Light clusters and pasta phases

Constança Providência

Universidade de Coimbra, Portugal

Challenges to Transport Theory for Heavy-Ion Collisions,
ECT*, May 20-24, 2019

UNIVERSIDADE DE COIMBRA
CFisUC
FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR
PHAROS
THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS
POPH QUALIFICAR É CRESCER.
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Motivation

- How can we describe the warm non-homogeneous stellar matter self-consistently?
- Which are the effects of including explicitly light clusters?
- What kind of information do we get from HIC to constrain neutron star matter?

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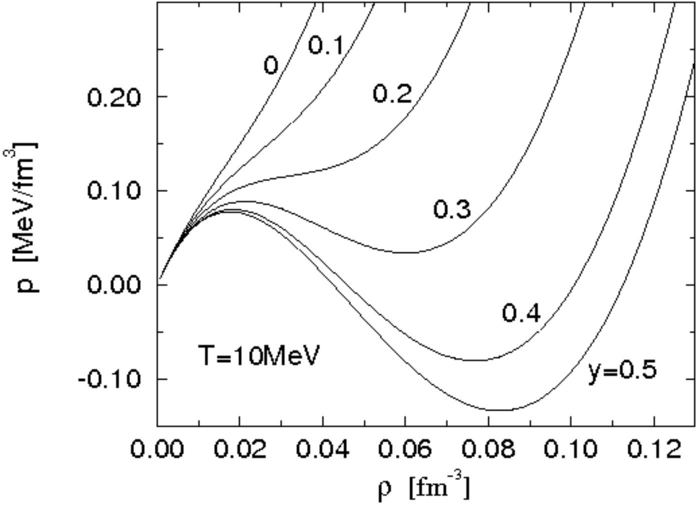
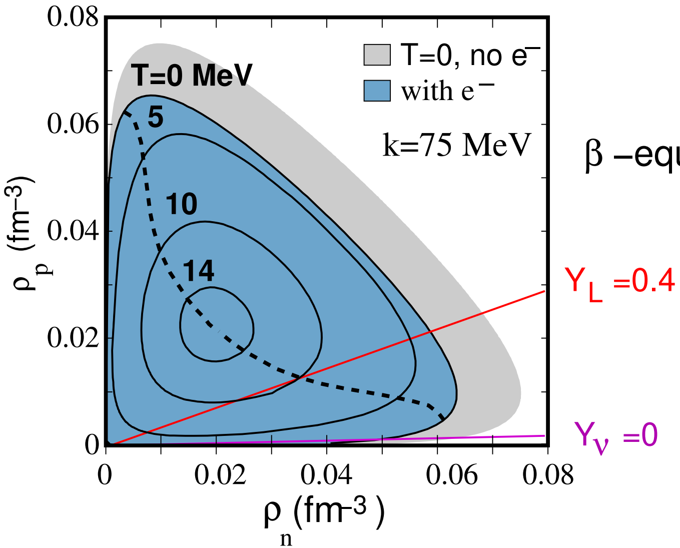
Nuclear Matter: Liquid-Gas Phase Transition



Dynamical Spinodal

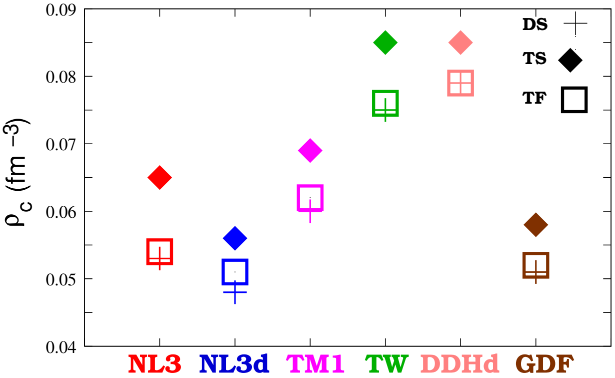
Dynamical versus thermodynamical Spinodal



β –equi

Transition density

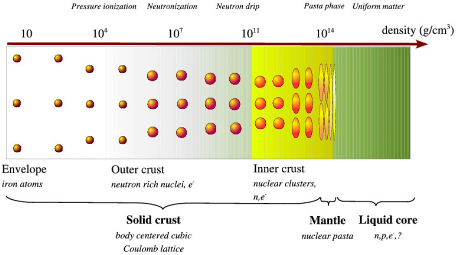
Pasta (TF) versus dynamical and thermodynamical spinodals, T=0



(Avancini *et al* PRC82(2010)

DS - Dynamical spinodal; TS - Thermodynamical spinodal; TF - Thomas Fermi

Crust



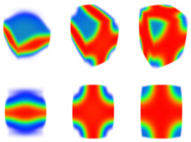
- Cold catalyzed matter
  (Chamel and Haensel, Living Reviews 2008)
  - Surface on NS: $p = 0$; ^{56}Fe
  - a bit deeper: nuclei embedded in a electron sea
  - $\rho > \rho_{drip}$: nucleons form clusters not necessarily spherical that take up lattice positions in a background gas of neutrons and electrons
- Above cristalization temperature: the crust melts and light clusters contribute to the equilibrium
- Light clusters play a role: neutron star cooling, accreting systems, and binary mergers.

