Nonequilibrium Green's Function (NGF) approach in 1D transport

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Why quantal anyway?

- Quantum effects missing from semiclassical transport:
  - **Exact antisymmetrization** [except for AMD]
    more than Pauli principle alone
  - **Quantum correlations**
    many-body state as a superposition of multiple Slater determinants
    different from n-body correlations inherited from the BBGKY hierarchy
    cannot be addressed fully classically (cf. Bell's theorem)

Nonequilibrium Green's Functions (NGF)

- System characterized by **Green's functions**

$$iG(x_1,t_1,x_2,t_2) = \langle \Phi | T[\psi(x_1,t_1)\psi^\dagger(x_2,t_2)] | \Phi \rangle$$

$$G^>, G^<, G^A, G^R, \text{etc. can also be defined.}$$

- Relations to other transport quantities

$$\rho(x_1,x_2;T) = -iG^<(x_1,T,x_2,T) = -\langle \Phi | \psi^\dagger(x_2,T)\psi(x_1,T) | \Psi \rangle$$

$$f(p;X,T) = \int dx\, e^{-ipx} \langle \Phi | \psi^\dagger(X - x/2,T)\psi(X + x/2,T) | \Psi \rangle$$

NGF: Kadanoff-Baym Equation

$$\left\{ i\hbar \frac{\partial}{\partial t_1} + \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_1^2} \right\} G^{\lessgtr}(\mathbf{1}\mathbf{1}') = \int \mathrm{d}x_{\bar{1}}\, \Sigma_{HF}(\mathbf{1}\bar{\mathbf{1}})\, G^{\lessgtr}(\bar{\mathbf{1}}\mathbf{1}')$$

$$+ \int_{t_0}^{t_1} \mathrm{d}\bar{\mathbf{1}}\, \Sigma^{+}(\mathbf{1}\bar{\mathbf{1}})\, G^{\lessgtr}(\bar{\mathbf{1}}\mathbf{1}') + \int_{t_0}^{t_{1'}} \mathrm{d}\bar{\mathbf{1}}\, \Sigma^{\lessgtr}(\mathbf{1}\bar{\mathbf{1}})\, G^{-}(\bar{\mathbf{1}}\mathbf{1}')$$

$$\left\{ -i\hbar \frac{\partial}{\partial t_{1'}} + \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_{1'}^2} \right\} G^{\lessgtr}(\mathbf{1}\mathbf{1}') = \int \mathrm{d}x_{\bar{1}}\, G^{\lessgtr}(\mathbf{1}\bar{\mathbf{1}})\, \Sigma_{HF}(\bar{\mathbf{1}}\mathbf{1}')$$

$$+ \int_{t_0}^{t_1} \mathrm{d}\bar{\mathbf{1}}\, G^{+}(\mathbf{1}\bar{\mathbf{1}})\, \Sigma^{\lessgtr}(\bar{\mathbf{1}}\mathbf{1}') + \int_{t_0}^{t_{1'}} \mathrm{d}\bar{\mathbf{1}}\, G^{\lessgtr}(\mathbf{1}\bar{\mathbf{1}})\, \Sigma^{-}(\bar{\mathbf{1}}\mathbf{1}')$$

Mean-Field Dynamics (TDHF)

+

Correlation integral (2-body interactions)

$$\Sigma_{\mathrm{HF}}(x_1, x_2) \approx U_{\mathrm{mf}}[\rho(x_1)]\delta(x_1 - x_2)$$

