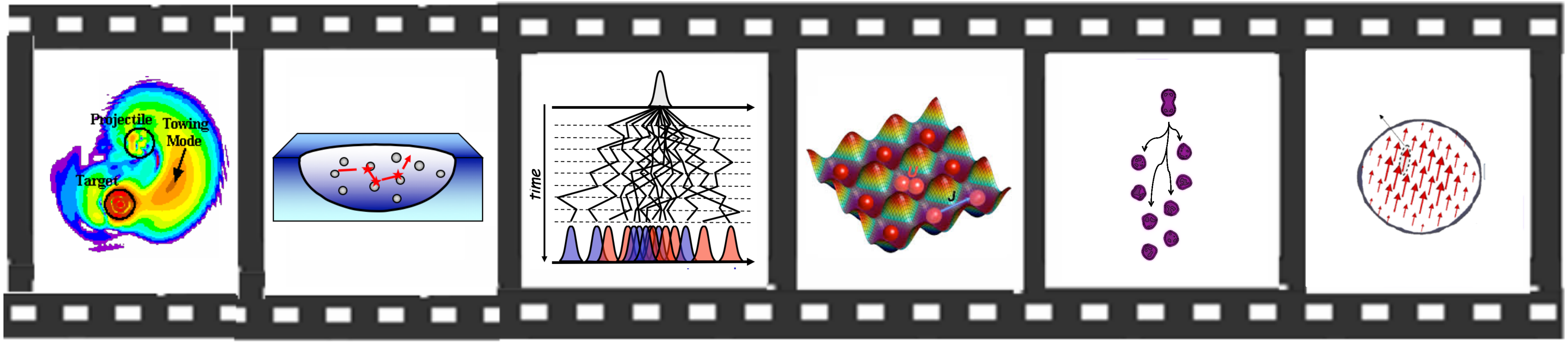


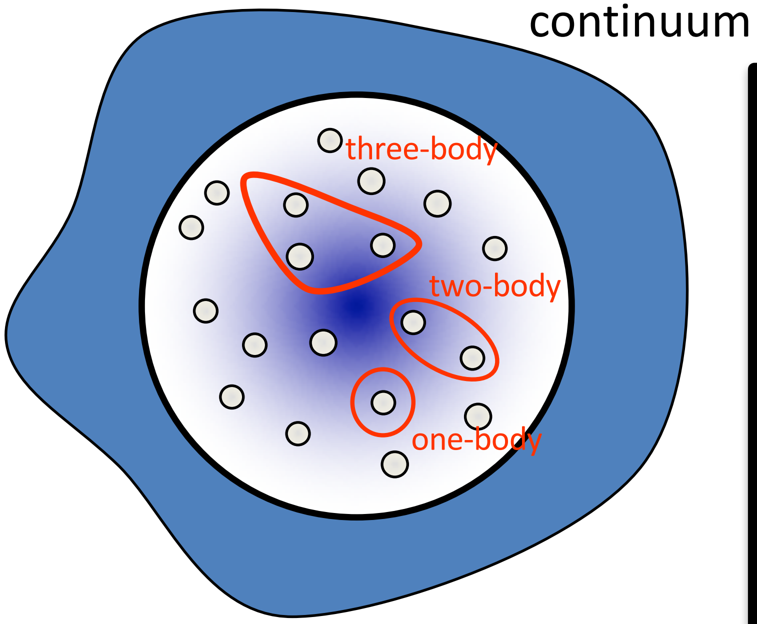
Stochastic extensions of quantum mean-field approaches

Denis Lacroix

INSTITUT DE PHYSIQUE NUCLÉAIRE
ORSAY

Coll: *S. Ayik*, B. Yilmaz, G. Scamps, Y. Tanimura

Goal: describe the dynamics of quantum many-body problem with complex interaction



The targeted physical situation:

-Number of particles [nucleons] (fermions):
from very few to several hundreds

-Quantum effects are important

-Particle are strongly correlated:
-superfluidity
-configuration mixing, nn collisions,…

-The system is open to the continuum:
-particle emission, resonances, …

-We also use DFT to get a « simple » description (also because TDHF does not make sense)

Born term $\mathcal{B}_{12} \Rightarrow (1-\rho_1)(1-\rho_2)\tilde{v}_{12}\rho_1\rho_2$

Pairing term $\mathcal{P}_{12} \Rightarrow \frac{1}{2}(1-\rho_1-\rho_2)\tilde{v}_{12}C_{12} - \frac{1}{2}$

Advantages

Several well-known theories can be Recovered; It is systematic

An illustration of use

Interacting Fermions in 1D

$$i\hbar\frac{\partial}{\partial t}\rho_1 = [h_1[\rho],\rho_1] + \frac{1}{2}\mathrm{Tr}_2\left[\bar{v}_{12}, C_{12}\right]$$

with

$$C_{12}(t) = -\frac{i}{\hbar}\int_{t_0}^{t} U_{12}(t,s)\,\underbrace{F_{12}(s)}_{(1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)}\,U^{\dagger}_{12}(t,s)\,ds + \delta C_{12}(t)$$

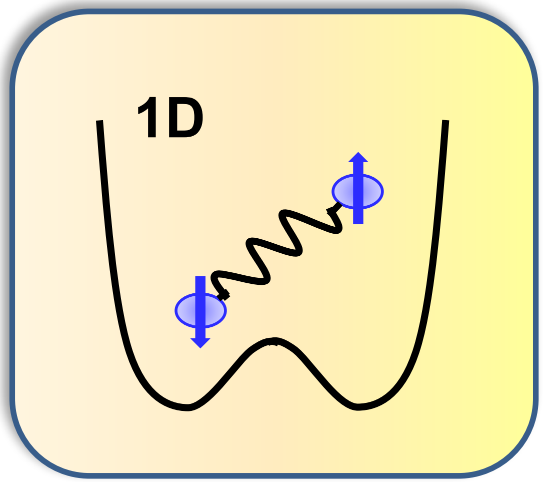
Non-Markovian master equation

$$\frac{d}{dt}n_\lambda(t) = \int_{t_0}^{t} dt' \left\{\bar{n}_\lambda(t')\,\mathcal{W}^{+}_{\lambda}(t,t') - n_\lambda(t')\,\mathcal{W}^{-}_{\lambda}(t,t')\right\}$$

Example: two interacting fermions in 1dimension

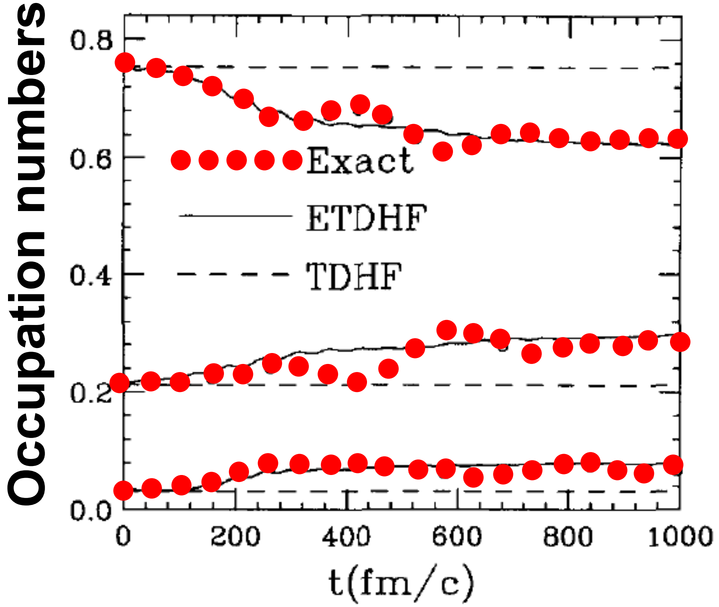
$$H = -\frac{\partial^2}{\partial x_1^2} + -\frac{\partial^2}{\partial x_2^2} + V(x_1 - x_2)$$

Difficulty:
memory effect!

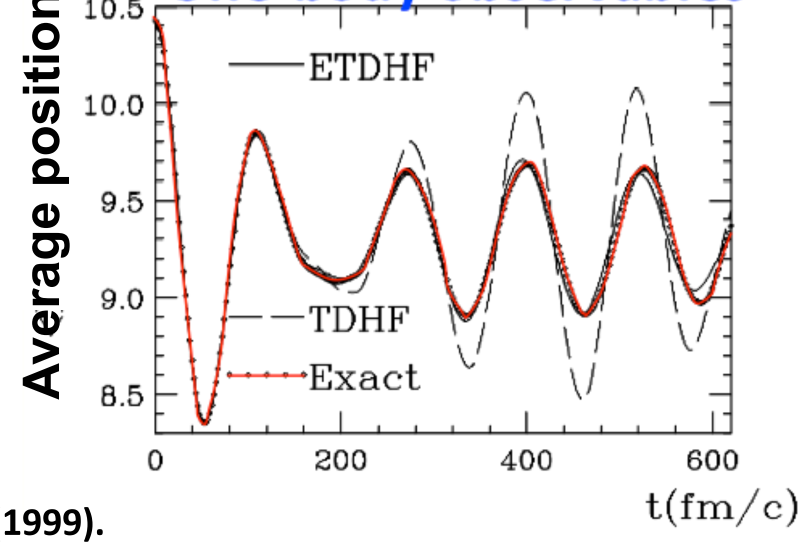


Lacroix, Chomaz, Ayik, Nucl. Phys. A (1999).

Occupation number evolution



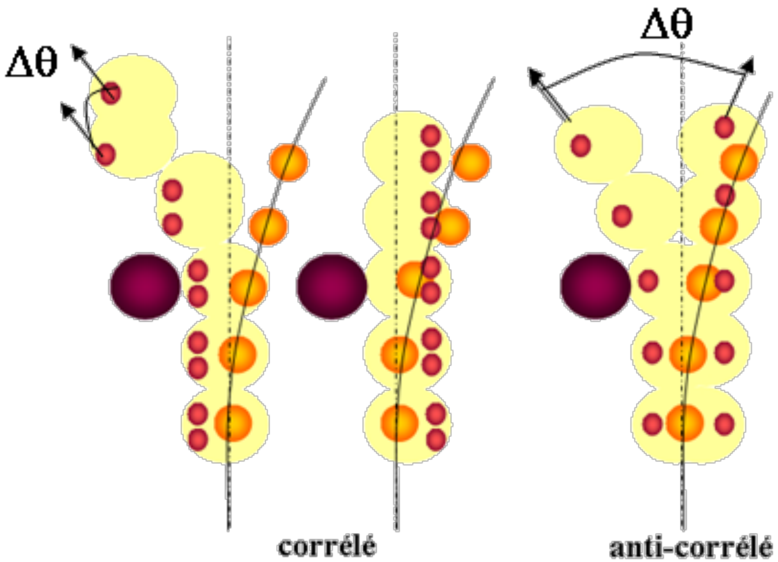
One-body observables



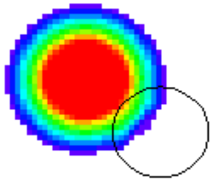
BBGKY strategy: two-particle emission with correlations

Assié, Lacroix, PRL 102 (2009)

Physical problem



h1



TD2RDM

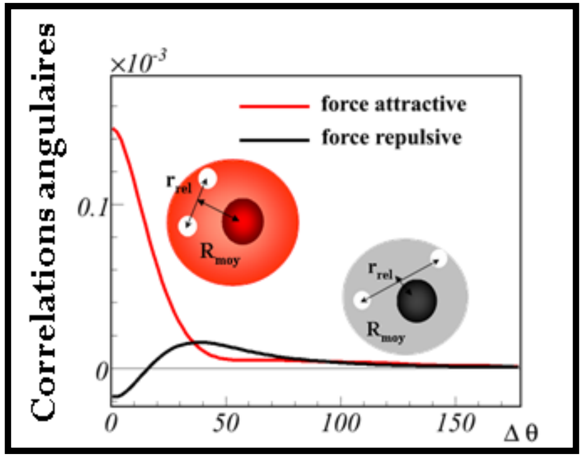
$$i\hbar\frac{\partial}{\partial t}\rho_1 = [h_1[\rho],\rho_1] + \frac{1}{2}\mathrm{Tr}_2\,[\bar{v}_{12}, C_{12}]$$

$$\begin{aligned}i\hbar\frac{\partial}{\partial t}C_{12} = & [h_1[\rho]+h_2[\rho], C_{12}] \\ & +\frac{1}{2}\Big\{(1-\rho_1)(1-\rho_2)\bar{v}_{12}\rho_1\rho_2 - \rho_1\rho_2\bar{v}_{12}(1-\rho_1)(1-\rho_2)\Big\} \\ & +\frac{1}{2}\Big\{(1-\rho_1-\rho_2)\,\bar{v}_{12}C_{12} - C_{12}\bar{v}_{12}\,(1-\rho_1-\rho_2)\Big\} \\ & \cancel{+\mathrm{Tr}_3\,[\bar{v}_{13},(1-P_{13})\,\rho_1 C_{23}\,(1-P_{12})]} \\ & \cancel{+\mathrm{Tr}_3\,[\bar{v}_{23},(1-P_{23})\,\rho_1 C_{23}\,(1-P_{12})]}\end{aligned}$$

Correlations dynamics is dominated by pairs of states that are initially time-reversed.

$$i\hbar\partial_t|\alpha\rangle = h[\rho]|\alpha\rangle; \qquad \dot{n}_\alpha = \frac{2}{\hbar}\sum_\gamma \mathrm{Im}(\mathbf{V}_{\alpha\gamma}\mathbf{C}_{\gamma\alpha})$$

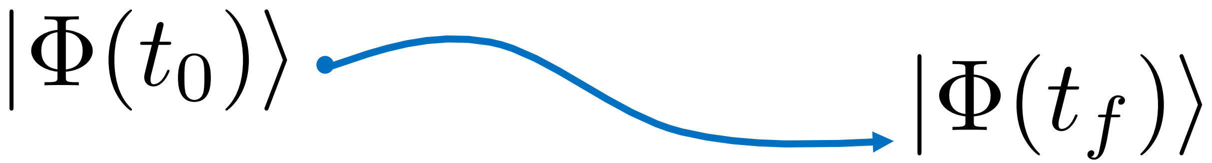
$$\begin{aligned}i\hbar\dot{\mathbf{C}}_{\alpha\beta} = & \mathbf{V}_{\alpha\beta}((1-n_\alpha)^2 n_\beta^2 - (1-n_\beta)^2 n_\alpha^2) \\ & + \sum_\gamma \mathbf{V}_{\alpha\gamma}(1-2n_\alpha)\mathbf{C}_{\gamma\beta} - \sum_\gamma \mathbf{V}_{\gamma\beta}(1-2n_\beta)\mathbf{C}_{\alpha\gamma}\end{aligned}$$



Goal of stochastic methods:

Can we replace a complicated problem by a set of “simpler” problem with fluctuations where complex effects are obtained through an average over fluctuating trajectories?

“simple” = mean-field



$$|\Phi(t_0)\rangle \longrightarrow |\Phi(t_f)\rangle$$

What are “complex correlations ”

⇨ Complexity might come from initial time $|\Psi(t_0)\rangle = \sum_n C_n(t_0)|\Phi_n(t_0)\rangle$

⇨ Mean-field is not properly quantized: missing zero point quantum fluctuations

⇨ Complexity comes from correlations beyond the mean-field that built up in time (ex: nucleon-nucleon collisions, action of quantum fluctuation one one-body DOFs,…)