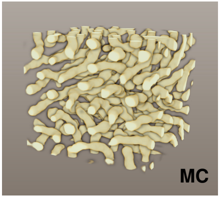
Inner crust and pasta phases

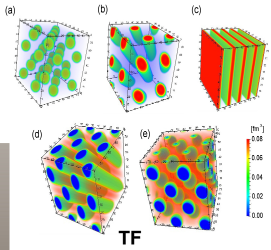
- In the inner crust the attractive nuclear and repulsive atomic length scales are comparable
  - leads to a complex ground state
  - gives rise to non-spherical shapes (rod, slabs, tubes, bubbles...)
  - should have unusual dynamical and transport properties
- First discussed: liqui drop model
  - Ravenhall, Pethick, Wilson, PRL27, 1983
  - Hashimoto, Seki, Yamada, PTP71 (1984)
- Developements
  - Liquid drop models
  - Thomas Fermi approximation: Skyrme and RMF models
  - Hartree-Fock (+BCS) approximation with the Skyrme forces
  - Classical molecular dynamics (CMD)
  - Quantum molecular dynamics (QMD)

Pasta phase

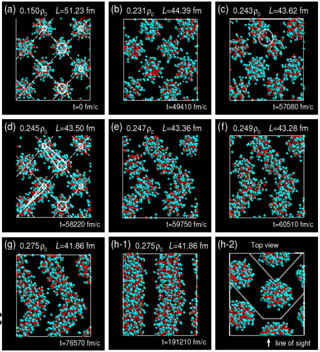
Different methods







QMC

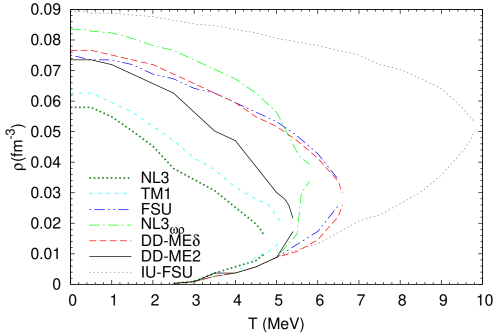


3D-HF (Newton, Stone, Pais),
CMD (Horowitz et. al),
QMD (Watanabe et al)

TF (Okomada et al)

Effect of T on pasta extension

β-equilibrium matter, no light clusters



- The critical temperature T_{crit} is model dependent

Light clusters in neutron stars

- Light clusters influence supernova matter properties (Arcones et al PRC78 (2008), Heckel et al PRC80 2009, Lalit et al EPJA 55:10 2019)
- Supernova EoS with clusters:
  - single nucleus approximation (Lattimer & Swesty(CLDM) , Shen et al (RMF)) Wigner Seitz approximation, a single nucleus in equilibrium with a gas of nucleons, electrons, α-particles (within excluded volume approximation)
  - nuclear statistical equilibrium: (Hempel & Schaffner-Bielich, Raduta& Gulminelli) all possible nuclear species in statisitical equilibrium, excluded volume describes interaction of clusters with background gas

Light clusters in neutron stars

- Supernova EoS with clusters:
  - quantum statistical approach (QS): quantum correlations with the medium, takes into account the excited states and temperature effect. But, the mass shifts available only for a few nuclear species and a limited density domain. Can be implemented only with approximations (Typel et al 2010)
  - RMF approach with light clusters
    - considered as new degrees of freedom.
    - characterized by a density, and possibly temperature, dependent effective mass,
    - interact with the medium via meson couplings.
    - In-medium effects are incorporated via the meson couplings, the effective mass shift, or both.
  - Constraints are needed to fix the couplings!

Constraining light clusters

- Cluster formation has been measured in heavy ion collisions (Qin et al PRL 108 (2012), Hagel et al PRL108,062702): equilibrium constants, Mott points and medium cluster binding energies
- Cluster formation in Supernova EOS constrained with equilibrium constants from HIC was studied in (Hempel et al PRC91, 045805 (2015))
  - the SN EoS should incorporate: mean-field interactions of nucleons, inclusion of all relevant light clusters, and a suppression mechanism of clusters at high densities

EOS

RMF Lagrangian for stellar matter

- Lagrangian density: $\mathcal{L} = \mathcal{L}_N + \mathcal{L}_c + \mathcal{L}_m + L_e$
  - nucleons, tritium, helion

$$\mathcal{L}_j = \bar{\psi}_j \left[\gamma_\mu i D_j^\mu - M_j^*\right] \psi_j \qquad i = p,\ n,\ t,\ 3he$$

  - alphas, deuterons

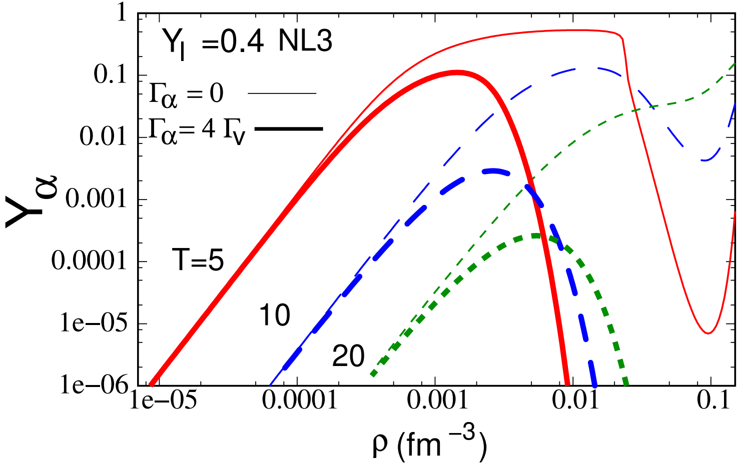
$$\begin{aligned}
\mathcal{L}_\alpha &= \frac{1}{2}(iD_\alpha^\mu \phi_\alpha)^*(iD_{\mu\alpha}\phi_\alpha) - \frac{1}{2}\phi_\alpha^* M_\alpha^2 \phi_\alpha, \\
\mathcal{L}_d &= \frac{1}{4}(iD_d^\mu \phi_d^\nu - iD_d^\nu \phi_d^\mu)^*(iD_{d\mu}\phi_{d\nu} - iD_{d\nu}\phi_{d\mu}) - \frac{1}{2}\phi_d^{\mu *} M_d^2 \phi_{d\mu},
\end{aligned}$$

$$\begin{aligned}
iD_j^\mu &= i\partial^\mu - g_{vj}\omega^\mu - \frac{g_{\rho j}}{2}\boldsymbol{\tau}\cdot\mathbf{b}^\mu, \\
M_j^* &= m_* = m - g_s\phi_0, \quad j = p, n \\
M_j^* &= M_j - g_{sj}\phi_0 - \mathbf{B}_j, \quad j = t, h, d, \alpha
\end{aligned}$$