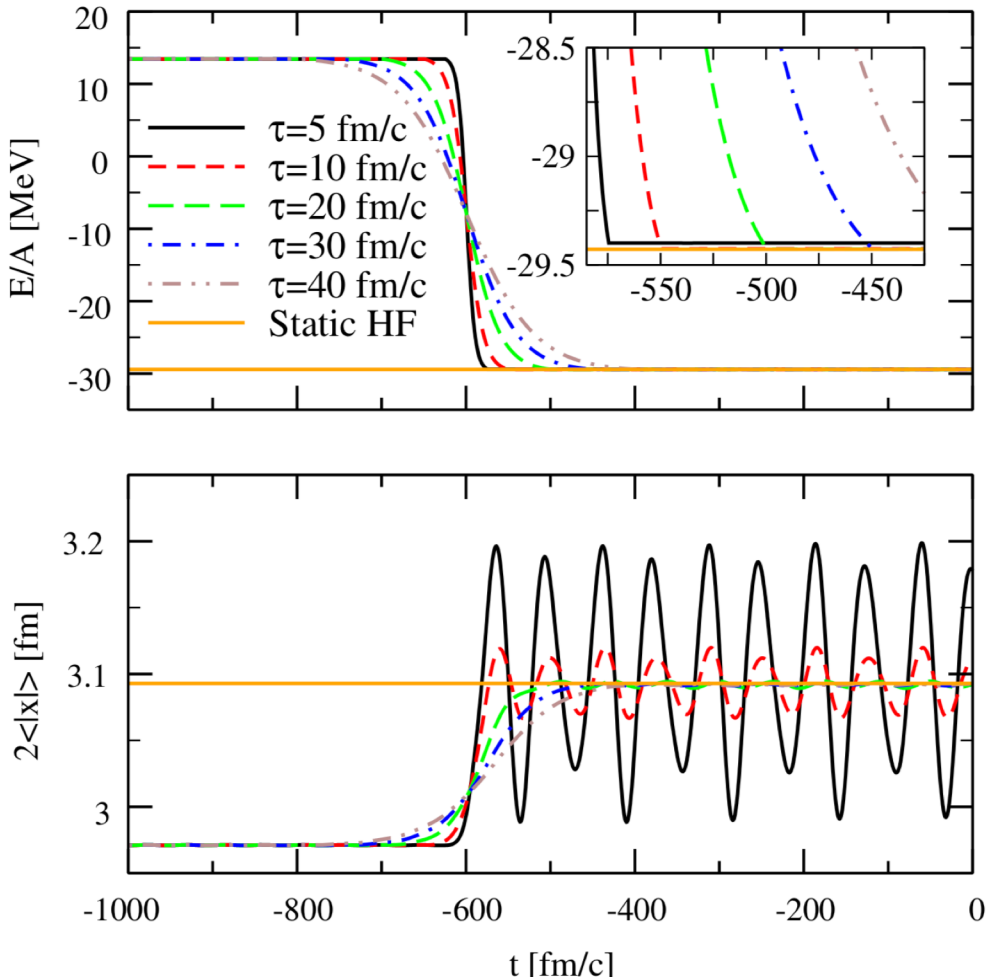
$$\Sigma \propto |V|^2$$

*Previous results from NGF **with mf***

Convergence to the HF ground state by turning on mf adiabatically

$$H(t) = (1 - f(t))H_{\mathrm{HO}} + f(t)H_{\mathrm{nucl}}$$

Adiabatic switching



$$f(t) : 0 \rightarrow 1$$

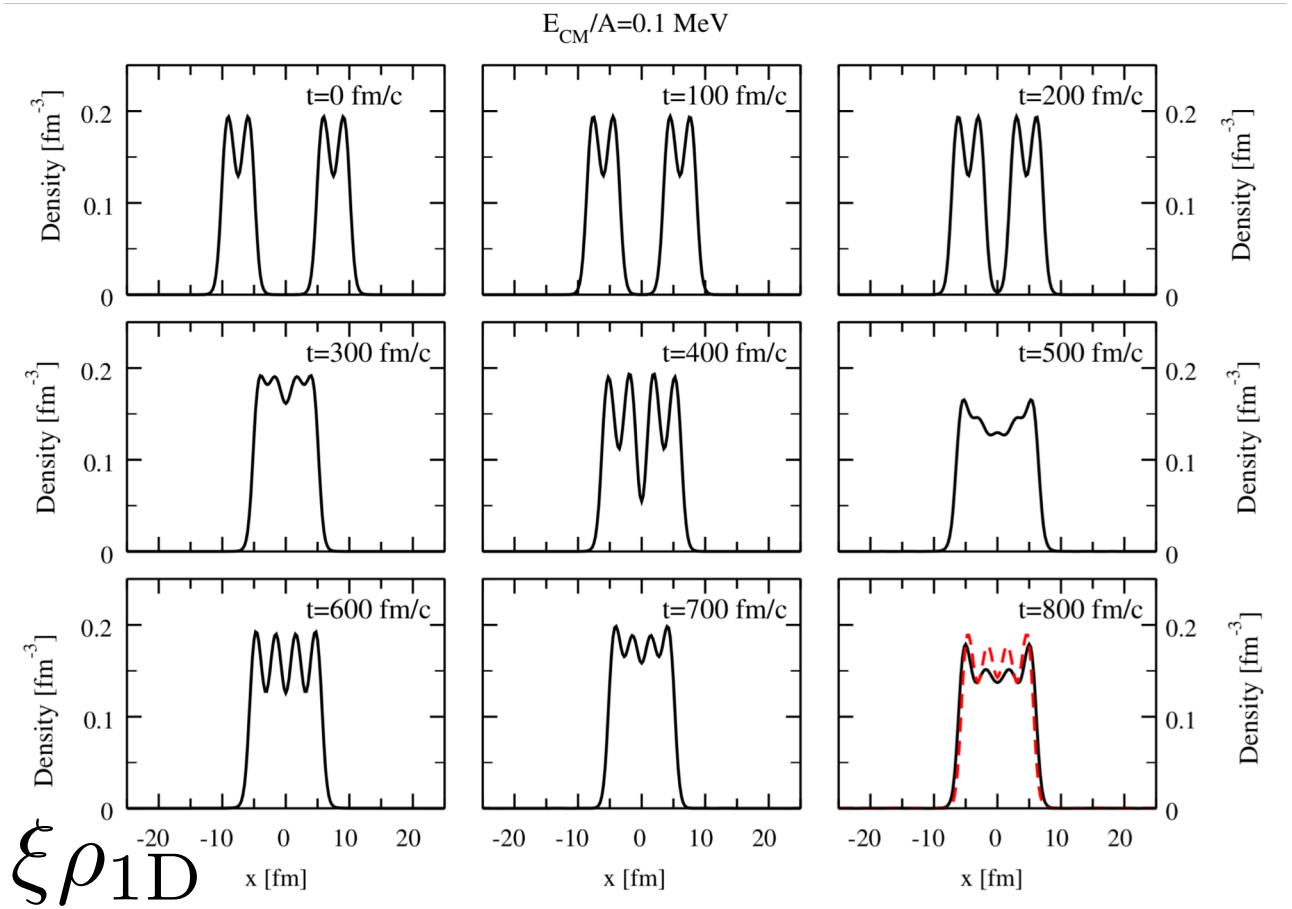
$$\Phi_0^{\mathrm{HO}} \rightarrow \Phi_0^{\mathrm{nucl}}$$

NGF builds a bridge between nuclear structure and nuclear dynamics.

A. Rios et al. / Annals of Physics 326 (2011) 1274–1319

*Previous results from NGF **with mf***

Collision of 1D slabs under mean field



$$\rho_{3\mathrm{D}} = \xi \rho_{1\mathrm{D}}$$

(Hugenholtz-van Hove Theorem)

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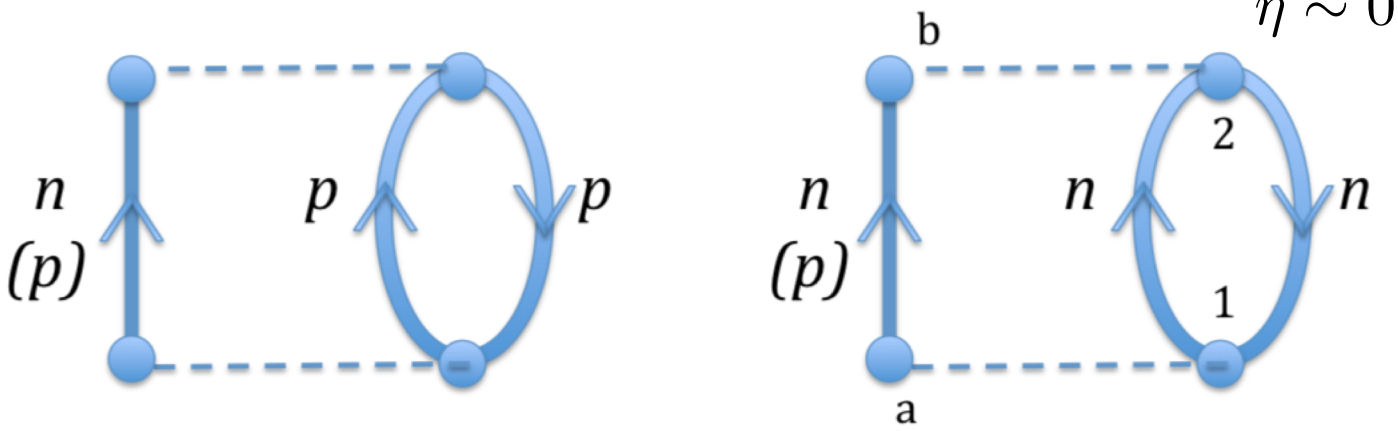
NGF: Born Approximation

