Relativistic Parton Transport at fixed shear viscosity η/s

V. Greco
UNIVERSITY of CATANIA
INFN-LNS

"We focus on energies and corresponding densities, where a hadronic representation is appropriate."
(from the abstract of the Workshop)

Outline

- Transport Theory at fixed η/s for QGP :
  - Motivations
  - How to fix locally η/s (Green-Kubo correlator)
  - Tests and comparisons
  - Study of the ∞ cross section limit ($\lambda \ll d$):

    → Ideal Hydro & viscous correction

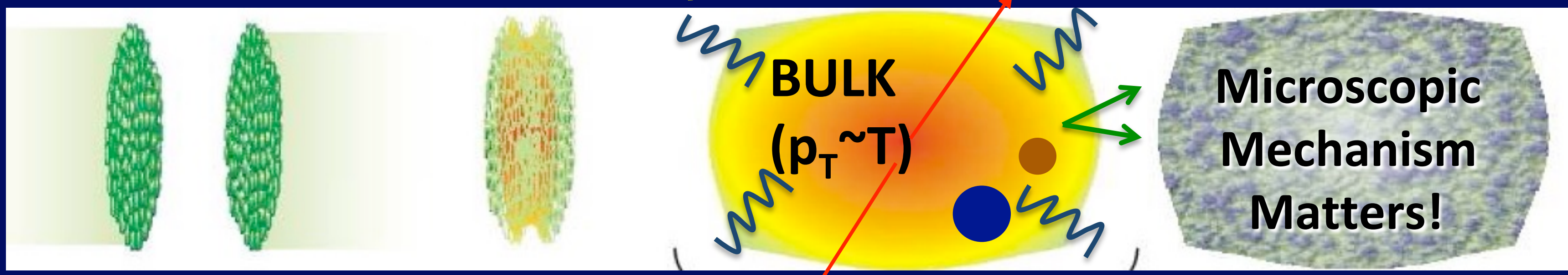
- Some results for HIC:
  - Hydro-like (equilibrium) study of v_n
  - Impact of non-equilibrium: initial stage & high-p_T

- Challenges and future directions:

Ultra-relativistic HIC

Going from $p_T \approx$ 1 to 500 Λ_{QCD} and $m_q \approx$ 1/20 to 20 Λ_{QCD} (700Λ_{QCD})

Initial Conditions — Quark-Gluon Plasma — Hadronization

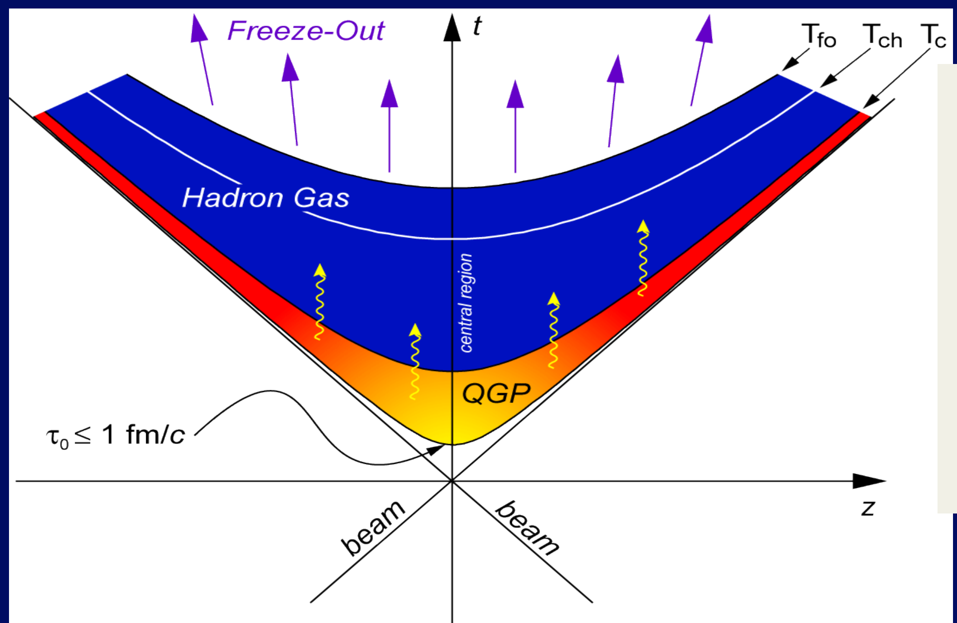


CGC (x<<1)
Gluon saturation?

MINIJETS
(p_T>>T,Λ_{QCD})

Heavy Quarks
(m_q>>T,Λ_{QCD})

+ photons &
+ dileptons



Freeze-out

Hadronization

Plasma phase

Pre-Equilibrium

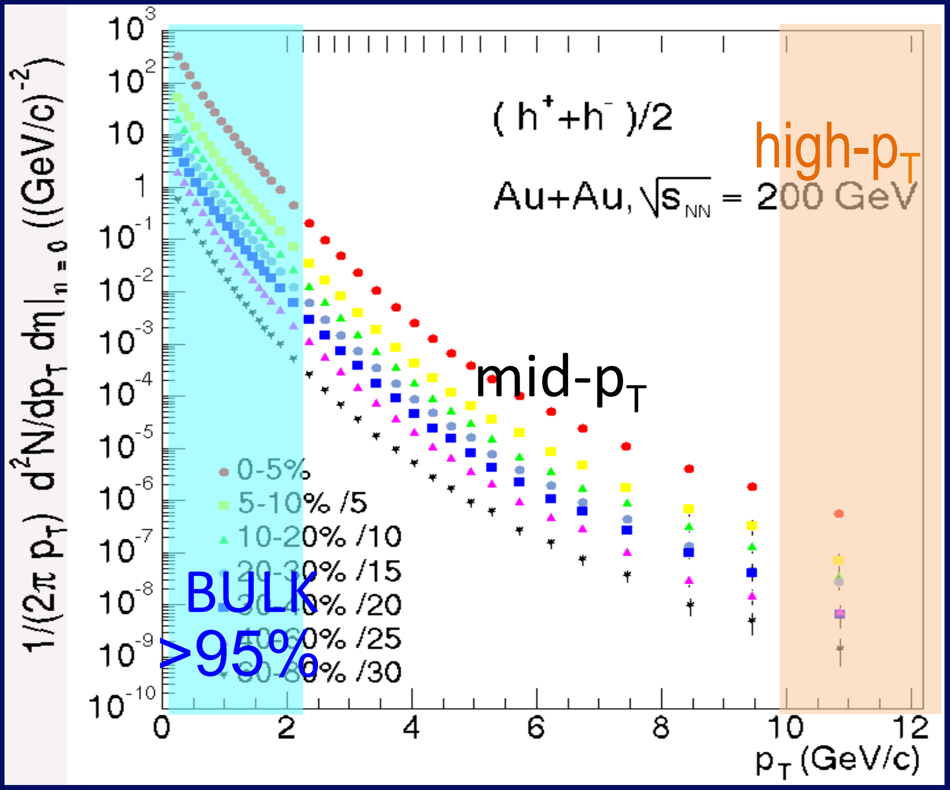
Scales in ultra-relativistic HIC

HARD ($p_T >>> \Lambda_{QCD}$)

PQCD APPLICABLE

jet quenching, heavy quarks

quarkonia, hard photons ...



SOFT ($p_T \sim \Lambda_{QCD}$, T)

DRIVEN BY *NON PERTURBATIVE QCD*

Hadron yields, collective modes of the bulk v_n,

strangeness enhancement, fluctuations ...

Ideal Hydrodynamics: a perfect fluid?

$$\begin{cases} \partial_\mu T^{\mu\nu}(x) = 0 \\ \partial_\mu j^\mu_B(x) = 0 \end{cases}$$

$$T^{\mu\nu}(x) = [\varepsilon + p]u^\mu u^\nu - p g^{\mu\nu}$$

$$T^* \approx T_f + \frac{1}{2} m \langle \beta_T^2 \rangle$$

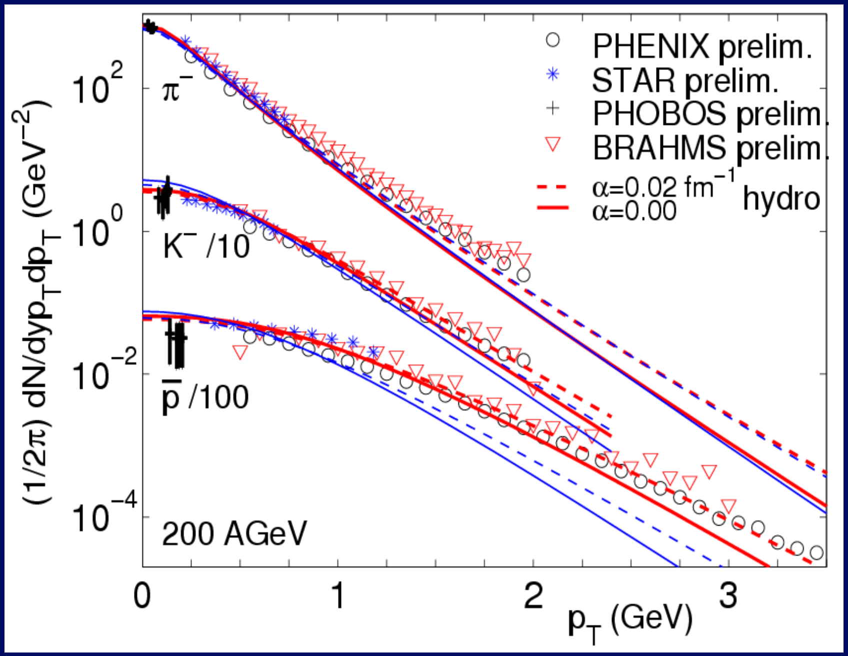
T_f ~ 120 MeV
$\langle\beta_T\rangle$ ~ 0.5

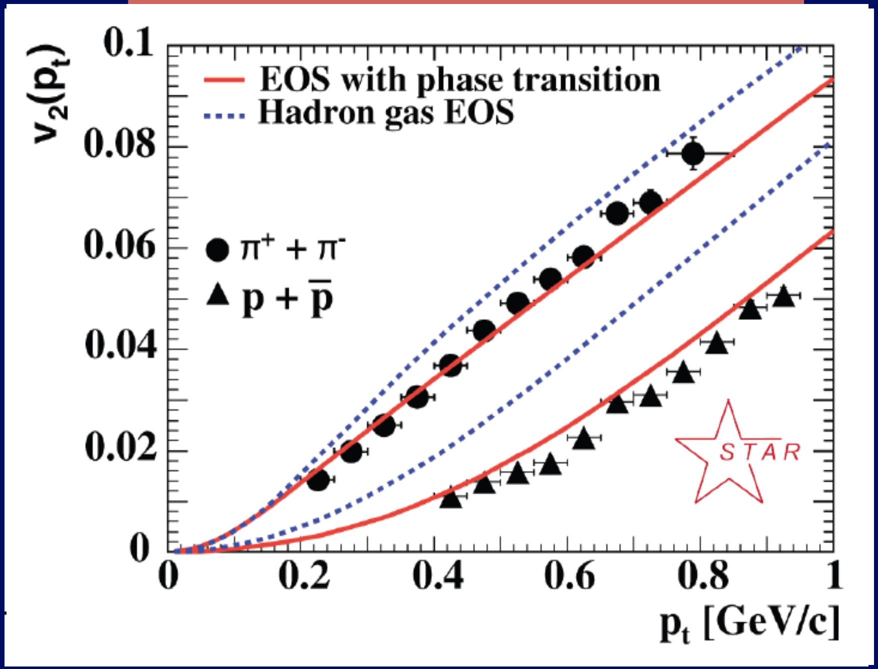
A $\tau_{th} \approx$ 0.5-1 fm/c just assumed!

No microscopic description (λ ->0), no dissipation,…only conservation laws!

- Blue shift of dN/dp_T hadron spectra
- Mass ordering of $v_2(p_T)$

For the first time very close to ideal Hydrodynamics





Ideal Hydrodynamics: a perfect fluid?

$$\begin{cases} \partial_\mu T^{\mu\nu}(x) = 0 \\ \partial_\mu j^\mu_B(x) = 0 \end{cases}$$

$$f_{eq}(x,p) \approx e^{-\frac{\gamma E - \vec{p}\cdot\vec{u} - \mu}{T}} \approx e^{-\frac{m_T}{T^*}}$$

$$T^* \approx T_f + \frac{1}{2} m \langle \beta_T^2 \rangle$$

T_f ~ 120 MeV

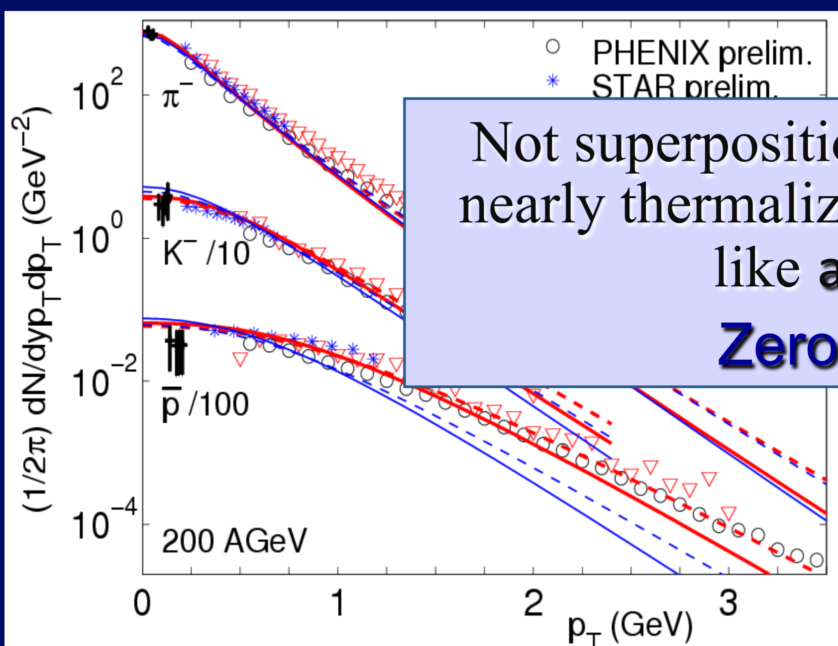
$\langle\beta_T\rangle$ ~ 0.5

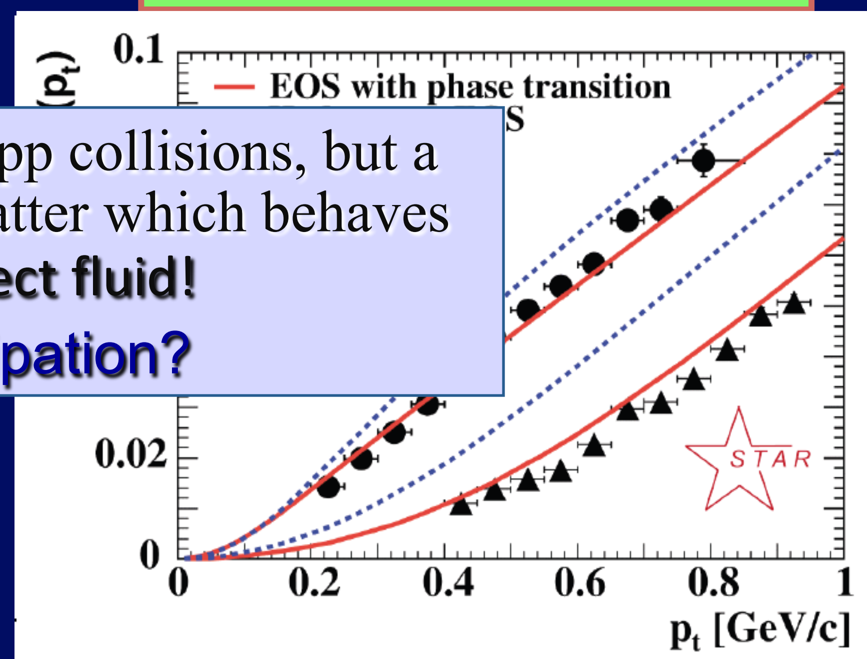
A $\tau_{th} \approx$ 0.5-1 fm/c just assumed!

No microscopic description (λ ->0), no dissipation,...only conservation laws!

- Blue shift of dN/dp_T hadron spectra
- Large v_2/ε
- Mass ordering of $v_2(p_T)$

For the first time very close to ideal Hydrodynamics

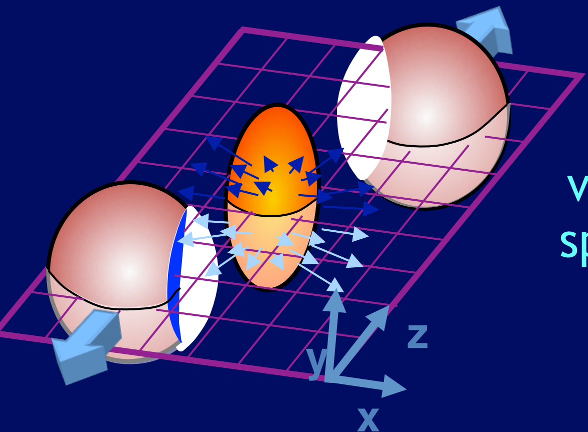




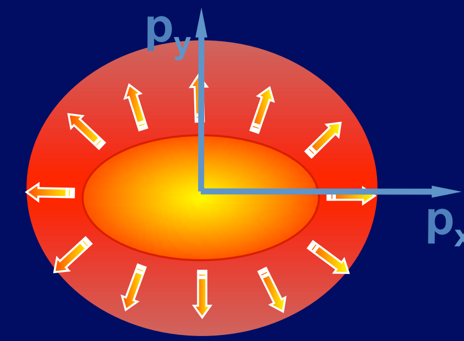
Not superposition of pp collisions, but a nearly thermalized matter which behaves like **a perfect fluid!**

Zero dissipation?

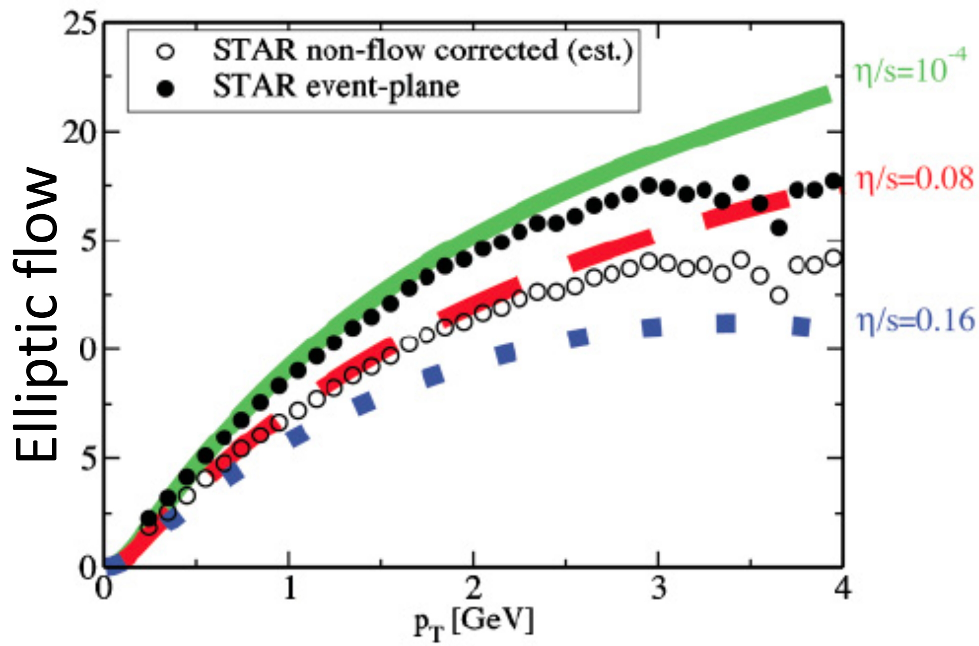
Success of viscous hydrodynamics for $v_2 \rightarrow \eta/s \approx 0.1$



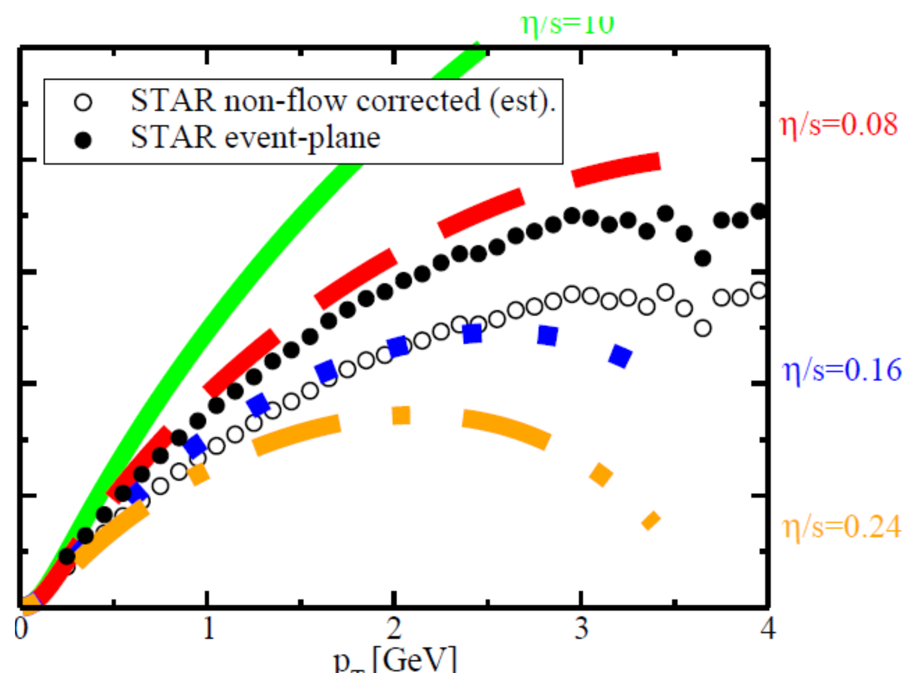
v_2/ε measures efficiency in converting space eccentricity to Momentum space



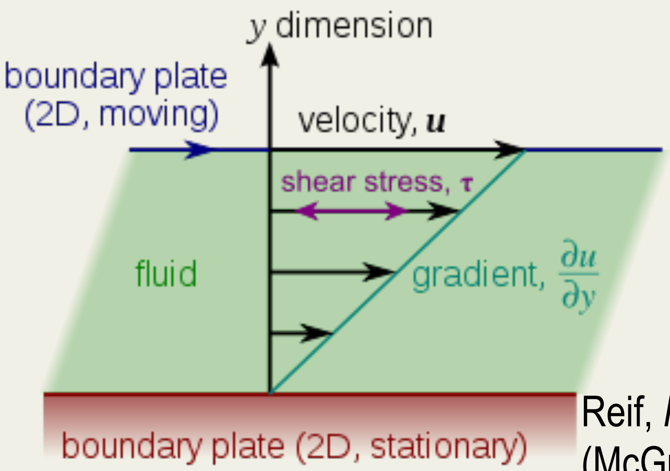
Glauber Init. Cond.



Color Glass Condensate- Init. Cond.



What is Shear Viscosity?



$$\frac{F_x}{A_{yz}} = -\eta \frac{\partial v_x}{\partial y}$$

Reif, *Fundamentals of Statistical and Thermal Physics* (McGraw-Hill, New York, 1965)

Danielewicz & Gyulassy, Phys.Rev. D31 (1985)

$\eta = (2.3 \pm 0.5) \cdot 10^8 Pa \cdot s$

8 drops 1932-2013

Text book kinetic theory

$$\frac{\eta}{s} \approx \frac{1}{15} < p > \cdot \lambda$$

Small η/s → small mean free path λ

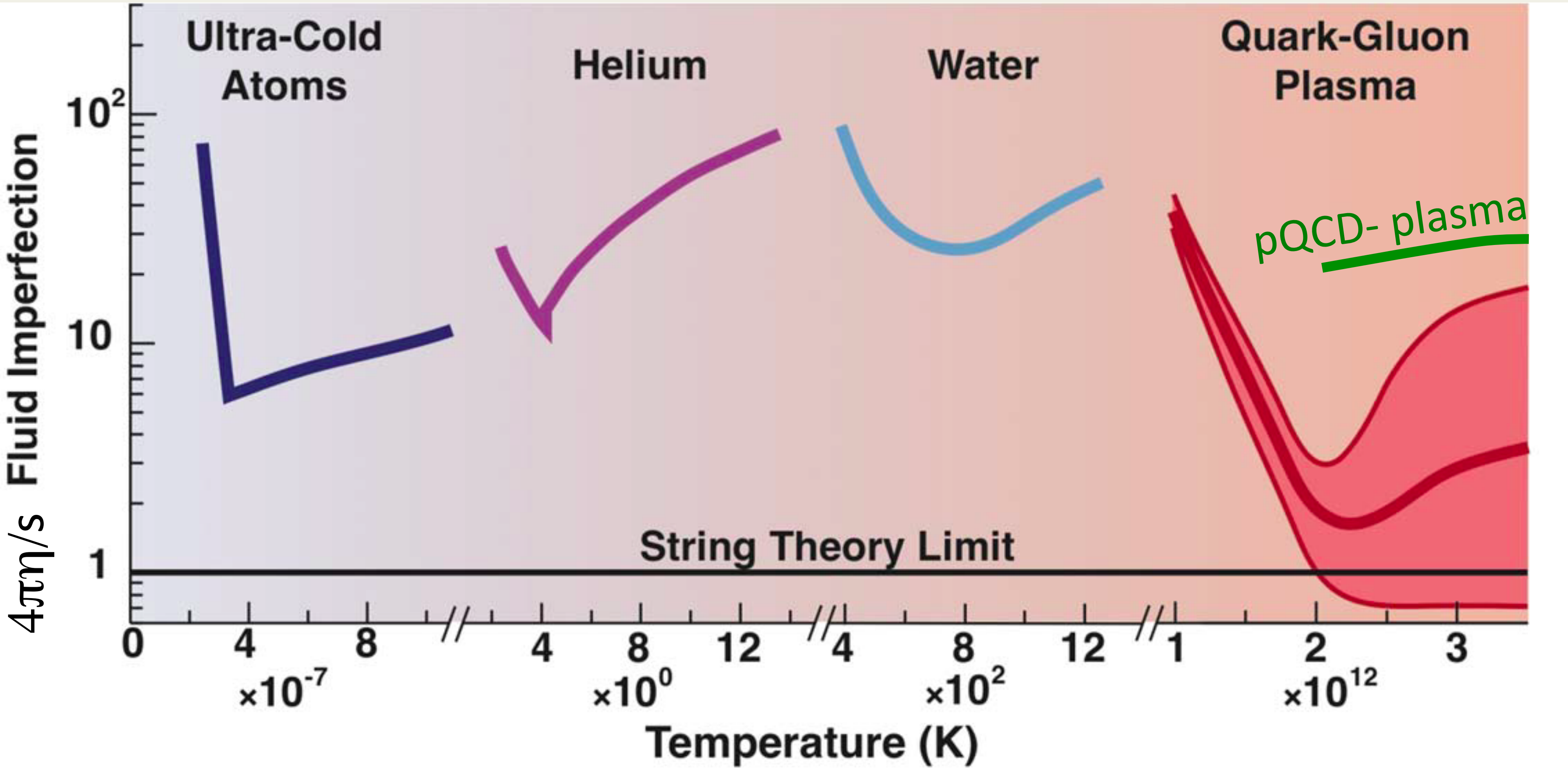
At limits of Quantum mechanism (<p>≈ ΔE, λ≈ cΔt)

$$\Delta E \cdot \Delta t \geq 1 \quad \rightarrow \eta / s > 1/15$$

which for QGP mean η> 10^{11} Pa•s

AdS/CFT, based on the conjecture that a Gauge theory in 4D (in the infinite coupling limit) is dual to a gravitational calculation in 5D gives **η/s> 1/4π**

Shear Viscosity for systems in 20 order of T magnitudes!



