \left\langle \frac{p_x}{p_T} \right\rangle$$

heavier system

↓

higher density, pressure

↓

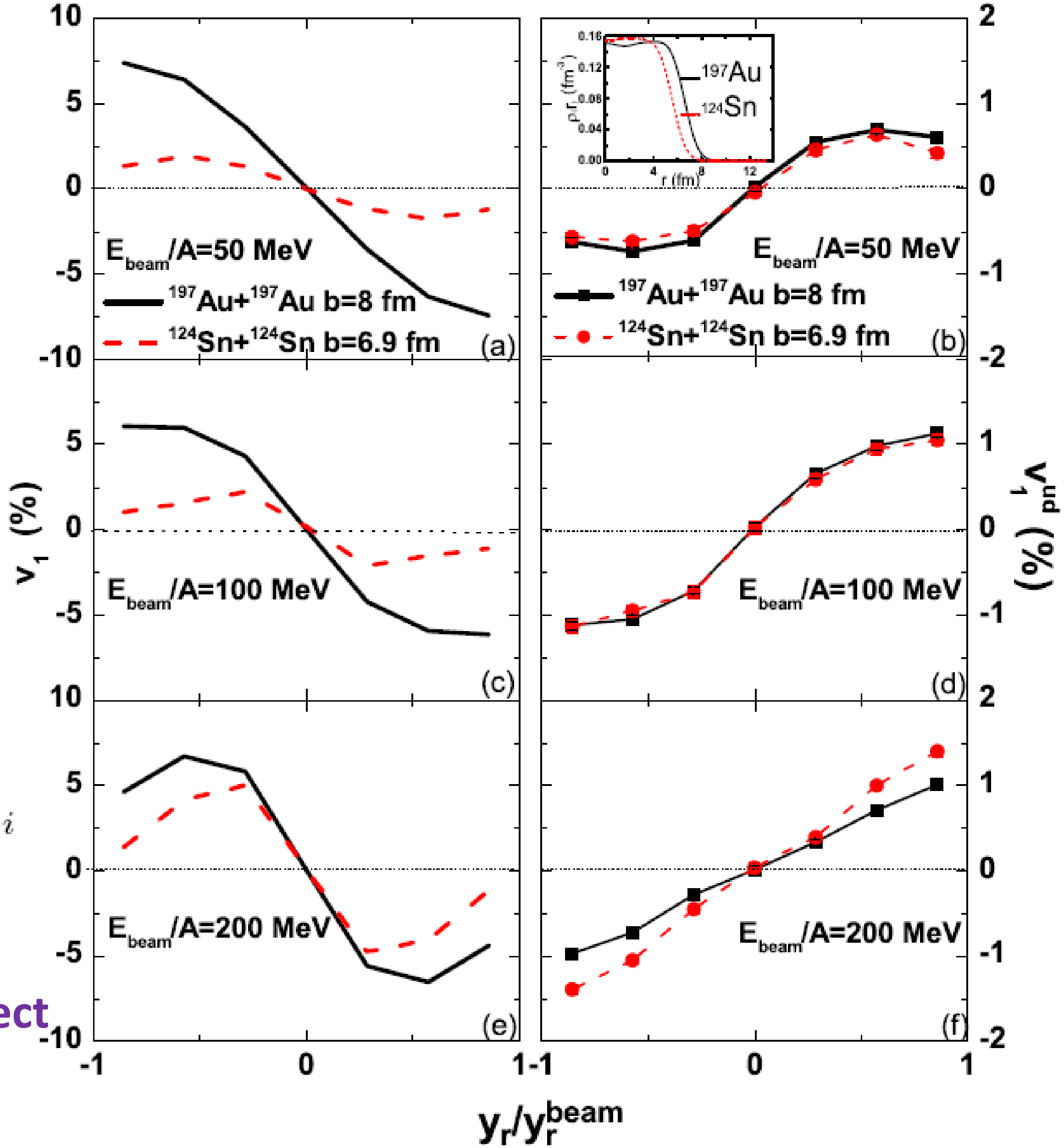
Larger v_1

Spin up-down differential directed flow:

$$v_1^{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left(\frac{p_x}{p_T} \right)_i$$

Surface effect

NN scatterings wash out spin effect



Effects of spin-orbit interaction on v_2

Elliptic flow:

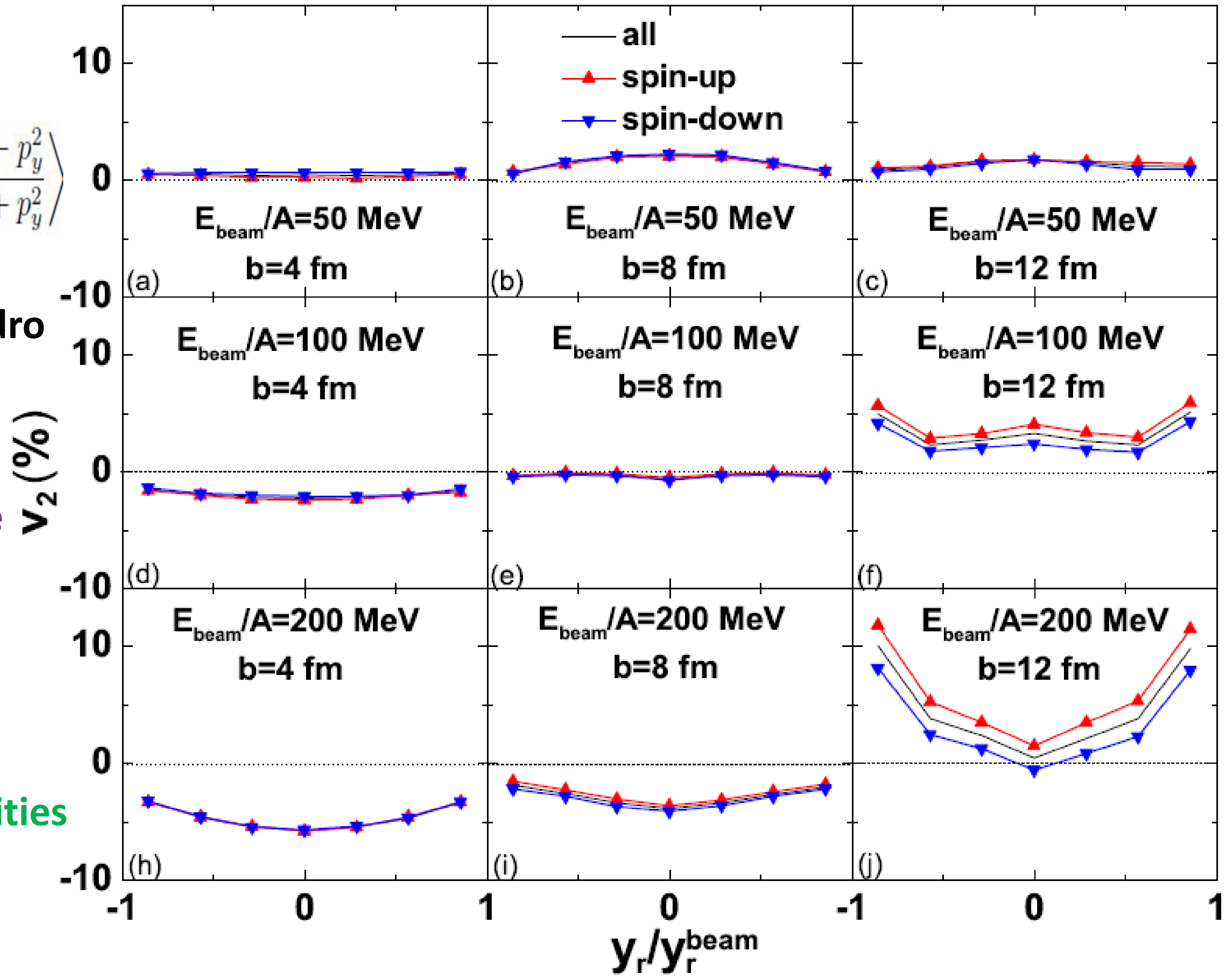
$$v_2 = \langle \cos(2\phi) \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

+: in-plane hydro

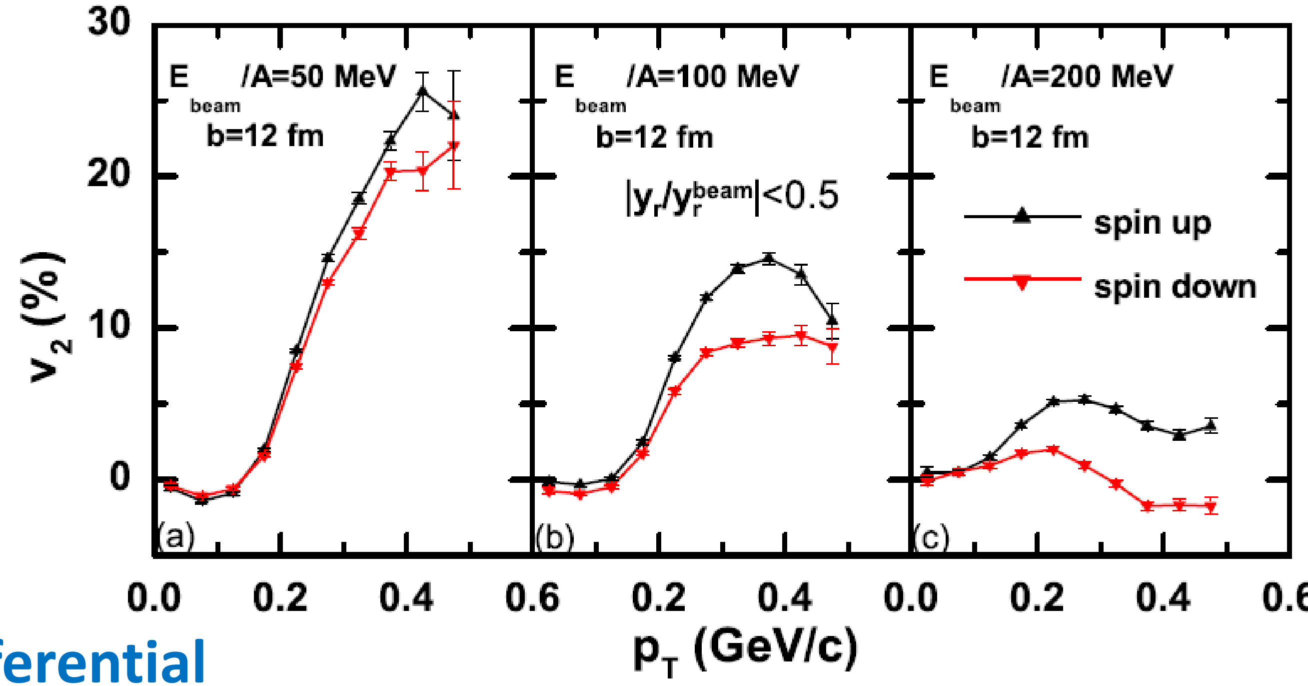
-: squeeze out

Dynamics more complicated than v_1

Spin splitting at large centralities



Spin splitting at higher p_T

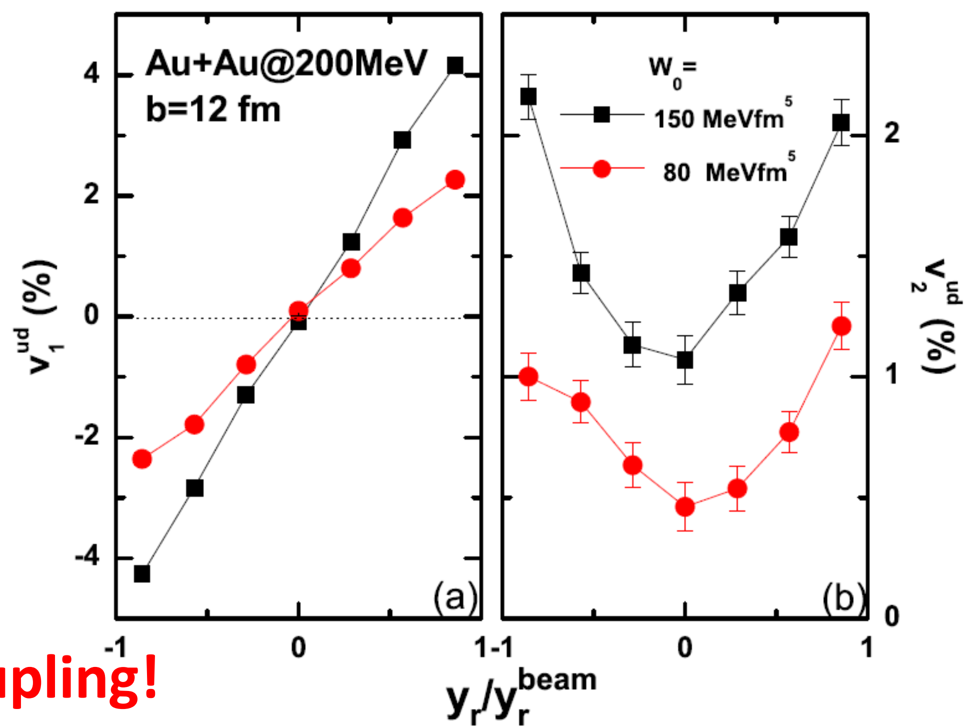


Spin up-down differential directed flow:

$$v_1^{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left(\frac{p_x}{p_T}\right)_i$$

Spin up-down differential elliptic flow:

$$v_2^{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left(\frac{p_x^2 - p_y^2}{p_T^2}\right)_i$$



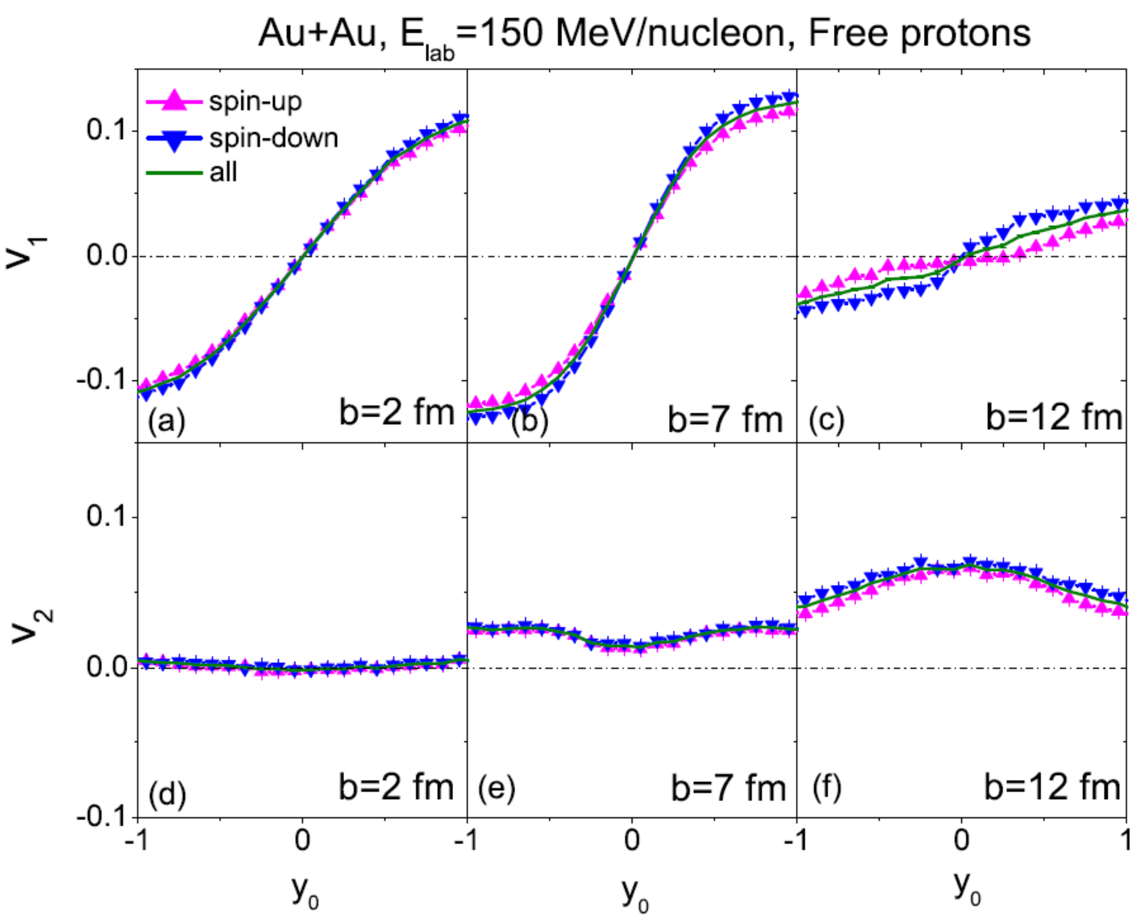
Both are sensitive probes of SO coupling!

