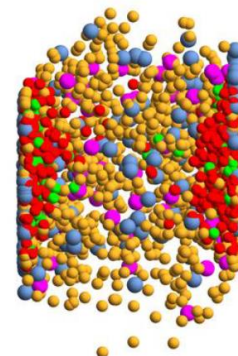


Description of strongly interacting nuclear matter within a microscopic off-shell transport approach

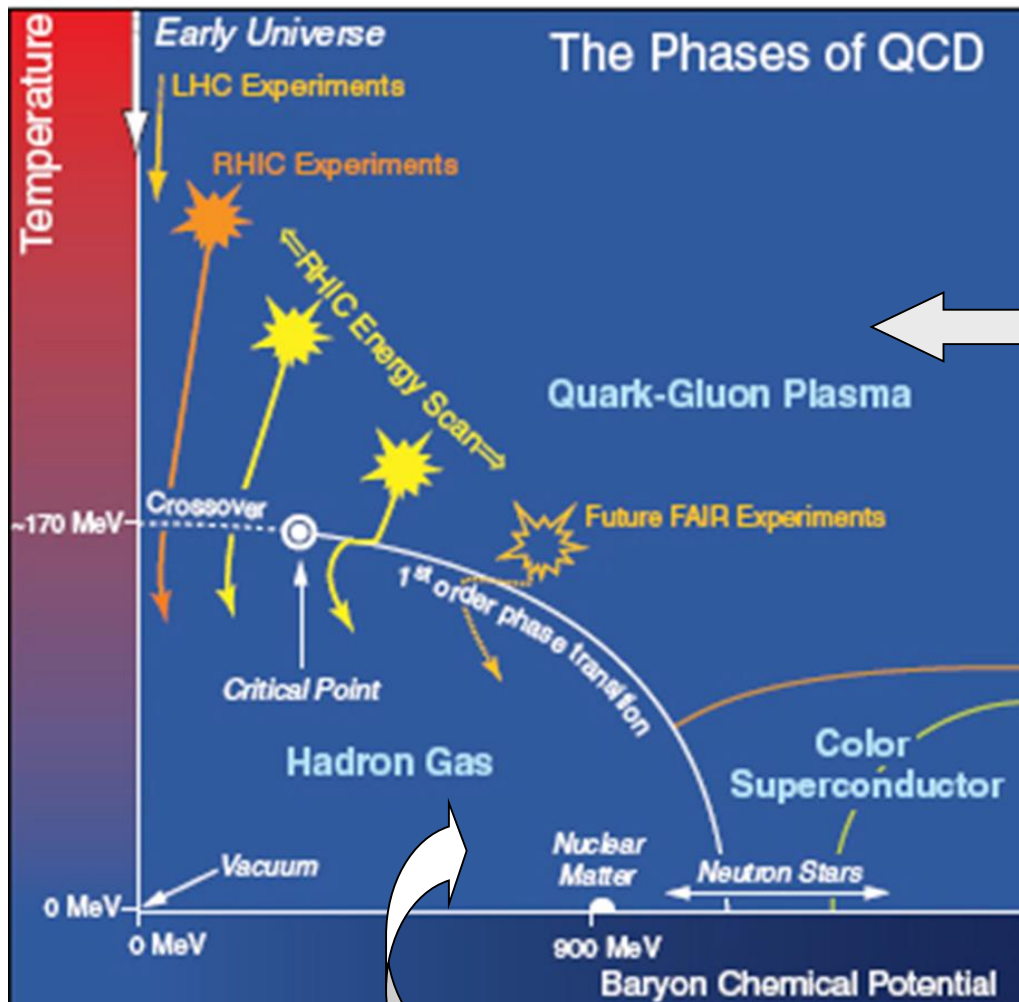
Elena Bratkovskaya
(GSI, Darmstadt & Uni. Frankfurt)
for the PHSD group



*Workshop 'Challenges to Transport Theory for Heavy-Ion
Collisions',
ECT*, Trento, Italy, May 20-24, 2019*

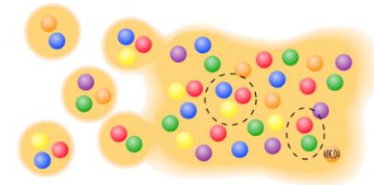


The ,holy grail' of heavy-ion physics:



The phase diagram of QCD

- Study of the **phase transition** from hadronic to partonic matter – **Quark-Gluon-Plasma**



- Search for the **critical point**
- Search for signatures of **chiral symmetry restoration**

- Study of the **in-medium** properties of hadrons at high baryon density and temperature

Dynamical description of heavy-ion collisions

The goal:

to study the properties of **strongly interacting matter** under extreme conditions from **a microscopic point of view**

Realization:

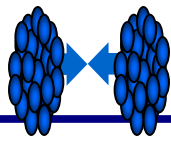
to develop a **dynamical many-body transport approach**

1) applicable for **strongly interacting systems**,
which includes:

2) **phase transition** from hadronic matter to QGP

3) **chiral symmetry restoration**





History: Semi-classical BUU equation



Boltzmann-Uehling-Uhlenbeck equation (non-relativistic formulation)
 - propagation of particles in the **self-generated Hartree-Fock mean-field potential** $U(\vec{r}, t)$ with an **on-shell collision term**:

$$\frac{\partial}{\partial t} f(\vec{r}, \vec{p}, t) + \frac{\vec{p}}{m} \vec{\nabla}_{\vec{r}} f(\vec{r}, \vec{p}, t) - \vec{\nabla}_{\vec{r}} U(\vec{r}, t) \vec{\nabla}_{\vec{p}} f(\vec{r}, \vec{p}, t) = \left(\frac{\partial f}{\partial t} \right)_{coll}$$

← **collision term:**
elastic and inelastic reactions

$f(\vec{r}, \vec{p}, t)$ is the **single particle phase-space distribution function**
 - probability to find the particle at position r with momentum p at time t

□ **self-generated Hartree-Fock mean-field potential:**

$$U(\vec{r}, t) = \frac{1}{(2\pi\hbar)^3} \sum_{\beta_{occ}} \int d^3 r' d^3 p V(\vec{r} - \vec{r}', t) f(\vec{r}', \vec{p}, t) + (Fock \text{ term})$$

□ **Collision term for 1+2→3+4 (let's consider fermions) :**

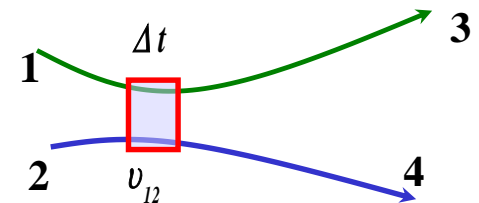
$$I_{coll} = \frac{4}{(2\pi)^3} \int d^3 p_2 d^3 p_3 \int d\Omega |v_{12}| \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \cdot \frac{d\sigma}{d\Omega} (1+2 \rightarrow 3+4) \cdot P$$

Probability including Pauli blocking of fermions:

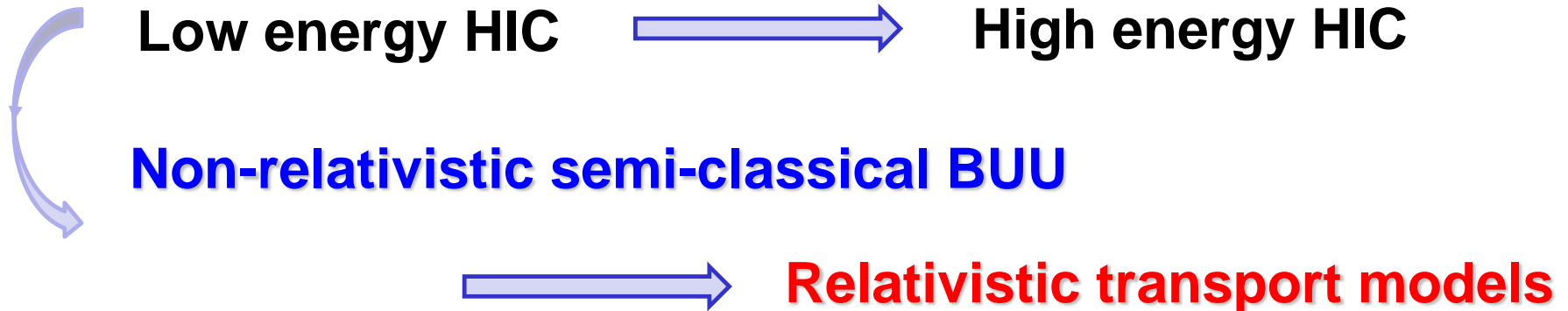
$$P = \underbrace{f_3 f_4 (1 - f_1) (1 - f_2)}_{\text{Gain term: } 3+4 \rightarrow 1+2} - \underbrace{f_1 f_2 (1 - f_3) (1 - f_4)}_{\text{Loss term: } 1+2 \rightarrow 3+4}$$

Gain term: 3+4→1+2

Loss term: 1+2→3+4



History: developments of relativistic transport models



‘Relativistic Vlasov-Uehling-Uhlenbeck model for heavy-ion collisions’

Che-Ming Ko, Qi Li, Phys.Rev. C37 (1988) 2270

‘Covariant Boltzmann-Uehling-Uhlenbeck approach for heavy-ion collisions’

Bernhard Blaettel, Volker Koch, Wolfgang Cassing, Ulrich Mosel, Phys.Rev. C38 (1988) 1767

‘Relativistic BUU approach with momentum dependent mean fields’

T. Maruyama, B. Blaettel, W. Cassing, A. Lang, U. Mosel, K. Weber, Phys.Lett. B297 (1992) 228

‘The Relativistic Landau-Vlasov method in heavy ion collisions’

C. Fuchs, H.H. Wolter, Nucl.Phys. A589 (1995) 732

■ ■ ■

Covariant transport equation



□ Covariant relativistic on-shell BUU equation :

from many-body theory by connected Green functions in phase-space + mean-field limit for the propagation part (VUU)

$$\left\{ \left(\Pi_\mu - \Pi_\nu (\partial_\mu^p U_\nu^\nu) - m^* (\partial_\mu^p U_S^\nu) \right) \partial_x^\mu + \left(\Pi_\nu (\partial_\mu^x U_\nu^\nu) + m^* (\partial_\mu^x U_S^\nu) \right) \partial_p^\mu \right\} f(x, p) = I_{coll}$$

$$I_{coll} \equiv \sum_{2,3,4} \int d2 d3 d4 [G^+ G]_{1+2 \rightarrow 3+4} \delta^4(\Pi + \Pi_2 - \Pi_3 - \Pi_4)$$

$$\times \{ f(x, p_3) f(x, p_4) (1 - f(x, p)) (1 - f(x, p_2)) - f(x, p) f(x, p_2) (1 - f(x, p_3)) (1 - f(x, p_4)) \}$$