Report to USA Nuclear Science Advisory Committee in 2013

Why we want to use a Boltzmann relativistic transport theory, if viscous Hydrodynamics works so well?

Also if viscosity is so low, mean free path is small

... QGP is strongly coupled

Does we are outside of the region of validity of Boltzmann?

$$\frac{\eta}{s} \cong \frac{1}{15} \langle p \rangle \cdot \lambda \rightarrow \lambda \cong \frac{5}{T} \frac{\eta}{s}$$

$$\rho_{QGP} \approx 4.5 T^3 \rightarrow \bar{d}_{QGP} \approx \frac{0.6}{T}$$

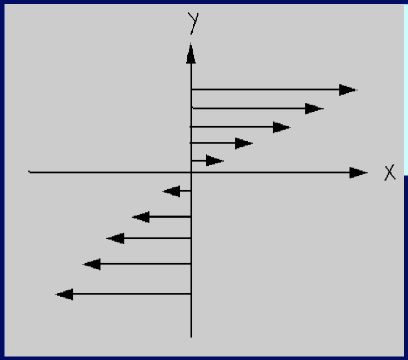
$$\lambda < \bar{d}$$

A relativistic fluid at small η/s ≈0.1 is not very dilute!

Viscous Hydrodynamics

Relativistic Navier-Stokes

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha)$$



$$\frac{F_x}{A_{yz}} = -\eta \frac{\partial v_x}{\partial y}$$

but it violates causality,

II^0 order expansion needed -> Israel-Stewart tensor based on entropy increase $\partial_\mu s^\mu > 0$

$$\pi^{\mu\nu} = \eta\nabla^{<\mu}u^{\nu>} + \tau_\pi\left[\Delta^\mu_\alpha\Delta^\nu_\beta D\pi^{\alpha\beta}\ldots\right]$$

τ_η,τ_ζ two parameters appears +

$\delta f \sim f_{eq}$ reduce the p_T validity range +

Full II° order has about 10 parameters

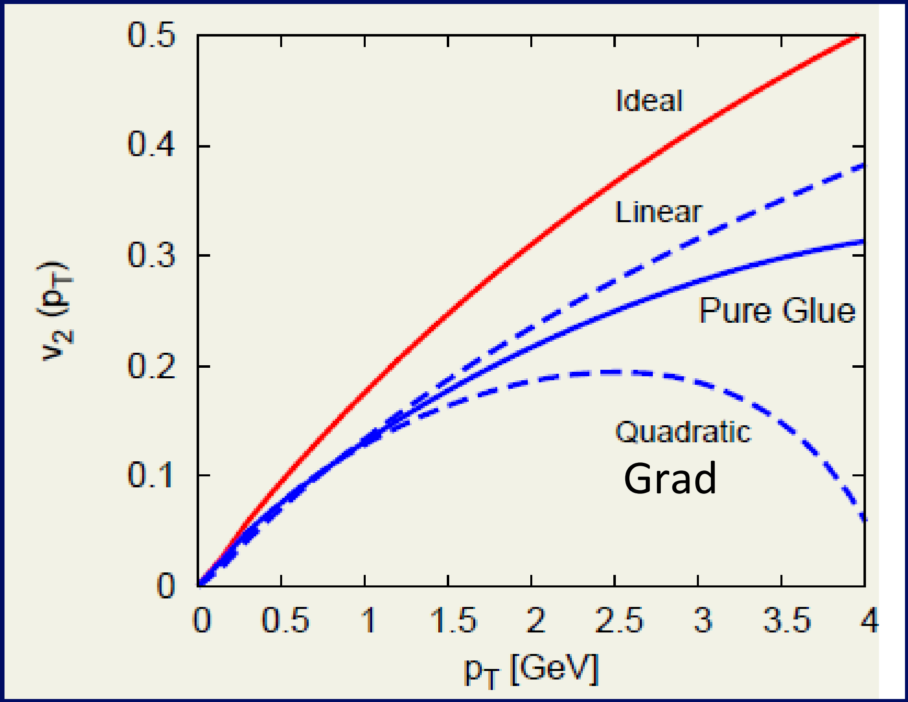
- -Dissipative correction to u^μ, T
- -Dissipative correction to f -> $f_{eq}+\delta f_{neq}$

There is no one to one correspondence!

$$T^{\mu\nu}_{eq} + \delta T^{\mu\nu} \Leftarrow f_{eq} + \delta f$$

An Asantz

$$\delta f = \frac{\pi^{\mu\nu}}{\varepsilon + P}\frac{p_\mu p_\nu}{T^2} f_{eq}$$



- p_T~3 GeV -> $\delta f/f \approx$ 1-4
- $\Pi^{\mu\nu}$ (t_0) =0 -> discard initial non-eq (ex. minijets)

Full Viscous Hydrodynamics

D. Rischke

$$
\begin{aligned}
\tau_\Pi \dot{\Pi} + \Pi &= \Pi_{\rm NS} + \tau_{\Pi q}\, q\cdot\dot{u} - \ell_{\Pi q}\,\partial\cdot q - \zeta\,\hat{\delta}_0\,\Pi\,\theta \\
&\quad + \lambda_{\Pi q}\, q\cdot\nabla\alpha + \lambda_{\Pi\pi}\,\pi^{\mu\nu}\sigma_{\mu\nu} \\
\tau_q\,\Delta^{\mu\nu}\dot{q}_\nu + q^\mu &= q^\mu_{\rm NS} - \tau_{q\Pi}\,\Pi\,\dot{u}^\mu - \tau_{q\pi}\,\pi^{\mu\nu}\,\dot{u}_\nu \\
&\quad + \ell_{q\Pi}\,\nabla^\mu\Pi - \ell_{q\pi}\,\Delta^{\mu\nu}\,\partial^\lambda\pi_{\nu\lambda} + \tau_q\,\omega^{\mu\nu}\,q_\nu - \frac{\kappa}{\beta}\,\hat{\delta}_1\,q^\mu\,\theta \\
&\quad - \lambda_{qq}\,\sigma^{\mu\nu}\,q_\nu + \lambda_{q\Pi}\,\Pi\,\nabla^\mu\alpha + \lambda_{q\pi}\,\pi^{\mu\nu}\,\nabla_\nu\alpha \\
\tau_\pi\,\dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} &= \pi^{\mu\nu}_{\rm NS} + 2\,\tau_{\pi q}\,q^{<\mu}\dot{u}^{\nu>} \\
&\quad + 2\,\ell_{\pi q}\,\nabla^{<\mu}q^{\nu>} + 2\,\tau_\pi\,\pi_\lambda{}^{<\mu}\omega^{\nu>\lambda} - 2\,\eta\,\hat{\delta}_2\,\pi^{\mu\nu}\,\theta \\
&\quad - 2\,\tau_\pi\,\pi_\lambda{}^{<\mu}\sigma^{\nu>\lambda} - 2\,\lambda_{\pi q}\,q^{<\mu}\nabla^{\nu>}\alpha + 2\,\lambda_{\pi\Pi}\,\Pi\,\sigma^{\mu\nu}
\end{aligned}
$$

Phys.Rev. D85 (2012)

It becomes quite complicated and the number of parameters increases significantly: τ_η, τ_ζ, δf, $\Pi^{\mu\nu}(\tau_0)$,…

Relativistic Boltzmann-Vlasov approach

$$\left\{p^{*\mu}\partial_\mu + \left[p^*_\nu F^{\mu\nu} + m^*\partial^\mu m^*\right]\partial^{p^*}_\mu\right\} f(x,p^*) = C[f]$$

Free streaming Field Interaction (EoS) Collisions -> $\eta \neq 0$

f(x,p) is the one-body distribution function

$$\begin{aligned} \mathcal{C}_{22} = & \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1+p'_2-p_1-p_2) \\ & -\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p'_1-p'_2) \end{aligned}$$

- $C[f_{eq}+\delta f] \neq 0$ deviation from ideal hydro (finite λ or η/s)
- We map with C[f] the phase space evolution of a fluid at fixed η/s !

One can expand over microscopic details (2<->2,2<->3…), but in a hydro language this is irrelevant only the global dissipative effect of C[f] is important!

In fact expanding C[f] one gets viscous hydrodynamics: Denicol, Rischke,…

Relativistic Boltzmann Equation

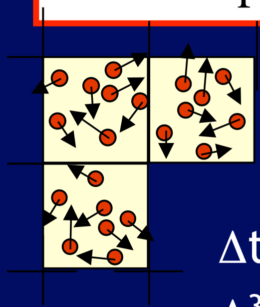
$$\left\{ p^{\mu}\partial_{\mu} + m^{*}\partial^{\mu}m^{*}\partial_{\mu}^{p} \right\} f(x,p) = C[\mathrm{f}]$$

Free streaming Field Interaction Collisions

$$C_{22} = \frac{1}{2E_p}\int \frac{d^3q}{(2\pi)^3 2E_q}\int \frac{d^3p'}{(2\pi)^3 2E'_p}\frac{d^3q'}{(2\pi)^3 2E'_q}\Big[f'(q')f'(p')\left|M_{gg\to gg}(p'q'\to pq)\right|^2 - f(q)f(p)\left|M_{gg\to gg}(pq\to p'q')\right|^2 \Big](2\pi)^4\delta^4(p+q-p'-q')$$

$$\boxed{\frac{(2\pi)^3\Delta N_{coll}}{\Delta t\Delta^3 x\Delta^3 p} = g\frac{\Delta^3 q}{(2\pi)^3} f_g(p) f_g(q) v_{rel}\sigma_{p,q\to p-k,q+k}}$$

Rate of collisions per unit time and phase space



Solved discretizing the space in $(\eta, x, y)_\alpha$ cells

$\Delta t \to 0$

$\Delta^3 x \to 0$

exact solution

$$f_i = \frac{\Delta N_i}{\frac{1}{(2\pi)^3}\Delta^3 x\,\Delta^3 p_i},$$

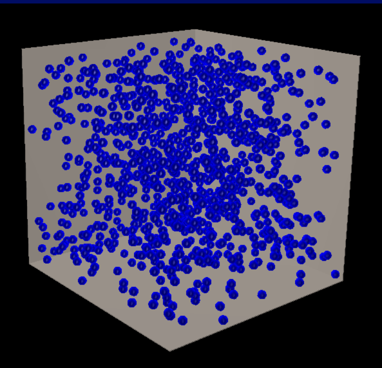
$$\boxed{P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}}$$

Xu & Greiner, PRC(2005)

Some test and check

of Boltzmann transport at ultrarelativistic limit

for thermalization time O(1fm/c)



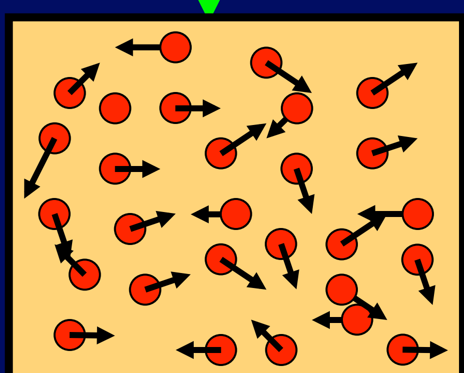
Simulation in a box

Test of equilabration in time scale of 1 fm/c
for ultra-relativistic particles

Highly non-equilibrated distributions

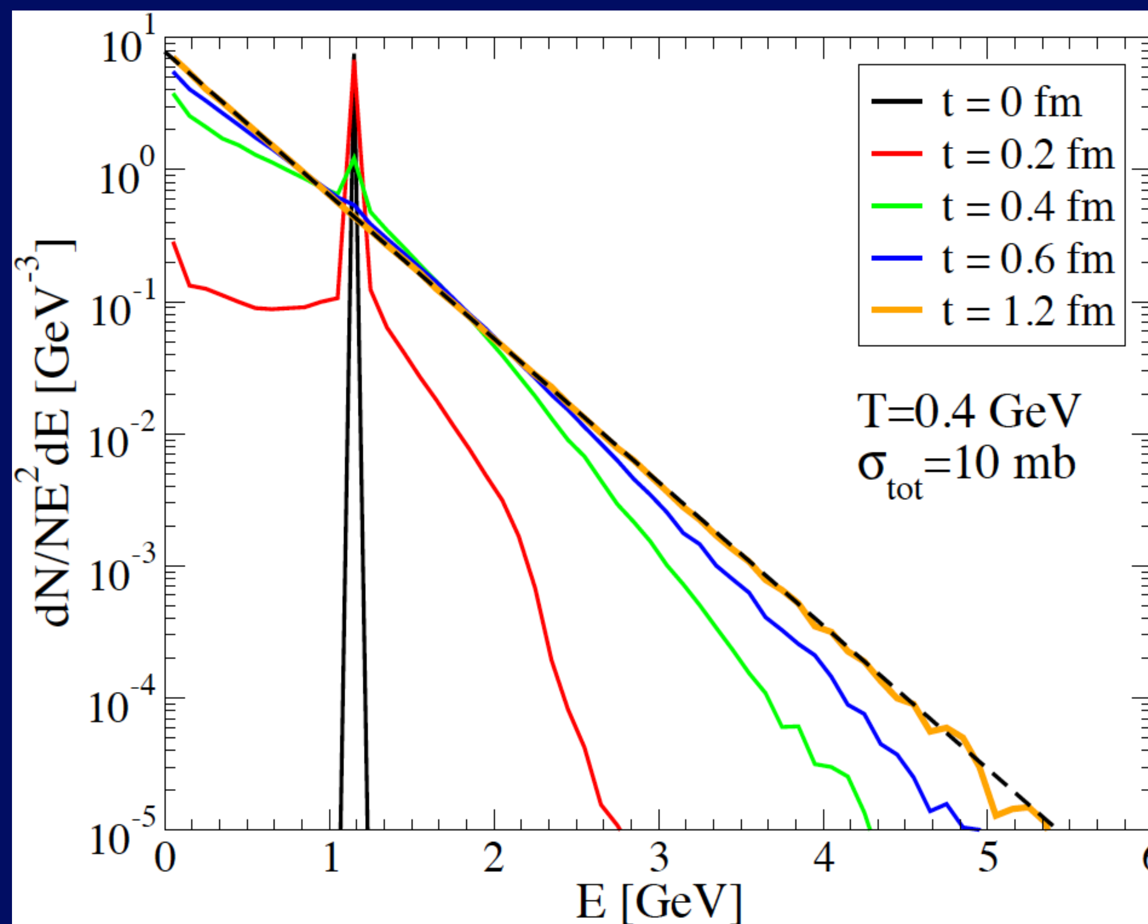
$$\frac{dN}{N dp_T dp_z} = \delta(p_T - p_0)\delta(p_z)$$

Going to equilibrium
E/N -> T=E/3N

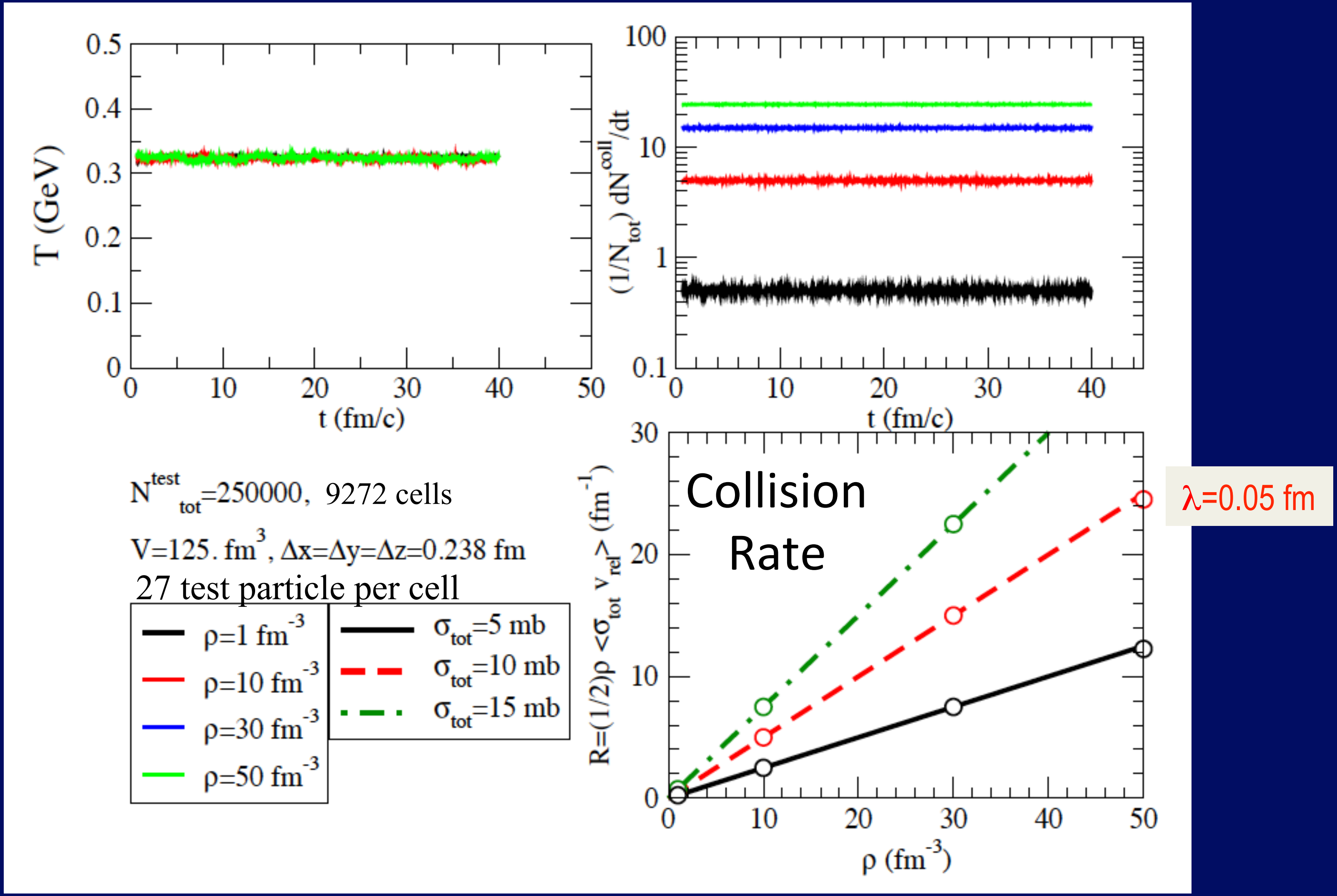


$$\frac{dN}{NE^2 dE} = \frac{1}{2T^3} e^{-E/T}$$

Particle off-equilibrium in a thermal bath at T=400 MeV



Some checks about the rate of collisions

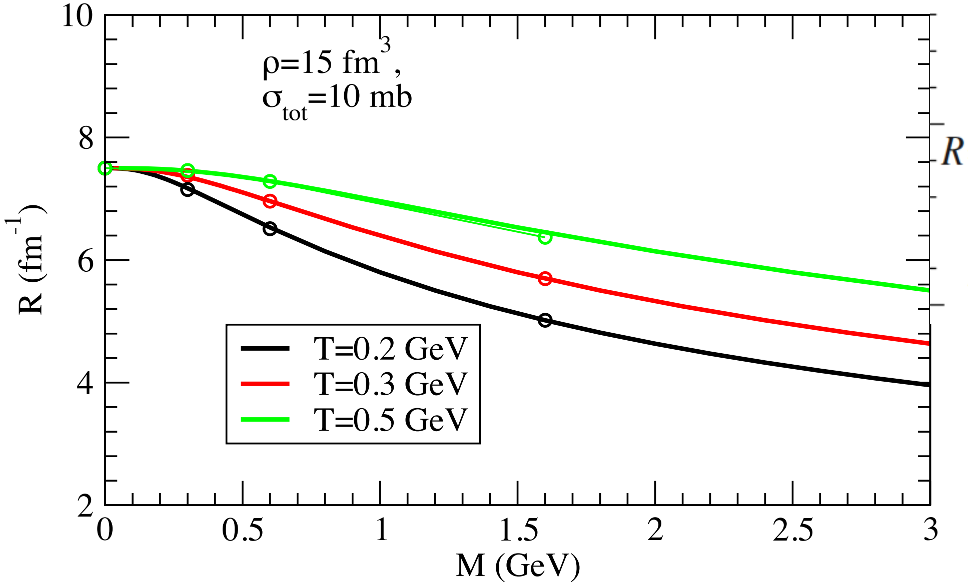


Stable in all the range of cross section and density of interest:

- A geometrical interpretation would have more trouble !

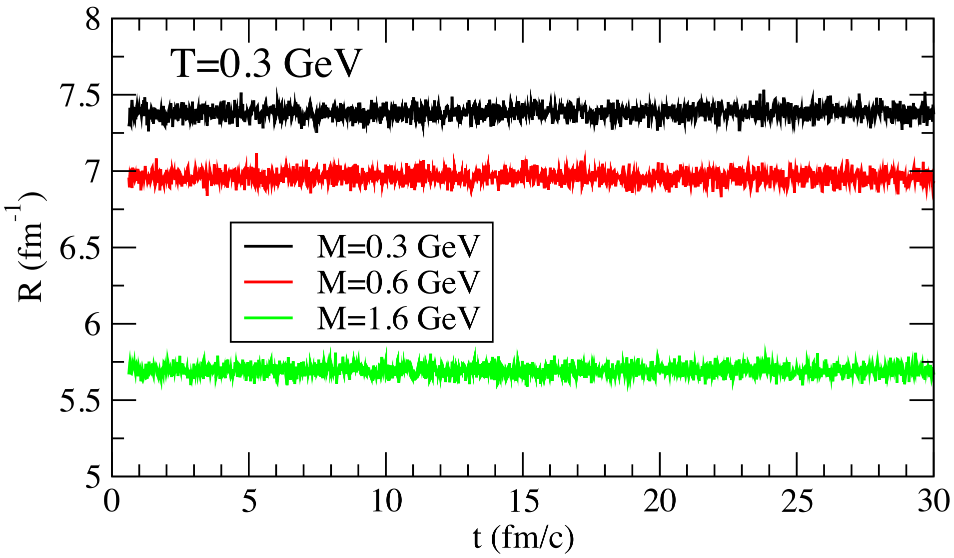
  Especially in the ultra-relativistic limit!

Some check at Finite Masses



$$R=\frac{1}{2}n_{tot}\langle\sigma v_{rel}\rangle=n_{tot}\frac{\beta}{8}\frac{\int_{\sqrt{s_0}}^{\infty} d\sqrt{s}\,\lambda(s)\sigma K_1(\beta\sqrt{s})}{M_a^2 M_b^2 K_2(\beta M_b)}$$

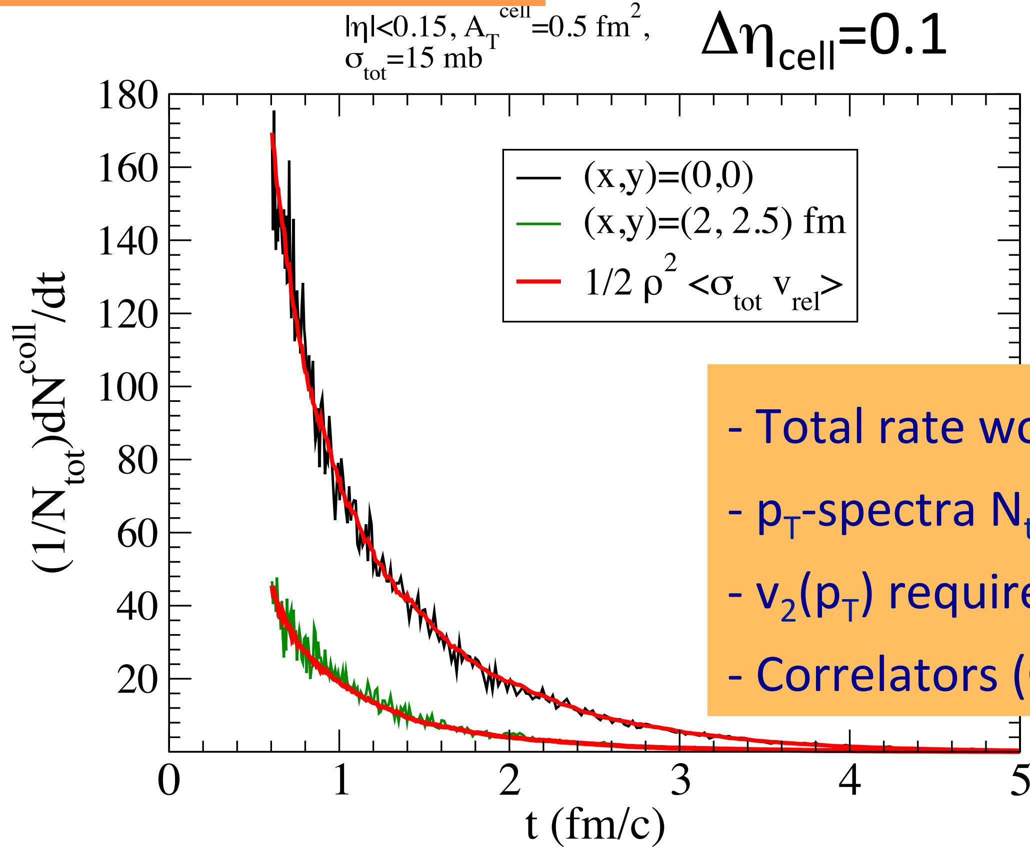
$$\lambda(s)=\left[s-(M_a+M_b)^2\right]\left[s-(M_a-M_b)^2\right]$$



Collision Rate precise at ≈ 0.1%

Test of collision rate locally in the expanding fireball

Au+Au@200AGeV, b=7.5 fm



- Total rate would converge also with N_{test}= 10-20
- p_T-spectra $N_{test} \approx 50$
- $v_2(p_T)$ require N_{test}> 100-200
- Correlators (Green-Kubo) N_{test}> 500-1000

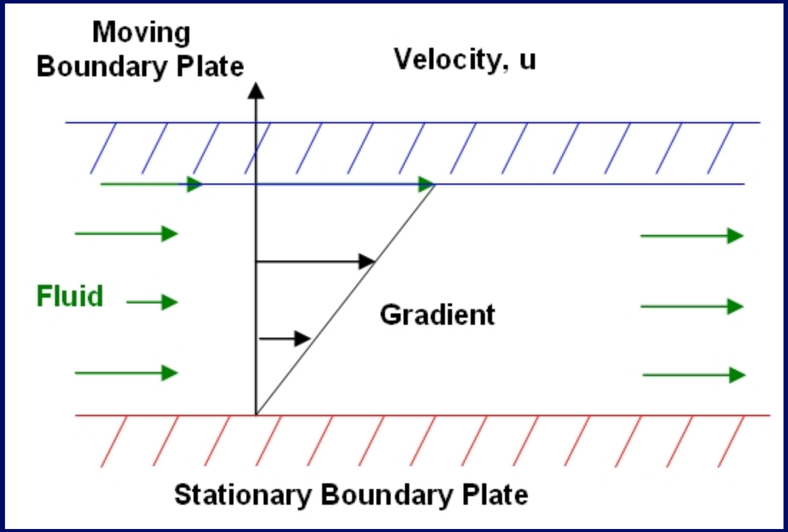
$$\Delta t\,(t) = 0.5\,\Delta z_{min}(t) = 0.5\,t\,\left[\tanh(\eta_m + \Delta\eta_c/2) - \tanh(\eta_m - \Delta\eta_c/2)\right]$$

Part I – Kinetic Theory at fixed η/s

Instead of starting from *cross-sections and fields*,
we reverse the process starting from η/s

What is the relation η <-> σ, dσ/dΘ, M, T, ρ?