Spin dynamics from QMD model

$$u_{so}^{even} = -\frac{1}{2}W_0(\rho\nabla\cdot\vec{J} + \rho_n\nabla\cdot\vec{J}_n + \rho_p\nabla\cdot\vec{J}_p)$$

$$u_{so}^{odd} = -\frac{1}{2}W_0[\vec{s}\cdot(\nabla\times\vec{j}) + \vec{s}_n\cdot(\nabla\times\vec{j}_n) + \vec{s}_p\cdot(\nabla\times\vec{j}_p)]$$

C.C. Guo, Y.J. Wang, Q.F. Li, and F.S. Zhang, Phys. Rev. C 90, 034606 (2014)



$$\rho(\vec{r}) = \sum_i \rho_i(\vec{r}) = \sum_i \frac{1}{(2\pi L)^{3/2}} e^{[-(\vec{r}-\vec{r}_i)^2/(2L)]}$$

$$\vec{s}(\vec{r}) = \sum_i \rho_i(\vec{r})\vec{\sigma}_i,$$

$$\vec{j}(\vec{r}) = \sum_i \rho_i(\vec{r})\vec{p}_i,$$

$$\vec{J}(\vec{r}) = \sum_i \rho_i(\vec{r})\vec{p}_i \times \vec{\sigma}_i,$$

Effects of SO coupling on spin dynamics are robust and can be observed in different approaches.

The multiplicity of a M-nucleon cluster

$$\frac{dN_M}{d^3K} = G\binom{A}{M}\binom{M}{Z}\frac{1}{A^M}\int\left[\prod_{i=1}^{Z} f_p(\mathbf{r}_i,\mathbf{k}_i)\right]\left[\prod_{i=Z+1}^{M} f_n(\mathbf{r}_i,\mathbf{k}_i)\right]$$
$$\times \rho^W(\mathbf{r}_{i_1},\mathbf{k}_{i_1},\ldots,\mathbf{r}_{i_{M-1}},\mathbf{k}_{i_{M-1}})\,\delta\big(\mathbf{K}-(\mathbf{k}_1+\cdots+\mathbf{k}_M)\big)\,d\mathbf{r}_1\,d\mathbf{k}_1\cdots d\mathbf{r}_M\,d\mathbf{k}_M$$

R. Mattiello et al., Phys. Rev. Lett 1995 Phys. Rev. C 1997.

ρ^W **the Wigner phase-space density of the M-nucleon cluster**

Spatial wave function: s-wave assumption

Statistical factor Only asymmetric spin-isospin allowed

- G **: coalescence with a given isospin**
- G'**: coalescence with a given spin and isospin**

3/8 ,d
1/12, t
1/12, ^{3}He

$^2_1H(S=1)$ G'	$^3_1H(S=1/2)$ G'	$^3_2He(S=1/2)$ G'
$p\uparrow \& n\uparrow \longrightarrow 1/2\ (S_z=1)$	$p\uparrow \& n\uparrow \& n\downarrow \longrightarrow 1/6(S_z=+1/2)$	$n\uparrow \& p\uparrow \& p\downarrow \longrightarrow 1/6(S_z=+1/2)$
$p\uparrow \& n\downarrow \longrightarrow 1/4\ (S_z=0)$		
$p\downarrow \& n\uparrow \longrightarrow 1/4\ (S_z=0)$	$p\downarrow \& n\uparrow \& n\downarrow \longrightarrow 1/6(S_z=-1/2)$	$n\downarrow \& p\uparrow \& p\downarrow \longrightarrow 1/6(S_z=-1/2)$
$p\downarrow \& n\downarrow \longrightarrow 1/2\ (S_z=-1)$		

Wigner phase-space density

deuteron

$$\rho_d^W(\mathbf{r},\mathbf{k}) = \int \phi\left(\mathbf{r}+\frac{\mathbf{R}}{2}\right)\phi^*\left(\mathbf{r}-\frac{\mathbf{R}}{2}\right)\exp(-i\mathbf{k}\cdot\mathbf{R})\,d\mathbf{R},$$

$$\mathbf{k} = (\mathbf{k}_1 - \mathbf{k}_2)/2 \qquad \mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)$$

Internal wave function $\phi(r)$ ⇒ root-mean-square radius of 1.96 fm

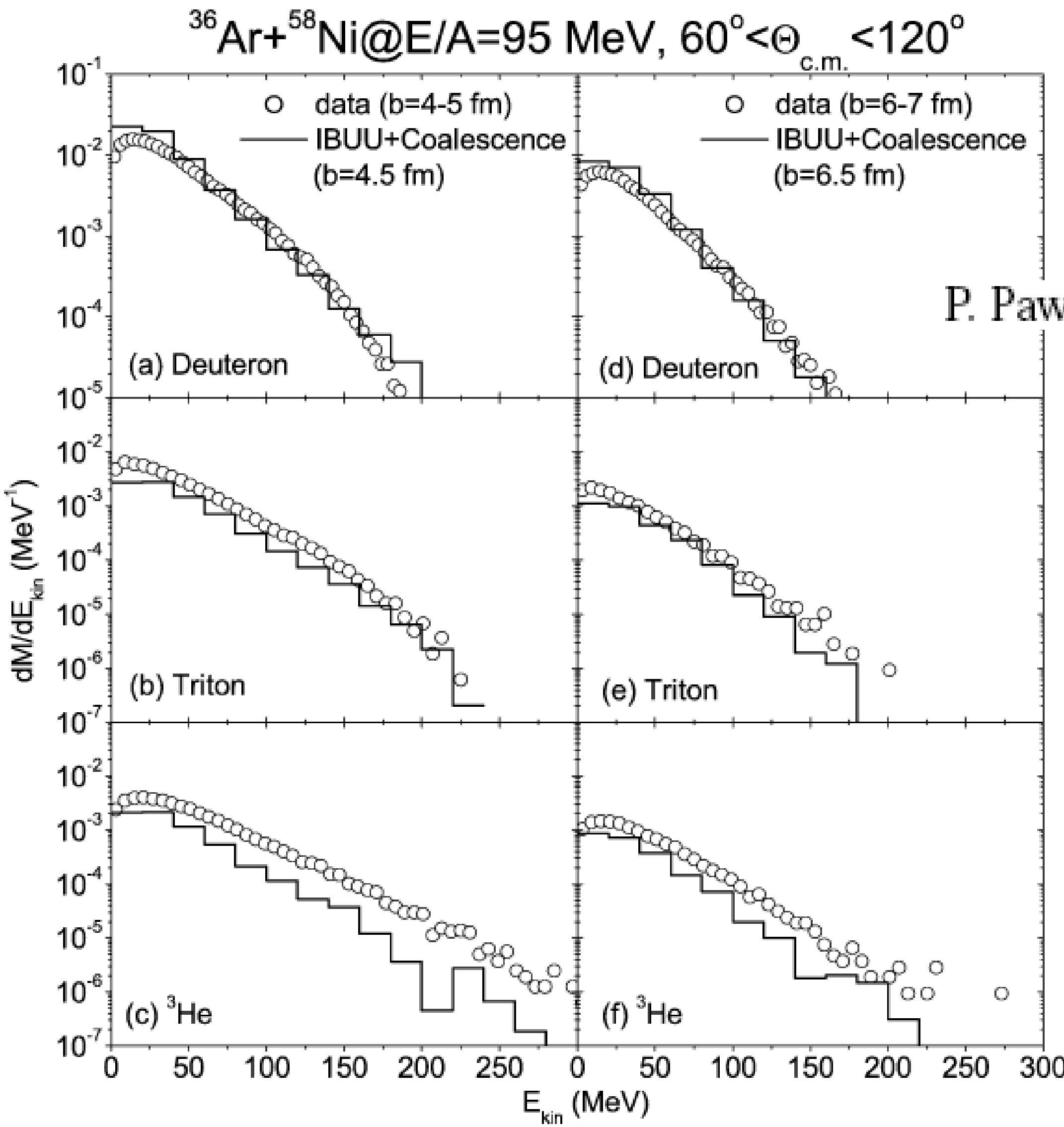
Triton or Helium3

$$\rho^W_{t(^3He)}(\rho,\lambda,\mathbf{k}_\rho,\mathbf{k}_\lambda) = \int \psi\left(\rho+\frac{\mathbf{R}_1}{2},\lambda+\frac{\mathbf{R}_2}{2}\right)\psi^*\left(\rho-\frac{\mathbf{R}_1}{2},\lambda-\frac{\mathbf{R}_2}{2}\right) \times \exp(-i\mathbf{k}_\rho\cdot\mathbf{R}_1)\exp(-i\mathbf{k}_\lambda\cdot\mathbf{R}_2)3^{3/2}\,d\mathbf{R}_1\,d\mathbf{R}_2$$

$$\begin{pmatrix}\mathbf{R}\\ \boldsymbol{\rho}\\ \boldsymbol{\lambda}\end{pmatrix} = J\begin{pmatrix}\mathbf{r}_1\\ \mathbf{r}_2\\ \mathbf{r}_3\end{pmatrix} \quad J = \begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}}\end{pmatrix} \quad \begin{pmatrix}\mathbf{K}\\ \mathbf{k}_\rho\\ \mathbf{k}_\lambda\end{pmatrix} = J^{-,+}\begin{pmatrix}\mathbf{k}_1\\ \mathbf{k}_2\\ \mathbf{k}_3\end{pmatrix} \quad J^{-,+} = \begin{pmatrix}1 & 1 & 1\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}}\end{pmatrix}$$

Internal wave function $\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ ⇒ **RMS radius** 1.61 and 1.74 fm for triton and ^{3}He

Light cluster production from coalescence using Wigner function method



reproduce experimental data from

P. Pawlowski, et al., Eur. Phys. J. A 9 (2000) 371

reasonably well

Suitable for loosely bound clusters

Suitable for rare particles:
Perturbative treatment

L.W. Chen, B.A. Li, and C.M. Ko, NPA (2003)

${}^{2}_{1}H$ wave function

S=1 T=0

$$\left|{}^{2}_{1}H\right\rangle \sim \left|spin\right\rangle\left|isospin\right\rangle$$

	S	T	
	$\uparrow\uparrow$	pp	
$S=1$	$\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)$	$\frac{1}{\sqrt{2}}(pn+np)$	$T=1$
	$\downarrow\downarrow$	nn	
$S=0$	$\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)$	$\frac{1}{\sqrt{2}}(pn-np)$	$T=0$

S_z = +1: $\psi_1 \sim \frac{1}{\sqrt{2}}(p\uparrow n\uparrow - n\uparrow p\uparrow)$

S_z = 0: $\psi_2 \sim \frac{1}{2}(p\uparrow n\downarrow + p\downarrow n\uparrow - n\uparrow p\downarrow - n\downarrow p\uparrow)$

S_z = -1: $\psi_3 \sim \frac{1}{\sqrt{2}}(p\downarrow n\downarrow - n\downarrow p\downarrow)$

$$\psi_4 \sim \frac{1}{2}(p\uparrow n\downarrow - p\downarrow n\uparrow - n\uparrow p\downarrow + n\downarrow p\uparrow)$$

$$\psi_5 \sim \frac{1}{\sqrt{2}}(p\uparrow n\uparrow + n\uparrow p\uparrow)$$

$$\psi_6 \sim \frac{1}{2}(p\uparrow n\downarrow + p\downarrow n\uparrow + n\uparrow p\downarrow + n\downarrow p\uparrow)$$

$$\psi_7 \sim \frac{1}{\sqrt{2}}(p\downarrow n\downarrow + n\downarrow p\downarrow)$$

$$\psi_8 \sim \frac{1}{2}(p\uparrow n\downarrow - p\downarrow n\uparrow + n\uparrow p\downarrow - n\downarrow p\uparrow)$$

Assign all many-nucleon states which are allowed from the Pauli principle the same weight.

8 wave function (considering the spin-isospin and antisymmetrization), 3 of 8 are feasible.