$d2 \equiv \frac{d^3 p_2}{E_2}$

Gain term \rightarrow $3+4 \rightarrow 1+2$ \leftarrow *Loss term* $1+2 \rightarrow 3+4$

$$m^*(x, p) = m + U_S(x, p) \quad - \text{effective mass}$$

$$\Pi_\mu(x, p) = p_\mu - U_\mu(x, p) \quad - \text{effective momentum}$$

where $\partial_\mu^x \equiv (\partial_t, \vec{\nabla}_r)$

$U_S(x, p)$, $U_\mu(x, p)$ are scalar and vector part of particle **self-energies**

$\delta(\Pi_\mu \Pi^\mu - m^{*2})$ – mass-shell constraint

Dynamical transport model: collision terms

□ BUU eq. for **different particles of type $i=1,...n$**

$$Df_i \equiv \frac{d}{dt} f_i = I_{coll} [f_1, f_2, ..., f_n]$$

Drift term=Vlasov eq. collision term

$i :$ *Baryons* : $p, n, \Delta(1232), N(1440), N(1535), ..., \Lambda, \Sigma, \Sigma^*, \Xi, \Omega; \Lambda_c$

Mesons : $\pi, \eta, K, \bar{K}, \rho, \omega, K^*, \eta', \phi, a_1, ..., D, \bar{D}, J / \Psi, \Psi', ...$

→ **coupled set of BUU equations** for different particles of type $i=1,...n$

$$\left\{ \begin{array}{l} Df_N = I_{coll} [f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ...] \\ Df_\Delta = I_{coll} [f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ...] \\ ... \\ Df_\pi = I_{coll} [f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ...] \\ ... \end{array} \right.$$

Elementary hadronic interactions

Consider **all possible interactions** – **elastic and inelastic collisions** - for the system of (N,R,m) , where N -nucleons, R - resonances, m -mesons, and **resonance decays**

Low energy collisions:

- binary $2 \leftrightarrow 2$ and $2 \leftrightarrow 3(4)$ reactions
- $1 \leftrightarrow 2$: formation and **decay** of baryonic and mesonic resonances

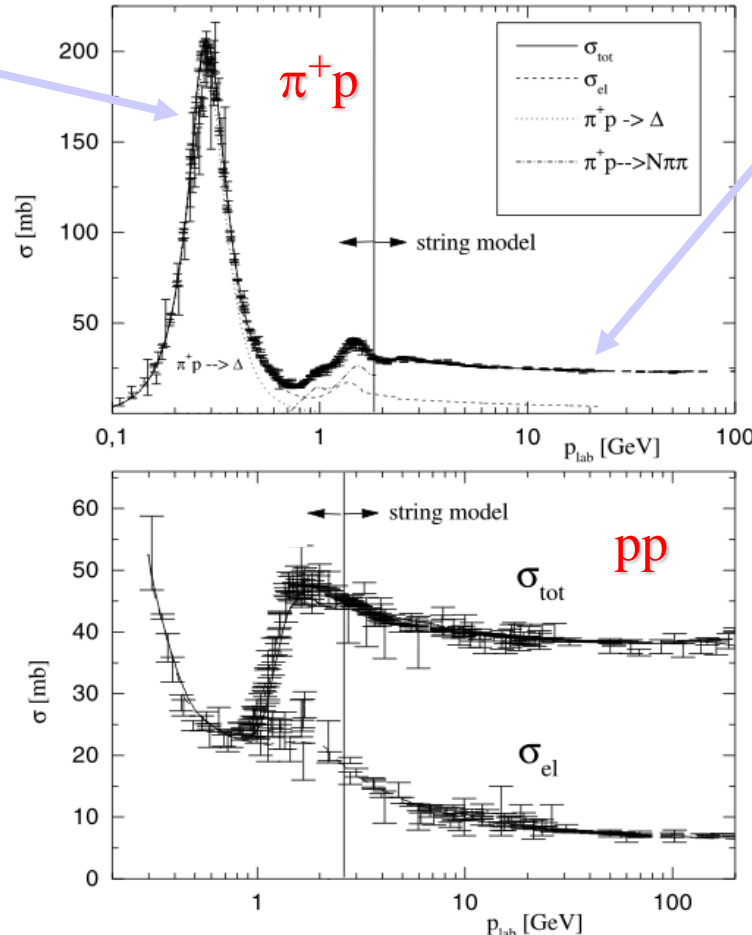
$BB \leftrightarrow B'B'$
 $BB \leftrightarrow B'B'm$
 $mB \leftrightarrow m'B'$
 $mB \leftrightarrow B'$
 $mm \leftrightarrow m'm'$
 $mm \leftrightarrow m'$...

Baryons:

$B = p, n, \Delta(1232),$
 $N(1440), N(1535), \dots$

Mesons:

$M = \pi, \eta, \rho, \omega, \phi, \dots$



High energy collisions: (above $s^{1/2} \sim 2.5$ GeV)

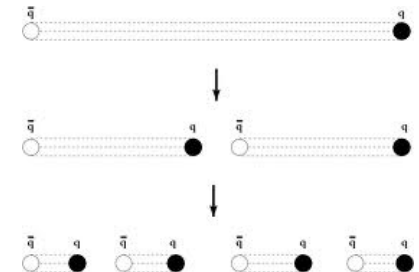
Inclusive particle production:

$BB \rightarrow X, mB \rightarrow X, mm \rightarrow X$

X = many particles

described by

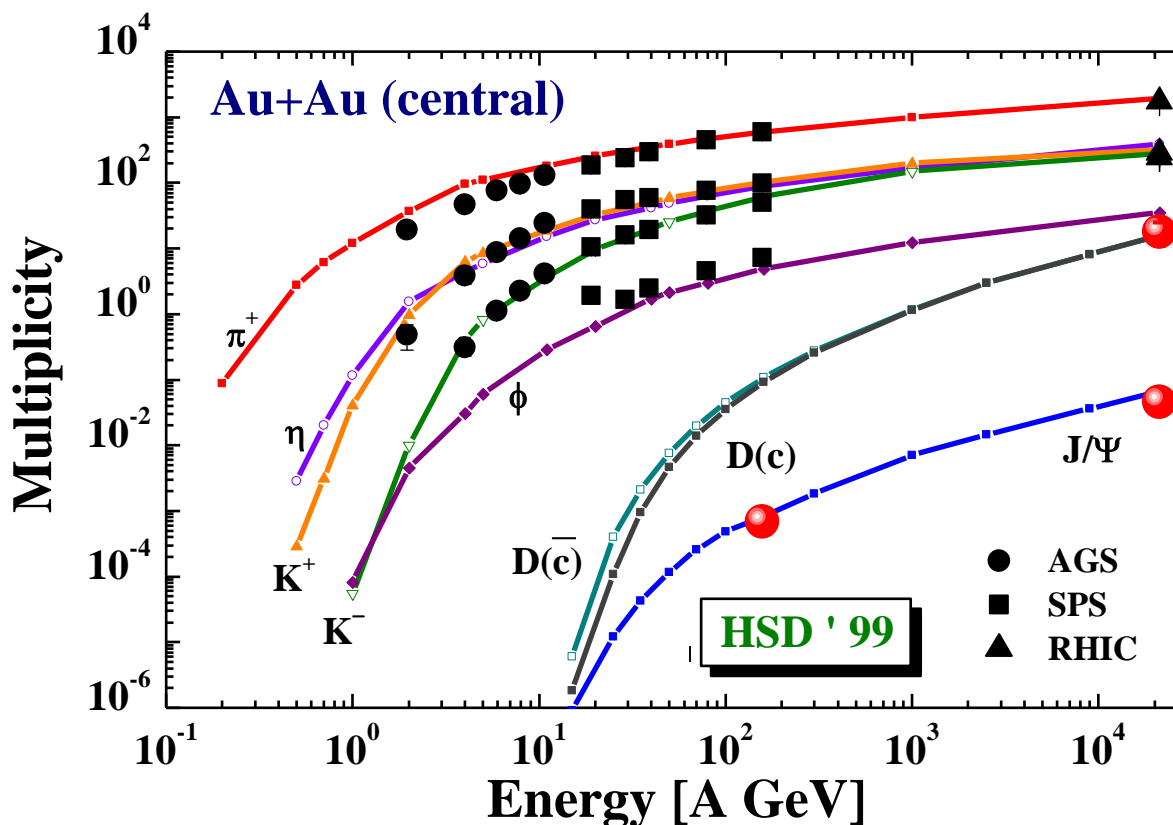
string formation and decay
 (string = excited color singlet states $q-q\bar{q}, q-q\bar{q}$)
 using **LUND string model**





Hadron-String-Dynamics – a microscopic transport model for heavy-ion reactions

- very good description of particle production in **pp, pA, pA, AA reactions**
- unique description of nuclear dynamics from **low** (~ 100 MeV) to **ultrarelativistic** (> 20 TeV) energies



From weakly to strongly interacting systems

In-medium effects (on hadronic or partonic levels!) = changes of particle properties in the hot and dense medium

Examples: **hadronic medium** - vector mesons, strange mesons
QGP – dressing of partons

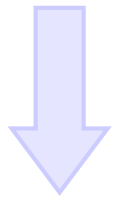
Many-body theory:

Strong interaction → **large width** → broad spectral function → **quantum object**

Semi-classical on-shell BUU: applies for small collisional width, i.e. for a weakly interacting systems of particles

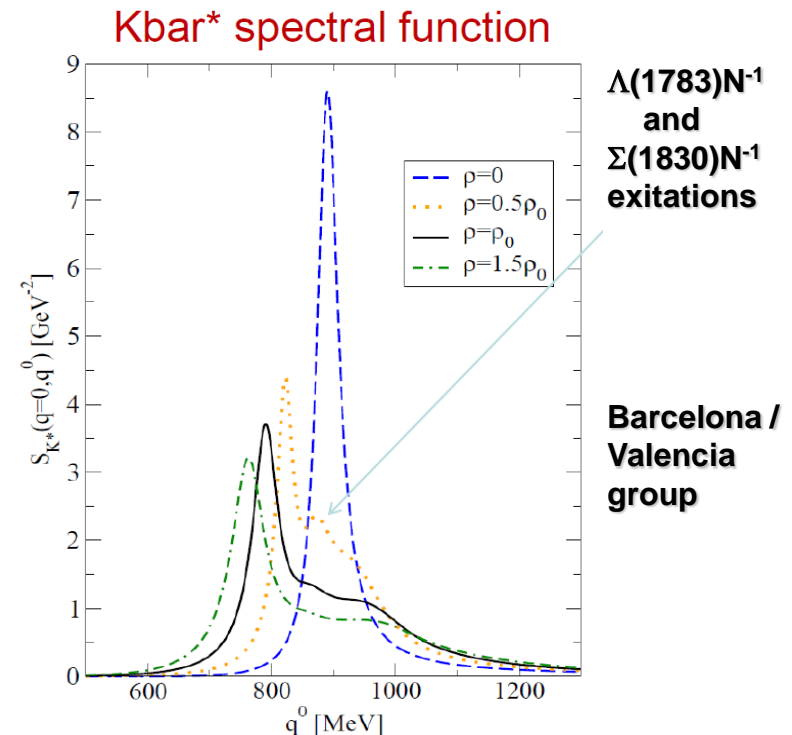
■ How to describe the dynamics of broad **strongly interacting quantum states** in transport theory?