$$V(p) \propto |p| \exp\left[-\left(\frac{\eta p}{2\hbar}\right)^2\right]$$

$\eta \sim 0.57 fm$



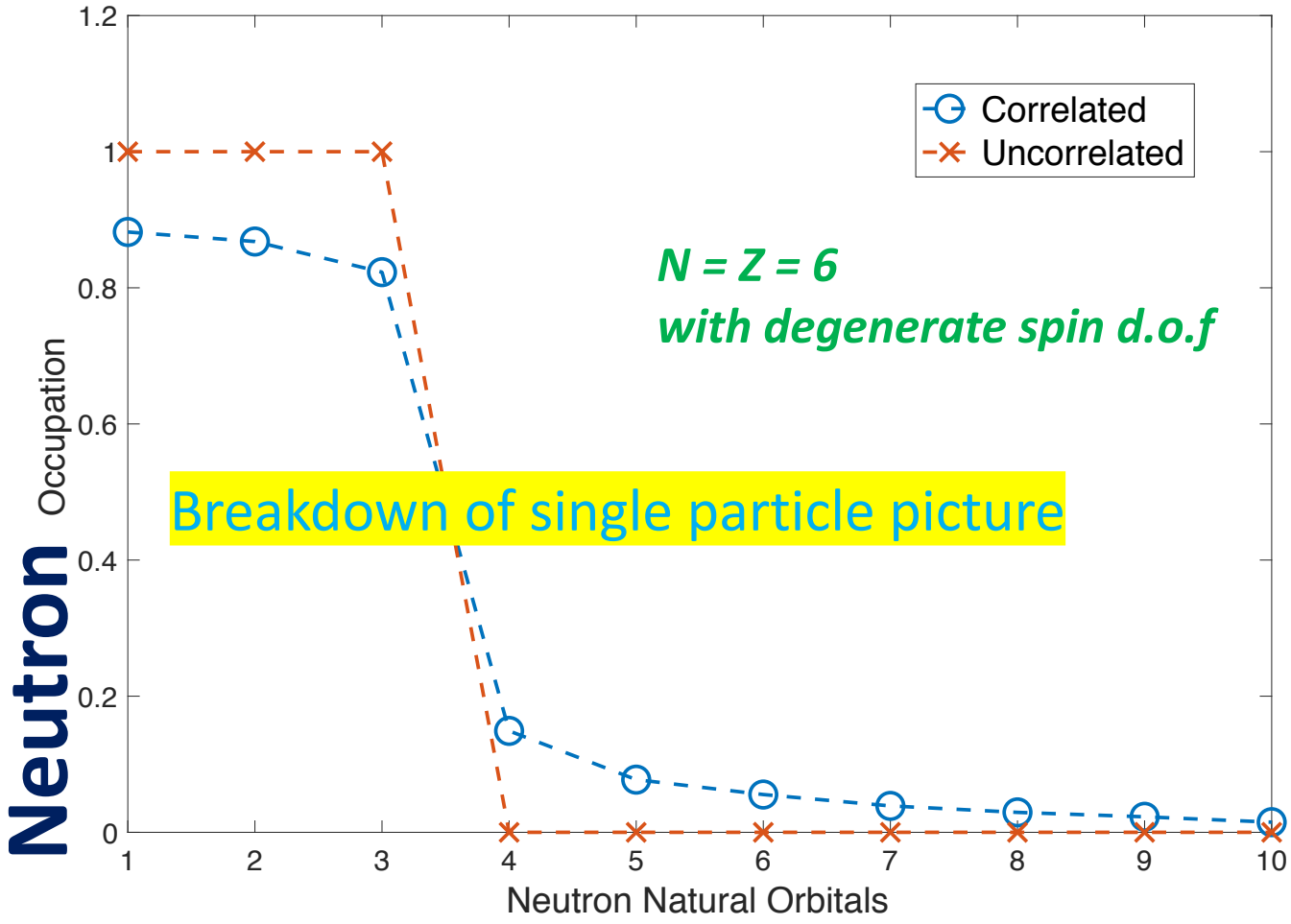
$$\Sigma^{\lessgtr}(p,t;p',t') = \int \frac{\mathrm{d}p_1}{2\pi}\frac{\mathrm{d}p_2}{2\pi} V(p-p_1)V(p'-p_2)G^{\lessgtr}(p_1,t;p_2,t')\Pi^{\lessgtr}(p-p_1,t;p'-p_2,t')$$

$$\Pi^{\lessgtr}(p,t;p',t') = \int \frac{\mathrm{d}p_1}{2\pi}\frac{\mathrm{d}p_2}{2\pi} G^{\lessgtr}(p_1,t;p_2,t')G^{\gtrless}(p_2-p',t';p_1-p,t)\,,$$

2-body interactions -> correlations

$$H(t) = (1 - f(t))H_{\mathrm{HO}} + f(t)H_{\mathrm{nucl}}$$

$$\Phi_0^{\mathrm{HO}}(\mathrm{uncorrelated}) \rightarrow \Phi_0^{\mathrm{nucl}}(\mathrm{correlated})$$



2-body interactions -> correlations

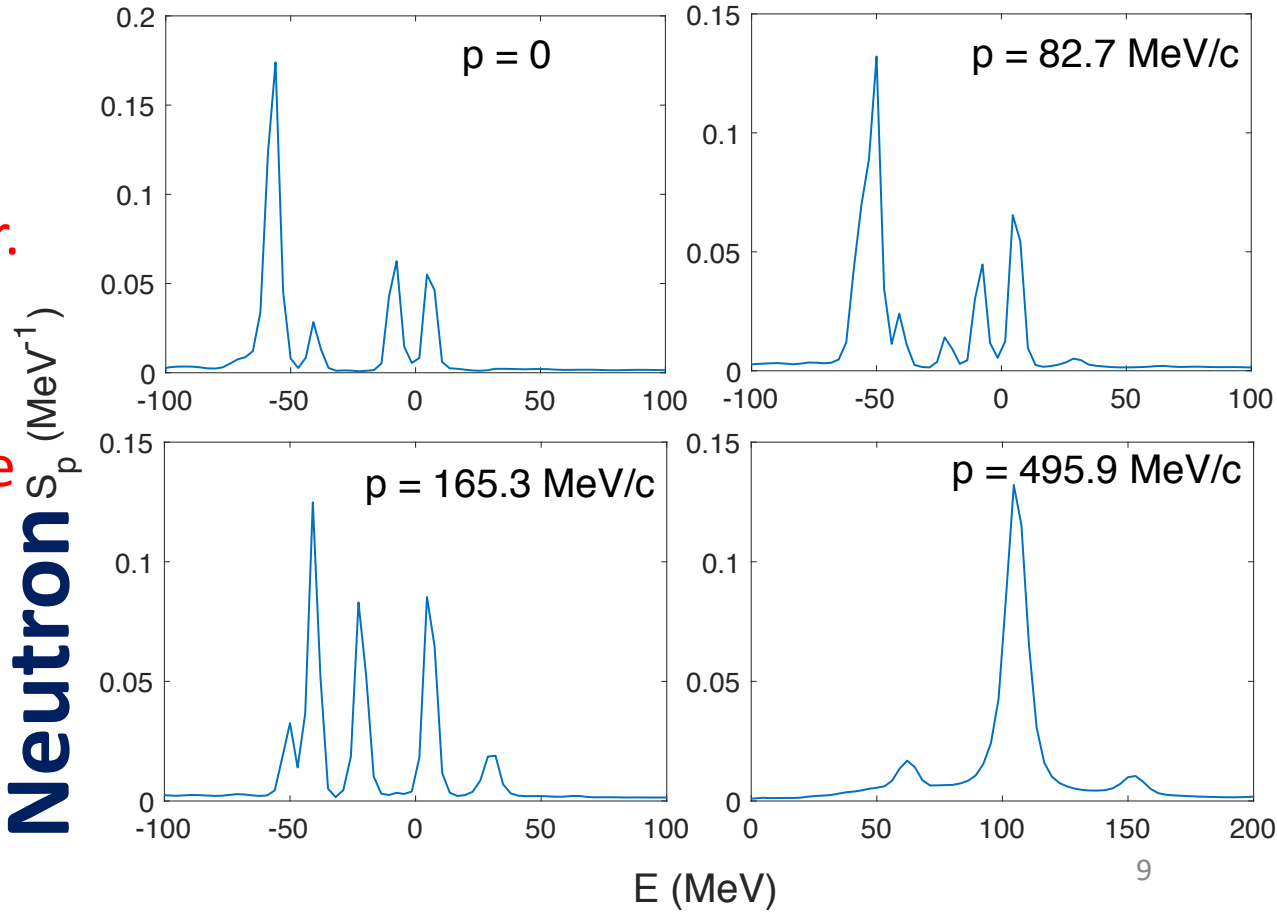
$$G^R = \sum_a \frac{|\langle \Phi_a^{(A+1)} | \psi^\dagger(p) | \Phi_0^{(A)} \rangle|^2}{E - (E_a^{(A+1)} - E_0^{(A)}) + i\eta} + \sum_b \frac{|\langle \Phi_b^{(A-1)} | \psi(p) | \Phi_0^{(A)} \rangle|^2}{E - (E_b^{(A-1)} - E_0^{(A)}) + i\eta}$$

$$S \propto -\mathrm{Im}(G^R)$$

Momentum is NOT a good quantum number.

Collisions between 2 particles need not have well-defined initial and final momenta.