- couplings: constrained by HIC data ou first principle calculations

Cluster-Vector meson coupling g_{vj}

α particles in homogeneous matter, β-equilibrium matter with trapped neutrinos



- $g_{vj} = A_j g_v$
- ω-meson: partially the classical Excluded Volume mechanism

Mass shift in clusters - g_{sj}

- Binding energy for each cluster: $B_j = A_j m^* - M_j^*$
- $m^* = m - g_s\phi_0$, nucleon effective mass
- $M_j^* = A_j m - g_{sj}\phi_0 - (B_j^0 + \delta B_j)$, cluster effective mass
- $g_{sj} = x_{sj} A_j g_s$, the cluster- scalar meson coupling
  - needs to be determined from experiments

Mass shift in clusters - δB_j

- Binding energy for each cluster $B_j = A_j m^* - M_j^*$
- $m^* = m - g_s\phi_0$, nucleon effective mass
- $M_j^* = A_j m - g_{sj}\phi_0 - (B_j^0 + \delta B_j)$, cluster effective mass
- $-\delta B_j = \frac{Z_j}{\rho_0}\left(m\rho_p^* - \epsilon_p^*\right) + \frac{N_j}{\rho_0}\left(m\rho_n^* - \epsilon_n^*\right)$

  ϵ_j^* and ρ_j^* associated with the gas lowest energy levels

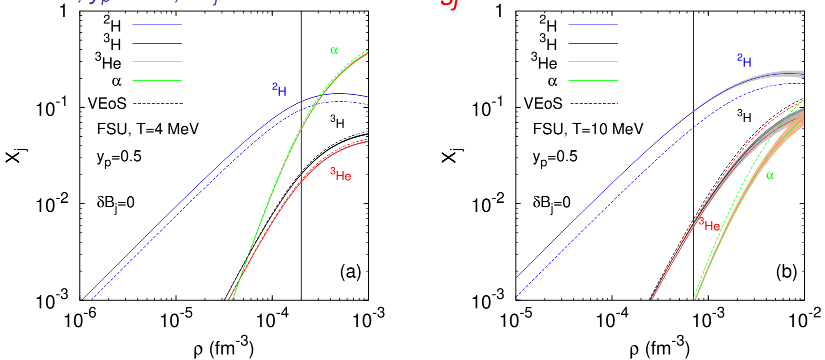
$$
\begin{aligned}
\epsilon_j^* &= \frac{1}{\pi^2}\int_0^{p_{F_j}(\text{gas})} p^2 e_j(p)(f_{j+}(p) + f_{j-}(p))dp \\
\rho_j^* &= \frac{1}{\pi^2}\int_0^{p_{F_j}(\text{gas})} p^2 (f_{j+}(p) + f_{j-}(p))dp\,,
\end{aligned}
$$

  - the energy states occupied by the gas are excluded: double counting avoided!
  - δB_j contributes to the ω and ρ-meson equations of motion

    ρ_n and $_p$ are written in terms of V_0 and b_0

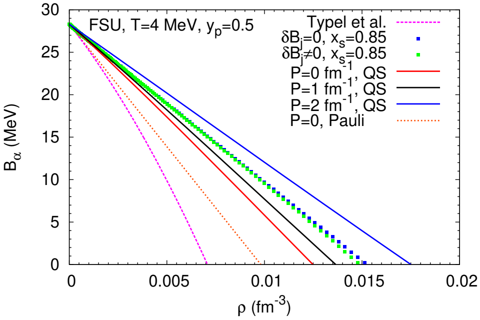
Determination of x_s: Virial EoS

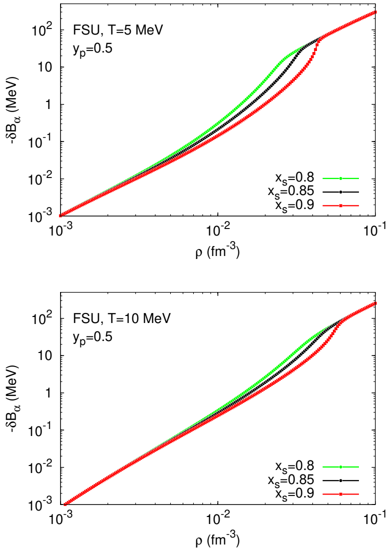
T = 4, 10 MeV, y_p = 0.5, δB_j = 0 - $x_{sj} = 0.85 \pm 0.05$



- VEoS: model independent constraint, only depends on experimental binding energies and scattering phase shifts
- Provides correct zero-density limit for finite T EoS
  - $\rho_j \lambda_j^3 << 1$, λ_j thermal wavelength of particle (black line $\rho_n \lambda^3 = 0.01$)
- Breaks down when the interaction with particles becomes stronger $\rightarrow \delta B_j$ takes action