Generalities: Many-body physics with stochastic methods

D. Lacroix and S. Ayik EPJA Review (2014)

Quantum or Auxiliary Field Monte-Carlo

$|\Phi\rangle\langle\Phi|$

$\overline{|\Phi_a\rangle\langle\Phi_b|}$

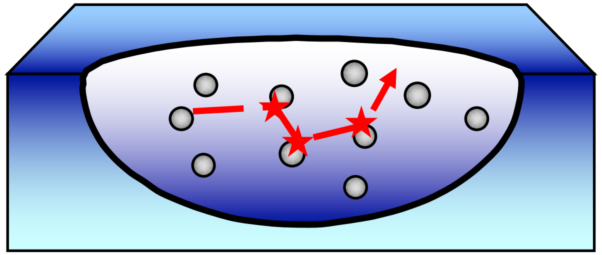
All Correlations

Stochastic TDHF like

$|\Phi\rangle\langle\Phi|$

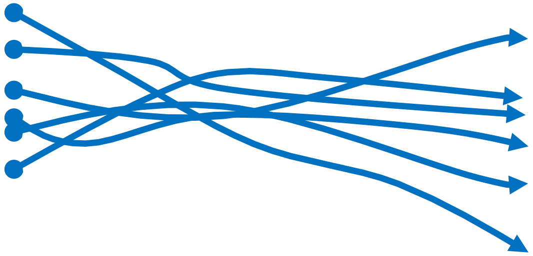
$\overline{|\Phi\rangle\langle\Phi|}$

Correlations that built up in time
Direct NN collisions



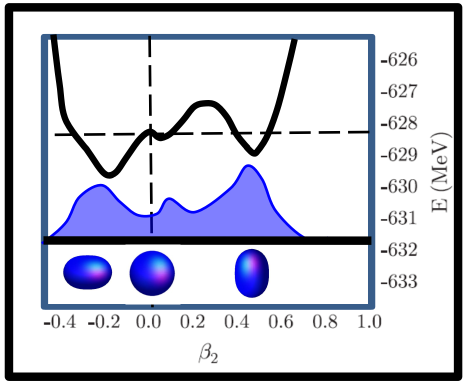
Stochastic Mean-Field

$\overline{\rho_{ij}(t_0)}$



$\overline{\rho_{ij}(t)}$

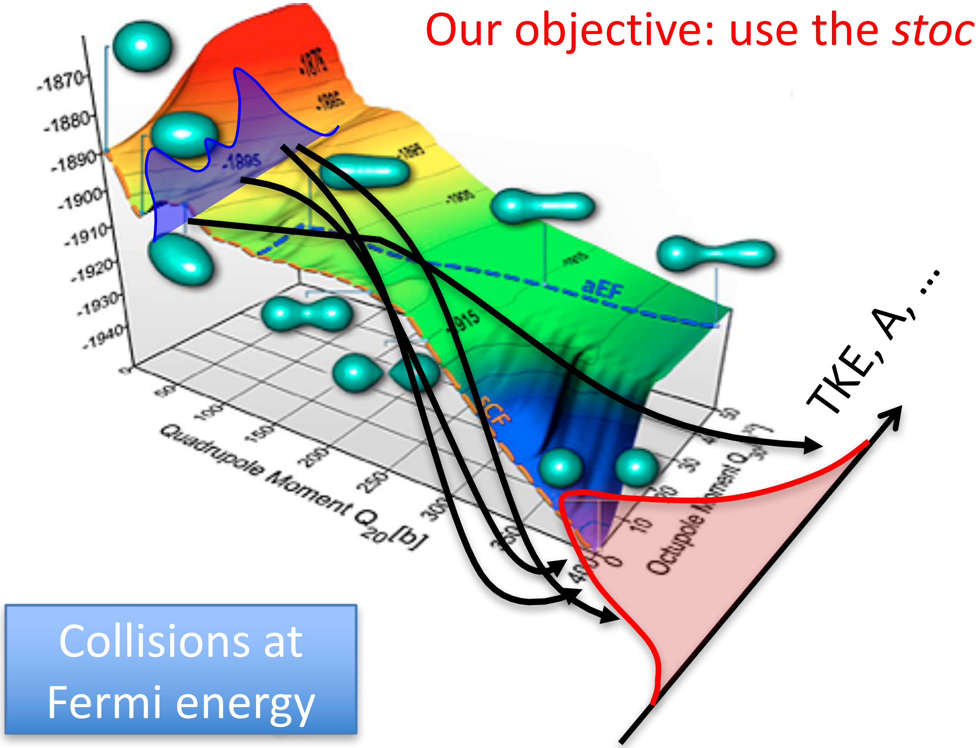
Initial fluctuations



Beyond the independent quasi-particle picture: ongoing work

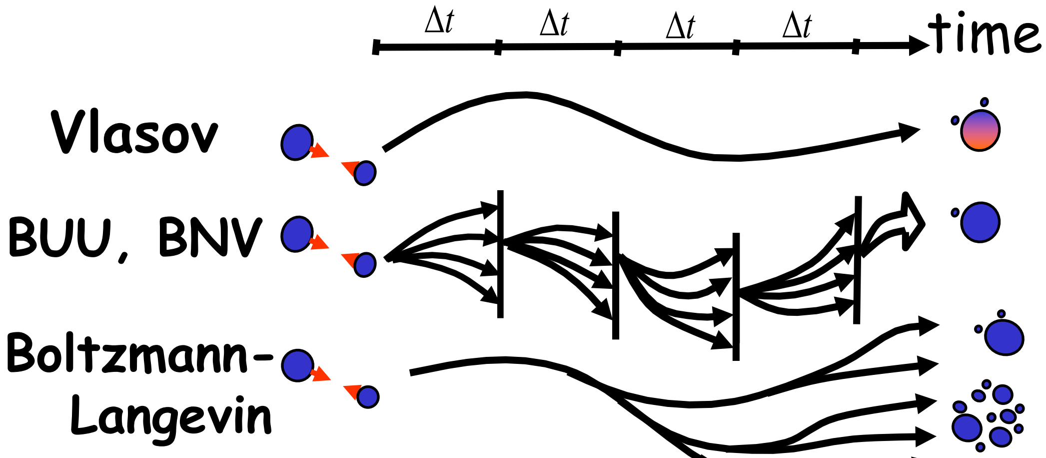
Our objective: use the *stochastic mean-field* approach to describe fission

Lacroix, Ayik, EPJA (Review) 50 (2014)



Quantum fluctuations can be treated approximately by sampling initial zero-point motion followed by classical trajectories (here classical=mean-field)

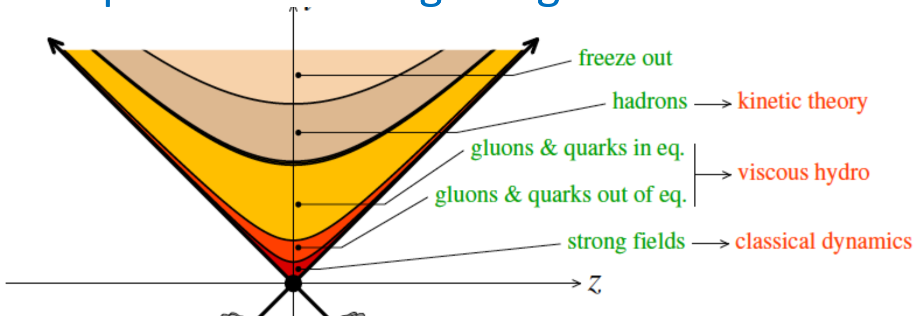
Collisions at Fermi energy



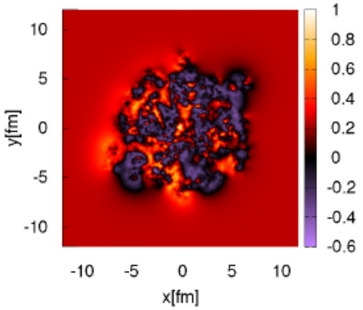
Adapted from J. Randrup et al, NPA538 (92).

Related approaches:

-description of little big-bang at RHIC or LHC



Gelis, Schenke, arxiv:1604:00335



Truncated Wigner theory

For Bose-Einstein condensates

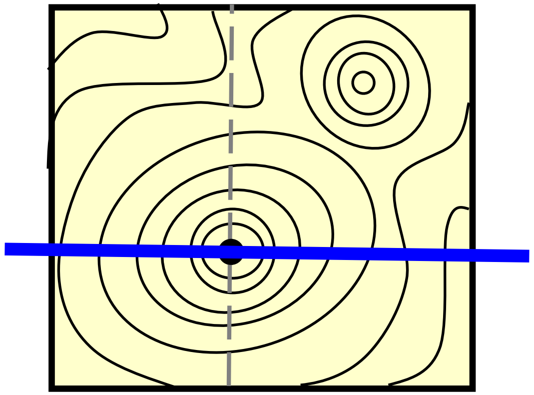
Sinatra, Lobo, and Castin, J. Phys. B 35 (2002)

Introduction on Phase-space methods

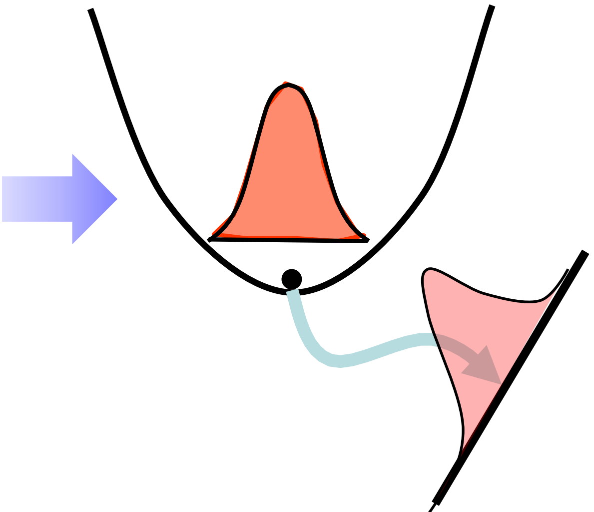
Goal: simulate quantum mechanics with quasi-classical evolutions

Illustration

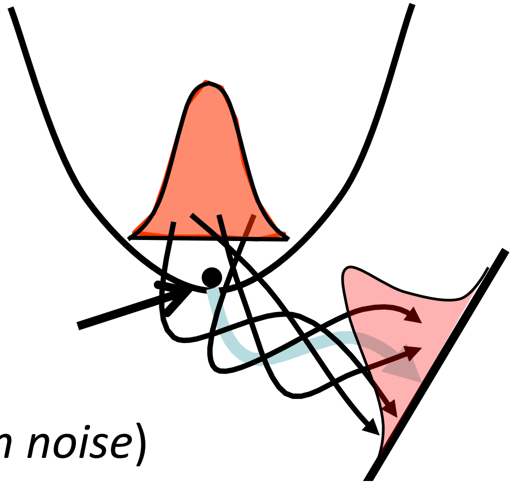
Collective energy landscape



Wave-evolution



Many-classical trajectories



NB: there are many Phase-space Methods, especially for Bosons

(see Gardiner, Zoller, *Quantum noise*)

Solution 1:
Schroedinger Eq.

$$i\hbar\frac{d|\phi\rangle}{dt} = h|\phi\rangle$$

Ex: Wigner transform

$$f(r, p, t)$$

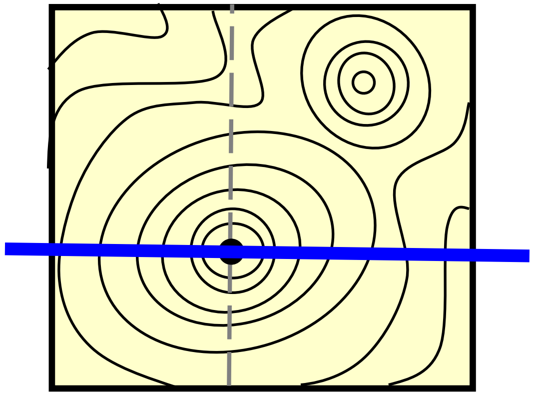
+ dynamical evolution

Classical mechanics
With random initial
fluctuations

$$\dot{r}^{\lambda} = p^{\lambda}/m$$
$$\dot{p}^{\lambda} = -\partial_r V(r^{\lambda})$$

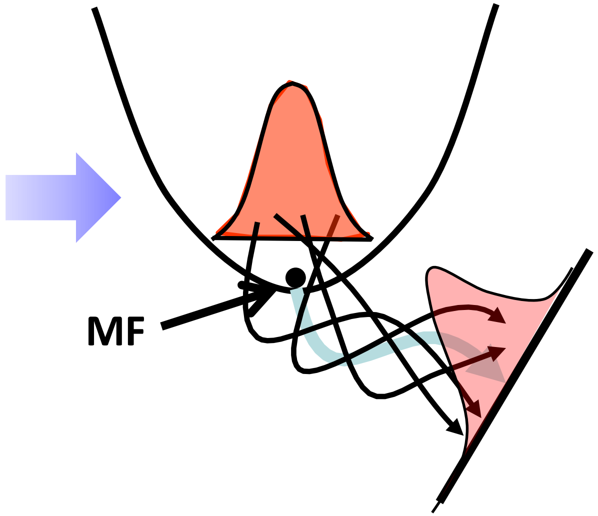
What do we call classical for Fermi systems?

Collective phase-space



Ayik, Phys. Lett. B 658, (2008).

Quantum fluctuations



The dynamics is described by a set of mean-field evolutions with random initial conditions

Mean-Field theory

$$\frac{d\langle A_\alpha\rangle}{dt} = \mathcal{F}\left(\{\langle A_\beta\rangle\}\right)$$ at all time $\sigma^2_Q = \langle A^2\rangle - \langle A\rangle^2$

Stochastic Mean-Field

$$\frac{dA^{(n)}_\alpha}{dt} = \mathcal{F}\left(\{A^{(n)}_\beta\}\right)$$

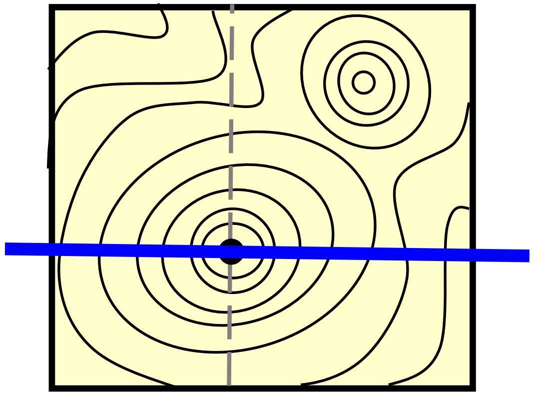
$$\{A^{(n)}_\alpha\}$$

at all time $\Sigma^2_C = \overline{A^{(n)}A^{(n)}} - \overline{A^{(n)}}^2$

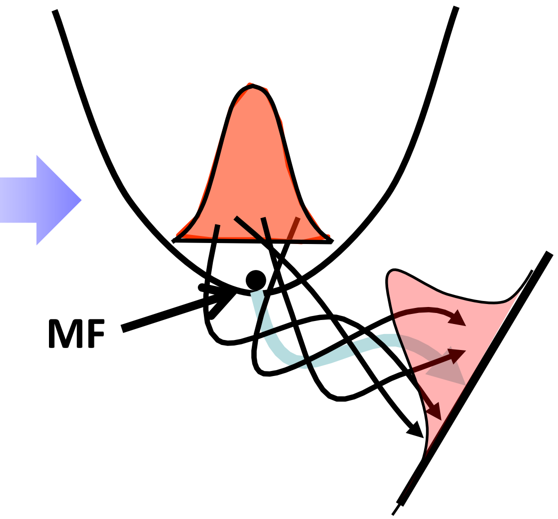
Constraint: $\Sigma^2_C(t=0) = \sigma^2_Q(t=0)$

The stochastic mean-field (SMF) concept applied to many-body problem

Collective phase-space



Quantum fluctuations



The dynamics is described by a set of mean-field evolutions with random initial conditions

Ayik, Phys. Lett. B 658, (2008).

The average properties of initial sampling should identify with properties of the initial state.

SMF in density matrix space

$$\rho(\mathbf{r},\mathbf{r}',t_0) = \sum_i \Phi_i^*(\mathbf{r},t_0) n_i \Phi_j(\mathbf{r}',t_0)$$

$$\Downarrow$$

$$\rho^\lambda(\mathbf{r},\mathbf{r}',t_0) = \sum_{ij} \Phi_i^*(\mathbf{r},t_0) \rho_{ij}^\lambda \Phi_j(\mathbf{r}',t_0)$$

$$\Rightarrow$$

$$\overline{\rho_{ij}^\lambda} = \delta_{ij} n_i$$

$$\overline{\delta\rho_{ij}^\lambda \delta\rho_{j'i'}^\lambda} = \frac{1}{2}\delta_{jj'}\delta_{ii'}\left[n_i(1-n_j) + n_j(1-n_i)\right].$$

SMF in collective space

$$Q(t_0)$$

$$\Downarrow$$

$$Q^\lambda(t_0)$$

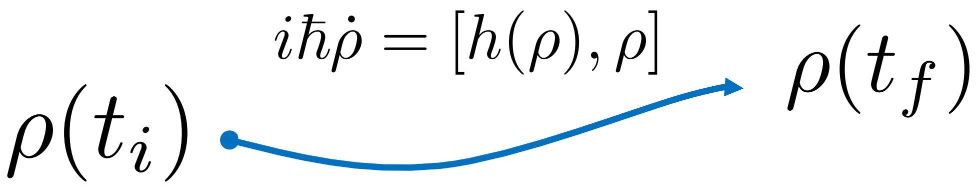
$$\Rightarrow$$

$$\overline{Q}^\lambda(t_0) = Q(t_0)$$

$$\sigma_Q(t_0) = \overline{(Q^\lambda(t_0) - \overline{Q^\lambda(t_0)}^2)}$$

How it works?

TDHF level



$$\rho(t_i) \qquad i\hbar\dot{\rho} = [h(\rho), \rho] \qquad \rho(t_f)$$

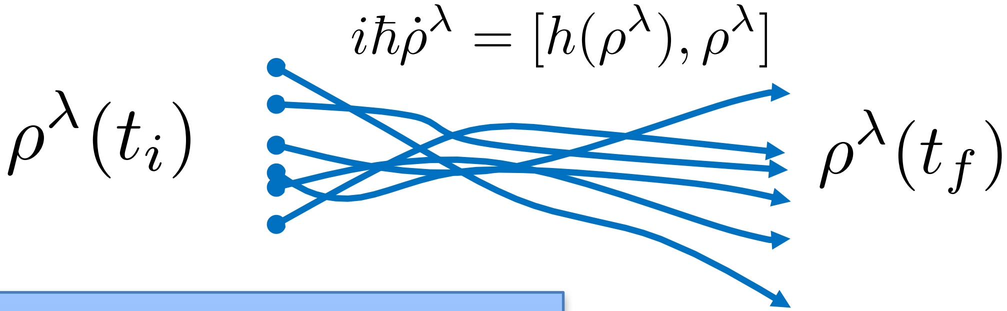
$$\rho = \begin{pmatrix} 1 & & & & & & & & \\ & 1 & & & & & & \mathbf{0} & \\ & & 1 & & & & & & \\ \text{holes} & & & 1 & & & & & \\ & & & & 0 & & & & \\ & & & & & 0 & \text{Part.} & & \\ & \mathbf{0} & & & & & 0 & & \\ & & & & & & & 0 & \\ & & & & & & & & 0 \end{pmatrix}$$

TDHF with initial fluctuations

$$\overline{\delta\rho^{\lambda}_{ij}\delta\rho^{\lambda}_{j'i'}} = \frac{1}{2}\delta_{jj'}\delta_{ii'}\left[n_i(1-n_j) + n_j(1-n_i)\right].$$

$$\rho = \begin{pmatrix} 1 & & & & & & & & \\ & 1 & & & & & & \boxed{\neq 0} & \\ & & 1 & & & & & & \\ & & & 1 & & & & & \\ & & & & 0 & & & & \\ & \boxed{\neq 0} & & & & 0 & & & \\ & & & & & & 0 & & \\ & & & & & & & 0 & \\ & & & & & & & & 0 \end{pmatrix}$$

Stochastic Mean-Field



$$\rho^{\lambda}(t_i) \qquad i\hbar\dot{\rho}^{\lambda} = [h(\rho^{\lambda}), \rho^{\lambda}] \qquad \rho^{\lambda}(t_f)$$

Some advantages

-Just N independent times something we know how to solve.

-Fluctuations can spontaneously break some symmetries.

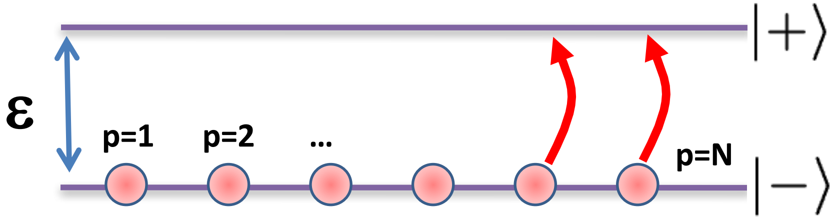
-Can be applied with initial thermal equilibrium too.

