# Insights on the EoS parameters from GW170817

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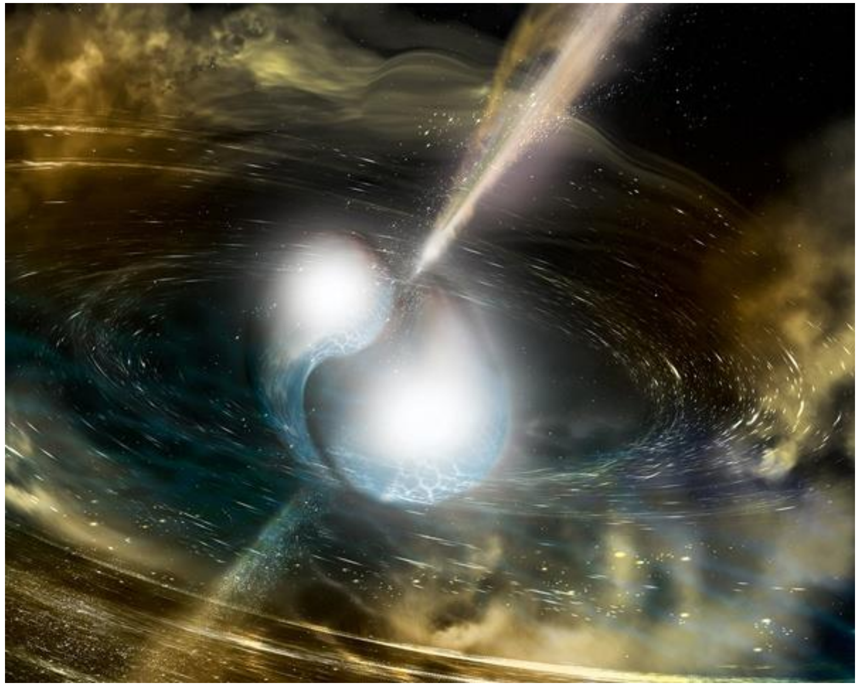
This material is based upon work supported by the U.S. Department of Energy Office of Science under Cooperative Agreement DE-SC0000661, the State of Michigan and Michigan State University. Michigan State University designs and establishes FRIB as a DOE Office of Science National User Facility in support of the mission of the Office of Nuclear Physics.

# NS calculation with nuclear EoS

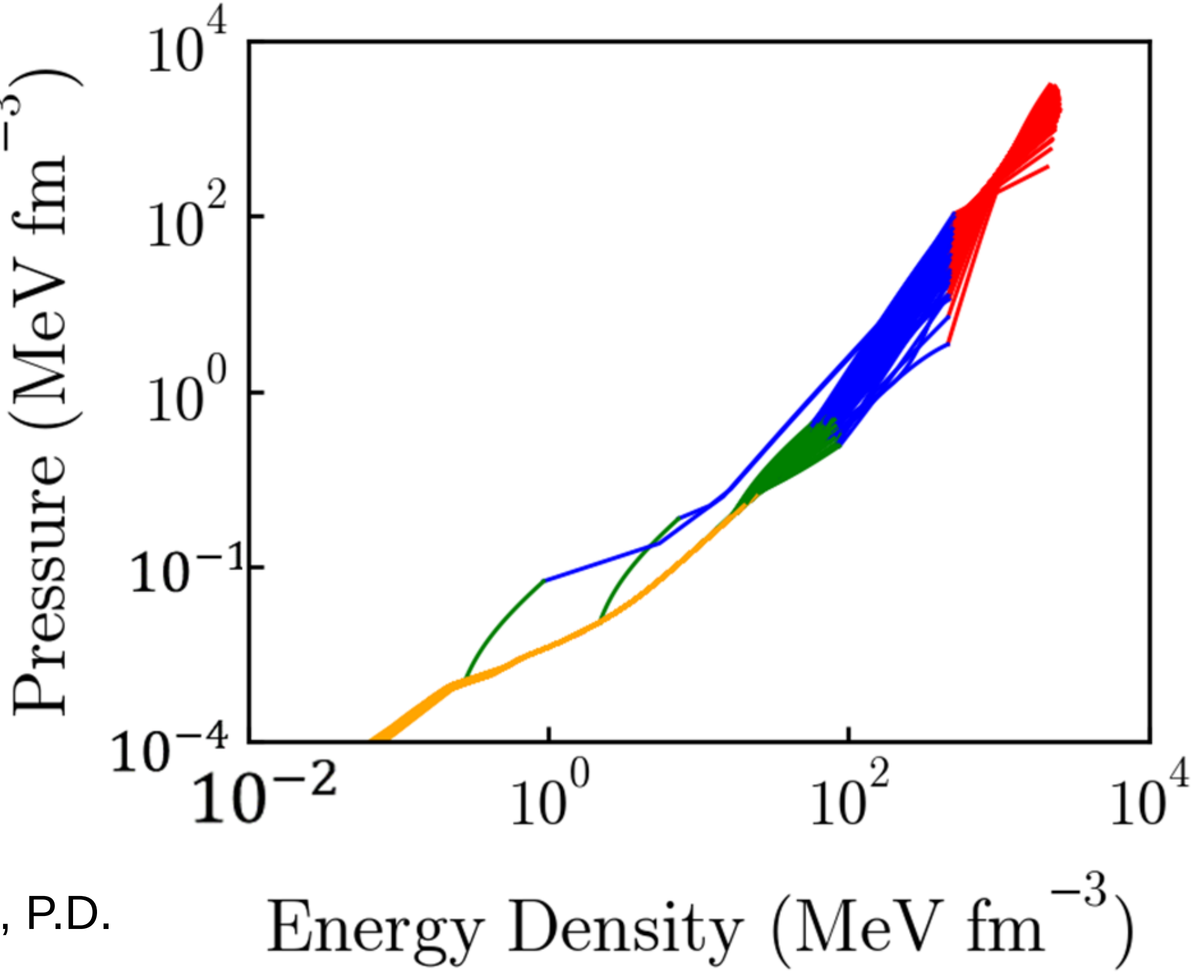
# Neutron star merger

- On 17th Aug, 2017, gravitational wave (GW) from a binary neutron star (NS) merger is observed for the first time.
- One of the major observables : Tidal deformability $\Lambda$
  - NS deform due to tidal force in close proximity
  - Deformation energy must come from orbital energy
  - GW period deviates from point mass calculation
  - The ease of deformation (neutron star equation of state) is given by **Tidal deformability $\Lambda$**
- Initial constraint of $\Lambda < 700$ from GW (B.P. Abbott et al. 2017)
- $\Lambda$ can also be calculated from NS EoS,
- **Can EoS obtained from nuclear physics describe neutron star properties and give reasonable $\Lambda$?**



# Nuclear Physics EoS: Skyrmes interactions

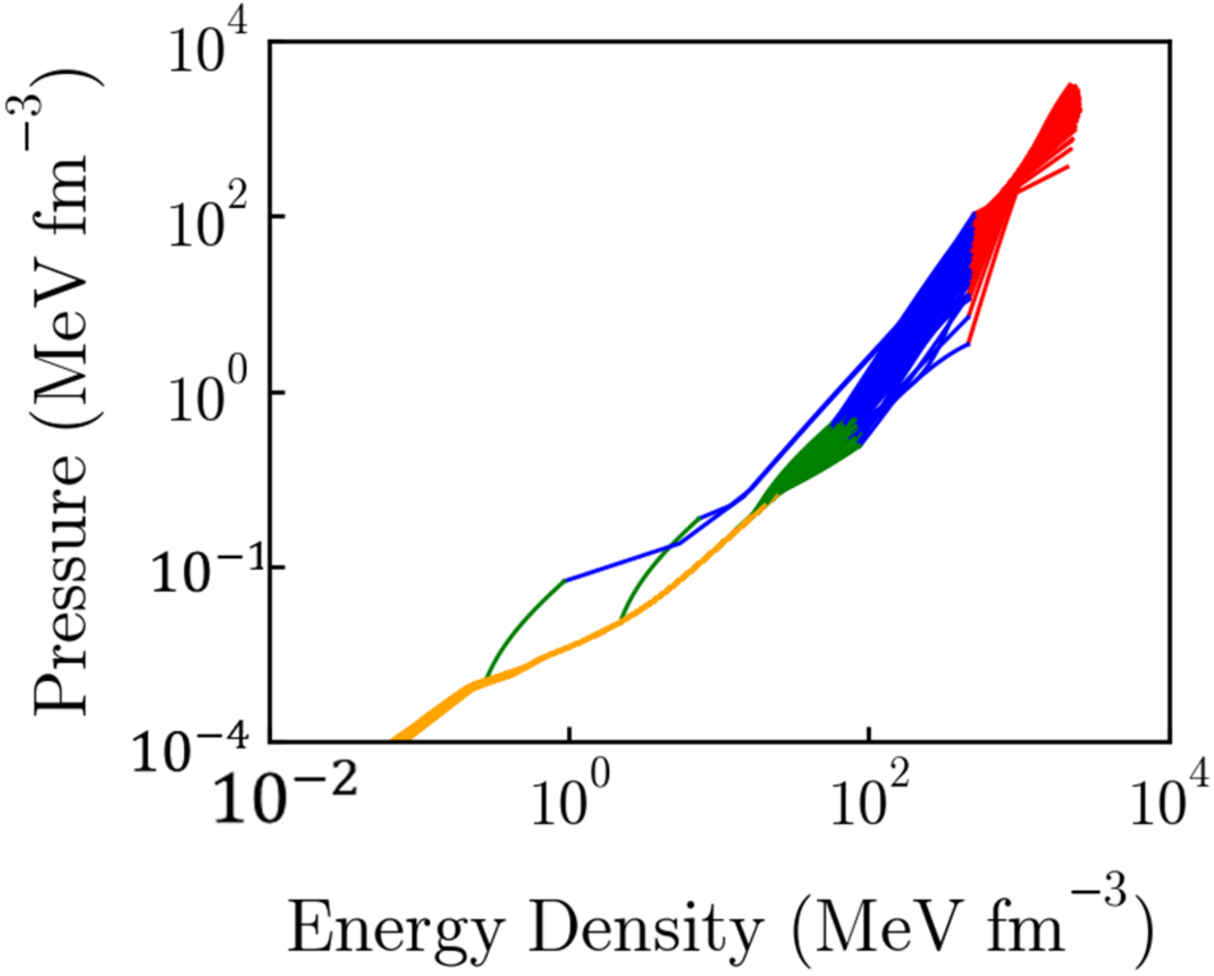
- 11 free parameters functions
- Parameters set varies when fitted to different experimental results
- (Dutra, 2012) is a summary paper including most Skyrme in the academia market. It listed Skyrme that are considered reasonable.
- We also used the extended sets from Pawel Danielewicz and Alex Brown.
- A total of 248 EoS were used.



