

Extracting information on nuclear EoS from low-energy reaction dynamics: a detailed study of low-lying dipole modes

Challenges to Transport Theory for Heavy-Ion Collisions



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Introduction: low-energy reactions and nuclear EoS

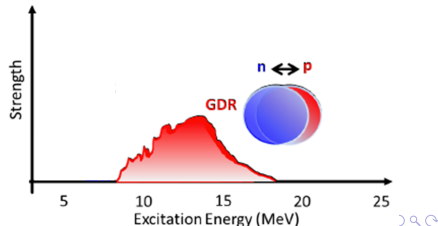
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- Dipole excitations in nuclei:
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 - Pygmy Dipole Resonance (PDR)
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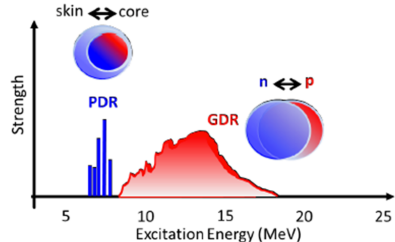
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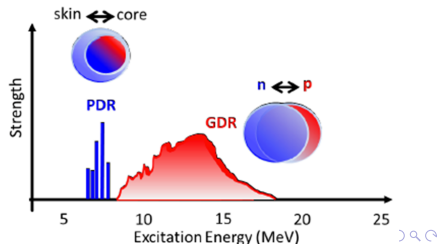
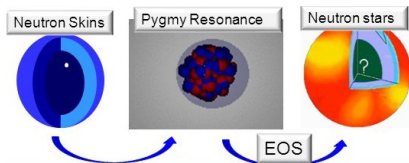
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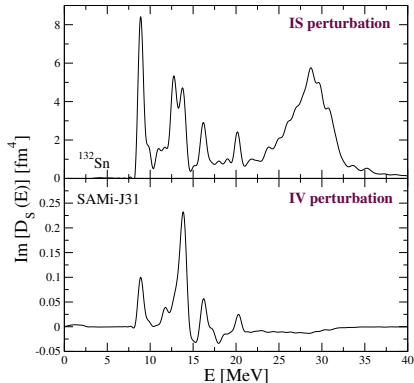
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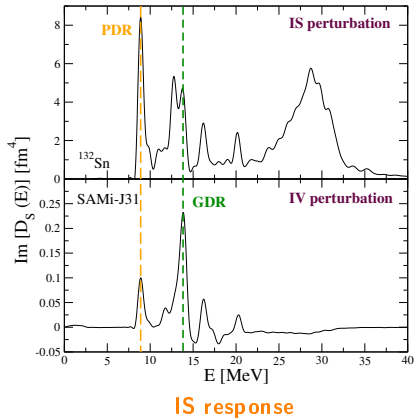
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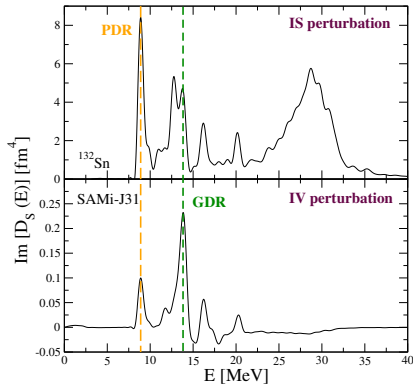
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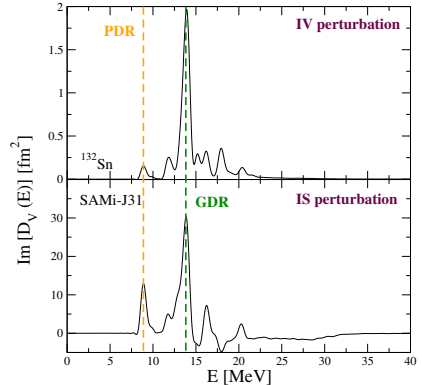


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IS response



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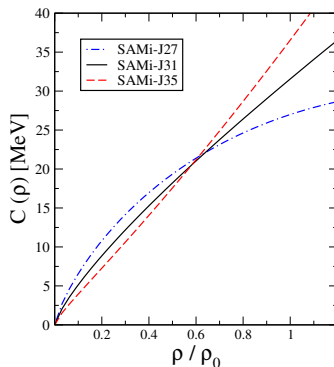
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[X. Roca-Maza et al., PRC87, (2013)]