- Check η with the Green-Kubo correlator



$$\frac{F}{A} = \eta \frac{\partial u}{\partial y}$$

$$\eta / s \cong \frac{1}{15} \frac{< p >}{\sigma \rho}$$?

Shear Viscosity in Box Calculation

Green-Kubo correlator

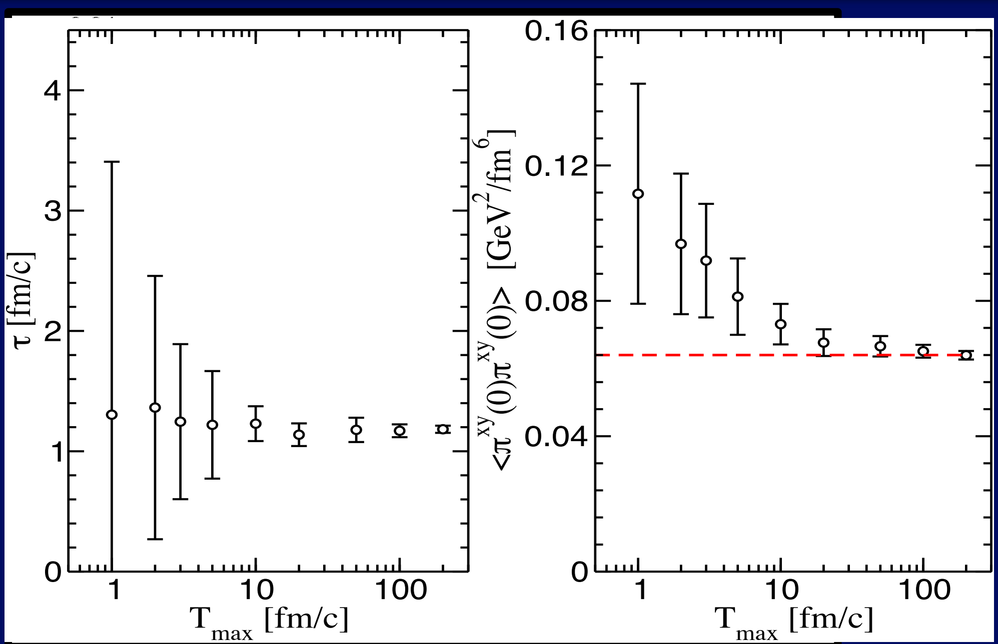
$$\eta = \frac{1}{T}\int_0^{\infty} dt \int_V d^3x \left\langle \Pi^{xy}(\vec{x},t)\,\Pi^{xy}(0,0)\right\rangle$$

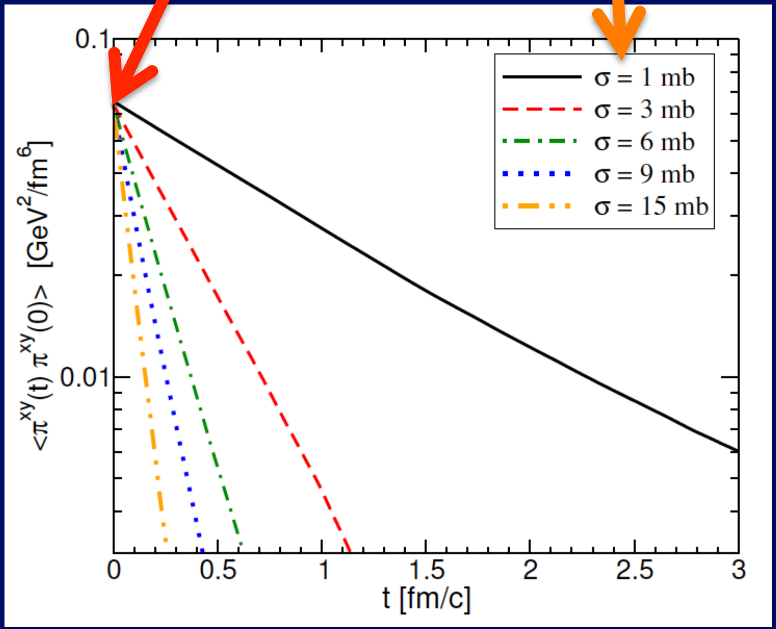
$$\left\langle \Pi^{xy}(\vec{x},t)\Pi^{xy}(0,0)\right\rangle = \left\langle \Pi^{xy}(0,0)\Pi^{xy}(0,0)\right\rangle \cdot e^{-t/\tau}$$

microscopic details

$$\eta = \frac{V}{T}\underbrace{\langle \pi^{xy}(0)\,\pi^{xy}(0)\rangle}_{\text{macroscopic observables}}\,\tau$$

$$= \frac{4}{15}\frac{\varepsilon T}{V}$$





S. Plumari et al., Phys. Rev. C86 (2012)
See also:
Wesp et al., Phys. Rev. C 84 (2011);

Needed very careful tests of convergency vs. N_{test}, Δx_{cell}, # time steps !

Shear Viscosity in Box Calculation

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microscopic details

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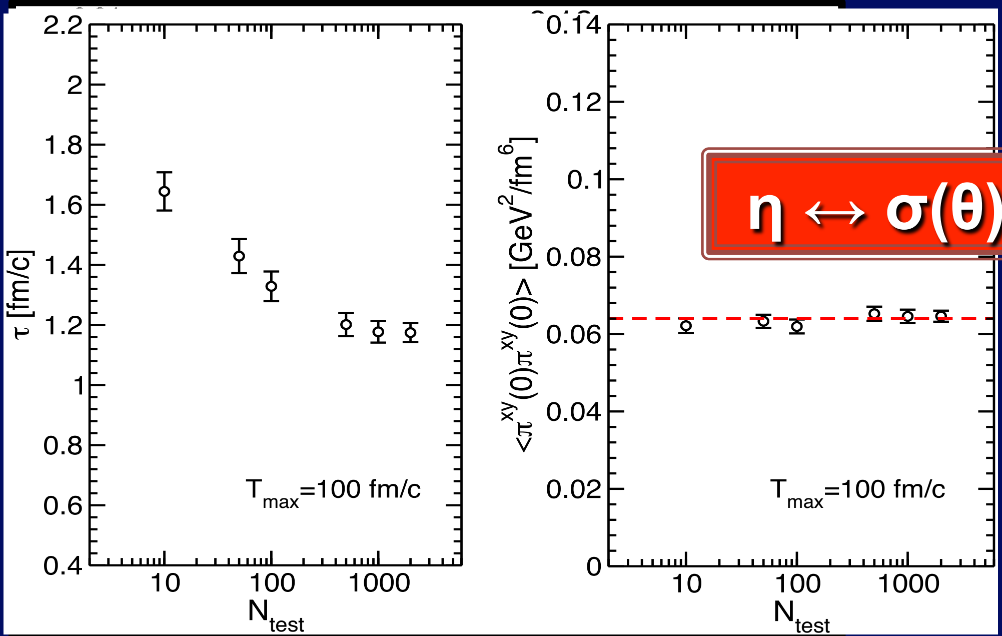

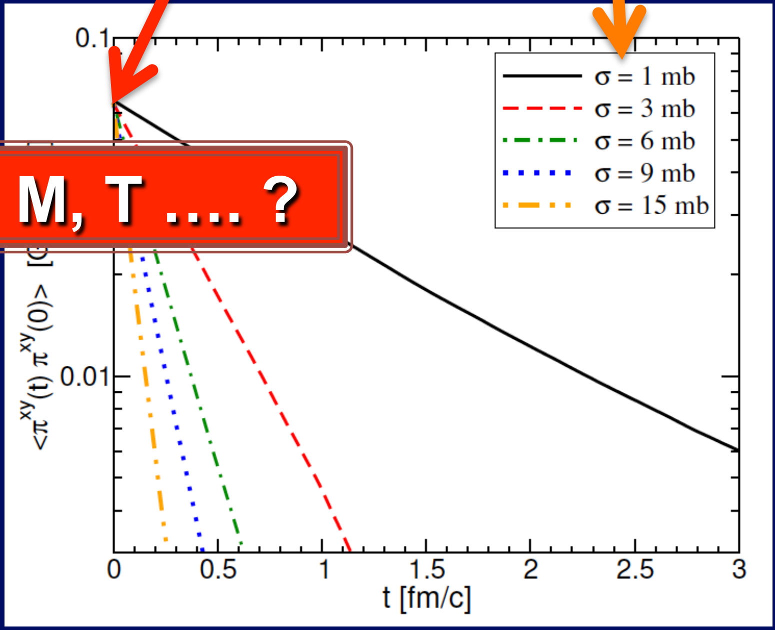

η ↔ σ(θ), ρ, M, T …. ?

S. Plumari et al., Phys. Rev. C86 (2012)
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Needed very careful tests of convergency vs. N_{test}, Δx_{cell}, # time steps !

Non Isotropic Cross Section - $\sigma(\theta)$

Relaxation Time Approximation

$$\eta_{RTA}/s = \frac{1}{15} < p > \tau_{tr} = \frac{1}{15} \frac{< p >}{\langle h(a) \rangle \sigma_{TOT} \rho}$$

$$h(a) = 4a(1+a)\left[(2a+1)\ln(1+a^{-1}) - 2\right] \quad , a = m_D^2 / s$$

$h(a)=\sigma_{tr}/\sigma_{tot}$ weights cross section by q^2

RTA is the one usually employed to make theroethical estimates: Gavin NPA(1985); Kapusta, PRC82(10); Redlich and Sasaki, PRC79(10), NPA832(10); Khvorostukhin PRC (2010) …

for a generic cross section:

$$\frac{d\sigma}{d\Omega} \propto \left(q^2(\theta) + m_D^2\right)^{-2}$$

m_D regulates the angular dependence

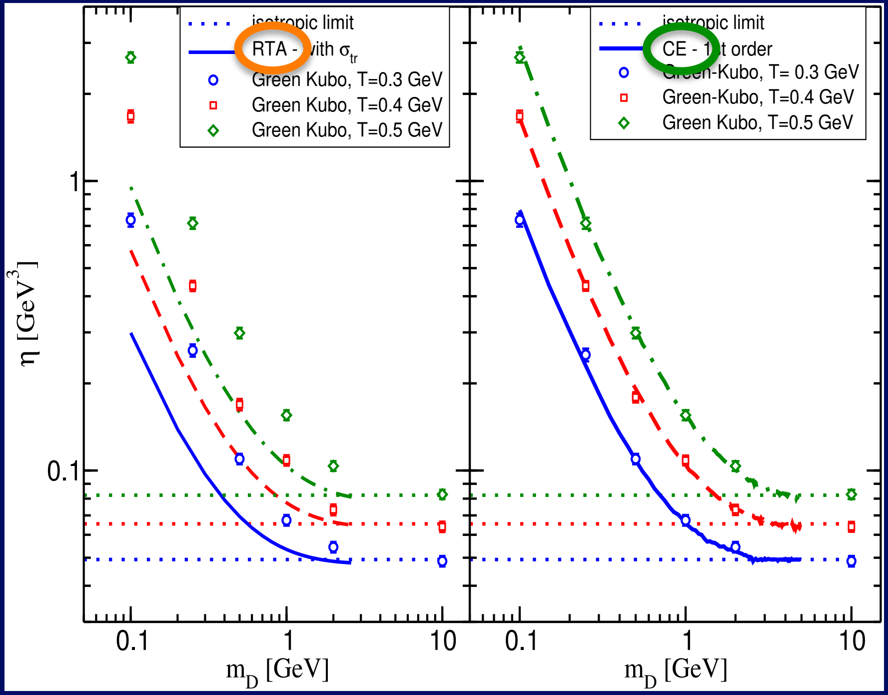
Chapmann-Enskog (CE)

$$\eta/s = \frac{1}{15} \langle p \rangle \tau_\eta = \frac{1}{15} \frac{\langle p \rangle}{g(a) \sigma_{tot} \rho}$$

$$g(a) = \frac{1}{50} \int dy y^6 \left[(y^2 + \frac{1}{3}) K_3(2y) - y K_2(2y)\right] h\left(\frac{a^2}{y^2}\right)$$

g(a) correct function that fix the momentum transfer for shear motion

Green-Kubo in a box - $\sigma(\theta)$



- CE and RTA can differ by about a factor 2
- Green-Kubo agrees with CE

S. Plumari et al., PRC86(2012)054902

Simulate a fixed shear viscosity

Usually input of a transport approach are *cross-sections and fields*, but here we reverse it and start from η/s with aim of creating a more direct link to viscous hydrodynamics

Chapmann-Enskog

$$\frac{\eta}{s} = \frac{1}{15}\langle p\rangle \cdot \tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{g(\frac{m_D}{T})\sigma_{TOT}\rho}$$

$$g(a) = \frac{1}{50}\int dy y^6 \left[(y^2+\frac{1}{3})K_3(2y) - yK_2(2y)\right] h\left(\frac{a^2}{y^2}\right)$$

$g(a=m_D/2T)$ correct function that fix the relaxation time for the shear motion

$0 < g(m_D/2T) < 2/3$

forward peaked

Isotropic $m_D \to \infty$

Transport code

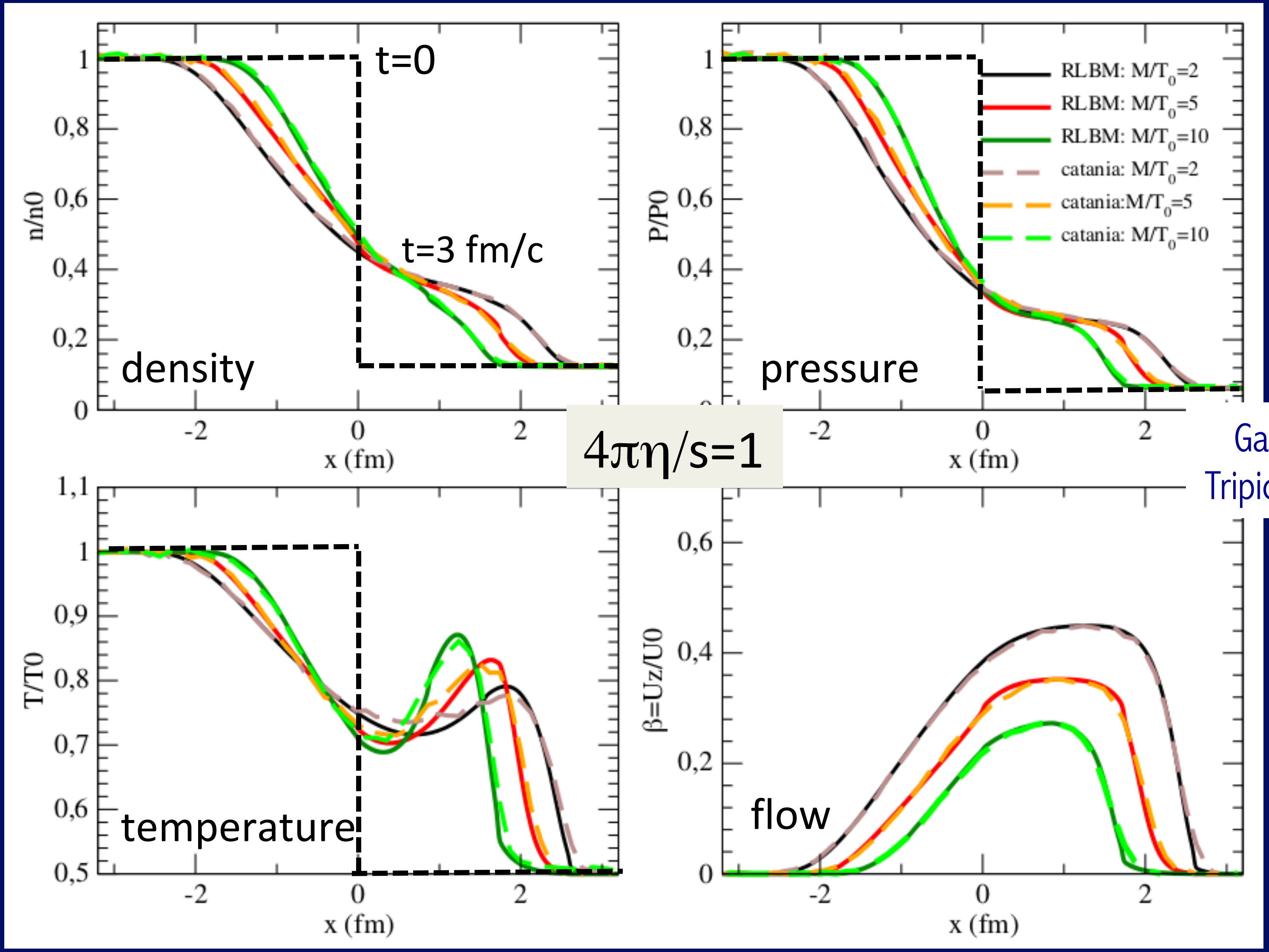
$$\sigma_{tot}(n(\vec{r}),T) = \frac{1}{15}\frac{\langle p_\alpha\rangle}{g(a)n_\alpha}\frac{1}{\eta/s}$$

Space-Time dependent cross section evaluated locally

M. Ruggieri et al., PLB727 (2013), PRC89(2014)

Comparison to Relativistic Lattice Boltzmann

Riemann problem: shock waves (extreme dynamics)



T_L=200 MeV

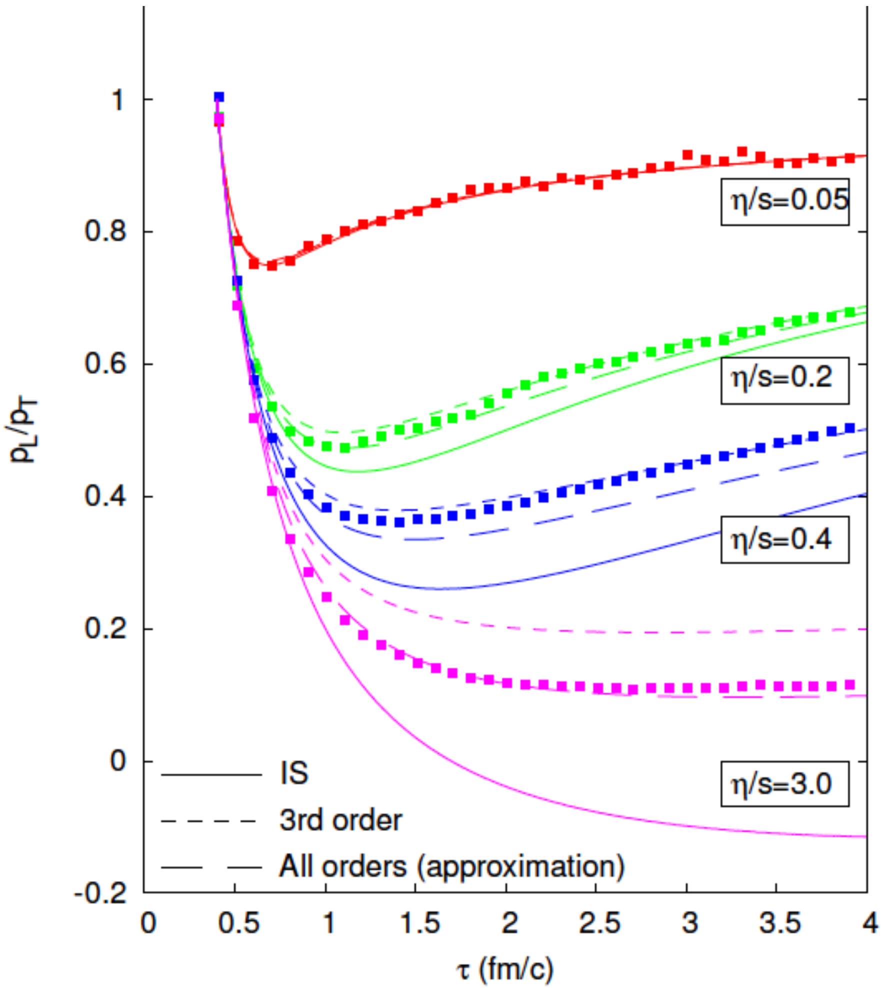
T_R=400 MeV

Gabbana, Plumari,VG, Tripiccione, in preparation

RLBM-Gabbana, Mendoza, Succi, Tripiccione, PRE95 (2017)
already tested against viscous hydro for M=0

Study from BAMPS-Frankfurt

El, Xu, Greiner, Phys.Rev. C81 (2010) 041901



- Convergency for small η/s of Boltzmann transport at fixed η/s with viscous hydro

- Better agreement with 3rd order viscous hydro for large η/s

$$s^\mu = -\int \frac{d^3p}{E} p^\mu f(\ln f - 1)\,. \qquad (3)$$

$\ln(f)$ will be expanded to the third order in $\phi \approx C_0 \pi_{\mu\nu} p^\mu p^\nu$ [see Eq.(1)]. We obtain

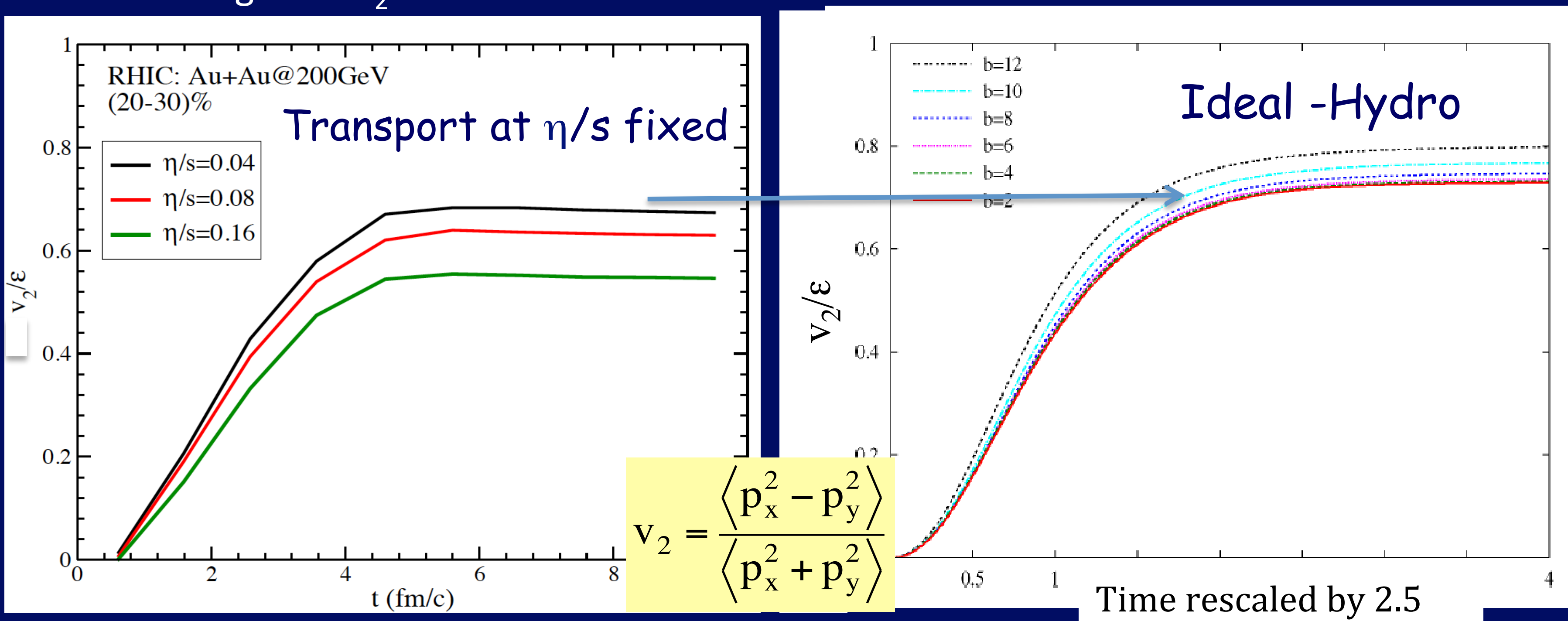
$$s^\mu \approx -\int \frac{d^3p}{E}\, f_0 p^\mu \left(\ln f_0 - 1 + \phi + \phi \ln f_0 + \frac{\phi^2}{2} - \frac{\phi^3}{6}\right)$$
$$= s_0 u^\mu - \frac{\beta_2}{2T}\pi_{\alpha\beta}\pi^{\alpha\beta}u^\mu - \frac{8}{9}\frac{\beta_2^2}{T}\pi_{\alpha\beta}\pi^{\alpha}_{\sigma}\pi^{\beta\sigma}u^\mu\,, \qquad (4)$$