M.Dutra, O. Lourenco, J.S. Sa Martins, A.Delfno, J.R. Stone, P.D. Stevenson, Phys Rev. C 85, 035201 (2012)

# NS model to bridge NP observable to properties of NS

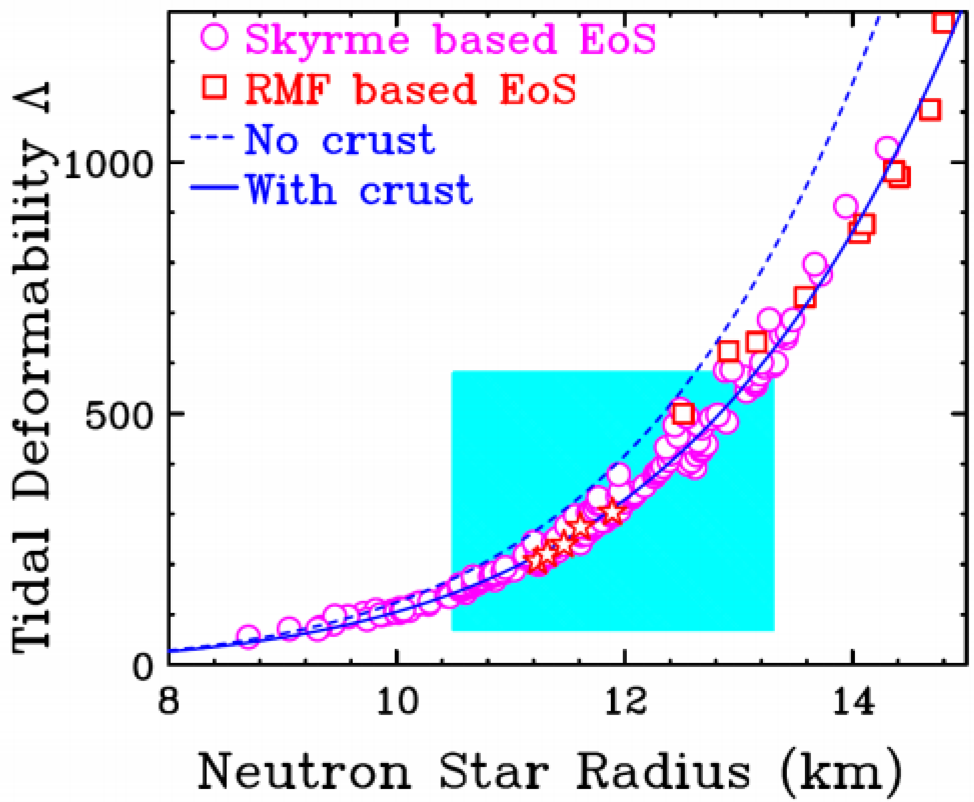
- NS is not a homogeneous nucleon matter across all densities. Need to stitch different EoSs from different density regions
- Yellow region: Crustal EoS from (G. Baym, 1971)
- Green region: Ultra relativistic gas equation to connect crustal EoS with nuclear physics EoS: Skyrme
- Blue region: **Beta equilibrated** Skyrme with electrons and muons.
- Red region: Polytropes ($P = K\rho^{\Gamma}$) at high density > $3\rho_0$. $\Gamma$ is chosen such that the EoS will support 2 solar mass NS.
- Code is developed to bridge nuclear physics experimental observable and properties of NS



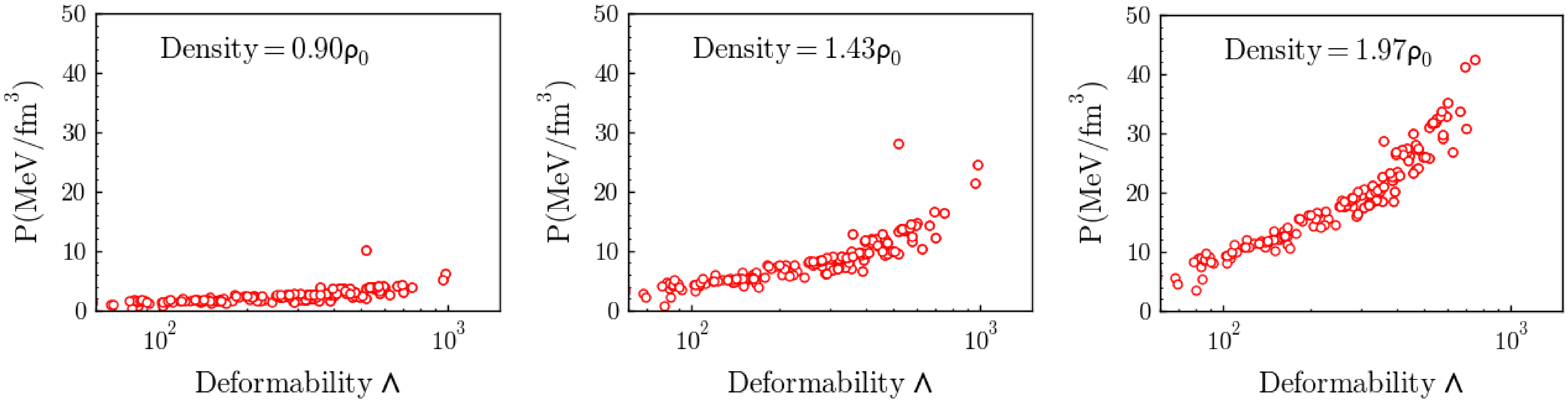
Gordon Baym, Christopher Pethick, Peter Sutherland, Astrophys J., 170:299-137 (1971)

# Neutron Star properties from the model

- A program was developed in collaboration with F.J. Fattoyev such that NS properties can be calculated from nuclear EoS.
- $\Lambda = \frac{2}{3} \mathrm{k}_2(\mathrm{R}) \left(\frac{c^2 R}{Gm}\right)^5$ vs R is plotted.
- Red points corresponds to FSU-gold EoS and relativistic mean field density functionals.
- Blue line corresponds to neutron star EoS of our model.
- Blue dash line corresponds to Skyrme calculation without crust.
- Can we improve our constraints with nuclear reaction experiments?
  - Different density and pressure regions can be created by selecting reaction systems, energy and impact parameters.
  - Pressures can be deduced from nuclear reactions at different energy and impact parameters.
  - Is **P** sensitive to **Λ? What is the optimum density to explore?**

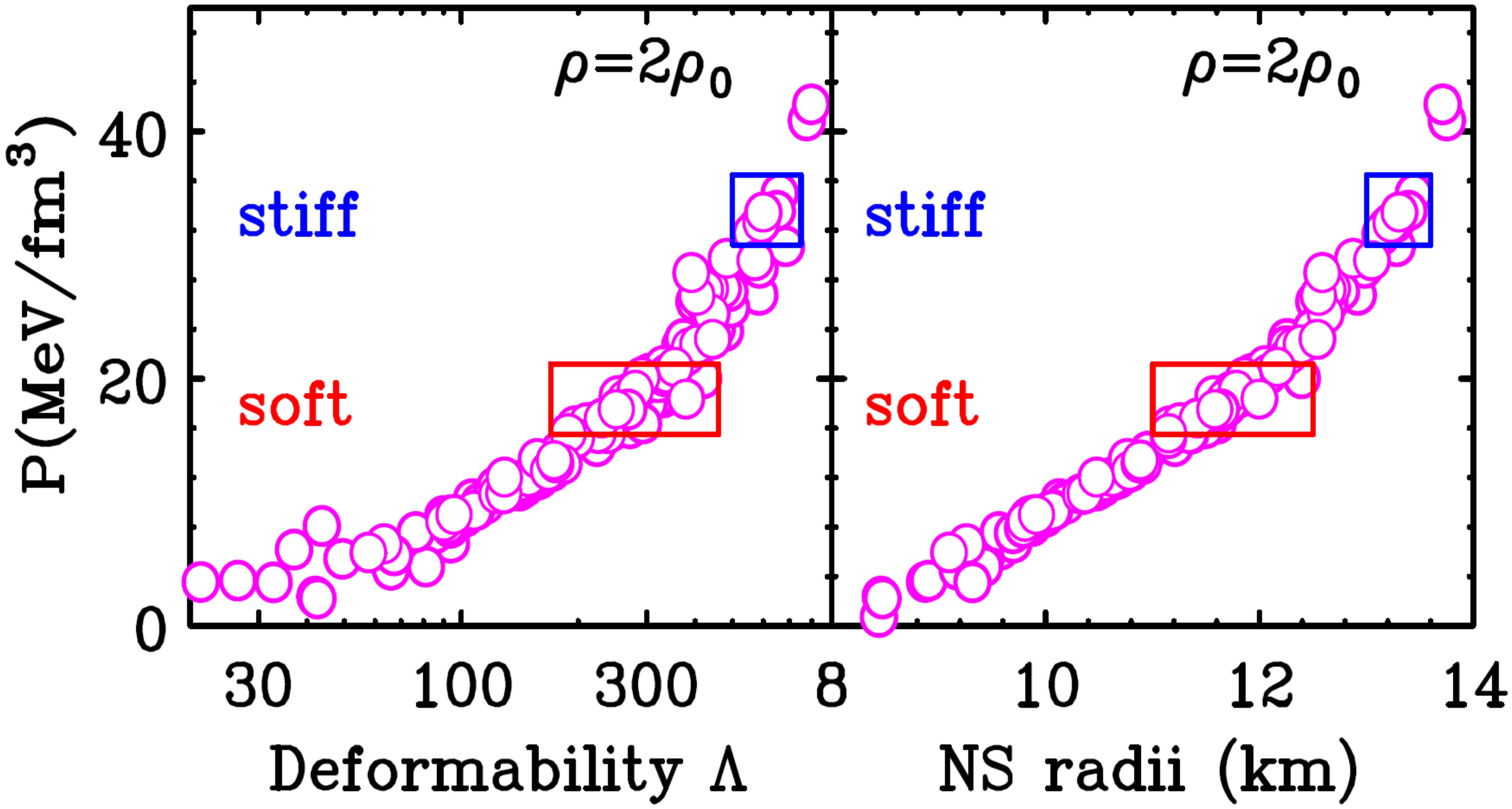


# P-Λ sensitivity test at various densities



1. Correlation between P and Λ increases with density
2. We need density > $1.5\rho_0$ for constraints on Λ to be useful

Example

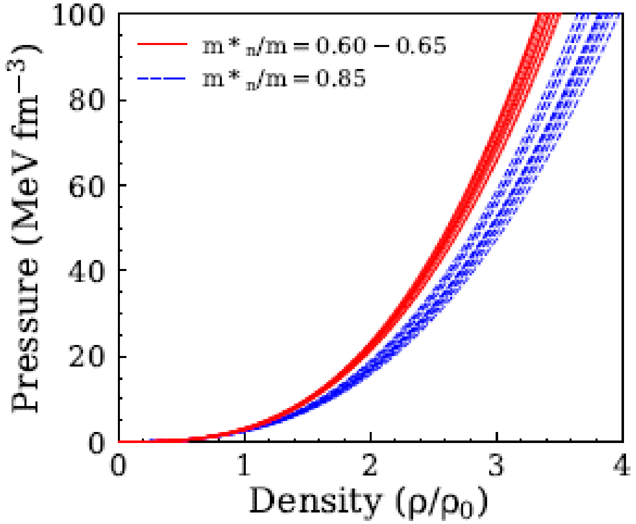


$\rho=2\rho_0$
$\rho=2\rho_0$
stiff
soft
stiff
soft
P(MeV/fm$^3$)
0
20
40
30
100
300
8
10
12
14
Deformability Λ
NS radii (km)