⇒ isolate influence of **IV channel**

$$E_{\text{sym}}(\rho) = C(\rho)I^2$$



- Sensitivity of $E_{\text{IV-GDR}}$ to E_{sym} at crossing
- Role of symmetry energy slope:
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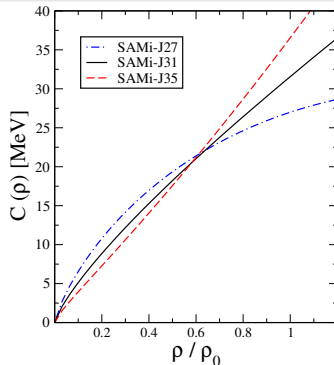
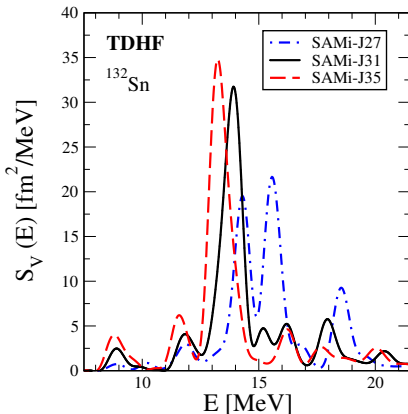
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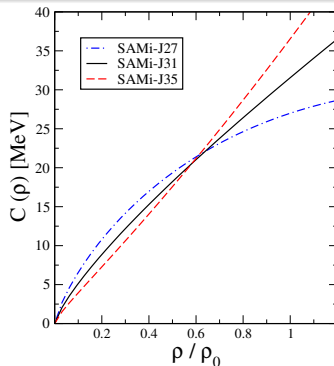
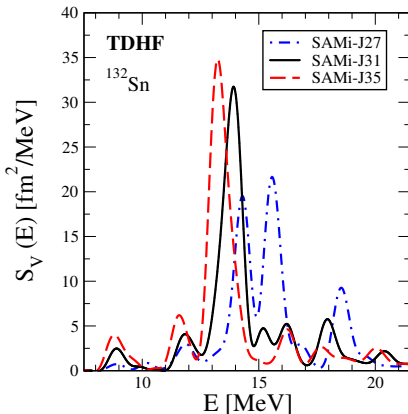
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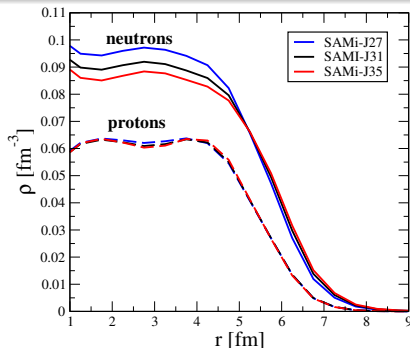
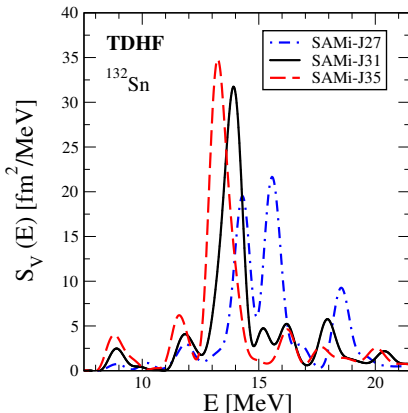
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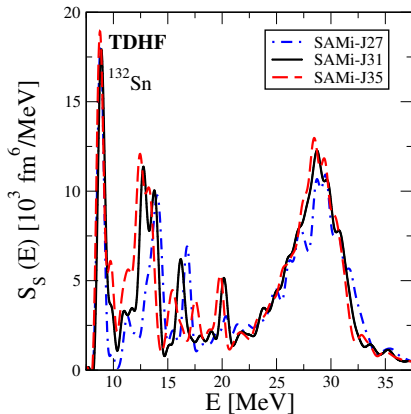
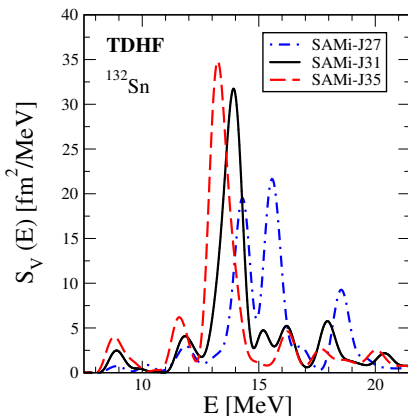
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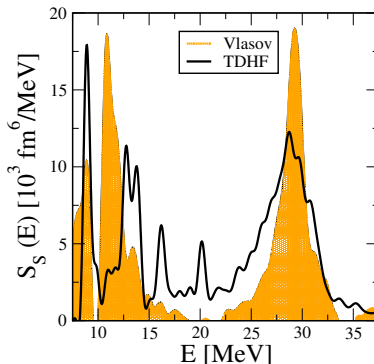
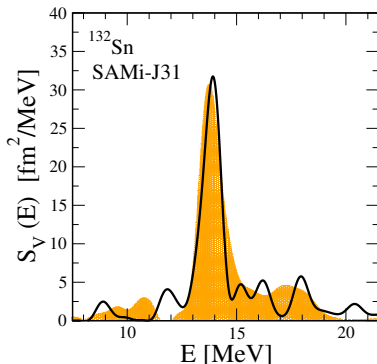
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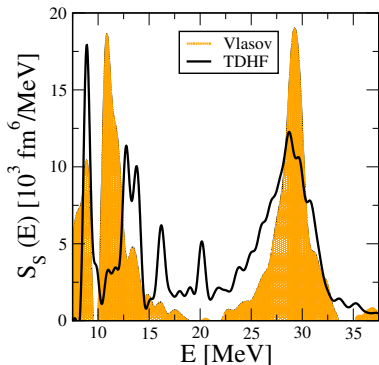
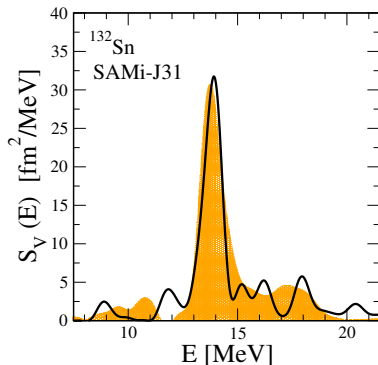