□ **semi-classical BUU**



first order gradient expansion of quantum **Kadanoff-Baym** equations

□ **generalized transport equations based on Kadanoff-Baym dynamics**



Dynamical description of strongly interacting systems

Quantum field theory →

Kadanoff-Baym dynamics for resummed single-particle Green functions $S^<$

$$\hat{S}_{0x}^{-1} S_{xy}^< = \Sigma_{xz}^{ret} \odot S_{zy}^< + \Sigma_{xz}^< \odot S_{zy}^{adv}$$

(1962)

Green functions $S^</math> / self-energies Σ :$

$$iS_{xy}^< = \eta \langle \{ \Phi^+(y) \Phi(x) \} \rangle$$

$$iS_{xy}^> = \langle \{ \Phi(y) \Phi^+(x) \} \rangle$$

$$iS_{xy}^c = \langle T^c \{ \Phi(x) \Phi^+(y) \} \rangle - \text{causal}$$

$$iS_{xy}^a = \langle T^a \{ \Phi(x) \Phi^+(y) \} \rangle - \text{anticausal}$$

Integration over the intermediate spacetime

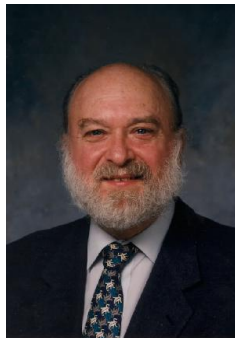
$$S_{xy}^{ret} = S_{xy}^c - S_{xy}^< = S_{xy}^> - S_{xy}^a - \text{retarded}$$

$$S_{xy}^{adv} = S_{xy}^c - S_{xy}^> = S_{xy}^< - S_{xy}^a - \text{advanced}$$

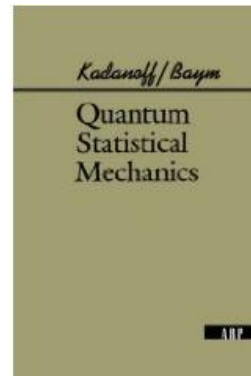
$$\eta = \pm 1 (\text{bosons} / \text{fermions})$$

$$T^a (T^c) - (\text{anti-})\text{time-ordering operator}$$

$$\hat{S}_{0x}^{-1} \equiv -(\partial_x^\mu \partial_\mu^x + M_0^2)$$



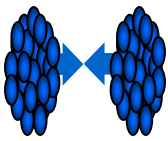
Leo Kadanoff



Gordon Baym

1st application for spacially homodeneous system with deformed Fermi sphere:

P. Danielewicz, Ann. Phys. 152, 305 (1984); ... H.S. Köhler, Phys. Rev. 51, 3232 (1995); ...



From Kadanoff-Baym equations to generalized transport equations

After the **first order gradient expansion of the Wigner transformed Kadanoff-Baym equations** and separation into the real and imaginary parts one gets:

Generalized transport equations (GTE):

$$\begin{array}{c} \text{drift term} \quad \text{Vlasov term} \quad \boxed{\text{backflow term}} \quad \text{collision term} = \text{'gain' - 'loss' term} \\ \diamond \{ P^2 - M_0^2 - \text{Re} \Sigma_{XP}^{ret} \} \{ S_{XP}^< \} - \boxed{\diamond \{ \Sigma_{XP}^< \} \{ \text{Re} S_{XP}^{ret} \}} = \frac{i}{2} [\Sigma_{XP}^> S_{XP}^< - \Sigma_{XP}^< S_{XP}^>] \end{array}$$

Backflow term incorporates the **off-shell** behavior in the particle propagation
! vanishes in the quasiparticle limit $A_{XP} \rightarrow \delta(p^2 - M^2)$

□ GTE: Propagation of the Green's function $iS_{XP}^< = A_{XP} N_{XP}$, which carries information not only on the **number of particles** (N_{XP}), but also on their **properties**, interactions and correlations (via A_{XP})

Botermans-Malfliet (1990)

□ **Spectral function:**

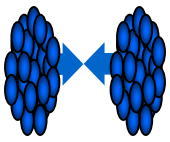
$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - \text{Re} \Sigma_{XP}^{ret})^2 + \Gamma_{XP}^2/4}$$

$\Gamma_{XP} = -\text{Im} \Sigma_{XP}^{ret} = 2 p_0 \Gamma$ — **'width' of spectral function**
= reaction rate of particle (at space-time position X)

4-dimentional generalization of the Poisson-bracket:

$$\diamond \{ F_1 \} \{ F_2 \} := \frac{1}{2} \left(\frac{\partial F_1}{\partial X_\mu} \frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu} \frac{\partial F_2}{\partial X^\mu} \right)$$

□ **Life time** $\tau = \frac{\hbar c}{\Gamma}$



General testparticle off-shell equations of motion

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

□ Employ **testparticle Ansatz** for the real valued quantity $i S_{XP}^<$

$$F_{XP} = A_{XP} N_{XP} = i S_{XP}^< \sim \sum_{i=1}^N \delta^{(3)}(\vec{X} - \vec{X}_i(t)) \delta^{(3)}(\vec{P} - \vec{P}_i(t)) \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine **equations of motion** !

→ **Generalized testparticle Cassing's off-shell equations of motion for the time-like particles:**

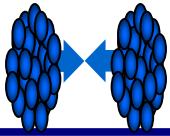
$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\vec{P}_i + \vec{\nabla}_{P_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} \text{Re}\Sigma_i^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial \text{Re}\Sigma_{(i)}^{\text{ret}}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$



Collision term in off-shell transport models

Collision term for reaction 1+2->3+4:

$$I_{coll}(X, \vec{P}, M^2) = Tr_2 Tr_3 Tr_4 \underbrace{A(X, \vec{P}, M^2) A(X, \vec{P}_2, M_2^2) A(X, \vec{P}_3, M_3^2) A(X, \vec{P}_4, M_4^2)}_{|G((\vec{P}, M^2) + (\vec{P}_2, M_2^2) \rightarrow (\vec{P}_3, M_3^2) + (\vec{P}_4, M_4^2))|_{\mathcal{A}, \mathcal{S}}^2} \delta^{(4)}(P + P_2 - P_3 - P_4)$$

$$[\underbrace{N_{X\vec{P}_3 M_3^2} N_{X\vec{P}_4 M_4^2} \bar{f}_{X\vec{P} M^2} \bar{f}_{X\vec{P}_2 M_2^2}}_{\text{,gain' term}} - \underbrace{N_{X\vec{P} M^2} N_{X\vec{P}_2 M_2^2} \bar{f}_{X\vec{P}_3 M_3^2} \bar{f}_{X\vec{P}_4 M_4^2}}_{\text{,loss' term}}]$$

with $\bar{f}_{X\vec{P} M^2} = 1 + \eta N_{X\vec{P} M^2}$ and $\eta = \pm 1$ for bosons/fermions, respectively.

The trace over particles 2,3,4 reads explicitly

for fermions

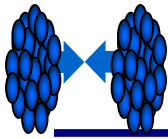
$$Tr_2 = \sum_{\sigma_2, \tau_2} \frac{1}{(2\pi)^4} \int d^3 P_2 \left(\frac{dM_2^2}{2\sqrt{\vec{P}_2^2 + M_2^2}} \right)$$

for bosons

$$Tr_2 = \sum_{\sigma_2, \tau_2} \frac{1}{(2\pi)^4} \int d^3 P_2 \left(\frac{dP_{0,2}^2}{2} \right)$$

additional integration

The transport approach and the particle spectral functions are fully determined once the **in-medium transition amplitudes G are known in their **off-shell dependence!****



In-medium transition rates: G-matrix approach

Need to know **in-medium transition amplitudes G** and their off-shell dependence

$$|G((\vec{P}, M^2) + (\vec{P}_2, M_2^2) \rightarrow (\vec{P}_3, M_3^2) + (\vec{P}_4, M_4^2))|_{\mathcal{A}, \mathcal{S}}^2$$

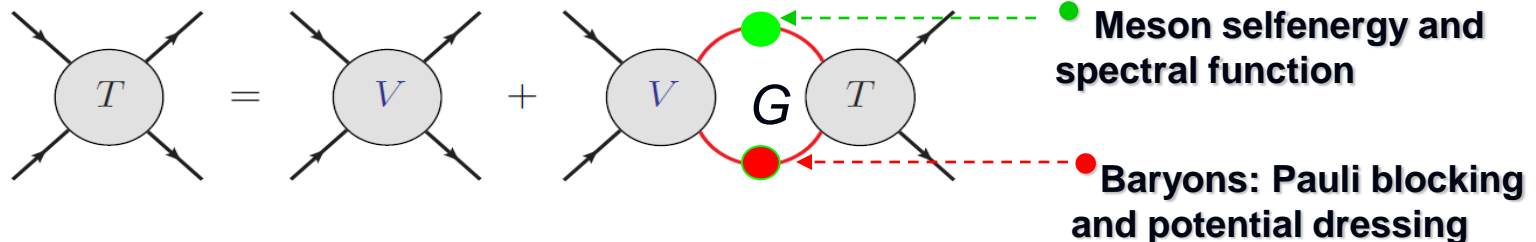


Coupled channel G-matrix approach

Transition probability :

$$P_{1+2 \rightarrow 3+4}(s) = \int d \cos(\theta) \frac{1}{(2s_1 + 1)(2s_2 + 1)} \sum_i \sum_{\alpha} G^{\dagger} G$$

with $G(p, \rho, T)$ - **G-matrix** from the solution of **coupled-channel equations**:



$$\blacksquare T_{ij}(\rho, T) = V_{ij} + V_{il} G_l(\rho, T) T_{lj}(\rho, T)$$

For strangeness:

D. Cabrera, L. Tolos, J. Aichelin, E.B., PRC C90 (2014) 055207; W. Cassing, L. Tolos, E.B., A. Ramos, NPA727 (2003) 59

Transition probabilities for $\pi Y \leftrightarrow K^- p$ ($Y = \Lambda, \Sigma$)