# NS calculation with a more restricted EDFs

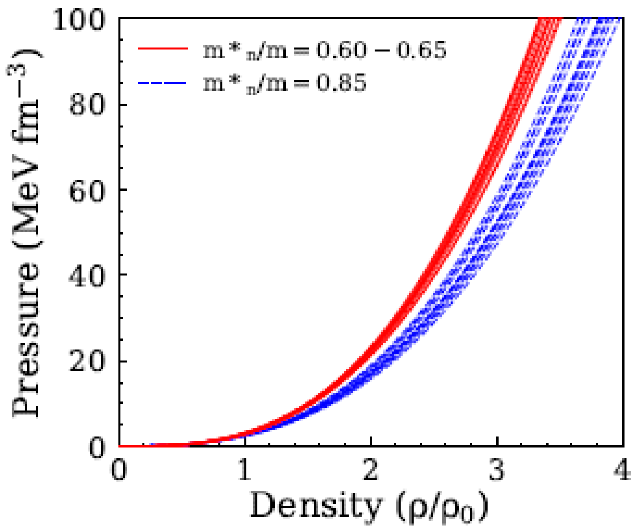
# Adding constraints from experiments

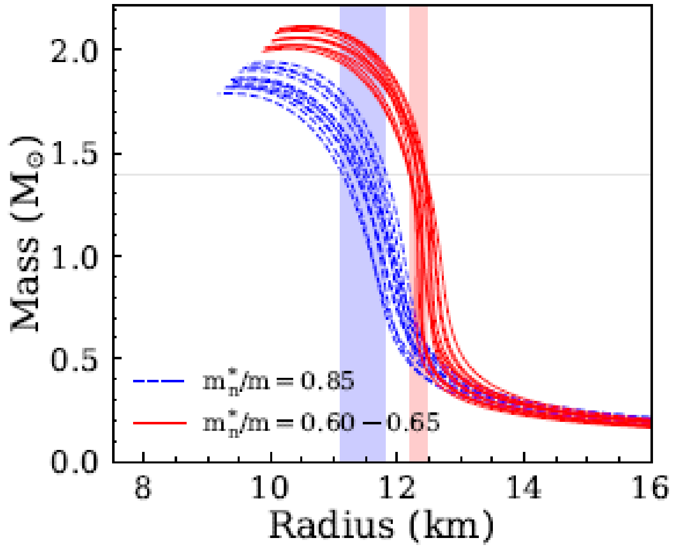
- Just to list a few…
- $K_0$, the nuclear incompressibility, is extracted from the energy of giant monopole resonances and $217 < K_0 < 230$ MeV
- Binding energies of various nuclei (e.g. 16O, 24O, 34Si, 40Ca, 48Ca, 48Ni…) have been measured
- RMS charge radii and single particle energies of the said nuclei have also been measured.
- Ab-inito calculation is performed for low-density neutron matter.
- Skyrme that satisfy them all?

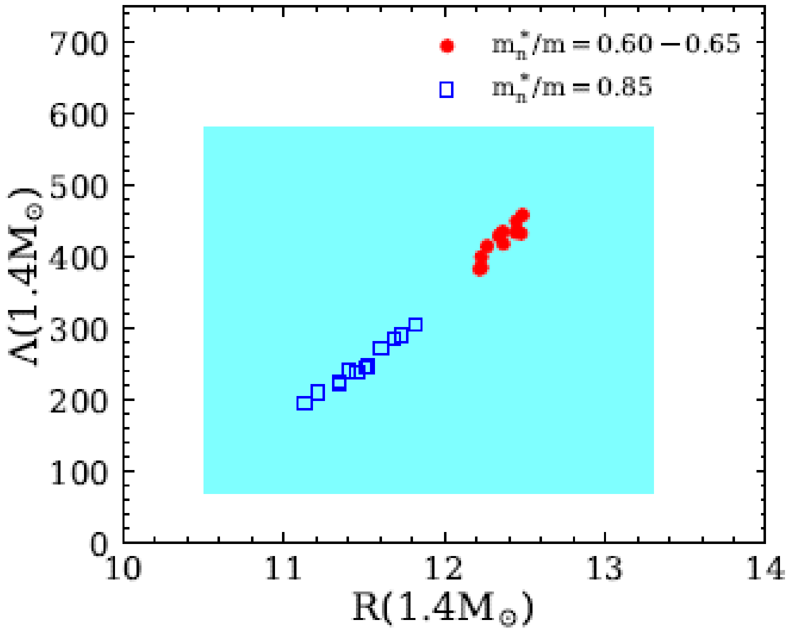


# NS sensitivity to neutron effective mass

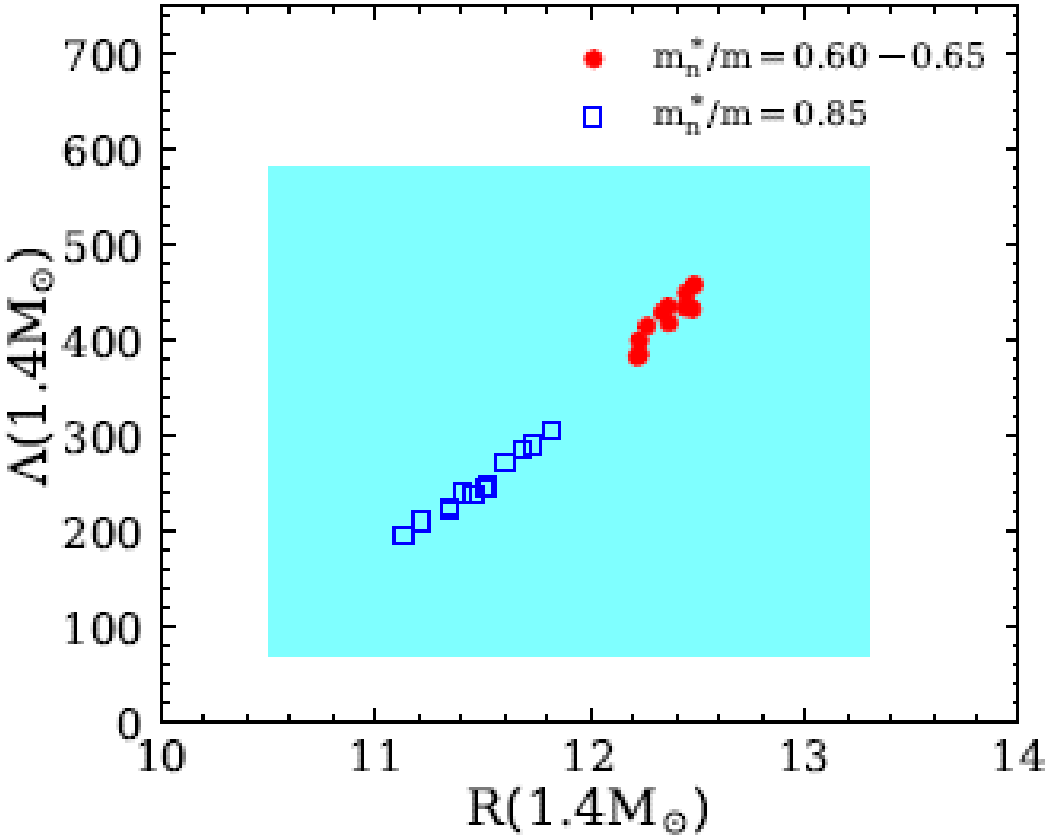
- It is possible to parameterize Skyrme such that they satisfy aforementioned conditions with wildly different neutron-matter effective mass:
  - $\frac{m_n^*(\rho)}{m} = \frac{c_n}{c_n + d_n\rho}$ where $\mathcal{E}(\rho) = a_n\rho^2 + b_n\rho^{2+\sigma} + c_n\rho^{5/3} + d_n\rho^{8/3}$
- However, neutron star calculation reveals sensitivity between maximum possible NS and effective mass:







# Final result with the restricted Skyrme



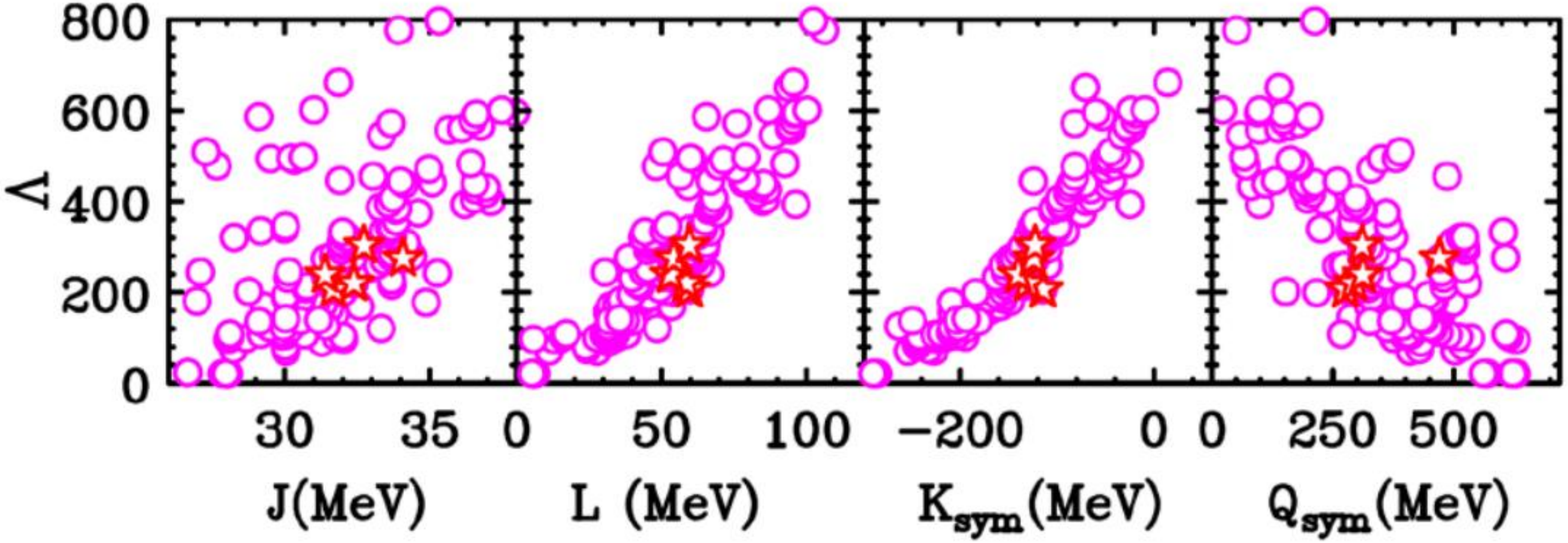
# Correlations between neutron star properties and nuclei properties