Boltzmann transport at fixed η/s for <u>non dilute systems</u> converge to hydrodynamics

Test in 3+1D: v_2/ε response for almost ideal case

EoS $c_s^2=1/3$ (dN/dy tuned to RHIC, geometry of Au+Au)

Integrated v_2 vs time

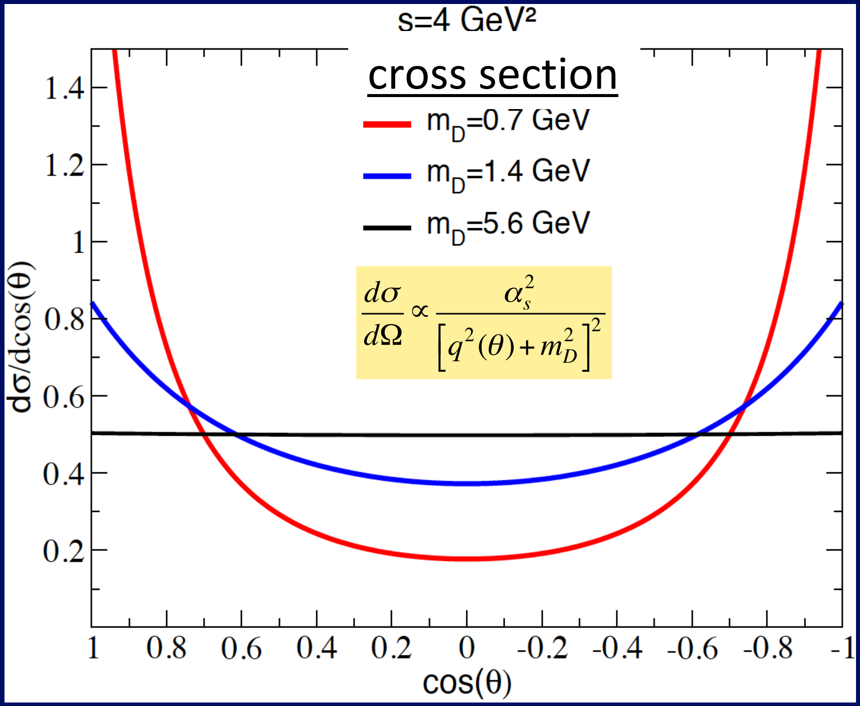


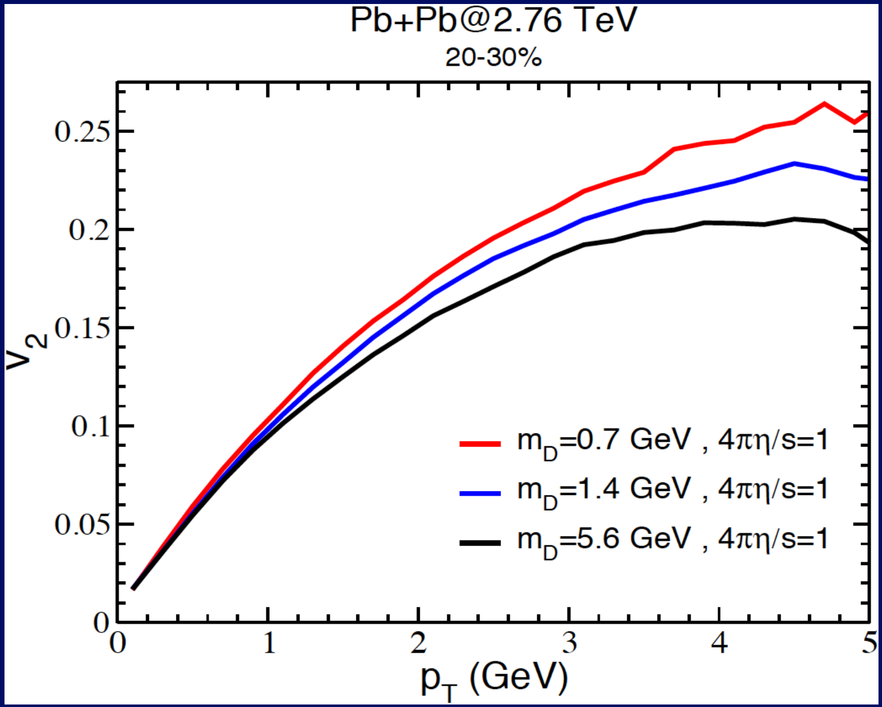
$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

Bhalerao et al., PLB627(2005)

In the bulk the transport has an hydro v_2/ε_2 response!

η/s or details of the cross section?





Keep same η/s means:

$$\frac{\eta}{s} = \frac{1}{15}\langle p\rangle \cdot \tau_\eta$$

$$\tau_\eta^{-1} = g\left(\frac{m_D}{T}\right)\sigma_{TOT}\rho$$

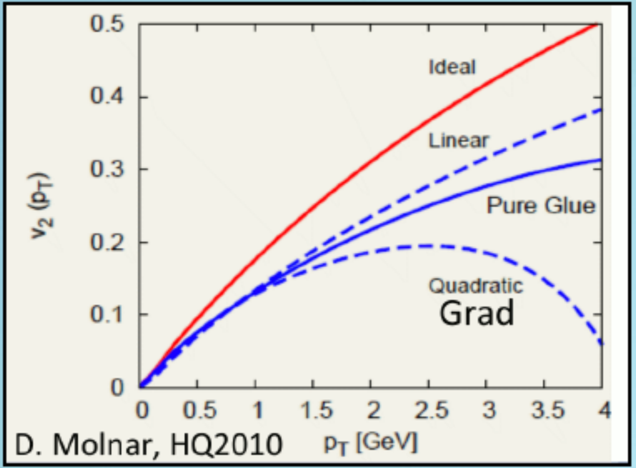
$$\frac{\sigma_{TOT}(m_{1D})}{\sigma_{TOT}(m_{2D})} = \frac{g(m_{2D})}{g(m_{1D})}$$

for m_D=0.7 GeV -> factor 2 larger σ_{tot} is needed respect to isotropic case

From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

$$f(x,p) = f^{(0)}(x,p) + \delta f(x,p)$$

$$T^{\mu\nu} = T^{(0)\mu\nu} + \delta T^{\mu\nu} \leftarrow f^{(0)} + \delta f$$



A common choice for δf – the Grad ansatz

$$\delta f \propto \Gamma_s f^{(0)} p^\alpha p^\beta \langle \nabla_\alpha u_\beta \rangle \propto p_T^2$$

BUT it doesn't care about the microscopic dynamics

In general in the limit σ→∞, f(σ) can be expanded in power of 1/σ.

$$f(\sigma) \underset{\sigma\to\infty}{\approx} f^{(0)} + \frac{1}{\sigma}\delta f + O\left(\frac{1}{\sigma^2}\right) \Longrightarrow v_n(p_T) \underset{\sigma\to\infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma}\delta v_n + O\left(\frac{1}{\sigma^2}\right)$$

PURPOSE: evaluate the ideal hydrodynamics limit $f^{(0)}$, $v_n^{(0)}$ and the viscous corrections δf and δv_n solving the Relativistic Boltzmann eq for large values of the cross section σ

From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

Coodinate space (x,y)

- We start with an initial azimuthally symmetric profile (optical Glauber model).
- Then we deform the initial distribution ($\alpha \ll 1$)

$$z = x + iy \rightarrow z + \delta z \equiv z - \alpha \bar{z}^{n-1}$$

2π/n symmetry

This Creates only

n=2 n=3 n=4 n=5 n=6

Momentum space

- Thermal distribution:

$$dN/d^3p \propto \exp(-p/T)$$

- Constant distribution:

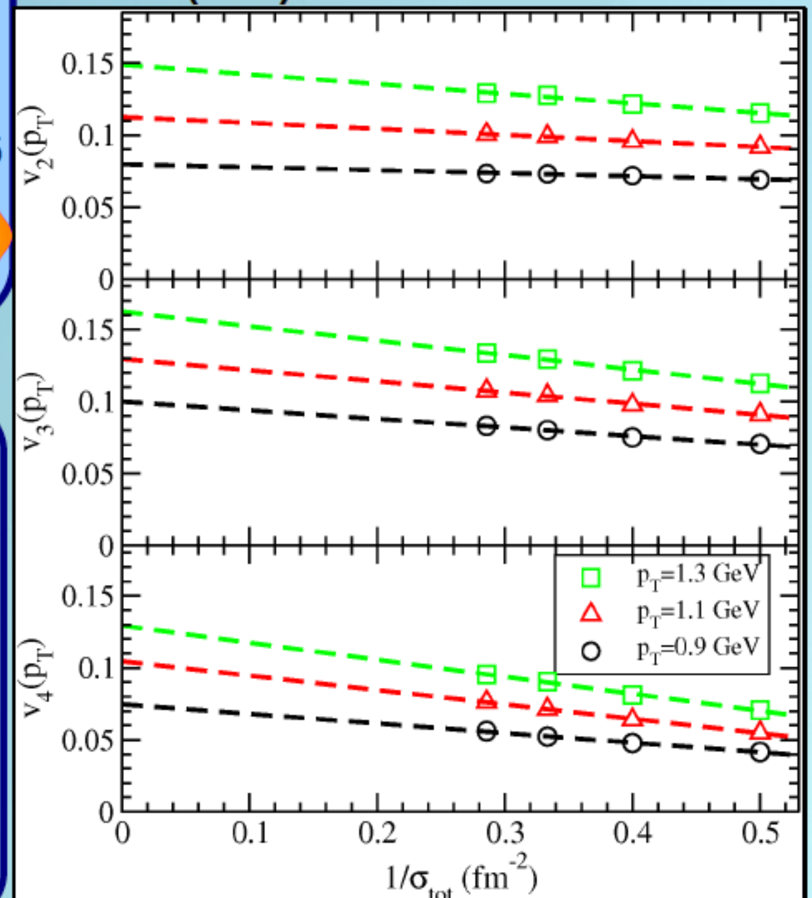
$$dN/d^3p \propto \theta(p_0 - p)$$

We assume initially the same local $T^{\mu\nu}(x)$

$$f(\sigma) \underset{\sigma\to\infty}{\approx} f^{(0)} + \frac{1}{\sigma}\delta f + O\left(\frac{1}{\sigma^2}\right)$$

$$v_n(p_T) \underset{\sigma\to\infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma}\delta v_n + O\left(\frac{1}{\sigma^2}\right)$$

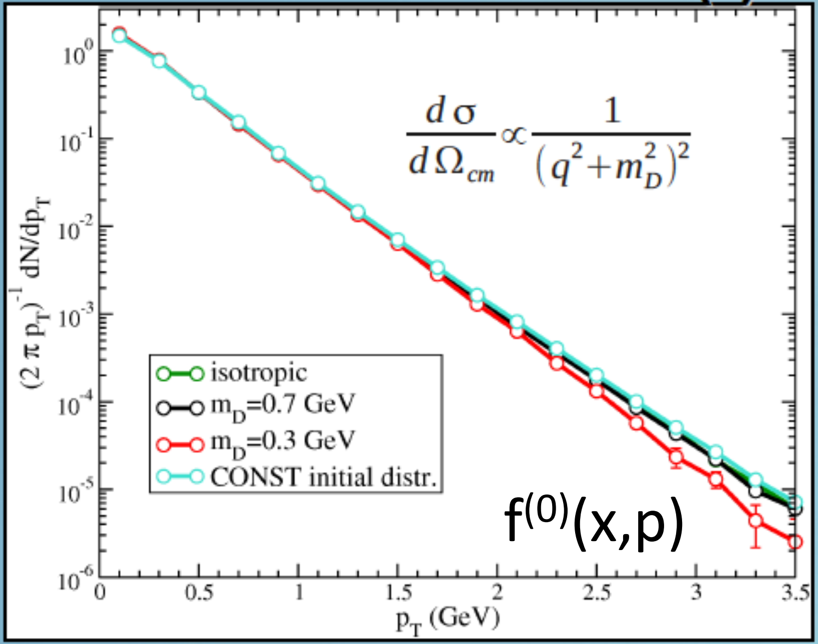
S.Plumari,G.L. Guardo,V. Greco,J.Y.Ollitrault NPA 941 (2015) 87

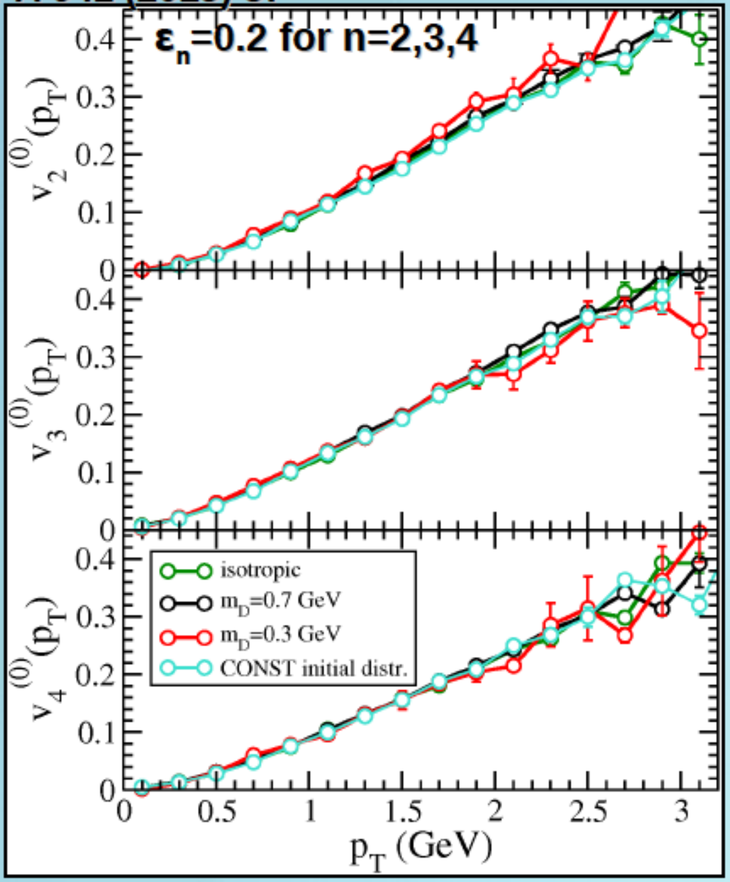


From Transport to Hydro: extraction of viscous corrections to $f(x,p)$ and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

S.Plumari,G.L. Guardo,V. Greco,J.Y.Ollitrault
NPA 941 (2015) 87

For the same initial local $T^{\mu\nu}(x)$:





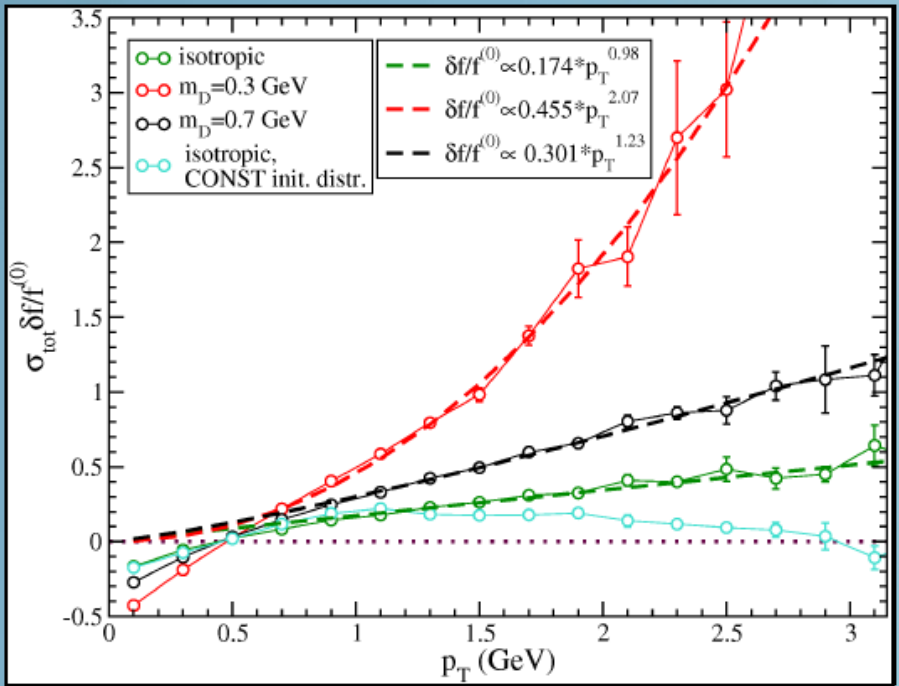
For $\sigma \to \infty$ we find the ideal Hydro limit:

- $f^{(0)}$ is an exponential decreasing function.
- $f^{(0)}$ doesn't depends on microscopical details (i.e. mD).

- Universal behavior of $v_n^{(0)}(p_T)$
- $v_n^{(0)}(p_T)/\varepsilon_n$ is approximatively the same for all n and p_T.

From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

S.Plumari,G.L. Guardo,V. Greco,J.Y.Ollitrault NPA 941 (2015) 87



In δf and δv_n it is encoded the information about the microscopical details

- **$\delta f(p_T)/f^{(0)} \propto p_T^{\alpha}$ with α = 1. - 2. and $\alpha(m_D)$.**

For isotropic σ similar to R.S. Bhalerao et al. PRC 89, 054903 (2014)

…but in strongly coupled system one does not expect a very forward peaked cross-section

Motivation for transport vs Hydrodynamics

- Starting from 1-body distribution function f(x,p) and not from $T_{\mu\nu}$:

Hydro $\xleftarrow[\eta/s<<1]{p_T \approx 3T}$ Transport fixed η/s

- Extension to mid- p_T (minijets): large $\delta f(p_T)$
- Initial pre-equilibrium
- Freeze-out consistent with η/s (Hydro weakness)
- large η/s and local large stress tensor (pA)
- Microscopic mechanism: Hadronization (beyond SHM)
- Heavy Quarks beyond Fokker-Planck

Now,
some examples of things where
one can go beyond Viscous Hydro:

I- initial stage off-equilibrium

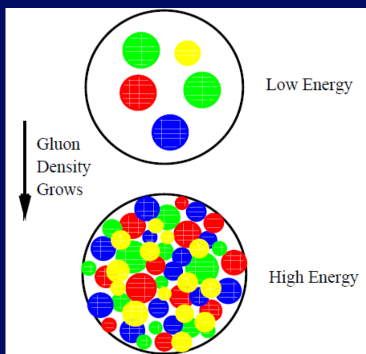
II- Initial State Fluctuations: v_2=v_3

III- From Chromo-magnetic fields to QGP

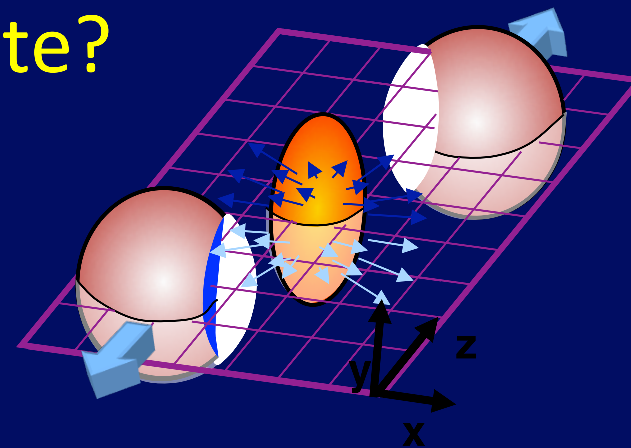
IV- Extension to pA collisions

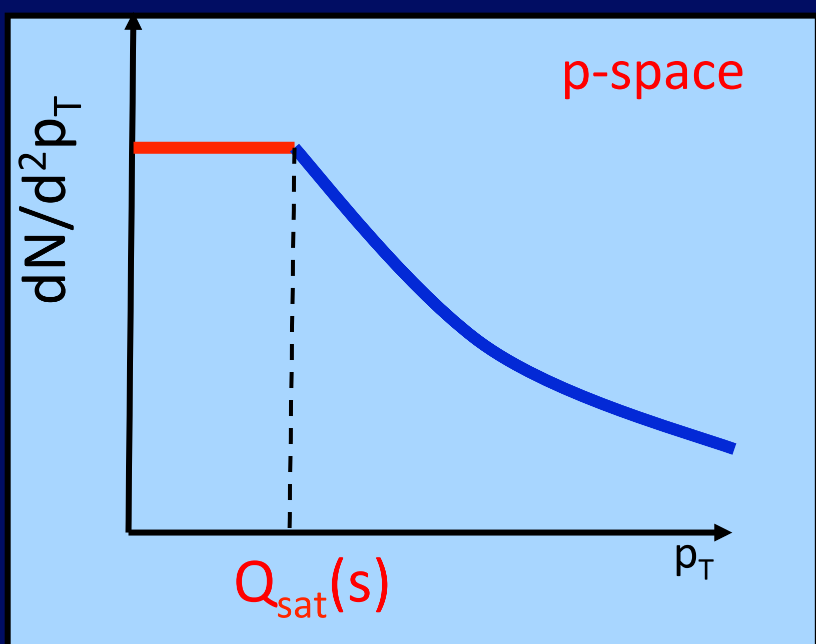
I - Transport at fixed η/s: initial off-equilibrium

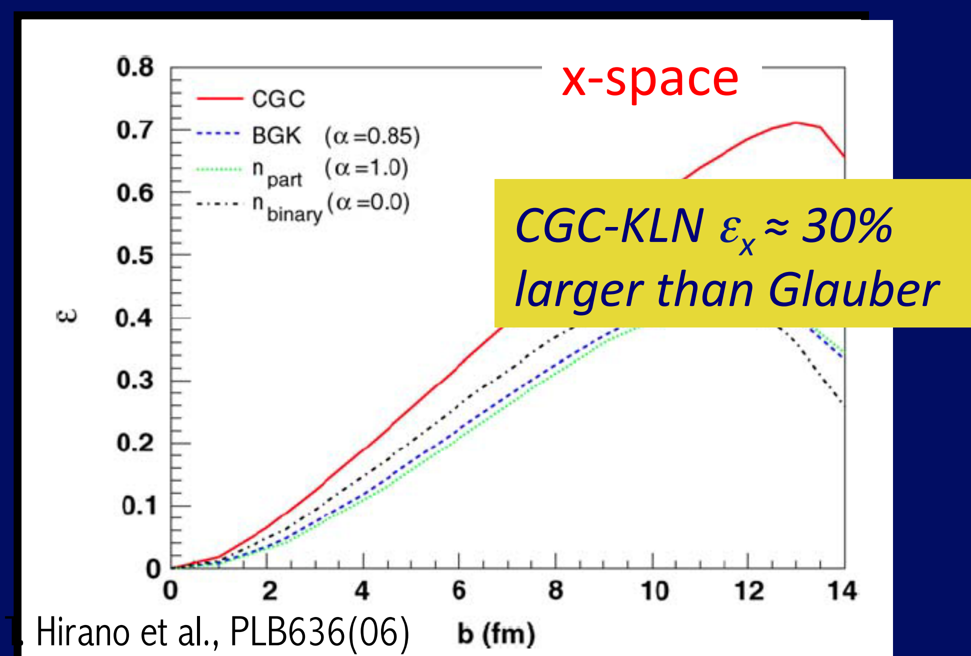
What is the impact of non-equilibrium Color Glass condensate initial state?



QCD high energy limit







CGC-KLN $\varepsilon_x \approx 30\%$ larger than Glauber

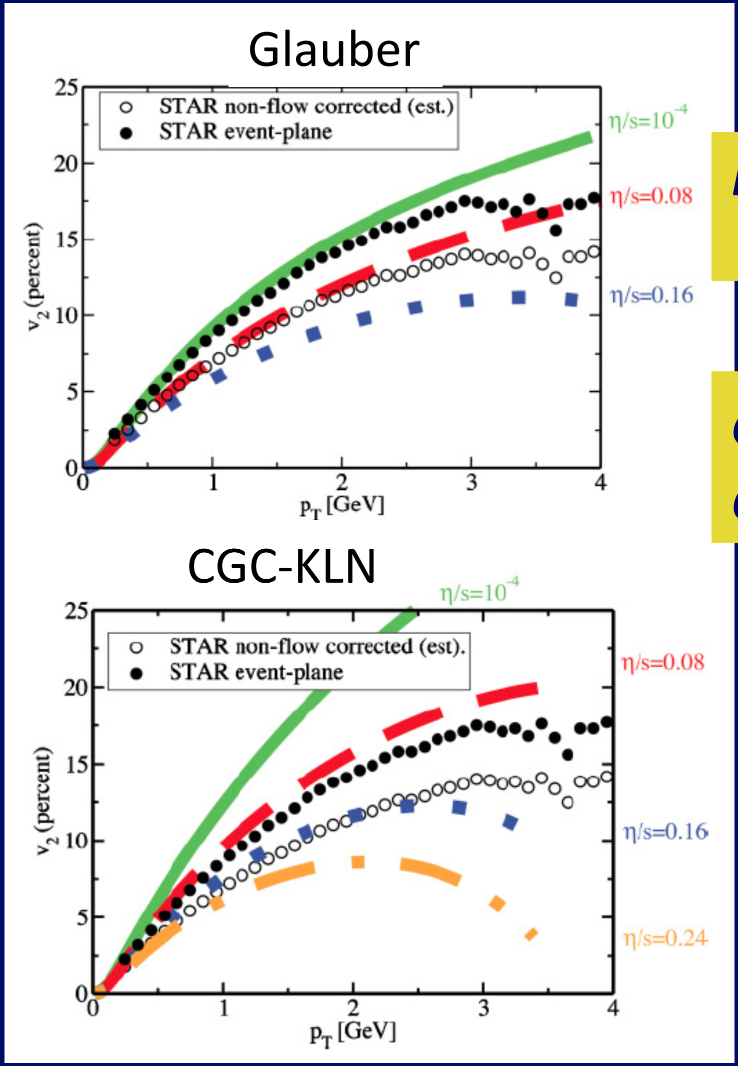
T. Hirano et al., PLB636(06)

V_2 from KLN (CGC) in Hydro

What does KLN in hydro?