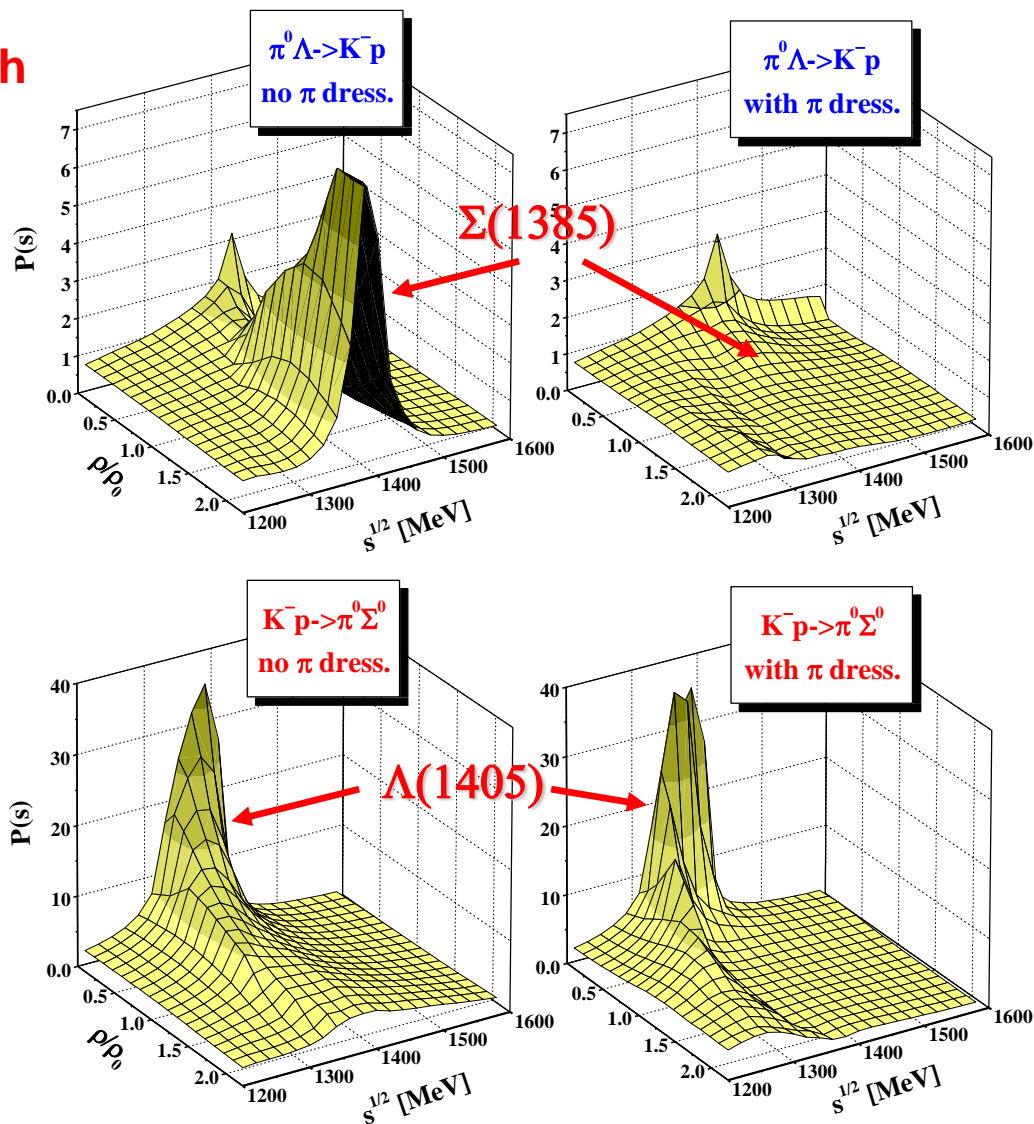
L. Tolos et al., NPA 690 (2001) 547

Coupled-channel G-matrix approach provides in-medium transition probabilities for different channels, e.g. $\pi Y \leftrightarrow K^- p$ ($Y = \Lambda, \Sigma$)

- **With pion dressing:**
 $\Lambda(1405)$ and $\Sigma(1385)$ melt away with baryon density

□ K^- absorption/production from πY collisions are strongly suppressed in the nuclear medium

□ πY is the **dominant channel** for K^- production in heavy-ion collisions !



W. Cassing, L. Tolos, E.L.B., A. Ramos, NPA 727 (2003) 59

Mean-field potential in off-shell transport models

- **Many-body theory:** Interacting relativistic particles have a **complex self-energy**:

$$\Sigma_{XP}^{ret} = Re \Sigma_{XP}^{ret} + i Im \Sigma_{XP}^{ret}$$

The neg. imaginary part $\Gamma_{XP} = -Im \Sigma_{XP}^{ret} = 2 p_0 \Gamma$ is related via the width $\Gamma = \Gamma_{coll} + \Gamma_{dec}$ to the inverse lifetime of the particle $\tau \sim 1/\Gamma$

- The **collision width** Γ_{coll} is determined from the **loss term** of the collision integral I_{coll}

$$-I_{coll}(loss) = \Gamma_{coll}(X, \vec{P}, M^2) N_{X\vec{P} M^2}$$

- By **dispersion relation** we get a contribution to the **real part of self-energy**:

$$Re \Sigma_{XP}^{ret}(p_0) = P \int_0^\infty dq \frac{Im \Sigma_{XP}^{ret}(q)}{(q - p_0)}$$

which gives a **mean-field potential** U_{XP} via:

$$Re \Sigma_{XP}^{ret}(p_0) = 2 p_0 U_{XP}$$

→ The **complex self-energy** relates in a self-consistent way to the **self-generated mean-field potential and collision width**

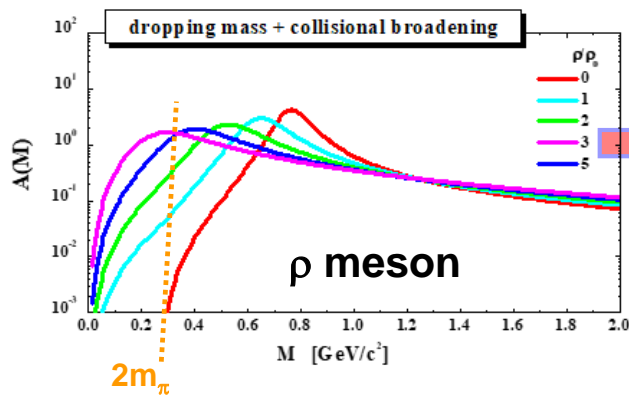
In-medium
 $\rho \gg \rho_0$

Off-shell vs. on-shell transport dynamics

Time evolution of the mass distribution of ρ and ω mesons for central C+C collisions ($b=1$ fm) at 2 A GeV for **dropping mass + collisional broadening scenario**

$$A(M, p, \rho) = \frac{2}{\pi} \frac{M^2 \Gamma_{\text{tot}}(M, p, \rho)}{(M^2 - M_0^2 - \text{Re} \Sigma^{\text{ret}}) + (M \Gamma_{\text{tot}}(M, p, \rho))^2},$$

width $\Gamma \sim -\text{Im} \Sigma^{\text{ret}} / M$



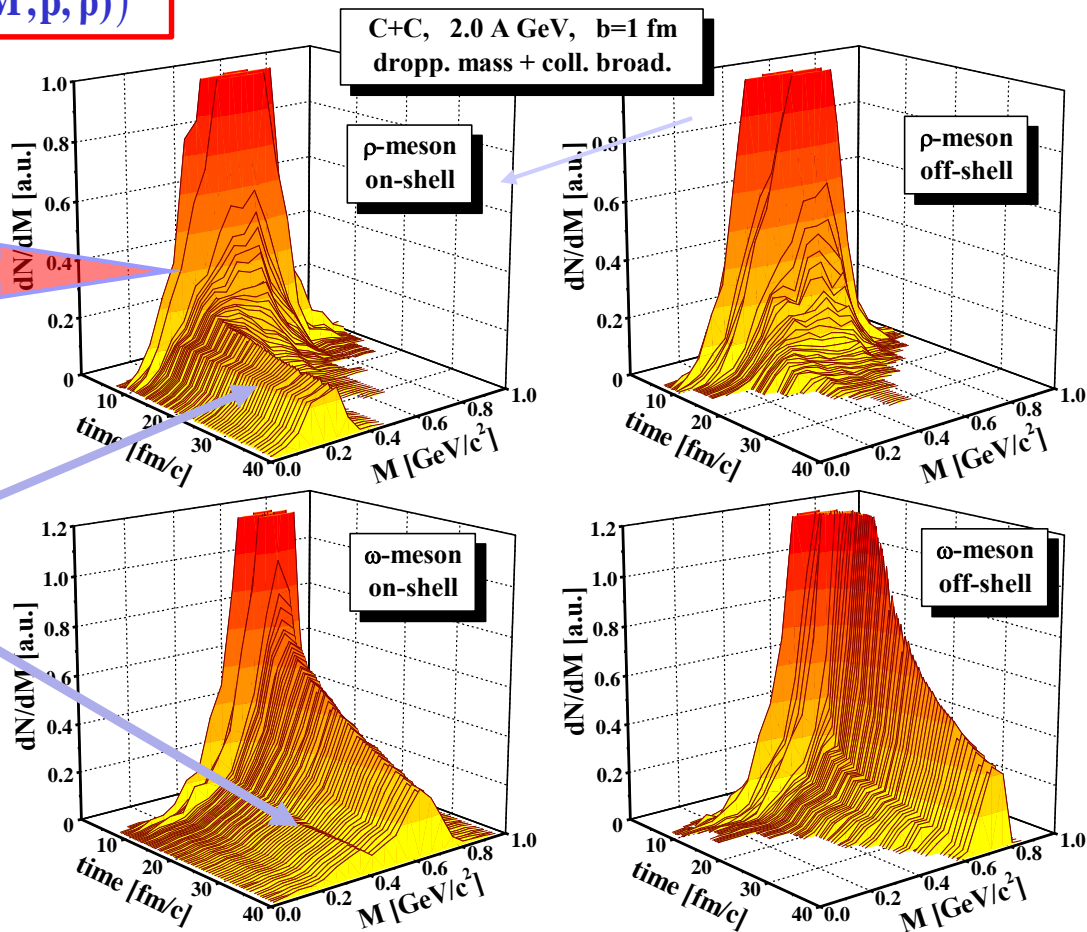
On-shell BUU:

low mass ρ and ω mesons live forever (and shine ,fake' dileptons)!

The off-shell spectral function becomes **on-shell** in the vacuum **dynamically** by propagation through the medium!