G= 3/8 (no information about spin)

$p\uparrow \& n\uparrow \longrightarrow G' = 1/2 (S_z = +1)$

$p\uparrow \& n\downarrow \longrightarrow G' = 1/4 (S_z = 0)$

$p\downarrow \& n\uparrow \longrightarrow G' = 1/4 (S_z = 0)$

$p\downarrow \& n\downarrow \longrightarrow G' = 1/2 (S_z = -1)$

S=1/2 T=1/2

${}^{3}_{1}H$ & ${}^{3}_{2}He$ wave function

$$\left|{}^{3}_{1}H / {}^{3}_{2}He\right\rangle \sim \left|spin\right\rangle \left|isospin\right\rangle \qquad S_\rho T_\lambda - S_\lambda T_\rho$$

	S	**T**	
$S=3/2$	$\uparrow\uparrow\uparrow$	ppp	$T=3/2$
	$\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow+\downarrow\uparrow\uparrow+\uparrow\downarrow\uparrow)$	$\frac{1}{\sqrt{3}}(ppn+npp+pnp)$	
	$\frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow+\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow)$	$\frac{1}{\sqrt{3}}(nnp+pnn+npn)$	
	$\downarrow\downarrow\downarrow$	nnn	
$S=1/2$ ρ	$\frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)$	$\frac{1}{\sqrt{6}}(2ppn-pnp-npp)$	$T=1/2$
	$\frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow-2\downarrow\downarrow\uparrow)$	$\frac{1}{\sqrt{6}}(pnn+npn-2nnp)$	
$S=1/2$ λ	$\frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)$	$\frac{1}{\sqrt{2}}(pnp-npp)$	$T=1/2$
	$\frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow-\downarrow\uparrow\downarrow)$	$\frac{1}{\sqrt{2}}(pnn-npn)$	

$(S_z=+1/2)$

$$\Psi_1({}^3_2\mathrm{He}) \sim \frac{1}{\sqrt{6}}(p^\uparrow n^\uparrow p^\downarrow - p^\downarrow n^\uparrow p^\uparrow - n^\uparrow p^\uparrow p^\downarrow$$
$$+ n^\uparrow p^\uparrow p^\downarrow - p^\uparrow p^\downarrow n^\uparrow + p^\downarrow p^\uparrow n^\uparrow),$$

$(S_z=-1/2)$

$$\Psi_2({}^3_2\mathrm{He}) \sim \frac{1}{\sqrt{6}}(p^\uparrow n^\downarrow p^\downarrow - p^\downarrow n^\downarrow p^\uparrow - n^\downarrow p^\uparrow p^\downarrow$$
$$+ n^\downarrow p^\uparrow p^\downarrow - p^\uparrow p^\downarrow n_\downarrow + p^\downarrow p^\uparrow n^\downarrow).$$

Another 5 states with same spin-isospin states but do not satisfy wave function antisymmetrization

$\{S({}^3_1H)=1/2 \,\&\, S({}^3_2He)=1/2\}$

24 wave function (considering the spin- isospin and antisymmetrization), 2 of 24 are feasible.

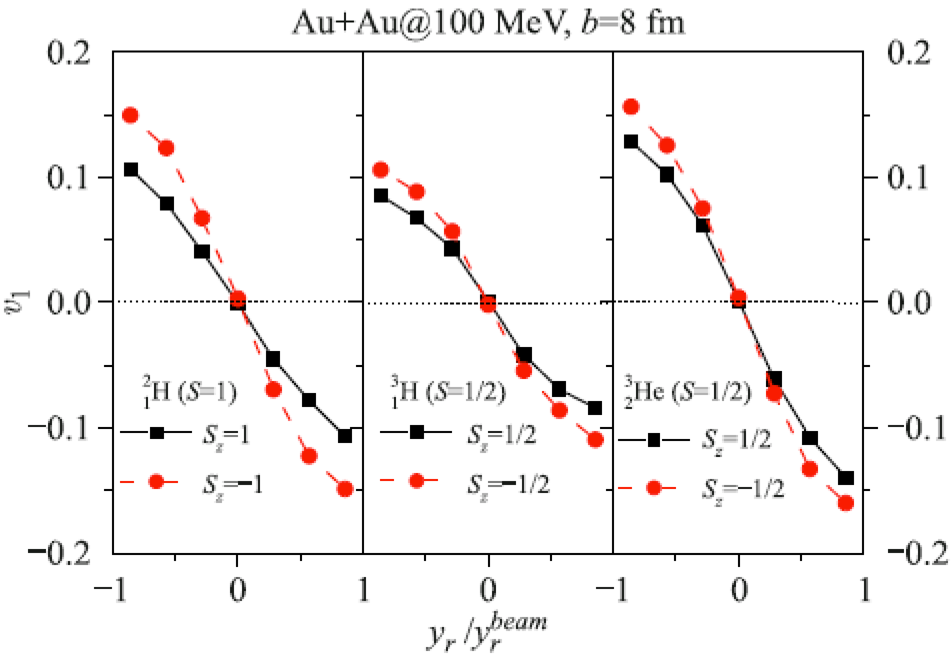
G= 1/12 (no information of spin)

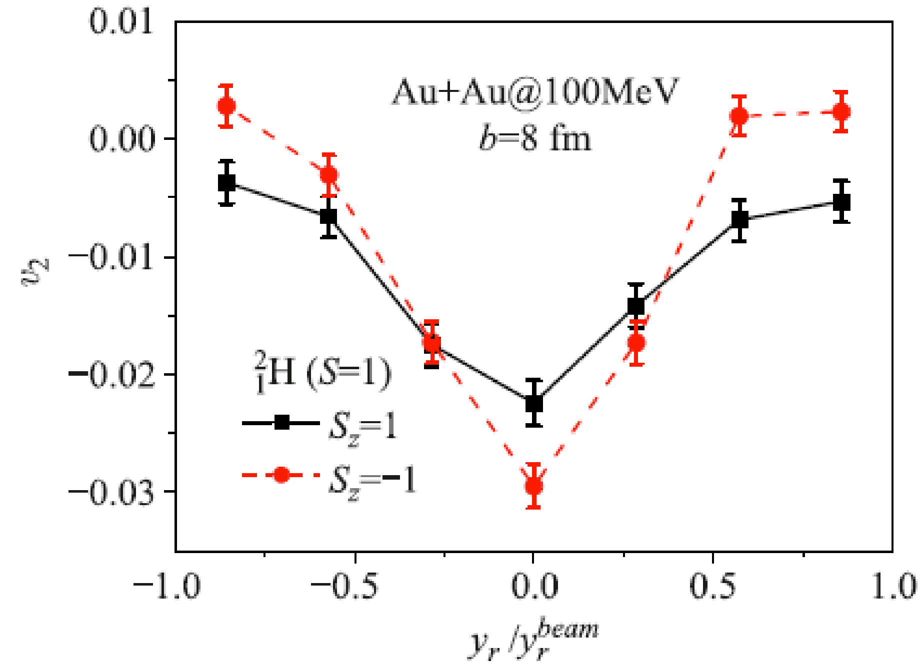
${}^3_2He$ $\qquad G'$

$n\uparrow \& p\uparrow \& p\downarrow \longrightarrow 1/6(S_z=+1/2)$

$n\downarrow \& p\uparrow \& p\downarrow \longrightarrow 1/6(S_z=-1/2)$

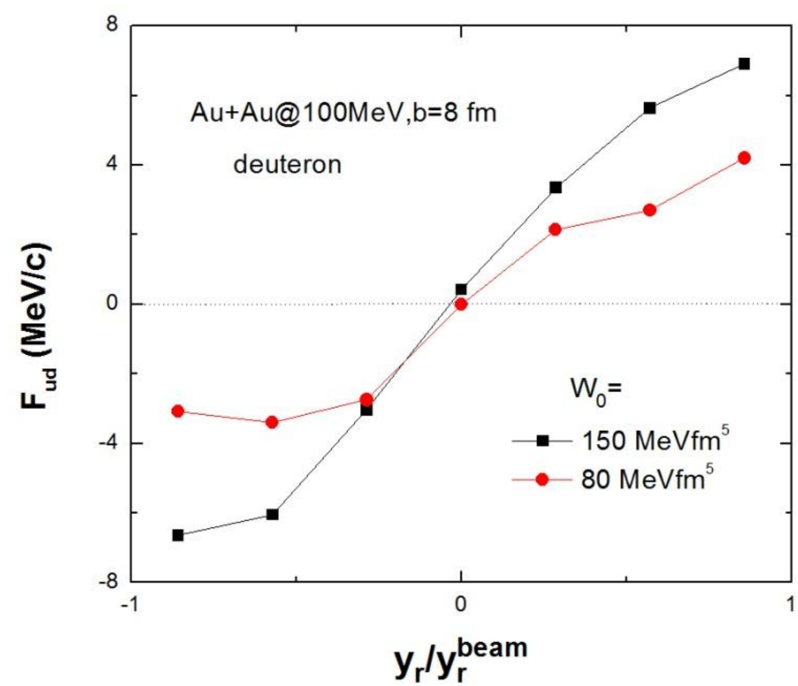
Similar for ${}^3_1He$



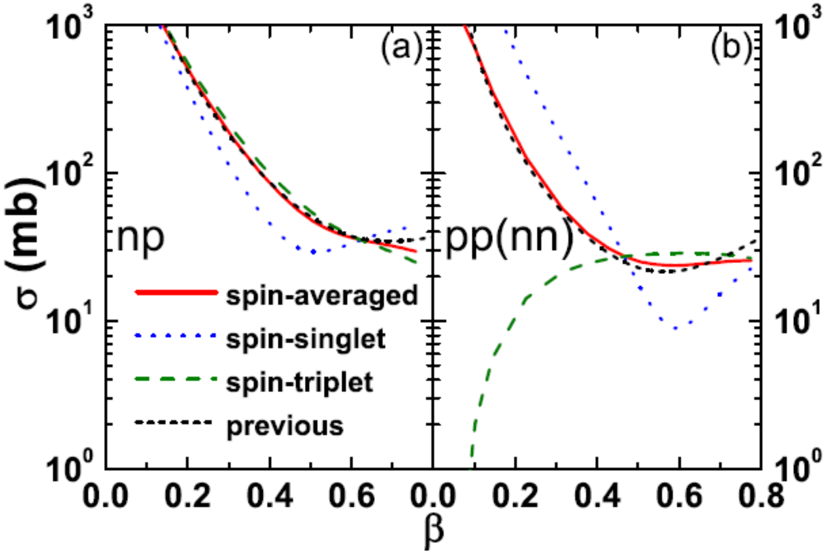


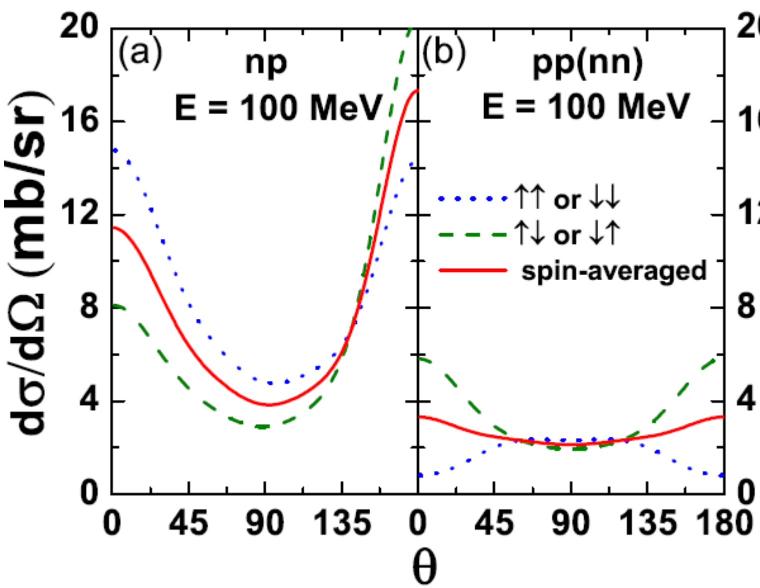
Spin splitting of light clusters collective flows observed

Useful probe of SO coupling

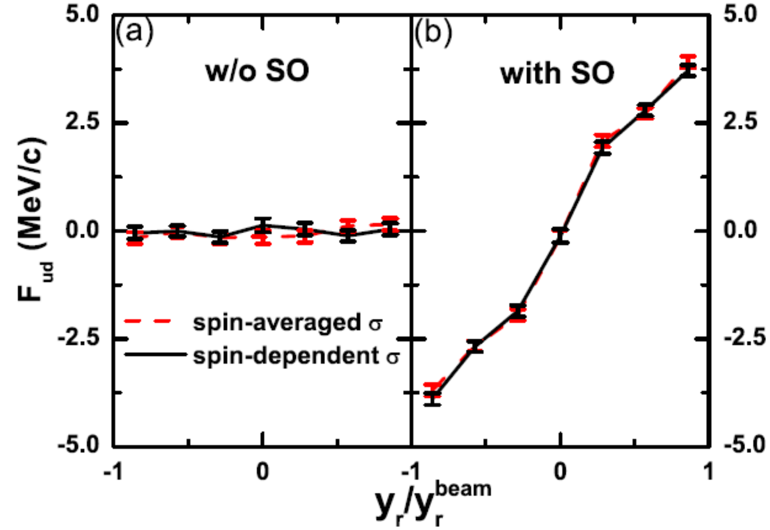


Spin-dependent cross section from phase-shift analysis of NN scatterings in free space

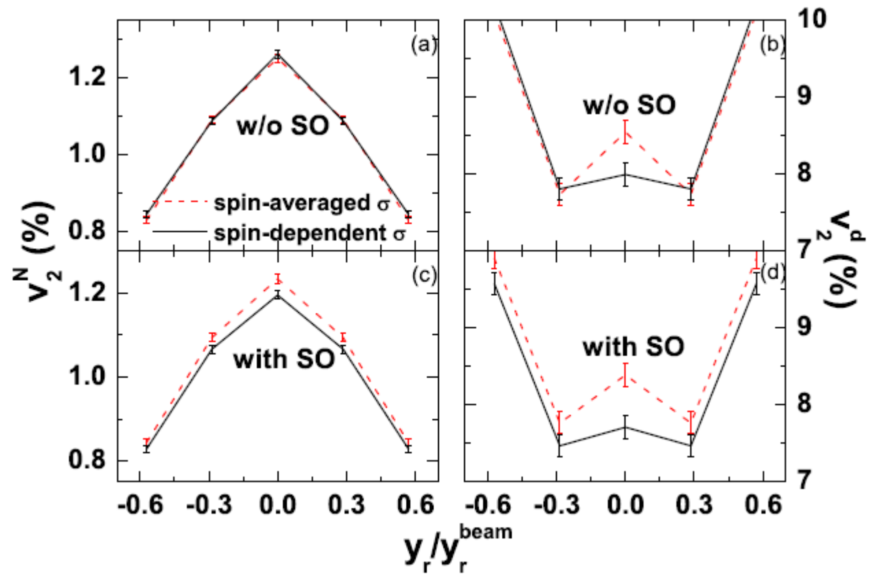




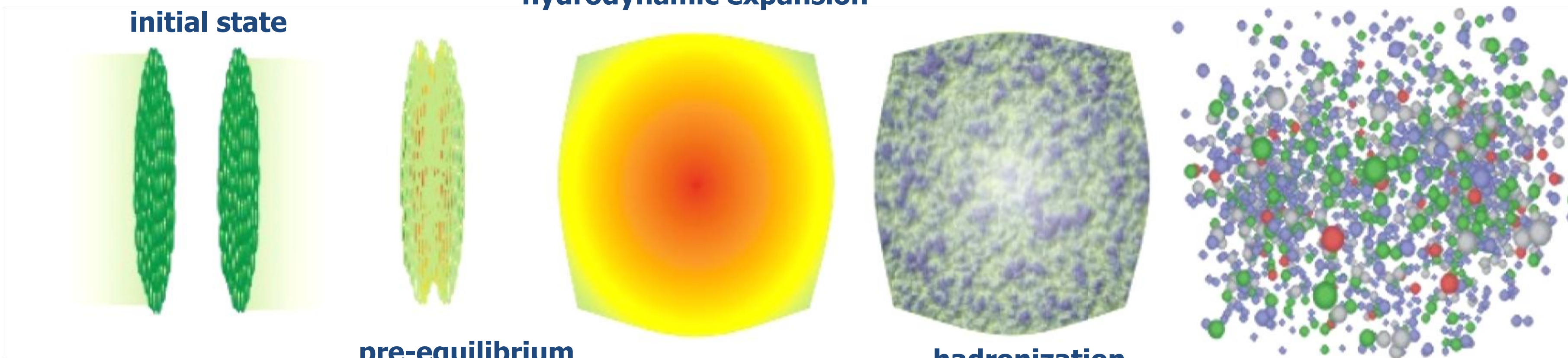
Spin up-down differential flow not affected