Comparison between Vlasov and TDHF model



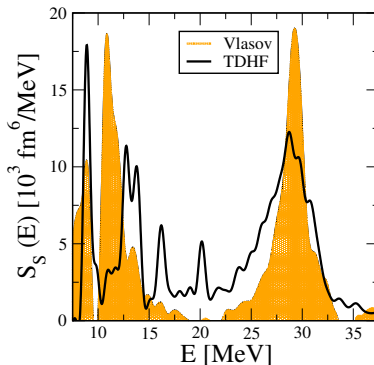
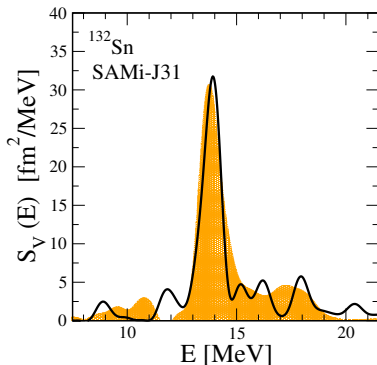
- Good **reproduction** of **IV GDR** and **IS GDR**
- Two contributions in **low-energy** region: [see M. Urban, PRC85, (2012)]
 - PDR mode (outer surface)
 - PDR mode (inner surface against bulk)
- Displacement of PDR peaks \Rightarrow numerical treatment of **surface**

