Nuclear giant dipole resonance in the lattice Hamiltonian method of BUU equation

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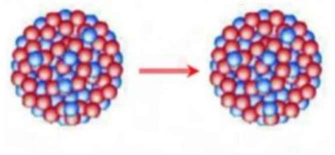
Transport model of heavy-ion collisions



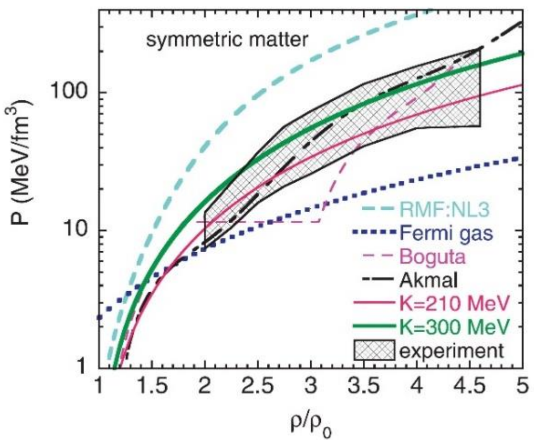
Wigner function $$f(\vec{r},\vec{p}) = \frac{1}{(2\pi\hbar)^3}\int \exp(-i\frac{\vec{p}}{\hbar}\cdot\vec{s})\rho(\vec{r}+\vec{s}/2,\vec{r}-\vec{s}/2)\mathrm{d}^3 s$$

Semi-classical phase space distribution

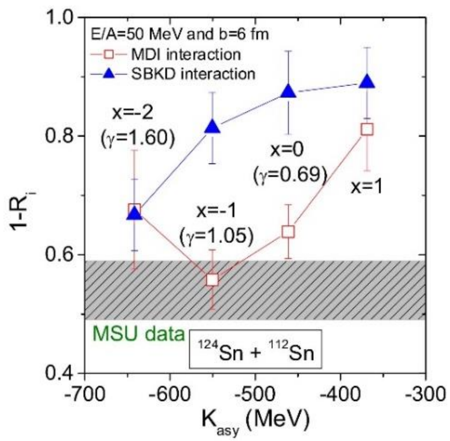
Boltzmann-Uehling-Uhlenbeck equation

$$\frac{\partial f(\vec{r},\vec{p})}{\partial t} + \frac{\vec{p}}{m}\cdot\vec{\nabla}_r f(\vec{r},\vec{p}) + \boxed{\vec{\nabla}_p U}\cdot\vec{\nabla}_r f(\vec{r},\vec{p}) - \boxed{\vec{\nabla}_r U}\cdot\vec{\nabla}_p f(\vec{r},\vec{p}) = I\left[f(\vec{r},\vec{p})\right]$$

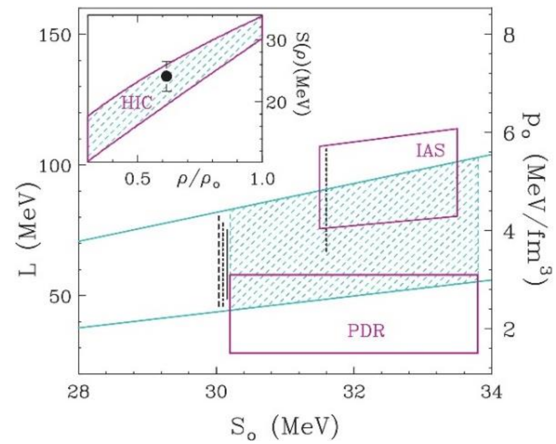
Related to the equation of state of nuclear matter



P. Danielewicz et al, Science298, 1592-1596 (2002)

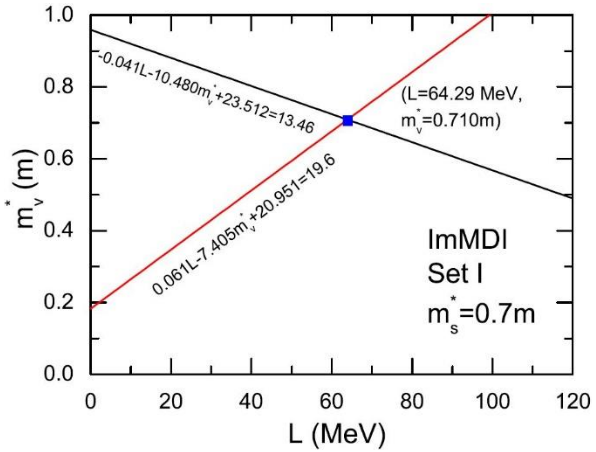


L.-W. Chen et al, PRL94, 032701 (2005)

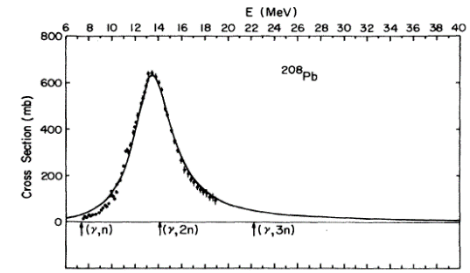


M. B. Tsang et al, PRL102, 122701 (2009)

Giant resonance in BUU equation



H.-Y. Kong et al, PRC95, 034324 (2017)



centroid energy and the electric dipole polarizability → Slope parameter of Symmetry energy and effective mass splitting

$$I(f) = -\int \frac{d\vec{p}_2}{(2\pi\hbar)^3} v_{\mathrm{rel}} \boxed{\sigma^*_{\mathrm{NN}}} \left[f_1 f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f_1)(1-f_2) \right]$$

In-medium cross section ↔ two particle-two hole correlation

↓

Collisional damping

Spreading width → In medium nucleon-nucleon collision

In order to calculate giant resonance within BUU

- The lattice Hamiltonian method are used to solve the BUU equation, which conserves the energy exactly and momentum to a high degree of accuracy

- The initial radial density distribution are obtained self-consistently through varying the same Hamiltonian used in the BUU calculation (Thomas-Fermi initialization)

- Stochastic method are used in dealing with the nucleon-nucleon collision term of BUU equation

- Technically, the present method are implemented based on the high performance GPU parallel computing, which enables the use of more test particle

1 Background and motivation

2 The lattice Hamiltonian method

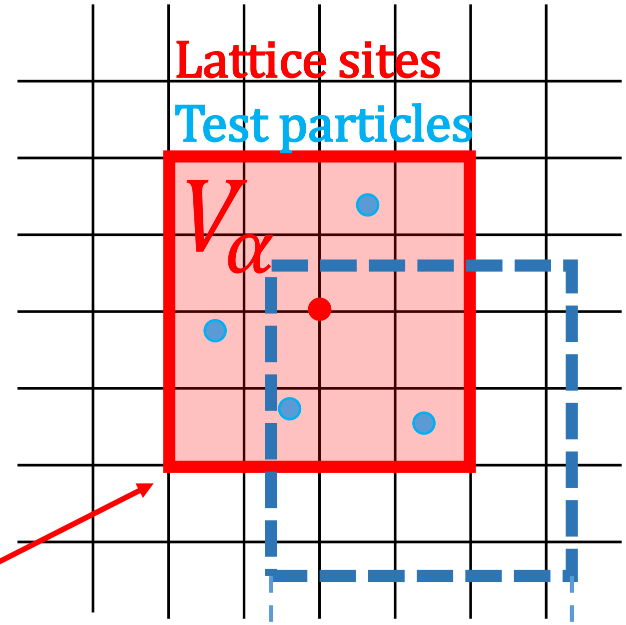
3 Giant dipole resonance with the lattice Hamiltonian method

The lattice Hamiltonian method

To study nuclear giant resonance in tranport model, we need long time evolution

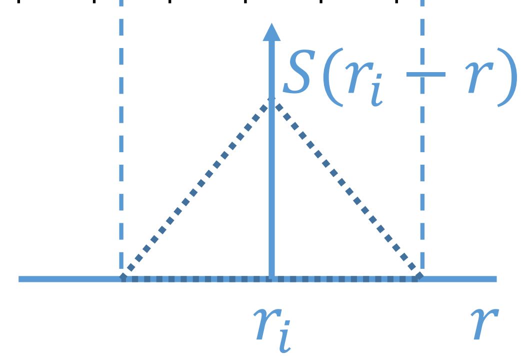
suitable

The lattice Hamiltonian method of solving BUU equation conserves the total energy



$$f(\vec{r}_\alpha, \vec{p}) = \frac{1}{2} \frac{1}{N_\mathrm{E}/(2\pi\hbar)^3} \sum_{\boxed{i \in V_\alpha}} S(\vec{r_i} - \vec{r}) \delta(\vec{p_i} - \vec{p})$$

$$\rho(\vec{r}_\alpha) = 2 \int f(\vec{r}_\alpha, \vec{p}) \frac{d\vec{p}}{(2\pi\hbar)^3} = \frac{1}{N_\mathrm{E}} \sum_{i \in V_\alpha} S(\vec{r_i} - \vec{r})$$



$$H \approx \sum_i \frac{p_i^2}{2m} + N_\mathrm{TP} \Delta V \sum_\alpha \left\{ \mathcal{H}^\mathrm{local}[\rho(r_\alpha)] + \mathcal{H}^\mathrm{DD}[\rho(r_\alpha)] + \mathcal{H}^\mathrm{Cou}[\rho(r_\alpha)] + \mathcal{H}^\mathrm{grad}(\vec{r}_\alpha) + \mathcal{H}^\mathrm{MD}(\vec{r}_\alpha) \right\}$$