1) r-space from KLN (larger ε_x)

2) p-space thermal at $t_0 \approx 0.6$-0.9 fm/c - No Q_s scale , We'll call it **fKLN-Th**



Larger ε_x - > higher η/s to get the same $v_2(p_T)$

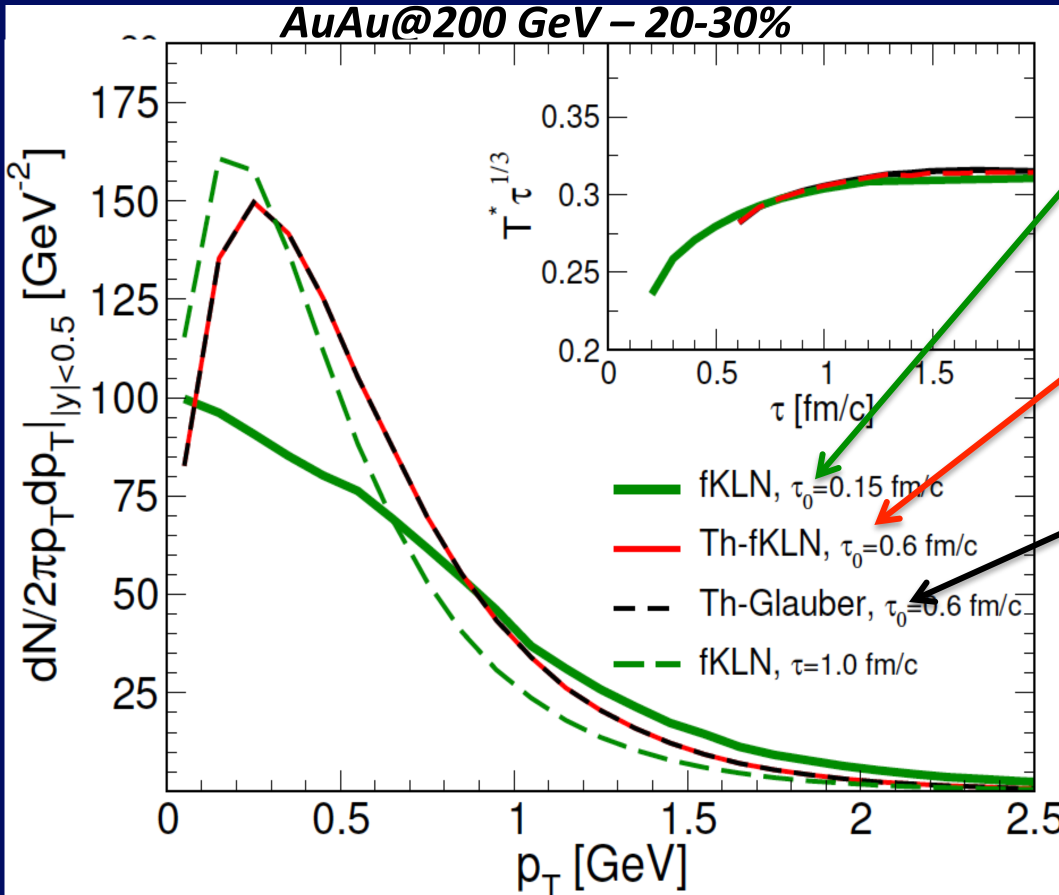
Glauber → $\eta/s = 0.08$

CGC-KLN → $\eta/s=0.16$

Luzum and Romatschke
PRC78(2008) 034915

See also:
Alver et al., PRC 82, 034913 (2010)
Heinz *et al.*, PRC 83, 054910 (2011)

Implementing KLN p_T distribution



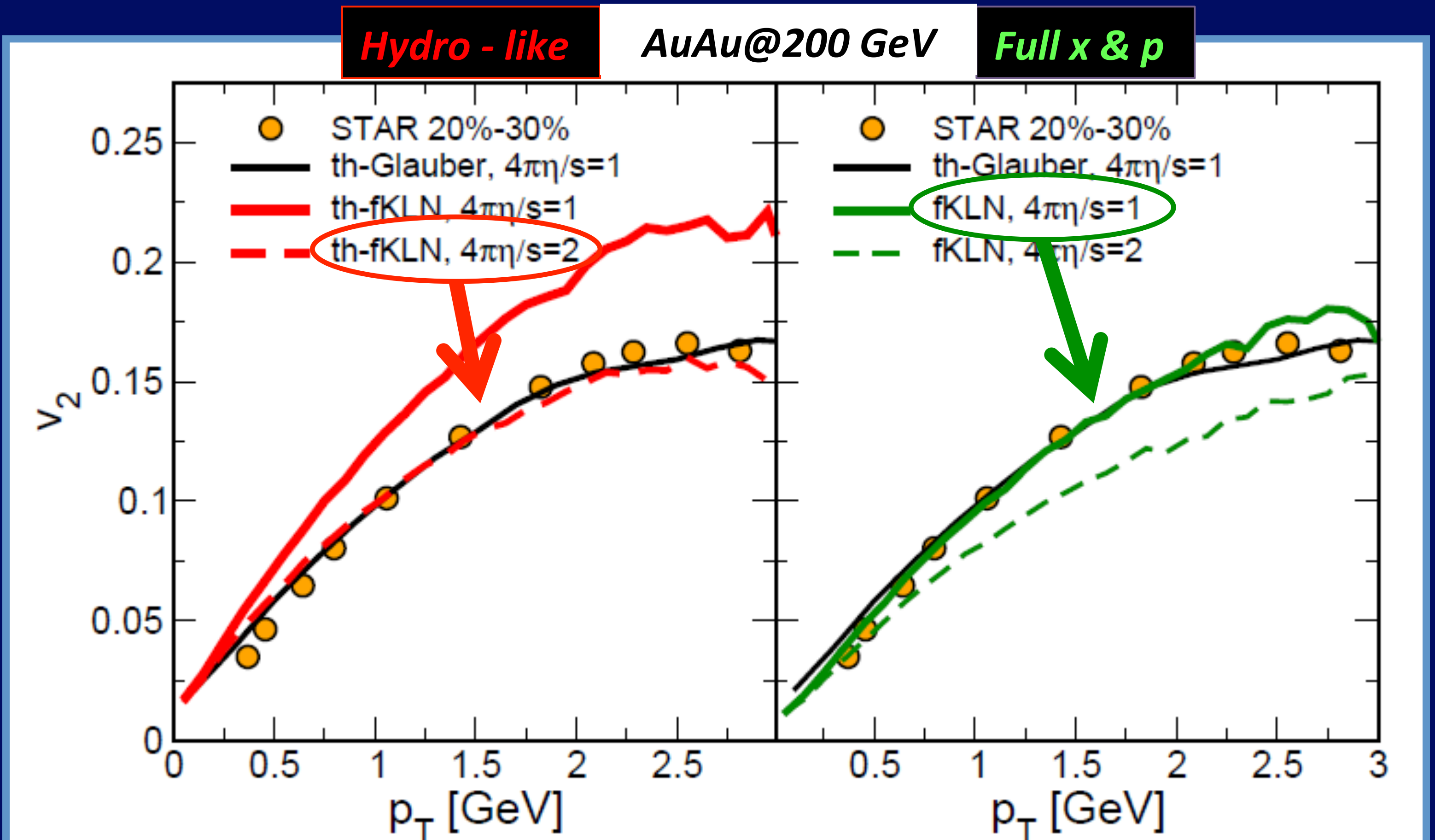
Using kinetic theory
we can implement full KLN
(x & p space) - ε_x=0.34, Qs =1.4 GeV

KLN only in x space (like in Hydro)
ε_x=0.341, Qs=0 -> Th-KLN

Glauber in x & thermal in p
ε_x=0.289 , Qs=0 -> Th-Glauber

M. Ruggieri *et al.*, Phys.Lett. B727 (2013) 177

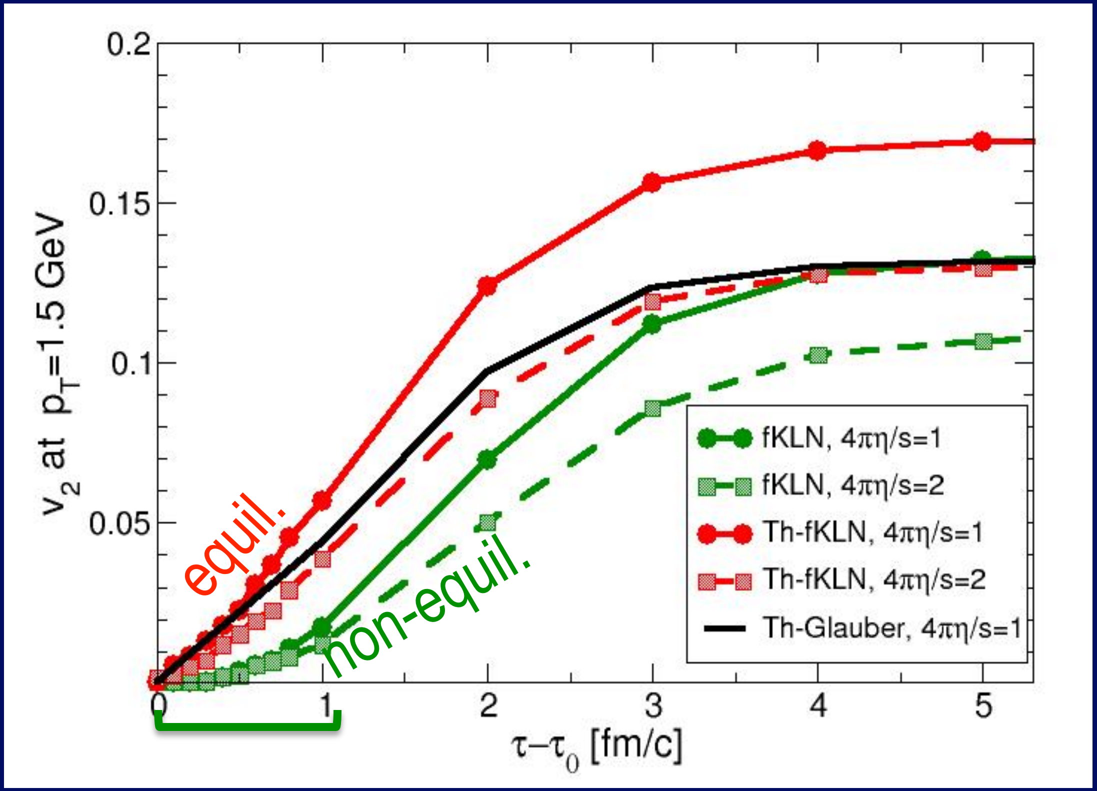
Results with kinetic theory



M. Ruggieri *et al.*, **Phys.Lett. B727 (2013) 177 -** 1303.3178 [nucl-th]

- When implementing KLN and Glauber like in Hydro we get the same of Hydro
- When implementing full KLN we get close to the data with $4\pi\eta/s = 1$:
  larger ε_x compensated by Q_s saturation scale (non-equilibrium distribution)

What is going on?



❖ We clearly see that when non-equilibrium distribution is implemented in the initial stage (≤ 1 fm/c) v_2 grows slowly with respect to thermal one

❖ Deformation of p_T distribution -> affects $v_2(p_T)$!!

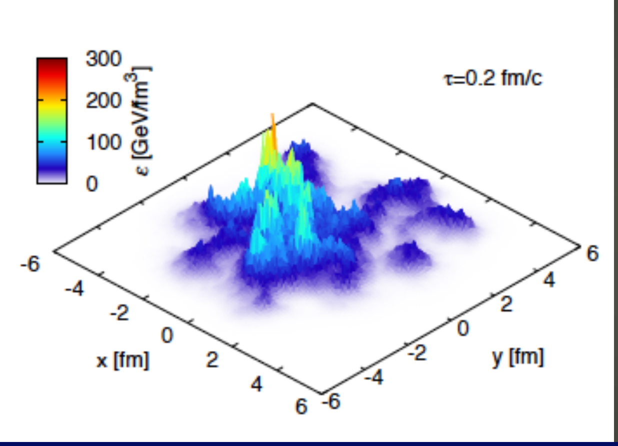
❖ Effect decrease with centrality and with beam energy!

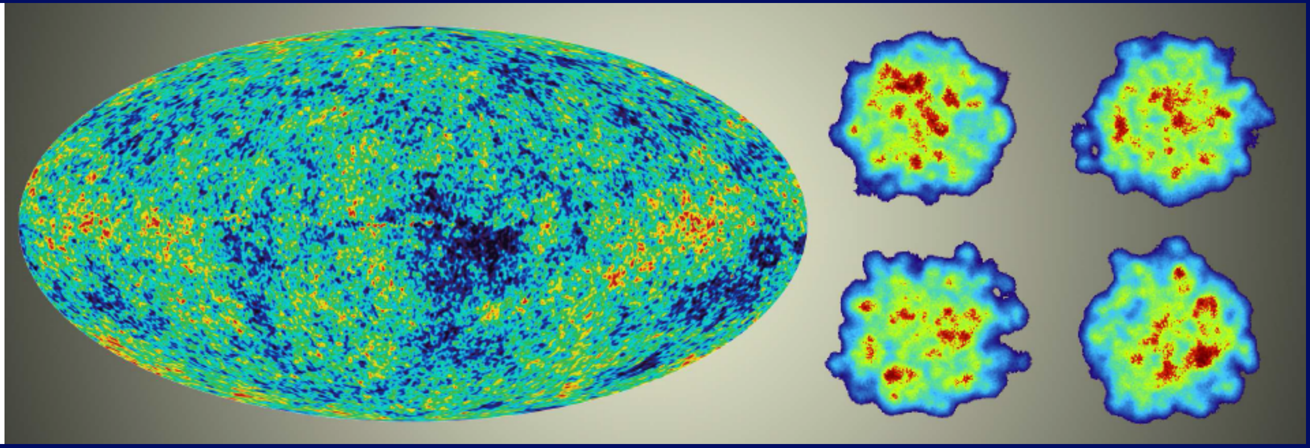
II – Initial State Fluctuations

What is the impact of Initial State Fluctuations?

Local large gradients against Hydro

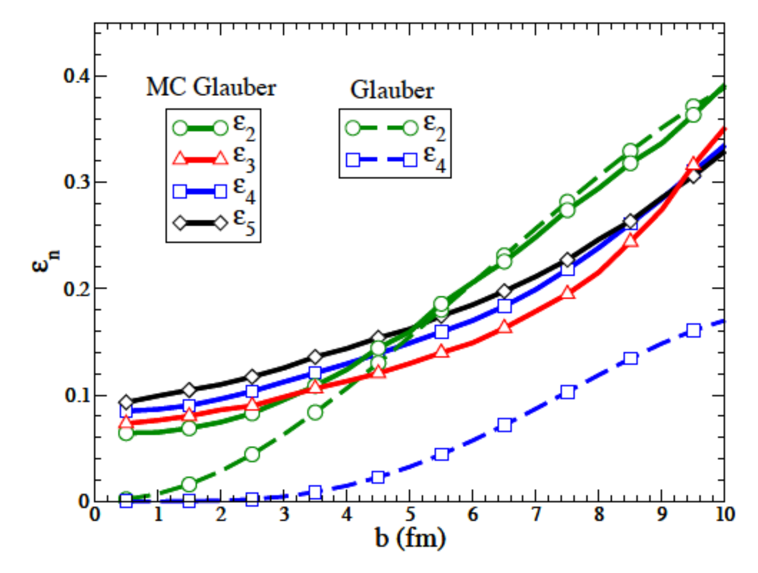
(indeed they are cut-off at t_0)





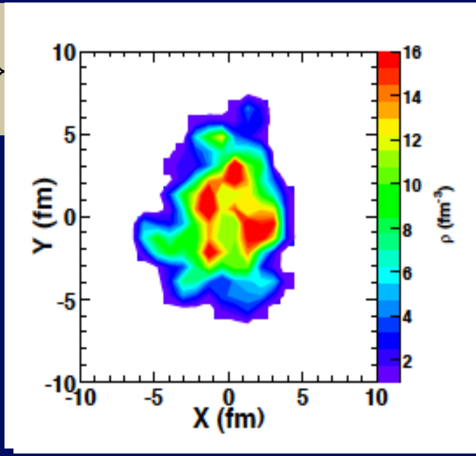
Include Initial State Fluctuations

MonteCarlo Glauber



$$\rho_\perp \propto \sum_{i=1}^{N_{part}} \exp\left\{-\left[(x-x_i)^2 + (y-y_i)^2\right]/2\sigma^2\right\}$$

σ = 0.5 fm



G-Y. Qin, H. Petersen, S.A. Bass and B. Muller, PRC82(2010)
H.Holopainen, H. Niemi and K.J. Eskola, PRC83(2011)

$$\varepsilon_n = \frac{\left\langle r_\perp^n \cos\left[n(\phi-\Phi_n)\right]\right\rangle}{\left\langle r_\perp^n \right\rangle}$$

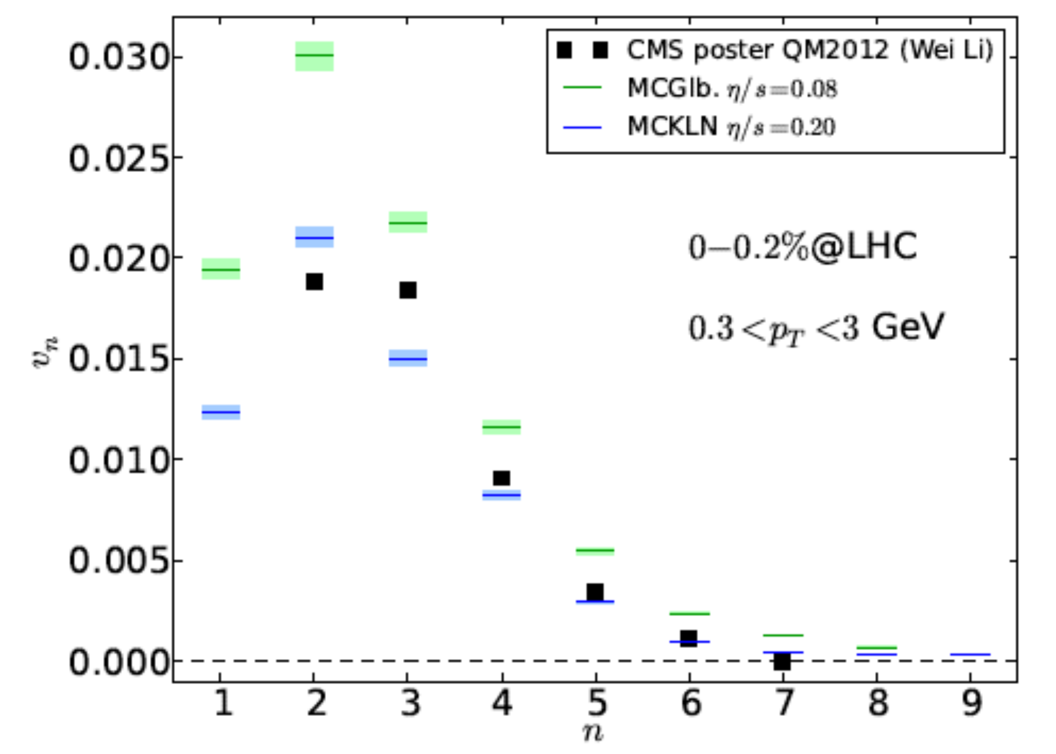
$$\Phi_n = \frac{1}{n}\arctan\frac{\left\langle r_\perp^n sen\left[n(\phi-\Phi_n)\right]\right\rangle}{\left\langle r_\perp^n \cos\left[n(\phi-\Phi_n)\right]\right\rangle}$$

Impact of Fluctuations as in hydro:

- Decrease of $v_{2\ (15\text{-}20\%)}$
- appeareance of a large $v_3 \approx v_2$ in ultra-central
- Enanhcement of v_4 about a factor 3

In ultra central collision, of course viscous hydro works better:

large source, smaller surface gradients,less corona and/ or hadronic contaminations



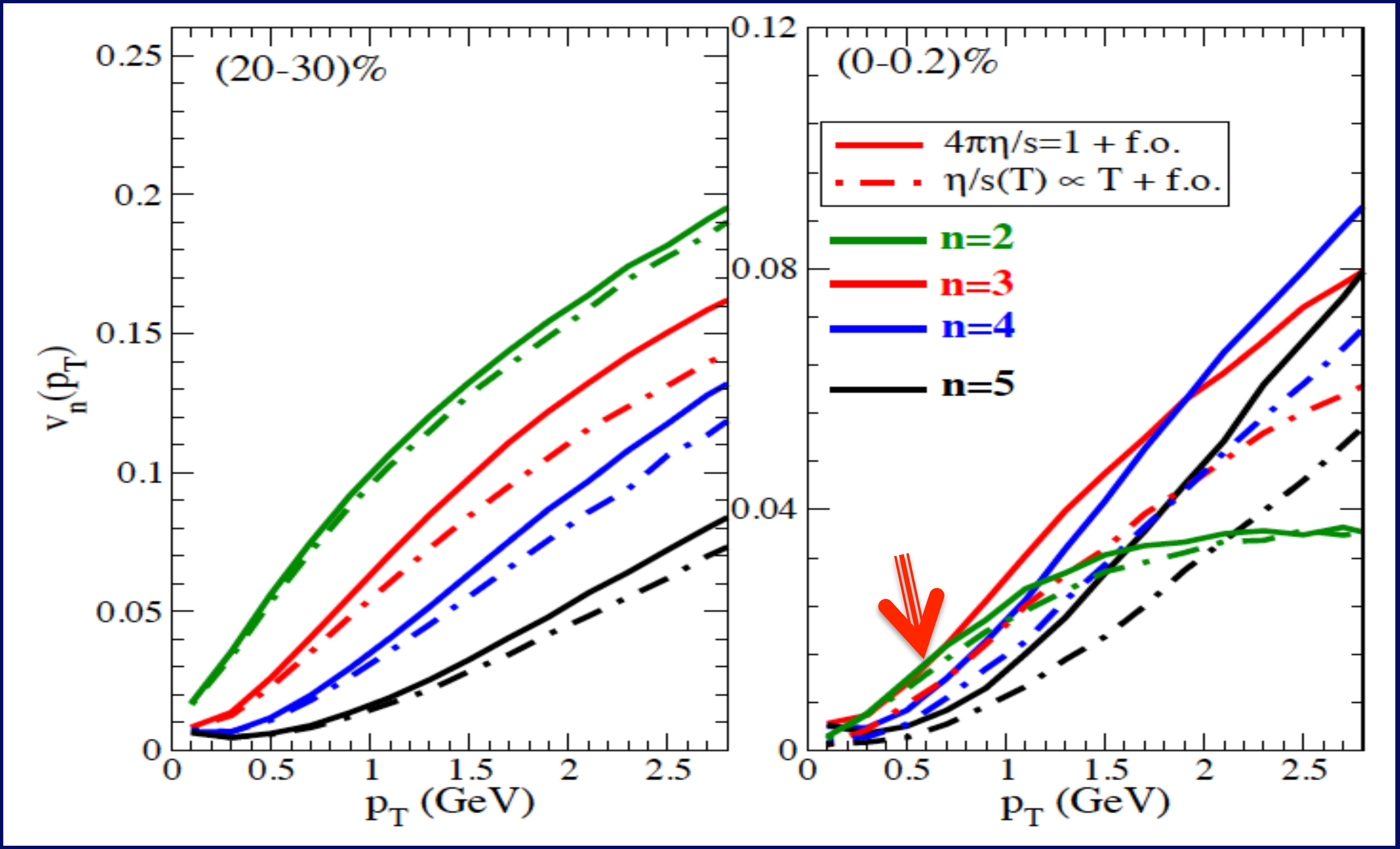
A significant failure of Hydro!

Where it should work the best!

Neither MC-Glb nor MC-KLN gives the correct initial power spectrum! † R.I.P.

Is it due to some non-equilibrium physics or freeze-out dynamics?

Include Initial State Fluctuations : $v_n(p_T)$ in ULTRA-central



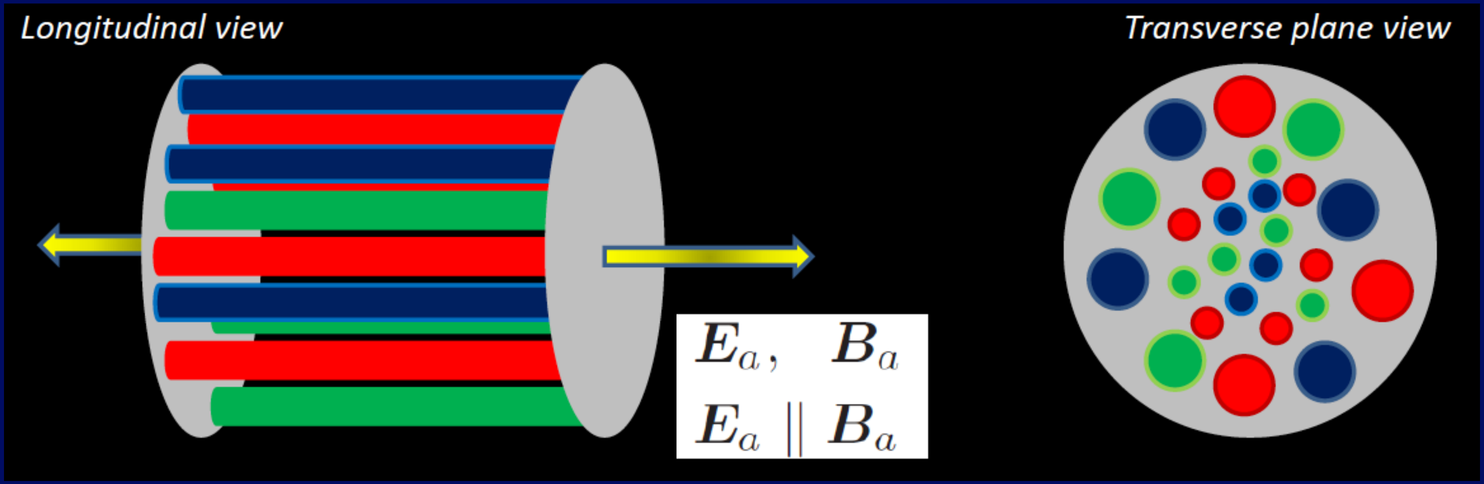
- For Ultra-central collisions there is quite larger sensitivity to $\eta/s(T)$
- Strong saturation of $v_2(p_T)$ with p_T , while $v_n \approx p_T^{\alpha}$ seen experimentally
- $V_3 \approx V_2$ in ultra-central collisions… woud solve a main puzzle!!!