On-shell

Off-shell



Detailed balance on the level of $2 \leftrightarrow n$: treatment of multi-particle collisions in transport approaches

W. Cassing, NPA 700 (2002) 618

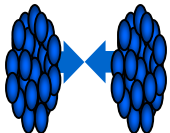
Generalized collision integral for $n \leftrightarrow m$ reactions:

$$I_{coll} = \sum_n \sum_m I_{coll}[n \leftrightarrow m]$$

$$\begin{aligned} I_{coll}^i[n \leftrightarrow m] = & \frac{1}{2} N_n^m \sum_\nu \sum_\lambda \left(\frac{1}{(2\pi)^4} \right)^{n+m-1} \int \left(\prod_{j=2}^n d^4 p_j A_j(x, p_j) \right) \left(\prod_{k=1}^m d^4 p_k A_k(x, p_k) \right) \\ & \times A_i(x, p) W_{n,m}(p, p_j; i, \nu \mid p_k; \lambda) (2\pi)^4 \delta^4(p^\mu + \sum_{j=2}^n p_j^\mu - \sum_{k=1}^m p_k^\mu) \\ & \times [\tilde{f}_i(x, p) \prod_{k=1}^m f_k(x, p_k) \prod_{j=2}^n \tilde{f}_j(x, p_j) - f_i(x, p) \prod_{j=2}^n f_j(x, p_j) \prod_{k=1}^m \tilde{f}_k(x, p_k)]. \end{aligned}$$

$\tilde{f} = 1 + \eta f$ is Pauli-blocking or Bose-enhancement factors;
 $\eta=1$ for bosons and $\eta=-1$ for fermions

$W_{n,m}(p, p_j; i, \nu \mid p_k; \lambda)$ is a **transition probability**



Antibaryon production in heavy-ion reactions

Multi-meson fusion reactions

$$m_1 + m_2 + \dots + m_n \leftrightarrow B + \bar{B}$$

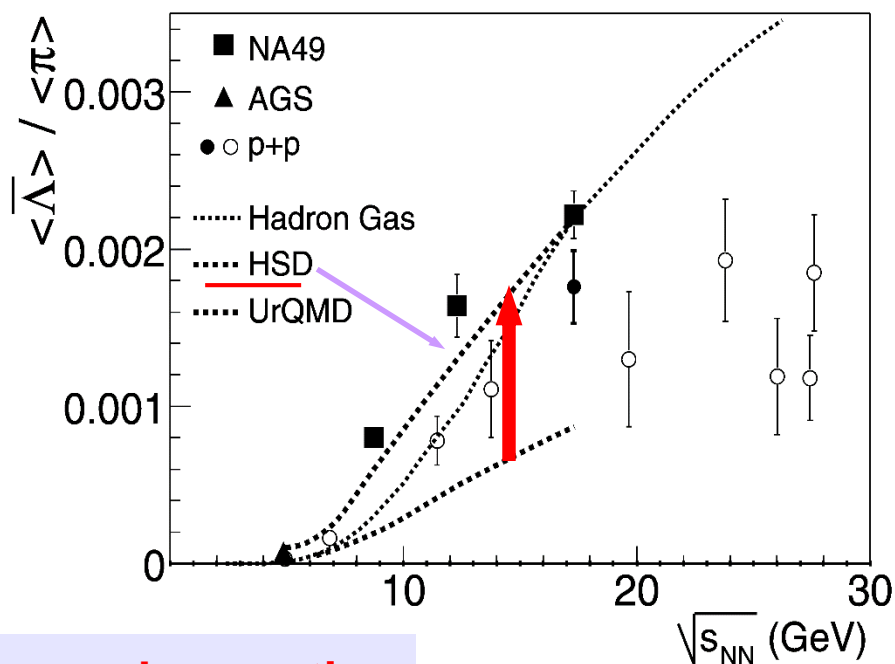
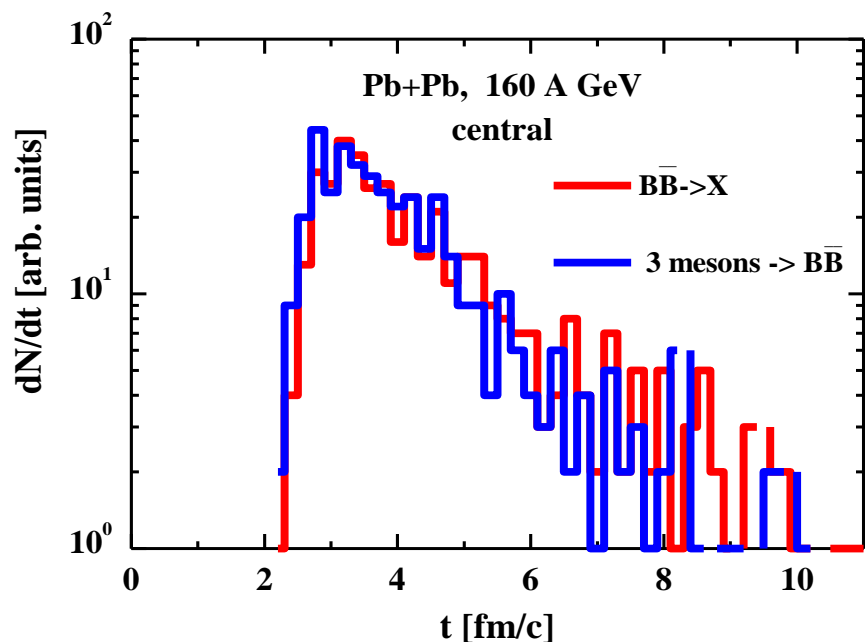
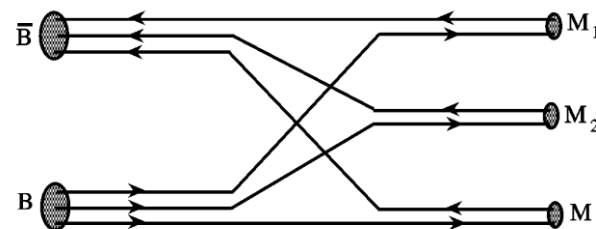
$m = \pi, \rho, \omega, \dots$ $B = p, \Lambda, \Sigma, \Xi, \Omega, \dots$ (>2800 channels)

□ important for anti-baryon (anti-p, anti- Λ , anti- Ξ , anti- Ω) dynamics !

W. Cassing, NPA 700 (2002) 618

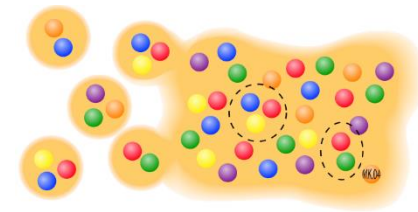
E. Seifert, W. Cassing, PRC 97, 024913 (2018);

PRC 97, 044907 (2018)



→ approximate equilibrium of annihilation and recreation

Goal: microscopic transport description of the **partonic** and **hadronic** phase



Problems:

- ❑ How to model a **QGP phase** in line with IQCD data?
- ❑ How to solve the **hadronization problem**?

Ways to go:

pQCD based models:

- **QGP phase**: pQCD cascade
 - **hadronization**: quark coalescence
- AMPT, HIJING, BAMPS

„Hybrid“ models:

- **QGP phase**: **hydro** with QGP EoS
 - **hadronic freeze-out**: after burner - hadron-string transport model
- Hybrid-UrQMD

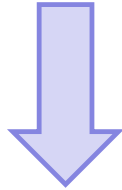
- **microscopic** transport description of the **partonic** and **hadronic phase** in terms of strongly interacting dynamical **quasi-particles** and off-shell hadrons

→ PHSD



Degrees-of-freedom of QGP

❖ IQCD gives QGP EoS at finite μ_B



! need to be interpreted in terms of **degrees-of-freedom**

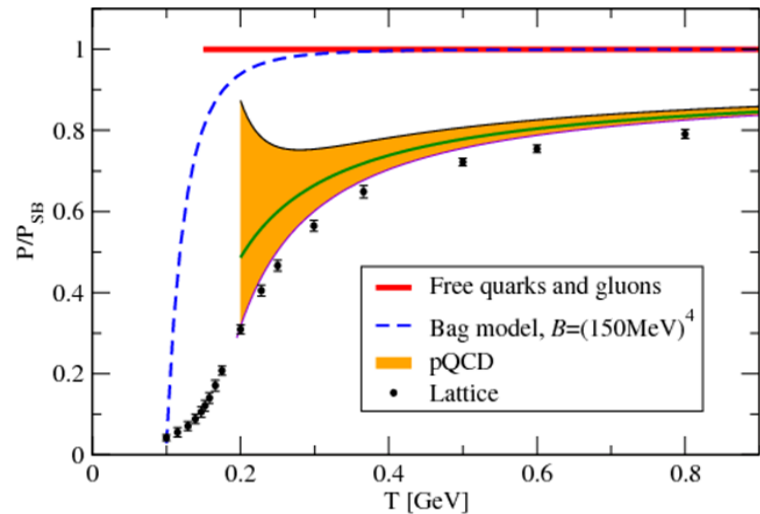
pQCD:

- ☐ weakly interacting system
- ☐ massless quarks and gluons



❖ How to learn about degrees-of-freedom of QGP?

Theory \leftrightarrow HIC experiments

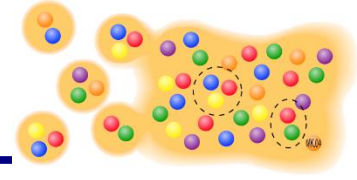


Non-perturbative QCD \leftarrow pQCD

Thermal QCD
= QCD at high parton densities:

- ☐ strongly interacting system
- ☐ massive quarks and gluons
- \rightarrow quasiparticles
- = effective degrees-of-freedom

From SIS to LHC: from hadrons to partons



The goal: to study of the phase transition from hadronic to partonic matter and properties of the Quark-Gluon-Plasma from a microscopic origin

→ need a consistent non-equilibrium transport model

- ❑ with explicit parton-parton interactions (i.e. between quarks and gluons)
- ❑ explicit phase transition from hadronic to partonic degrees of freedom
- ❑ IQCD EoS for partonic phase (‘crossover’ at small μ_q)
- ❑ Transport theory: off-shell Kadanoff-Baym equations for the Green-functions $S_h^<(x,p)$ in phase-space representation for the partonic and hadronic phase



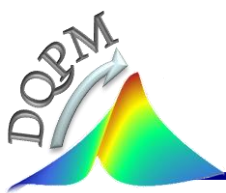
→ **Parton-Hadron-String-Dynamics (PHSD)**

QGP phase described by

**Dynamical QuasiParticle Model
(DQPM)**

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;
NPA831 (2009) 215;
W. Cassing, EPJ ST 168 (2009) 3

A. Peshier, W. Cassing, PRL 94 (2005) 172301;
W. Cassing, NPA 791 (2007) 365; NPA 793 (2007)



Dynamical QuasiParticle Model (DQPM) - Basic ideas:

DQPM describes **QCD** properties in terms of **,resummed' single-particle Green's functions** (propagators) – in the sense of a two-particle irreducible (2PI) approach:

$$\begin{aligned} \text{gluon propagator: } \Delta^{-1} &= P^2 - \Pi & \& \quad \text{quark propagator } S_q^{-1} &= P^2 - \Sigma_q \\ \text{gluon self-energy: } \Pi &= M_g^2 - i2\gamma_g\omega & \& \quad \text{quark self-energy: } \Sigma_q &= M_q^2 - i2\gamma_q\omega \end{aligned}$$

(scalar approximation)

- the resummed properties are specified by **complex (retarded) self-energies**:
- the **real part of self-energies** (Σ_q, Π) describes a **dynamically generated mass** (M_q, M_g);
- the **imaginary part** describes the **interaction width** of partons (γ_q, γ_g)
- **Spectral functions** : $A_q \sim \text{Im} S_q^{\text{ret}}$, $A_g \sim \text{Im} \Delta^{\text{ret}}$

□ Entropy density of interacting bosons and fermions in the quasiparticle limit (2PI)

(G. Baym 1998):

QGP

$$\begin{aligned} s^{dqp} = & -d_g \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \frac{\partial n_B}{\partial T} (\text{Im} \ln(-\Delta^{-1}) + \text{Im} \Pi \text{Re} \Delta) & \text{gluons} \\ & - d_q \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \frac{\partial n_F((\omega - \mu_q)/T)}{\partial T} (\text{Im} \ln(-S_q^{-1}) + \text{Im} \Sigma_q \text{Re} S_q) & \text{quarks} \\ & - d_{\bar{q}} \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \frac{\partial n_F((\omega + \mu_q)/T)}{\partial T} (\text{Im} \ln(-S_{\bar{q}}^{-1}) + \text{Im} \Sigma_{\bar{q}} \text{Re} S_{\bar{q}}) & \text{antiquarks} \end{aligned}$$