R.J. Lenk and V.R. Pandharipande, PRC39,2242 (1989)

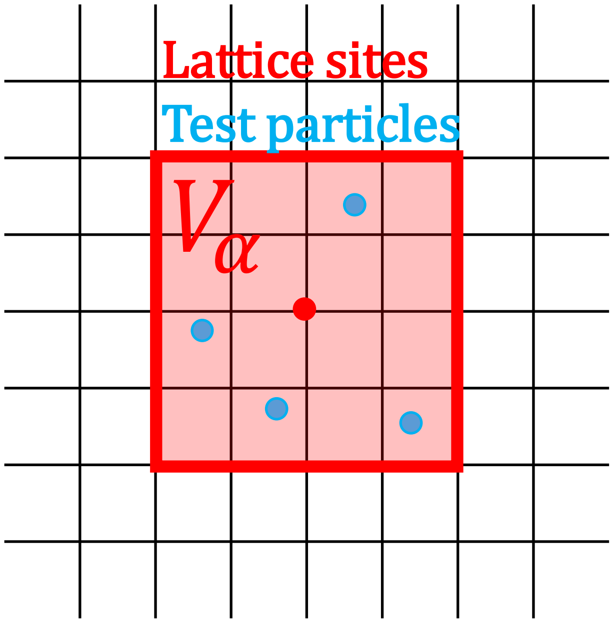
The lattice Hamiltonian method

The treatment of momentum dependent part

$$\varepsilon_{MD}(\vec{r}_\alpha) = \int \sum_{n=1,2,3} C_n(\vec{p}_1 - \vec{p}_2)^{2n} f(\vec{r}_\alpha, \vec{p}_1) f(\vec{r}_\alpha, \vec{p}_2) d^3\vec{p}_1 d^3\vec{p}_2 + \delta\ terms$$

$$= \sum_{i \in V_\alpha} S(\vec{r}_\alpha - \vec{r}_i) \sum_{j \in V_\alpha} \sum_{n=1,2,3} C_n(\vec{p}_i - \vec{p}_j)^{2n} S(\vec{r}_\alpha - \vec{r}_j) + \delta\ terms$$

When derivatived with respect to coordinate and momentum of test particles

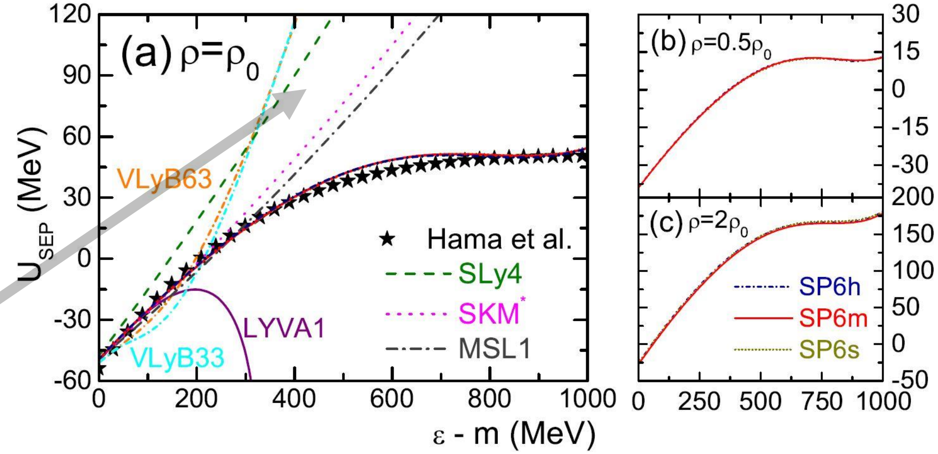


$$\frac{\partial \varepsilon_{MD}(\vec{r}_\alpha)}{\partial \vec{r}_i} = 2S'(\vec{r}_i - \vec{r}_\alpha) \times \sum_{j \in V_\alpha} \sum_{n=1,2,3} \left\{ C^{[n]}(\vec{p}_i - \vec{p}_j)^2 n + \delta\ terms \right\}$$

$$\frac{\partial \varepsilon_{MD}(\vec{r}_\alpha)}{\partial \vec{p}_i} = 2S(\vec{r}_i - \vec{r}_\alpha) \sum_{j \in V_\alpha} \sum_{n=1,2,3} \left\{ 2nC^{[n]}(\vec{p}_i - \vec{p}_j)(\vec{p}_i - \vec{p}_j)^{2(n-1)} + \delta\ term \right\}$$

Skyrme pseudopotential

$$v_{Sk} = t_0^{(0)}(1 + x_0^{(0)}\hat{P}_\sigma) + \frac{1}{6}t_3^{(0)}(1 + x_3^{(0)}\hat{P}_\sigma)\rho^\alpha(\vec{R})$$
$$+ \frac{1}{2}t_1^{(2)}(1 + x_1^{(2)}\hat{P}_\sigma)[\vec{k}'^2 + \vec{k}^2]$$
$$+ t_2^{(2)}(1 + x_2^{(2)}\hat{P}_\sigma)\vec{k}' \cdot \vec{k}$$



R. Wang, L.-W. Chen, and Y. Zhou, PRC98, 054618 (2018)

N3LO local nuclear energy density functional → Corresponding N3LO Skyrme pseudopotential

B.G. Carlsson et al PRC78,044326 (2008)

F. Raimondi et al PRC83,054311 (2011)

4^{th} and 6^{th} order of momentum

$$+\tfrac{1}{4}t_1^{(4)}(1 + x_1^{(4)}\hat{P}_\sigma)[(\vec{k}'^2 + \vec{k}^2)^2 + 4(\vec{k}' \cdot \vec{k})^2] + t_2^{(4)}(1 + x_2^{(4)}\hat{P}_\sigma)(\vec{k}' \cdot \vec{k})(\vec{k}'^2 + \vec{k}^2)$$

$$+\tfrac{1}{2}t_1^{(6)}(1 + x_1^{(6)}\hat{P}_\sigma)(\vec{k}'^2 + \vec{k}^2)^2[(\vec{k}'^2 + \vec{k}^2)^2 + 12(\vec{k}' \cdot \vec{k})^2]$$

$$+t_2^{(6)}(1 + x_2^{(6)}\hat{P}_\sigma)(\vec{k}' \cdot \vec{k})[3(\vec{k}'^2 + \vec{k}^2)^2 + 4(\vec{k}' \cdot \vec{k})^2]$$

plus spin-orbit and tensor terms

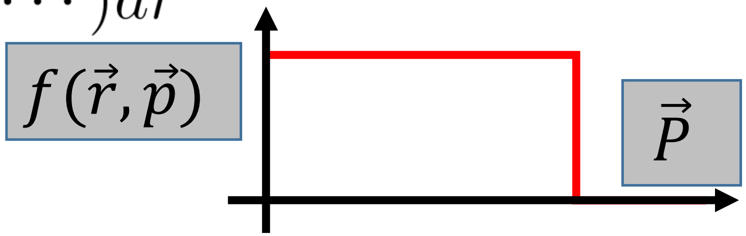
an overall $\delta(\vec{r}_1 - \vec{r}_2)$ is implicit

Initialization of test particles

Thomas-Fermi initialization

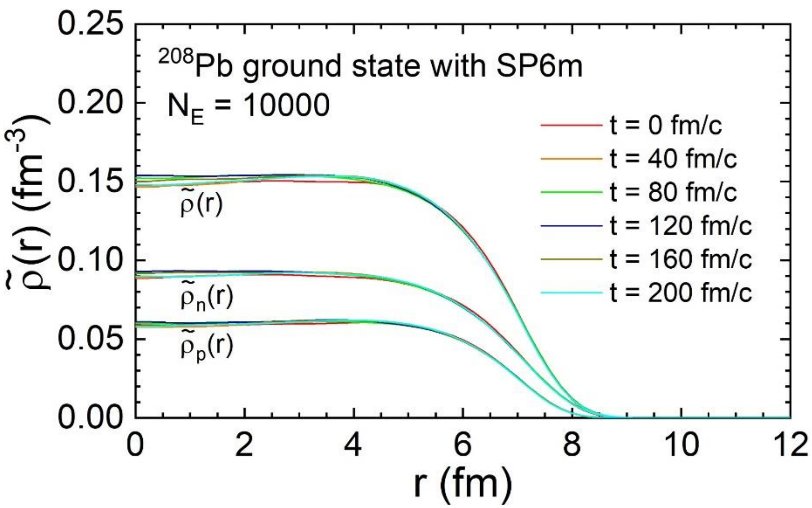
$$E = \int \mathcal{H}(r, \rho_\tau(r), \nabla\rho_\tau(r), \nabla^2\rho_\tau(r)\cdots)dr$$