S. Plumari et al., PRC92(2015)

III- From Chromo-magnetic fields to QGP

A first tentative: Color electric flux tubes

Initial stage starting from chromoeletric fields
then matched to parton transport at fixed $\eta/s(T)$



A possible approach color fields decay via vacuum instability toward pair creation (Schwinger mechanism, 1951)

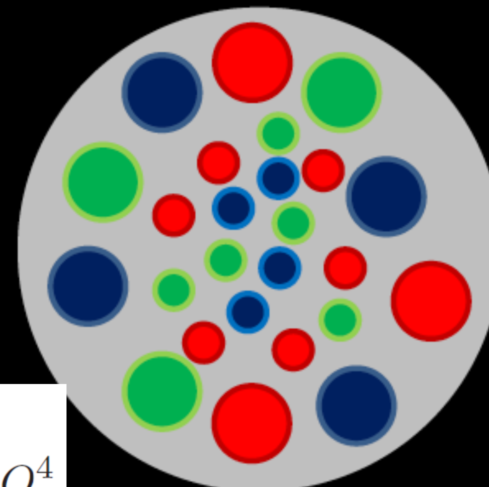
Schwinger effect in Chromodynamics

Abelian Flux Tube Model

Longitudinal view

$$\boldsymbol{E}_a\,, \quad \boldsymbol{B}_a$$
$$\boldsymbol{E}_a \parallel \boldsymbol{B}_a$$

Transverse plane view



$$R \propto 1/Q_s$$
$$\boldsymbol{E}\cdot\boldsymbol{E},\ \boldsymbol{B}\cdot\boldsymbol{B} \propto Q_s^4$$

Focus on a single flux tube:

(.) neglect color-magnetic fields;
(.) assume abelian dynamics for ***color-electric fields***;
(.) initial field is ***longitudinal;***
(.) assume ***Schwinger effect*** takes place:
Color-eletric color field decays into quark-antiquark as well as gluon pairs

Abelian Flux Tube Model

In order to permit ***particle creation*** from the vacuum we need to add a ***source term*** to the rhs of the Boltzmann equation:

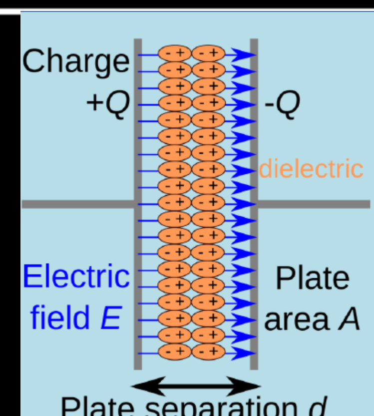
$$\left(p_\mu \partial^\mu + g Q_{jc} F^{\mu\nu} p_\mu \partial_\nu^p\right) f_{jc} = p_0 \frac{\partial}{\partial t}\frac{dN_{jc}}{d^3x d^3p} + \mathcal{C}[f]$$

Florkowski and Ryblewski, PRD 88 (2013)

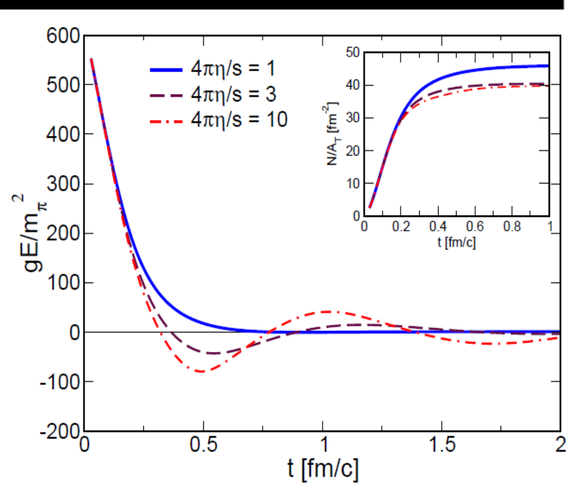
Invariant source term

Invariant source term: change of ***f*** due to particle creation in the volume at ***(x,p)***.

In our model, particles are created by means of the Schwinger effect, hence



10^{25} Volt/m



$$\frac{dN_{jc}}{d\Gamma} \equiv p_0 \frac{dN_{jc}}{d^4x d^2p_T dp_z} = \mathcal{R}_{jc}(p_T)\delta(p_z)p_0$$

$$\mathcal{R}_{jc}(p_T) = \frac{\mathcal{E}_{jc}}{4\pi^3}\left|\ln\left(1 \pm e^{-\pi p_T^2/\mathcal{E}_{jc}}\right)\right|$$

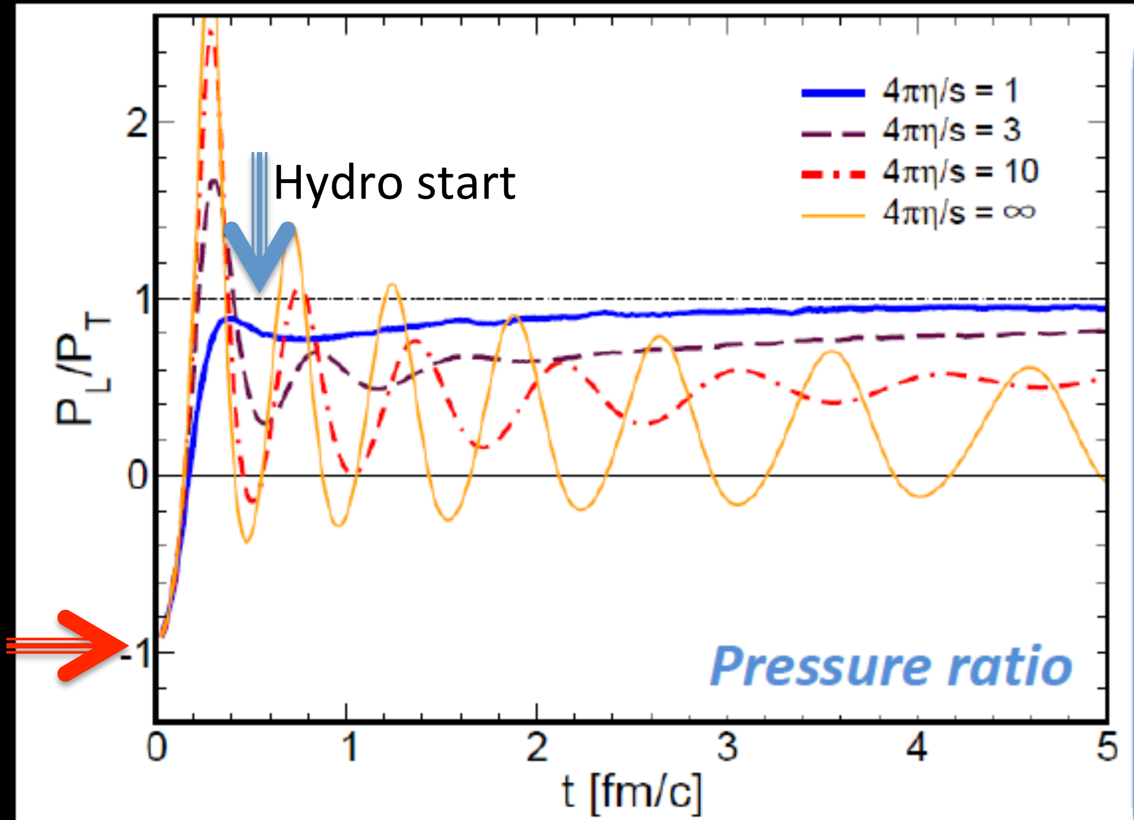
$$\mathcal{E}_{jc} = \left(g|Q_{jc}E| - \sigma_j\right)\theta\left(g|Q_{jc}E| - \sigma_j\right)$$

See also:
Gelis and Tanji, PRD 87 (2013)

$\mathcal{E}_{jc}$ effective force on pairs
Q_{jc} color flavor charges

Massless quanta

Pressure isotropization



M. Ruggieri et al., PRC92(2016)

$$T^{\mu\nu}_{field} = \mathrm{diag}\left(\varepsilon, P_T, P_T, P_L\right)$$
$$\propto \mathrm{diag}\left(\mathcal{E}^2, \mathcal{E}^2, \mathcal{E}^2, -\mathcal{E}^2\right)$$

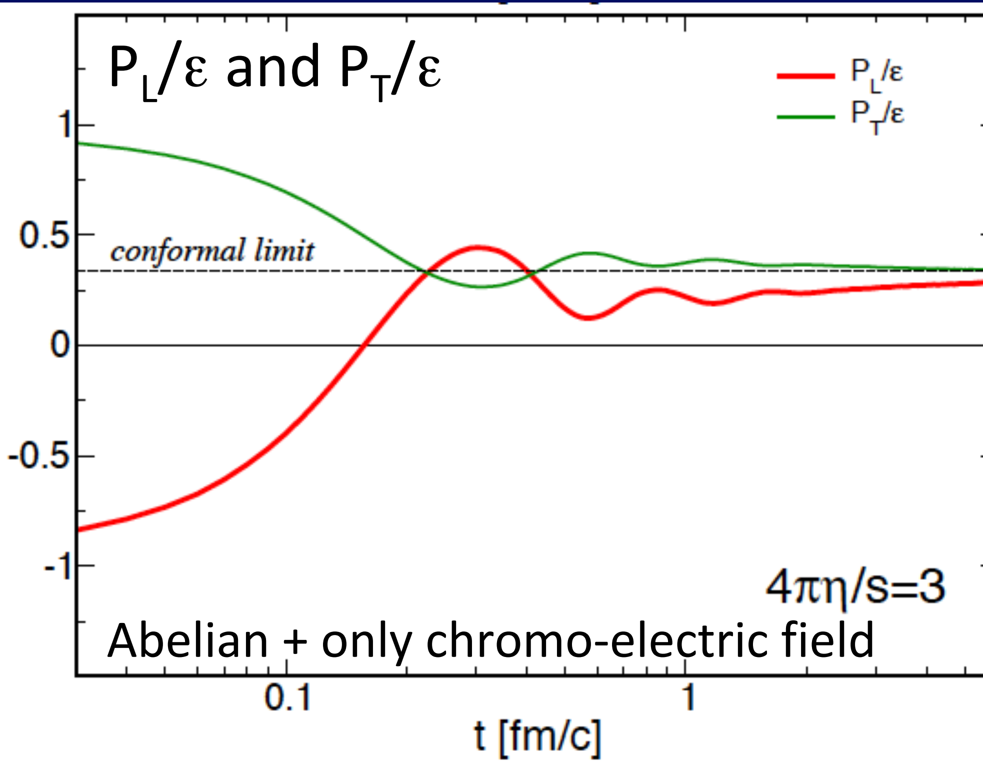
$$T^{\mu\nu}_{particles} = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{p^\mu p^\nu}{E} f(\boldsymbol{x}, \boldsymbol{p})$$

$$T^{\mu\nu} = T^{\mu\nu}_{particles} + T^{\mu\nu}_{field}$$

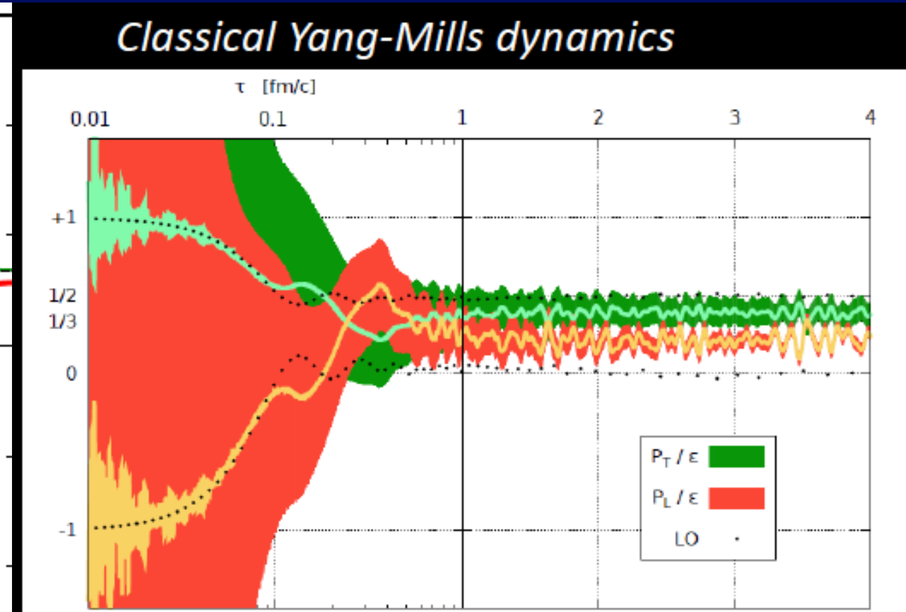
$$P_L = T_{zz}$$
$$P_T = \frac{T_{xx} + T_{yy}}{2}$$

- t=0 pure field with negative field P_L
- t=0.2 fm/c → $P_L > 0$ (particles pop-up) independently of η/s
- t≈0.5-1 fm/c nearly isotropization for $4\pi\eta/s<3$

Color flux tubes coupled to transport at fixed η/s(T)



M. Ruggieri, L. Oliva, S.Plumari, VG, PRC92(2015)



Epelbaum and Gelis, PRL 88 (2013)

(.) Classic Yang-Mills calculation, 3+1D

(.) Quantum fluctuations rather than Schwinger effect

$$H = \frac{1}{2}\sum_{x,a,i} E_i^a(x)^2 + \frac{1}{4}\sum_{x,a,i,j} F_{ij}^a(x)^2, \quad (14)$$

where the magnetic part of the field strength tensor is

$$F_{ij}^a(x) = \partial_i A_j^a(x) - \partial_j A_i^a(x) - \sum_{b,c} f^{abc} A_i^b(x) A_j^c(x); \quad (15)$$

$$\frac{dA_i^a(x)}{dt} = E_i^a(x), \quad (16)$$

$$\frac{dE_i^a(x)}{dt} = \sum_j \partial_j F_{ji}^a(x) - \sum_{b,c,j} f^{abc} A_j^b(x) F_{ji}^c(x). \quad (17)$$

The challenge will be to coupled to
non-Abelian Yang-Mills fields
to transport at fixed η/s

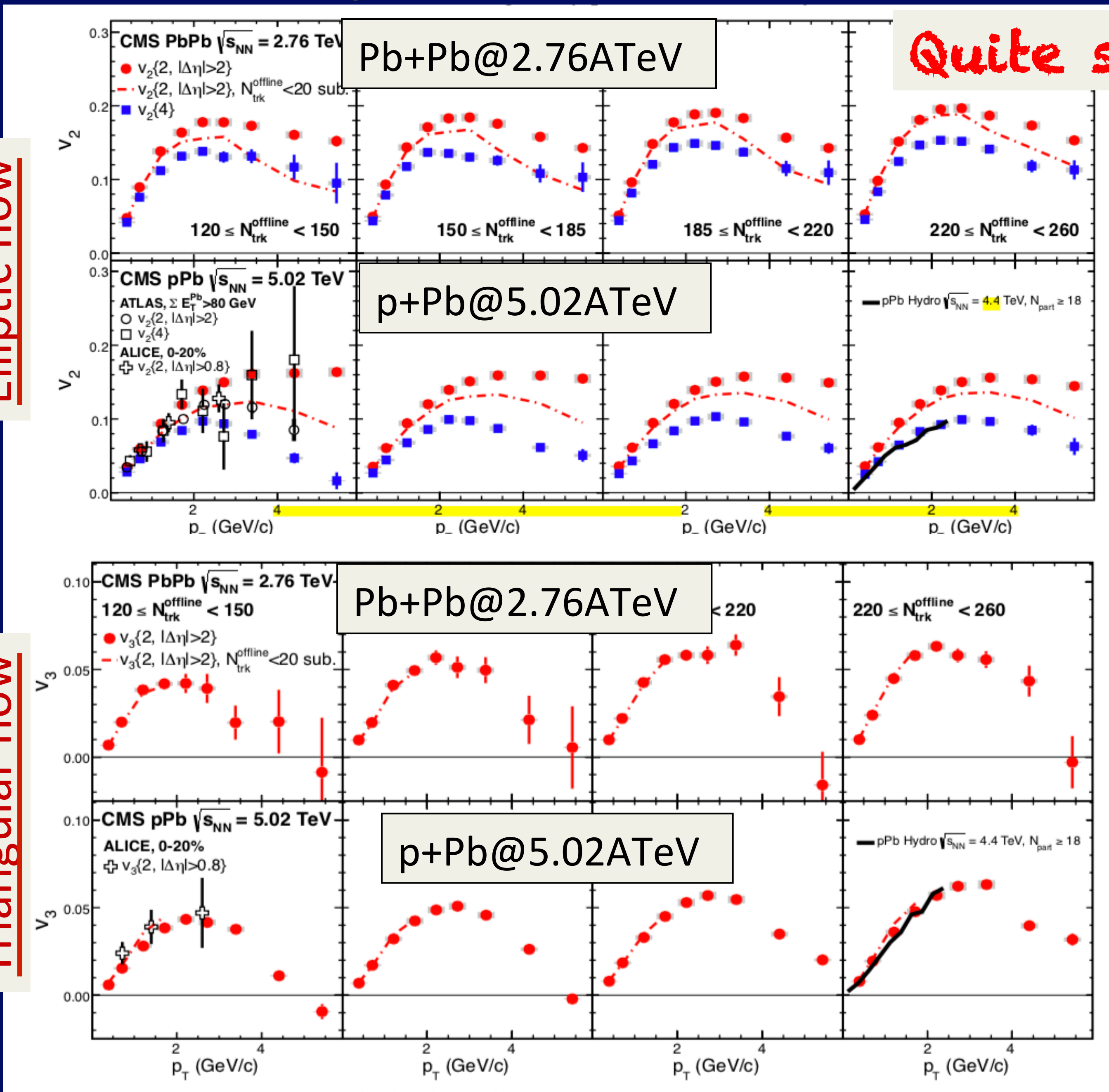
M. Ruggieri, L. Oliva, VG, PRD97(2018)

Is pA the baseline for AA?

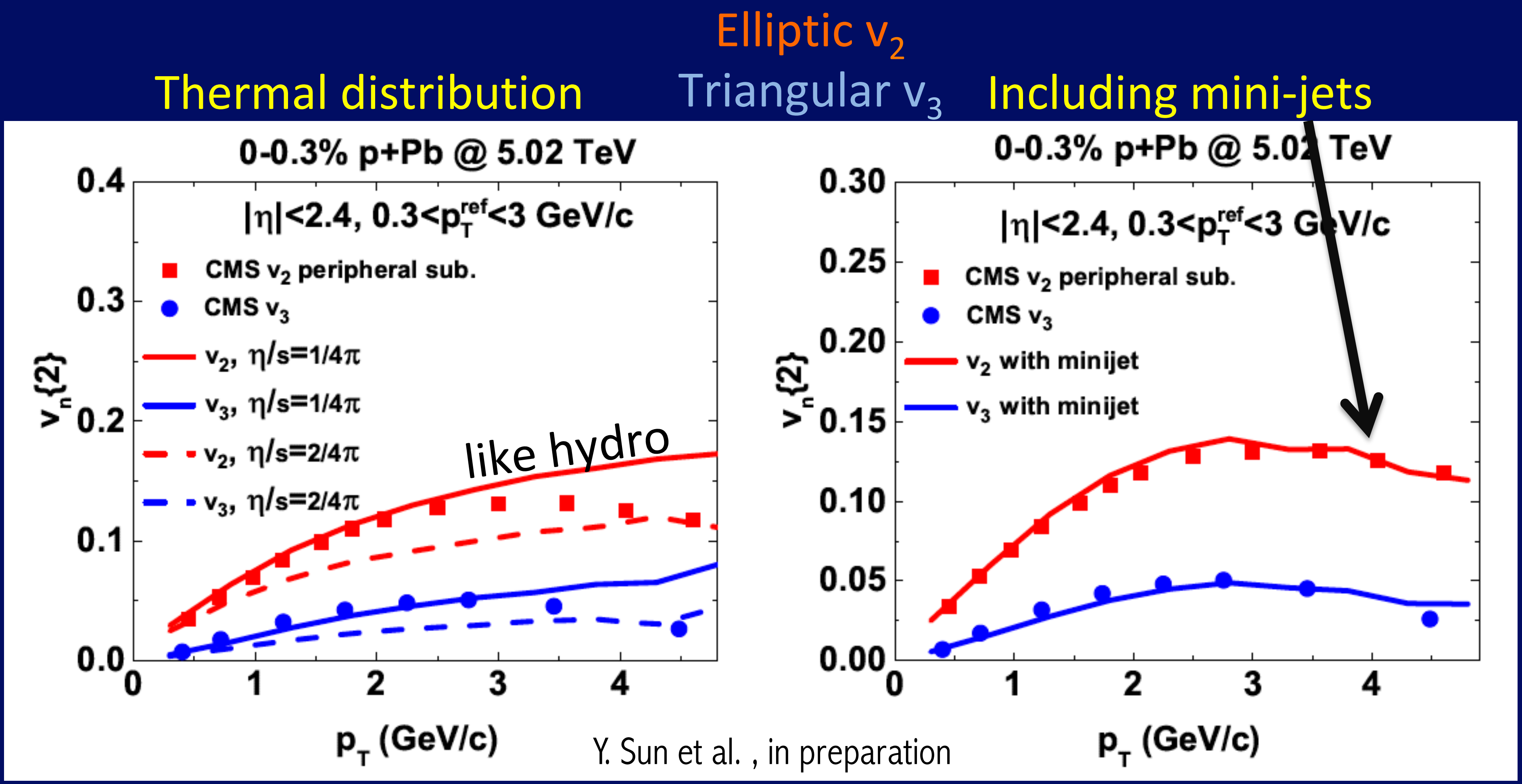
Quite surprisingly!

Elliptic flow

Triangular flow



Preliminary Results for pA with parton transport



However results with different initial state fluctuation w.r.t. AA

And comparing partons with charged hadrons

Work to be done and further physics to be included...

Challenges and future directions:

- Pre-equilibrium from Yang-Mills field dynamics
  [→ Color dynamics (Wong's Equation)]
- Extension to pA collisions → AA and pA unified description
- Hadronization: statistical model vs coalescence (+ fragm.)
- Understanding relevance of freeze-out (depends on previous point)
- Contribute to develop 3+1D anisotropic viscous hydrodynamics

Drawbacks of transport w.r.t. Hydrodynamic

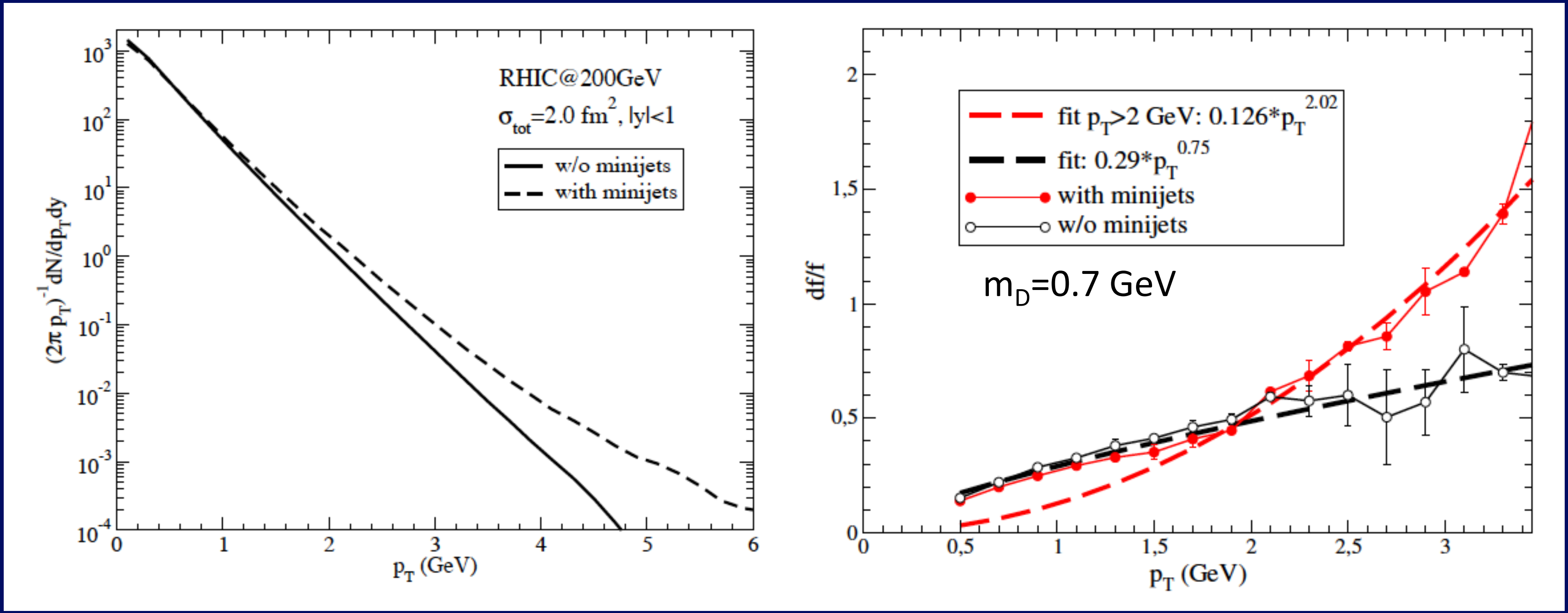
$p_T \approx 3T$

Hydro $\leftarrow$ Transport

$\eta/s \ll 1$

- Fixed relation between τ <--> η/s, but…
- Bulk viscosity not completely indipendent,
- Computational time: (Bayesan analysis)
- ….