-predicting power is remarkably good (see below)

Description of large amplitude collective motion with SMF

The case of spontaneous symmetry breaking

Lipkin Model



$$H = \varepsilon J_0 - \frac{V}{2}(J_+J_+ + J_-J_-)$$

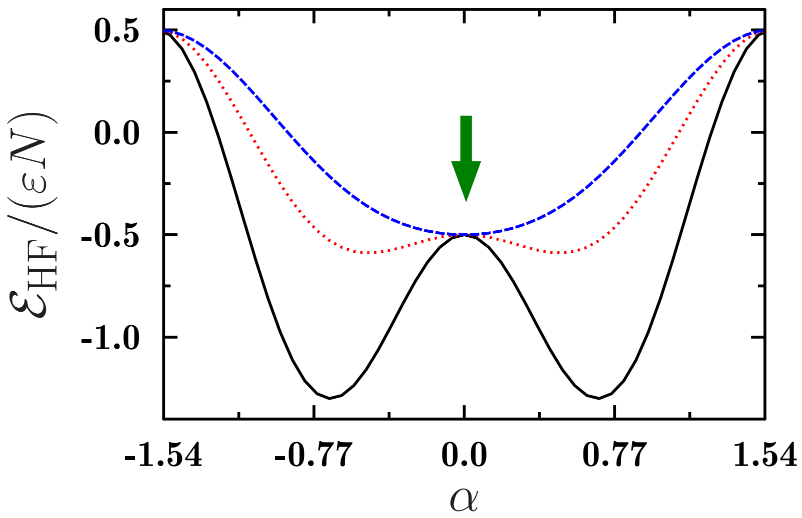
$$J_0 = \frac{1}{2}\sum_{p=1}^{N}\left(c^\dagger_{+,p}c_{+,p} - c^\dagger_{-,p}c_{-,p}\right) \qquad J_y = \frac{1}{2i}(J_+ - J_-)$$

$$J_+ = \sum_{p=1}^{N} c^\dagger_{+,p}c_{-,p}, \quad J_- = J_+^\dagger, \qquad J_x = \frac{1}{2}(J_+ + J_-)$$

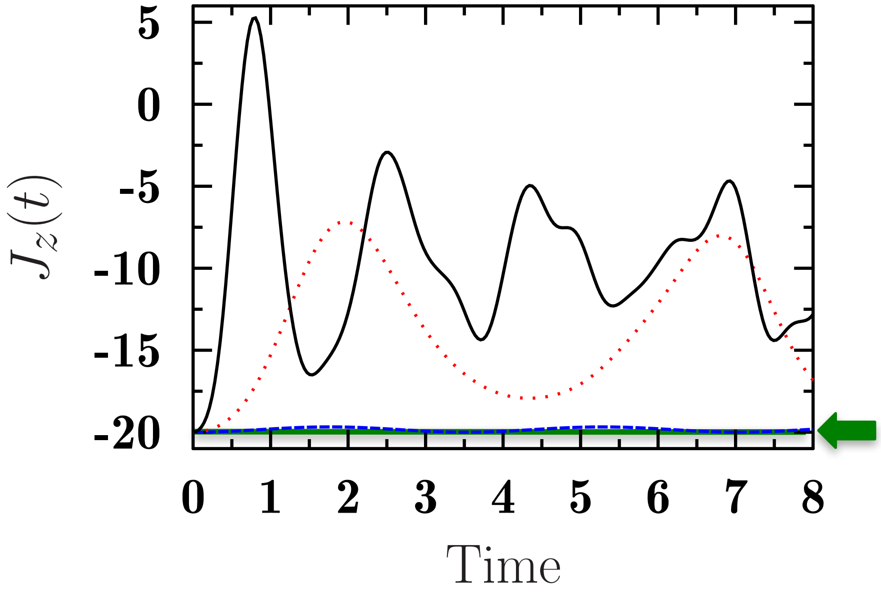
See for instance : Ring and Schuck book

Severyukhin, Bender, Heenen, PRC74 (2006)

N=40 particles



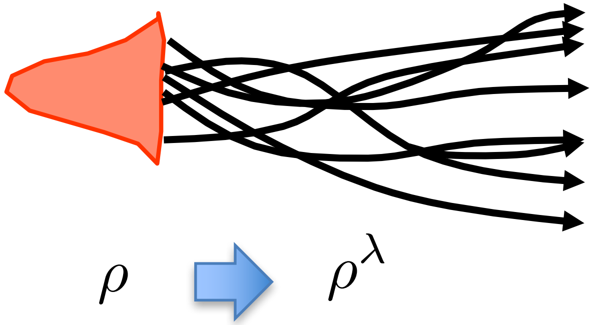
Exact dynamics

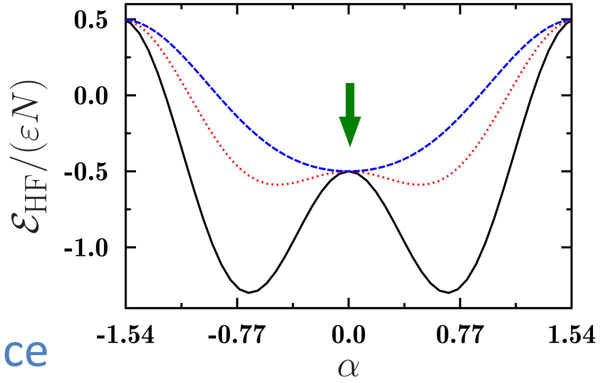


Mean-field is stationary

Description of large amplitude collective motion with SMF

The stochastic mean-field solution





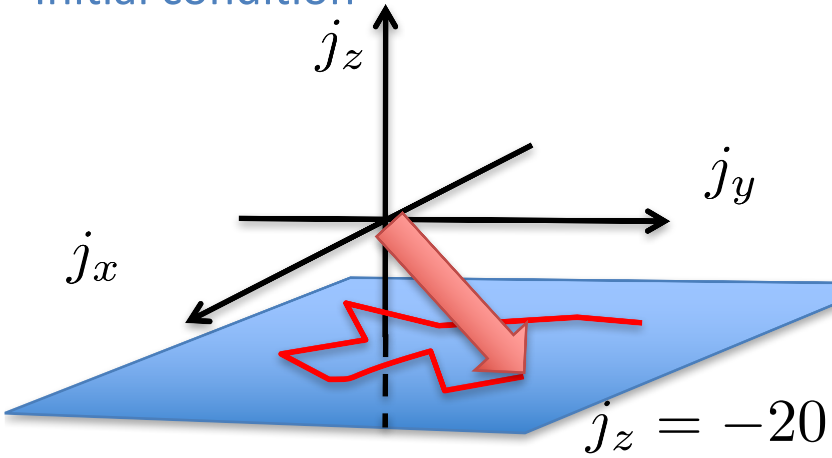
Formulation in quasi-spin space

$$j_i \equiv \langle J_i \rangle / N \Rightarrow j_i^\lambda$$

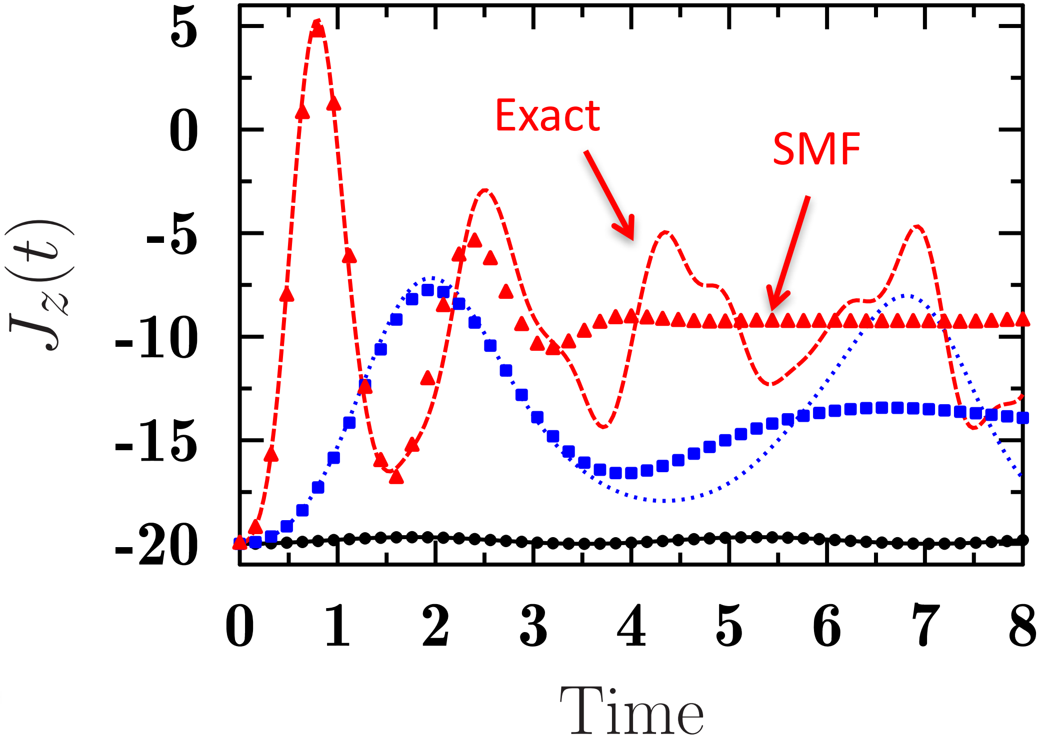
$$\overline{j_i^\lambda(t_0)} = 0$$

$$\overline{j_x^\lambda(t_0) j_x^\lambda(t_0)} = \overline{j_y^\lambda(t_0) j_y^\lambda(t_0)} = \frac{1}{4N}.$$

Initial condition



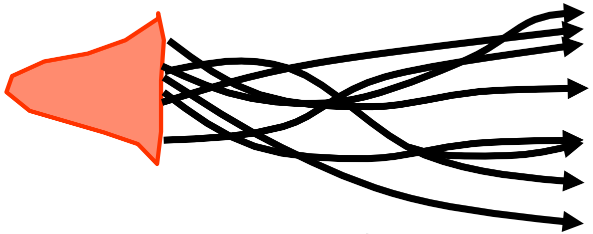
One-body observables



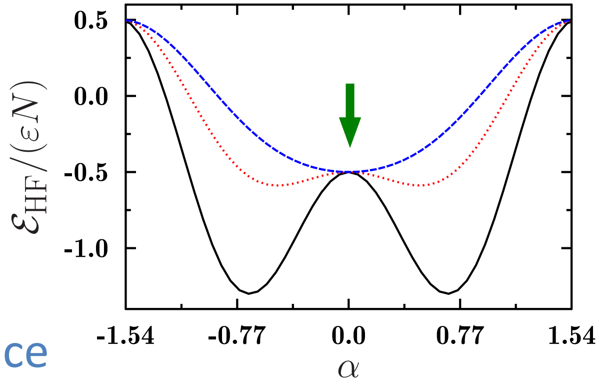
Lacroix, Ayik, Yilmaz, PRC 85 (2012)

Description of large amplitude collective motion with SMF

The stochastic mean-field solution



$\rho \;\Rightarrow\; \rho^{\lambda}$



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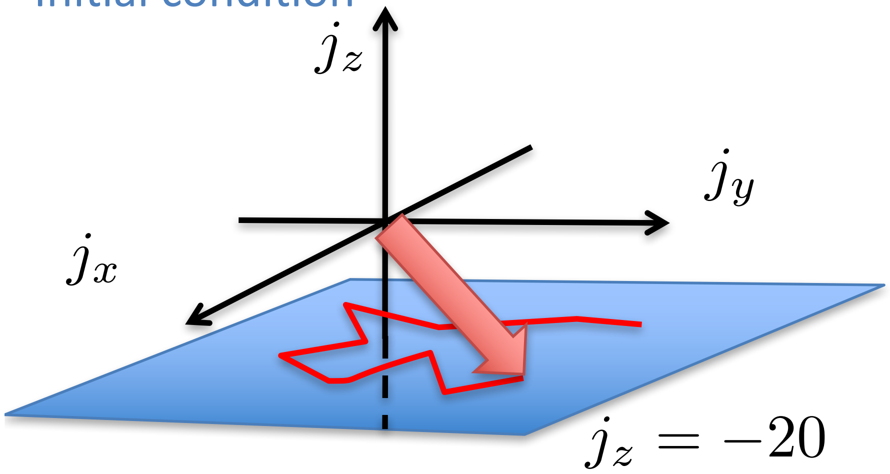
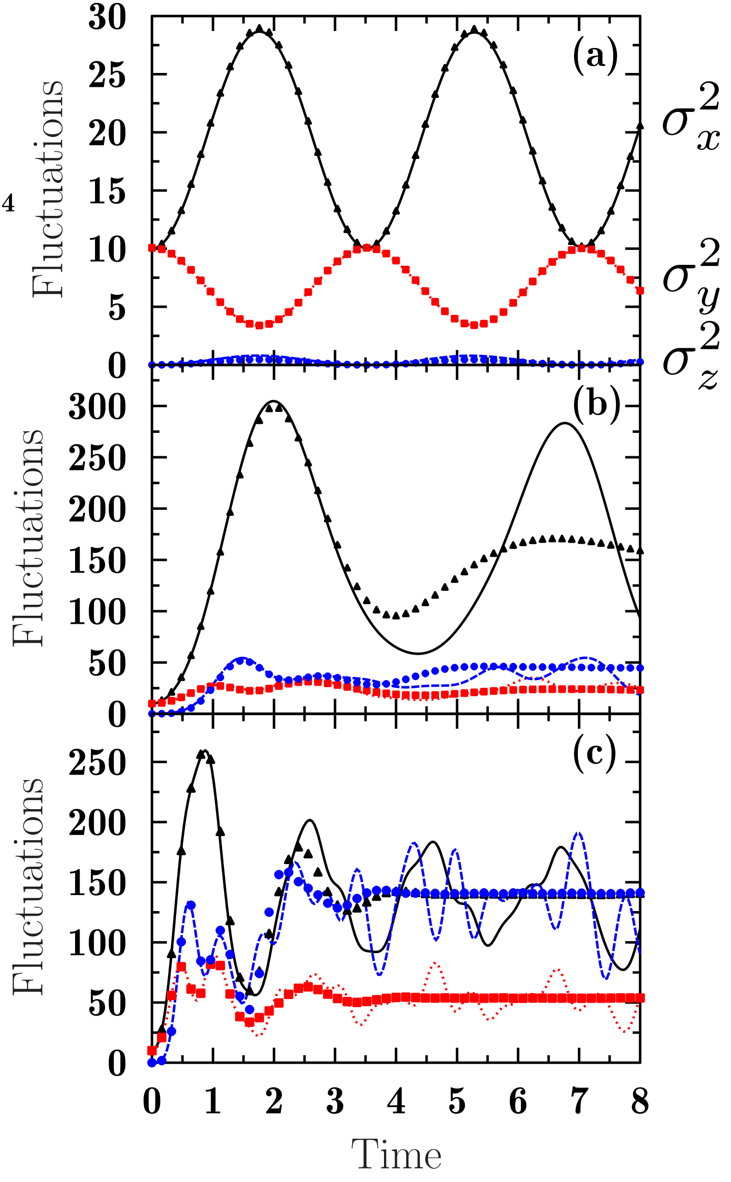
Initial condition

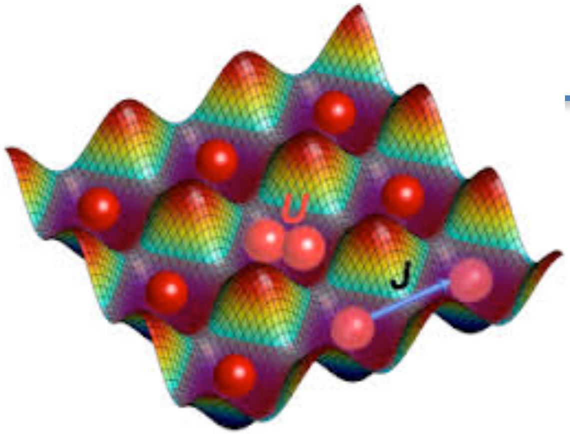


Lacroix, Ayik, Yilmaz, PRC 85 (2012)

Fluctuations

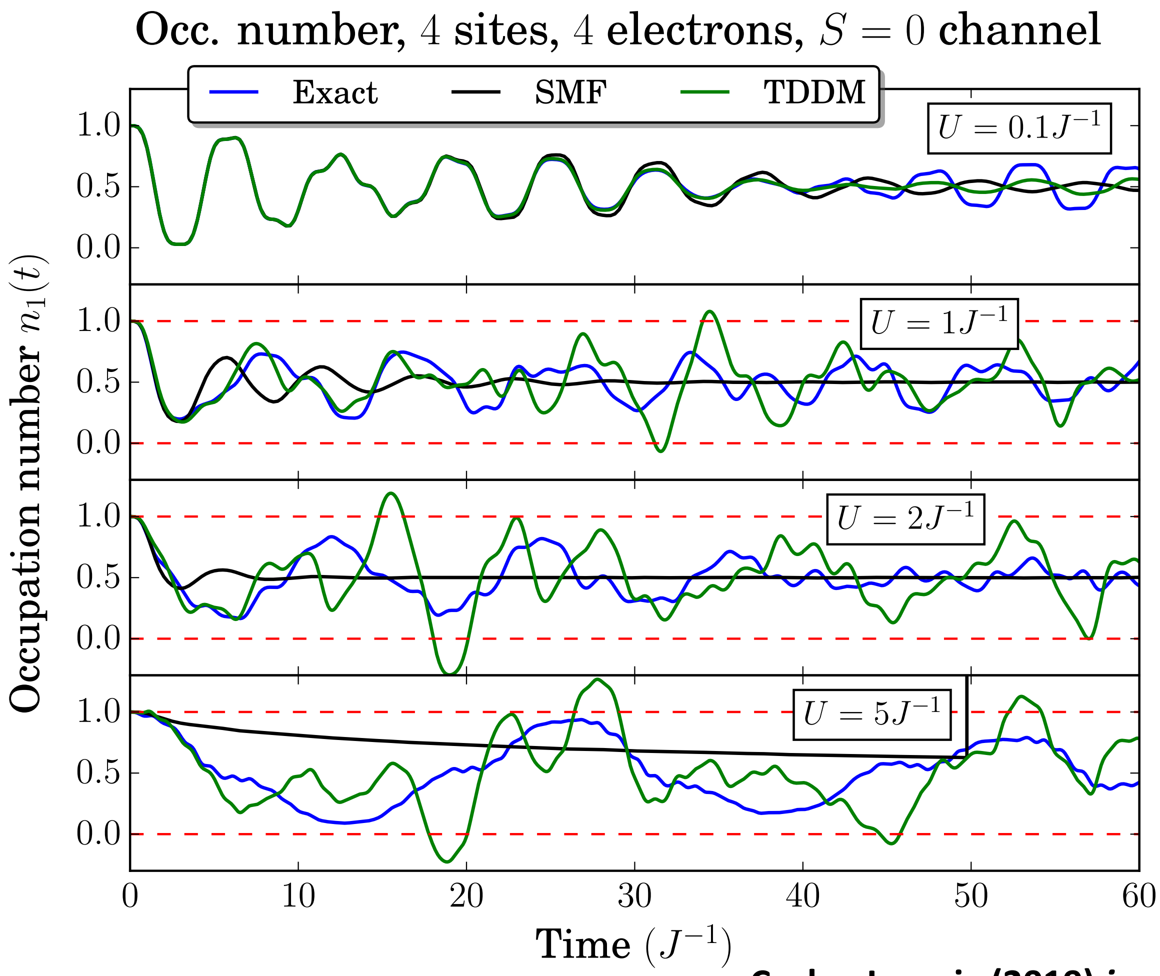


Another example: application to systems on lattice

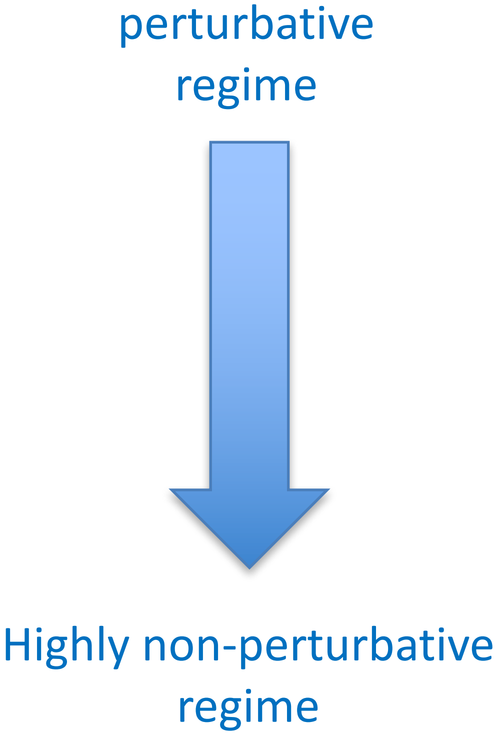


Lacroix, Hermanns, Hinz, Bonitz, PRB90 (2014)

$$H = -J \sum_{i,j}^{N_s} \sum_{\sigma} \delta_{\langle i,j\rangle} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i}^{N_s} c^{\dagger}_{i\uparrow} c_{i\uparrow} c^{\dagger}_{i\downarrow} c_{i\downarrow},$$



TDDM: propagation of
One and two-body density.



perturbative
regime

Highly non-perturbative
regime

Czuba, Lacroix (2019) *in preparation*

Why it works so well?