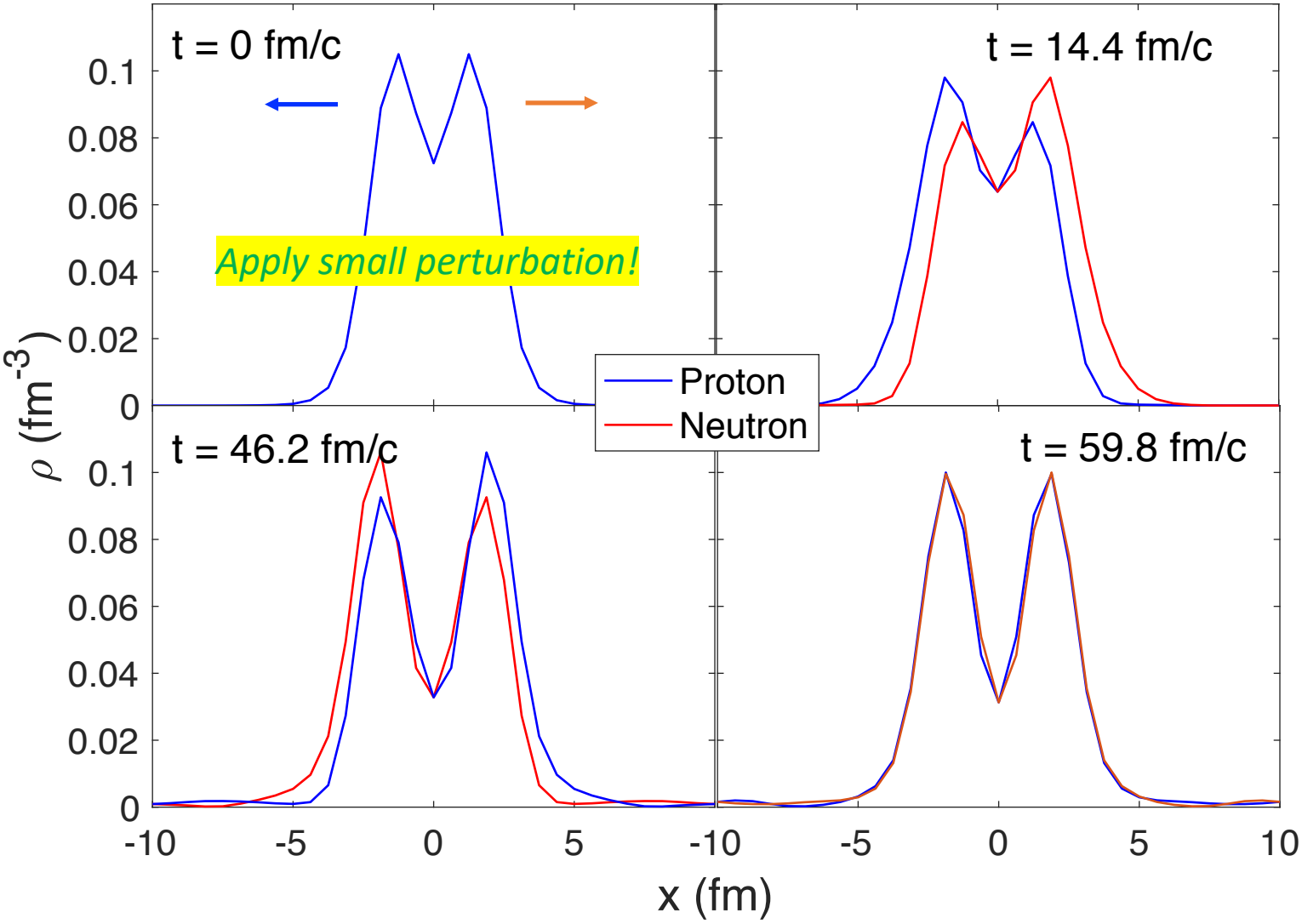
Spectral functions (p, E)



Dissipation from correlation integrals

Oscillation of neutron-proton slab (N = Z = 4)

[isovector dipole oscillation]