- Dutra’s Skyrmes set is limited in numbers. Bias in EoS selection cannot be quantified.
- Uncertainties in correlation is hard to quantified for the same reason
- **How do we handle those problems?**

$$S(\rho)=J+Lx+\frac{1}{2}K_{sym}x^2+\frac{1}{6}Q_{sym}x^3+\mathcal{O}(x^4)$$

$$L=3\rho\frac{dS(\rho)}{d\rho};\ K_{sym}=27\rho^2\frac{d^2S(\rho)}{d\rho^2}$$

$$Q_{sym}=27\rho^3\frac{d^3S(\rho)}{d\rho^3}$$



# NS calculation with Meta EOS

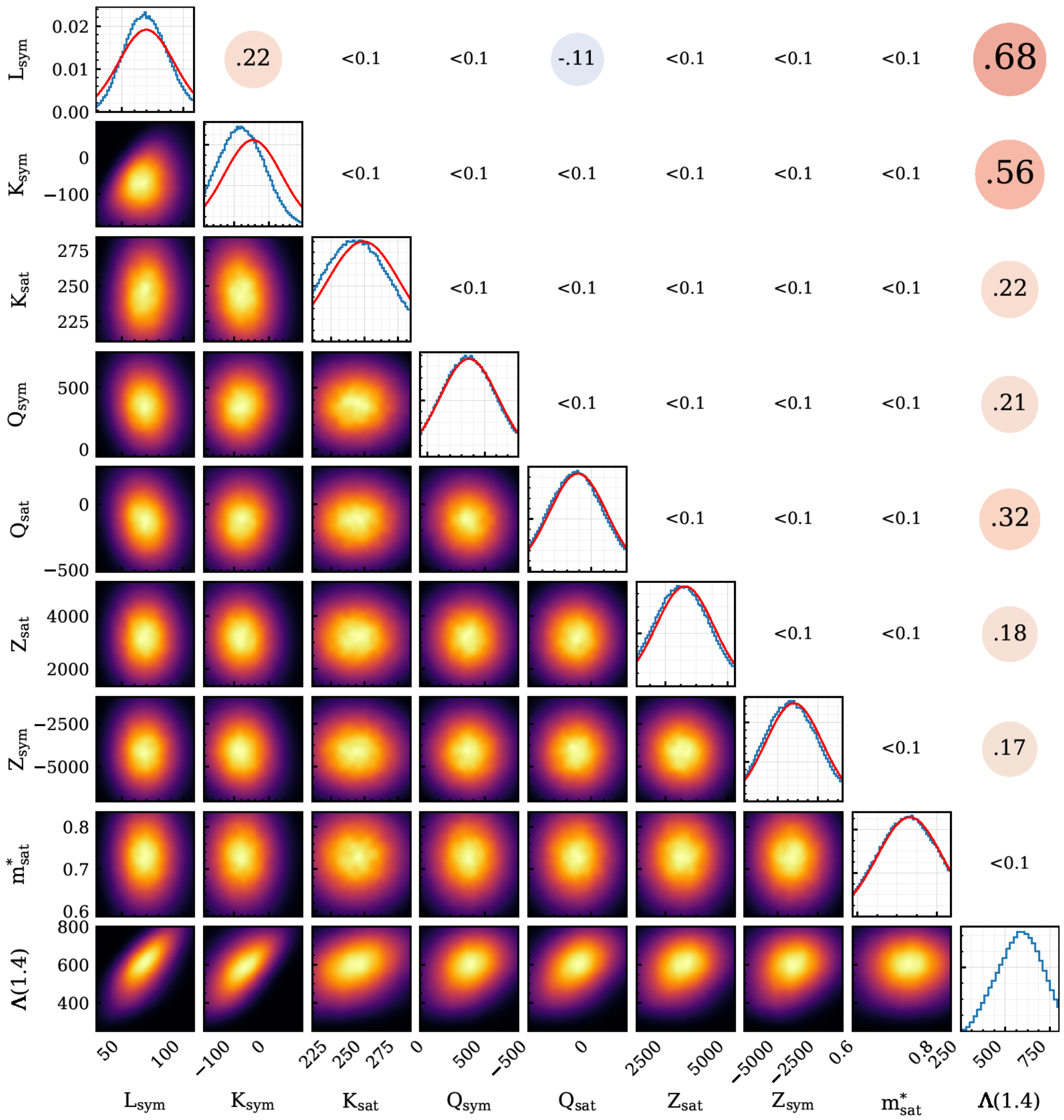
# What is a Meta EOS

Unllike Skyrme, A Meta EOS takes $E_0$, $K_{sat}$, $Q_{sat}$, $Z_{sat}$, $S_0$, $L$, $K_{sym}$, $Q_{sym}$, $Z_{sym}$ as free parameters

In Skyrme these terms are all correlated. Metamodelling EOS ensure independence.

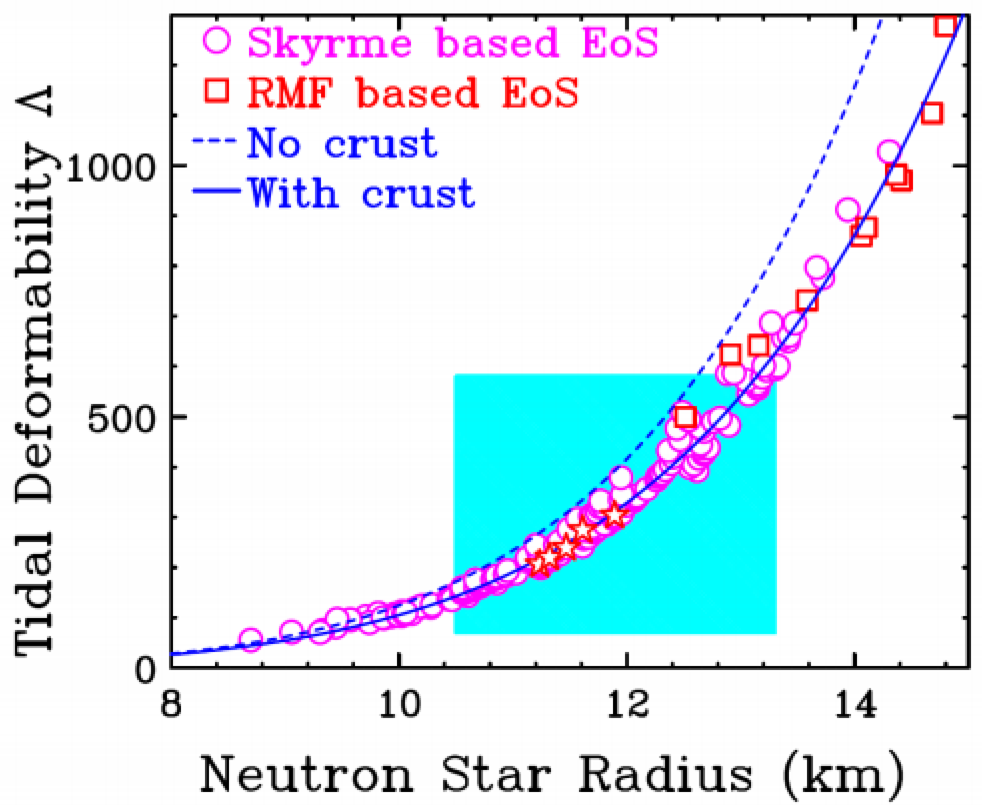
# Preliminary result

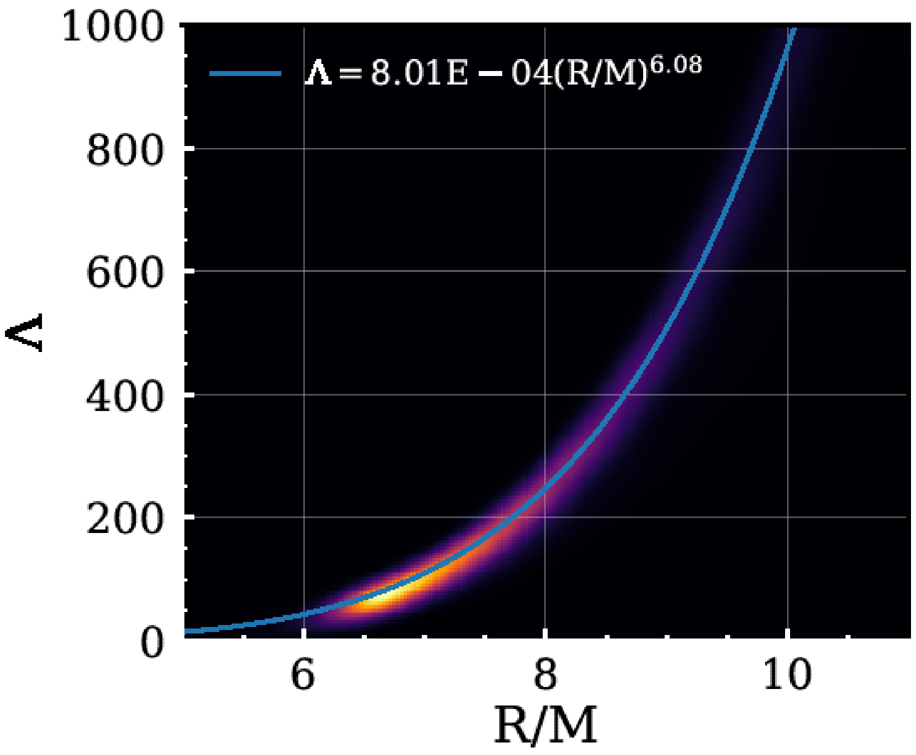
- Red line in the diagonal plots: Prior
- Blue line in the diagonal plots: Posterior
- Change in shape between red and blue line is the result of comparison with measured deformability.
- Only L, Ksym and Ksat show changes between prior and posterior distribution.