Link with a non-truncated simplified BBGKY hierarchy

Lacroix, Tanimura, Ayik and Yimaz, EPJA (2016)

From $$i\hbar\frac{d\rho^{(n)}}{dt} = \left[h(\rho^{(n)}), \rho^{(n)}\right]$$

One can obtain a set of coupled equations for: $C_{1\ldots k} = \overline{\delta\rho_1^{(n)}\ldots\delta\rho_k^{(n)}}.$

The first two equations is:
$$i\hbar\frac{d}{dt}\overline{\rho}(t) = [h(\overline{\rho}(t)), \overline{\rho}(t)] + \mathrm{Tr}_2\left[\tilde{v}_{12}, C_{12}\right]$$

$$\begin{aligned} i\hbar\frac{d}{dt}C_{12} &= [h_1[\overline{\rho}] + h_2[\overline{\rho}], C_{12}] \\ &+ \mathrm{Tr}_3\left[\tilde{v}_{13} + \tilde{v}_{23}, C_{13}\overline{\rho}_2 + C_{23}\overline{\rho}_1\right] \\ &+ \mathrm{Tr}_3\left[\tilde{v}_{13} + \tilde{v}_{23}, C_{123}\right], \end{aligned}$$

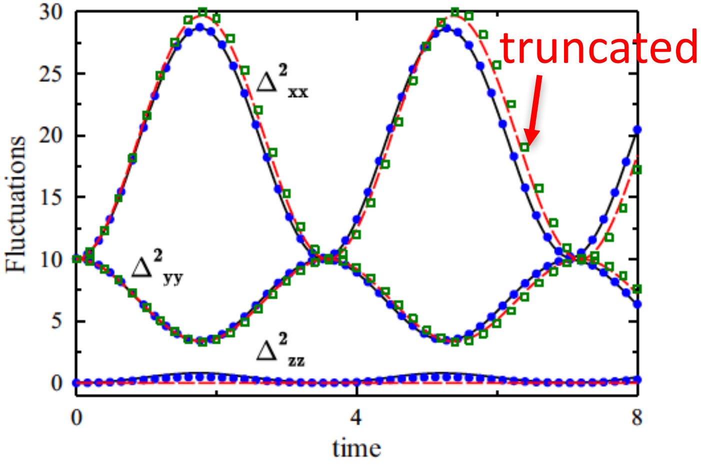
Here starts
the approximation.

And more generally:

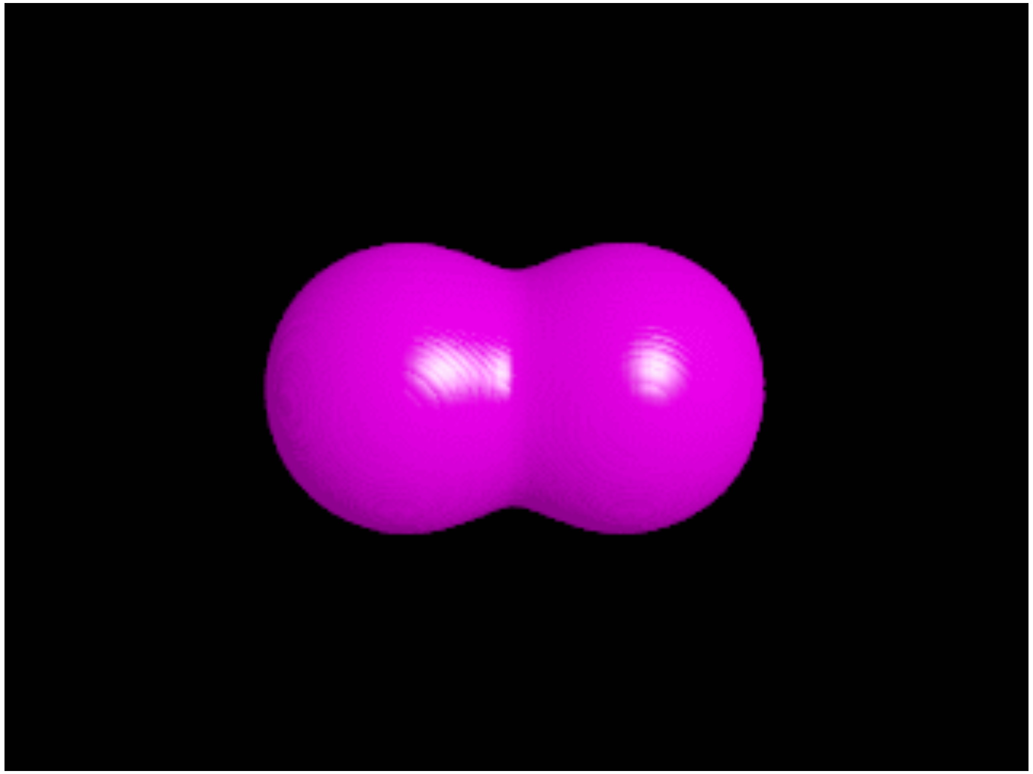
$$\begin{aligned} i\hbar\frac{d}{dt}C_{1\ldots k} &= \left[\sum_{\alpha\leq k} t_\alpha, C_{1\ldots k}\right] \\ &+ \sum_{\alpha=1}^{k}\mathrm{Tr}_{k+1}[\tilde{v}_{\alpha k+1}, C_{1\ldots k}\overline{\rho_{k+1}}] \\ &+ \sum_{\alpha=1}^{k}\mathrm{Tr}_{k+1}\left[\tilde{v}_{\alpha k+1}, C_{1\ldots(\alpha-1)(\alpha+1)\ldots(k+1)}\overline{\rho_\alpha}\right] \\ &+ \sum_{\alpha=1}^{k}\mathrm{Tr}_{k+1}\left[\tilde{v}_{\alpha k+1}, C_{1\ldots(\alpha-1)(\alpha+1)\ldots k}C_{\alpha k+1}\right] \\ &+ \sum_{\alpha=1}^{k}\mathrm{Tr}_{k+1}\left[\tilde{v}_{\alpha k+1}, C_{1\ldots(k+1)}\right]. \end{aligned}$$

But no truncation…

Lipkin model again



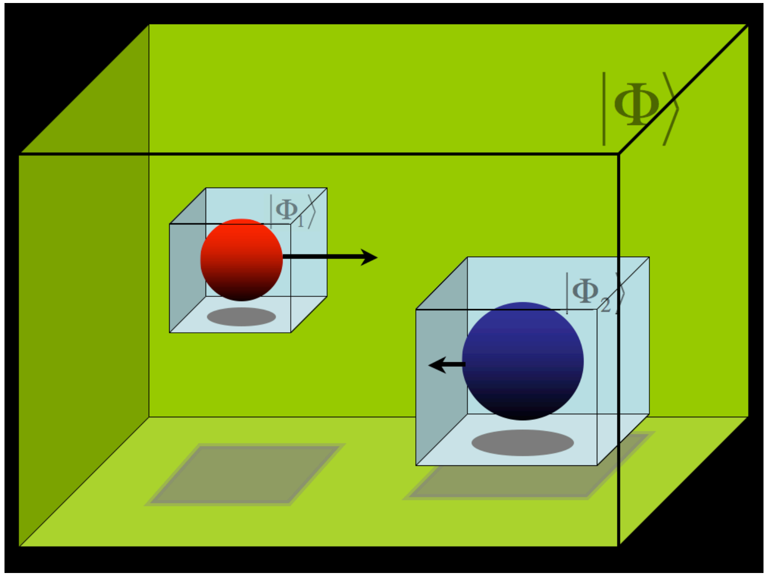
Recent applications in nuclear physics



Dynamical description of superfluid nuclei

Recent progress

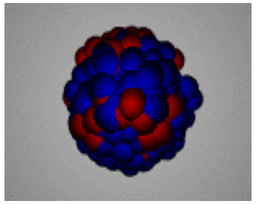
Nuclear motion of superfluid nuclei on a mesh (here within TDHF+BCS [TDDFT with superfluidity])



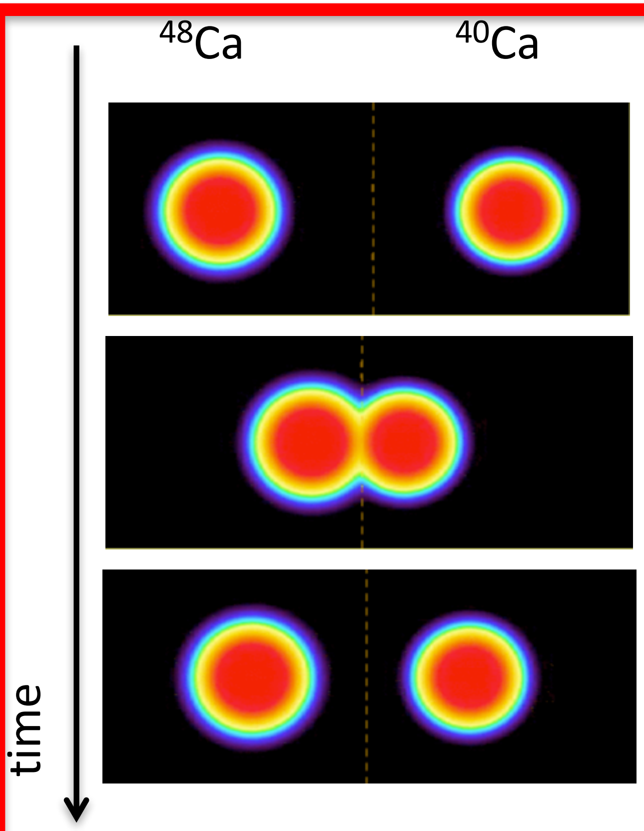
Scamps, Tanimura, Lacroix (2012-2017)

Applied to a number of physical process

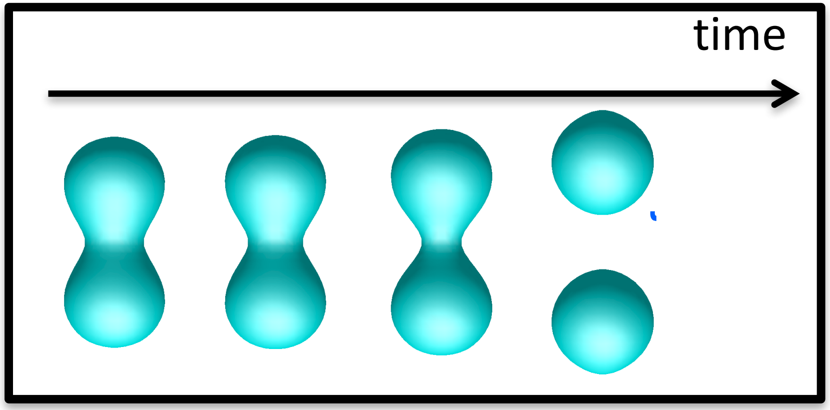
Vibrations



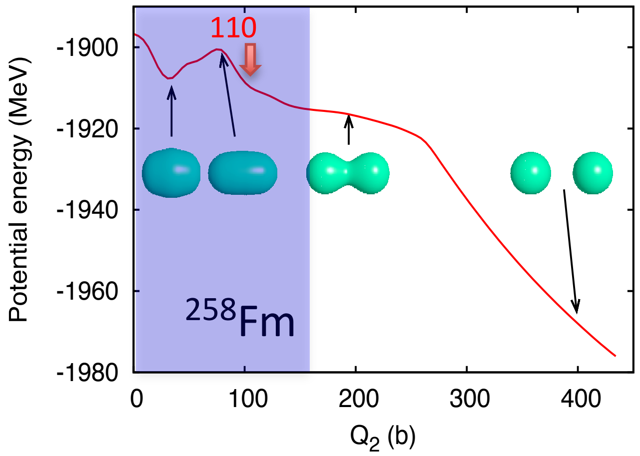
Fusion/Transfert



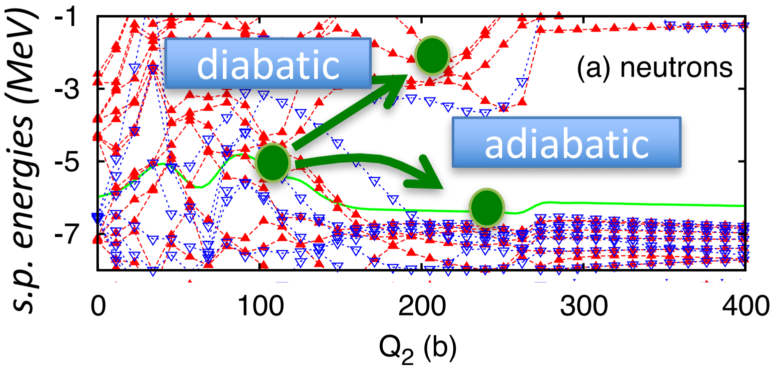
Fission



Time-dependent mean-field with pairing
Accounting for non-adiabaticity



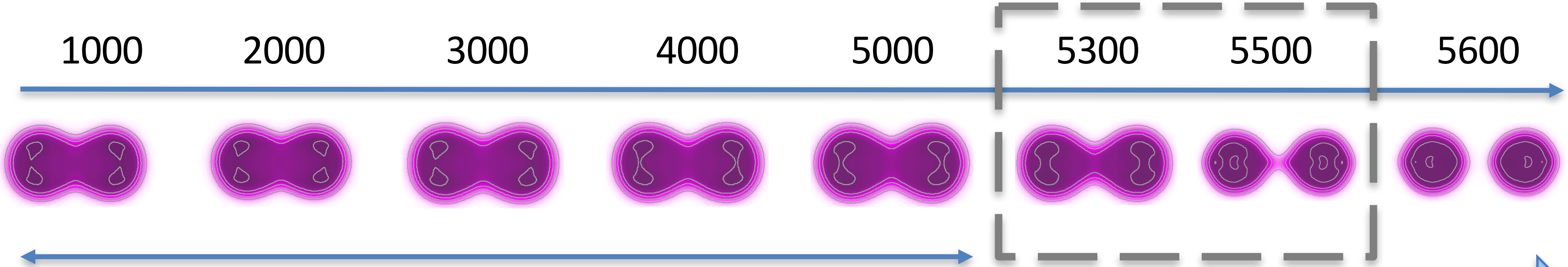
Without pairing the system do not fission: Mean-field without pairing is too diabatic!