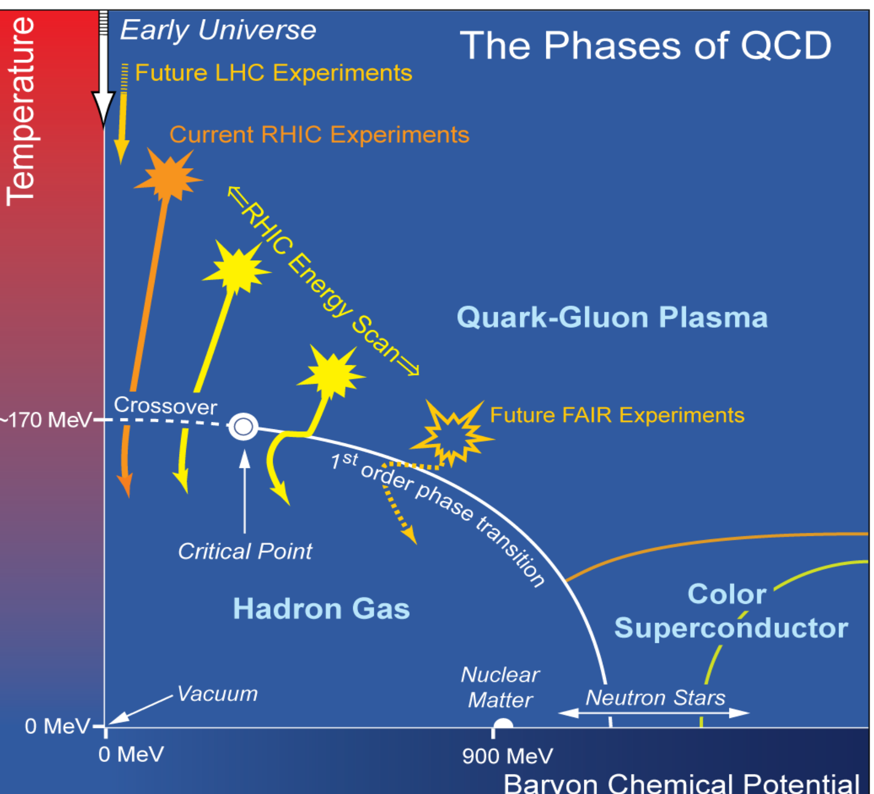


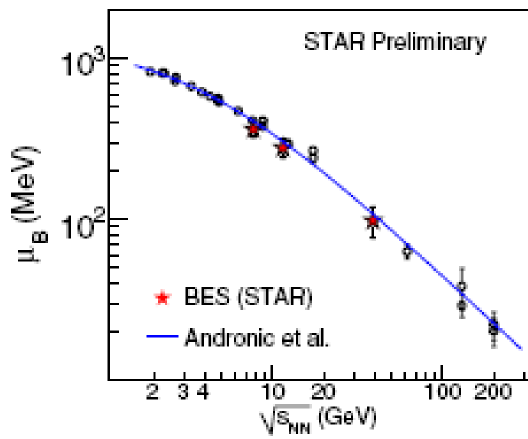
Slightly affect the overall v_2, especially for clusters

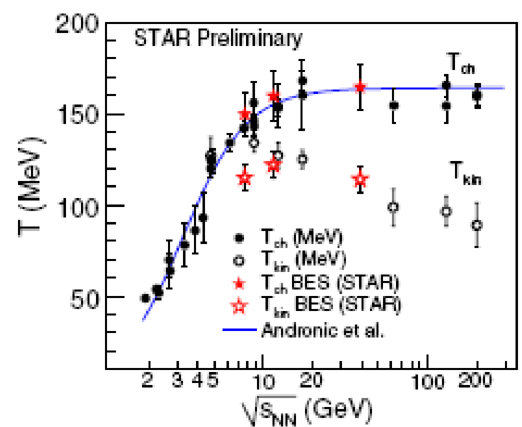


Y. Xia, JX, B.A. Li, and W.Q. Shen, PRC (2017)









Top RHIC:

$\sqrt{s_{NN}} = 200$ **GeV**

RHIC-BES:

$\sqrt{s_{NN}}$ **~7.7-39 GeV**

RHIC-BES:

Search for signals of critical point at finite μ_B!

Spin effects on relativistic heavy-ion collisions - chirality

Dirac equation for massless particles

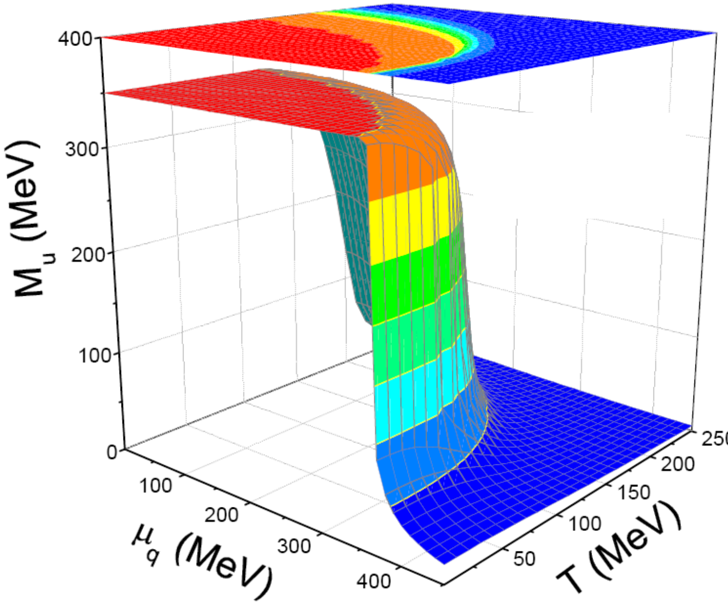
$$\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi = \overline{\psi}(\gamma^0\partial_t - \gamma^k\partial_k)\psi = 0$$

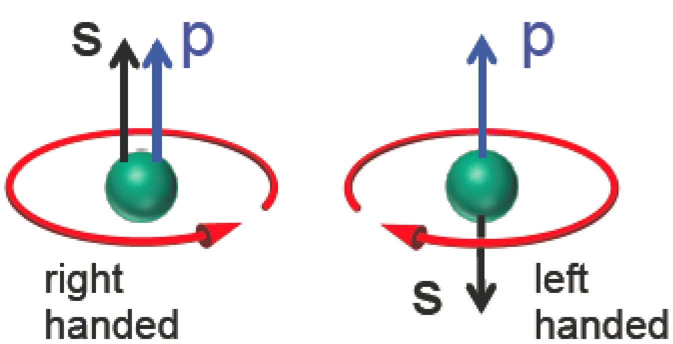
$$\overline{\psi}\left[\begin{pmatrix} 0 & h \\ h & 0 \end{pmatrix} + \begin{pmatrix} 0 & \vec{\sigma}\cdot\vec{p} \\ -\vec{\sigma}\cdot\vec{p} & 0 \end{pmatrix}\right]\psi = 0$$

Weyl SOC

$$h \sim +\vec{\sigma}\cdot\vec{p} \quad \text{for} \quad \psi_L = \frac{1}{2}(1-\gamma^5)$$

$$h \sim -\vec{\sigma}\cdot\vec{p} \quad \psi_R = \frac{1}{2}(1+\gamma^5)$$





Chiral symmetry is restored at high μ and/or T

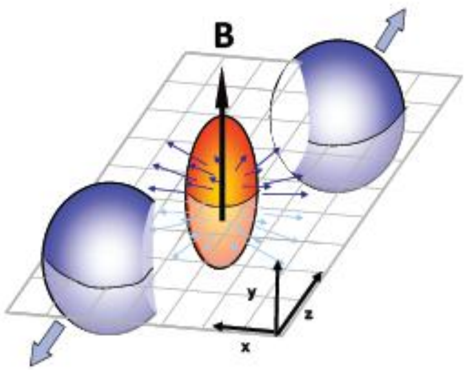
	E	B	ω
J_V	σ Ohm's law	$\frac{N_C e}{2\pi^2}\mu_A$ Chiral magnetic effect	$\frac{N_C}{\pi^2}\mu_V\mu_A$ Vector chiral vortical effect
J_A	$\propto \frac{\mu_V\mu_A}{T^2}\sigma$ Chiral electric separation effect	$\frac{N_C e}{2\pi^2}\mu_V$ Chiral separation effect	$N_C\left(\frac{T^2}{6} + \frac{\mu_V^2+\mu_A^2}{2\pi^2}\right)$ Axial chiral vortical effect

D.E. Kharzeev, Prog. Part. Nucl. Phys., 2014

Equations of motion for massless particles

$$h = \pm\vec{\sigma}\cdot(\vec{p}-\vec{A}) = c\vec{\sigma}\cdot\vec{k}$$

$$\vec{B} = \nabla\times\vec{A}$$ **Under a vector potential**



using $$\vec{\sigma} \approx c\hat{k} - \frac{\hbar}{2k^2}\hat{k}\times\frac{d\hat{k}}{dt}$$

Adiabatic approximation

$$c\vec{\sigma}\cdot\vec{k} = k$$

E. van der Bijl and R.A. Duine, PRL (2011)
X.G. Huang, Scientific Report (2016)

Spin kinetic equations of motion (SEOM)

$$\frac{d\vec{r}}{dt} = c\vec{\sigma}$$

$$\frac{d\vec{k}}{dt} = c\vec{\sigma}\times\vec{B}$$

$$\frac{d\vec{\sigma}}{dt} = 2c\vec{k}\times\vec{\sigma}$$

chiral kinetic equations of motion (CEOM)

$$\sqrt{G}\frac{d\vec{r}}{dt} = \hat{k} + c\frac{\hbar}{2k^2}\vec{B}$$

$$\sqrt{G}\frac{d\vec{k}}{dt} = \vec{k}\times\vec{B}$$

$$\sqrt{G} = 1 + c\frac{\vec{B}\cdot\vec{k}}{2k^3}$$

Phase-space volume changed

D. Xiao, J. Shi, and Q. Niu, PRL (2005)

$$d^3r d^3k/(2\pi\hbar)^3 \Rightarrow \sqrt{G}d^3r d^3k/(2\pi\hbar)^3$$

$$\langle A\rangle = \sum_i A_i\sqrt{G_i}/\sum_i\sqrt{G_i}$$

M.A. Stephenov and Y. Yin, PRL (2012)
J.W. Chen, S. Pu, Q. Wang, and X.N. Wang, PRL (2013)
D.T. Son and N. Tamamoto, PRD (2013)

CME, CSE, and CMW

4 types of particles： q=±1, c=±1

$$\mu_{qc} = q\mu + c\mu_5$$

$$\rho_R = \rho_{q(+)c(+)} + \rho_{q(-)c(-)}$$
$$\rho_L = \rho_{q(+)c(-)} + \rho_{q(-)c(+)}$$
$$\vec{J}_R = \vec{J}_{q(+)c(+)} - \vec{J}_{q(-)c(-)}$$
$$\vec{J}_L = \vec{J}_{q(+)c(-)} - \vec{J}_{q(-)c(+)}$$

Number density
$$\rho_{qc} = qN_c \int \frac{d^3k}{(2\pi\hbar)^3}\sqrt{G}\, f\left(\frac{k-\mu_{qc}}{T}\right),$$

Current density
$$\vec{J}_{qc} = N_c \int \frac{d^3k}{(2\pi\hbar)^3}\sqrt{G}\dot{\vec{r}}\, f\left(\frac{k-\mu_{qc}}{T}\right),$$

$$\rho = \rho_R + \rho_L, \quad \rho_5 = \rho_R - \rho_L$$
$$\vec{J} = \vec{J}_R + \vec{J}_L, \quad \vec{J}_5 = \vec{J}_R - \vec{J}_L$$

Isotropic Fermi-Dirac distribution f →

$$\vec{J} = \frac{N_c}{2\pi^2\hbar^2}\mu_5 e\vec{B},$$ Chiral magnetic effect (CME)

$$\vec{J}_5 = \frac{N_c}{2\pi^2\hbar^2}\mu e\vec{B}.$$ Chiral separation effect (CSE)

K.E. Kharzeev and H.-U. Yee, PRD (2011)

$\mu/T \ll 1$ and $\mu_5/T \ll 1$,

$$\rho \approx \frac{N_c T^2}{3\hbar^3}\mu,$$
$$\rho_5 \approx \frac{N_c T^2}{3\hbar^3}\mu_5.$$