The Dynamical QuasiParticle Model (DQPM)



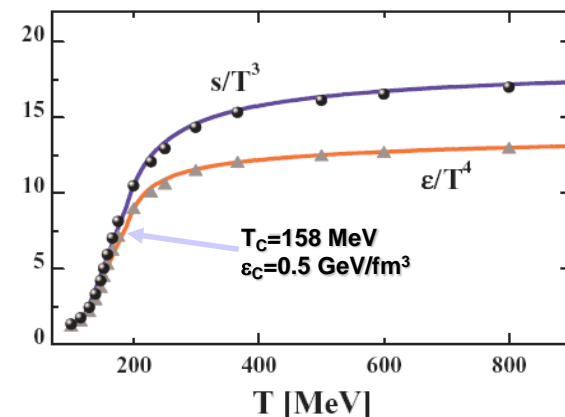
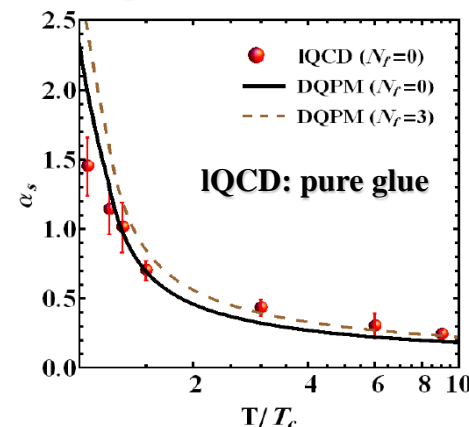
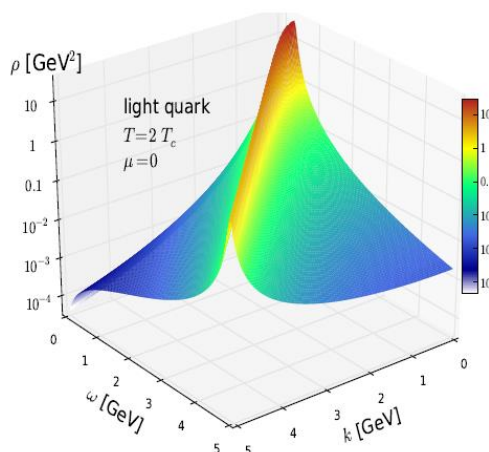
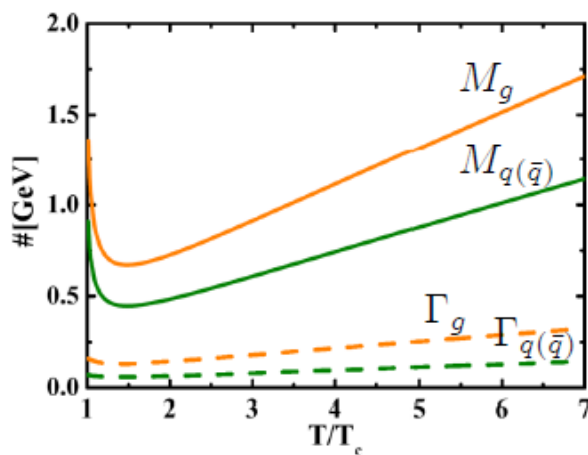
- Basic idea: **interacting quasi-particles: massive quarks and gluons** (g, q, q_{bar}) with **Lorentzian spectral functions** :

$$\rho_i(\omega, T) = \frac{4\omega\Gamma_i(T)}{\left(\omega^2 - \vec{p}^2 - M_i^2(T)\right)^2 + 4\omega^2\Gamma_i^2(T)} \quad (i = q, \bar{q}, g)$$

- Modeling of the **quark/gluon masses and widths** \rightarrow HTL limit at high T with 3 model parameters – fitted to lattice QCD data

\rightarrow Quasi-particle properties:

large width and mass for gluons and quarks



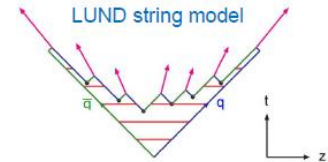
- DQPM** provides **mean-fields (1PI)** for gluons and quarks as well as **effective 2-body interactions (2PI)**
- DQPM** gives **transition rates** for the formation of hadrons \rightarrow **PHSD**



Parton-Hadron-String-Dynamics (PHSD)

Initial A+A collisions :

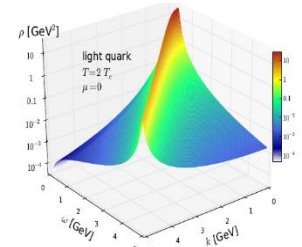
$N+N \rightarrow$ string formation \rightarrow decay to pre-hadrons



Formation of QGP stage if $\varepsilon > \varepsilon_{\text{critical}}$:

dissolution of pre-hadrons \rightarrow (DQPM) \rightarrow

\rightarrow massive quarks/gluons + mean-field potential U_q



Partonic stage – QGP :

based on the **D**ynamical **Q**uasi-**P**article **M**odel (DQPM)

(quasi-) elastic collisions:

$$q + q \rightarrow q + q \quad g + q \rightarrow g + q$$

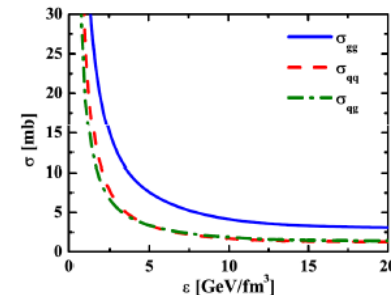
$$q + \bar{q} \rightarrow q + \bar{q} \quad g + \bar{q} \rightarrow g + \bar{q}$$

$$\bar{q} + \bar{q} \rightarrow \bar{q} + \bar{q} \quad g + g \rightarrow g + g$$

inelastic collisions:

$$q + \bar{q} \rightarrow g \quad q + \bar{q} \rightarrow g + g$$

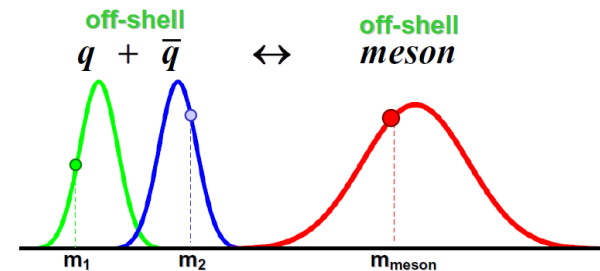
$$g \rightarrow q + \bar{q} \quad g \rightarrow g + g$$



Hadronization (based on DQPM):

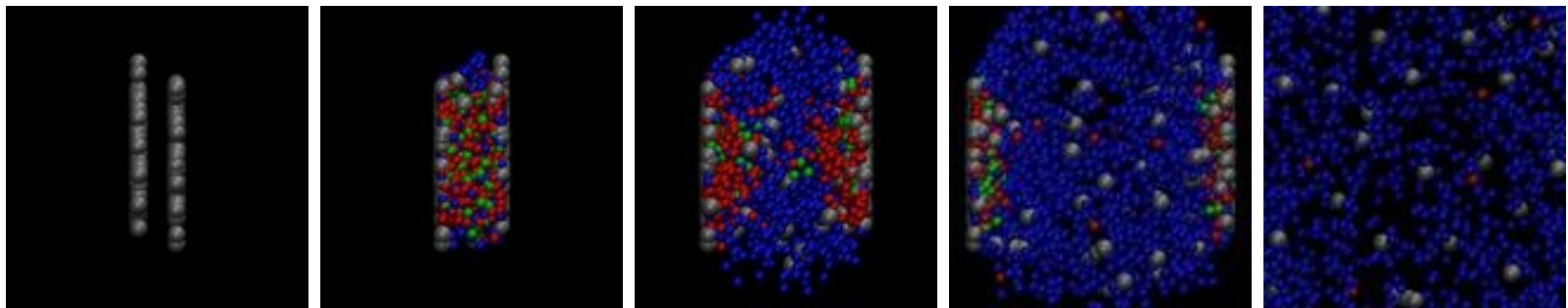
$$g \rightarrow q + \bar{q}, \quad q + \bar{q} \leftrightarrow \text{meson (or 'string')}$$

$$q + q + q \leftrightarrow \text{baryon (or 'string')}$$



Hadronic phase: hadron-hadron interactions – off-shell HSD

Traces of the QGP in observables in high energy heavy-ion collisions



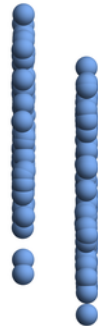
Stages of a collision in PHSD

$t = 0.05 \text{ fm/c}$



$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$ – Section view



 Baryons (394)

 Antibaryons (0)

 Mesons (0)

 Quarks (0)

 Gluons (0)

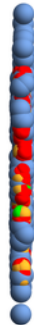
Stages of a collision in PHSD

$t = 1.6512 \text{ fm}/c$



$\text{Au} + \text{Au} \quad \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$ – Section view



 Baryons (394)

 Antibaryons (0)

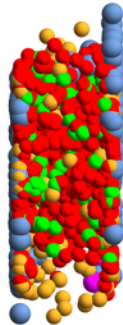
 Mesons (1523)

 Quarks (4553)

 Gluons (368)

Stages of a collision in PHSD

$t = 3.91921 \text{ fm}/c$



$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$ – Section view

 Baryons (426)

 Antibaryons (29)

 Mesons (1189)

 Quarks (4459)

 Gluons (783)

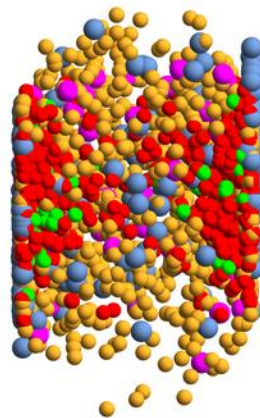
Stages of a collision in PHSD

$t = 7.31921 \text{ fm}/c$



$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$ – Section view



 Baryons (540)

 Antibaryons (120)

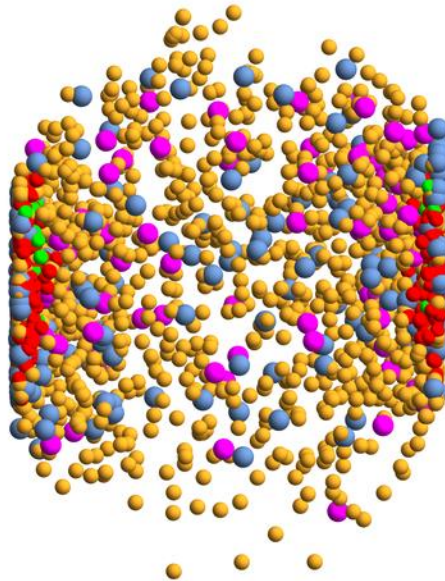
 Mesons (2481)