Contribution of δB_j

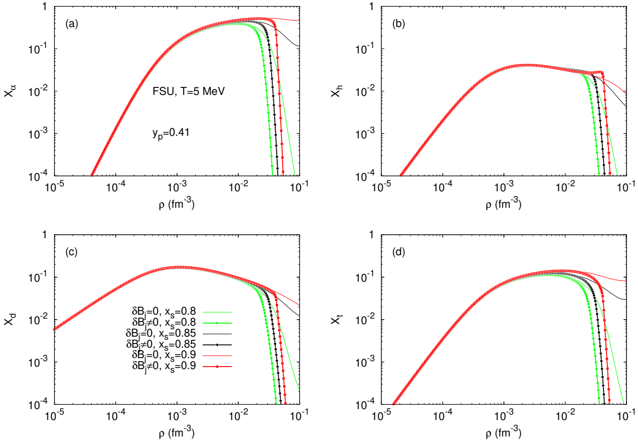




- δB_j completely negligible in the VEoS range of densities
- but rises fast for larger densities

Cluster fractions: effect of δB_j

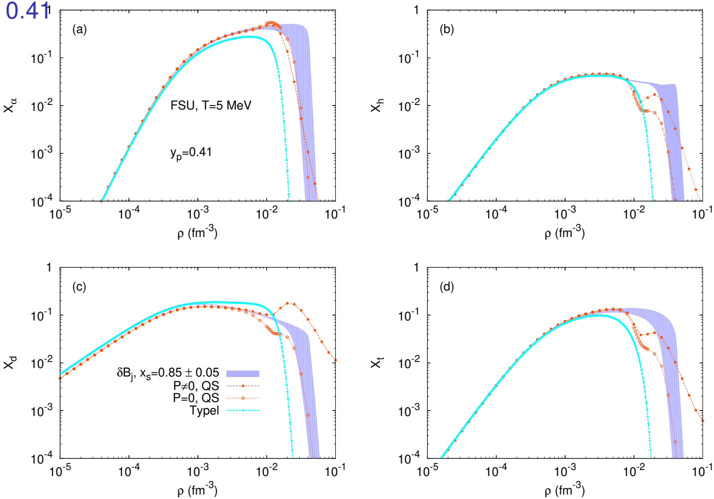
FSU, T = 5 MeV, y_p = 0.41 with $g_{vj} = A_j g_v$



- δB_j is important for dissolution of clusters!
- The higher x_s the higher the dissolution density: $0.04 < \rho_{dis} < 0.06$ fm^{-3}

Cluster fractions: comparing approaches

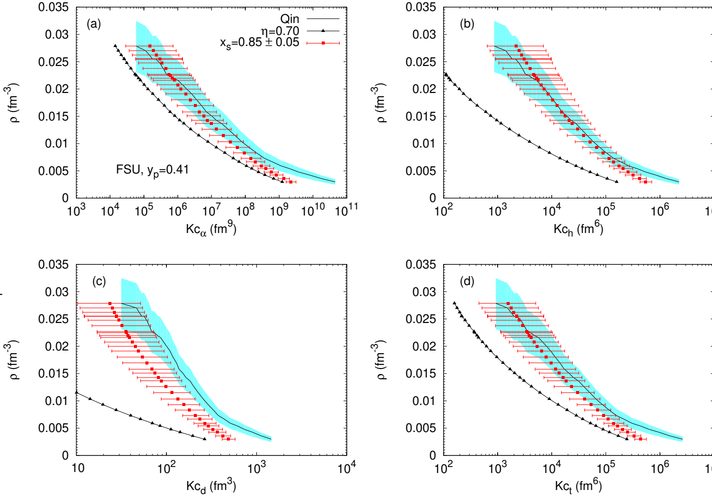
FSU, T = 5 MeV, y_p = 0.41



- blue band: $0.8 \leq g_{sj} \leq 0.9$
- Typel et al: empirical quadratic term in ρ: too much suppresion at high ρ
- QS: Pauli blocking strongly dependent on CM motion, correlations shift Mott density to larger densities

Equilibrium constants

Qin et al, PRL 108, 172701 2012



- Kc from HIC:

$$K_c[j] = \frac{\rho_j}{\rho_n^{N_j}\,\rho_p^{Z_j}}$$

- Unique existing constraint on in-medium modifications of light clusters at finite T
- with $g_{sj} = (0.85 \pm 0.05)\, A_j\, g_s$

EoS for HM with light clusters

- The total baryonic density:

$$\rho = \rho_p + \rho_n + \sum_{clusterj} A_j \rho_j$$

- Global proton fraction

$$Y_p = y_p + \sum_j \frac{Z_i}{A_i} y_i$$

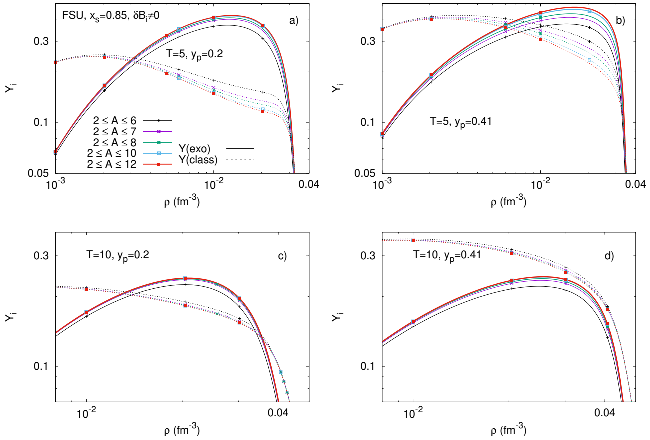
with $y_i = A_i \rho_i / \rho$ the mass fraction of cluster i