# Preliminary result

$$\Lambda = \frac{2}{3} k_2(R) \left(\frac{c^2 R}{Gm}\right)^5$$



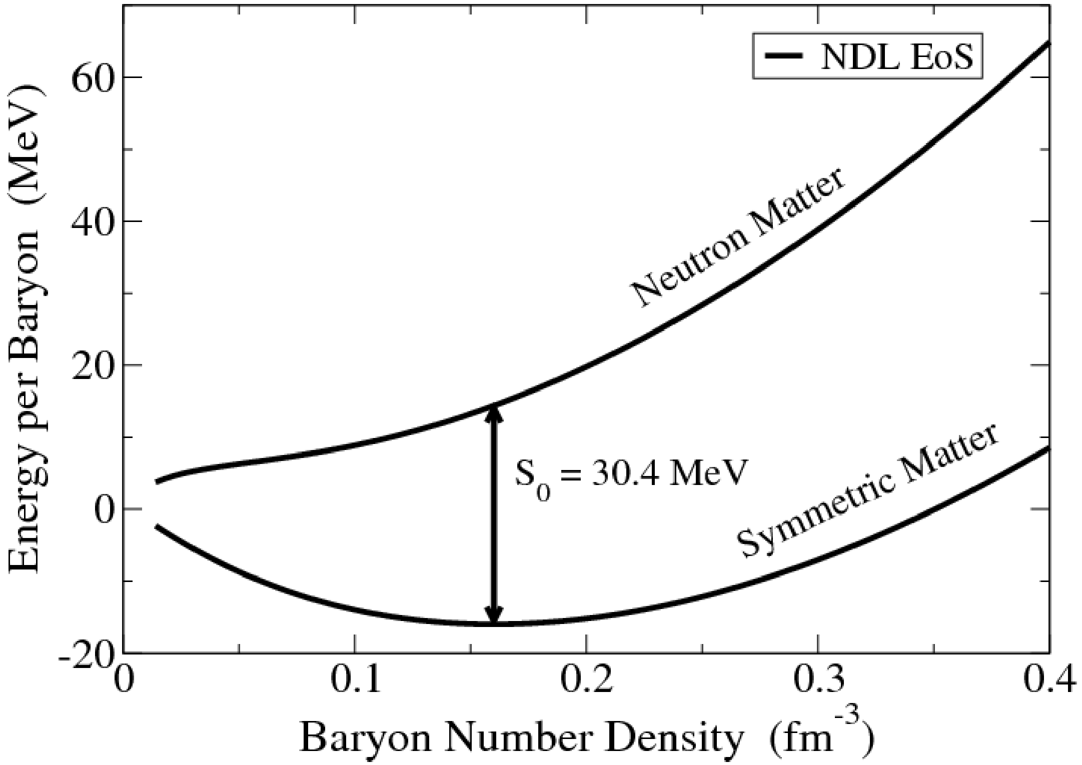


# Back up slides

# 1. Introduction to Nuclear EoS

# What is Equation of state (EoS)

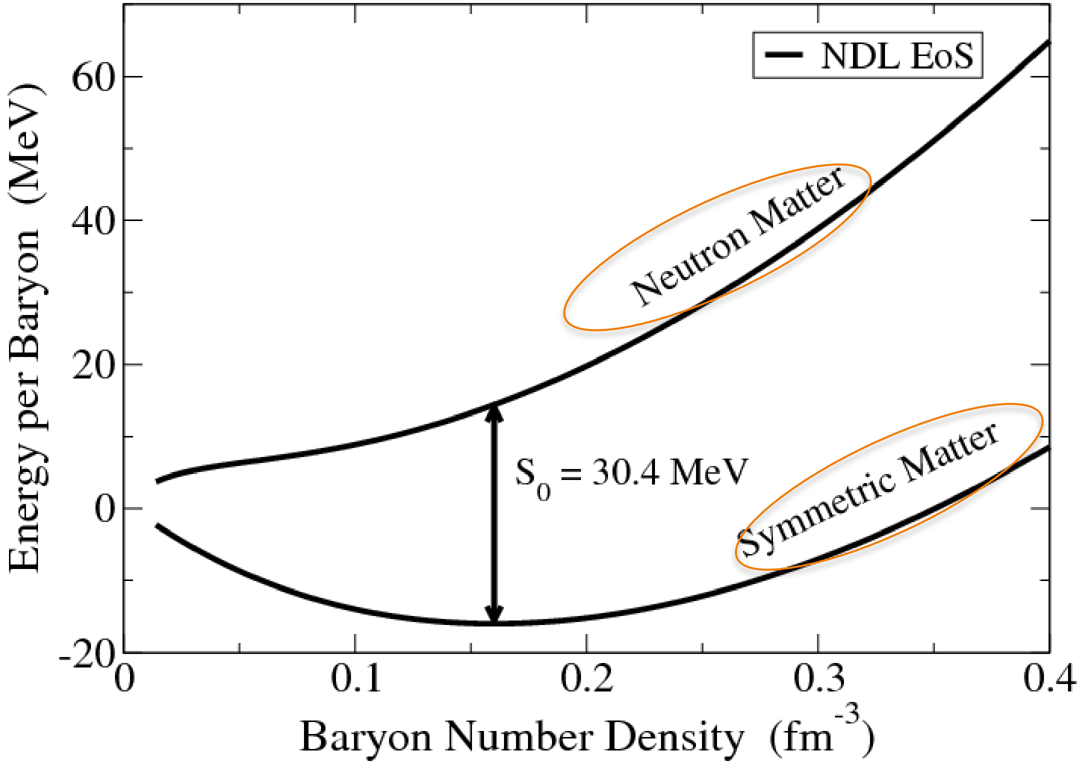
- Thermodynamic equation that relates variables that describe the matter. (e.g. Pressure, Volume, etc.)
- EoS describes bulk properties of matter
  - Soft/hard
  - Compact/sparse
  - Temperature
  - Pressure
  - etc.
- Different EoSs describe different matter
- Matter of interest: Nuclear matter
  - Matter that is made up of homogeneous nucleons
  - Examples: Core of heavy nuclei, neutron stars.
  - The nuclear EoS should be the same for heavy nuclei & neutron stars



Meixner, M. et al. arXiv:1303.0064v1 [astro-ph.HE] (2013)

# Symmetric energy term : transition from hadronic EoS to Neutron star (NS) EoS

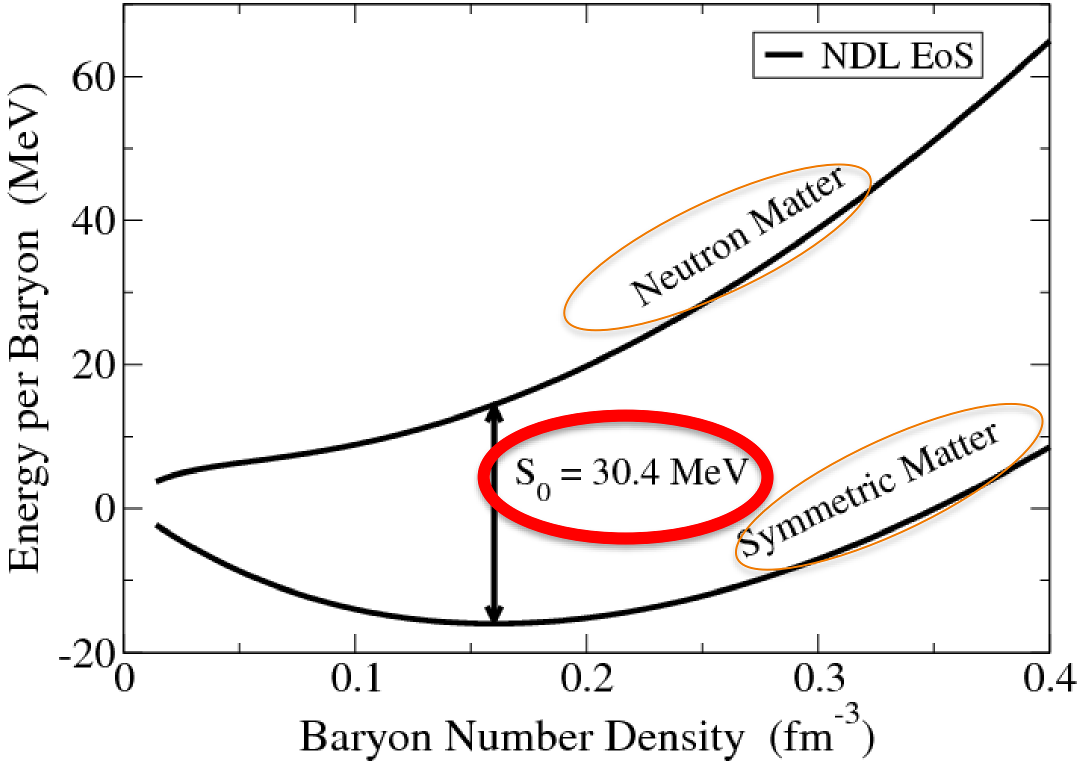
- Nuclei and NS are in different region of isospin space.
- NS consists of mostly neutrons while nuclei consists of (almost) equal number of protons and neutrons (more similar to symmetric nuclear matter)



Meixner, M. et al. arXiv:1303.0064v1 [astro-ph.HE] (2013)

# Symmetric energy term : transition from hadronic EoS to Neutron star (NS) EoS

- Nuclei and NS are in different region of isospin space.
- NS consists of mostly neutrons while nuclei consists of (almost) equal number of protons and neutrons (more similar to symmetric nuclear matter)
- The 2 regions are connected through **symmetry energy term** $S(\rho)$
- $\Delta E_{neutron\ excess} = S(\rho)\delta^2$, where $S(\rho)$ is often called **symmetry energy term.**
- $S(\rho)$ is the focus of the HiRA group research.
- Study observables across nuclei with different asymmetry $\delta = \frac{N-Z}{A}$, extrapolate hadronic EoS to pure neutron EoS.



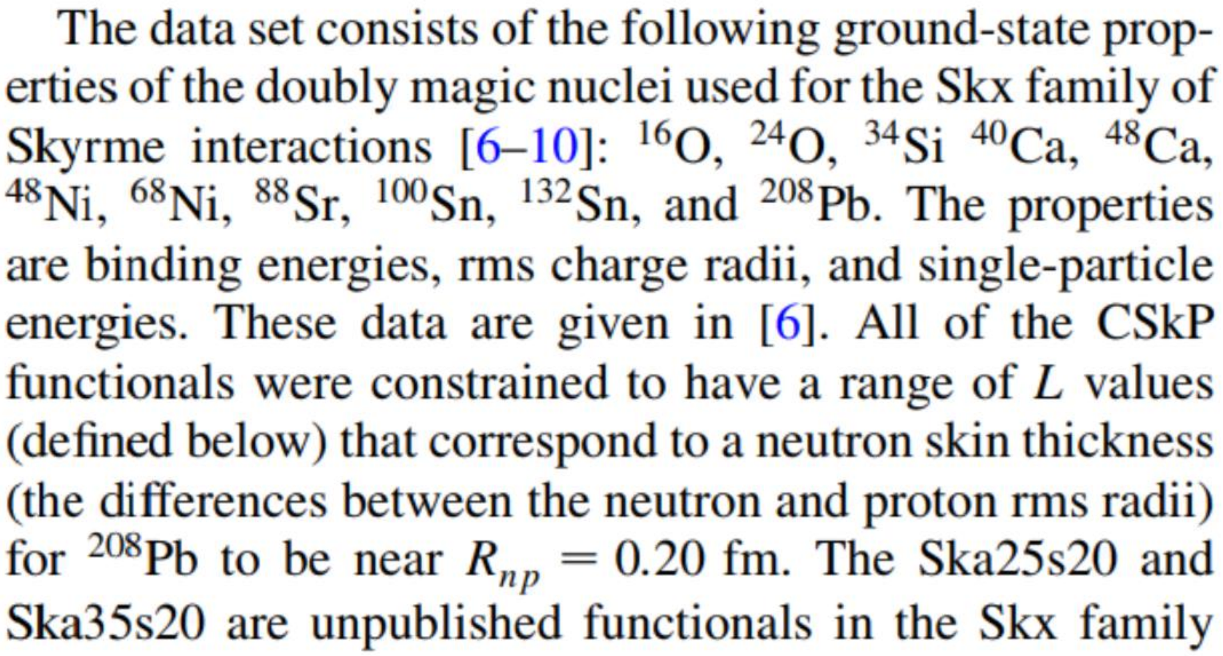
Meixner, M. et al. arXiv:1303.0064v1 [astro-ph.HE] (2013)

# Meta EOS

- It is a 12-free parameters equation with the following form:
  - $\mathrm{E}_{EFLc}(\rho,\delta) = t^{FG*}(\rho,\delta) + v^{N}_{EFLc}(\rho,\delta)$
  - Where
  - $t^{FG*}(\rho,\delta) = \frac{t^{FG}_{sat}}{2}\left(\frac{\rho}{\rho_0}\right)^{\frac{2}{3}}\left[\left(1+\frac{\kappa_{sat}\rho}{\rho_0}\right)\left((1+\delta)^{\frac{5}{3}}+(1-\delta)^{\frac{5}{3}}\right)+\frac{\kappa_{sym}\rho}{\rho_0}\delta\left((1+\delta)^{\frac{5}{3}}-(1-\delta)^{\frac{5}{3}}\right)\right]$
  - $v^{N}_{EFLc}(\rho,\delta) = \sum_{i=0}^{4}\frac{1}{i!}\left(v^{is}_{i}+v^{iv}_{i}\delta^2\right)\left(1-(-3x)^{5-i}\right)e^{-b\rho/\rho_0}x^i$
  - Such that:

$$
\begin{aligned}
v^{is}_0 &= E_{sat} - t^{FG}_{sat}(1+\kappa_{sat})\\
v^{is}_1 &= -t^{FG}_{sat}(2+5\kappa_{sat})\\
v^{is}_2 &= K_{sat} - 2t^{FG}_{sat}(-1+5\kappa_{sat})\\
v^{is}_3 &= Q_{sat} - 2t^{FG}_{sat}(4-5\kappa_{sat})\\
v^{is}_4 &= Z_{sat} - 8t^{FG}_{sat}(-7+5\kappa_{sat})
\end{aligned}
$$