TDHFB or TDHF+BCS solve this problem

Time (fm/c)

Fast fission

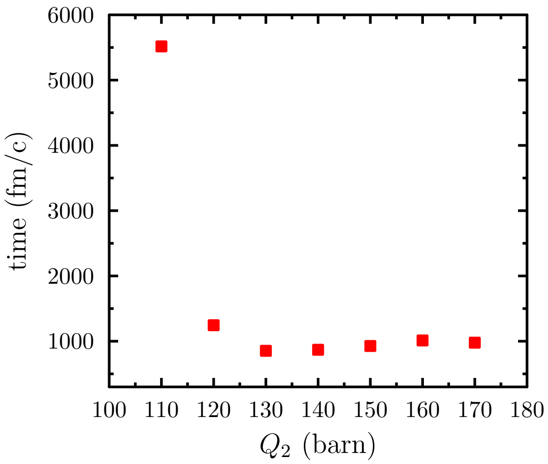


Extremely slow pairing/dissipation dominated motion

Coulomb boost

Fission time with TDHF+BCS

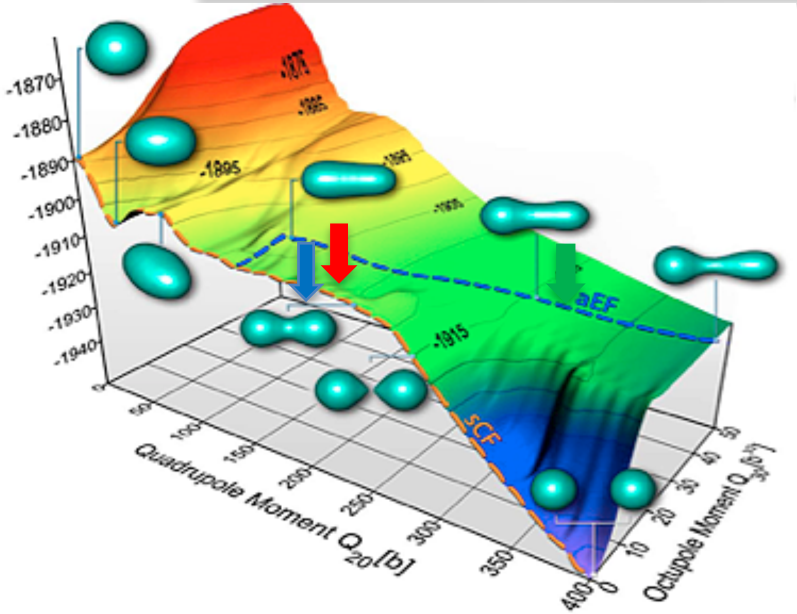
Scamps, Simenel, DL PRC 92 (2015)
Tanimura, DL, Ayik, PRL 118 (2017)

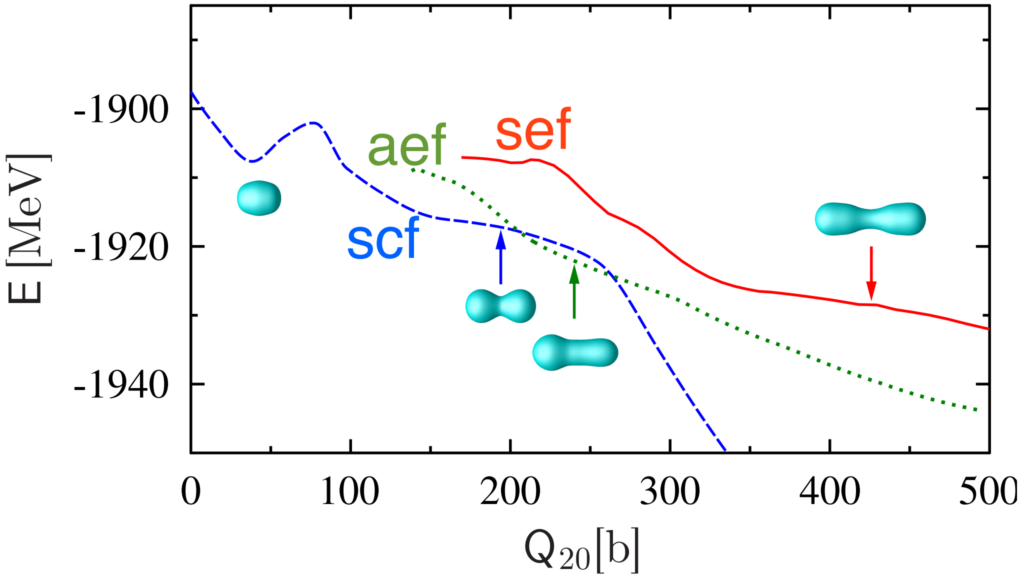


Confirms the finding of:

Bulgac, Magierski, Roche, and Stetcu
Phys. Rev. Lett. 116, 122504 (2016)

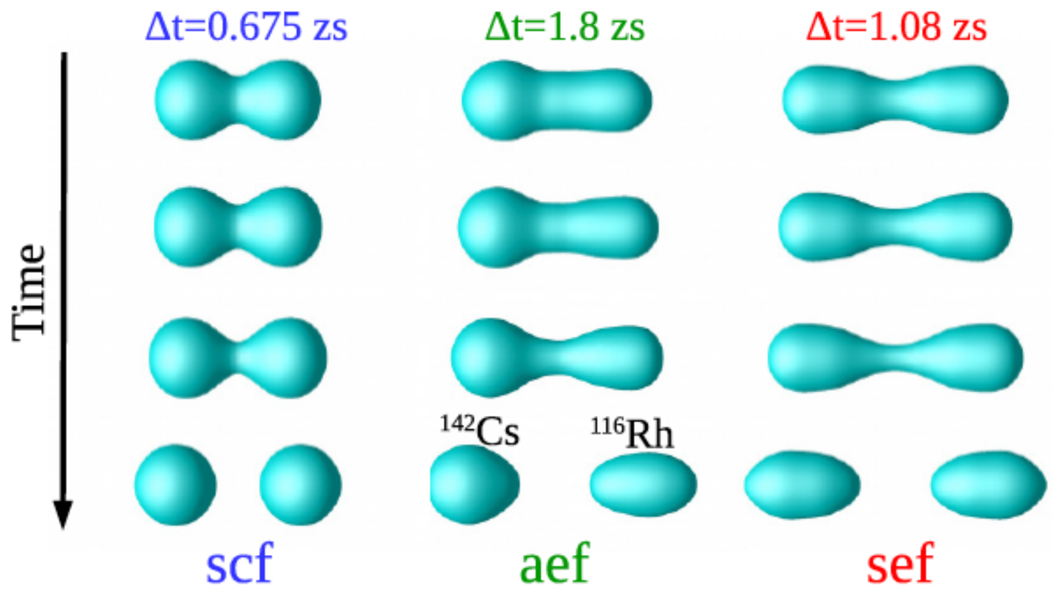
Fission of superfluid ^{258}Fm

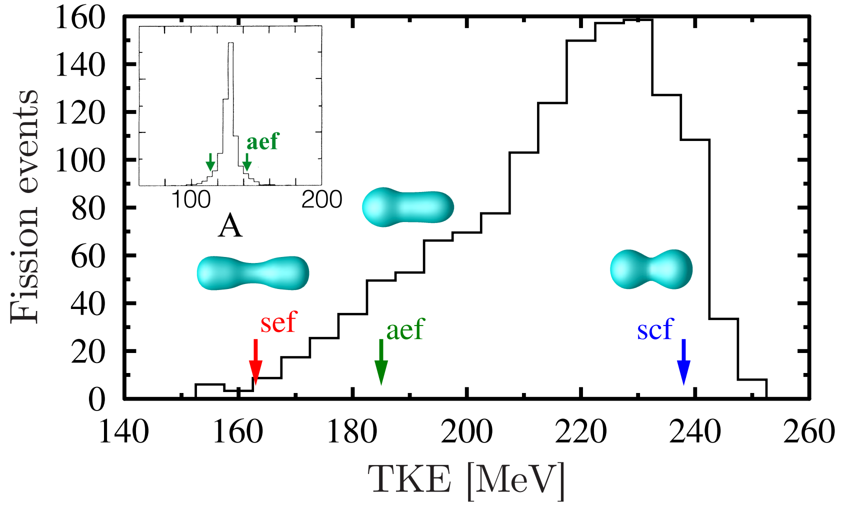




Identification of main fission paths

1 zs = 10^{-21} s





SMF in density matrix space

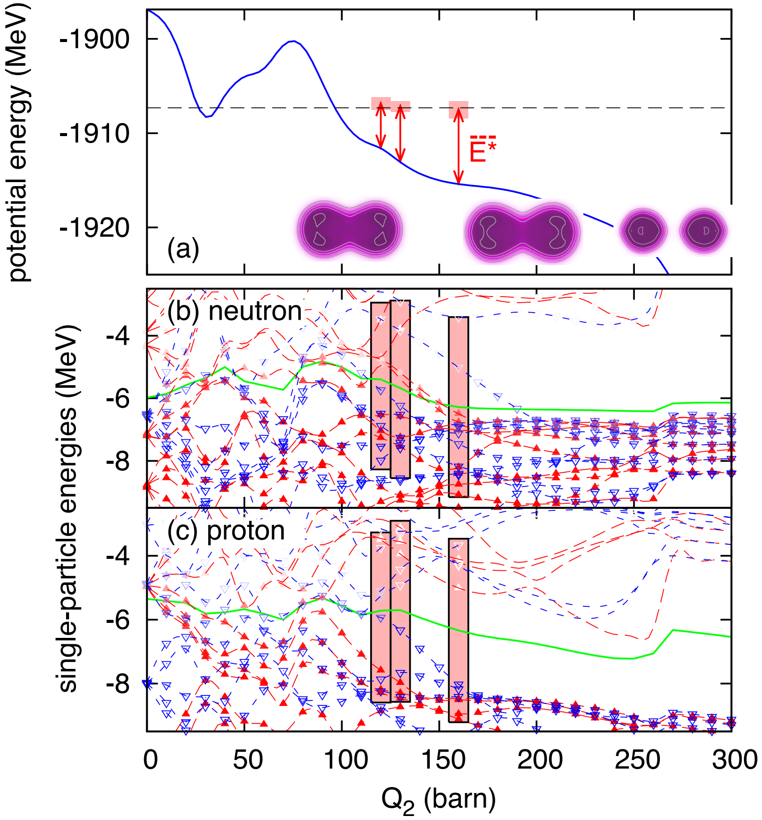
$$\rho(\mathbf{r},\mathbf{r}',t_0) = \sum_i \Phi_i^*(\mathbf{r},t_0) n_i \Phi_j(\mathbf{r}',t_0)$$

$$\rho^\lambda(\mathbf{r},\mathbf{r}',t_0) = \sum_{ij} \Phi_i^*(\mathbf{r},t_0) \rho_{ij}^\lambda \Phi_j(\mathbf{r}',t_0)$$

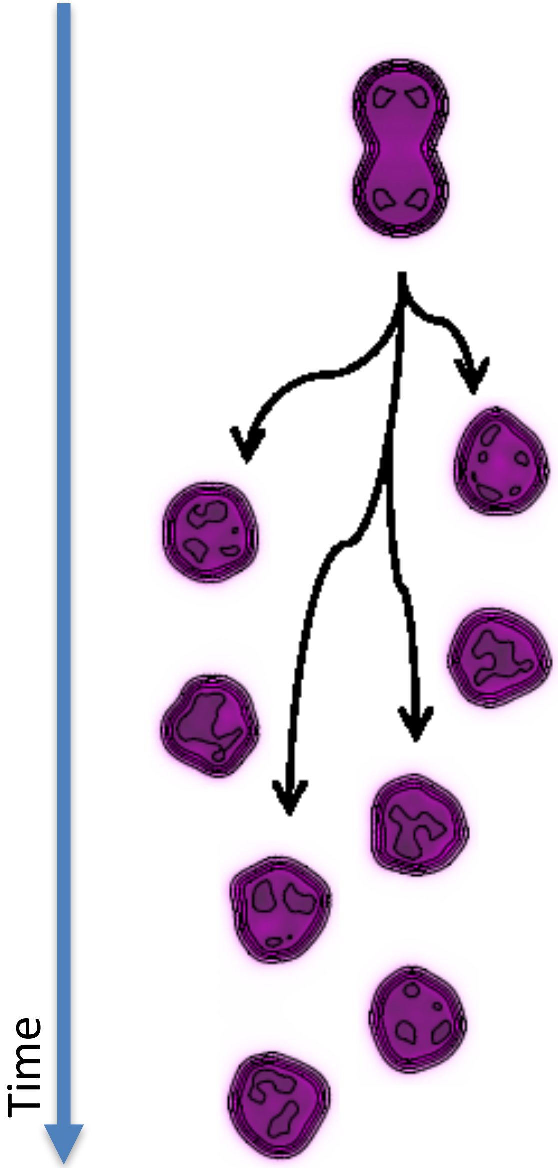
$$\overline{\rho_{ij}^\lambda} = \delta_{ij} n_i$$

$$\overline{\delta\rho_{ij}^\lambda \delta\rho_{j'i'}^\lambda} = \frac{1}{2}\delta_{jj'}\delta_{ii'}\left[n_i(1-n_j) + n_j(1-n_i)\right].$$

Range of fluctuation fixed by energy cons.



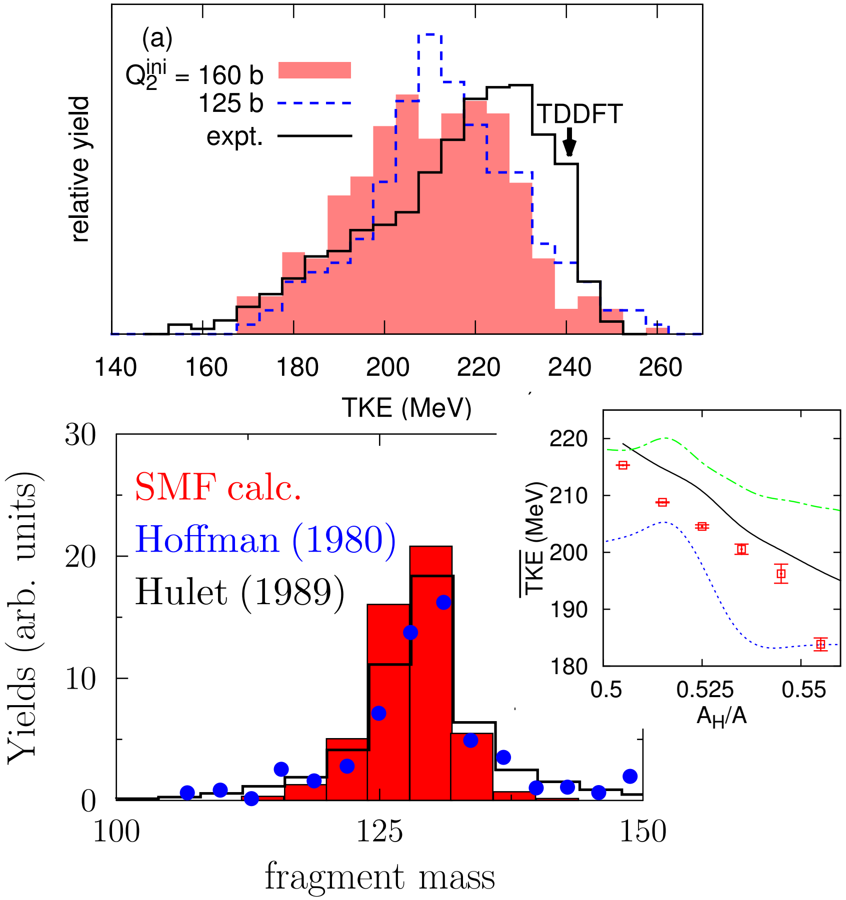
How to conceal microscopic deterministic approach and randomness ?



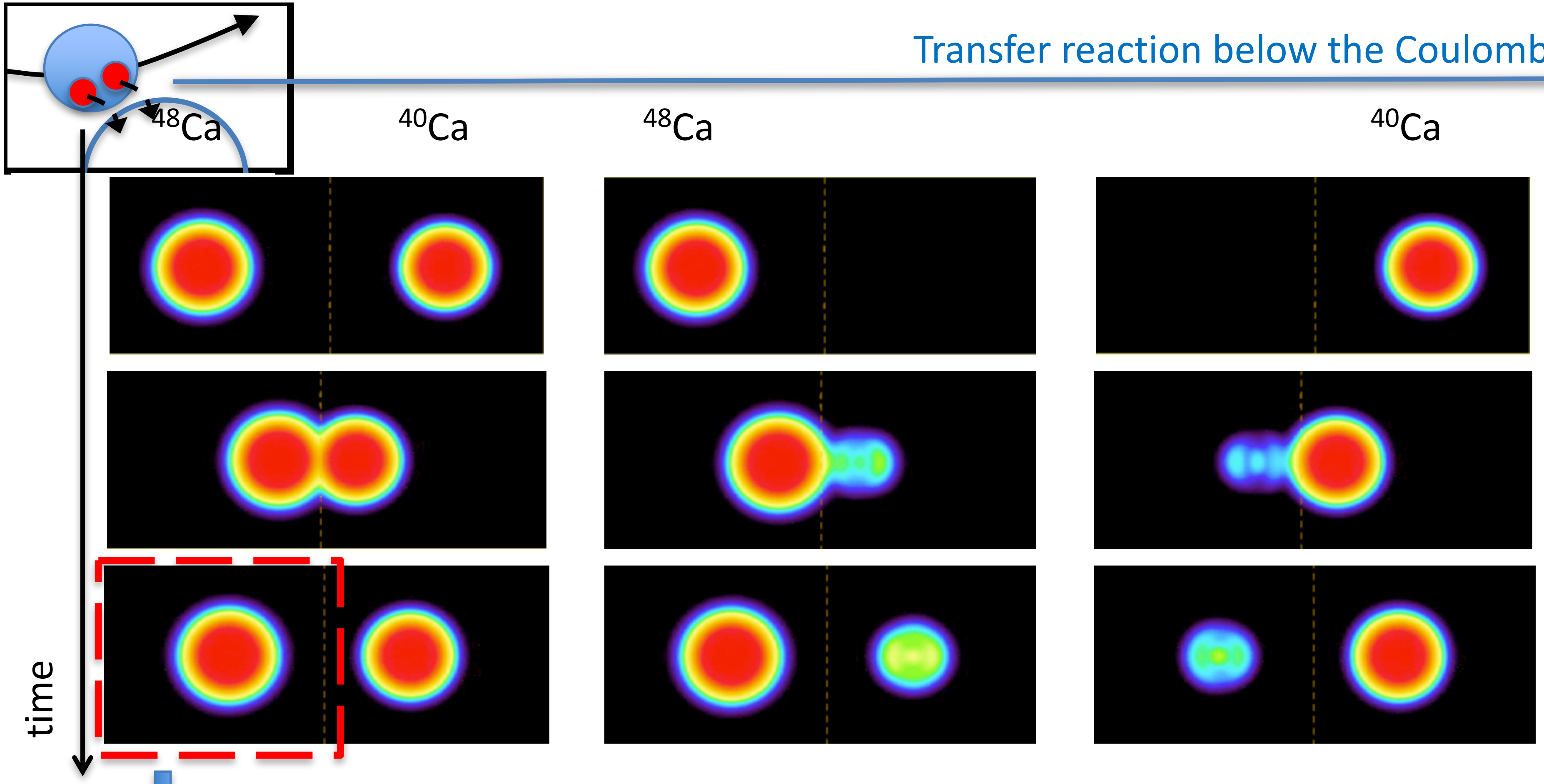
Tanimura, Lacroix, Ayik, PRL (2017)

From deterministic to statistical approach

Theory vs experiment



Transfer reaction below the Coulomb barrier



Extract one, two, …
nucleons transfer probabilities
$P_{1n},\ P_{2n},\dots$

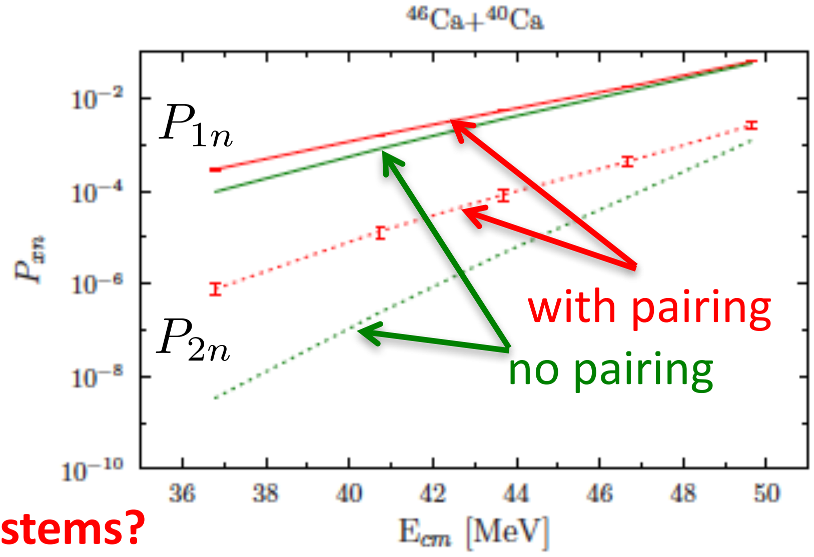
Two normal nuclei

Simenel, PRL 105 (2010).