- Charge neutrality: $\rho_e = Y_p \rho$
- Light clusters are in chemical ewuilibrium

$$\mu_i = N_i \mu_n + Z_i \mu_p$$

Light clusters: including “exotic” clusters, $A > 4$

Pais Phys Rev C 99, 055806

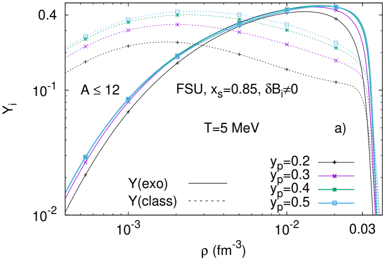


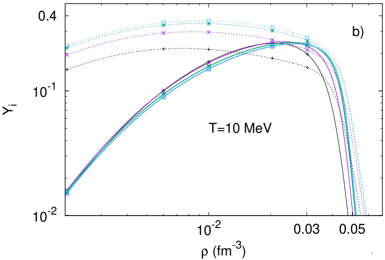
- classic clusters: d, t, h, α
- $Y_{light} = \sum_{i=2}^{A_{max}} Y_i \qquad Y_{exo} = Y_{light} - Y_{class}$
- The largest contribution of the “exotic" clusters occurs for intermediate densities, when the total distribution of clusters has a peak

Exotic clusters and proton fraction

Pais Phys Rev C 99, 055806

- “exotic" clusters do not play a role at small densities and close to the transition to homogeneous matter
- but at the maximum: “exotic" clusters are more abundant if T not too high (differences disappear at $T > 10$
- at the maximum distribution, the difference between “exotic" and “classical" clusters increases as the proton fraction decreases





Exotic clusters: decays and effective densities

Pais Phys Rev C 99, 055806

Decay modes:

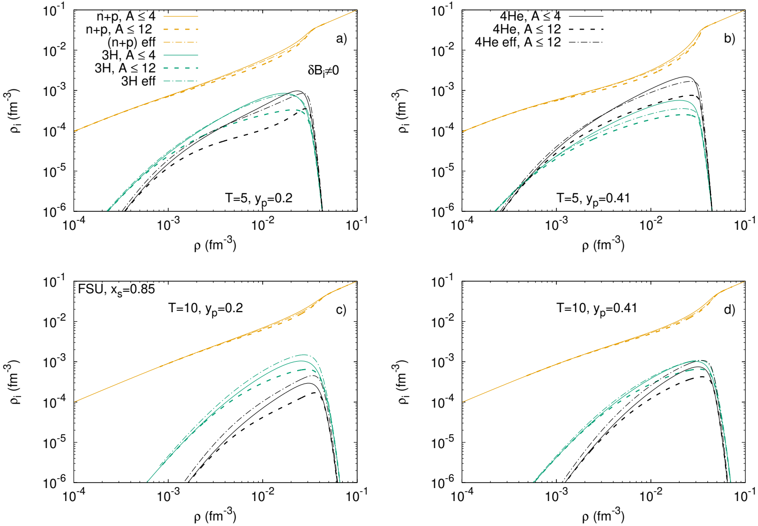
$$
\begin{aligned}
{}^5\mathrm{He} &\longrightarrow {}^4\mathrm{He} + \mathrm{n} \\
{}^4\mathrm{H} &\longrightarrow {}^3\mathrm{H} + \mathrm{n} \\
{}^7\mathrm{He} &\longrightarrow {}^6\mathrm{Li} + \mathrm{n} \\
{}^6\mathrm{H} &\longrightarrow {}^3\mathrm{H} + 3\mathrm{n} \\
{}^5\mathrm{H} &\longrightarrow {}^3\mathrm{H} + 2\mathrm{n} \\
{}^5\mathrm{Li} &\longrightarrow {}^4\mathrm{He} + \mathrm{p} \\
{}^8\mathrm{Be} &\longrightarrow 2({}^4\mathrm{He}) \\
{}^7\mathrm{Be} &\longrightarrow {}^7\mathrm{Li} \\
{}^9\mathrm{He} &\longrightarrow 2({}^4\mathrm{He}) + \mathrm{n} \\
{}^7\mathrm{H} &\longrightarrow {}^3\mathrm{H} + 4\mathrm{n}
\end{aligned}
$$

Effective densities, $\widetilde{\rho}_i$:

$$
\begin{aligned}
\widetilde{\rho}_{^4\mathrm{He}} &= \rho_{^4\mathrm{He}} + \rho_{^5\mathrm{He}} + \rho_{^5\mathrm{Li}} + 2\rho_{^8\mathrm{Be}} + 2\rho_{^9\mathrm{He}} \\
\widetilde{\rho}_{^3\mathrm{H}} &= \rho_{^3\mathrm{H}} + \rho_{^4\mathrm{H}} + \rho_{^5\mathrm{H}} + \rho_{^6\mathrm{H}} + \rho_{^7\mathrm{H}} \\
\widetilde{\rho}_{^6\mathrm{Li}} &= \rho_{^6\mathrm{Li}} + \rho_{^7\mathrm{He}} \\
\widetilde{\rho}_{^7\mathrm{Li}} &= \rho_{^7\mathrm{Li}} + \rho_{^7\mathrm{Be}} \\
\widetilde{\rho}_{\mathrm{n}} &= \rho_{\mathrm{n}} + \rho_{^5\mathrm{He}} + \rho_{^4\mathrm{H}} + \rho_{^7\mathrm{He}} + 3\rho_{^6\mathrm{H}} \\
&\quad +2\rho_{^5\mathrm{H}} + \rho_{^9\mathrm{He}} + 4\rho_{^7\mathrm{H}} \\
\widetilde{\rho}_{\mathrm{p}} &= \rho_{\mathrm{p}} + \rho_{^5\mathrm{Li}}
\end{aligned}
$$