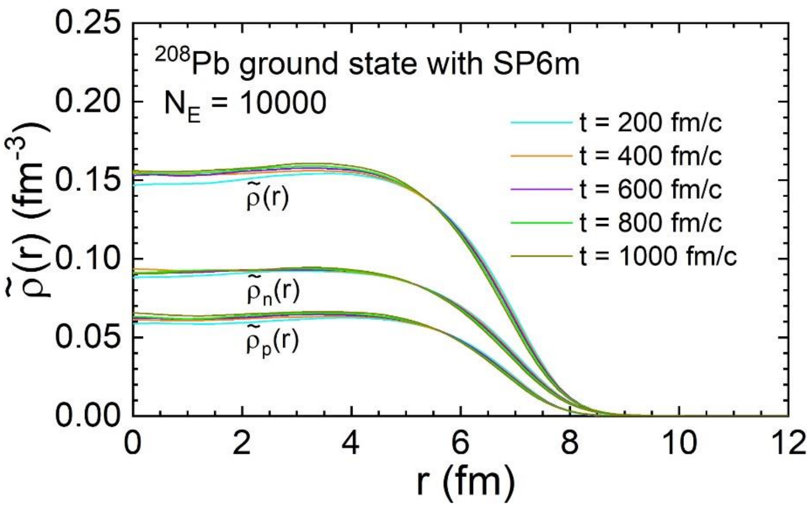
$$\frac{\partial H}{\partial \rho} - \nabla \frac{\partial H}{\partial(\nabla\rho)} + \nabla^2 \frac{\partial H}{\partial(\nabla^2\rho)} = 0$$



$$\boxed{\frac{1}{2m}\{p_\tau^F[\rho(r)]\}^2 + U_\tau\{p_\tau^F[\rho(r)], r\} = \mu_\tau}$$

The density profile in lattice Hamiltonian method

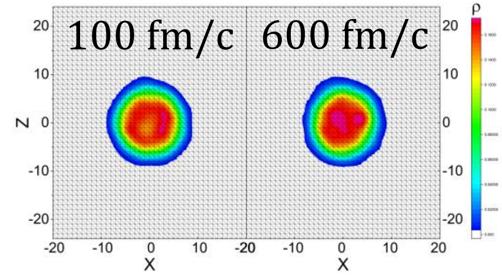


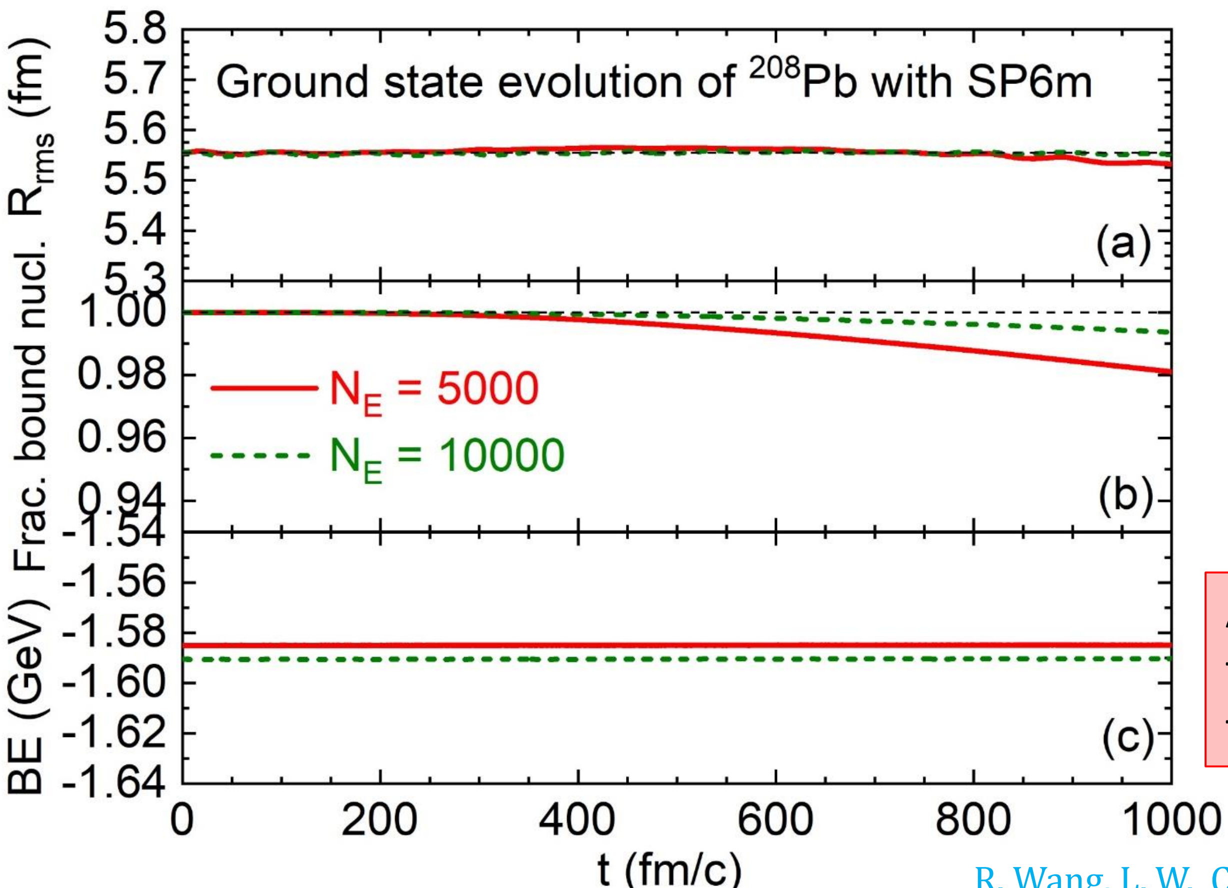


R. Wang, L. W. Chen, and Z. Zhang, PRC99, 044609 (2019)

Ground state LHV Calculations

The rms radius, fraction of bound nucleons, binding energy of ground state evolution





Free test nucleons are those whose form factor do not overlap with that of other test nucleons

$$\langle r^2 \rangle = N_{\rm nucl}^{-1} \int_{\rm bound} d^3 r^2 \rho(r)$$

At 1000 fm/c, only about 2% of test nucleons evaporated from the nucleus for $N_E = 5000$ case

R. Wang, L. W. Chen, and Z. Zhang, PRC99, 044609 (2019)

1 Background and motivation

2 The lattice Hamiltonian method

3 Giant dipole resonance with the lattice Hamiltonian method

Giant resonance in LH method

Small excitation of the external field $\hat{H}(t) = \hat{H}_0 + \lambda \hat{Q} \delta(t - t_0)$

In linear response theory

$$\delta \langle \hat{Q} \rangle (t) = \langle \hat{Q} \rangle (t) - \langle 0 | \hat{Q} | 0 \rangle (t) = -\frac{2\lambda\theta(t)}{\hbar} \sum_f |\langle f | \hat{Q} | 0 \rangle|^2 \sin \frac{(E_f - E_0)t}{\hbar}$$

Under the assumption that $\hat{Q}$ is one-body operator $\langle \hat{Q} \rangle (t) = \sum_i^N \hat{q}_i$

$$\langle \hat{Q} \rangle (t) = \langle \Phi | \hat{Q} | \Phi \rangle = \int \langle \Phi | \vec{r}_1 \rangle \langle \vec{r}_1 | \hat{Q} | \vec{r}_2 \rangle \langle \vec{r}_2 | \Phi \rangle d\vec{r}_1 d\vec{r}_2 = \int \langle \vec{r}_2 | \Phi \rangle \langle \Phi | \vec{r}_1 \rangle \langle \vec{r}_1 | \hat{Q} | \vec{r}_2 \rangle d\vec{r}_1 d\vec{r}_2$$

$$\langle \hat{Q} \rangle (t) = \int f(\vec{r}, \vec{p}) q(\vec{r}, \vec{p}) d\vec{r} d\vec{p} \qquad q(\vec{r}, \vec{p}) \equiv \int \exp(-i \frac{\vec{p}}{\hbar} \cdot \vec{s}) Q(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2}) d\vec{s}$$

Strength function

$$S(E) \equiv \sum_f |\langle f | \hat{Q} | 0 \rangle \delta(E - E_f - E_0) = -\frac{1}{\pi\lambda} \int_0^\infty dt \, \delta \langle \hat{Q} \rangle (t) \sin \frac{Et}{\hbar}$$