Viscous correction: Impact of minijets

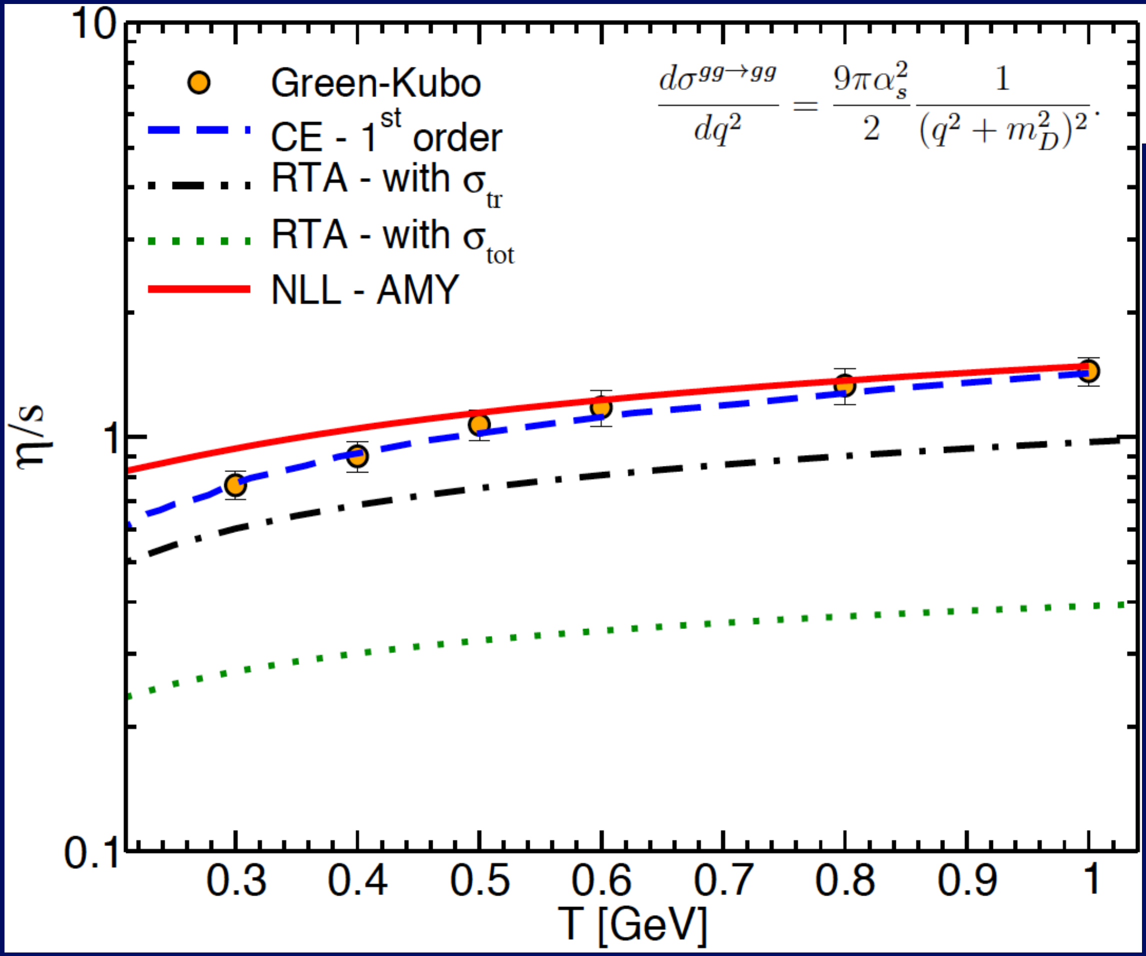


The Grad'slike correction comes from minijets

not included in a hydro approach

Viscosity of a pQCD gluon plasma

Agreement with AMY, JHEP 0305 (2003) 051



$$\frac{d\sigma^{gg\to gg}}{dq^2} = \frac{9\pi\alpha_s^2}{2}\frac{1}{(q^2+m_D^2)^2}.$$

$$\alpha_s(T) = \frac{4\pi}{11\ln\left(\frac{2\pi T}{\Lambda}\right)^2}$$

$$m_D = T\sqrt{4\pi\alpha_s}$$

Relaxation Time Approximation

$$\eta_{RTA}/s = \frac{1}{15} < p > \tau_{tr} = \frac{1}{15}\frac{< p >}{\sigma_{tr}\rho}$$

Chapmann-Enskog (CE)

$$\eta/s = \frac{1}{15}\langle p\rangle\,\tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{g(a)\sigma_{tot}\rho}$$

$g(a=m_D/2T)$ correct function that fix the momentum transfer for shear motion

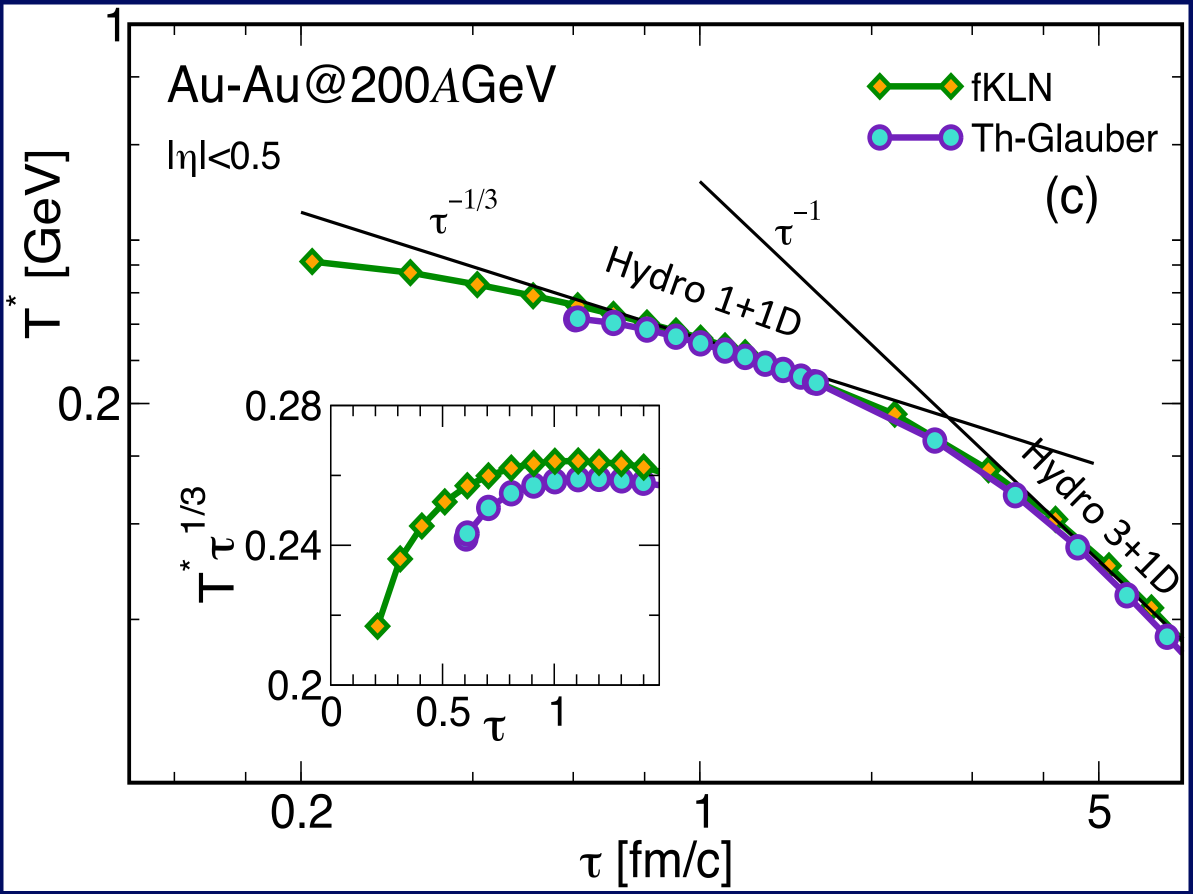
$0 < g(m_D/2T) < 2/3$

∞ forward peaked

Isotropic $m_D \to \infty$

close to AMY result JHEP(2003),
but there is a significant simplification:
only direct u & t channels
with simplified HTL propagator

Temperature evolution



$T \propto \tau^{-\delta}$

$\delta = P_L/\varepsilon$ – 1D boost invariance

$\delta = 1/3$ - 1D ideal expansion

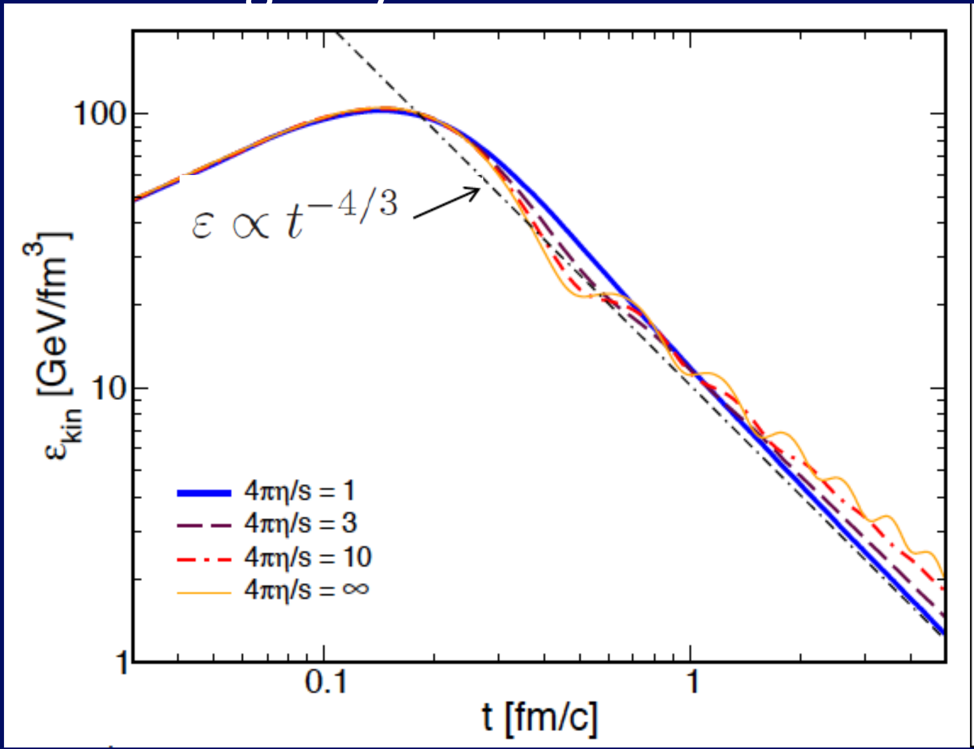
$\delta = 1$ - 3D expansion

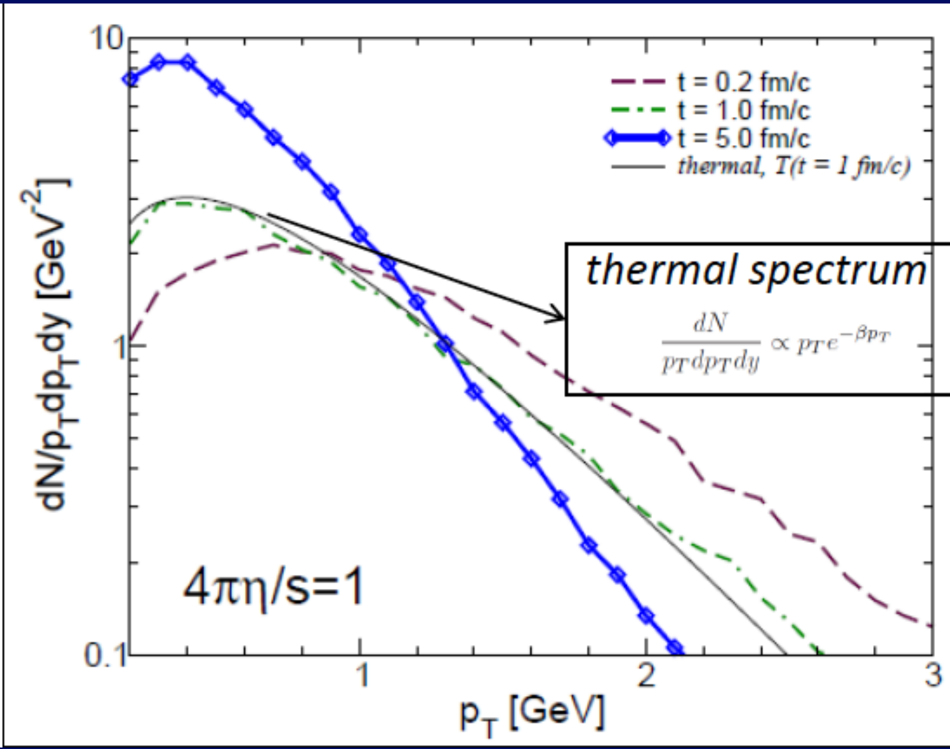
$\tau_{therm} \approx 0.8$ fm/c

$T^* = E/N$, in the local rest frame

Energy Density and p_T- spectra evolution

No divergency at $t \to 0$





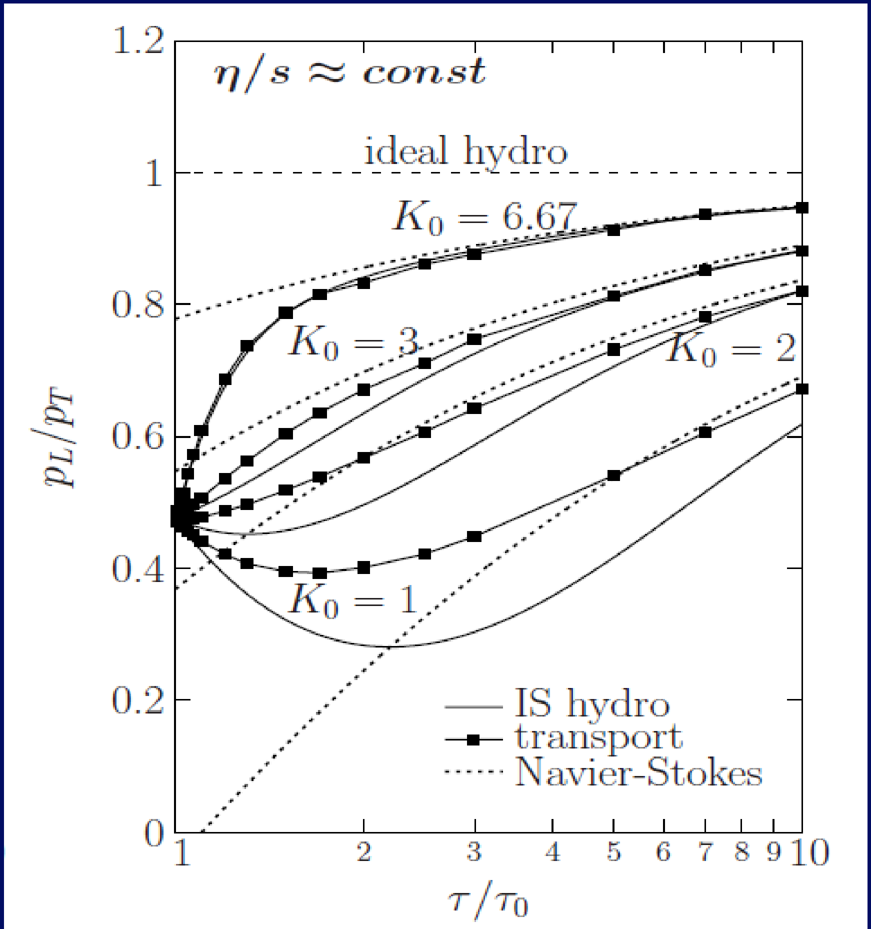
M. Ruggieri et al., PRC92(2015)

Does and when Boltzmann transport
at fixed shear viscosity
gives hydrodynamics?

Transport at fixed η/s vs Viscous Hydro in 1+1D

Comparison for the relaxation of pressure anisotropy P_L/P_T

Huovinen and Molnar, PRC79(2009)



Knudsen number^{-1}

$$K = \frac{L}{\lambda} \rightarrow \frac{\tau}{\lambda}$$

Large K small η/s

$$K_0 = \frac{1}{5}\frac{T_0\tau_0}{\eta/s}$$

$$\frac{\eta}{s} = \frac{1}{5} T \cdot \lambda$$

K increase with $(\tau/\tau_0)^{2/3}$

In the limit of small η/s (<0.16) transport converge to viscous hydro at least for the evolution P_L/P_T

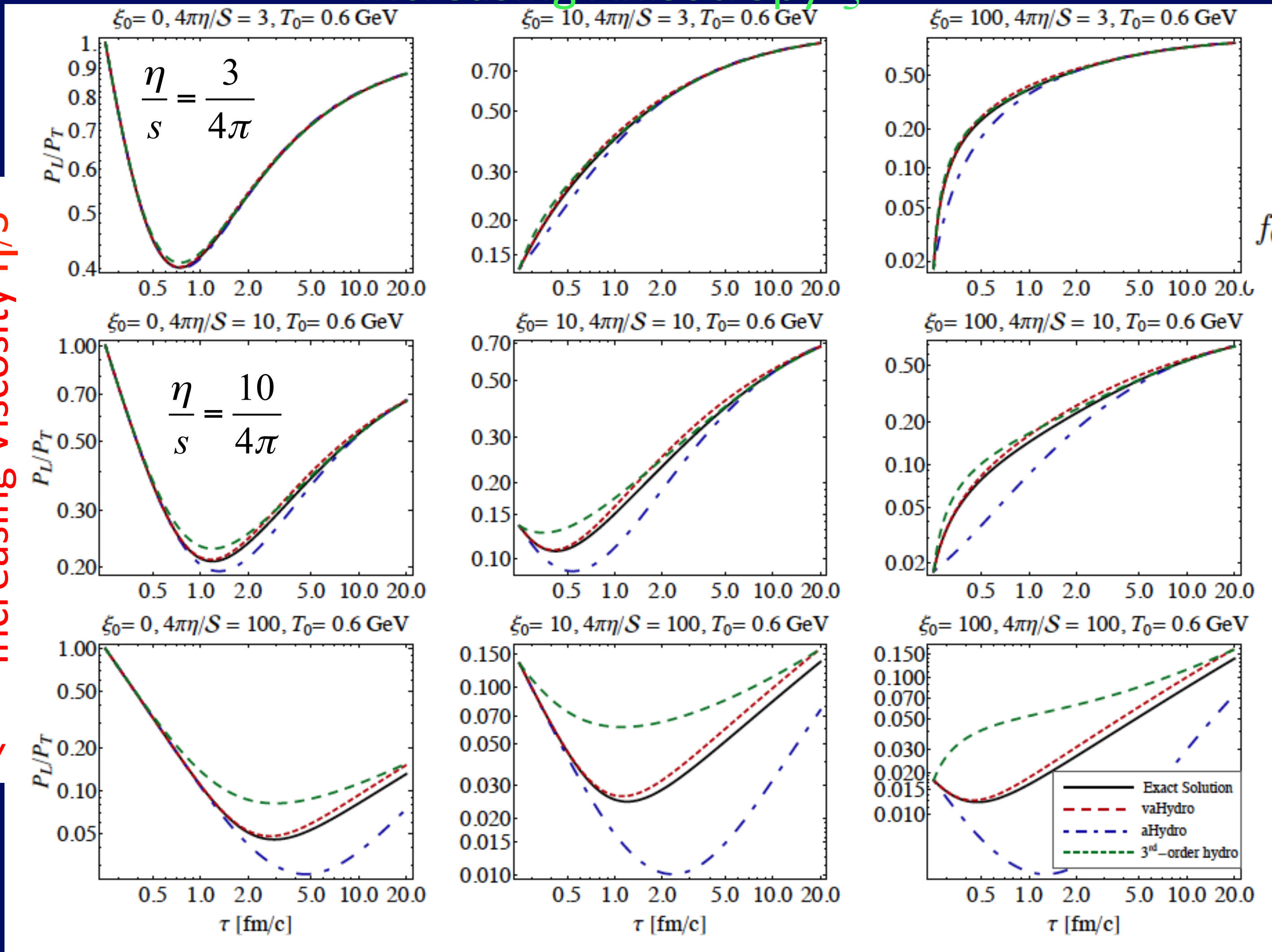
Denicol et al. have studied derivation of viscous hydro from Boltzmann kinetic theory: PRD85 (2012) 114047

Test of vaHydro in 0+1 D –Heinz, Strickland

Use Boltzmann at fixed η/s in 1+1D to improve viscous hydro – U. Heinz (HP2015)

Increasing Anisotropy ξ -->

<--- Increasing Viscosity η/S



$$f_0\left(\frac{\sqrt{\mathrm{p}_\perp^2+(1+\xi)p_z^2+m^2}}{\Lambda};\frac{\mu}{\Lambda}\right)$$

long. anisotropy param.

"Exact" Solution means Boltzmann Eq.

Bazow, Strickland, Heinz: arXiv:1311.6720

in 1+1D: Denicol et al., PRL(2014)

Hydrodynamics for strongly anisotropic expansion:

Account for large viscous flows by including their effect already at leading order in the Chapman-Enskog expansion:

Expand the solution $f(x,p)$ of the Boltzmann equation as

$$f(x,p) = f_0(x,p) + \delta f(x,p) \qquad \Big(\big|\delta f/f_0\big| \ll 1\Big),$$

$$f_0(x,p) = f_0\left(\frac{\sqrt{p_\mu \Omega^{\mu\nu}(x) p_\nu} - \tilde{\mu}(x)}{\tilde{T}(x)}\right),$$

where $\quad p_\mu \Omega^{\mu\nu}(x) p_\nu = m^2 + (1{+}\xi_\perp(x))p^2_{\perp,\mathrm{LRF}} + (1{+}\xi_L(x))p^2_{z,\mathrm{LRF}}$

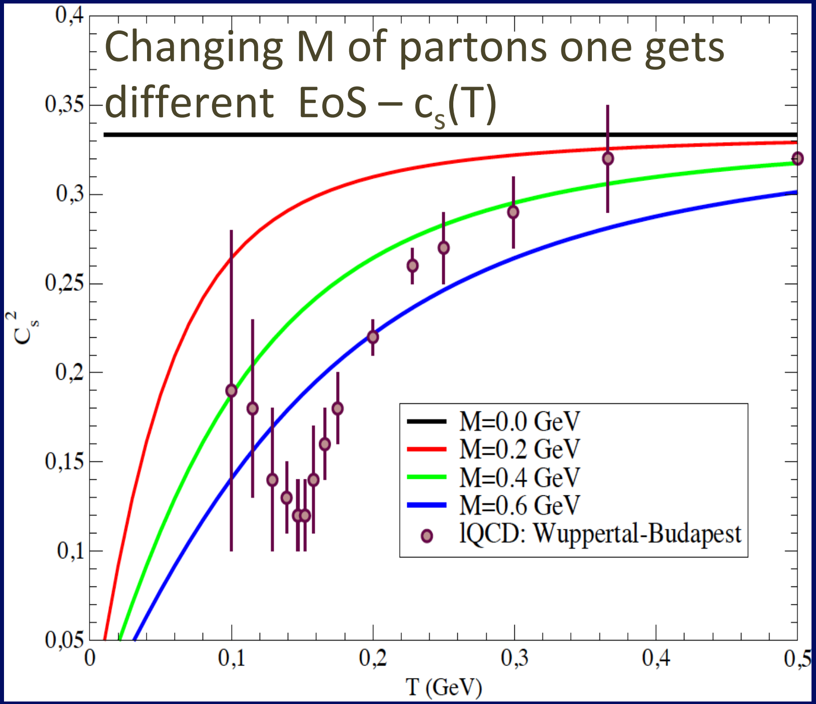
- $\tilde{T}(x)$, $\tilde{\mu}(x)$ are the effective temperature and chemical potential in the LRF, Landau matched to energy and particle density, e and n.
- $\xi_{\perp,L}$ parametrize the momentum anisotropy in the LRF, Landau matched to the transverse and longitudinal pressures, $P_\perp$ and P_L. (McNelis, Bazow, UH, arXiv:1803.01810)
- $P_\perp$ and P_L encode the bulk viscous pressure, $\Pi = (2P_\perp{+}P_L)/3 - P_{\mathrm{eq}}$, and the largest shear stress component, $P_L{-}P_\perp$.

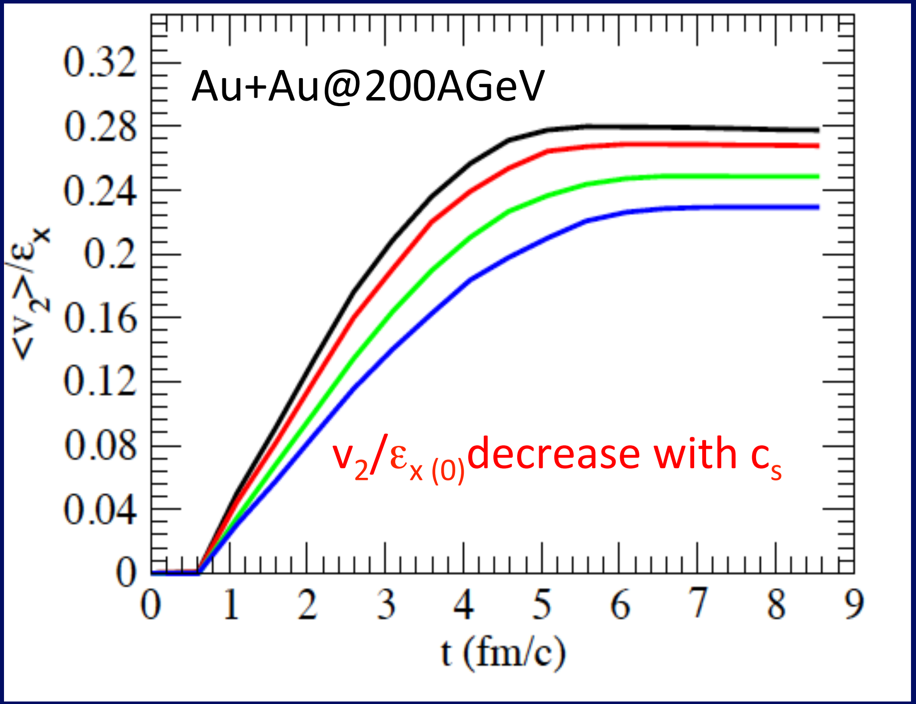
A variety of hydrodynamic approximations:

Different hydrodynamic approaches can be characterized by the different assumptions they make about the dissipative corrections and/or the different approximations they use to derive their dynamics from the underlying Boltzmann equation:

- **Ideal hydro:** local momentum isotropy ($\xi_{\perp,L} = 0$), $\Pi^{\mu\nu} = V^\mu = 0$.
- **Navier-Stokes (NS) theory:** local momentum isotropy ($\xi_{\perp,L} = 0$), ignores microscopic relaxation time by postulating instantaneous constituent relations for $\Pi^{\mu\nu}$, V^μ.
- **Israel-Stewart (IS) theory:** local momentum isotropy ($\xi_{\perp,L} = 0$), evolves $\Pi^{\mu\nu}$, V^μ dynamically, keeping only terms linear in $\mathrm{Kn} = \lambda_{\mathrm{mfp}}/\lambda_{\mathrm{macro}}$
- **Denicol-Niemi-Molnar-Rischke (DNMR) theory:** improved **IS theory** that keeps nonlinear terms up to order Kn^2, $\mathrm{Kn} \cdot \mathrm{Re}^{-1}$ when evolving $\Pi^{\mu\nu}$, V^μ.
- **Third-order Chapman-Enskog expansion (Jaiswal 2013):** local momentum isotropy ($\xi_{\perp,L} = 0$), keeping terms up to third order when evolving $\Pi^{\mu\nu}$, V^μ.
- **Anisotropic hydrodynamics (aHydro):** allows for leading-order local momentum anisotropy ($\xi_{\perp,L} \neq 0$), evolved according to equations obtained from low-order moments of BE, but ignores residual dissipative flows: $\Pi^{\mu\nu} = V^\mu = 0$.
- **Viscous anisotropic hydrodynamics (vaHydro):** improved **aHydro** that additionally evolves residual dissipative flows $\Pi^{\mu\nu}$, V^μ with **IS** or **DNMR theory**.