 Quarks (2901)

 Gluons (492)

Stages of a collision in PHSD

$t = 12.0192 \text{ fm/c}$



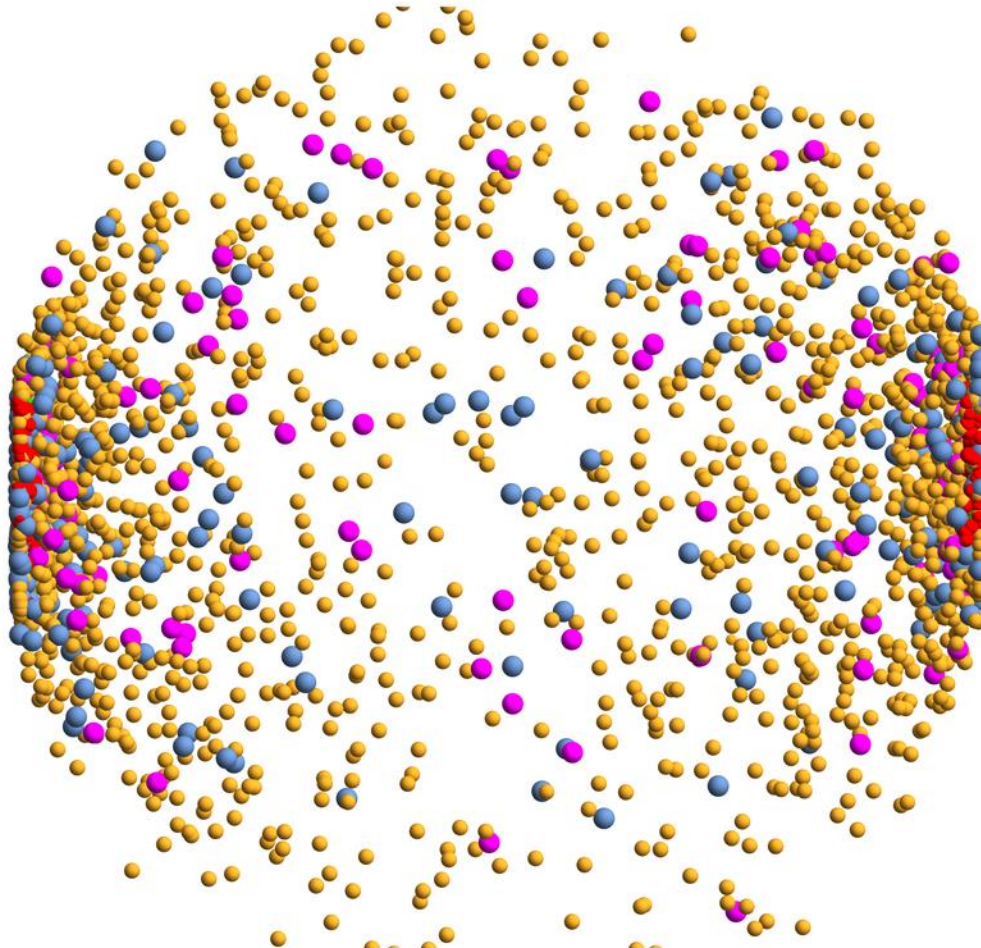
$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$ – Section view

-  Baryons (626)
-  Antibaryons (202)
-  Mesons (3357)
-  Quarks (1835)
-  Gluons (269)

Stages of a collision in PHSD

$t = 25.5191 \text{ fm/c}$



$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$ – Section view

 Baryons (710)

 Antibaryons (272)

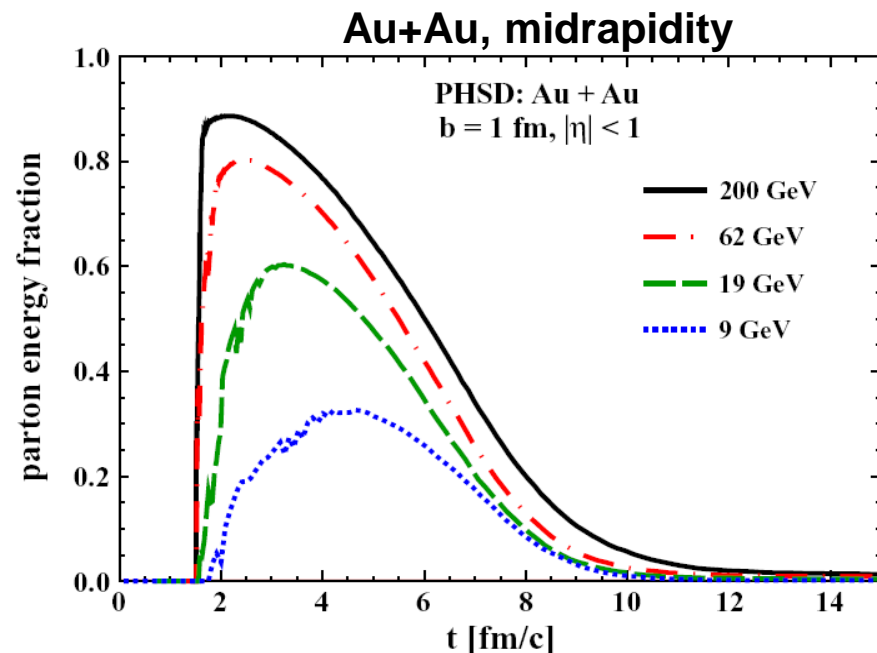
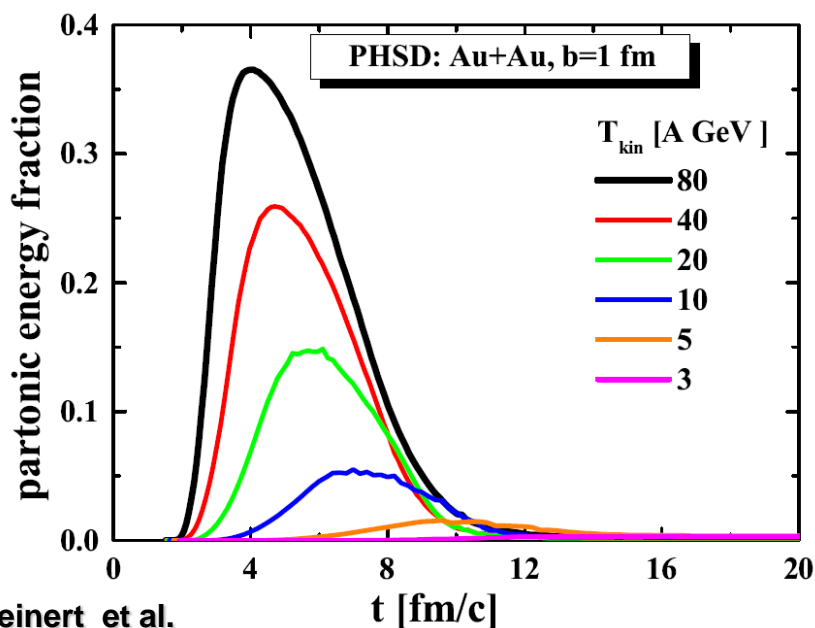
 Mesons (4343)

 Quarks (899)

 Gluons (46)

Partonic energy fraction in central A+A

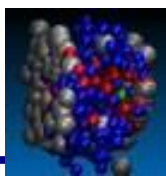
Time evolution of the partonic energy fraction vs energy



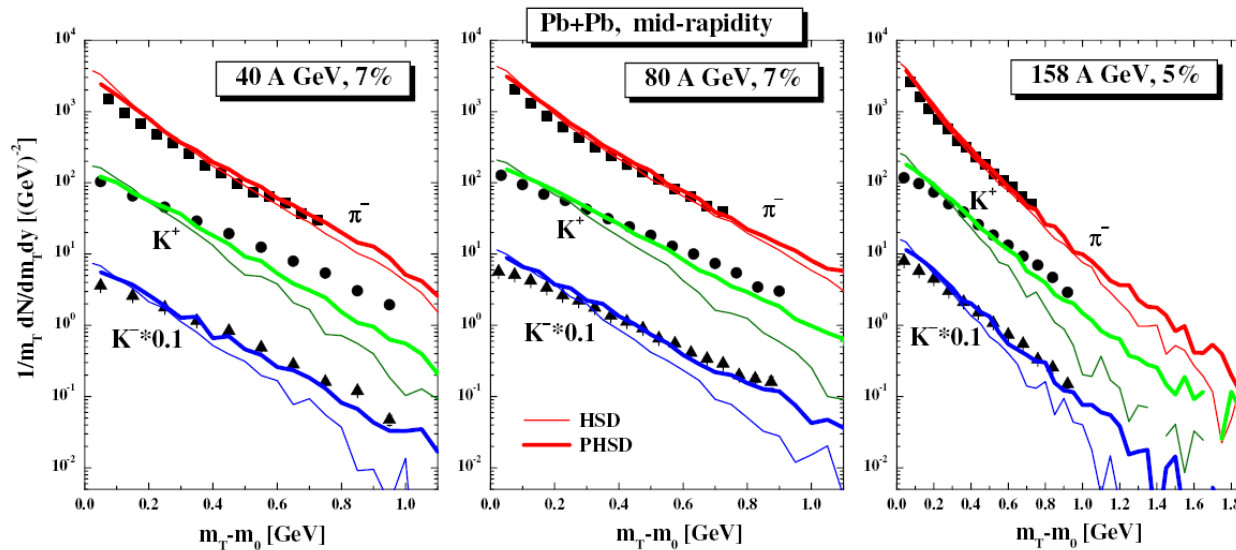
❑ Strong **increase of partonic phase with energy** from AGS to RHIC

❑ **SPS**: Pb+Pb, 160 A GeV: only about **40%** of the converted energy goes to partons; the rest is contained in the **large hadronic corona and leading partons**

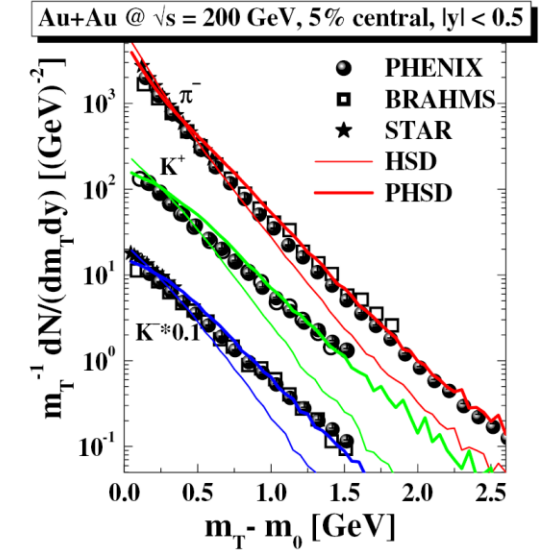
❑ **RHIC**: Au+Au, 21.3 A TeV: up to **90%** - **QGP**