Light clusters: accounting for exotic cluster decays

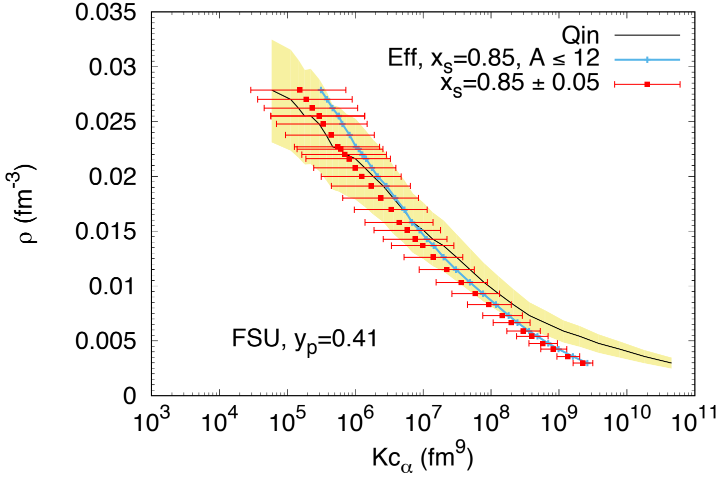
Pais et al PRC99, 055806



- "exotic" clusters: non-negligible role at intermediate densities
- "classical" light clusters only takes into account, in a reasonable way, the distribution of light clusters with $A \leq 12$

Exotic cluster decays and equilibrium constants K_c

How important are clusters with $A \leq 12$?



Equilibrium constant: $K_c[i] = \rho_i / (\rho_p^{Z_i} \rho_n^{N_i})$

Light clusters: average isospin, charge, mass number

Role of isospin, charge and mass

Define

- the average isospin of the light clusters, $< I >$

$$< I >= \frac{\sum_i I_i \rho_i}{\sum_i \rho_i}$$

- the average charge $< Z >$

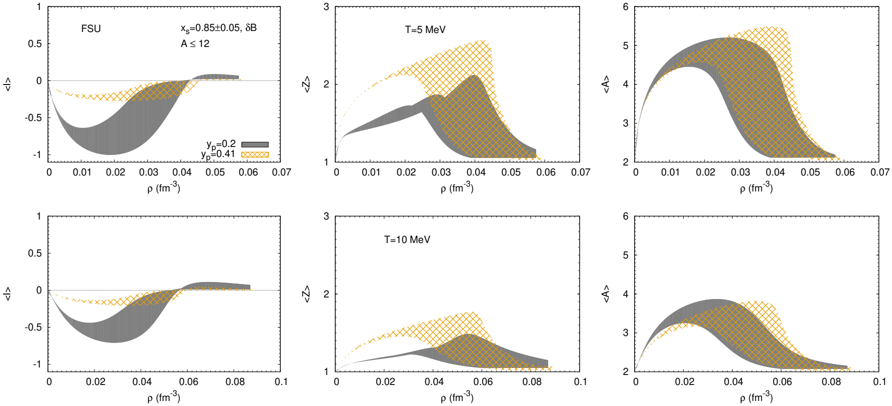
$$< Z >= \frac{\sum_i Z_i \rho_i}{\sum_i \rho_i}$$

- the average number of nucleons in the light clusters $< A >$

$$< A >= \frac{\sum_i A_i \rho_i}{\sum_i \rho_i}$$

Light clusters: average isospin, charge, mass number

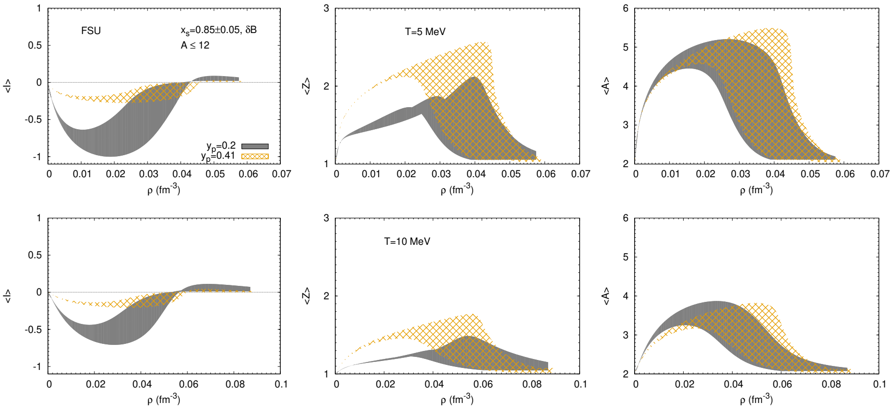
How important are clusters with $A \leq 12$?



- Important to include in the calculation for very asymmetric matter exotic neutron rich clusters.
- Even for $T = 10$ MeV, the maximum of the average isospin is above $-\frac{1}{2}$ for $y_p = 0.2$.

Light clusters: average isospin, charge, mass number

How important are clusters with $A \leq 12$?



- The presence of clusters with a large charge, i.e $Z > 2$, is more important in symmetric matter and low temperature.
- $\langle A \rangle$: sentitive to the temperature:
  - the larger T, the smaller the contribution from the most massive clusters.