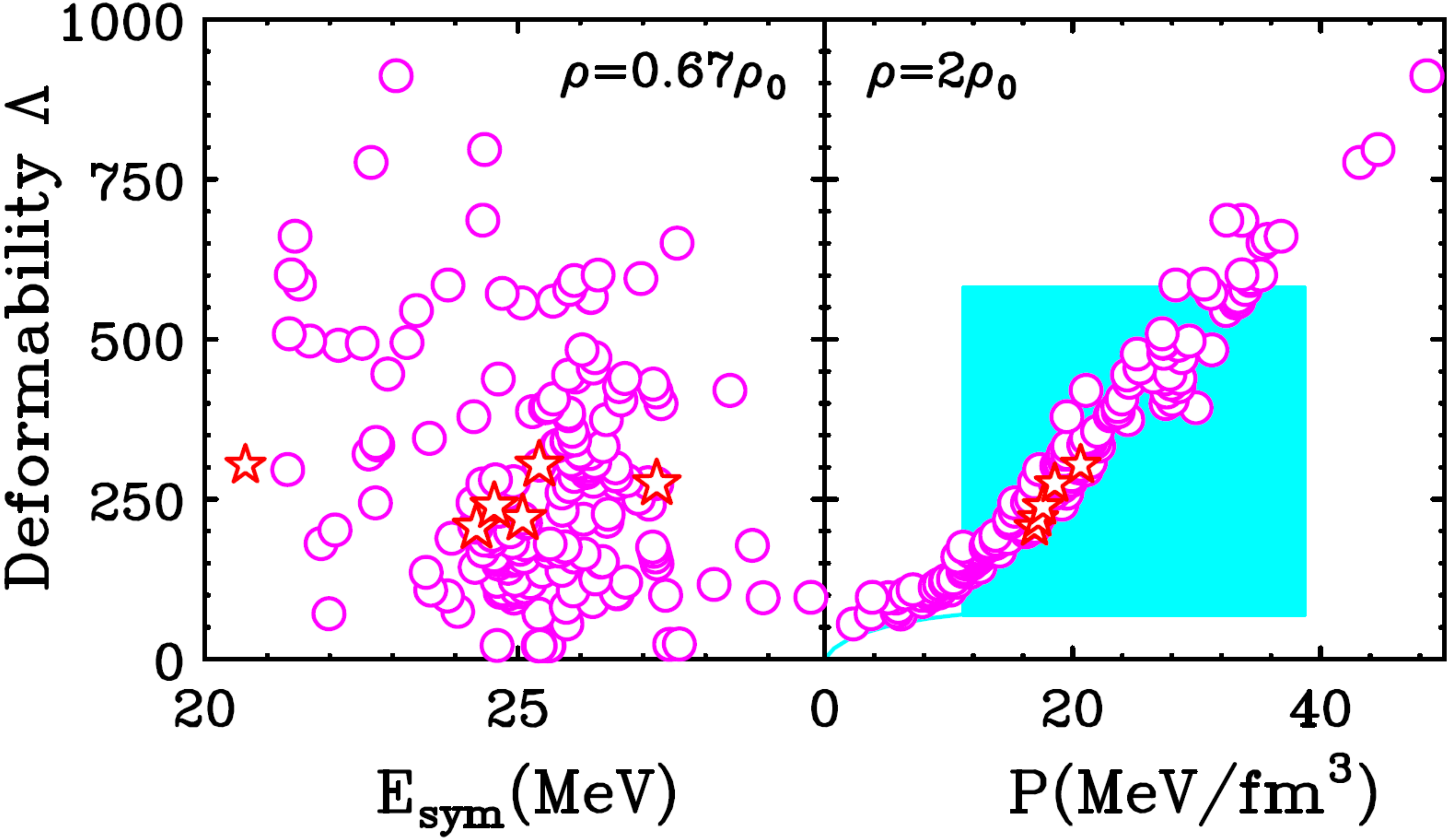
$$
\begin{aligned}
v^{iv}_0 &= S_0 - \frac{5}{9}t^{FG}_{sat}\left(1+\left(\kappa_{sat}+3\kappa_{sym}\right)\right)\\
v^{iv}_1 &= L - \frac{5}{9}t^{FG}_{sat}(2+5(\kappa_{sat}+3\kappa_{sat}))\\
v^{iv}_2 &= K_{sym} - \frac{10}{9}t^{FG}_{sat}(-1+5(\kappa_{sat}+3\kappa_{sym})\\
v^{iv}_3 &= Q_{sym} - \frac{10}{9}t^{FG}_{sat}\left(4-5\left(\kappa_{sat}+3\kappa_{sym}\right)\right)\\
v^{iv}_4 &= Z_{sym} - \frac{40}{9}t^{FG}_{sat}(-7+5\left(\kappa_{sat}+3\kappa_{sym}\right))
\end{aligned}
$$



The data set consists of the following ground-state properties of the doubly magic nuclei used for the Skx family of Skyrme interactions [6–10]: $^{16}$O, $^{24}$O, $^{34}$Si $^{40}$Ca, $^{48}$Ca, $^{48}$Ni, $^{68}$Ni, $^{88}$Sr, $^{100}$Sn, $^{132}$Sn, and $^{208}$Pb. The properties are binding energies, rms charge radii, and single-particle energies. These data are given in [6]. All of the CSkP functionals were constrained to have a range of $L$ values (defined below) that correspond to a neutron skin thickness (the differences between the neutron and proton rms radii) for $^{208}$Pb to be near $R_{np} = 0.20$ fm. The Ska25s20 and Ska35s20 are unpublished functionals in the Skx family

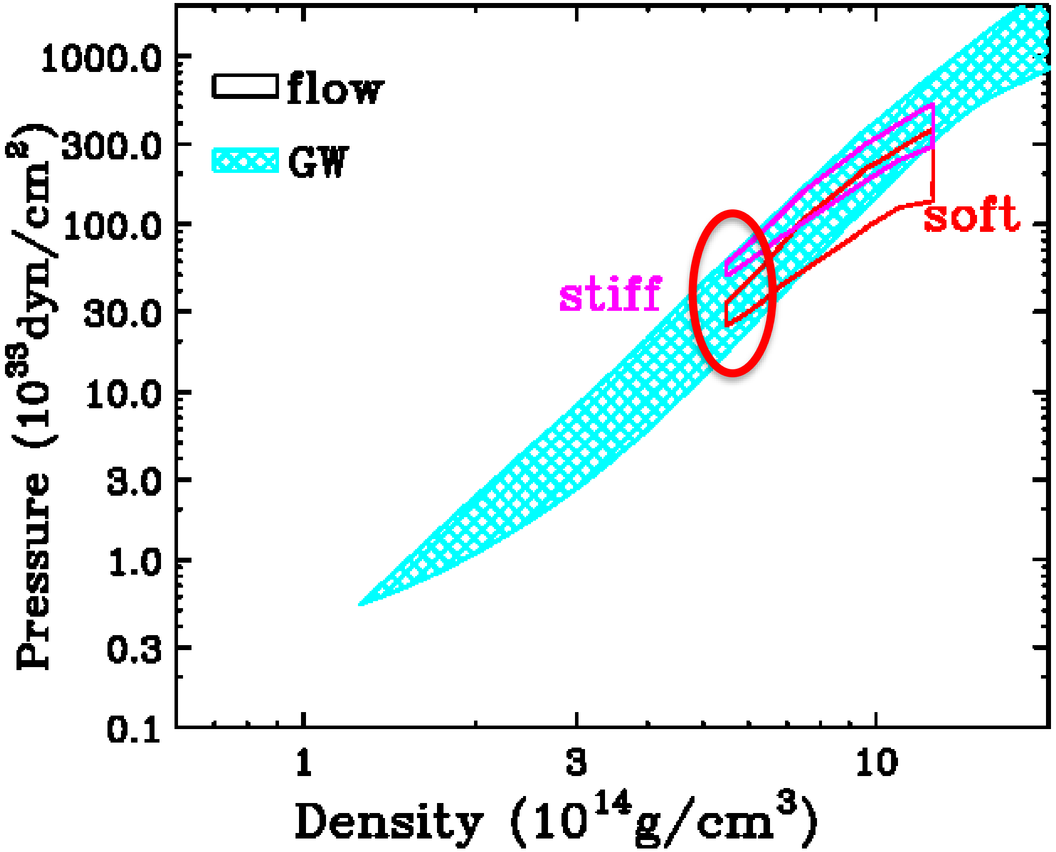
Brown, A., Phys. Rev. Lett. 111, 232502
PRC 58, 220 (July 1998)

Example


Deformability Λ
1000
750
500
250
0
20
25
0
20
40
$\rho=0.67\rho_0$
$\rho=2\rho_0$
$E_{sym}$(MeV)
P(MeV/fm$^3$)

# Result from the flow experiment

- Purple and red boxes show the constraints obtained from flow data with stiff and soft symmetry energy term respectively.
- Discrimination of stiff and soft symmetry energy term is good ~ $2\rho_0$.
- Recall that pressure at $1.5\rho_0$ - $2\rho_0$ provides optimal discrimination for Λ
- Furthermore, P(~$2\rho_0$) correlates strongly with NS radius.
- Recent EoS constraints from (LIGO, 2018) agrees well with HIC constraints from (P. Danielewicz, 2002)
- Pion ratios from SpiRIT experiments could place tighter constraints