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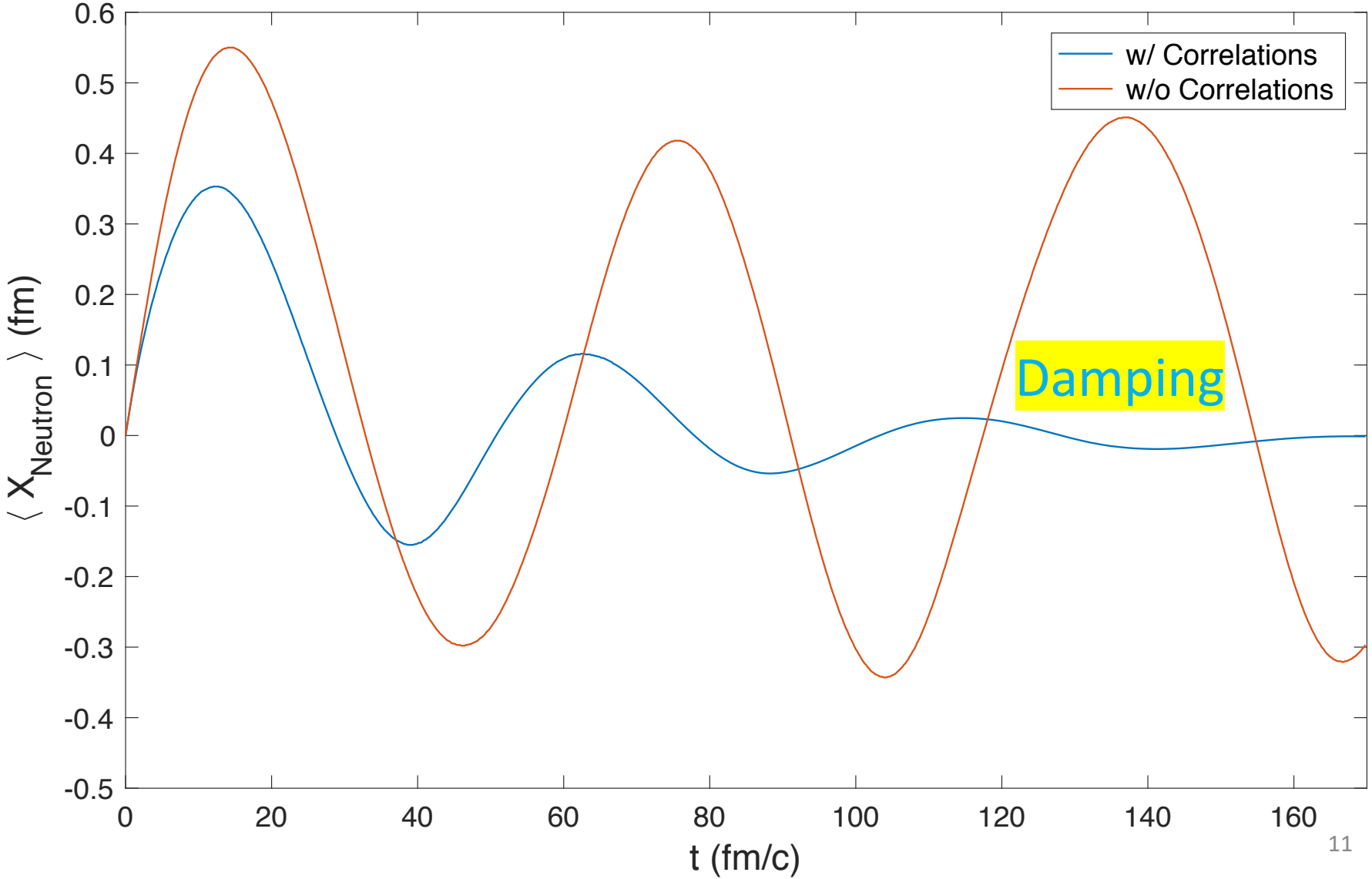
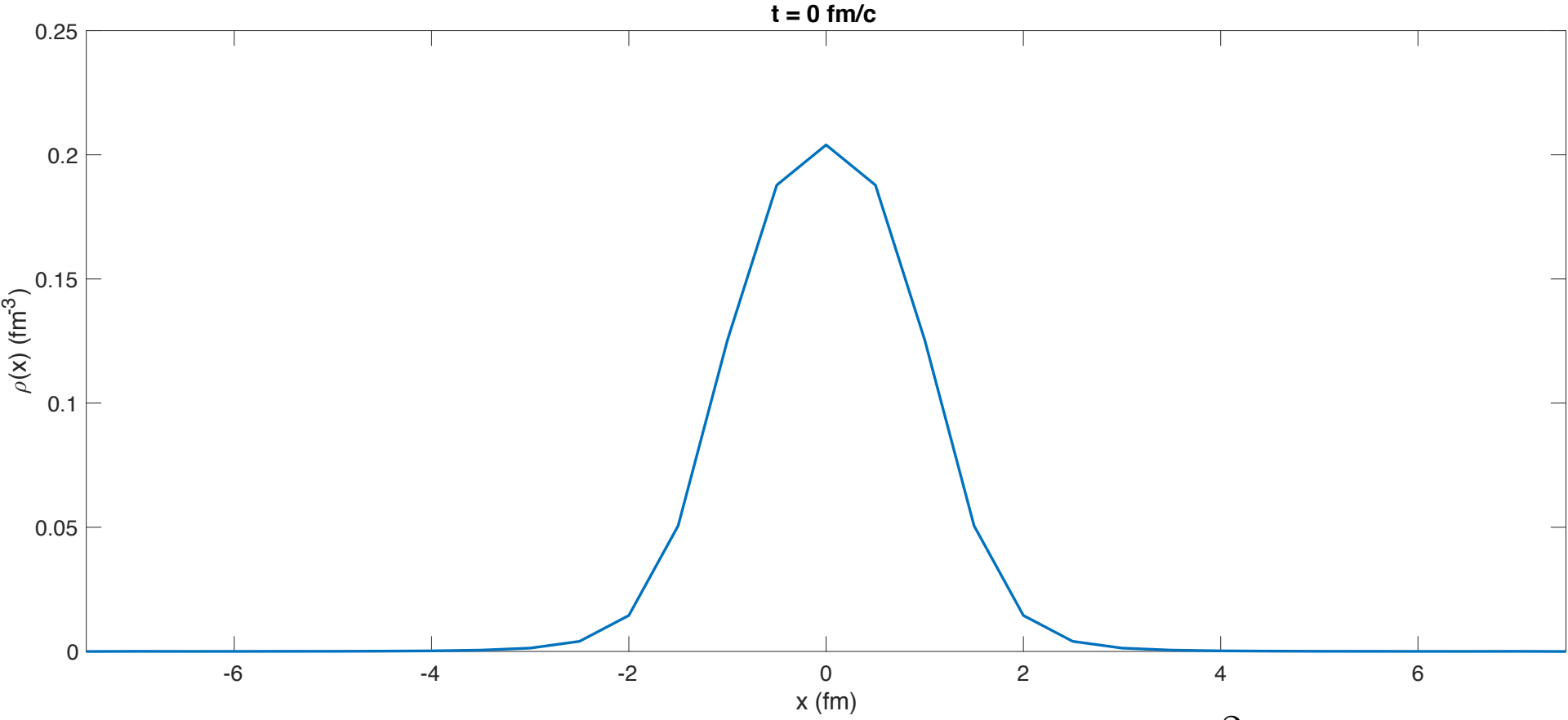


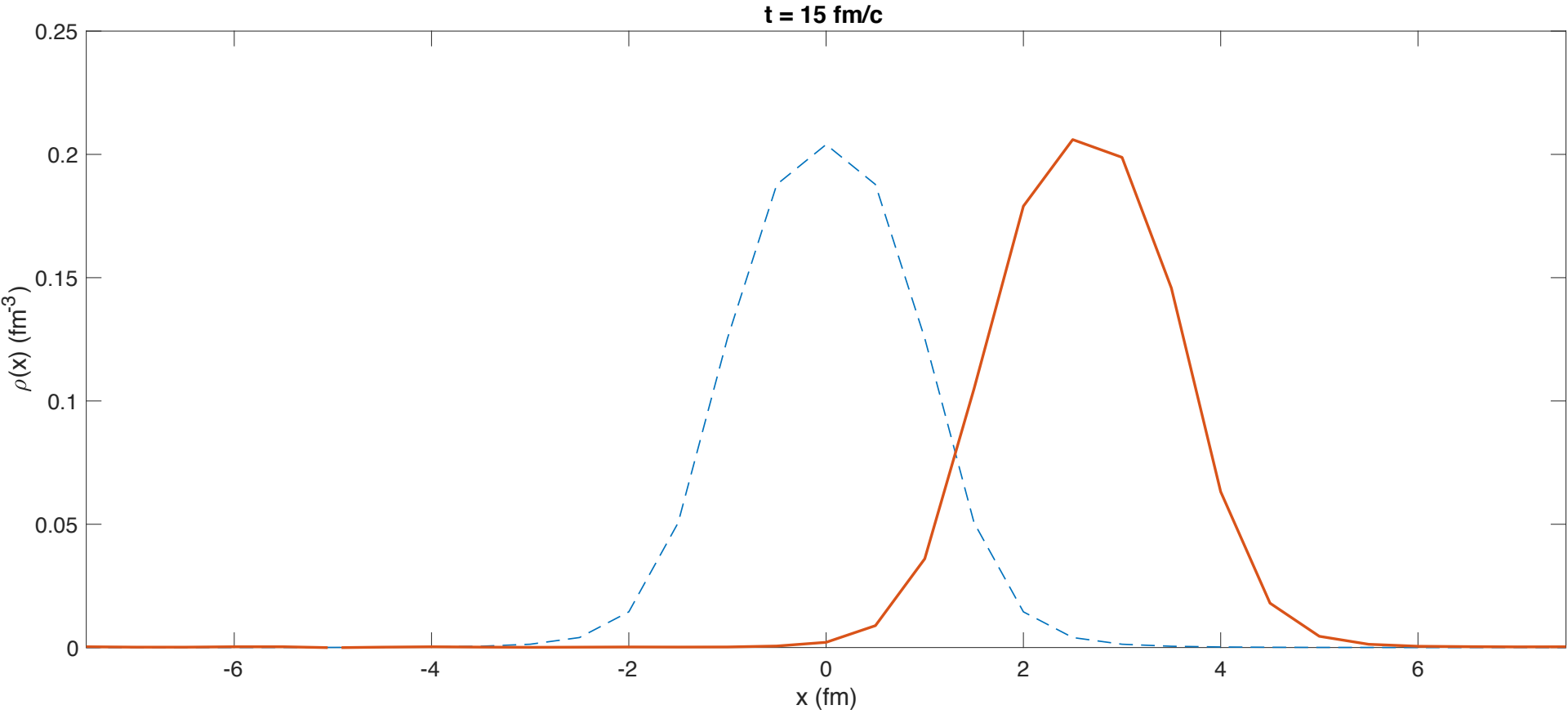
Slab at constant velocity



$$\text{Galilean transformation } G(p,t) = \exp(i(p \cdot r - \frac{p^2}{2m} \cdot t))$$

Applied to the entire preparation history! Non-markovian effect!