Central Pb + Pb at SPS energies



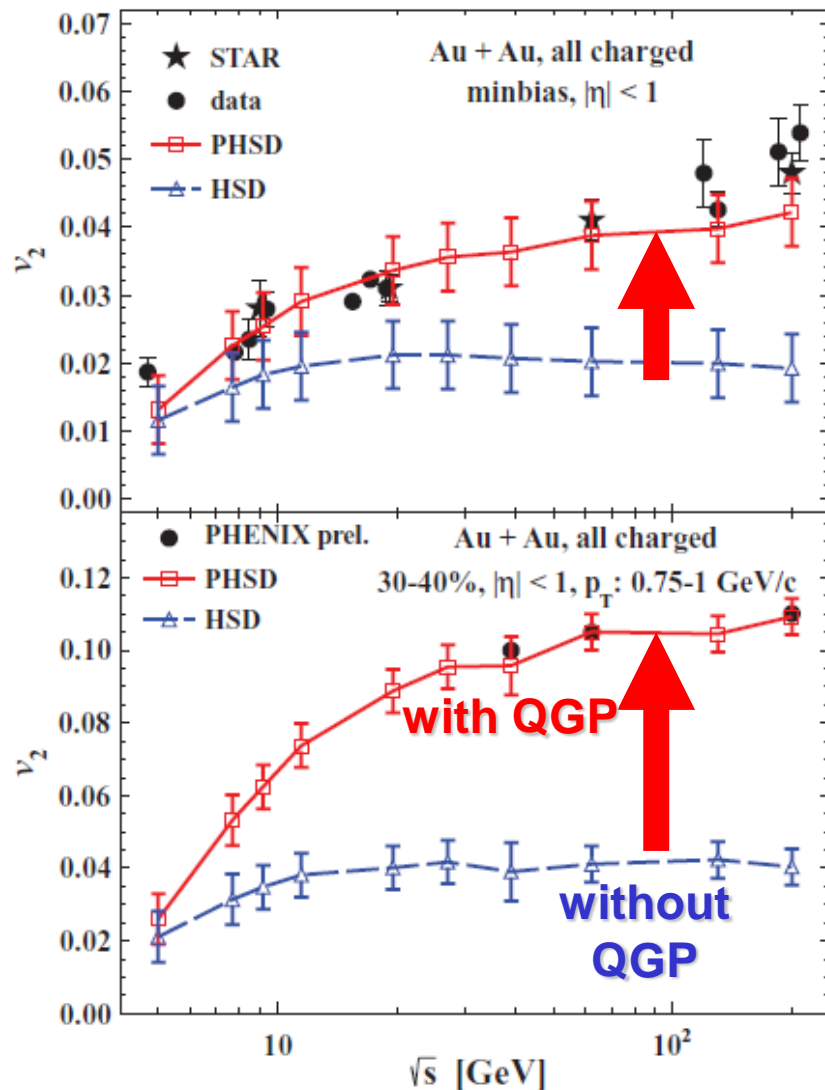
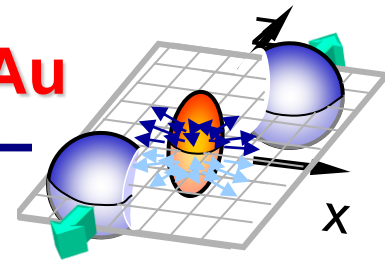
Central Au+Au at RHIC



- ❑ **PHSD** gives **harder m_T spectra** and works better than HSD (wo QGP) at high energies – RHIC, SPS (and top FAIR, NICA)
- ❑ however, at **low SPS** (and low FAIR, NICA) energies the **effect of the partonic phase decreases** due to the decrease of the partonic fraction

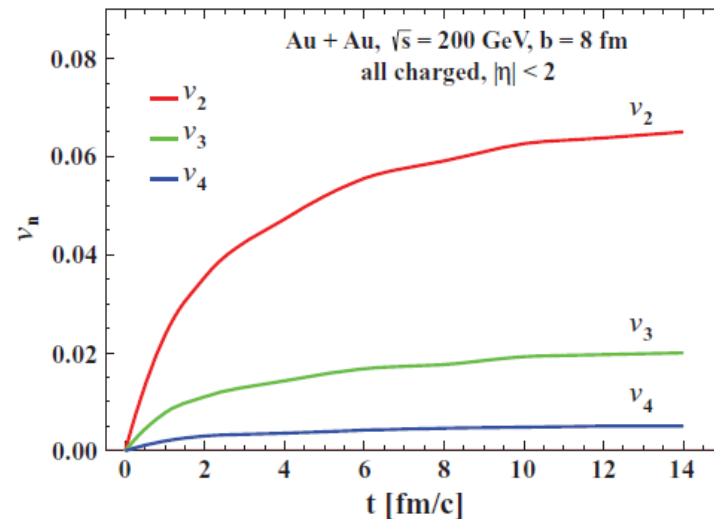


Elliptic flow v_2 vs. collision energy for Au+Au



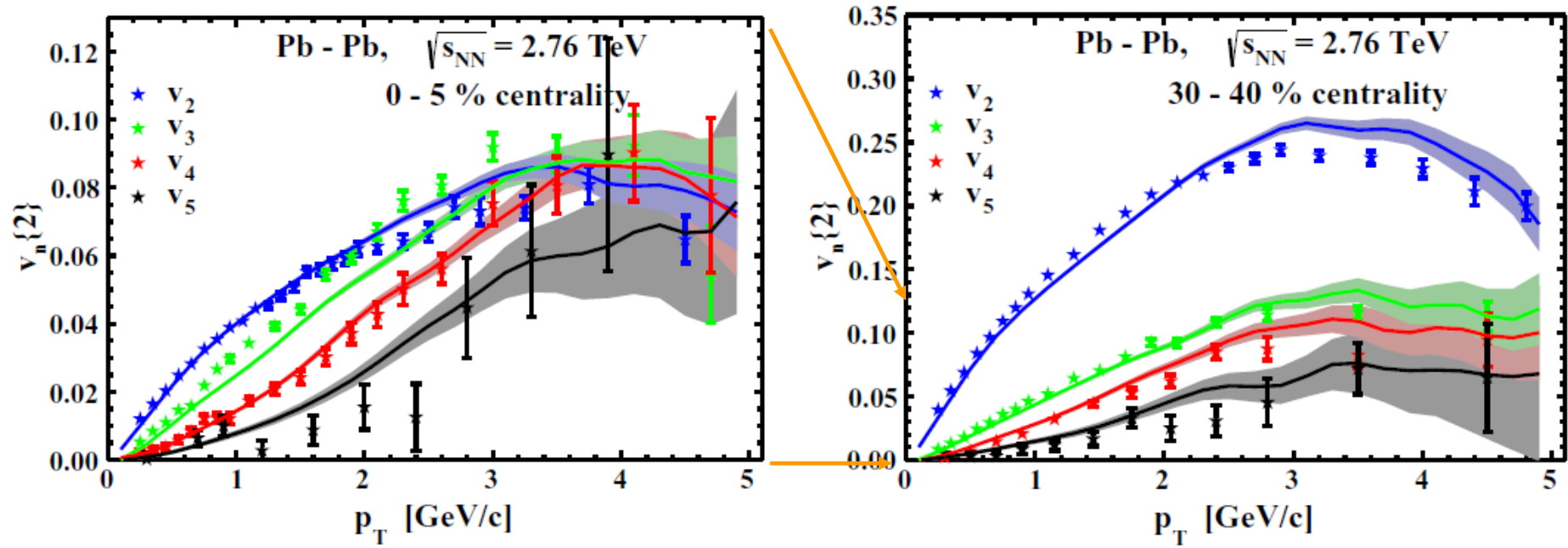
$$\frac{dN}{d\varphi} \propto \left(1 + 2 \sum_{n=1}^{+\infty} v_n \cos[n(\varphi - \psi_n)] \right)$$

$$v_n = \left\langle \cos n(\varphi - \psi_n) \right\rangle, \quad n = 1, 2, 3, \dots$$



- v_2 in PHSD is larger than in HSD due to the repulsive scalar mean-field potential $U_s(\rho)$ for partons
- v_2 grows with bombarding energy due to the increase of the parton fraction

V_n ($n=2,3,4,5$) of charged particles from PHSD at LHC



- PHSD: increase of v_n ($n=2,3,4,5$) with p_T
- v_2 increases with decreasing centrality
- v_n ($n=3,4,5$) show weak centrality dependence

symbols – ALICE

PRL 107 (2011) 032301

lines – PHSD (e-by-e)

v_n ($n=3,4,5$) develops by interaction in the QGP and in the final hadronic phase

Summary



The **PHSD** is a microscopic off-shell transport approach

- applicable for the description of **strongly-interaction hadronic and partonic matter** created in heavy-ion collisions
- based on the solution of **generalized transport equations** derived from **Kadanoff-Baym theory**
- applicable from **SIS to LHC energies**

Outlook:

Extention of the PHSD for **cluster formation** → **PHQMD**



Modeling of clusters and hypernucleus formation

The goal: Dynamical modeling of cluster formation by a combined model

PHQMD = (PHSD & QMD) & SACA

Nantes & GU & GSI & JINR collaboration:

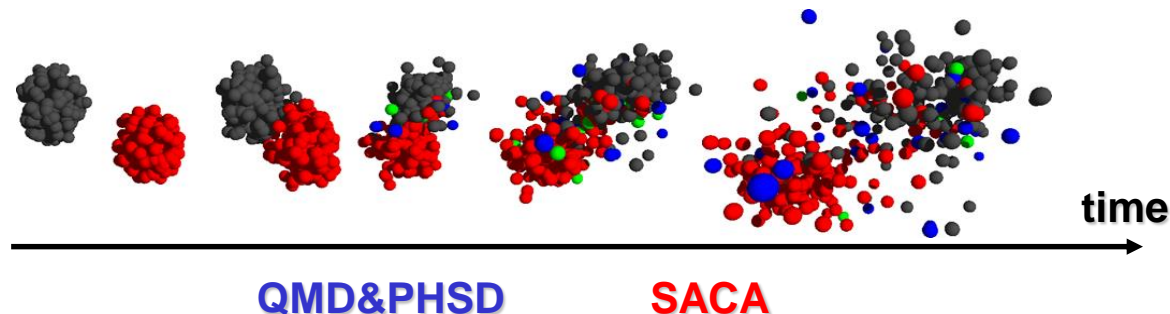
J. Aichelin, E. Bratkovskaya, A. Le Fèvre, Y. Leifels, V. Kireyeu, V. Kolesnikov and V. Voronyuk

□ **Parton-Hadron-Quantum-Molecular-Dynamics** -

a non-equilibrium microscopic transport model which describes **n-body dynamics** based on

QMD propagation with **collision integrals from PHSD** (Parton-Hadron-String Dynamics) and **cluster recognition by the SACA model** as well as by the Minimum Spanning Tree model (**MST**). MST can determine clusters at the end of the reaction.

□ **Simulated Annealing Clusterization Algorithm** – cluster recognition according to the largest binding energy (extension of the SACA model → **FRIGA** which includes hypernuclei). SACA allows to identify fragments very early during the reaction.



Thanks to:

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Viktor Kireev

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Barcelona University:

Laura Tolos

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Steffen Bass



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ECT*

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