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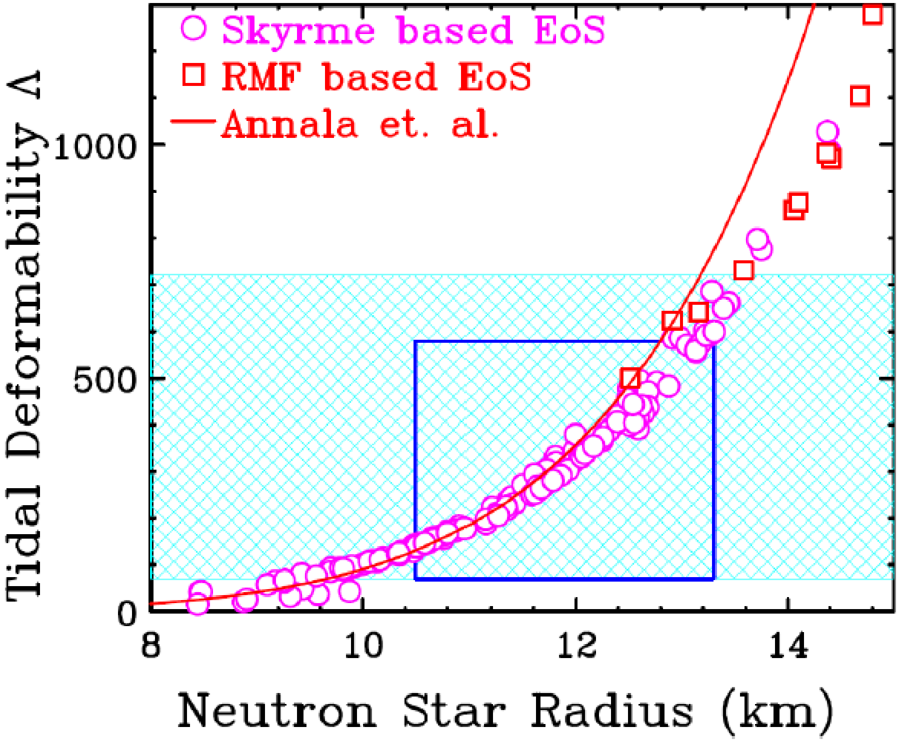
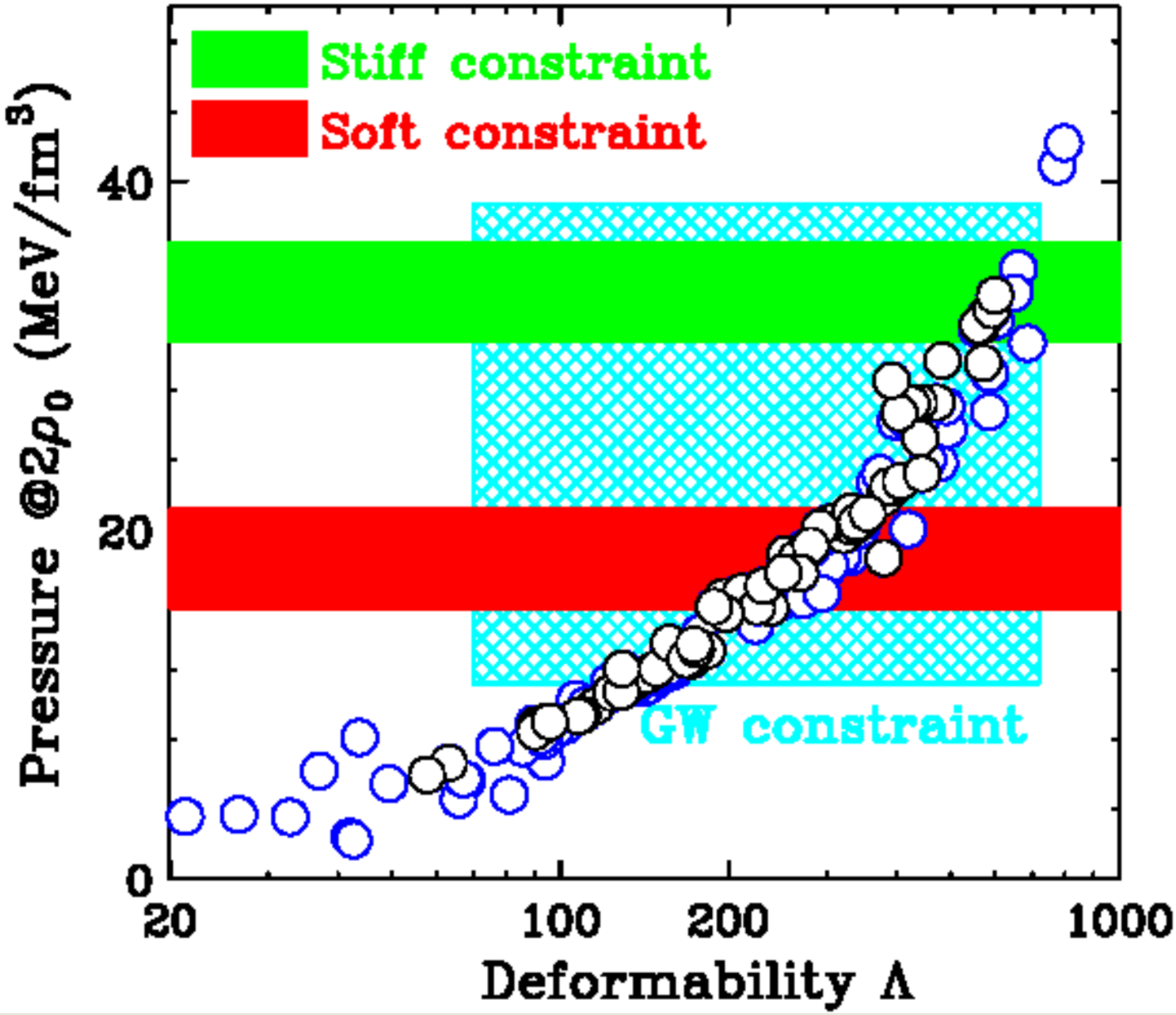
# Validation of the deformability calculation

- Λ vs R is plotted against other EoSs for validation
- $\Lambda = \frac{2}{3}\mathrm{k}_2(\mathrm{R})\left(\frac{c^2 R}{Gm}\right)^5$
- Red points corresponds to FSU-gold EoS and relativistic mean field density functionals.
- Red line corresponds to neutron star EoS constructed with piece-wise polytrope EoS
- Our calculation with Skyrme based EoS seems to agree with other calculations.
- How can we improve our constraints with nuclear reaction data?
  - Pressures can be deduced from nuclear reactions at different energy and impact parameters.
  - Is **P** sensitive to **Λ? What is the optimum?**



# Putting all constraints together

- Pressure at $2\rho_0$ as a function of $\Lambda$
- Open circles corresponds to theoretical calculation from Skyrmes.
- Green and red region corresponds to constraints from the flow experiment
- Blue region is the GW constraints
- **Conclusion: Observables at P($2\rho_0$) can improve the symmetry EoS constraint further beyond the GW constraint.**



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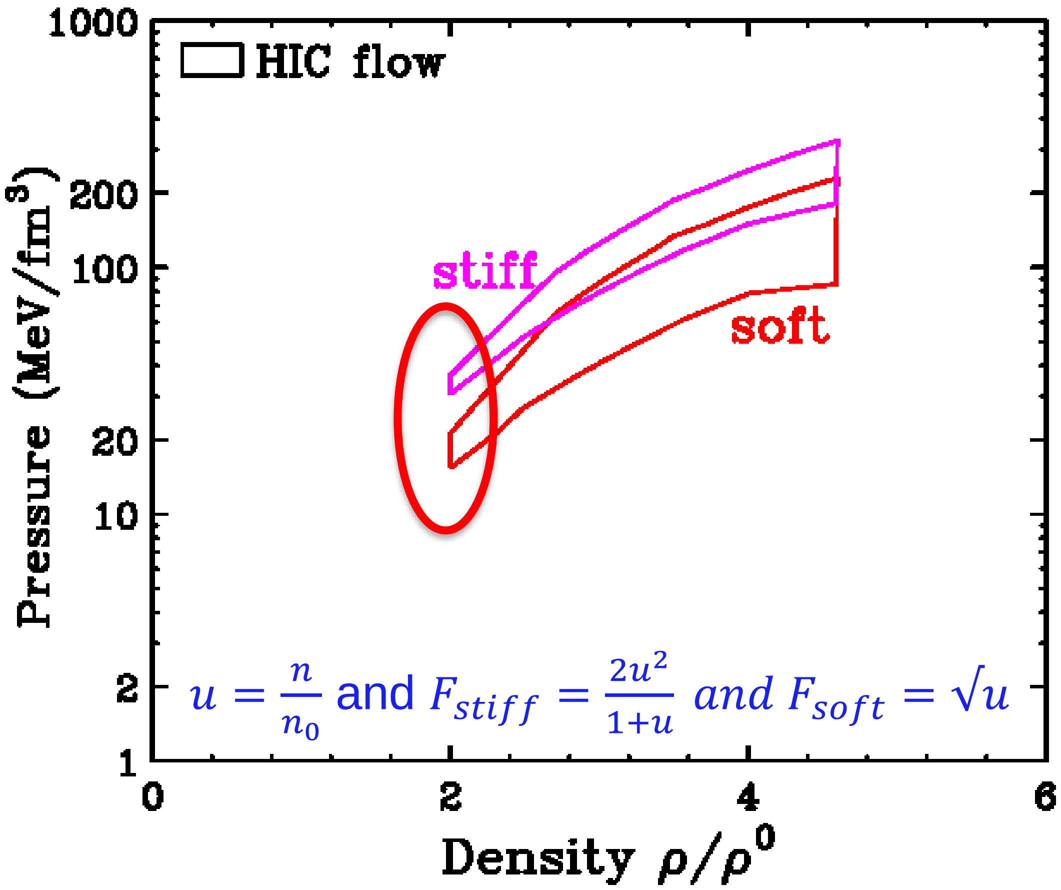
# Result from the flow experiment

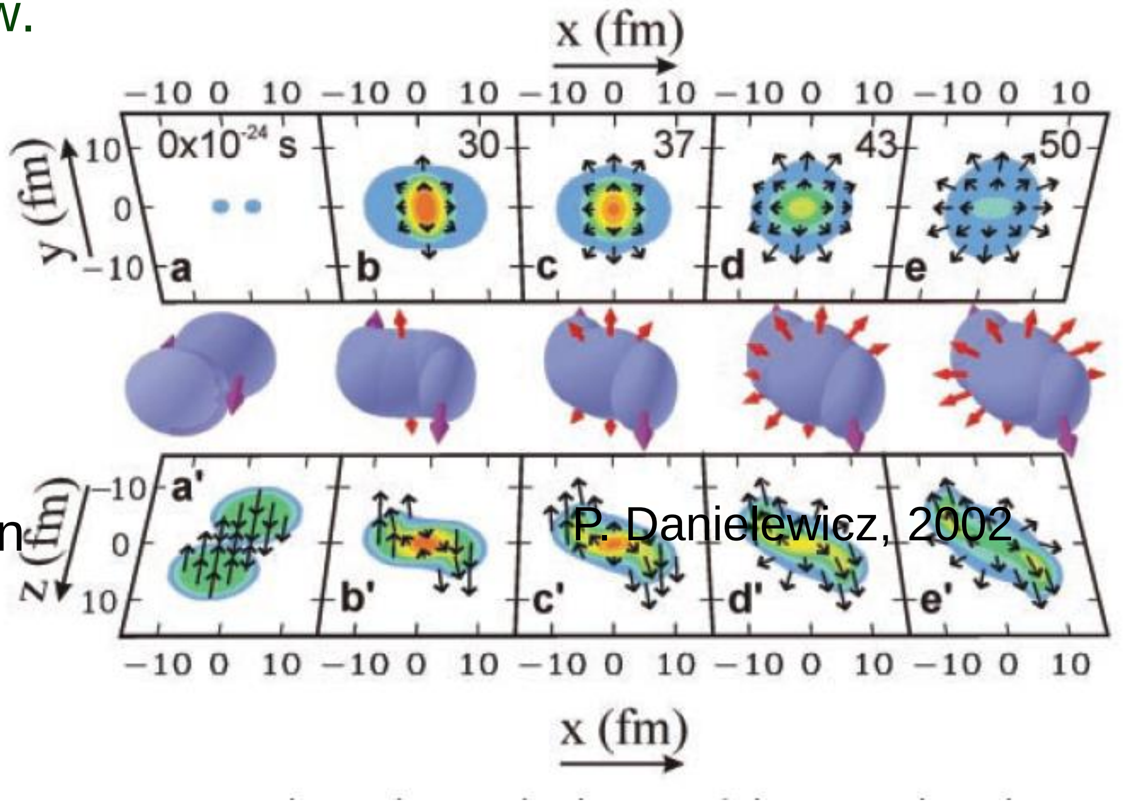
- Purple and red contours show the constraints obtained from flow data with stiff and soft symmetry energy term respectively.
- Discrimination of stiff and soft symmetry energy term is good ~ $2\rho_0$.
- P(~$2\rho_0$) also correlates strongly with NS radius.
- Recall that pressure at $1.5\rho_0$ - $2\rho_0$ provides good discrimination for Λ
- Pion ratios from SpiRIT experiments and future experiments could place tighter constraints



$$u = \frac{n}{n_0} \text{ and } F_{stiff} = \frac{2u^2}{1+u} \; and \; F_{soft} = \sqrt{u}$$

# Previous measurement: Flow experiment

- Flow experiment: Measure pressure by calculating ratio of in-plane and out-of-plane emission and transverse flow.
- Run down of a nuclear collision:
  1. Part of projectile overlaps target nuclei
  2. Overlapped part is squeezed and expand
  3. The non-overlapped part (spectator nucleons) is pushed and deflect sideway.
  4. The non-overlapped part also blocks in-plane emission
  5. Higher pressure -> Preferential out-of-plane emission
- The Au+Au experiment only measured pressure for symmetric matter. Need a **symmetry energy term** to convert it to NS pressure
  - A **Stiff** and **Soft** energy term (S($\rho$)) is employed.
  - 2 extremum. Everything in between is allowed by theory.