Giant resonance in LH method

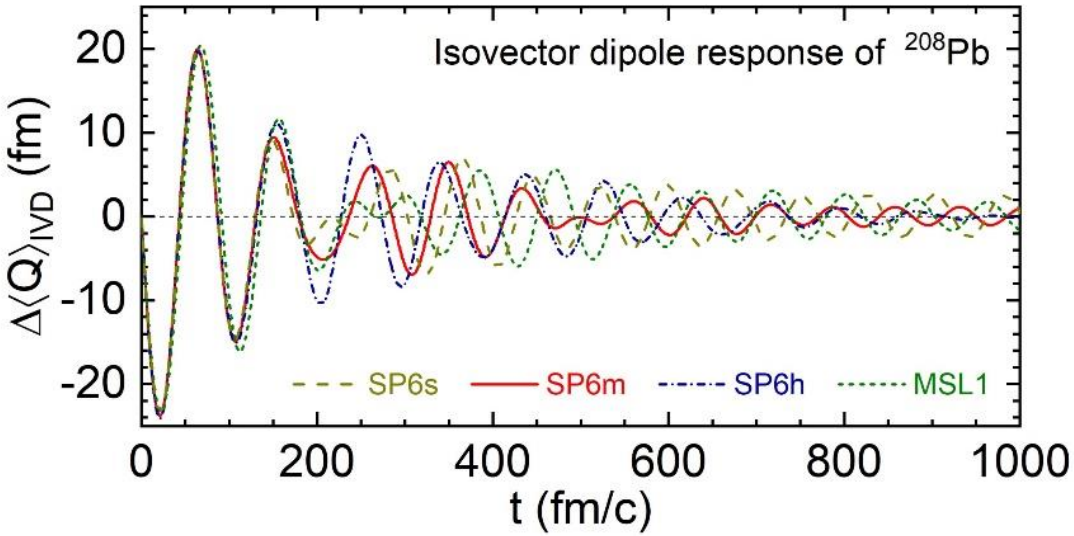
Vlasov result

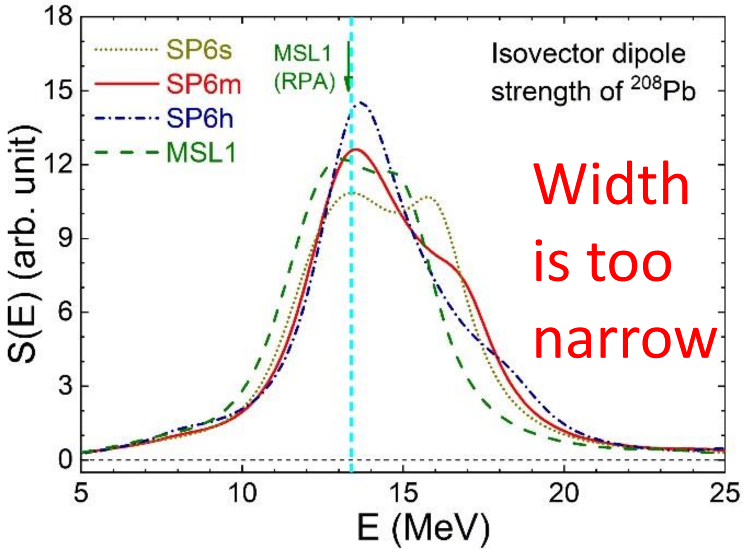
$$\vec{r}_i \longrightarrow \vec{r}_i + \lambda \frac{\partial q(\vec{r}_i, \vec{p}_i)}{\partial \vec{p}_i}, \qquad \vec{p}_i \longrightarrow \vec{p}_i + \lambda \frac{\partial q(\vec{r}_i, \vec{p}_i)}{\vec{r}_i}$$

Isovector dipole mode

$$\hat{Q}_{\mathrm{IVD}} = \frac{N}{A}\sum_i^{Z} \hat{z}_i - \frac{Z}{A}\sum_i^{N} \hat{z}_i$$

The response is multiplied by an artificial attenuation: $\Delta\langle Q\rangle(t)e^{-\gamma t/2\hbar}$





We can not describe the GDR width with Vlasov calculation alone

R. Wang, L. W. Chen, and Z. Zhang, PRC99, 044609 (2019)

Stochastic collision in LH method

For phase space volume $(\vec{p}_1 \pm \Delta^3 p_1)$ and $(\vec{p}_2 \pm \Delta^3 p_2)$ at $\vec{r}_\alpha$

$$\Delta N_{22}^{\text{coll}}(\vec{r}_\alpha) = \frac{\Delta^3 p_1}{(2\pi\hbar)^3} \left| \frac{df(\vec{r}_\alpha, \vec{p}_1)}{dt} \right|_{\vec{p}_2}^{\text{coll}} l^3 \Delta t$$

$$\left| \frac{df(\vec{r}_\alpha, \vec{p}_1)}{dt} \right|_{\vec{p}_2}^{\text{coll}} = \frac{\Delta^3 p_1}{(2\pi\hbar)^3} f(\vec{r}_\alpha, \vec{p}_1) f(\vec{r}_\alpha, \vec{p}_2)$$

$$\times \boxed{\int \frac{\mathrm{d}\vec{p}_3}{(2\pi\hbar)^3} \frac{\mathrm{d}\vec{p}_4}{(2\pi\hbar)^3} |\mathcal{M}_{12\to 34}|^2 (2\pi)^4 \delta^4(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)}$$

$$= \frac{\Delta^3 p_1}{(2\pi\hbar)^3} f(\vec{r}_\alpha, \vec{p}_1) f(\vec{r}_\alpha, \vec{p}_2) \boxed{v_{\text{rel}} \sigma_{\text{NN}}^*}$$

by definition

In lattice Hamiltonian method $$f(\vec{r}_\alpha, \vec{p}_i) = \frac{\Delta N_i S(\vec{r}_i - \vec{r}_\alpha)}{\frac{1}{(2\pi\hbar)^3} \Delta^3 p_i}$$

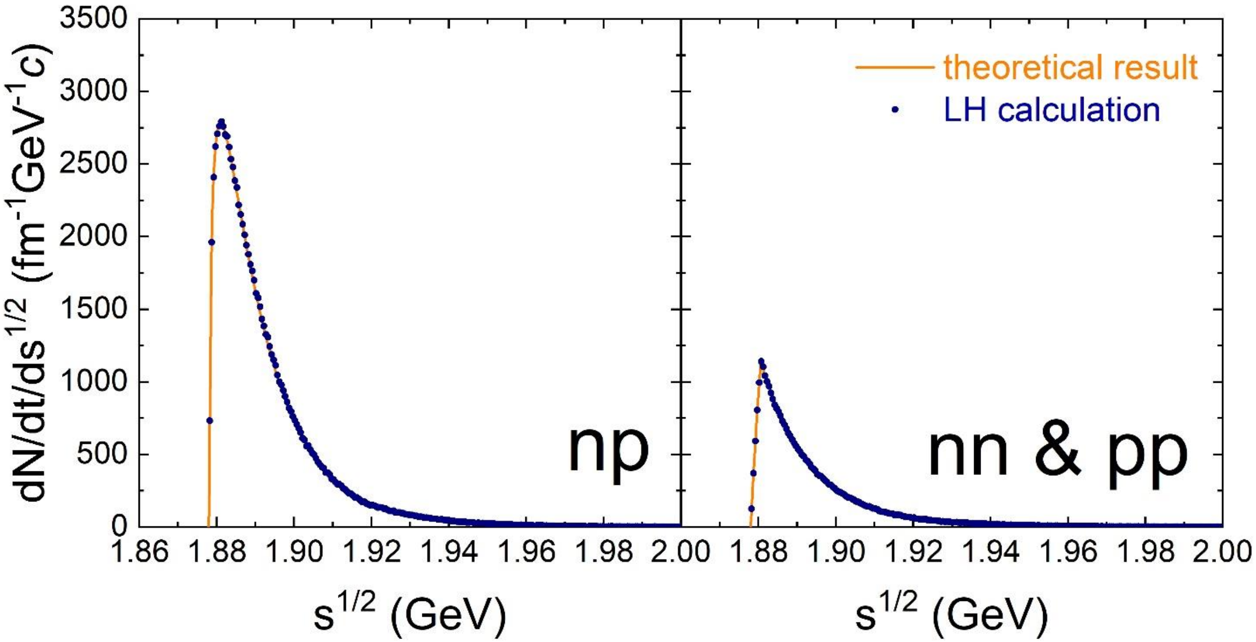
$$P22 = \frac{\Delta N_{22}^{\text{coll}}(\vec{r}_\alpha)}{\Delta N_1 \Delta N_2} = v_{\text{rel}} \frac{\sigma_{\text{NN}}^*}{N_{\text{E}}} S(\vec{r}_1 - \vec{r}_\alpha) S(\vec{r}_2 - \vec{r}_\alpha) l^3 \Delta t$$

Stochastic collision in LH method

For Boltzmann distribution at temperature T,
the number of collision per unit time and $\sqrt{s}$ satisfies

$$\frac{\mathrm{d}N_{22}^{\mathrm{coll}}}{\mathrm{d}t\mathrm{d}\sqrt{s}} = \frac{1}{2}N\rho\frac{s(s-4m^2)K_1(\sqrt{s}/T)\boxed{\sigma_{22}(\sqrt{s})}}{4m^4T_{\mathrm{B}}K_2^2(m/T)}$$