Pasta phase EOS

- β-equilibrium non-homgeneous matter within a TF calculation
- assumed a preferred single geometry (least free energy) for a given T, ρ and y_p
- only five possible shapes are considered: droplets, rods, slabs, tubes and bubbles
- β-equilibrium, $T = 0$: y_P is very small and only three shapes are energetically favorable: droplets, rods and slabs.
- a regular lattice in the Wigner-Seitz approximation is considered, the WS cell having the shape of the clusters
- a fixed Z and N number at a given density determines the WS volume, β-equilibrium condition determines N
- electrons: charge neutrality is imposed

Pasta phase: Thomas Fermi

Avancini et al PRC78,015802, PRC95,045804

- Meson equations

$$\nabla^2\phi = m_s^2\phi + \frac{1}{2}\kappa\phi^2 + \frac{1}{3!}\lambda\phi^3 - g_s\rho_s,$$

$$\nabla^2 V_0 = m_v^2 V_0 + \frac{1}{3!}\xi g_v^4 V_0^3 - g_v\rho_B,$$

$$\nabla^2 b_0 = m_\rho^2 b_0 - \frac{g_\rho}{2}\rho_3,$$

$$\nabla^2 A_0 = -e(\rho_p - \rho_e)$$

- Minimization of thermodynamic potential under the constraint of a fixed number of protons, neutrons, and electrons

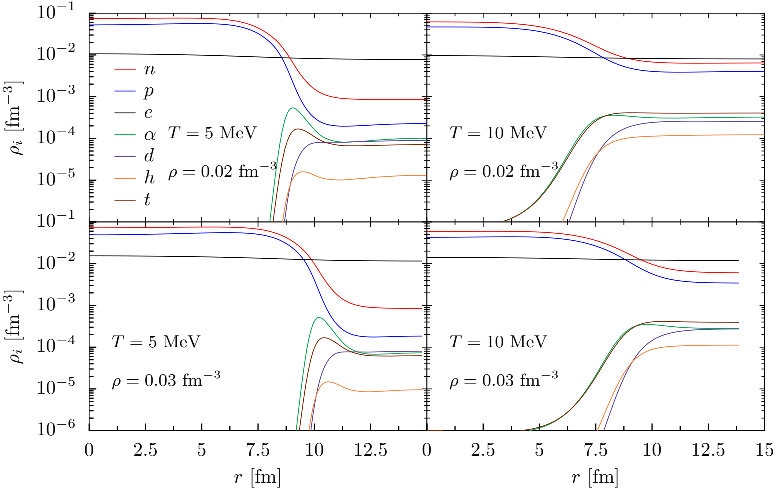
$$(p_{F_p}^2(\mathbf{r}) + M^{\star 2}(\mathbf{r}))^{1/2} + g_v V_0(\mathbf{r}) + \frac{1}{2}g_\rho b_0(\mathbf{r}) + eA_0(\mathbf{r}) = \mu_p,$$

$$(p_{F_n}^2(\mathbf{r}) + M^{\star 2}(\mathbf{r}))^{1/2} + g_v V_0(\mathbf{r}) - \frac{1}{2}g_\rho b_0(\mathbf{r}) = \mu_n,$$

$$(p_{F_e}^2(\mathbf{r}) + m_e^2)^{1/2} - eA_0(\mathbf{r}) = \mu_e.$$

Thomas-Fermi Density profiles

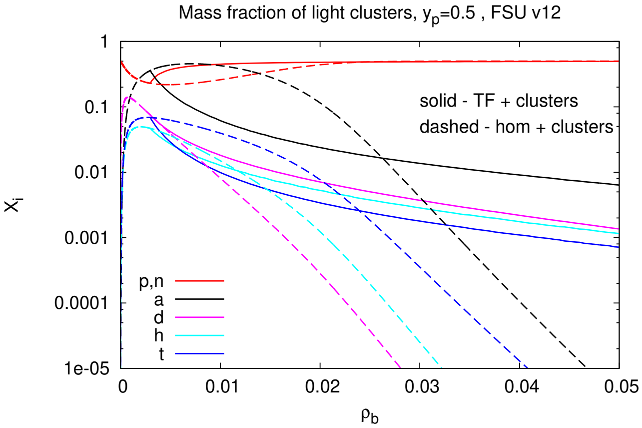
FSU, T = 5 MeV and Y_p = 0.5.



- light clusters: concentration at the surface of the pasta clusters, and presence in the neutron drip gas

Mass fraction of light clusters

FSU, $T = 5$ MeV and $Y_p = 0.5$



- light clusters with pasta:
  - the presence of pasta clusters reduce the contribution of light clusters at for densities 0.01-0.03 fm^{-3}
  - for intermediate densities ($\rho \gtrsim 0.03$ fm^{-3}: larger fractions of light clusters in pasta matter, smoother reduction

Pasta phases - calculation

Pais et al PRC 91, 055801 2015

- Compressible Liquid Drop (CLD) approximation:
  - The total free energy density is minimized, including the surface and Coulomb terms.
  - Separated regions of higher (pasta phases, f volume fraction) and lower density (background nucleon gas).
  - Minimization with respect to r_d, ρ_B^I, y_p^I, f
  - The Gibbs equilibrium conditions for $T = T^I = T^{II}$

$$\begin{aligned}
\mu_n^I &= \mu_n^{II}, \\
\mu_p^I &= \mu_p^{II} - \frac{\varepsilon_{surf}}{f(1-f)(\rho_p^I - \rho_p^{II})}, \\
P^I &= P^{II} - \varepsilon_{surf}\left(\frac{1}{2\alpha} + \frac{1}{2\Phi}\frac{\partial \Phi}{\partial f} - \frac{\rho_p^{II}}{f(1-f)(\rho_p^I - \rho_p^{II})}\right).
\end{aligned}$$