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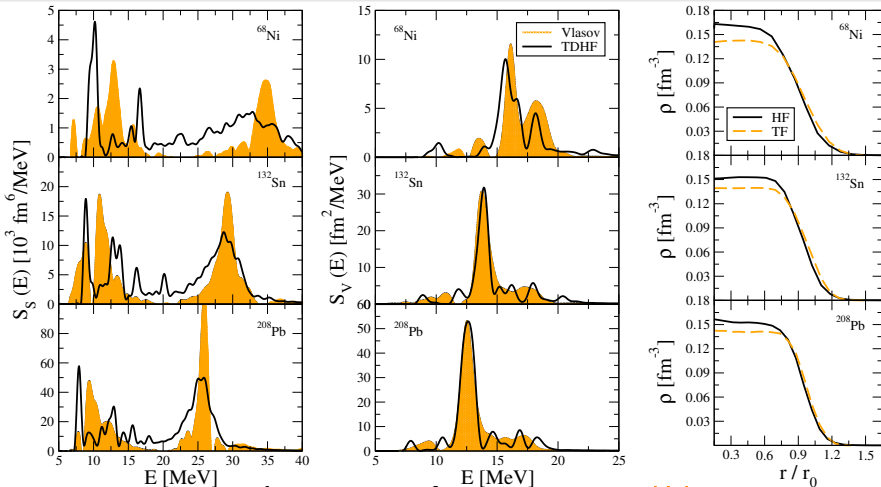
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Link between nuclear response and density profiles



- **Sharper evolution** from **bulk** to **surface** region favor **toroidal** mode

Smoother density profile leads to **robust PDR** oscillations

[S. Burrello et al., Phys. Rev. C 99, 054314 (2019)]

Sn isotope chain: N/Z evolution of PDR

- Dipole response **evolution** with the **neutron/proton content** \Rightarrow Sn **isotopes chain**
- **Question:** Why IV PDR **fraction of EWSR** does not grow from N=70 to N=82?
[S. Ebata, T. Nakatsukasa, T. Inakura, Phys. Rev. C 90, 024303 (2014)]
- **Explanation:** it reflects the **decrease** in the **IS fraction** and IS dipole strength
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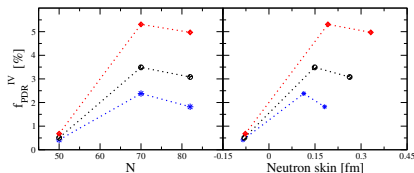
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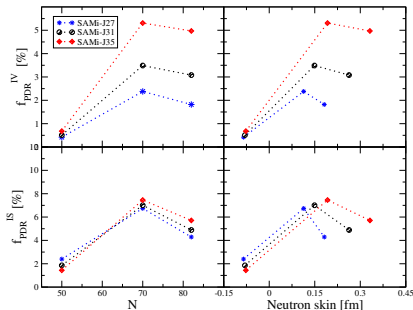
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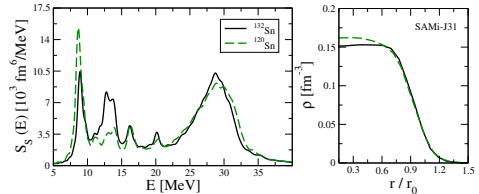
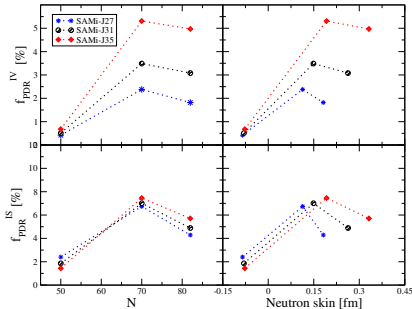