One normal-One superfluid

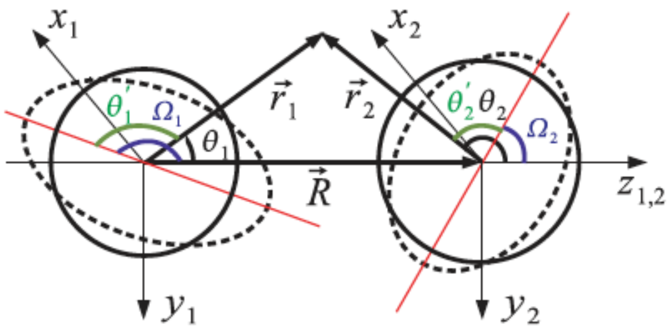
Scamps, DL, PRC 87 (2013).

What happens between two superfluid systems?

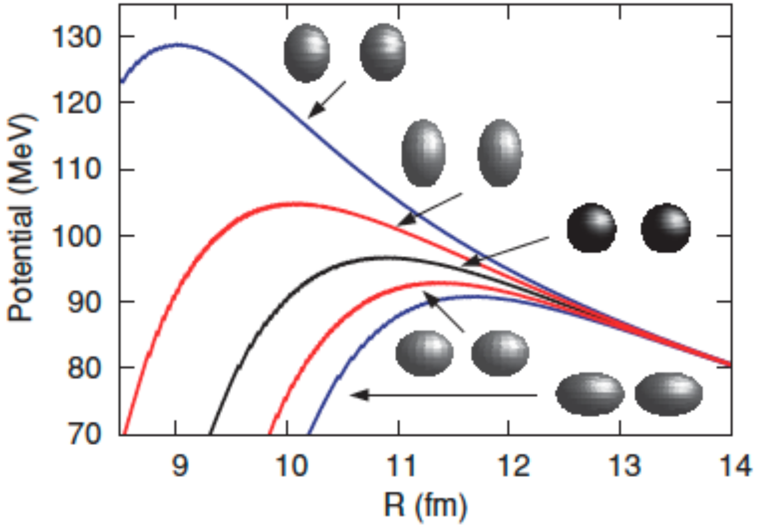


Emergent dynamical pairing phenomena close to the Coulomb barrier

Deformation effect in nuclear fusion



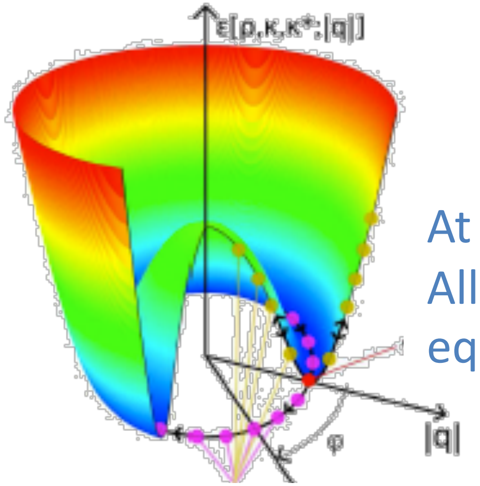
Orientation modifies the barrier



Gauge-angle deformation effect

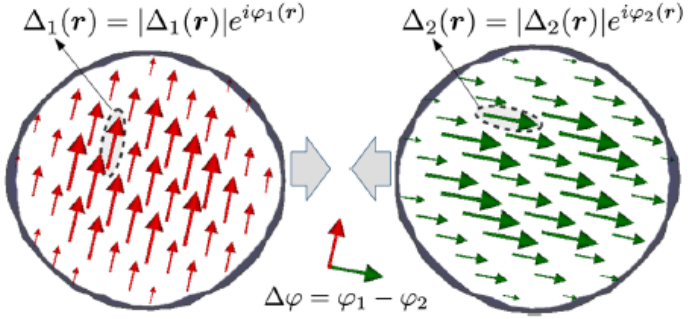
The observation is the result of all possible orientations (symmetry restoration)

The equivalent for pairing is the spontaneous breaking of particle number symmetry



At mean-field level
All orientation are
equivalent

Restoration of symmetry=averaging



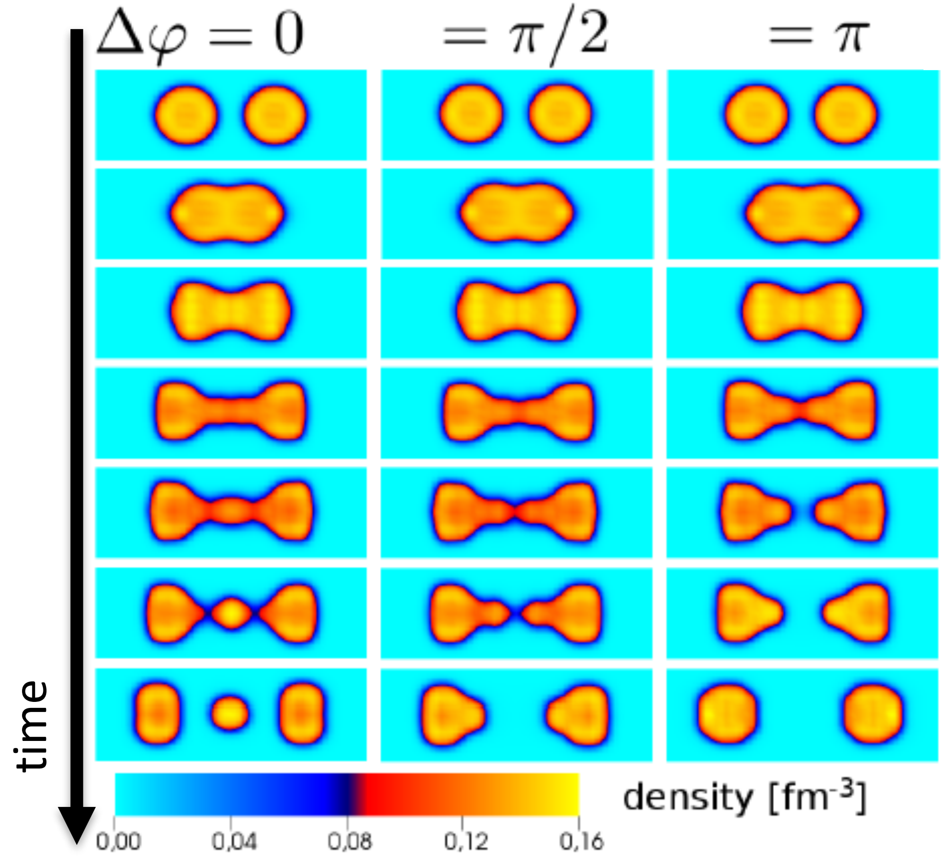
Magierski et al, PRL 119 (2017)

Collision between two superfluid nuclei:
Is there a visible aspects of gauge-angle orientation?

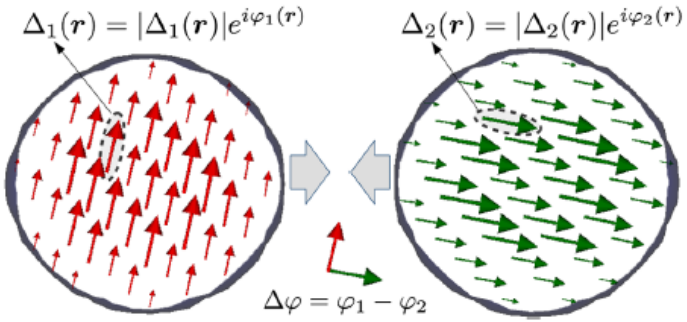
Emergent dynamical pairing phenomena close to the Coulomb barrier

Collisions with same energy but different orientations

^{240}Pu+^{240}Pu @ E=1.1 (E-Barrier)



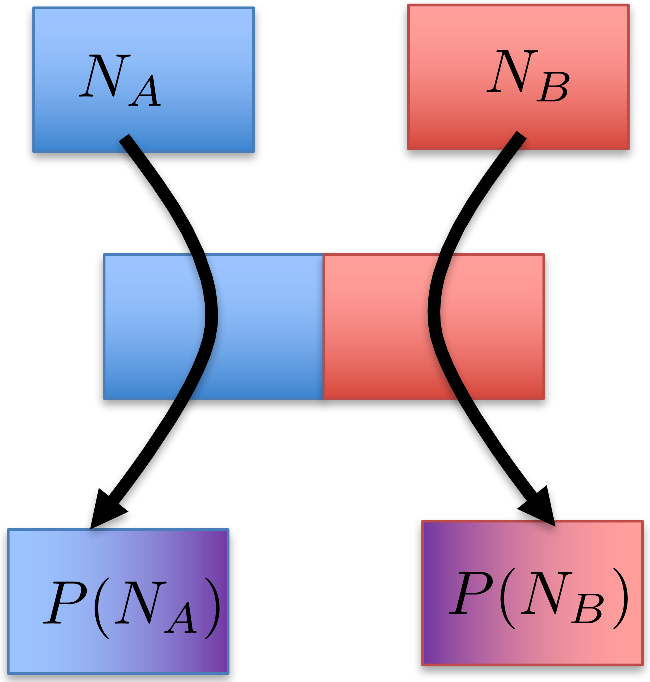
Magierski et al, PRL 119 (2017) – supplement material

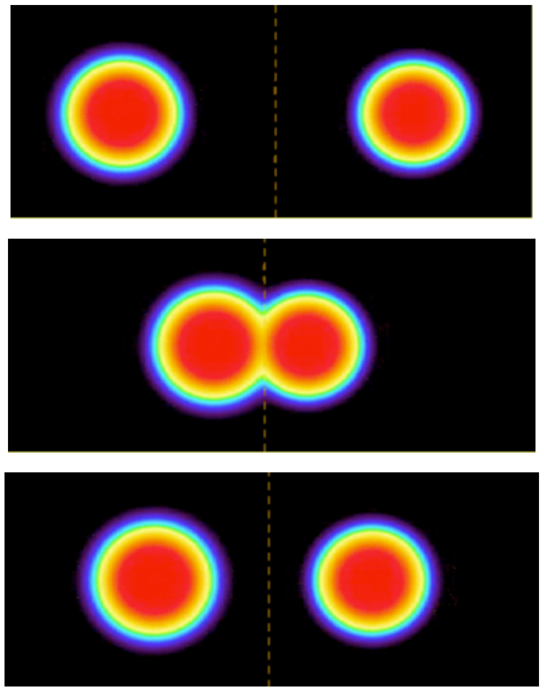


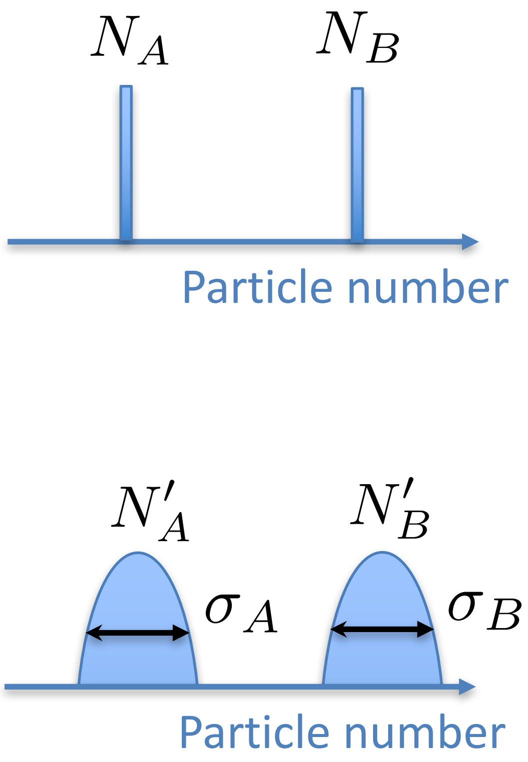
It was predicted that
Gauge angle
has a huge effect
in reaction between two
Superfluids!
(in a phase-space picture)

A minimal reaction model

Dietrich, PLB 32 (1970).

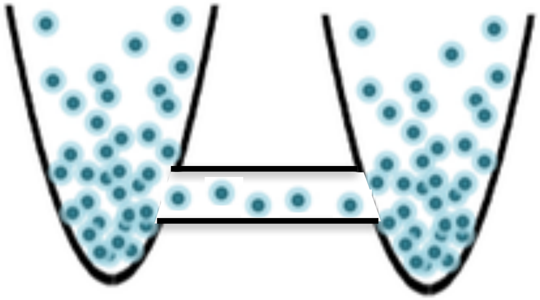


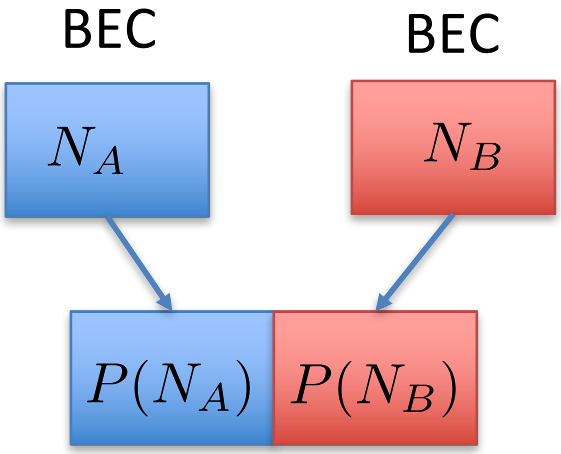




Effects beyond the mean-field

Interferences between 2 Bose-Einstein Condensate

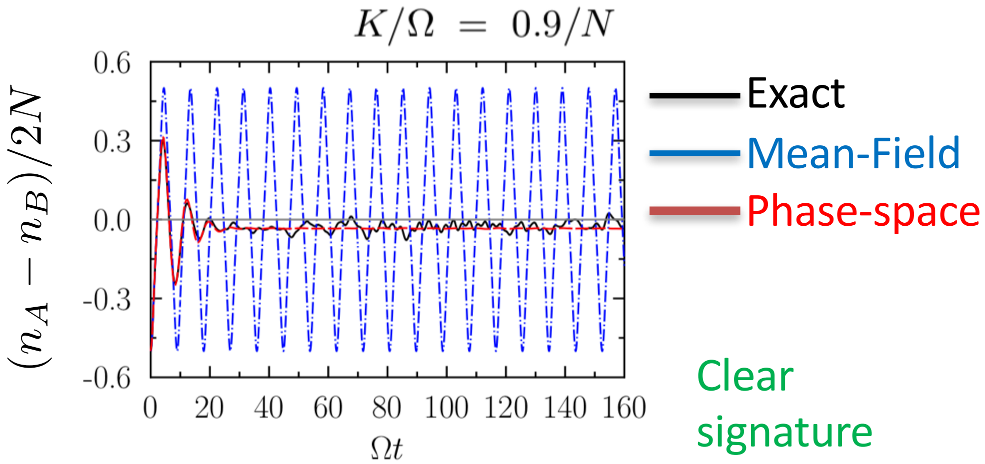


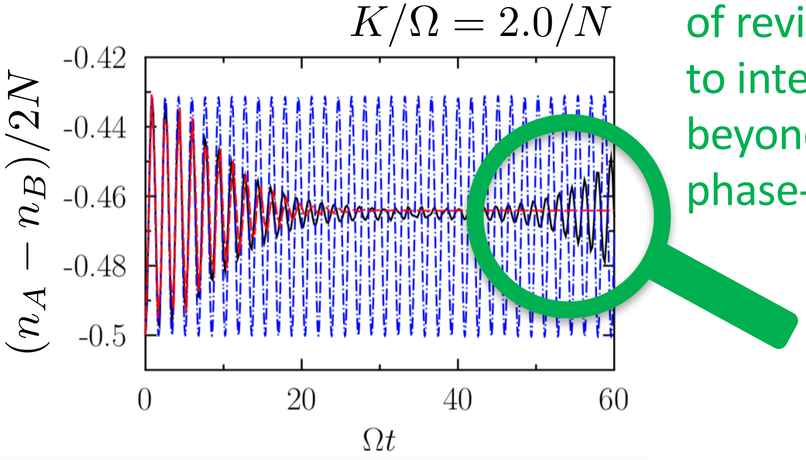


See for instance Castin, Dalibard, PRA 55 (1997).

$$H = \frac{\hbar\Omega}{2}\left(c_1^\dagger c_2 + c_2^\dagger c_1\right) + \hbar K\left[(c_1^\dagger)^2 c_1^2 + (c_2^\dagger)^2 c_2^2\right]$$

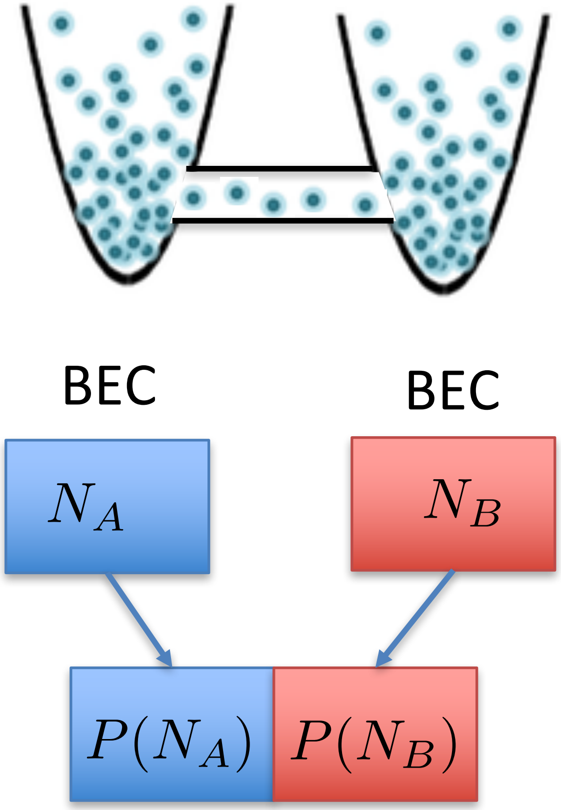
- Exact solution possible
- Application of phase-space method