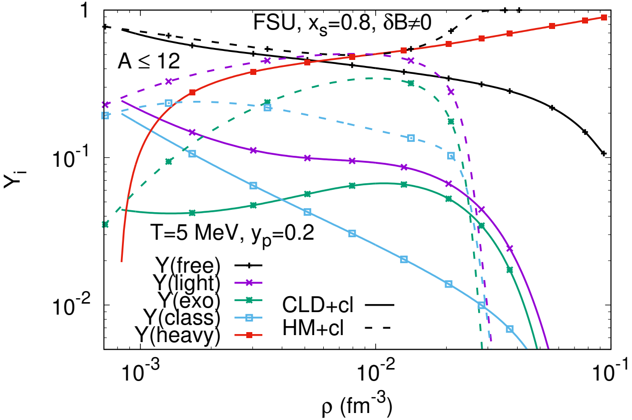
- Total free energy density $\mathcal{F}$ and total ρ_p of the system:

$$\begin{aligned}
\mathcal{F} &= f\,\mathcal{F}^I + (1-f)\,\mathcal{F}^{II} + F_e + \varepsilon_{surf} + \varepsilon_{Coul}, \\
\rho_p &= \rho_e = y_p\rho = \rho_p^I + (1-f)\rho_p^{II},
\end{aligned}$$

$$\varepsilon_{surf} = 2\varepsilon_{Coul}$$

Cluster fractions: including pasta phases

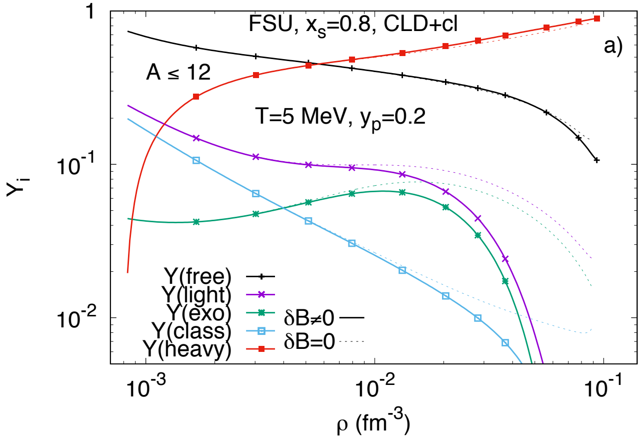
T = 5 MeV, y_p = 0.2



- The heavy cluster (CLD+cl calculation): light clusters less abundant but increases their melting density
- Increasing T $\rightarrow$ onset of both heavy and light clusters moves to larger densities.

Cluster fractions with pastas: effect of δB_j

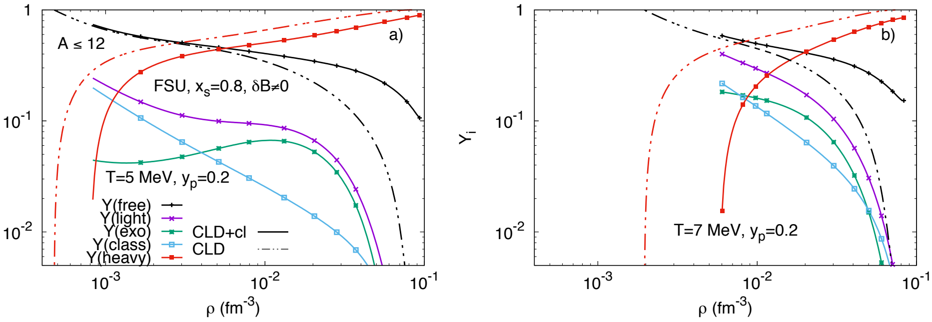
T = 5 MeV, y_p = 0.2



- δB_j has no effect at the onset of the heavy cluster, only close to the melting density of the light clusters.
- light clusters abundances are reduced and dissolve at lower densities.

Pasta versus Cluster fractions with pastas

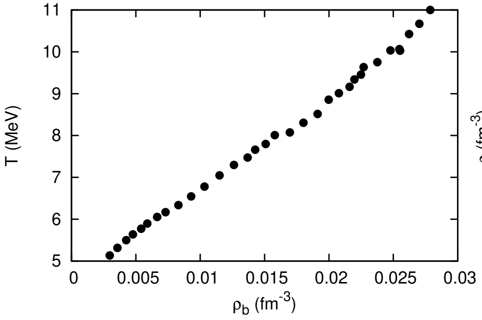
T = 5 and 10 MeV, y_p = 0.2

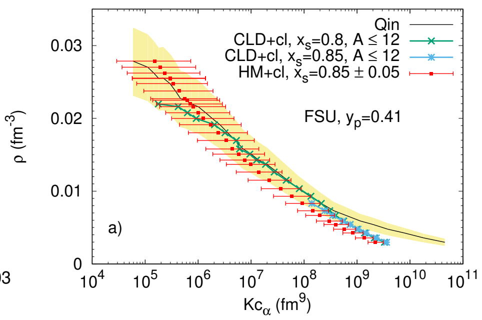


- inclusion of light clusters
  - moves the onset of the heavy cluster to larger densities
  - reduces the size of the heavy cluster
  - reduces the mass fraction of nucleons in the heavy clusters
  - increases the fraction of free nucleons in the background
  - if too restrictive concerning competing degrees of freedom, overestimation of the role of the heavy cluster

Equilibrium constants K_c: light and heavy clusters

How important are clusters with $A \leq 12$?





Equilibrium constant: $K_c[i] = \rho_i / (\rho_p^{Z_i} \rho_n^{N_i})$

Conclusions

- A self-consistent method to include in-medium effects acting on light clusters is proposed in a RMF framework and inclusion of a density dependent mass shift.
- Interactions of clusters with medium described by modification of sigma-meson coupling constant.
- Clusters dissolution obtained by the density-dependent extra term on the binding energy.
- reproduces both virial limit and K_c from HIC.
- Including the light clusters with $4 < A < 12$ has visible effects on the particle abundances and equilibrium constants
- Heavy cluster reduces the contribution of light clusters, but also shifts the melting density to higher densities.
- Extra constraints from experimental data are needed

Thank you !