Two body elastic cross section from
J. Cugnon et al,
NIMB 111 (1996) 215-220



In practice, we choose randomly N_{22} pairs and scale the collision probability.

$$P'_{22} = P_{22}\frac{N_\alpha(N_\alpha-1)/2}{N_{22}}$$

with N_α being the number of test nucleons in lattice site α

R. Wang, L.-W. Chen, C. M. Ko, Y.-G. Ma, and Z. Zhang, in preparation

Giant dipole resonance width

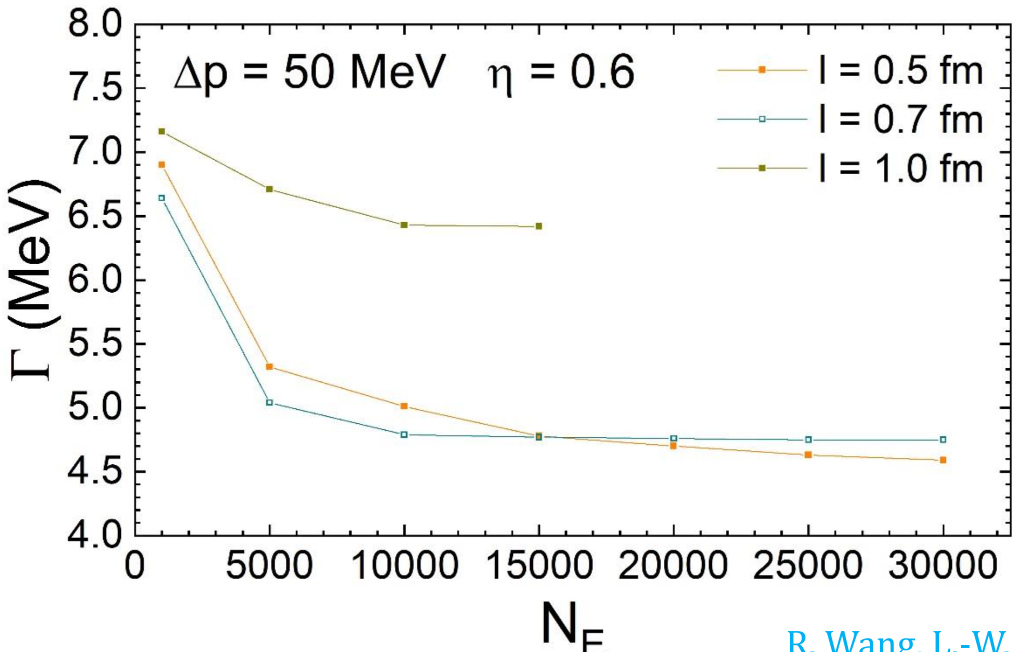
Pauli blocking

$$[1 - f(\vec{r}_\alpha, \vec{p}_3)] \times [1 - f(\vec{r}_\alpha, \vec{p}_4)]$$

Origins of uncertainty in calculating width

- Numerical fluctuation of $f(\vec{r}, \vec{p})$ caused by insufficient N_E
- Nonlocal effect in momentum space caused by finite Δp
- The finite lattice spacing l causes the average of different local density at nuclear surface, thus leads to diffusion in local momentum space

$$f_\tau(\vec{r}_\alpha, \vec{p}) \approx \frac{1}{\boxed{N_\mathrm{E}}} \sum_{\vec{p}_j \in \boxed{\Delta V_p}}^{\tau_j=\tau} \boxed{S}[\vec{r}_j(t) - \vec{r}_\alpha]$$



For larger lattice spacing, fluctuations of $f(\vec{r}, \vec{p})$ is smaller and the width converges rapidly as N_E increases.

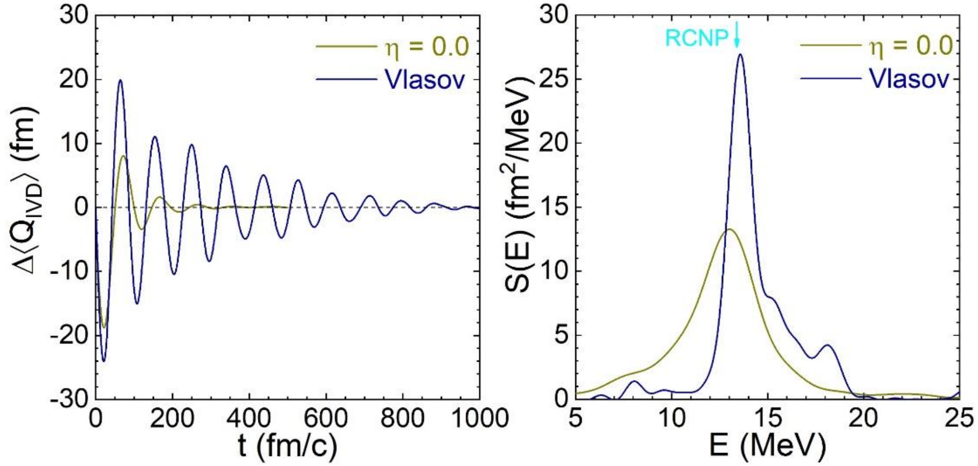
If we continue decreases l to below 0.5 fm, the converged value of width changes very little.

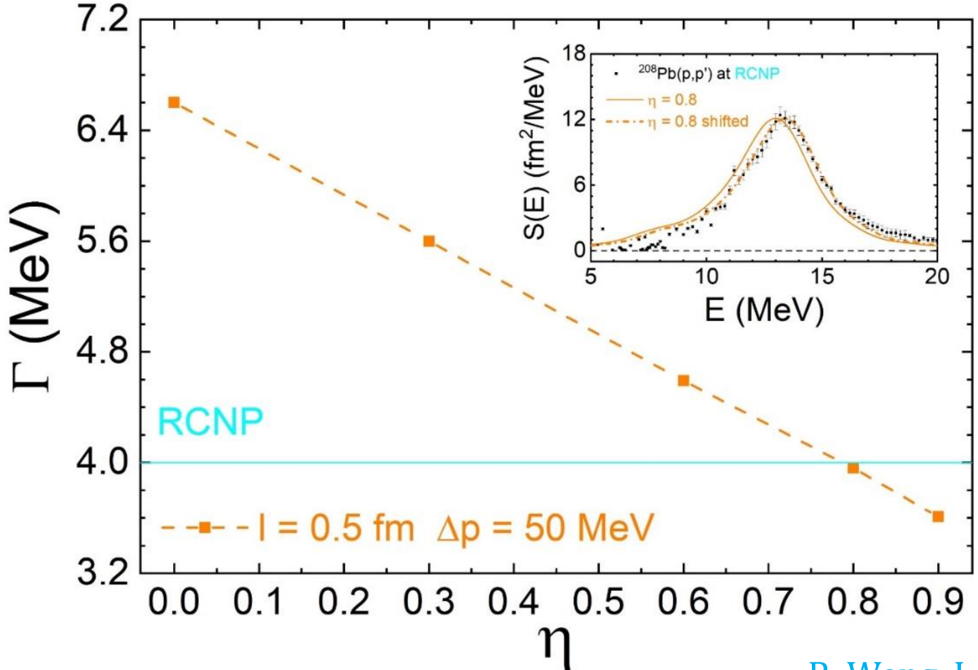
$$l = 0.5\ \mathrm{fm}, \Delta p = 50\ \mathrm{MeV}$$
$$N_E = 30000$$

R. Wang, L.-W. Chen, C. M. Ko, Y.-G. Ma, and Z. Zhang, in preparation

Giant dipole resonance width

With the inclusion of nucleon-nucleon collisions, the response of giant dipole resonance damps rapidly





Simple medium correction of nucleon-nucleon cross section

$$\sigma_{\rm NN}^{*} = \sigma_{\rm NN}^{\rm free}\left(1 - \eta\frac{\rho}{\rho_0}\right)$$

Since the damping is directly related to the N-N collisions, the width shows strong dependence on the in-medium N-N cross section