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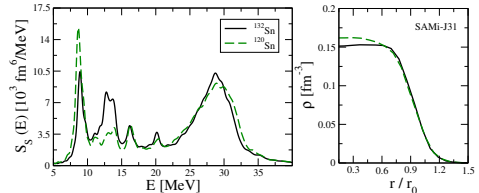
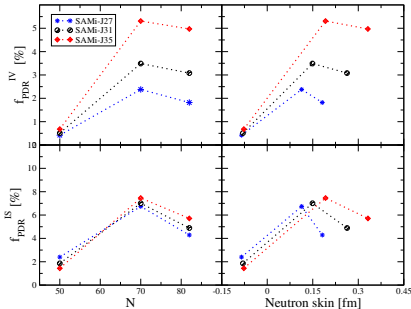
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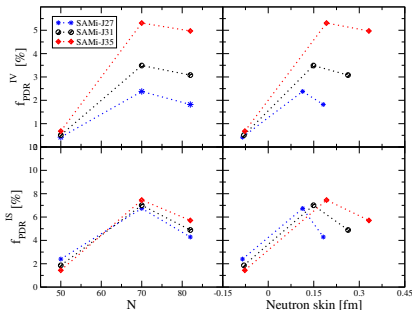
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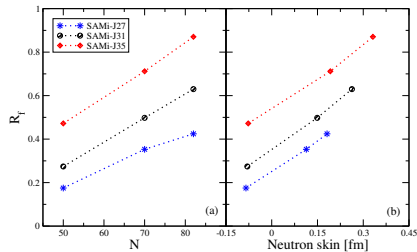
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- TDHF and RPA **equivalent** in zero-amplitude limit, despite **technical procedures**
- Question: which **numerical parameters** ensure the best agreement?
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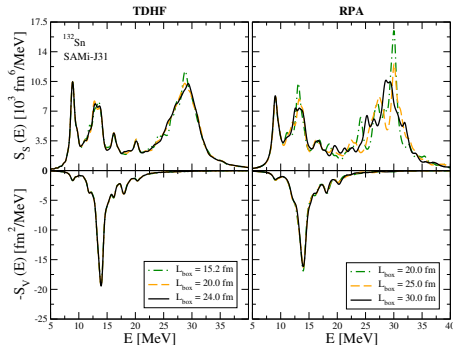
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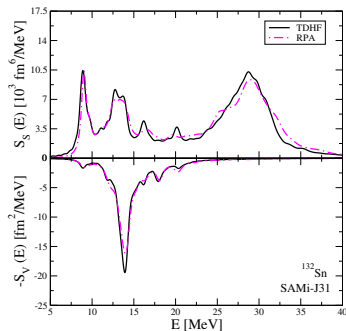
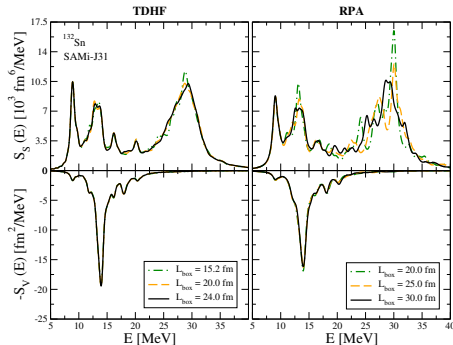


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Final remarks and conclusions

Summary

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- General **agreement** between **Vlasov**, **TDHF** and **RPA** calculations
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Further developments and outlooks

- Full study of **isotopes chain** to understand also **deformation** and **pairing** role
- Look at other **multipole** response channels (**quadrupole** resonances)
- Investigate **other mechanisms** to constraint the **effective interaction** and **EoS**

Thanks to all collaborators

TDHF model

D. Lacroix (IPN, IN2P3-CNRS, Orsay, France),

G. Scamps (Université libre de Bruxelles (ULB), Bruxelles, Belgium)

RPA calculations

G. Coló, X. Roca-Maza (University and INFN Sezione, Milano, Italy)

Semi-classical model

M. Colonna (LNS - INFN, Catania, Italy)

H. Zheng (Shaanxi Normal University, Xi'an, China)

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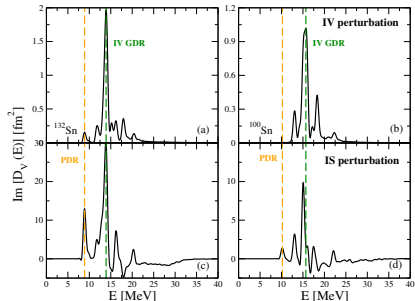
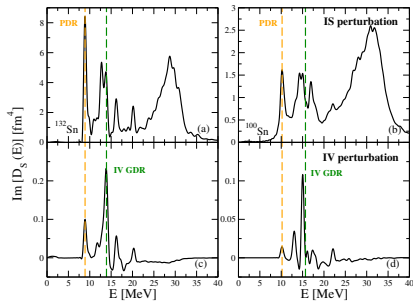
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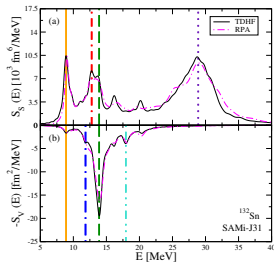
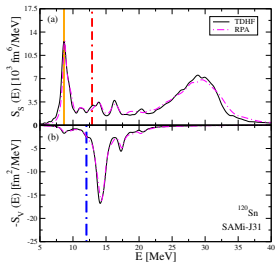
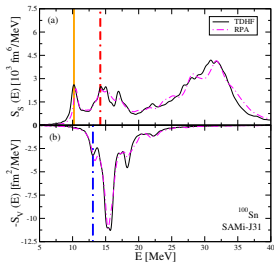
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THANK YOU FOR YOUR KIND ATTENTION!