→

$$\vec{J}_{R/L} = \pm\frac{3\hbar e\vec{B}}{2\pi^2 T^2}\rho_{R/L}$$

$$\partial_t \rho_{R/L} + \nabla\cdot\vec{J}_{R/L} = 0$$

dissipation →

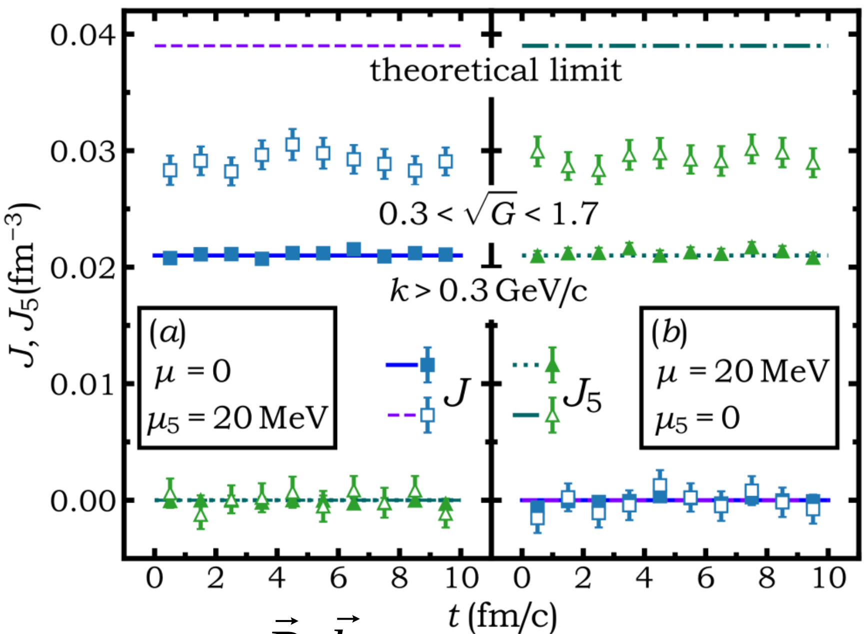
Chiral magnetic wave (CMW)

$$\left(\partial_t \pm \vec{v}_p\cdot\nabla - D_L\nabla^2\right)\rho_{R/L} = 0$$

$$v_p = \frac{3\hbar e B}{2\pi^2 T^2}$$

Box simulation of CME and CSE

CME **CSE**



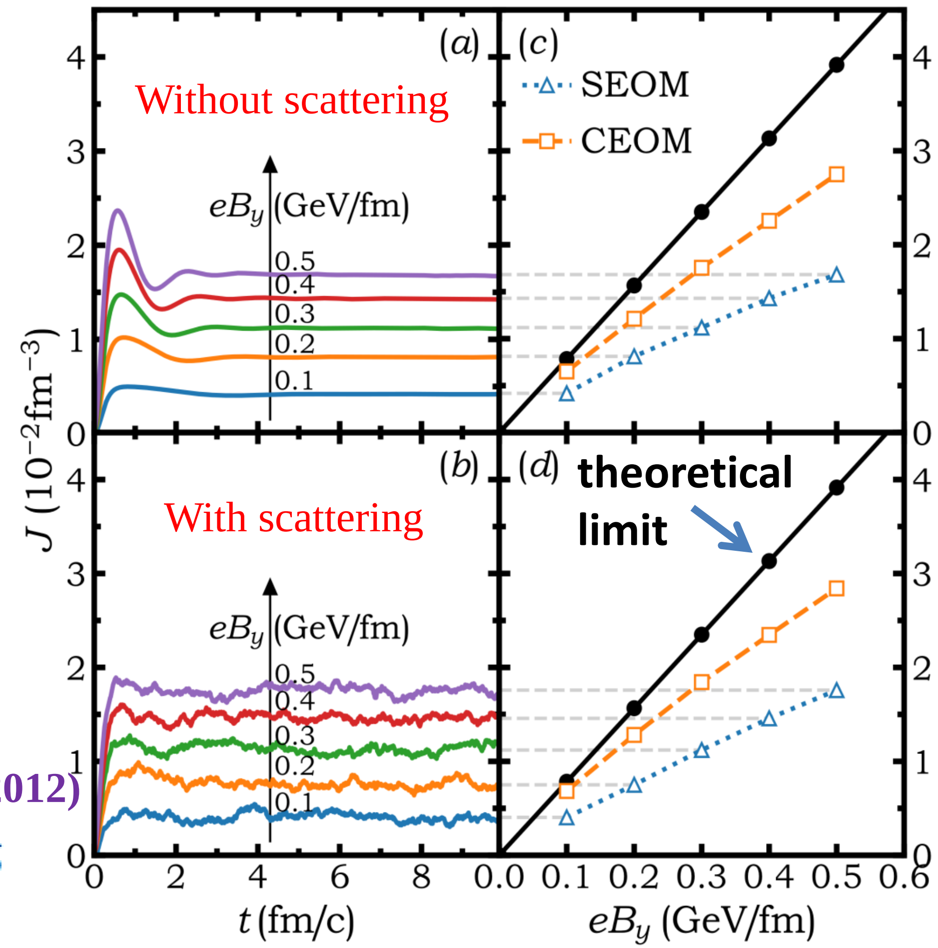
$$\sqrt{G} = 1 + c\frac{\vec{B}\cdot\vec{k}}{2k^3}$$

Divergent at too small k

M.A. Stephenov and Y. Yin, PRL (2012)

$$\vec{J} \approx \alpha\frac{N_c}{2\pi^2\hbar^2}\mu_5 e\vec{B} \qquad \vec{J}_5 \approx \alpha\frac{N_c}{2\pi^2\hbar^2}\mu e\vec{B}$$

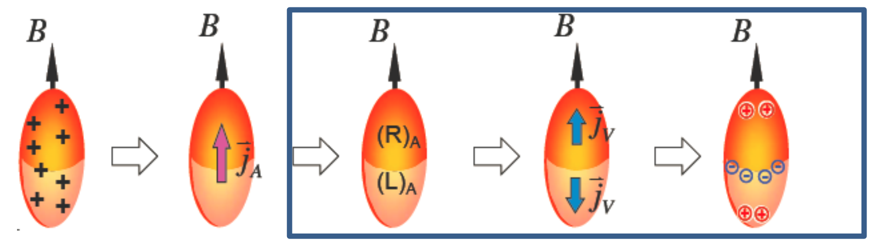
W.H. Zhou and JX, Phys. Rev. C 98, 044904 (2018)

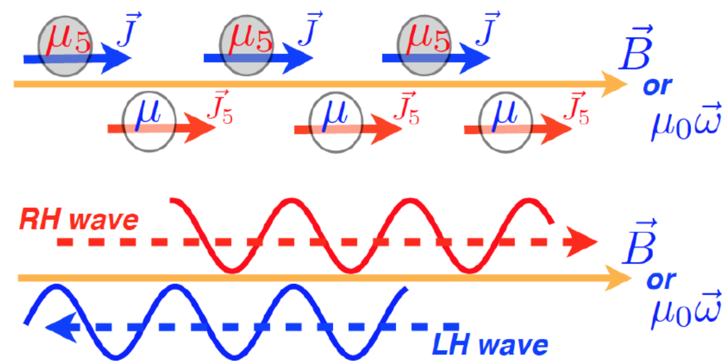


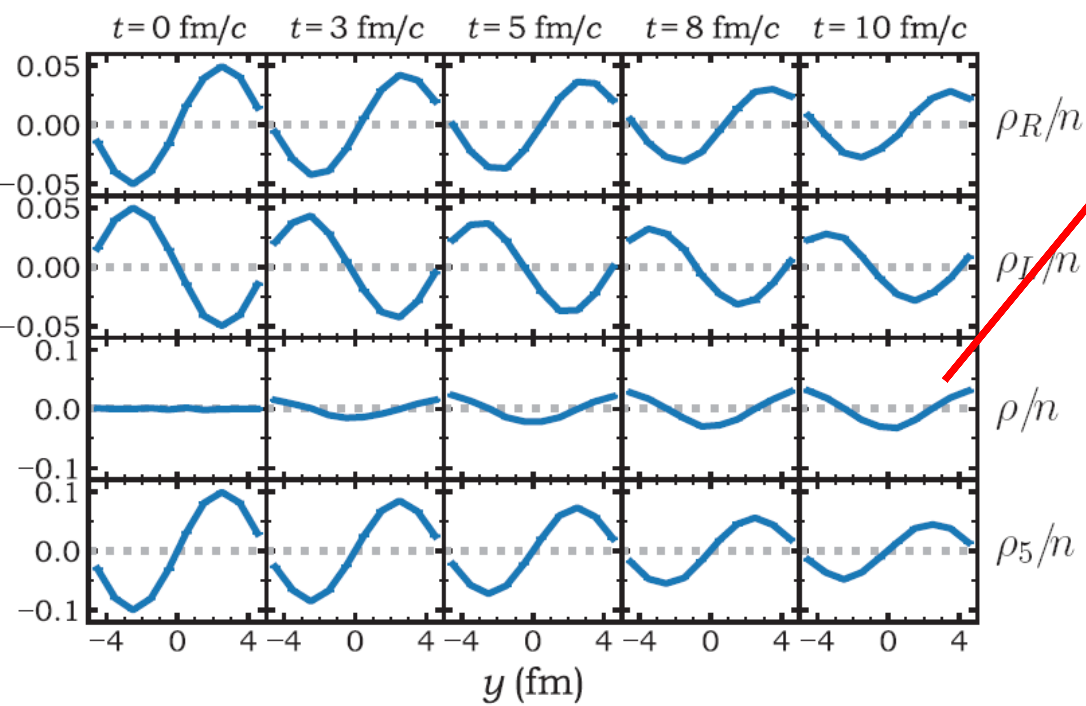
W.H. Zhou and JX, arXiv: 1904.01834 [nucl-th]

Box simulation of CMW with CEOM

$$j_A = \frac{N_c e}{2\pi^2}\mu_V B \qquad j_V = \frac{N_c e}{2\pi^2}\mu_A B$$

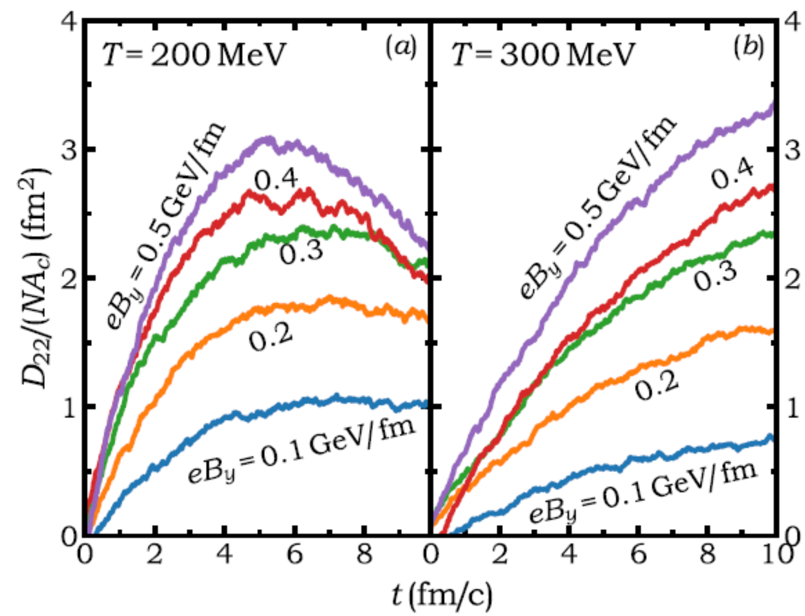






Electric quadruple moment

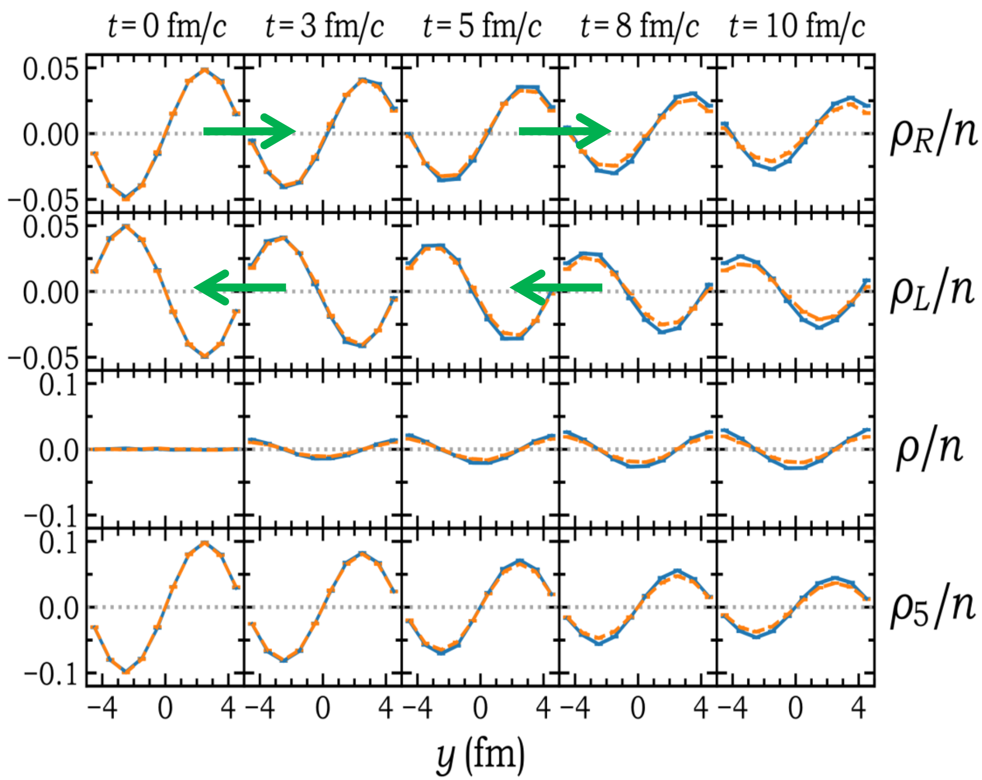
$$\mathcal{D}_{ij} = \int \rho(\vec{r})(3r_i r_j - r^2\delta_{ij})d^3r$$



W.H. Zhou and JX, Phys. Rev. C 98, 044904 (2018)