R. Wang, L.-W. Chen, C. M. Ko, Y.-G. Ma, and Z. Zhang, in preparation

Summary and outlook

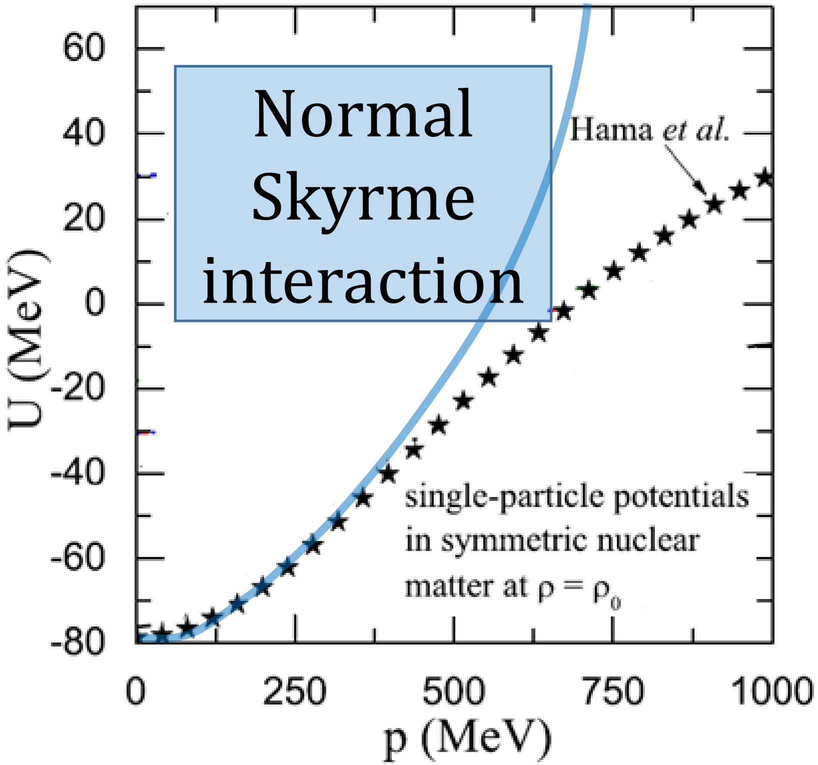
- We have developed a lattice Hamitonian method of BUU equation with Skyrme pseudopotential and stochastic nucleon-nucleon collision term.

- The isovector giant dipole resonance of ^{208}Pb have been studied based the above LH method, and we can decribe the experimental width and strength function with the present LH method.

- The GDR width shows strong dependence on the in-medium nucleon-nucleon cross section, and might be used to constrain the in-medium nucleon-nucleon cross section in the future.

Thank you

Extended Skyrme interaction



Normal Skyrme interaction

local nuclear energy density functional up to N3LO
B.G. Carlsson et al
PRC78,044326 (2008)

Corresponding Skyrme pseudo-potential up to N3LO
F. Raimondi et al
PRC83,054311 (2011)

high order terms of momentum

Local energy density functional

local nuclear energy density functional up to N3LO

B.G. Carlsson et al PRC78,044326 (2008)

$$H(\vec{r}) = f[\rho_{nLvJ}(\vec{r})] \quad \rho_{nLvJ}(\vec{r}) = \{[K_{nL}\rho_v(\vec{r},\vec{r}')]_J\}_{\vec{r}'=\vec{r}}$$

$$\rho_{00}(\boldsymbol{r},\boldsymbol{r}') = \rho(\boldsymbol{r},\boldsymbol{r}')$$

$$s_{1,\mu=\{-1,0,1\}}(\boldsymbol{r},\boldsymbol{r}') = -i\left\{\frac{1}{\sqrt{2}}[s_x(\boldsymbol{r},\boldsymbol{r}') - is_y(\boldsymbol{r},\boldsymbol{r}')], s_z(\boldsymbol{r},\boldsymbol{r}'), \frac{-1}{\sqrt{2}}[s_x(\boldsymbol{r},\boldsymbol{r}') + is_y(\boldsymbol{r},\boldsymbol{r}')]\right\}$$

$$\nabla_{1,\mu=\{-1,0,1\}} = -i\left\{\frac{1}{\sqrt{2}}(\nabla_x - i\nabla_y), \nabla_z, \frac{-1}{\sqrt{2}}(\nabla_x + i\nabla_y)\right\}$$

$$k_{1,\mu=\{-1,0,1\}} = -i\left\{\frac{1}{\sqrt{2}}(k_x - ik_y), k_z, \frac{-1}{\sqrt{2}}(k_x + ik_y)\right\}$$

No.	Tensor D_{nL}	Order n	Rank L
1	1	0	0
2	∇	1	1
3	$[\nabla\nabla]_0$	2	0
4	$[\nabla\nabla]_2$	2	2
5	$[\nabla\nabla]_0\nabla$	3	1
6	$[\nabla[\nabla\nabla]_2]_3$	3	3
7	$[\nabla\nabla]_0^2$	4	0
8	$[\nabla\nabla]_0[\nabla\nabla]_2$	4	2
9	$[\nabla[\nabla[\nabla\nabla]_2]_3]_4$	4	4
10	$[\nabla\nabla]_0^2\nabla$	5	1
11	$[\nabla\nabla]_0[\nabla[\nabla\nabla]_2]_3$	5	3
12	$[\nabla[\nabla[\nabla[\nabla\nabla]_2]_3]_4]_5$	5	5
13	$[\nabla\nabla]_0^3$	6	0
14	$[\nabla\nabla]_0^2[\nabla\nabla]_2$	6	2
15	$[\nabla\nabla]_0[\nabla[\nabla[\nabla\nabla]_2]_3]_4$	6	4
16	$[\nabla[\nabla[\nabla[\nabla[\nabla\nabla]_2]_3]_4]_5]_6$	6	6

terms up to N3LO

B.G. Carlsson et al PRC78,044326 (2008)

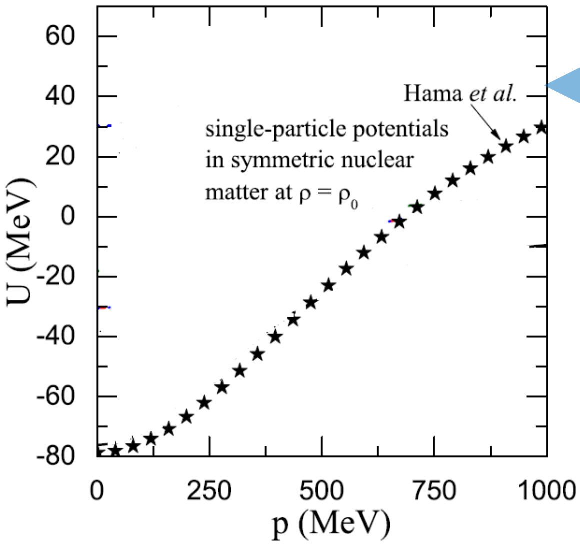
Skyrme pseudopotential

local nuclear energy density functional up to N3LO-PRC78

Code on spherical nuclei
B.G. Carlsson et al
CPC181, 1641-1657 (2010)

Corresponding Skyrme pseudopotential up to N3LO - PRC83

D. Davesne et al
developed various Skyrme effective pseudo-potentials up to N3LO which can be used to describe nuclear matter
PRC91,064303 (2015)
A&A585,A83 (2016)……



But can't reproduce the single particle potential

Hamiltonian density

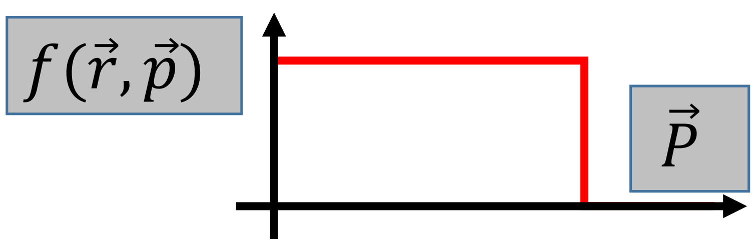
In the case of nuclear matter

$$\mathcal{H}(\vec{r}, f(\vec{r},\vec{p})) = \int dp \frac{\vec{p}^2}{2m} f(\vec{r},\vec{p}) + \mathcal{H}^{\text{local}}(\rho(\vec{r}))$$
$$+ \int dp dp' \sum_{n=1,2,3} C^{(i)} (\vec{p}-\vec{p}')^{2n} f(\vec{r},\vec{p}) f(\vec{r},\vec{p}')$$
$$+ \int dp dp' \sum_{n=1,2,3} D^{(i)} (\vec{p}-\vec{p}')^{2n} f_n(\vec{r},\vec{p}) f_n(\vec{r},\vec{p}') + n \to p$$
$$+ \mathcal{H}^{\text{grad}}(\rho(\vec{r}))$$

Momentum dependent part

$$C^{(i)} = \frac{1}{16\hbar^i}[t_1^{(i)}(2+x_1^{(i)}) + t_2^{(i)}(2+x_2^{(i)})]$$

$$D^{(i)} = \frac{1}{16\hbar^i}[-t_1^{(i)}(2x_1^{(i)}+1) + t_2^{(i)}(2x_2^{(i)}+1)]$$



Single nucleon potential

$$U(\vec{r},\vec{p}) = \varepsilon^0_{p\sigma\tau} = V\frac{\delta\mathcal{H}(\vec{r})}{\delta f_{p\sigma\tau}}$$

Equation of state

$$\boldsymbol{E = E(\rho)}$$

Single particle potential

In non-relativistic framework, the single nucleon energy $E\left(\rho,\delta,\vec{k}\right)$ can be expressed generally using the following dispersion relation:

$$E_{n/p}\left(\rho,\delta,\vec{k}\right)=\frac{\vec{k}^2}{2m}+\boxed{U_{n/p}\left(\rho,\delta,\boxed{\vec{k},f_{n/p}(\vec{r},\vec{k})}\right)}$$