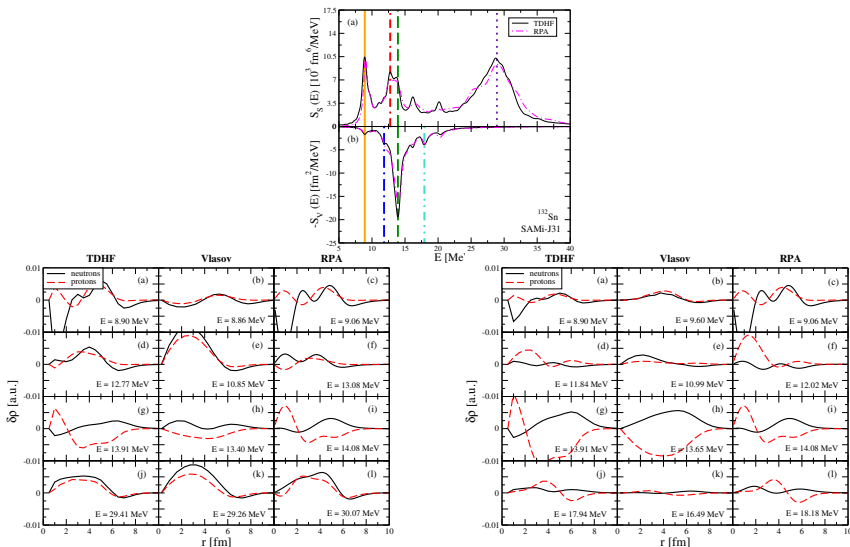
Back-up slides: focus on IS/IV mixing



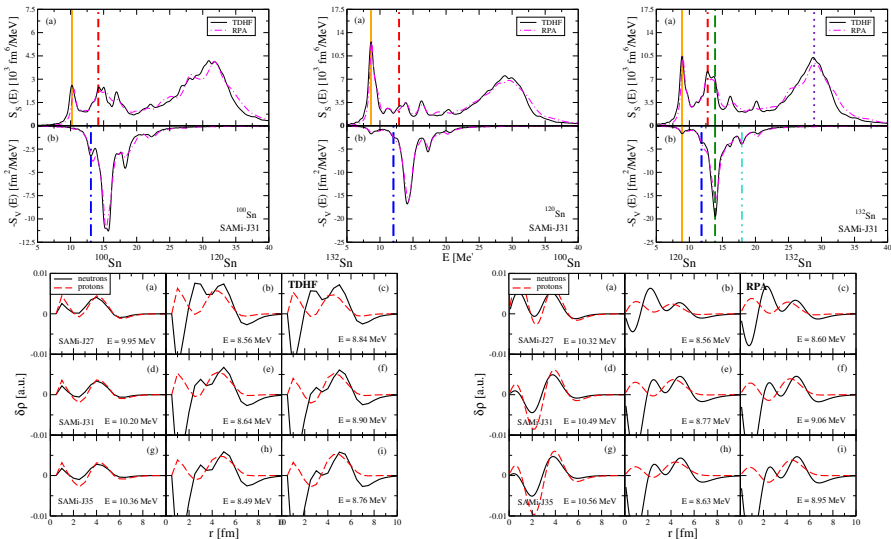
Back-up slides: dipole strength in Sn isotopes



Back-up slides: transition densities comparison



Back-up slides: transition densities of PDR



Back-up slides: torodail mode and 2nd IV peak