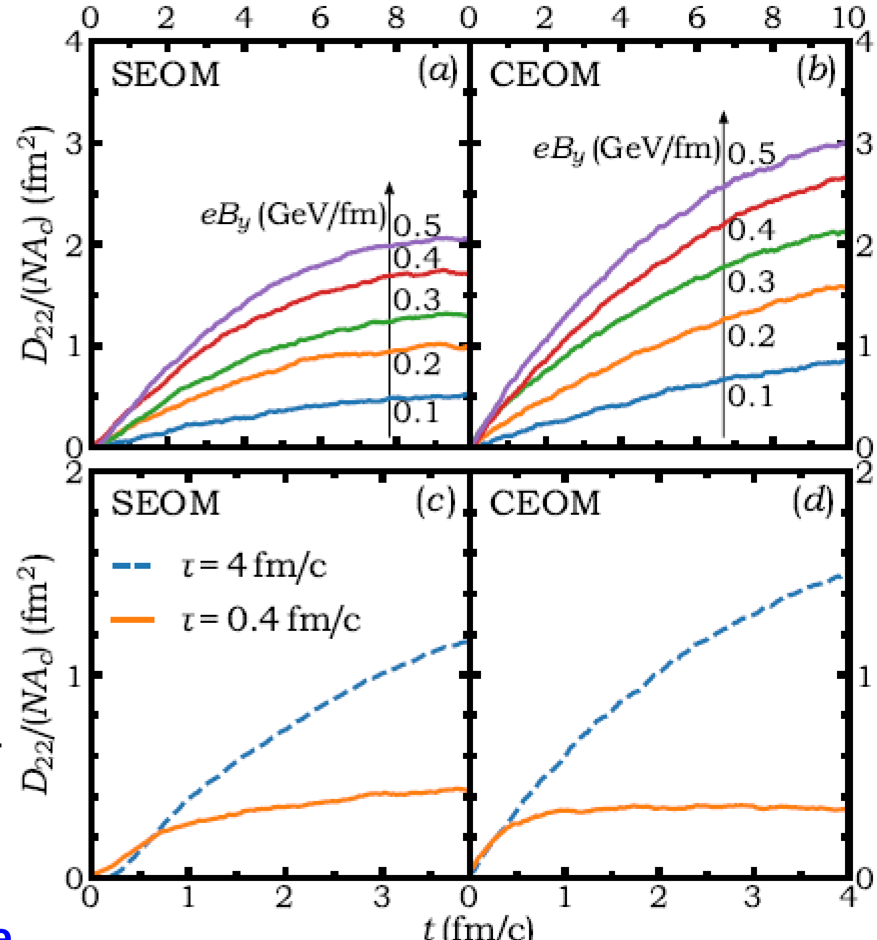
Box simulation of CMW with SEOM vs CEOM

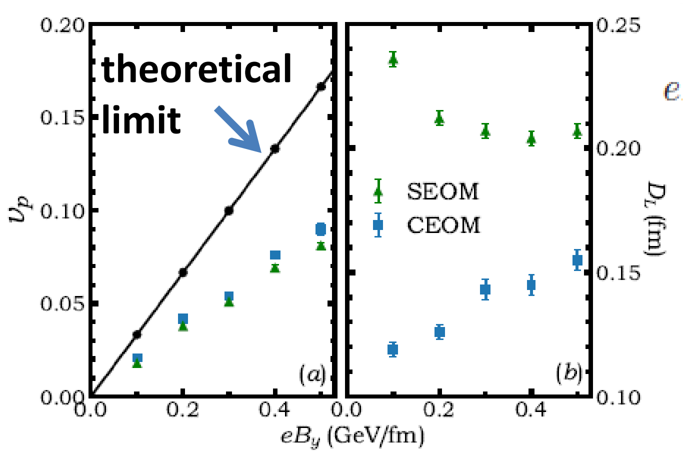
dashed: SEOM solid: CEOM



Electric quadruple moment

$$\mathcal{D}_{ij} = \int \rho(\vec{r})(3r_i r_j - r^2\delta_{ij})d^3r$$



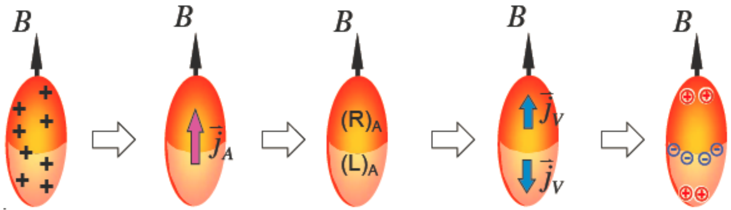


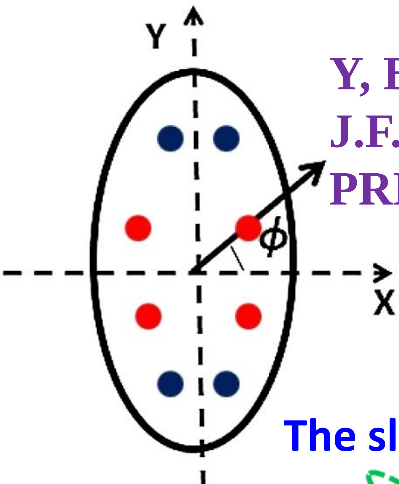
$$eB_y(t) = \frac{eB_y^0}{1+(t/\tau)^2}$$

SEOM is less sensitive to the fast decay of eBy compared with CEOM.

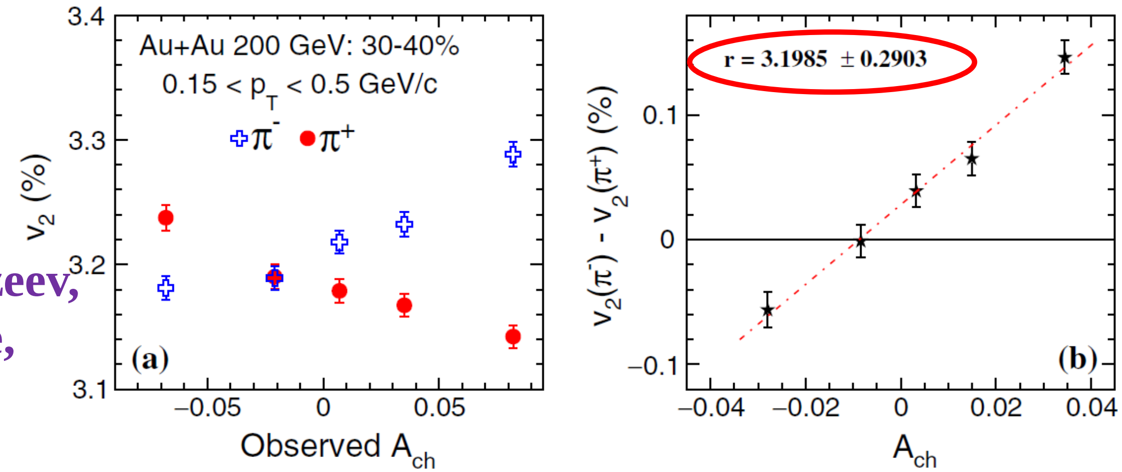
W.H. Zhou and JX, arXiv: 1904.01834 [nucl-th]

CMW and $v_2(\pi^-)$-$v_2(\pi^+)$ ~ A_{ch}





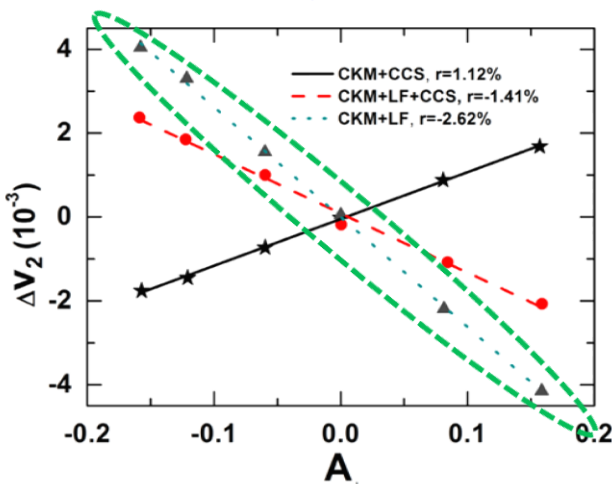
Y, Burnier, D.E. Kharzeev, J.F. Liao, and H.U. Yee, PRL (2011)



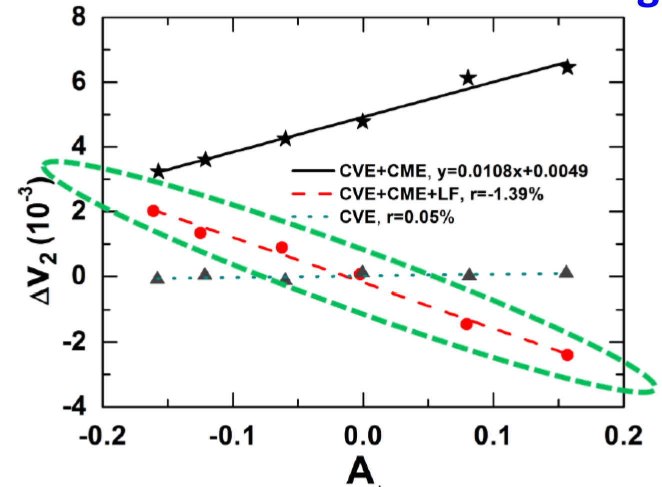
STAR, PRL (2015)

$$A_\pm \equiv (\bar{N}_+ - \bar{N}_-)/(\bar{N}_+ + \bar{N}_-)$$

The slope is negative with full CKM. **CVE with full CKM doesn't change the slope.**



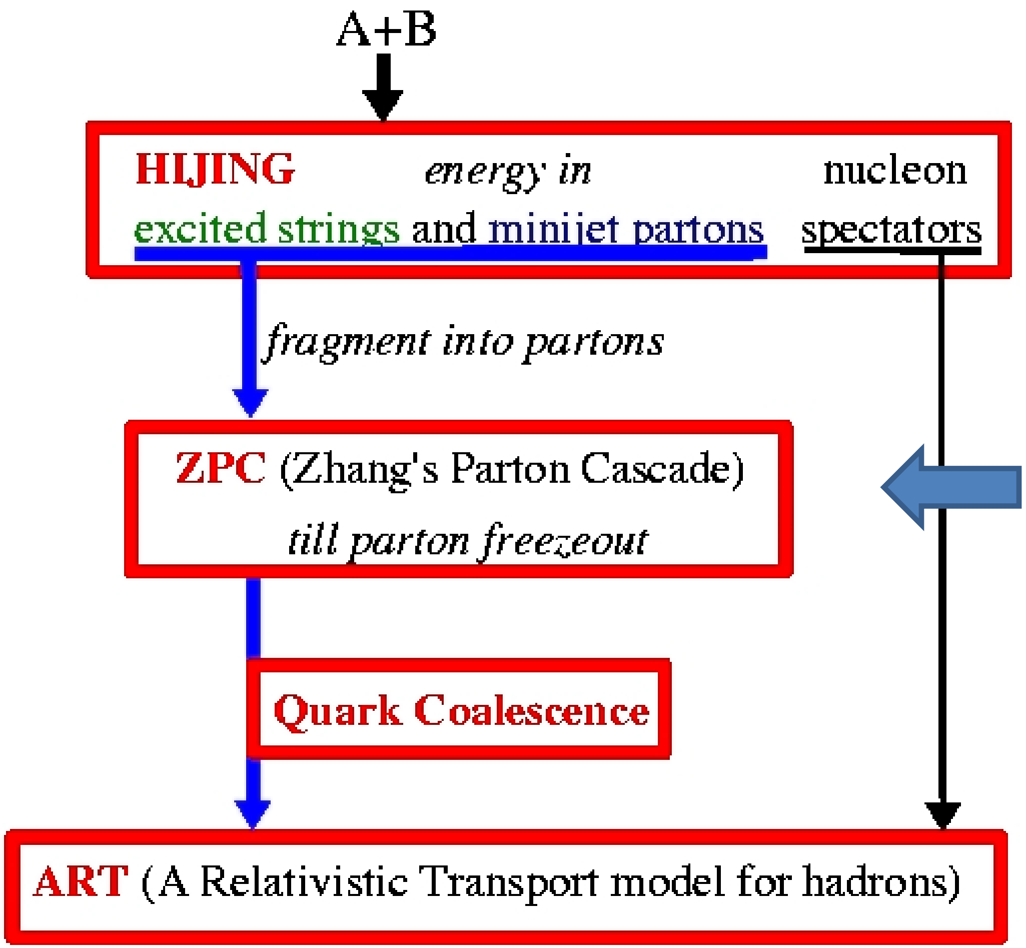
Y.F. Sun and C.M. Ko, PRC (2016)



Y.F. Sun and C.M. Ko, PRC (2017)

An extended AMPT with chiral dynamics

Structure of AMPT model with string melting



(electro)magnetic field from spectator protons

Space-time evolution

QGP response

Chiral dynamics for partons under vector potentials

$$\sqrt{G}\frac{d\vec{r}}{dt} = \hat{k} + c\frac{\hbar}{2k^2}\vec{B} + c\frac{\hbar}{2k^3}\vec{E}\times\vec{k}$$

$$\sqrt{G}\frac{d\vec{k}}{dt} = \vec{k}\times\vec{B} + c\frac{\hbar\vec{k}}{2k^3}(\vec{E}\cdot\vec{B}) + \vec{E}$$

$$\sqrt{G} = 1 + c\frac{\vec{B}\cdot\vec{k}}{2k^3}$$

To be done:

1) **E with QGP response**
2) **EOM with vorticity**

Space-time evolution of the magnetic field

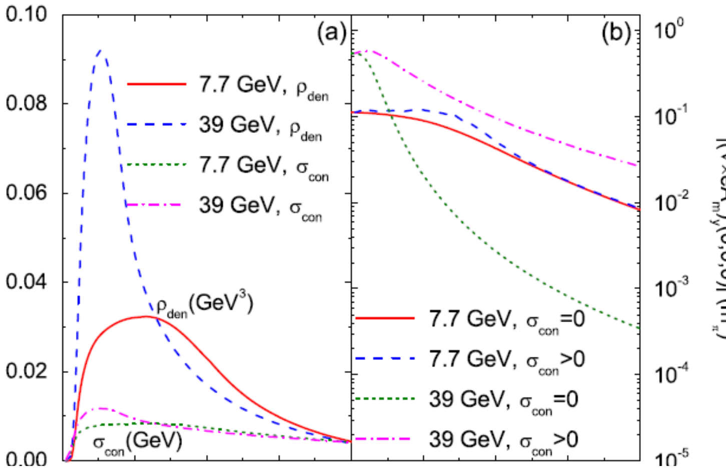
In vacuum (σ_{con}=0):

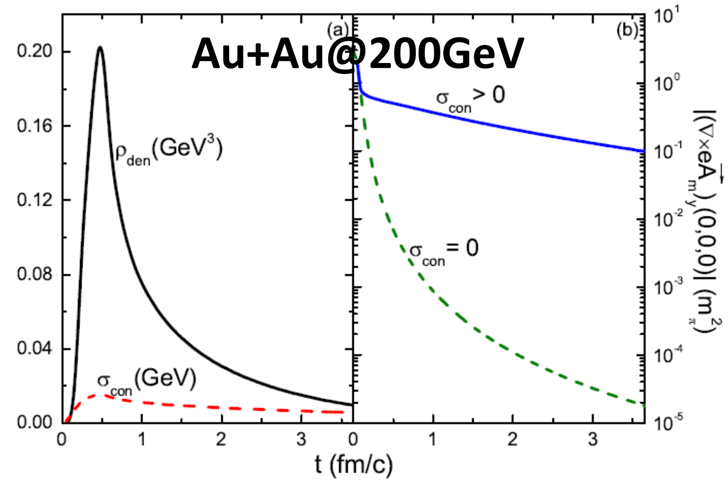
$$\vec{A}_m(t,\vec{r}) = \frac{e}{4\pi}\sum_n Z_n \frac{\vec{v}_n}{R_n - \vec{v}_n \cdot \vec{R}_n}$$