Single particle potential

Distribution function

$$U_{n/p}\left(\rho,\delta,\vec{k},f(\vec{r},\vec{k})\right)$$
$$=U_0\left(\rho,\vec{k},f(\vec{r},\vec{k})\right)+\sum_{i=1}\boxed{U_{sym,i}\left(\rho,\vec{k},f_{n/p}(\vec{r},\vec{k})\right)}\tau_3{}^i\delta^i$$

Symmetry potential

Effective mass $$\frac{\hbar^2}{2{m^*}_{n/p}}=\frac{\hbar^2}{2m}+\frac{1}{p}\frac{\partial U_{n/p}\left(\rho,\delta,\vec{p},f_{n/p}(\vec{r},\vec{k})\right)}{\partial p}$$

Requirements of new interactions

- Satisfies all possible constrains from the nuclear matter;

- Realistic single particle potential or Hama potential to higher momentum $\sim \mathbf{1.5\ GeV/c^2}$ (corresponding to incident energy of 1 GeV), at $\boldsymbol{\rho = \rho_0}$;

- Qualitatively reproduce the first order symmetry potential calculated by microscopic calculation up to momentum $\sim \mathbf{1.5\ GeV/c^2}$, at $\boldsymbol{\rho = \rho_0}$.

Landau Fermi liquid theory

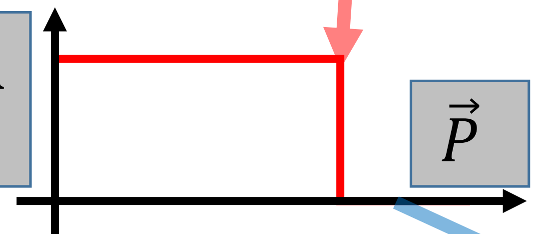
Real particles

distribution of ground state unknown

Qusai-particles

distribution of ground state

Distribution $f(\vec{r},\vec{p})$



$\vec{P}$

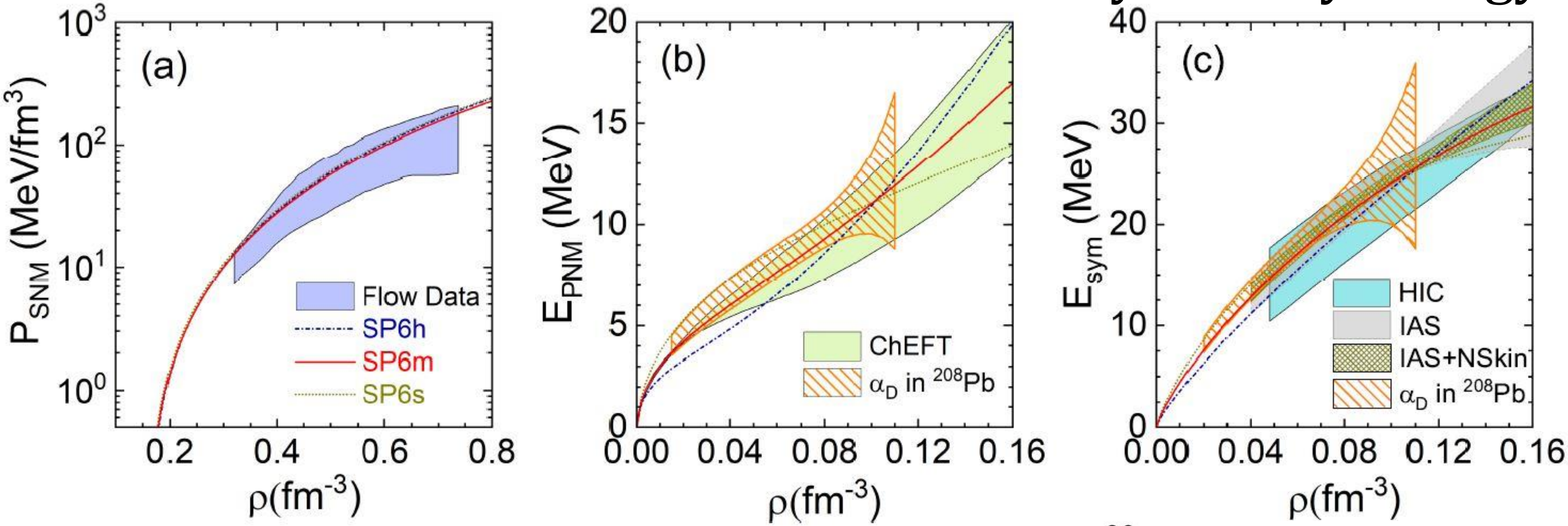
$$\int \varepsilon(\vec{r}) d^3r = \sum_i \langle i|\frac{p^2}{2m}|i\rangle + \frac{1}{2}\sum_{i,j}\langle ij|v_{Sk}|ij\rangle \sim a\ function\ of\ \boxed{f(\vec{r},\vec{p})}$$

$$\varepsilon(\vec{r}, n_{p\sigma\tau}) = \varepsilon_0(\vec{r}) + \frac{1}{V}\sum_{p\sigma} \boxed{\varepsilon^0_{p\sigma\tau}(\vec{r})} \delta f_{p\sigma\tau}(\vec{r}) + high\ order\ terms$$

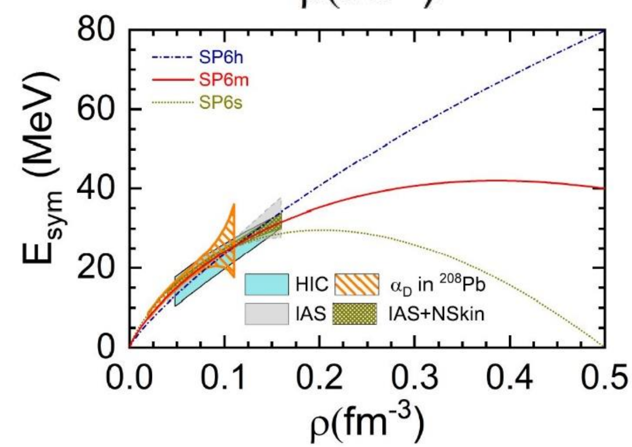
Single particle potential $\varepsilon^0_{p\sigma\tau} = V \dfrac{\delta\varepsilon(\vec{r}, f_{p\sigma\tau})}{\delta f_{p\sigma\tau}}$

Properties of Nuclear Matter

flow data of SNM　　EoS for PNM　　symmetry energy



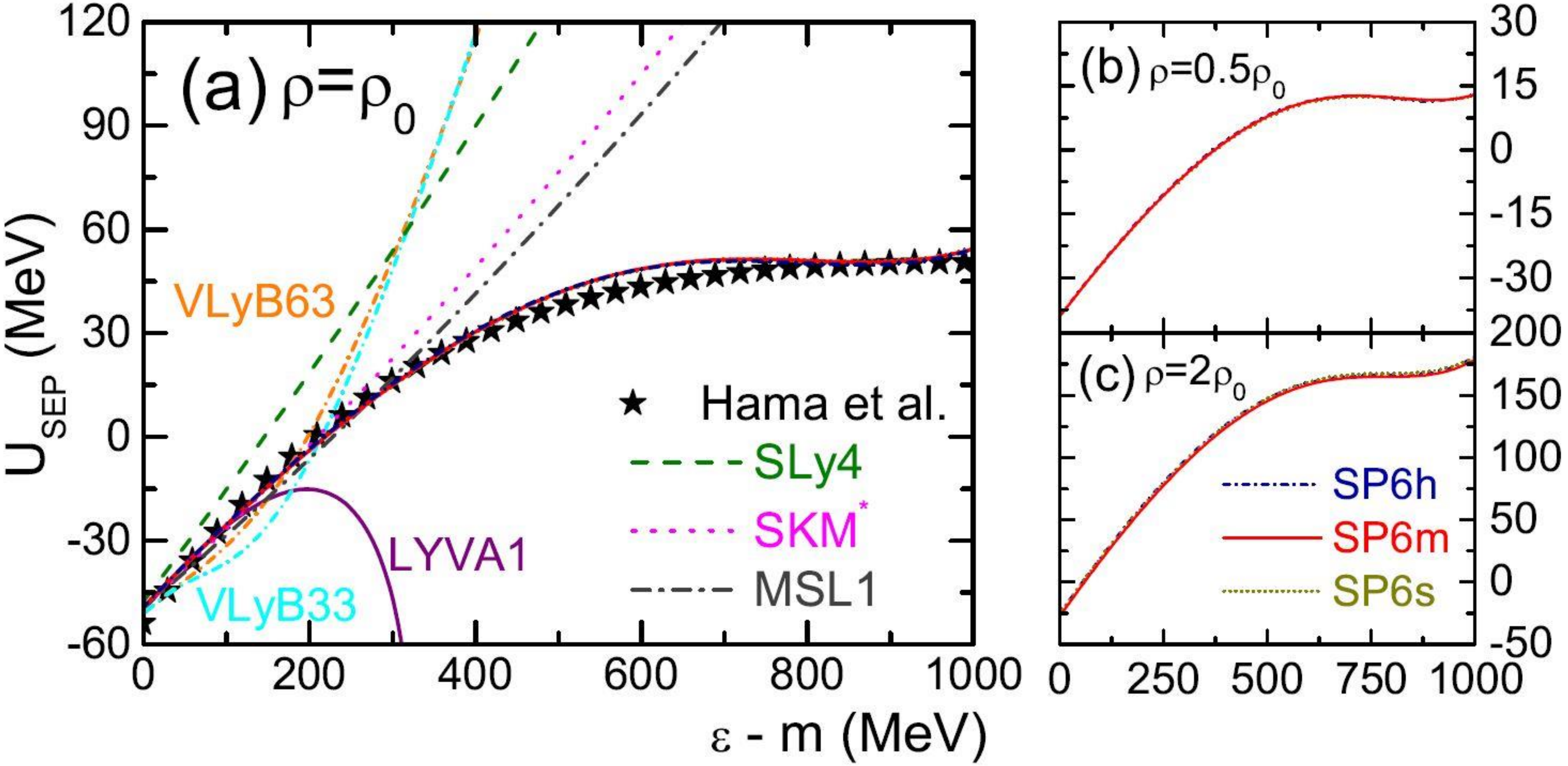
	ρ_0	E_0	K_0	$E_{sym}(\rho_0)$	$L(\rho_0)$	$E_{sym}(\rho_{sc})$	$L(\rho_{sc})$
	fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV
SP6s	**0.163**	**-16.1**	**216.9**	**28.84**	**18.28**	**25.60**	**31.42**
SP6m	**0.162**	**-16.0**	**216.1**	**32.02**	**47.89**	**26.09**	**45.94**
SP6h	**0.161**	**-15.9**	**230.7**	**34.54**	**77.62**	**25.97**	**59.92**