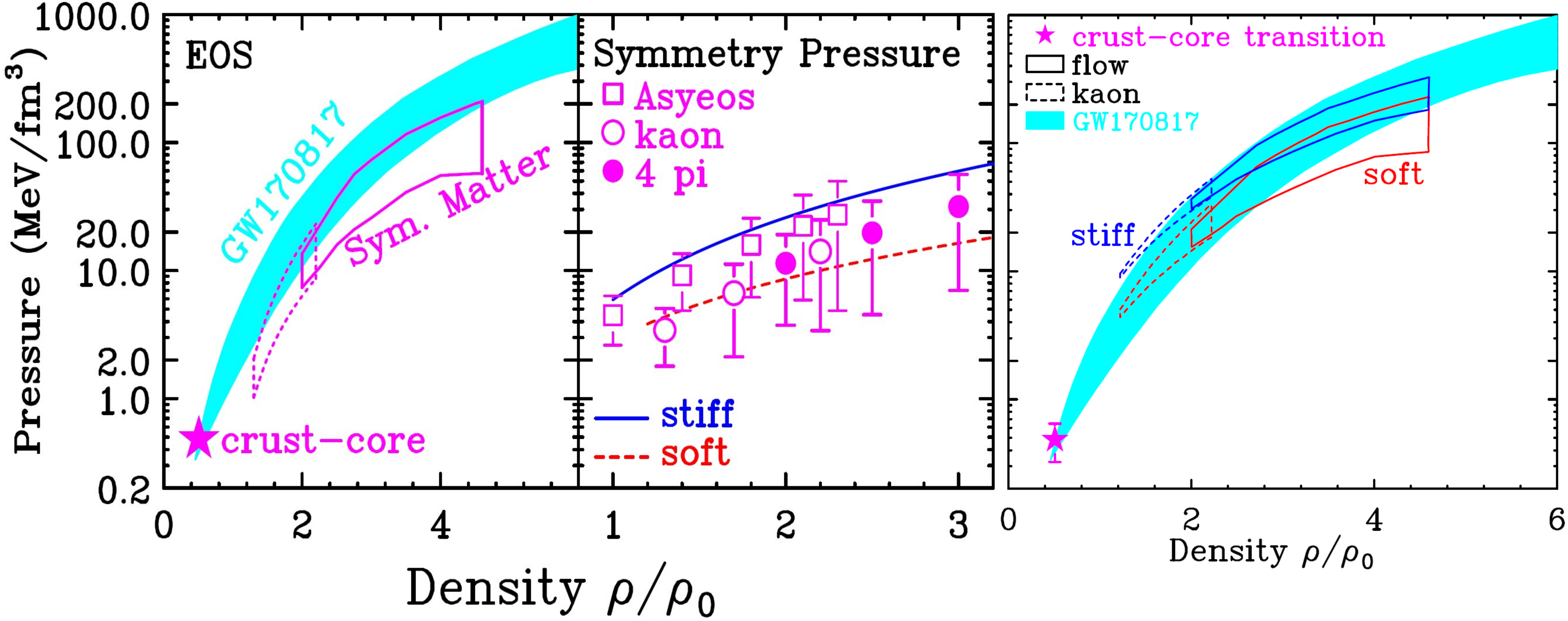


P. Danielewicz, 2002

$u = \frac{n}{n_0}$ and $F_{stiff} = \frac{2u^2}{1+u}$ $and$ $F_{soft} = \sqrt{u}$

Pawel Danielewicz, Roy Lacey, and William G. Lynch, Science 298, 1592 (2002).

arXiv:1901.07673



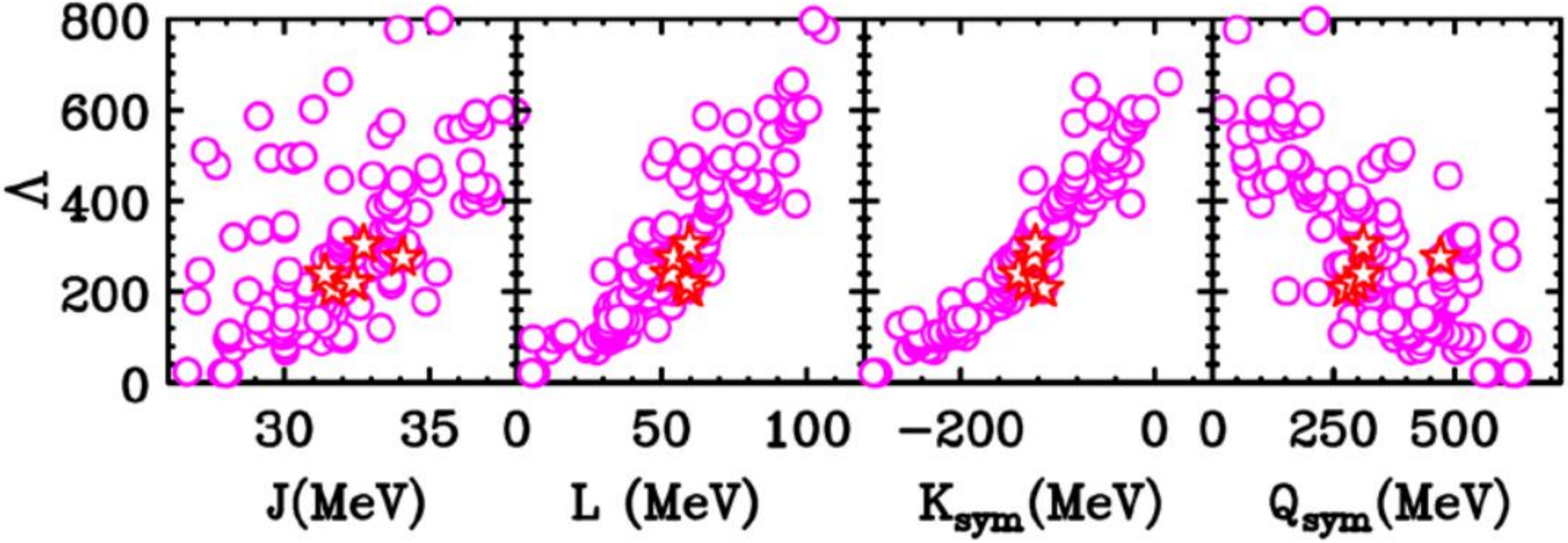
Symmetric matter constraints from 4pi flow data obtained in 2002.
GW170817 analysis obtained in 2018. Goal is to reduce the uncertainties.

- Dutra’s Skyrmes set is limited in numbers. Bias in EoS selection cannot be quantified.
- Uncertainties in correlation is hard to quantified for the same reason
- **In progress, will come back to it…**

$$S(\rho)=J+Lx+\frac{1}{2}K_{sym}x^2+\frac{1}{6}Q_{sym}x^3+O(x^4)$$

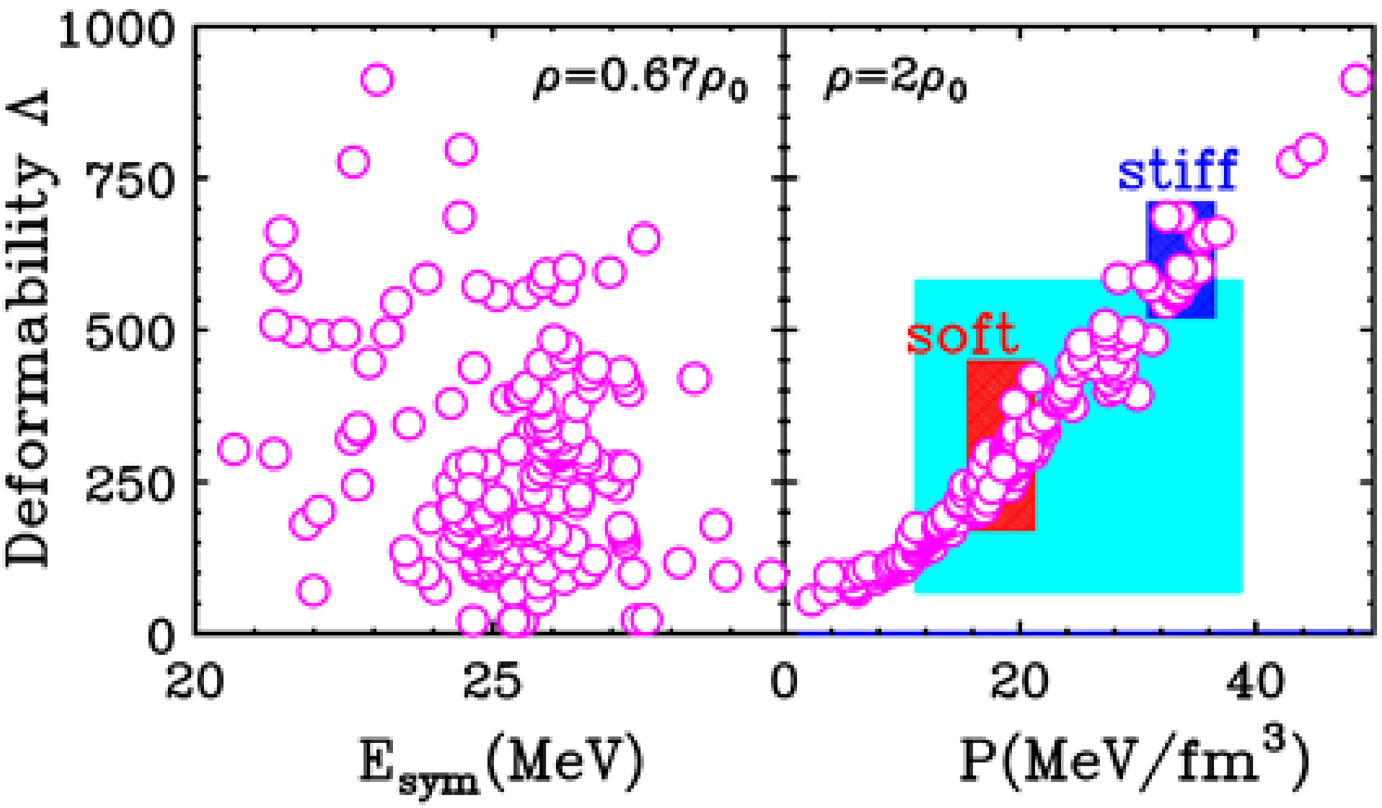
$$L=3\rho\frac{dS(\rho)}{d\rho};\ K_{sym}=27\rho^2\frac{d^2S(\rho)}{d\rho^2}$$

$$Q_{sym}=27\rho^3\frac{d^3S(\rho)}{d\rho^3}$$



# Putting all constraints together

- Pressure as a function of $\Lambda$
- Open circles correspond to theoretical calculation from Skyrme. No correlation to symmetry energy at $<\rho_0$.
- Green and red regions correspond to constraints from the flow experiment
- Blue region is the GW constraints
- **Conclusion: Observables at P(~2$\rho_0$) can improve the symmetry EoS constraint further beyond the GW constraint**
- **The new code will give us a better macroscopic understanding of NS structure.**



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