In QGP (σ_{con}>0):

$$\vec{A}^e_m = \frac{\hat{z}e}{4\sigma_{con}[(z-z_0)/v]} \times \frac{\exp\left\{\frac{-b^2}{4\{\lambda(t)-\lambda[-(z-z_0)/v]\}}\right\}}{4\{\lambda(t)-\lambda[-(z-z_0)/v]\}} \times \theta[vt-(z-z_0)]\theta[(z-z_0)-vt_0] + \frac{\hat{z}ev\gamma}{4\pi}\int_0^{+\infty} dk_\perp J_0(k_\perp b)\exp[-k_\perp^2\lambda(t) - k_\perp\gamma|(z-z_0)-vt_0|]$$

K. Tuchin, PRC (2016)



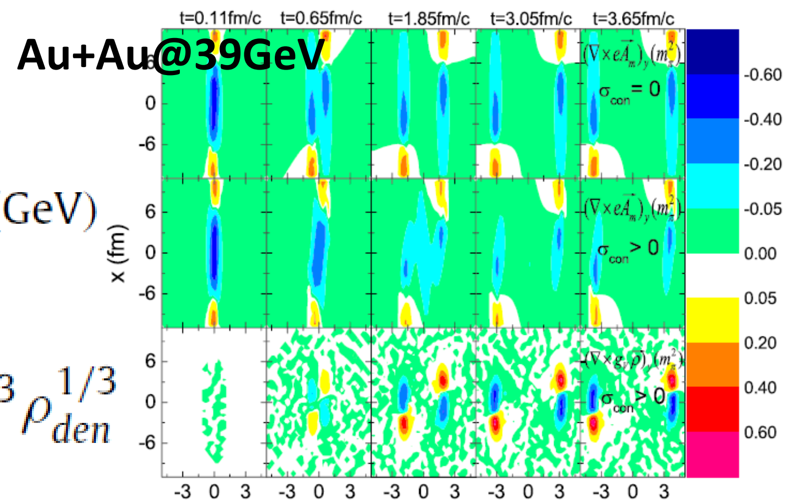


LQCD result:

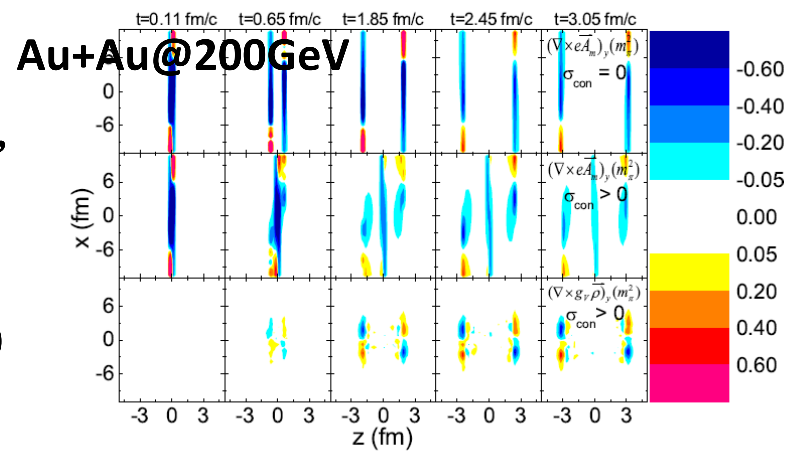
$$\sigma_{con} = 0.0058\frac{T}{T_c}(\text{GeV})$$

assume

$$T \approx (\pi^2/24)^{1/3}\rho_{den}^{1/3}$$



Z.Z. Han and JX, Phys. Lett. B 786, 255 (2018) ; Phys. Rev. C 99, 044915 (2019)



$v_2(\bar{x})-v_2(x) \sim A_{ch}$

$$H = c\vec{\sigma} \cdot \vec{k} + A_0$$

$$A_0 = b_i g_V \rho_0 + q_i e\varphi$$

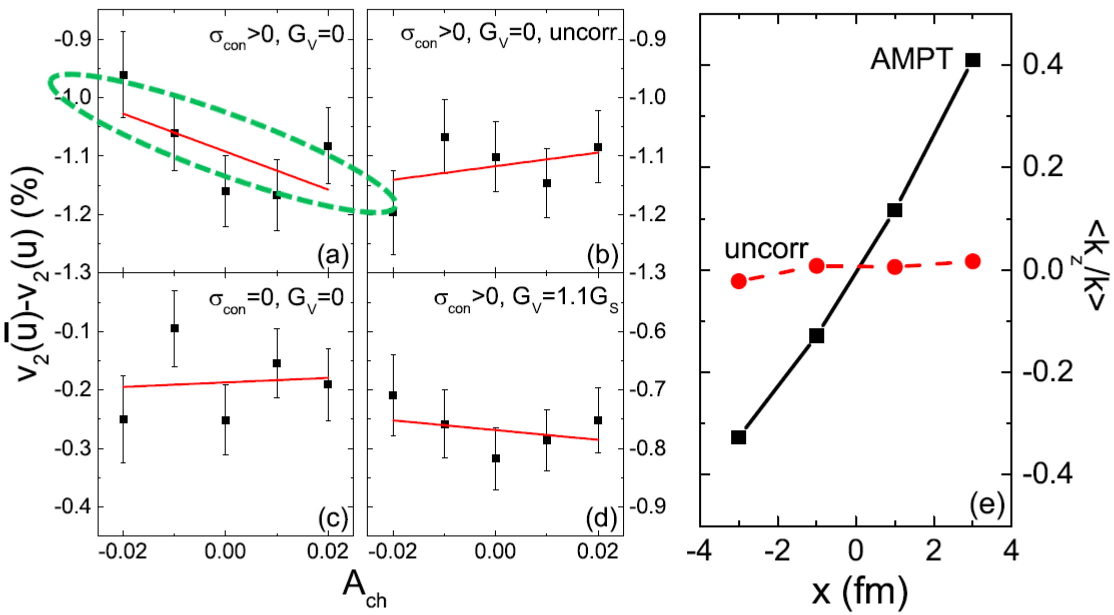
$$\vec{A} = b_i g_V \vec{\rho} + q_i e\vec{A}_m$$

Effective EB real EB

$$\sqrt{G}\frac{d\vec{r}}{dt} = \hat{k} + c\frac{\hbar}{2k^2}\vec{B}$$

$$\sqrt{G}\frac{d\vec{k}}{dt} = \vec{k} \times \vec{B}$$ Lorentz Force

$$\sqrt{G} = 1 + c\frac{\vec{B}\cdot\vec{k}}{2k^3}$$



Negative slope due to the Lorentz force originated from the initial $<k_z/k>$~x correlation

Linear fit around A_{ch}~0

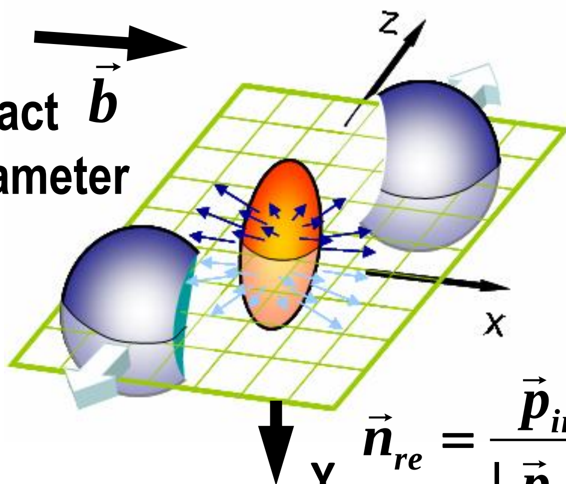
Slope also affected by σ_{con} and G_V

	$\sigma_{con} > 0,\ G_V = 0$	$\sigma_{con} > 0,\ G_V = 0$, uncorr	$\sigma_{con} = 0,\ G_V = 0$	$\sigma_{con} > 0,\ G_V = 1.1G_S$
$r(\%)$ for freeze-out $v_2(\bar{u}) - v_2(u)$	-3.244 ± 2.139	1.161 ± 2.073	0.385 ± 2.112	-0.828 ± 1.909
$r(\%)$ for initial $v_2(\pi^-) - v_2(\pi^+)$	2.488 ± 2.113	-0.475 ± 2.076	-2.699 ± 2.073	0.819 ± 1.999
$r(\%)$ for freeze-out $v_2(\pi^-) - v_2(\pi^+)$	0.301 ± 1.154	0.422 ± 1.139	0.861 ± 1.14	-0.596 ± 1.037

We can not obtain a positive slope as large as 3% observed experimentally.

Z.Z. Han and JX, PRC (2019)

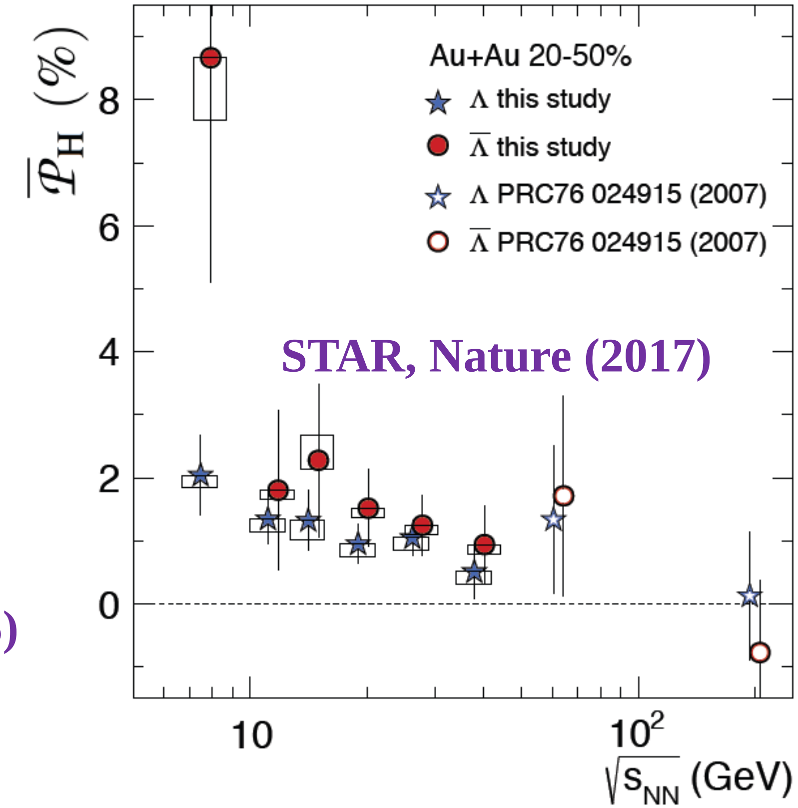
Spin polarization in relativistic heavy-ion collisions



$$\vec{n}_{re} = \frac{\vec{p}_{in} \times \vec{b}}{|\vec{p}_{in} \times \vec{b}|}$$

perpendicular to the reaction plane

Z. T. Liang and X. N. Wang, PRL (2005); PLB (2005)



Λ polarization

$$\frac{dN}{d\cos\theta^*} \propto 1 + \alpha_H p_H \cos\theta^*$$

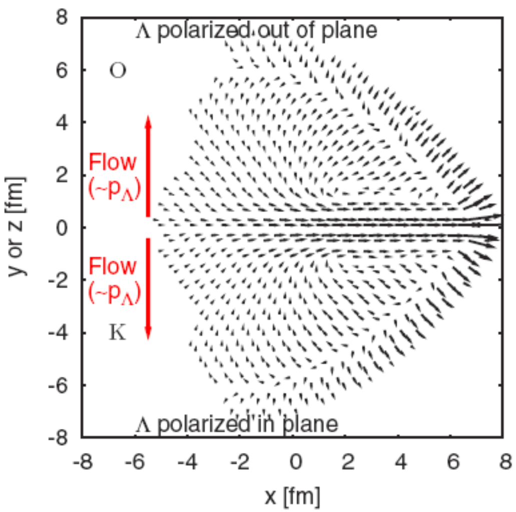
$$P_H = -\frac{8}{\pi\alpha_H}\langle\sin(\phi_P^* - \Psi_{RP})\rangle = \frac{\Lambda^\uparrow - \Lambda^\downarrow}{\Lambda^\uparrow + \Lambda^\downarrow} \neq 0$$

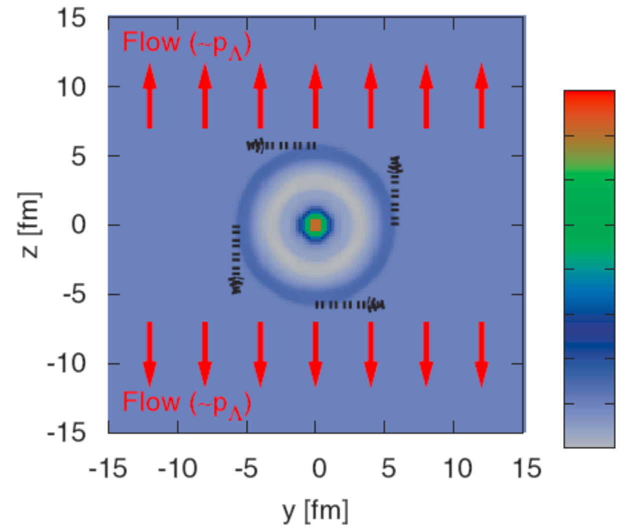
$$P_\Lambda \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$$