R. Wang, L. W. Chen and Y. Zhou, arXiv, 1806:03278

Single particle properties

The main improvement of the new interactions SP6s SP6m and SP6h is the realistic single nucleon potential to energy of about 1 GeV.

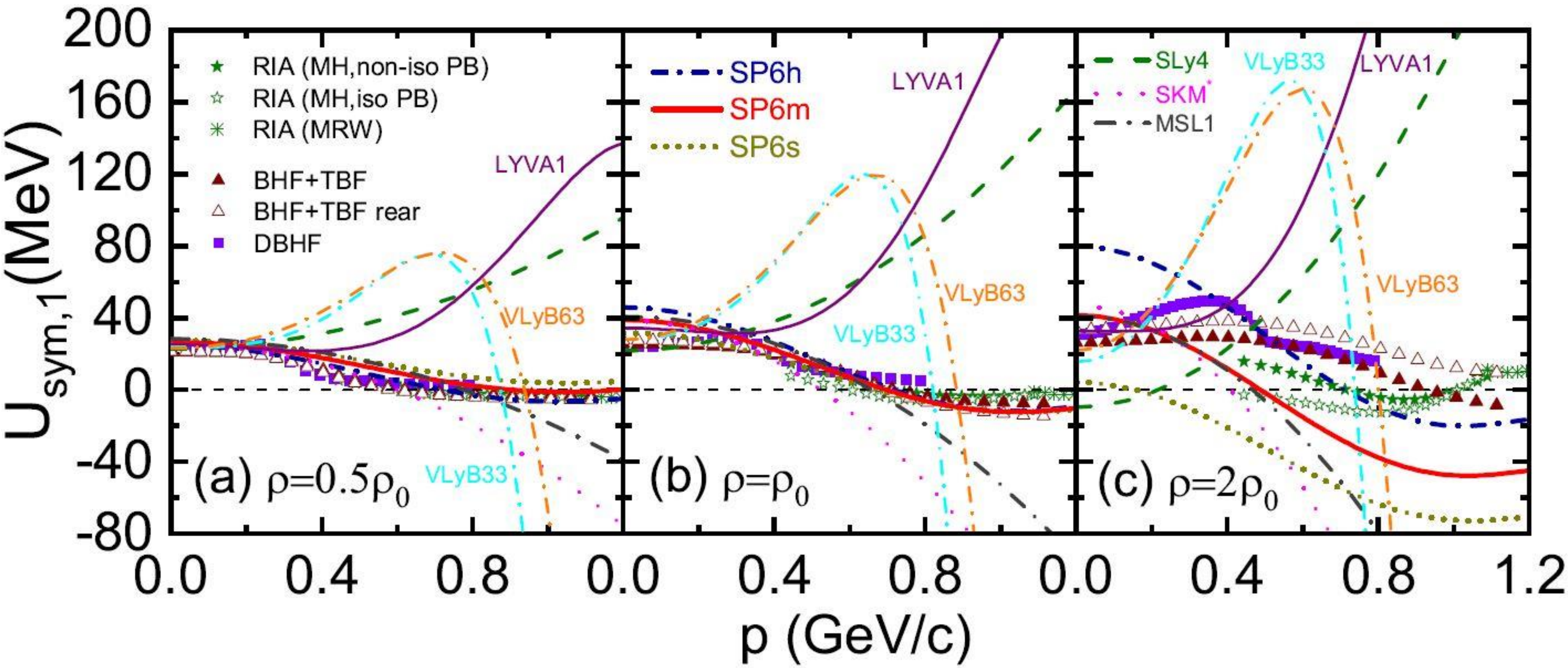


R. Wang, L. W. Chen and Y. Zhou, arXiv, 1806:03278

Single particle properties

The comparison of first order symmetry potential to microscopic calculations

relativistic impulse approximation (RIA)
Brueckner-Hartree-Fock (BHF)

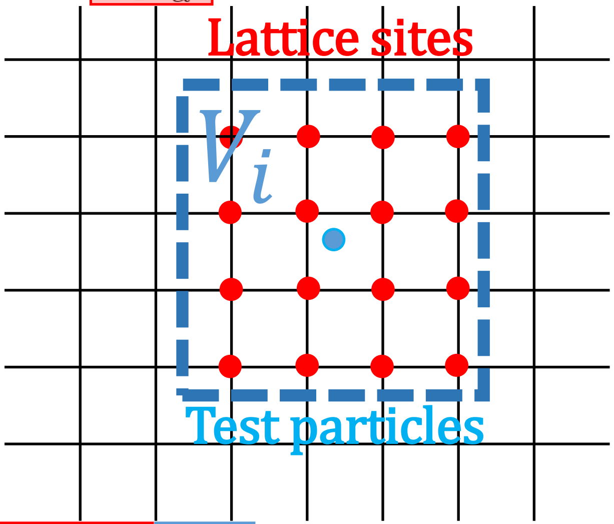


R. Wang, L. W. Chen and Y. Zhou, arXiv, 1806:03278

Lattice Hamiltonian Vlasov framework

The equation of motion

$$f(\vec{r}_\alpha, \vec{p}) = \sum_{\boxed{i \in V_\alpha}} S(\vec{r}_i - \vec{r})\delta(\vec{p}_i - \vec{p})$$



Hamilton equations, with the r_i and p_i of i-th test particle treated as the canonical coordinate and momentum of total Hamiltonian $\boldsymbol{H}$

$$\frac{d\vec{r}_i}{dt} = \frac{\partial H}{\partial \vec{p}_i} \approx \frac{\vec{p}_i}{E_i} + N_{TP}\Delta V \sum_{\boxed{\alpha \in V_i}} \frac{\partial \mathcal{H}^{\mathrm{MD}}(\vec{r}_\alpha)}{\partial \vec{p}_i}$$

$$\frac{d\vec{p}_i}{dt} = \frac{\partial H}{\partial \vec{r}_i} \approx N_{TP}\Delta V \sum_{\boxed{\alpha \in V_i}} \left\{ \boxed{\left[\frac{\partial \mathcal{H}^{\mathrm{local}}(\vec{r}_\alpha)}{\partial \rho} + \frac{\partial \mathcal{H}^{\mathrm{nl}}(\vec{r}_\alpha)}{\partial \rho}\right]} \boxed{\frac{\partial \rho_\alpha}{\partial \vec{r}_i}} + \frac{\partial \mathcal{H}^{MD}(\vec{r}_\alpha)}{\partial \vec{r}_i} \right\}$$

$$\boxed{U_{local}(\vec{r}_\alpha) + U_{nl}(\vec{r}_\alpha)} \qquad \approx \nabla_{\vec{r}} U(\vec{r})$$

$$\boxed{\frac{\partial \rho_\alpha}{\partial \vec{r}_i} = \frac{\partial}{\partial \vec{r}_i} \sum_{i \in V_\alpha} S(\vec{r}_i - \vec{r}) = S'(\vec{r}_i - \vec{r}_\alpha)}$$

Summing over all lattice sites that TP contribute