Clear signature of revival due to interference beyond the phase-space

Interferences between 2 Bose-Einstein Condensate



BEC

BEC

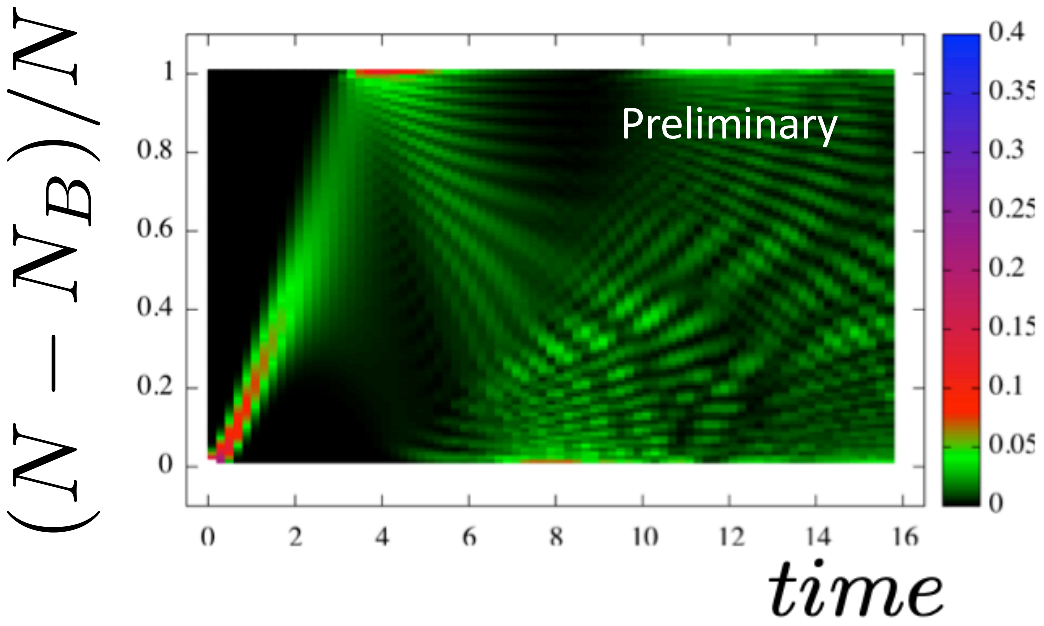
N_A

N_B

$P(N_A)$ $P(N_B)$

$$H = \frac{\hbar\Omega}{2}\left(c_1^\dagger c_2 + c_2^\dagger c_1\right) + \hbar K\left[(c_1^\dagger)^2 c_1^2 + (c_2^\dagger)^2 c_2^2\right]$$

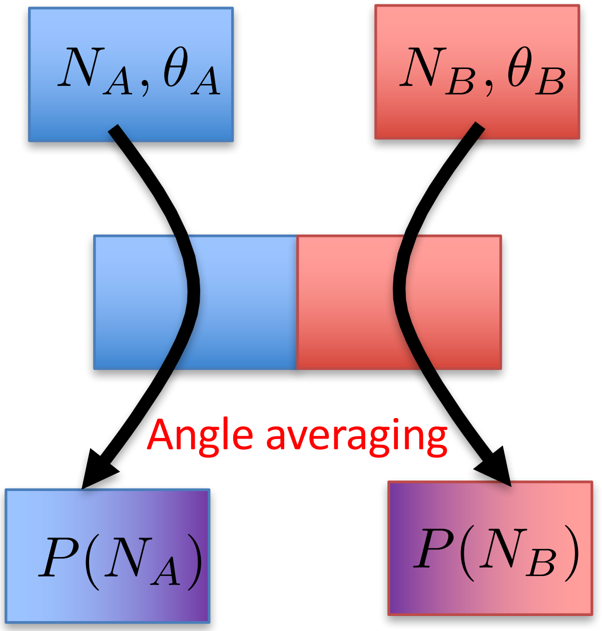
→ Exact solution



The case of fermion systems

A minimal reaction model

K. Dietrich, PLB 32 (1970).



N_A, θ_A

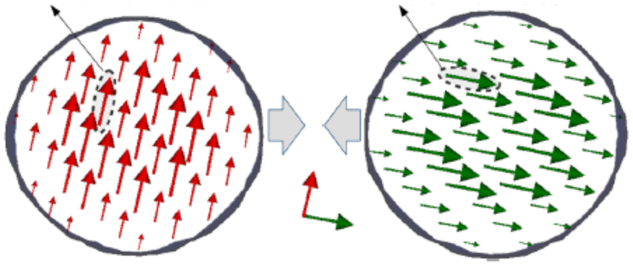
N_B, θ_B

Angle averaging

$P(N_A)$

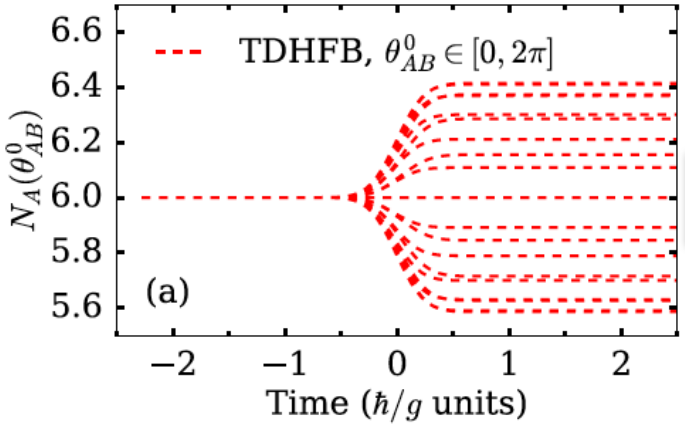
$P(N_B)$

$$\Delta_A = |\Delta_A| e^{i\theta_A} \qquad \Delta_B = |\Delta_B| e^{i\theta_B}$$

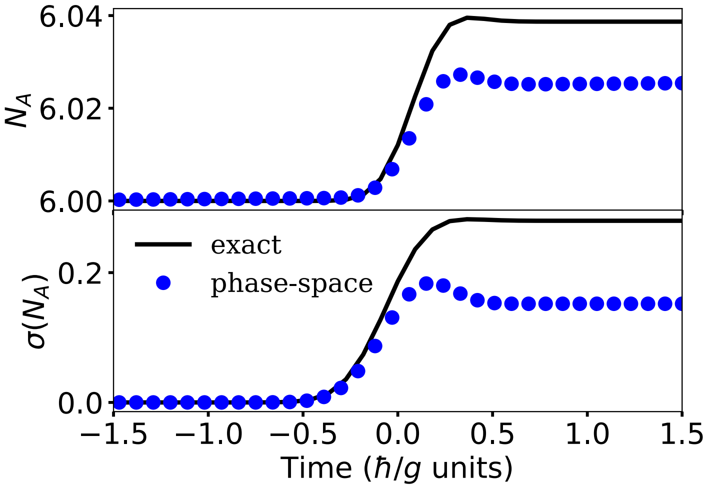


$$\theta^0_{AB} = \theta_A - \theta_B$$

Regnier, Lacroix, Phys. Rev. C 97 (2018)



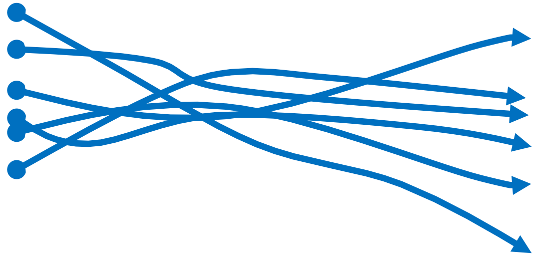
Semi-classical Phase-space distribution



The naïve phase-space picture does not work so well

Phase-space methods

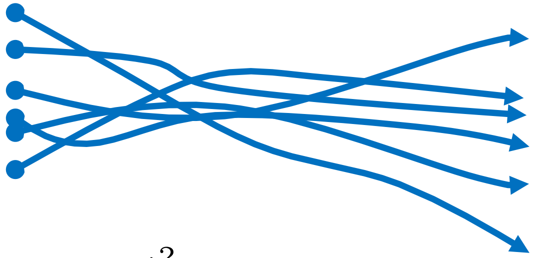
$\overline{\rho_{ij}(t_0)}$



$\overline{\rho_{ij}(t)}$

Different options beyond pure phase-space average

$$|\Psi(t_0)\rangle = \sum_\alpha C_\alpha(t_0)|\Phi_\alpha(t_0)\rangle$$



$$|\Psi(t)\rangle = \sum_\alpha C_\alpha(t)|\Phi_\alpha(t)\rangle$$

Equation of motion: $\delta \mathcal{S} = \delta \int_{t_1}^{t^2} dt \, \langle \Psi \, | H - I\hbar\partial_t | \, \Psi \rangle$

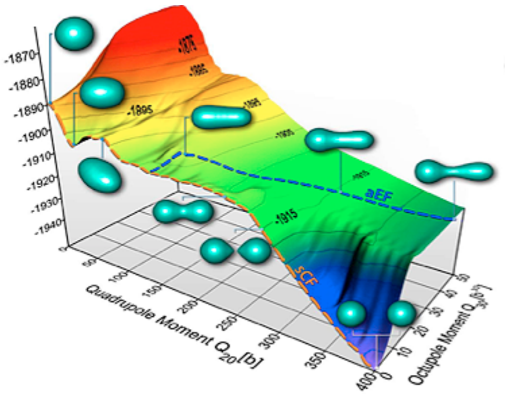
$$\Rightarrow \quad i\hbar\partial_t|\Psi\rangle = i\hbar\sum_\alpha \dot{C}_\alpha|\Phi_\alpha(t)\rangle + i\hbar\sum_\alpha C_\alpha(t)|\dot{\Phi}_\alpha(t)\rangle$$

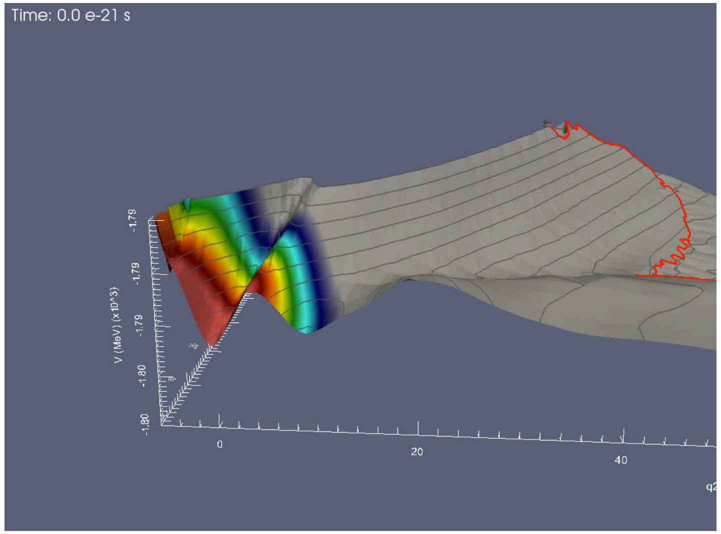
With coupled equations between $C_\alpha(t)$ and $|\Phi_\alpha(t)\rangle$

Towards Multi-Configutration TDHFB: possible simplifications

$$i\hbar\partial_t|\Psi\rangle = i\hbar\sum_\alpha \dot{C}_\alpha|\Phi_\alpha(t)\rangle + \cancel{i\hbar\sum_\alpha C_\alpha(t)|\dot{\Phi}_\alpha(t)\rangle}$$

Development on a fixed basis





Regnier, et al, Comp. Phys. Com. 200 (2016), ibid 225, (2018).

Development on given trajectories

$$i\hbar\partial_t|\Psi\rangle = i\hbar\sum_\alpha \dot{C}_\alpha|\Phi_\alpha(t)\rangle + i\hbar\sum_\alpha C_\alpha(t)\underline{|\dot{\Phi}_\alpha(t)\rangle}$$

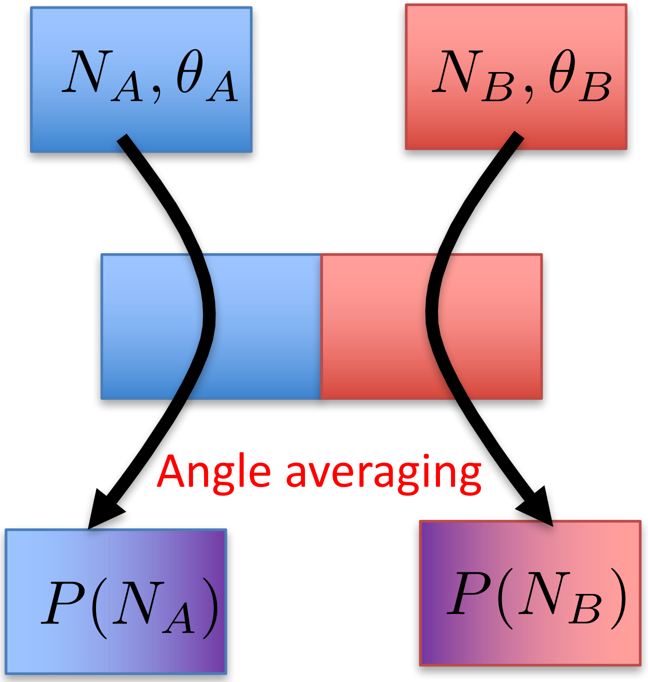
Assumed to be independent TDHF or TDHFB trajectories

Reinhard, Cusson, Goeke, Nucl. Phys. A398 (1983).

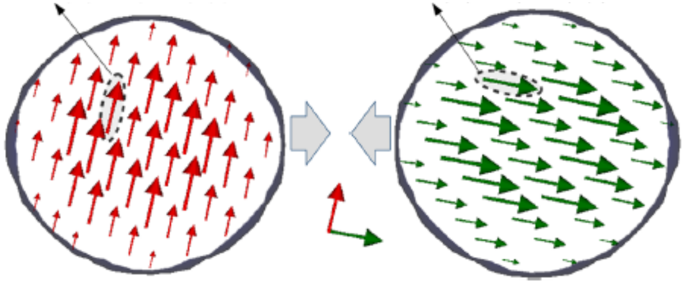
Back to the transfer between two superfluid systems

A minimal reaction model

K. Dietrich, Phys. Lett. B 32 (1970).

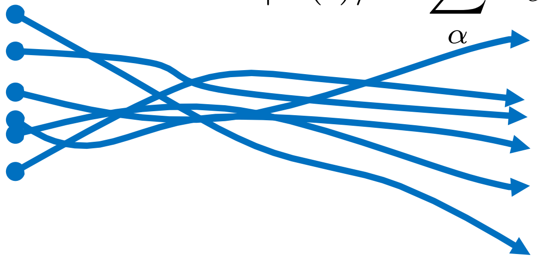


$$\Delta_A = |\Delta_A| e^{i\theta_A} \qquad \Delta_B = |\Delta_B| e^{i\theta_B}$$



$$\theta_{AB}^0 = \theta_A - \theta_B$$

$$|\Psi(t_0)\rangle = \sum_\alpha C_\alpha(t_0) |\Phi_\alpha(t_0)\rangle$$



$$|\Psi(t)\rangle = \sum_\alpha C_\alpha(t) |\Phi_\alpha(t)\rangle$$

Initial states

$$|\Phi(t_0)\rangle = \prod (U_L + \underset{\substack{\Uparrow \\ e^{2i\theta_A}}}{V_L a_R^\dagger a_R}) \otimes \prod (U_R + \underset{\substack{\Uparrow \\ e^{2i\theta_B}}}{V_R b_L^\dagger b_L}) |0\rangle$$

$$|\Psi(t_0)\rangle = P_{N_A} P_{N_B} |\Phi(t_0)\rangle$$

$$\Downarrow$$

$$|\Psi(t_0)\rangle = \iint d\theta_A d\theta_B \underbrace{C_{\theta_A \theta_B}(t_0)}_{\text{Coupled equation}} \underbrace{|\Phi(\theta_A, \theta_B)\rangle}_{\text{Independent TDHFB evolution}}$$

Regnier, Lacroix, *arXiv:1902.06491*

Back to the transfer between two superfluid systems

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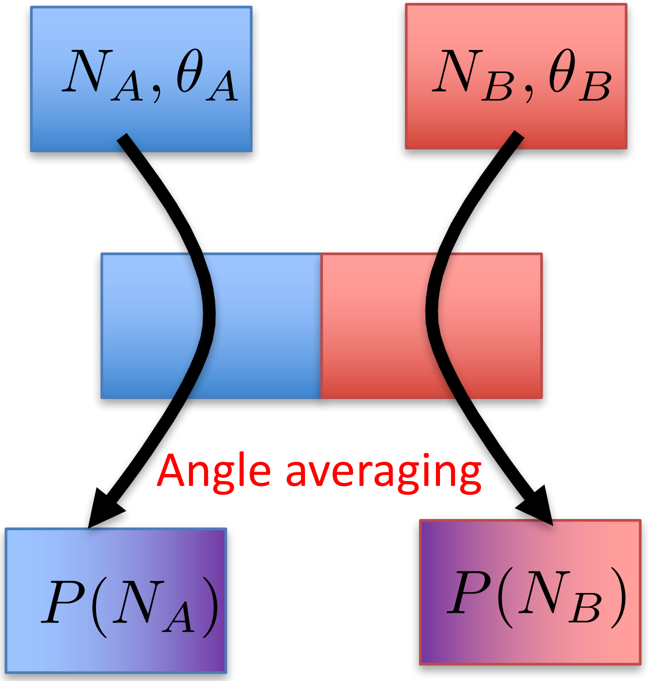
K. Dietrich, Phys. Lett. B 32 (1970).