Slab at constant velocity



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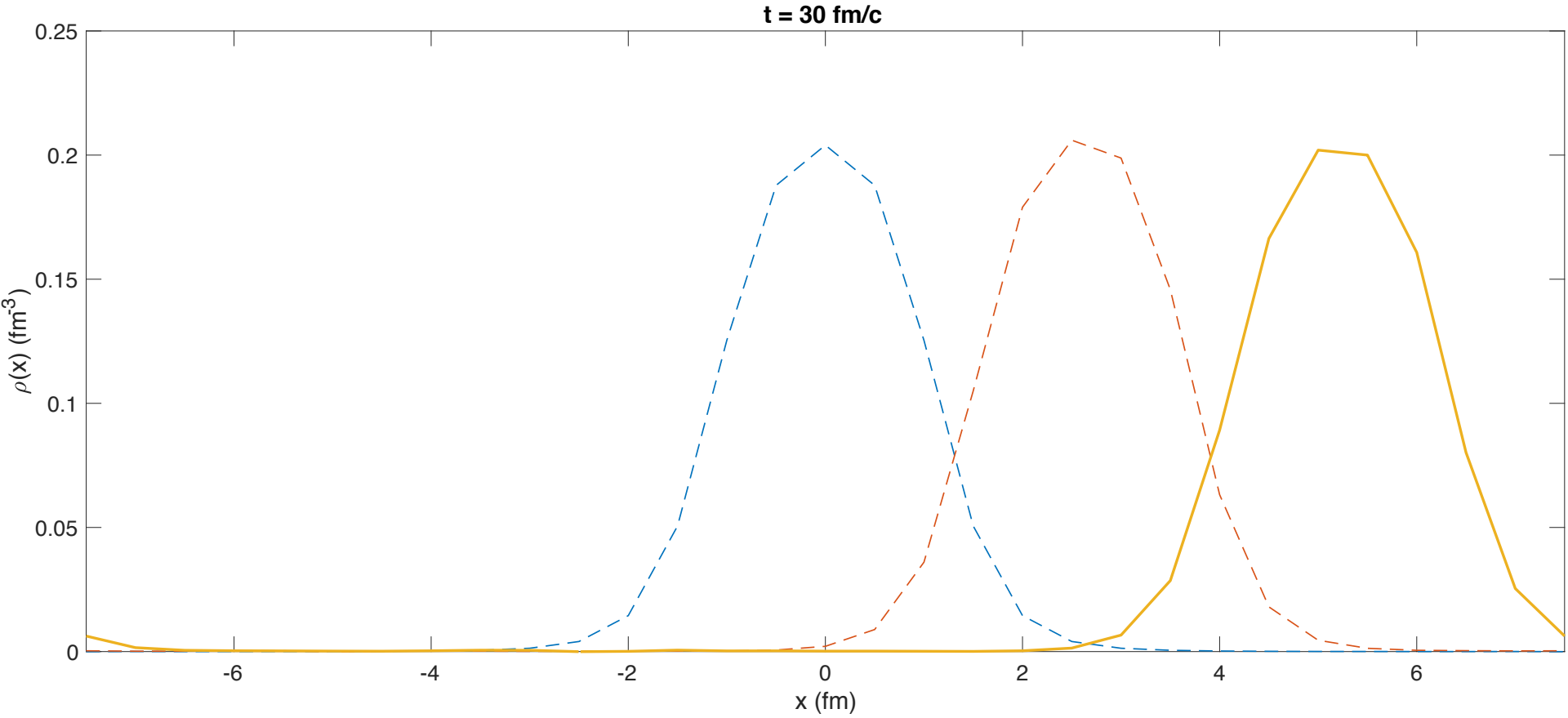
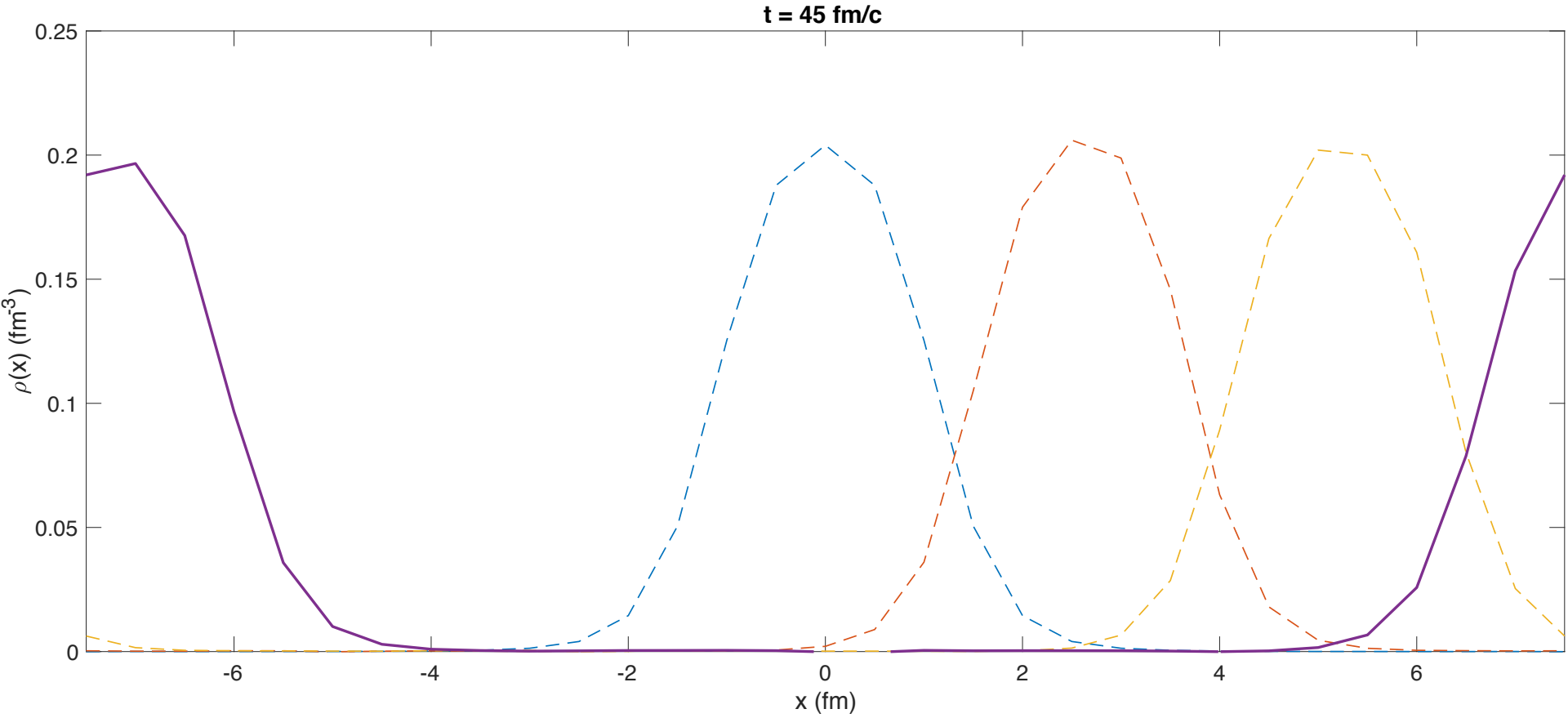
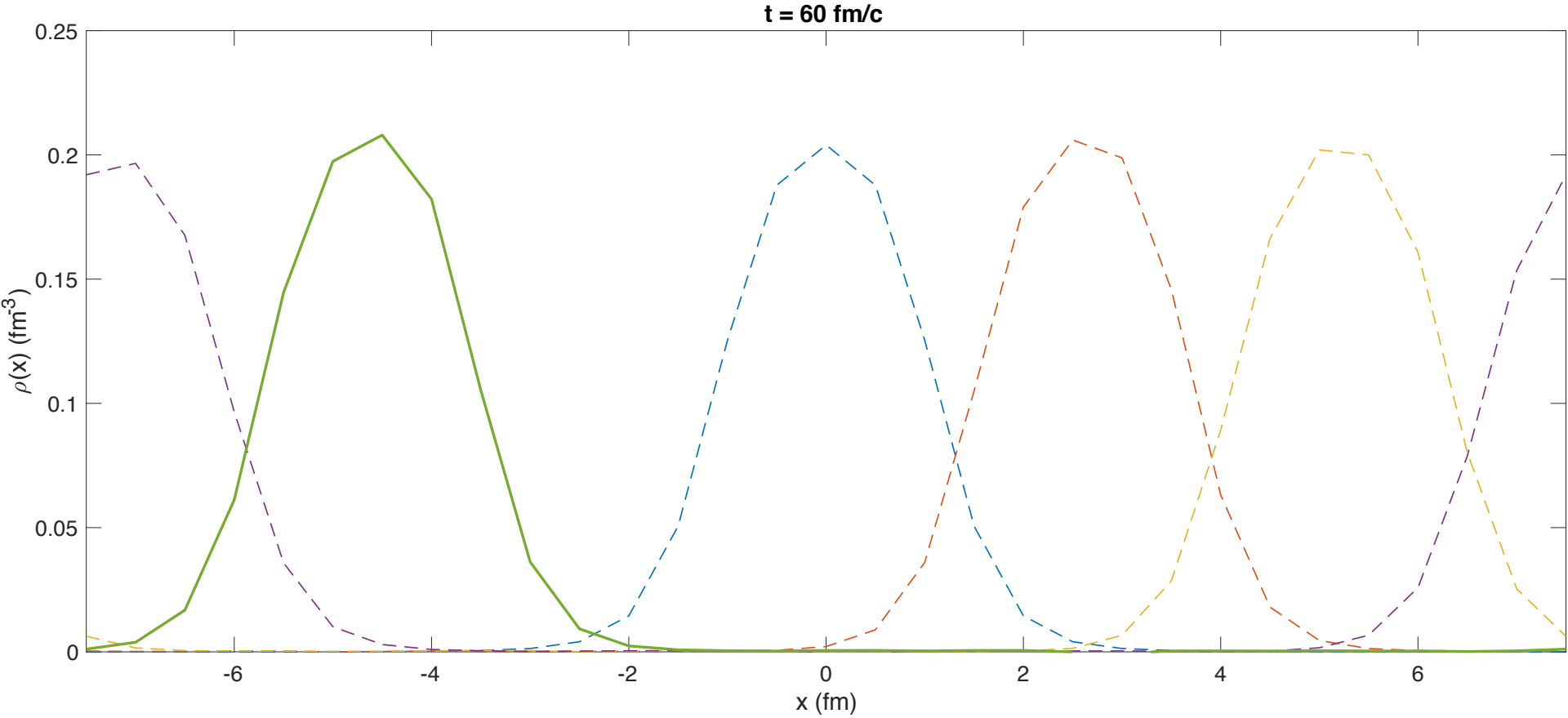