$$P_{\bar{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T}$$

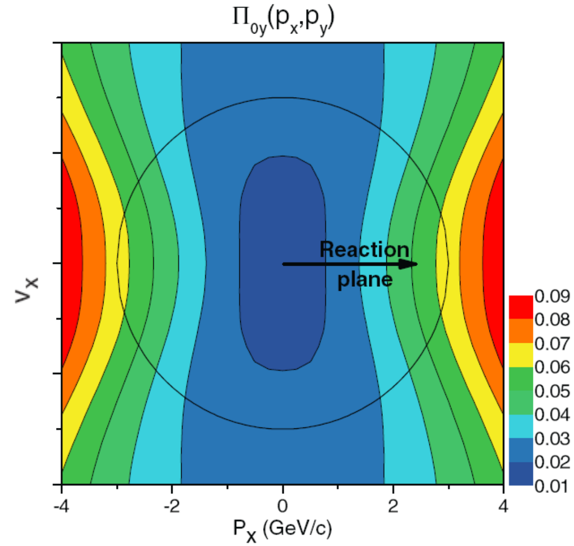
Vorticity
+
magnetic field
（？）

vorticity lead to same $\Lambda(\bar{\Lambda})$ polarization

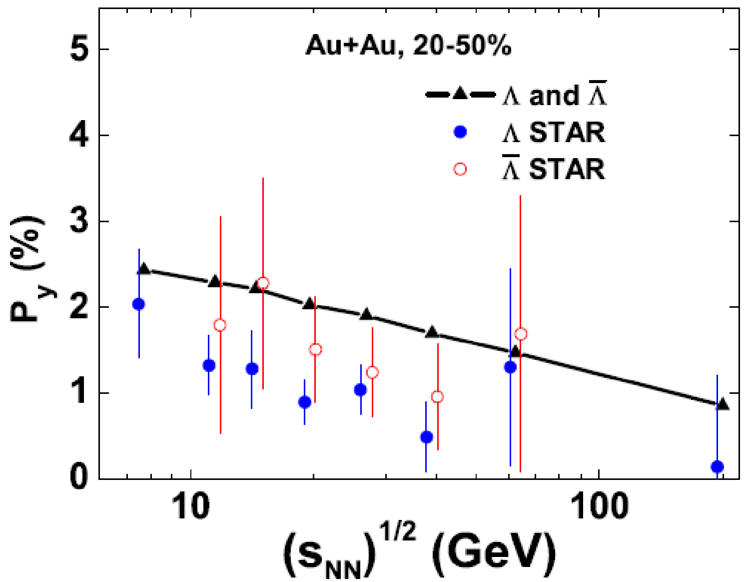




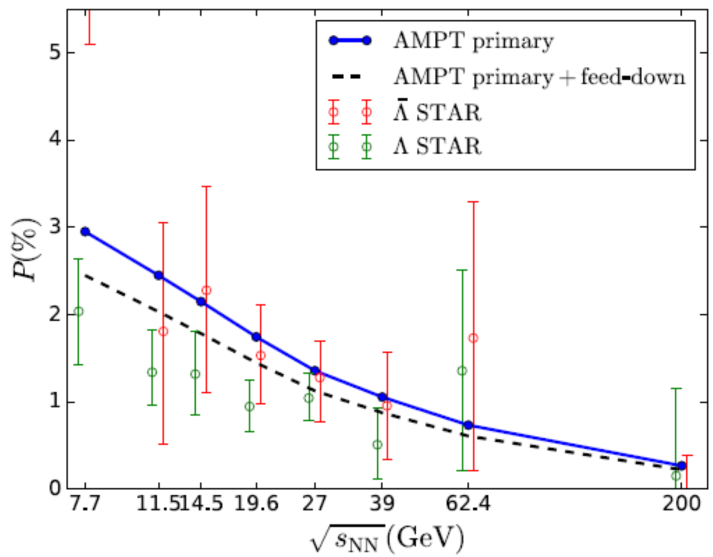
B. Betz et al., Phys. Rev. C, 2007



F. Becattini et al., Phys. Rev. C, 2013



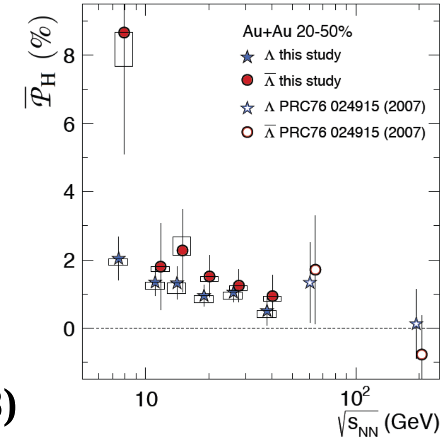
Y.F. Sun and C.M. Ko, Phys. Rev. C, 2017



H. Li, L.G. Pang, X.N. Wang, and X.L. Xia,
Phys. Rev. C, 2018

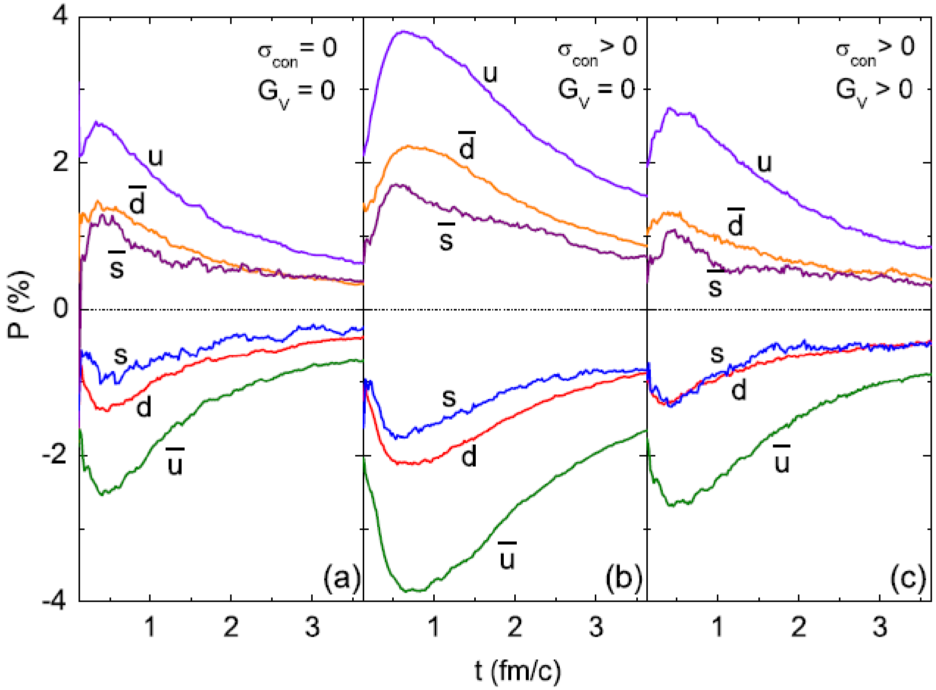
Splitting of quark-antiquark spin polarizations from vector potential

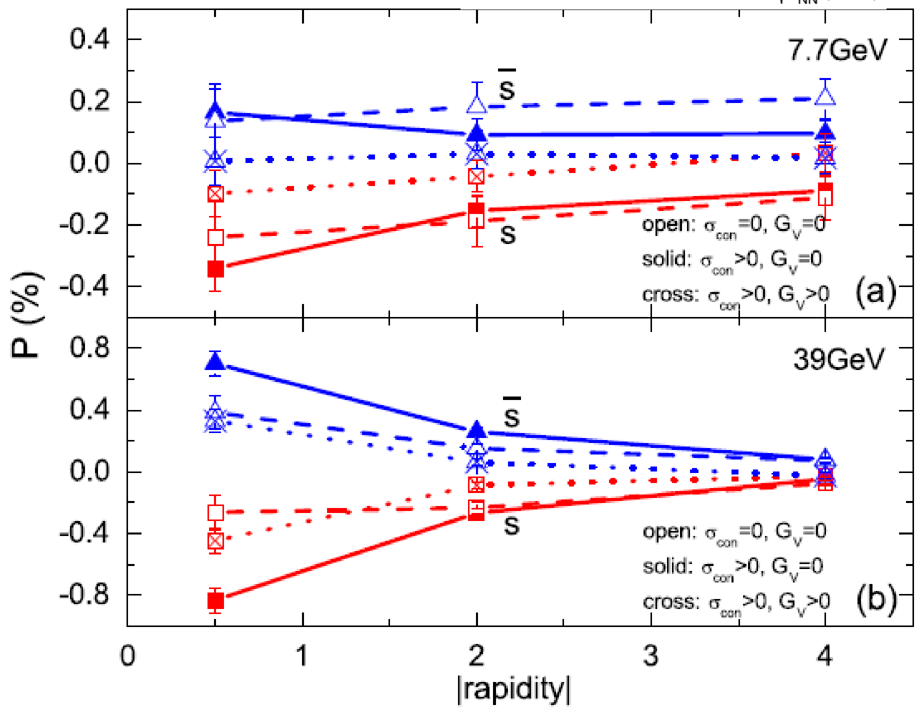
Thermal limit: $$\langle \vec{P} \rangle = \frac{\int \frac{d^3\vec{k}}{(2\pi)^3} c\dot{\vec{r}} \sqrt{G} \exp(-k/T)}{\int \frac{d^3\vec{k}}{(2\pi)^3} \sqrt{G} \exp(-k/T)} = \frac{\hbar \vec{B}}{4T^2}$$



$$\Lambda^{\uparrow} \sim s^{\uparrow}, \Lambda^{\downarrow} \sim s^{\downarrow}, \overline{\Lambda}^{\uparrow} \sim \bar{s}^{\uparrow}, \overline{\Lambda}^{\downarrow} \sim \bar{s}^{\downarrow}$$

Z.Z. Han and JX, Phys. Lett. B 786, 255 (2018)

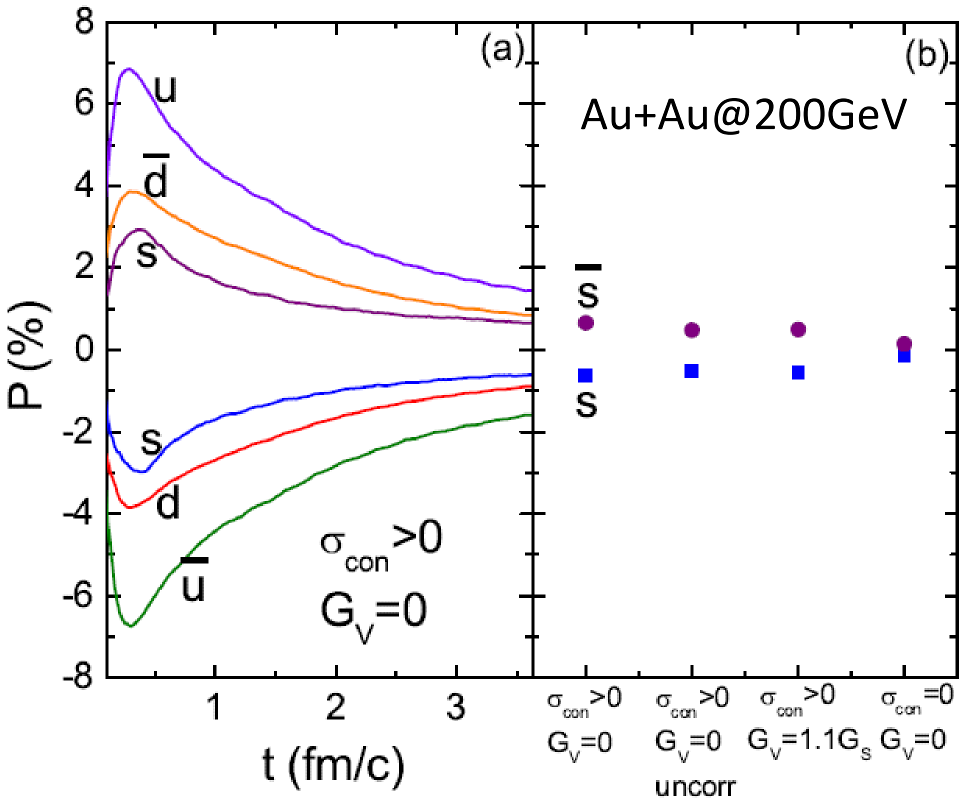


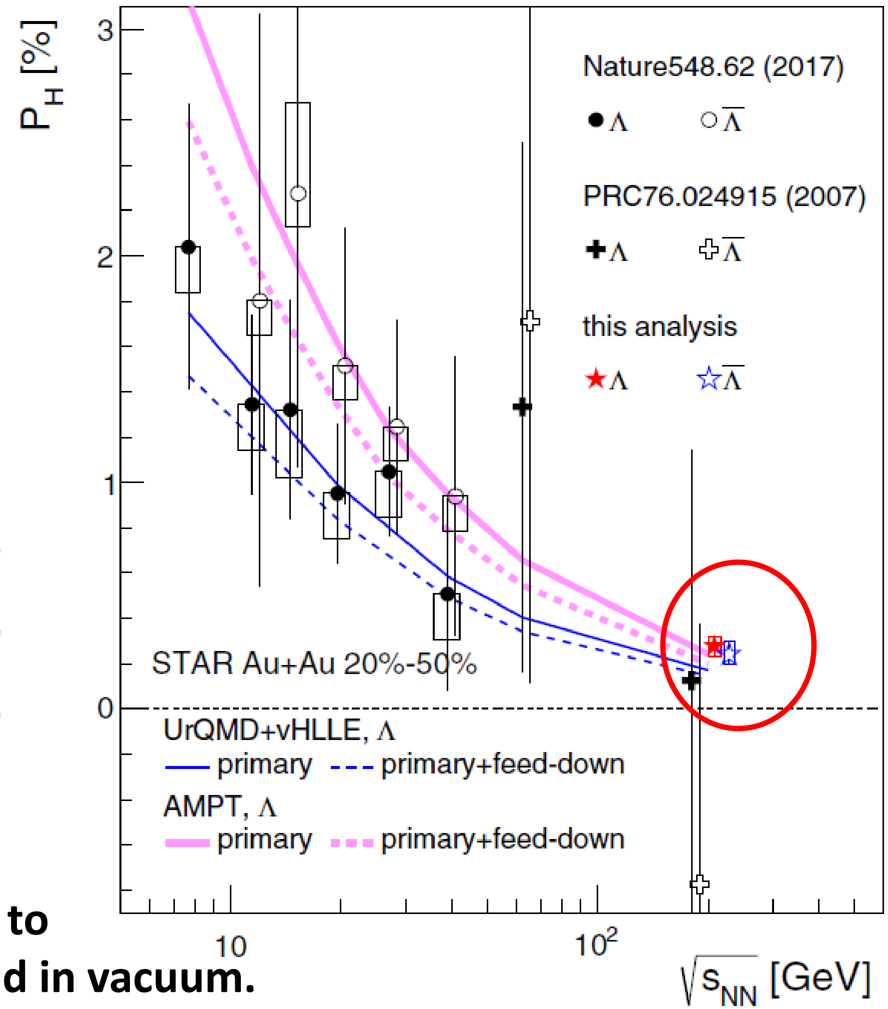


Smaller $\Lambda(s)$ spin polarization than $\overline{\Lambda}(\bar{s})$, consistent with exp data. Their splitting is sensitive not only to eB_y but also to G_V.

The large splitting at 7.7 GeV can not be obtained in the thermal limit under maximum/initial eB_y.

Splitting of quark-antiquark spin polarizations at 200 GeV





The experimentally observed small splitting seems to favor the space-time evolution of the magnetic field in vacuum.

Z.Z. Han and JX, PRC (2019)

STAR, PRC (2018)

Summary of spin dynamics in HIC

- **Low energy (TDHF):**
  dissipation and internal spin excitation
- **Intermediate energy (SIBUU, QMD):**
  spin-dependent potential/flow
- **Relativistic energy (AMPT, hydro):**
  chiral dynamics, polarization

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Thank you!

xujun@sinap.ac.cn, xujun@zjlab.org.cn