N_A, θ_A

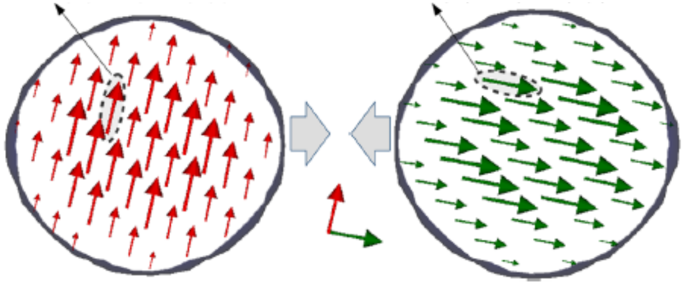
N_B, θ_B

Angle averaging

$P(N_A)$

$P(N_B)$

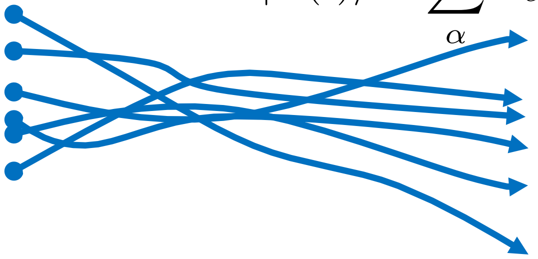
$\Delta_A = |\Delta_A| e^{i\theta_A}$ $\Delta_B = |\Delta_B| e^{i\theta_B}$



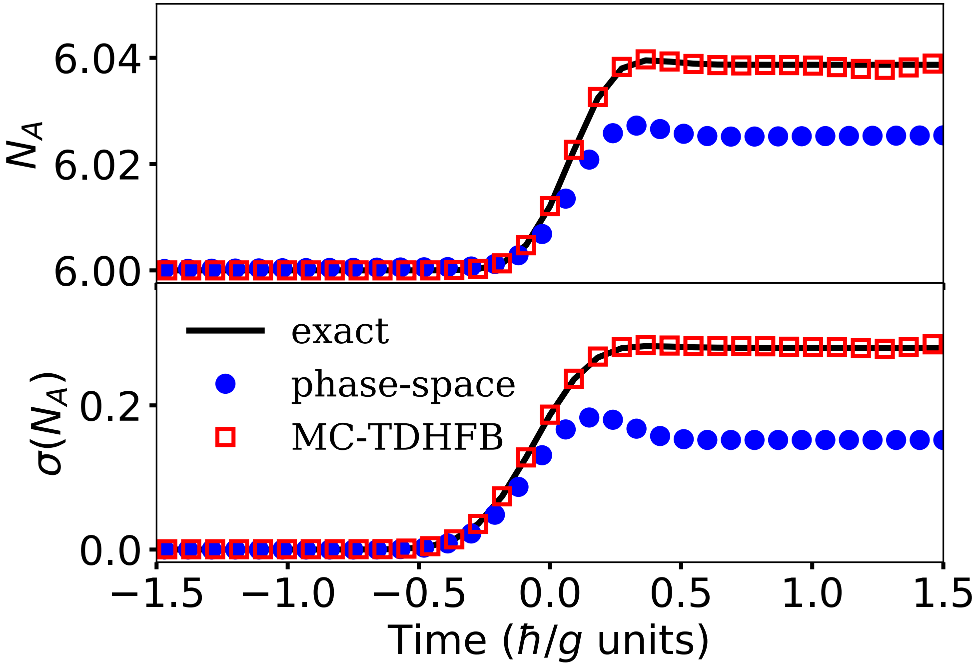
$$\theta^0_{AB} = \theta_A - \theta_B$$

$$|\Psi(t_0)\rangle = \sum_\alpha C_\alpha(t_0)|\Phi_\alpha(t_0)\rangle$$

$$|\Psi(t)\rangle = \sum_\alpha C_\alpha(t)|\Phi_\alpha(t)\rangle$$



Results



Regnier, Lacroix, *in preparation*

D. Lacroix and S. Ayik EPJA Review (2014)

Quantum or Auxiliary Field Monte-Carlo

$|\Phi\rangle\langle\Phi|$

$\overline{|\Phi_a\rangle\langle\Phi_b|}$

All Correlations

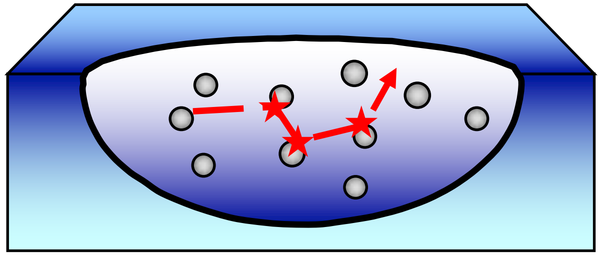
Stochastic TDHF like

$|\Phi\rangle\langle\Phi|$

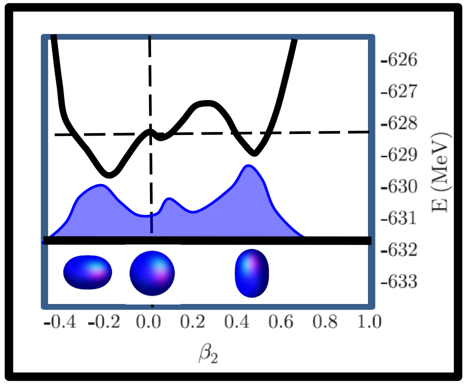
$\overline{|\Phi\rangle\langle\Phi|}$

Correlations that built up in time

Direct NN collisions

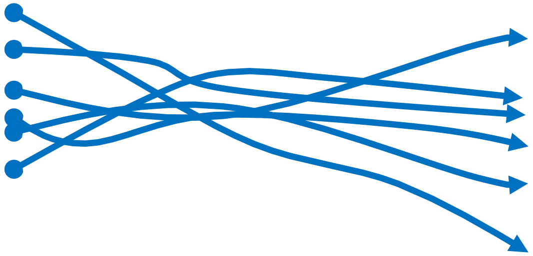


Initial fluctuations



Stochastic Mean-Field

$\overline{\rho_{ij}(t_0)}$



$\overline{\rho_{ij}(t)}$

Markovian limit, quantum-diffusion and stochastic Schrödinger Equation

GOAL: Restarting from an uncorrelated state $D = |\Phi_0\rangle\langle\Phi_0|$ we should:

1-have an estimate of $D = |\Psi(t)\rangle\langle\Psi(t)|$

2-interpret it as an average over jumps between “simple” states

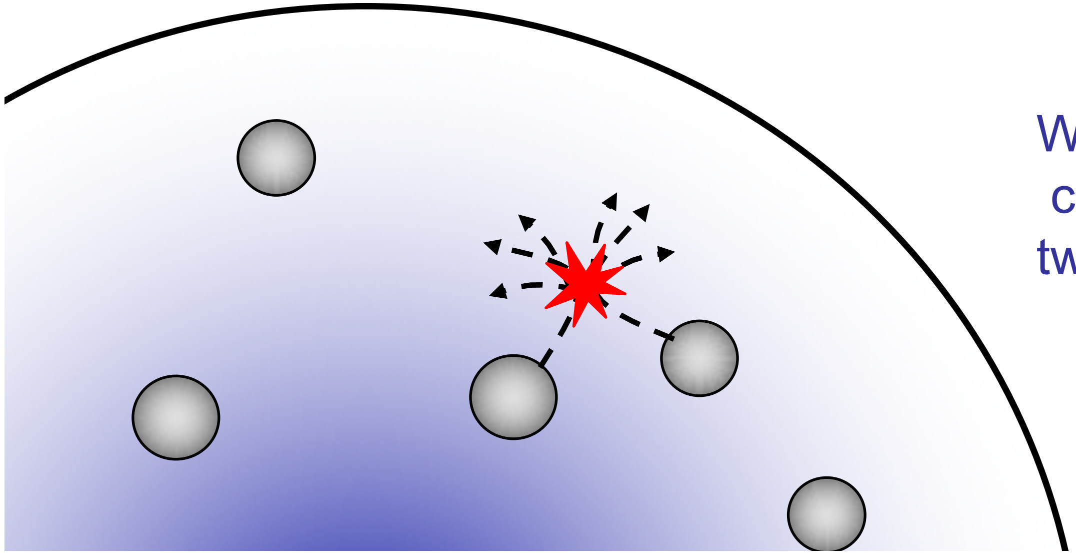
Weak coupling approximation : perturbative treatment

Reinhard and Suraud, Ann. of Phys. 216 (1992)

$$|\Psi(t')\rangle = |\Phi(t')\rangle - \frac{i}{\hbar}\int \delta v_{12}(s)|\Phi(s)\rangle\, ds - \frac{1}{2\hbar^2}T\left(\int\int \delta v_{12}(s)\delta v_{12}(s')\, ds ds'\right)|\Phi(s)\rangle$$

Residual interaction in the mean-field interaction picture

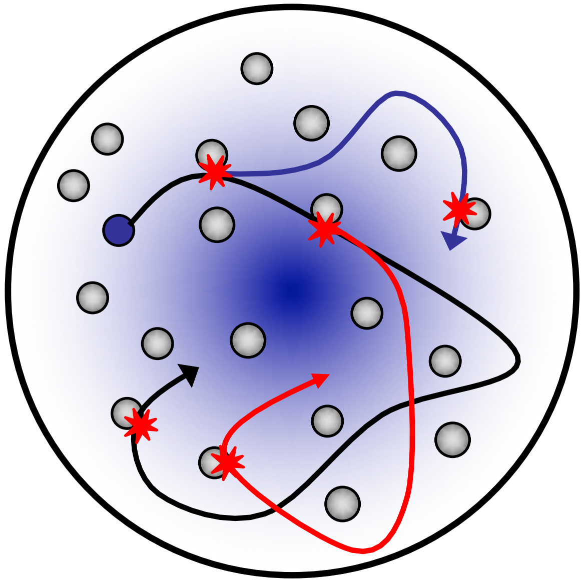
Statistical assumption in the Markovian limit :

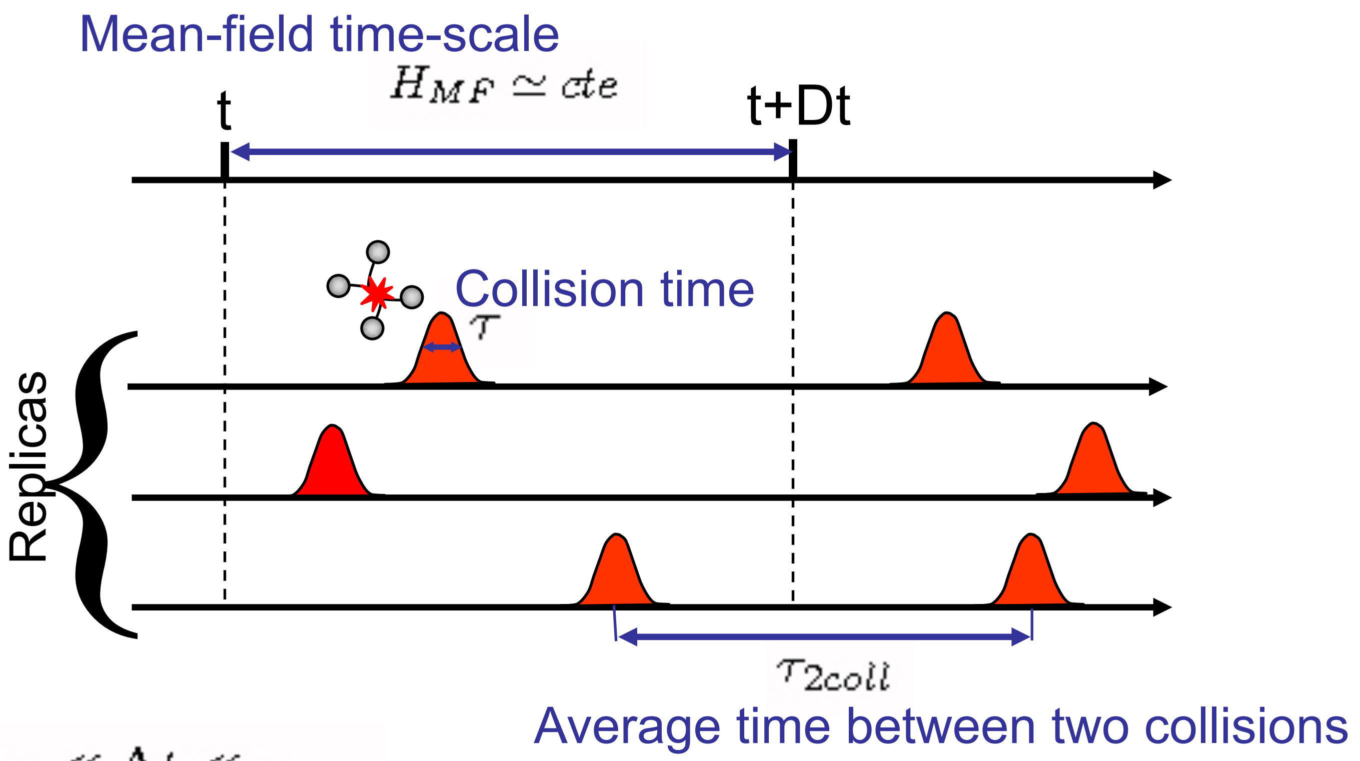


We assume that the residual interaction can be treated as an ensemble of two-body interaction:

$$\begin{cases} \overline{\delta v_{12}(s)} = 0 \\ \overline{\delta v_{12}(s)\delta v_{12}(s')} \propto \overline{\delta v_{12}^2(s)}\, e^{-(s-s')^2/2\tau^2} \end{cases}$$

Time-scale and Markovian dynamic





Hypothesis : $\tau \ll \Delta t \ll \tau_{2coll}$

Average Density Evolution:

➡ $$\overline{\Delta D} = \frac{\Delta t}{i\hbar}[H_{MF}, D] - \frac{\tau \Delta t}{2\hbar^2}\overline{[\delta v_{12}, [\delta v_{12}, D]]}$$

Dissipation: link between Extended TDHF and Lindblad Eq.

One-body density
Master equation
step by step

$$\overline{\Delta D} = \frac{\Delta t}{i\hbar}[H_{MF}, D] - \frac{\tau \Delta t}{2\hbar^2}\overline{[\delta v_{12},[\delta v_{12}, D]]}$$

Initial simple state

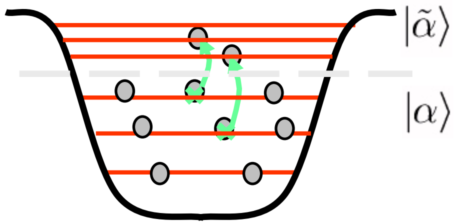
$$D = |\Phi\rangle\langle\Phi|$$

$$\rho = \sum_\alpha |\alpha\rangle\langle\alpha|$$

$$i\hbar\frac{d}{dt}\rho = [h_{MF},\rho] - \frac{\tau}{2\hbar^2}\mathcal{D}(\rho)$$

with $\langle j|\mathcal{D}|i\rangle = \overline{\left\langle\left[\left[a_i^+ a_j, \delta v_{12}\right], \delta v_{12}\right]\right\rangle}$

2p-2h nature
of the interaction



$$\mathcal{D}(\rho) = Tr_2\left[v_{12}, C_{12}\right]$$

with
$$\begin{aligned} C_{12} &= (1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 \\ &\quad -\rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2) \end{aligned}$$

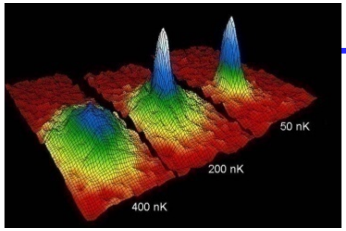
Separability of the interaction $v_{12} = \sum_\lambda O_\lambda(1)O_\lambda(2)$

$$\mathcal{D}(\rho) = \sum_k \gamma_k\left(A_k A_k \rho + \rho A_k A_k - 2A_k\rho A_k\right)$$

- Dissipation contained in Extended TDHF is included
- The master equation is a Lindblad equation
- Associated SSE

Lacroix, PRC73 (2006)

Application to Bose-Einstein condensates



1D bose condensate with gaussian two-body interaction

N-body density: $D = |N:\alpha\rangle\langle N:\alpha|$

SSE on single-particle state :

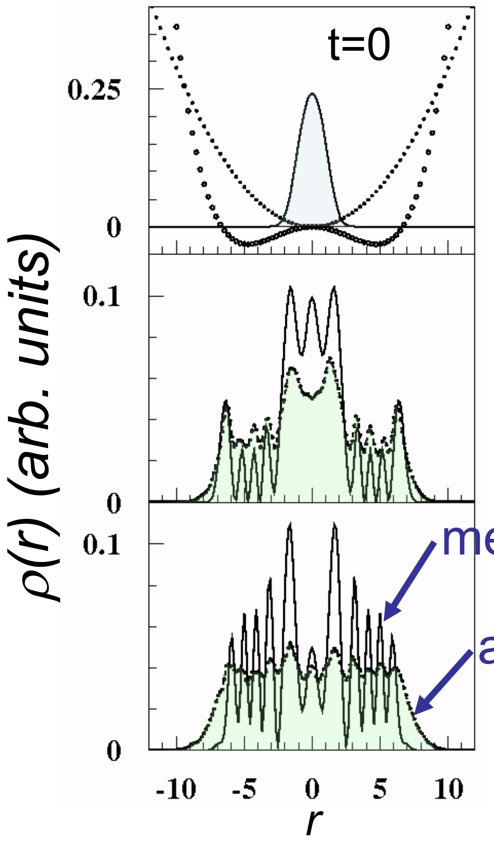
$$d|\alpha\rangle = \left\{\frac{dt}{i\hbar}h_{MF}(\rho) + \sum_k dW_k(1-\rho)A_k - \frac{dt\tau}{2\hbar^2}\sum_k \gamma_k\left[A_k^2\rho + \rho A_k\rho A_k - 2A_k\rho A_k\right]\right\}|\alpha\rangle$$

with $dW_k dW_{k'} = -\frac{dt\tau}{\hbar^2}\gamma_k\delta_{kk'}$

➡ The numerical effort is fixed by the number of A_k

Density evolution

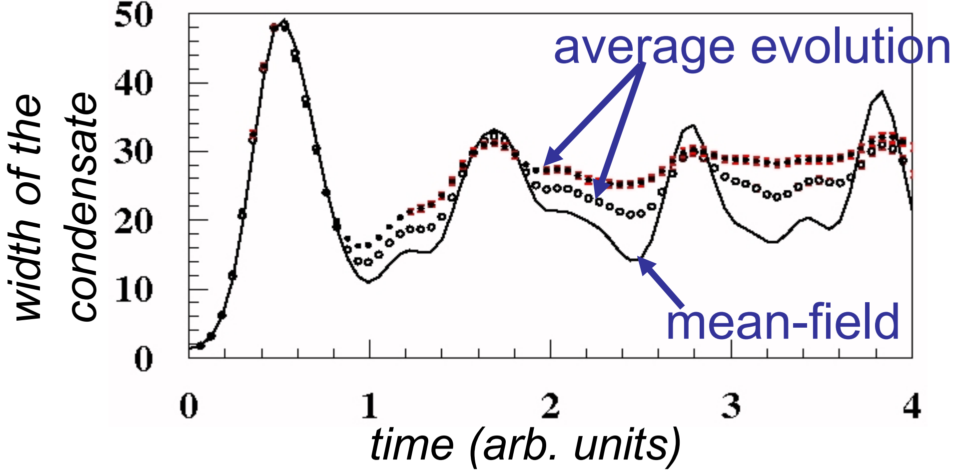
Condensate size



t>0

mean-field

average evolution



Summary of different approaches presented

BBGKY like approaches

- TD2RDM (with pairing approx.) or ETDHF with memory

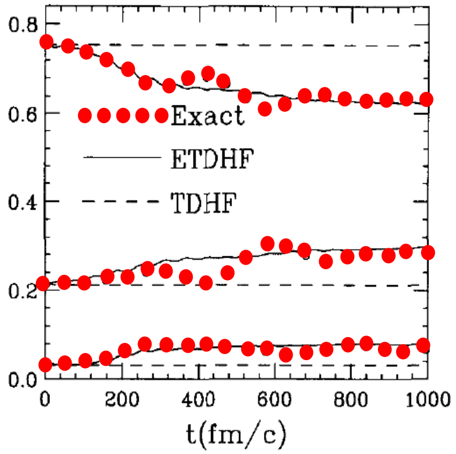
Phase-space methods

- Quite Successful application to different systems (normal and superfluid)
- Application to some nuclear physics cases

Introduction of interference beyond the phase-space approach

- Ongoing projects: application of MC-TDHF or MC-TDHFB approaches
- Applications in some specific situations

Dissipation



Correlation effects on emission