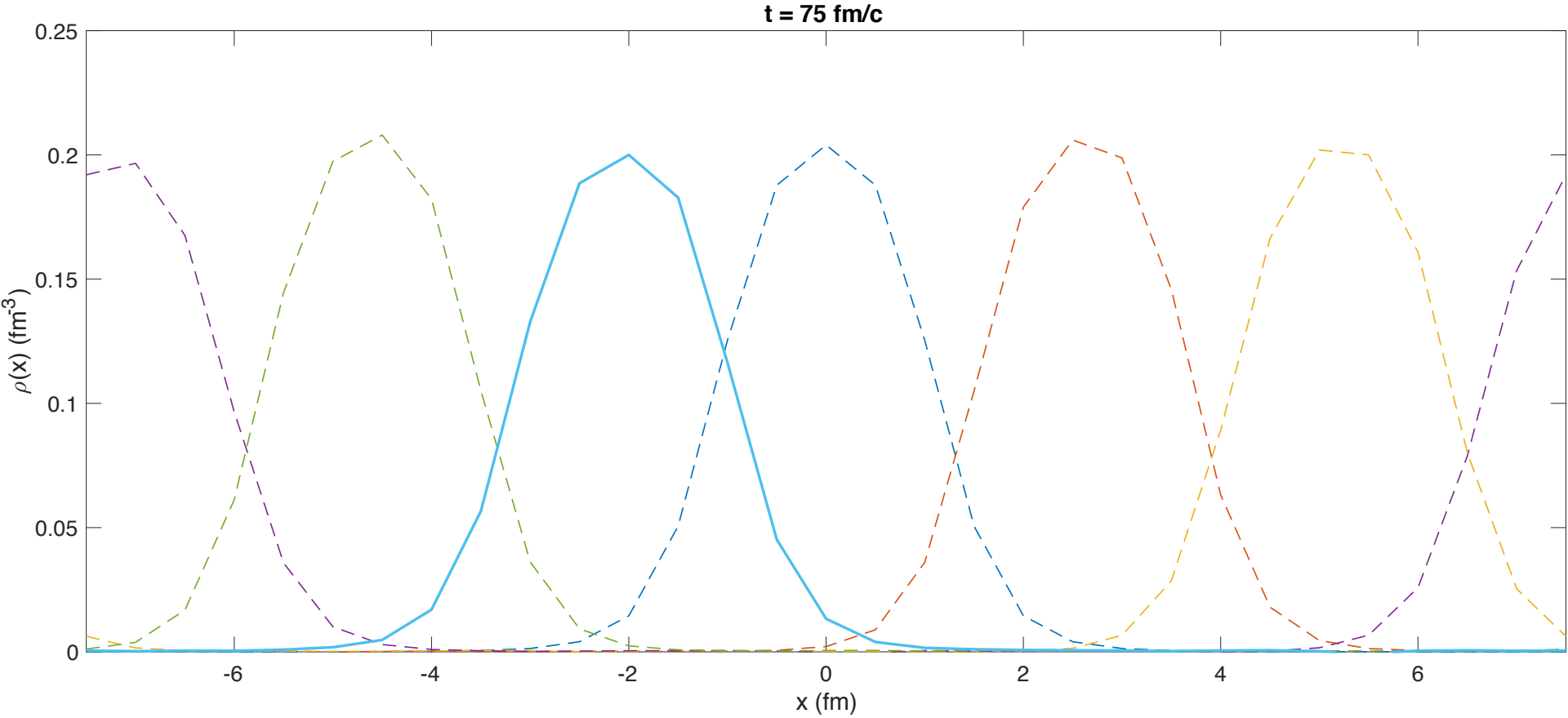
Slab at constant velocity



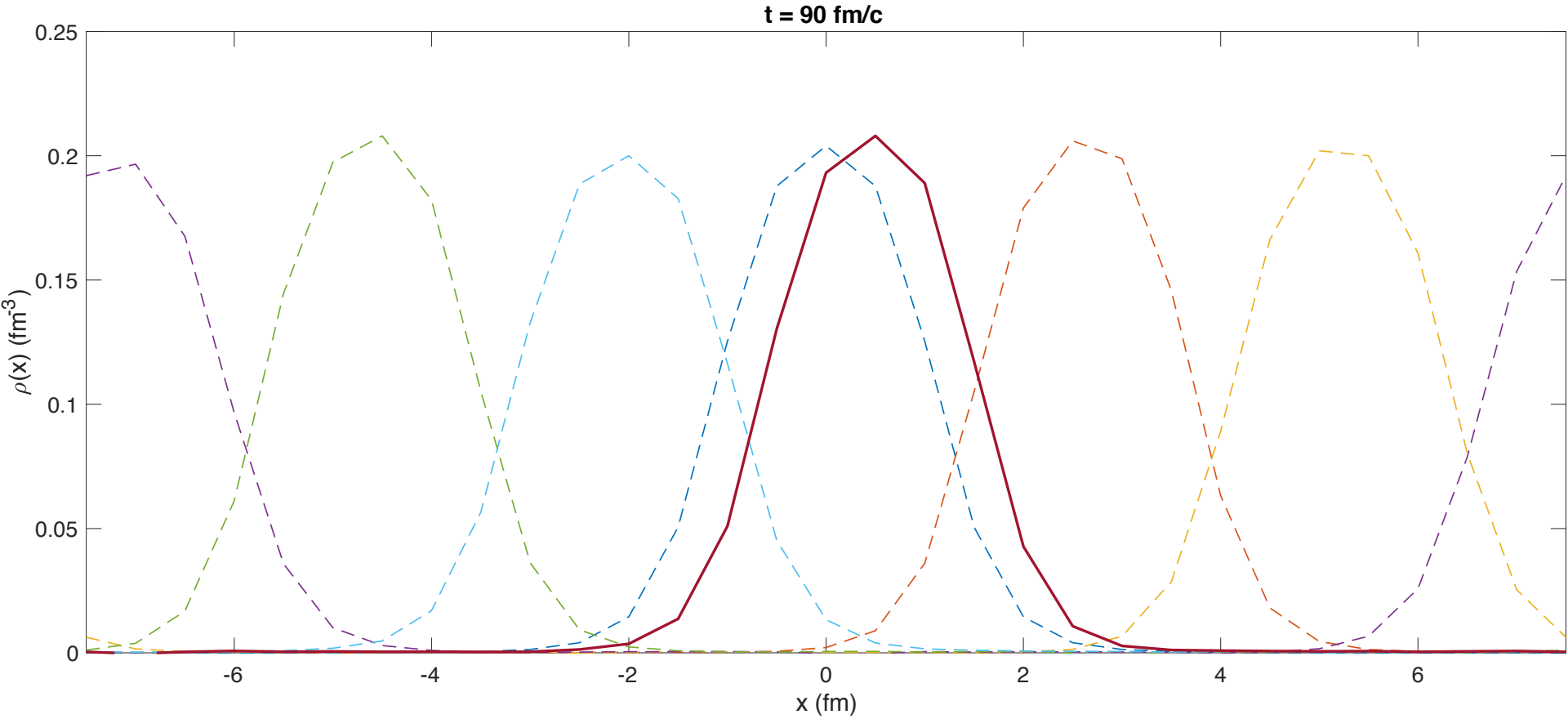
Slab at constant velocity



Slab at constant velocity



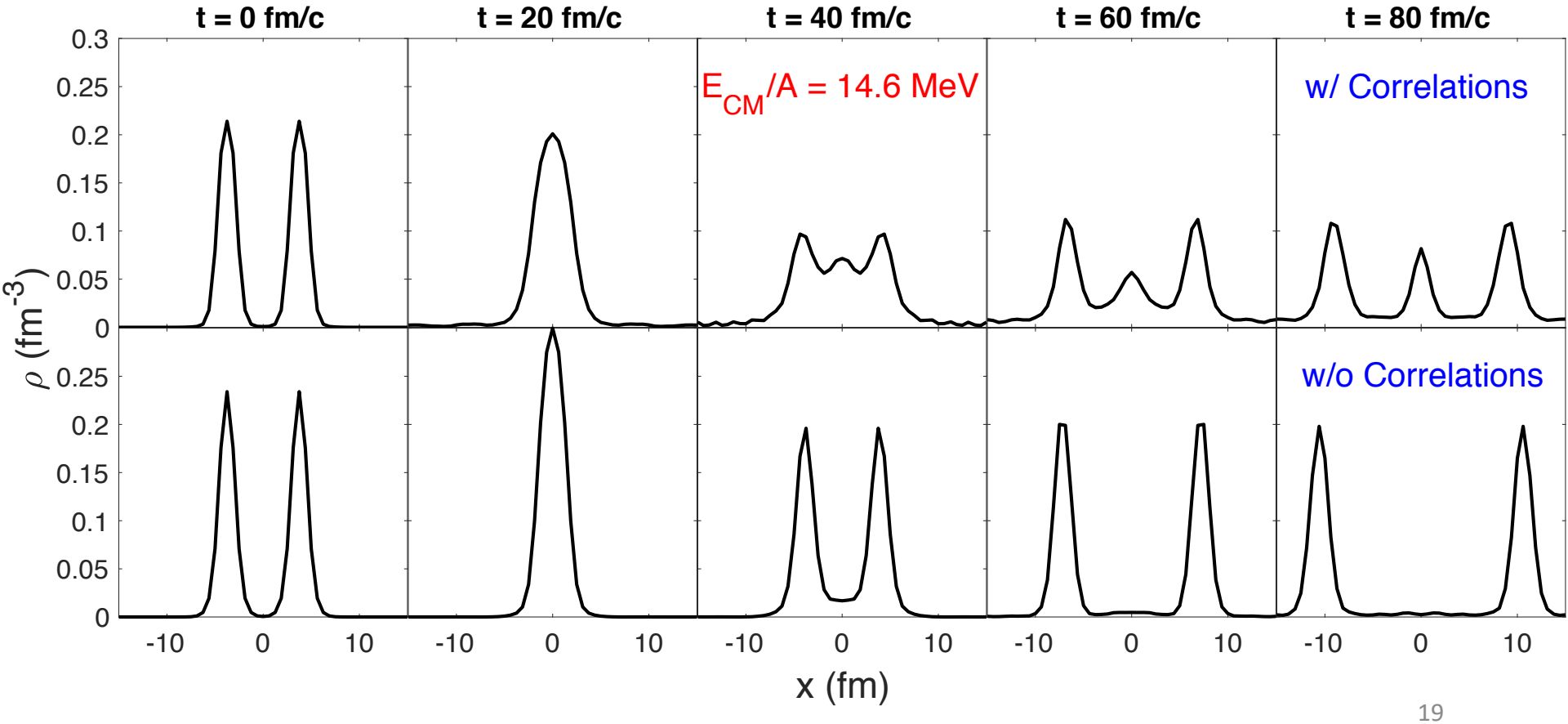
Slab at constant velocity



No spurious friction from the correlation integrals!

Collisions of 1D slabs (N = Z = 2)

- Stopping from short-ranged two-body interactions
- Central lump - Signal of multifragmentation? Artefact?



Summary

- Compared to *semi-classical transport models*, the NGF approach contains the *quantum correlations (up to 2nd order Born approximation)* due to short-ranged two-body correlations.
- Compared to *TDHF*, the NGF approach provides *two-body dissipation*, essential for descriptions of collisions at intermediate or high energies.
- NGF can be consistently applied to the *low-energy limit* (cf. TDHF) and the *high-energy limit* (cf. semi-classical transport).