Transport at fixed η/s vs Viscous Hydro a test in 3+1D





- Time scales, trends and value quite similar to hydro evolution
- An exact comparison under the same conditions has not been done

σ_{tot}=15 mb

Initial Conditions

✧ r-space: standard Glauber model

✧ p-space: Boltzmann-Juttner T_{max}=1.7-3.5 T_c [p_T<2 GeV]+ $\underbrace{\text{minijet}}$ [p_T>2-3GeV]

Discarded in viscous

We fix maximum initial T at RHIC 200 AGeV

T_{max0} = 340 MeV

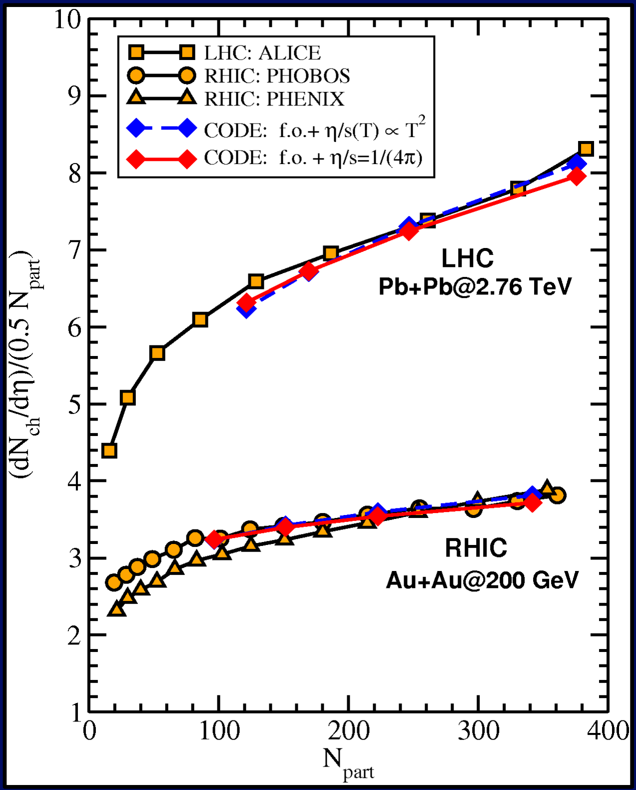
$T_0\,\tau_0$ =1 -> τ_0=0.6 fm/c

Typical hydro condition

Then we scale it according to initial ε

$$\frac{1}{\tau A_T}\frac{dN_{ch}}{d\eta} \propto T^3$$

	62 GeV	200 GeV	2.76 TeV
T_0	290 MeV	340 MeV	580 MeV
τ_0	0.7 fm/c	0.6 fm/c	0.3 fm/c

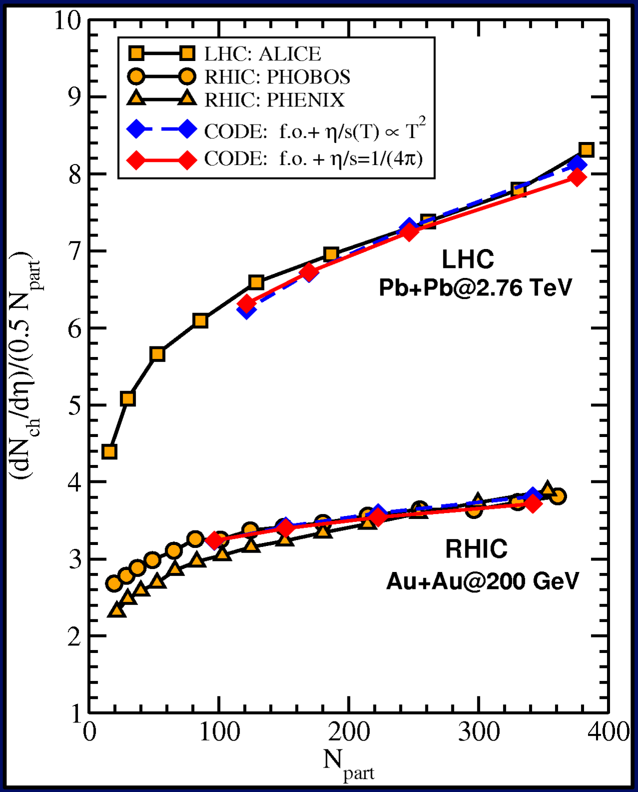


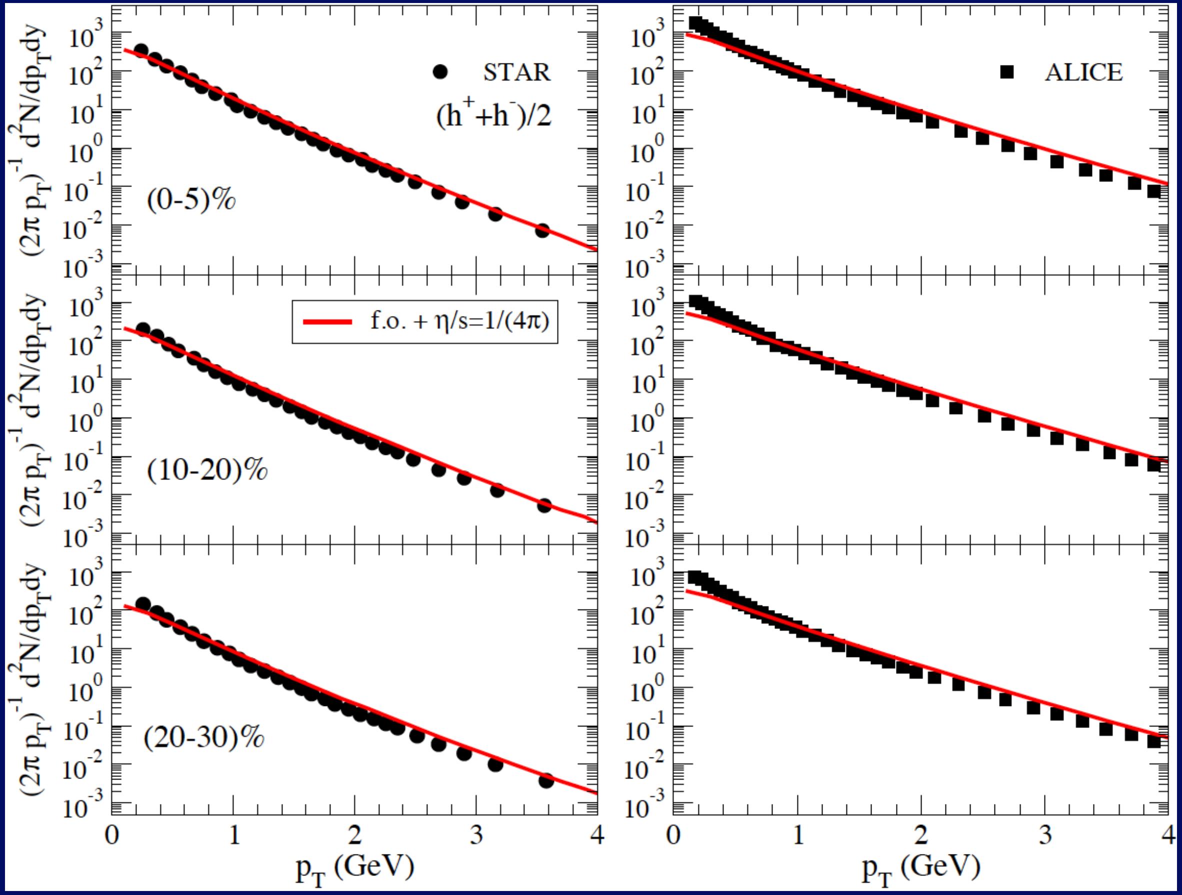
Multiplicity & Spectra

✧ r-space: standard Glauber condition

✧ p-space: Boltzmann-Juttner T_{max}=2(3) T_c [p_T<2 GeV]+ minijet [p_T>2-3GeV]

No fine tuning





Simulate a fixed shear viscosity

Usually input of a transport approach are *cross-sections and fields*, but here we reverse it and start from η/s with aim of creating a more direct link to viscous hydrodynamics

Chapmann-Enskog

$$\frac{\eta}{s} = \frac{1}{15}\langle p\rangle \cdot \tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{g(\frac{m_D}{T})\sigma_{TOT}\rho}$$

$$g(a) = \frac{1}{50}\int dy y^6 \left[(y^2+\frac{1}{3})K_3(2y) - yK_2(2y)\right] h\left(\frac{a^2}{y^2}\right)$$

$g(a=m_D/2T)$ correct function that fix the relaxation time for the shear motion

Transport code

$$\sigma_{tot}(n(\vec{r}),T) = \frac{1}{15}\frac{\langle p_\alpha\rangle}{g(a)n_\alpha}\frac{1}{\eta/s}$$

Space-Time dependent cross section evaluated locally

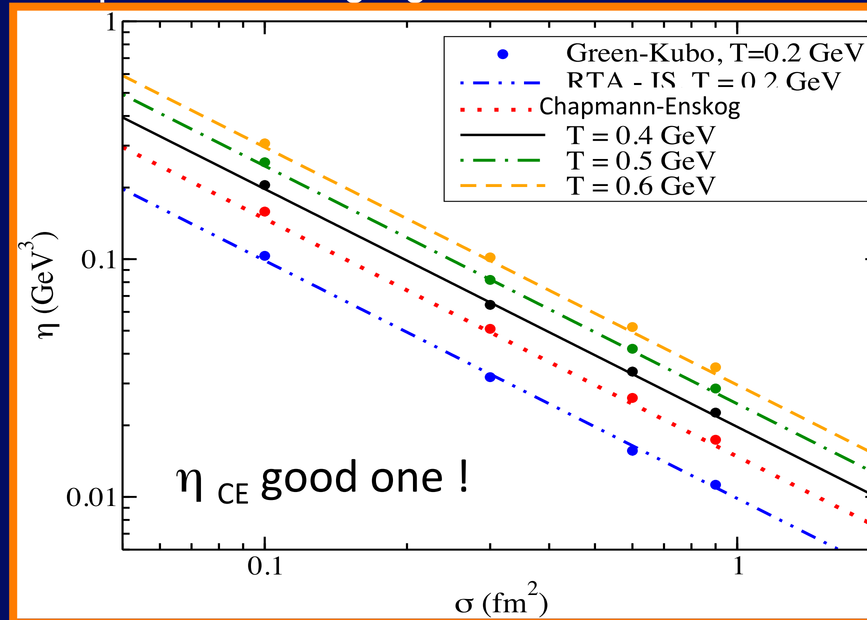
M. Ruggieri et al., PLB727 (2013), PRC89(2014)

$0 < g(m_D/2T) < 2/3$

forward peaked

Isotropic $m_D \to \infty$

Chapman-Enskog agrees with Green-Kubo



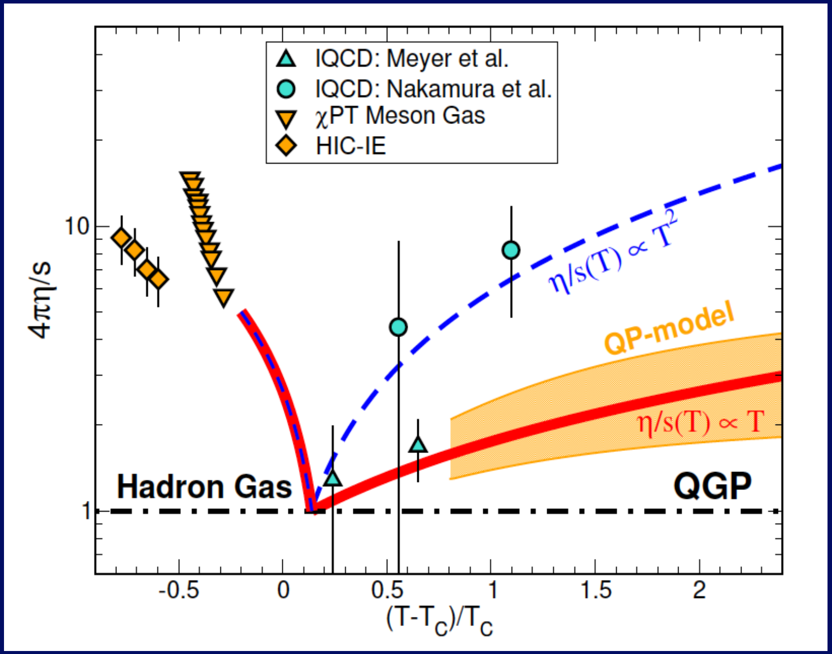
Green-Kubo Relation

$$\eta = \frac{1}{T}\int_0^\infty dt \int_V d^3x \left\langle \Pi^{xy}(\vec{x},t)\,\Pi^{xy}(0,0)\right\rangle$$

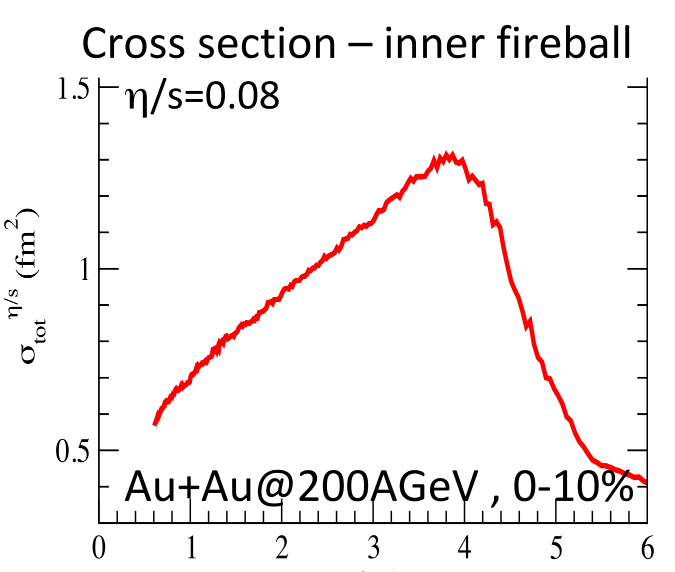
S.Plumari et al., PRC86(2012)

Cross section and freeze-out

Freeze-out is a smooth process: scattering rate < expansion rate



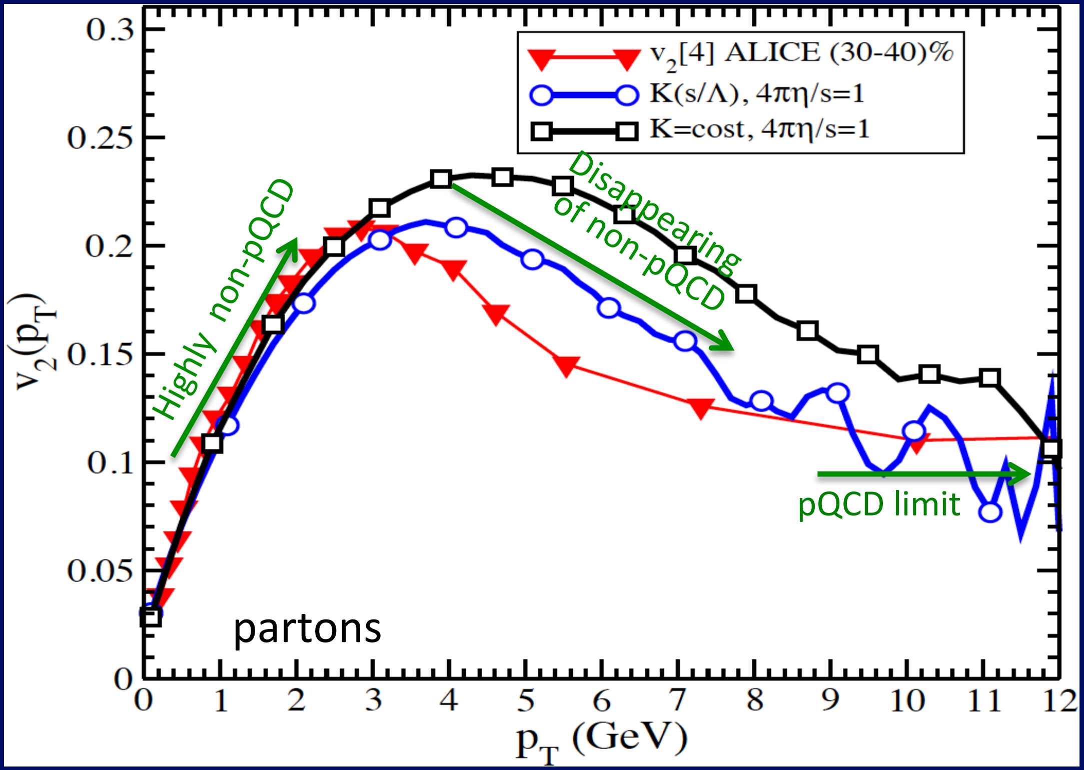
- ✓ η/s increases in the cross-over region, realizing a smooth f.o. self-consistently dependent on h/s:
- ✓ Different from hydro that is a sudden cut of expansion at some $T_{f.o.}$.
- ✓ By definition freeze-out ≠ Hydro



$$\sigma^* = g(a)\sigma_{tot} \approx \frac{1}{15}\frac{\bar{p}}{\rho}\frac{1}{\eta/s}$$

$\rho(\tau_0)$=23 fm^{-3}, η/s=0.08 → σ_{ToT}= 6 mb

Natural extension from low to high p_T



α_s=0.3 and m_D=0.7 GeV

Renormalize σ to fix η/s

$$\sigma^*(s) = K\,\sigma_{pQCD}(s)$$

$$K(s) = 1 + \gamma\, e^{-s/\Lambda^2}$$

$\Lambda \approx$ 4-5 GeV, γ fixed by η/s

pQCD limit, but with α_s=0.3
- No Q^2 dependence &
- No radiative part

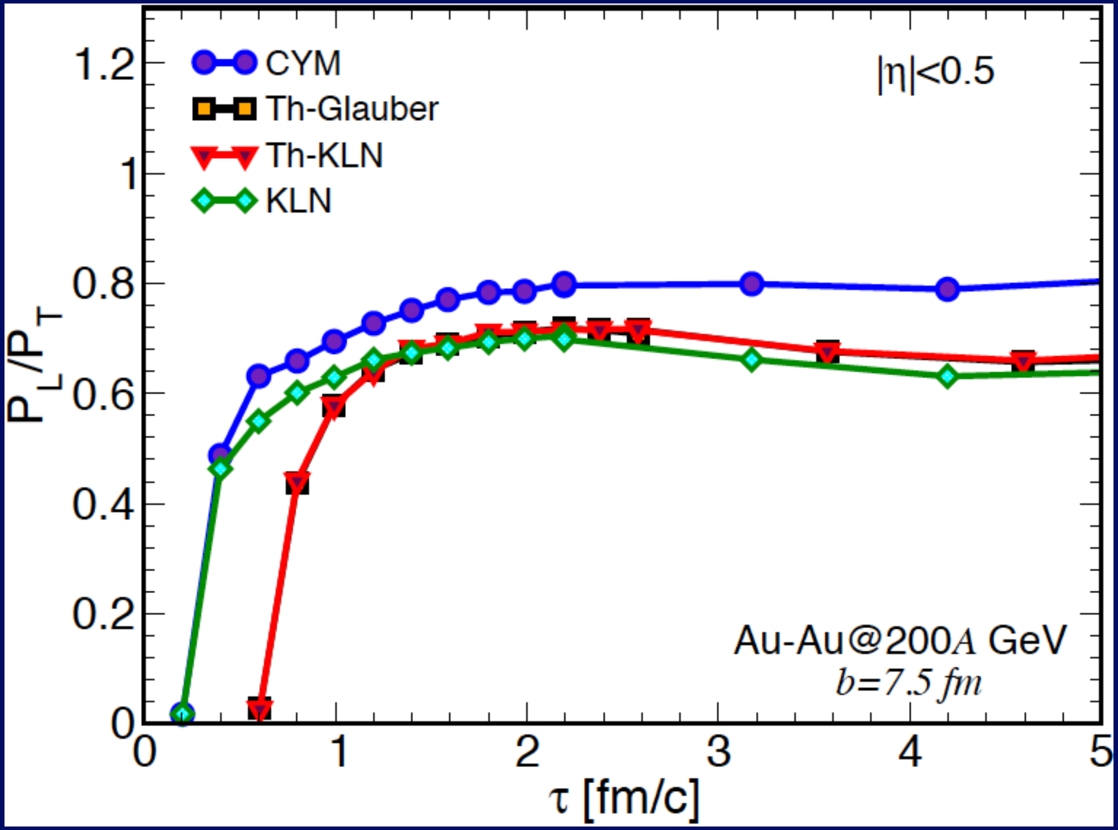
Boltzmann transport describes rise and fall of $v_2(p_T)$

Transition between low and high p_T in a unified framework!

No Fine tuning! Employed the relaxation time approximation!

S. Plumari and VG, EPIC@LHC, AIP1422(2012)- arXiV:1110.4138 [hep-ph]

Longitudinal and transverse pressure



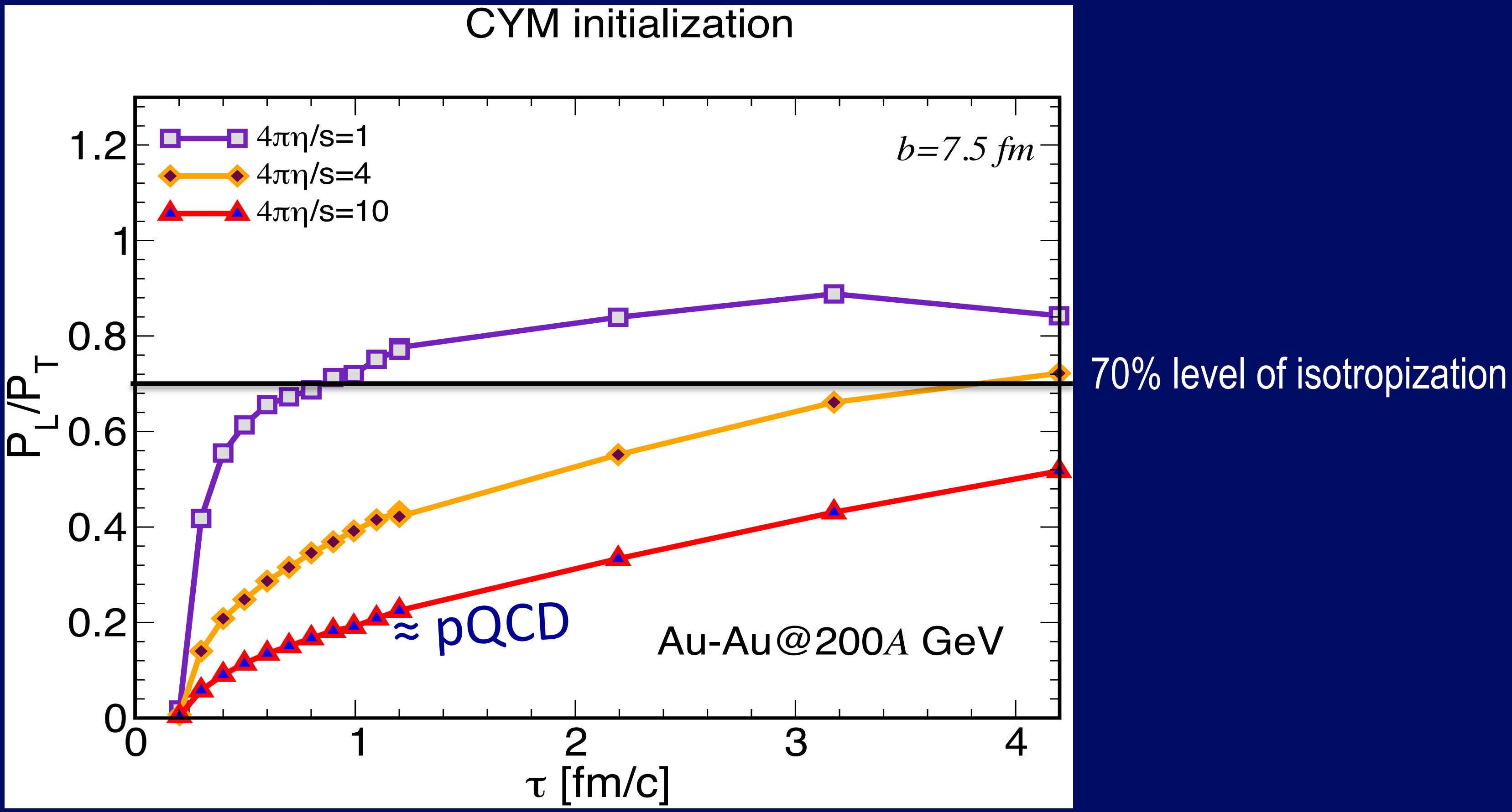
t=1/Qs≈0.1-0.2 fm/c
-> $P_L/P_T > 0$
Gelis & Epelbaum
arXiV:1307.2214

CYM (IP-Glasma)
Courtesy of B. Schenke
& R. Venugopalan

✧ P_L/P_T show also a very fast equilibration ($\Delta\tau_{isotr}$≈0.5 fm/c)!

✧ However it is not this that makes a difference for v_2:
isotropization time quite similar for all the cases

Longitudinal and transverse pressure



- ✧ For η/s > 0.3 one misses fast isotropization in P_L/P_T (τ ≥ 2-3 fm/c)
- ✧ For η/s ≈ pQCD no isotropization
- ✧ Semi-quantitative agreement with Florkowski et al., PRD88 (2013) 034028

our is 3+1D not in relax.time but full integral but *no gauge field*

Stochastic approach

$$p_\mu \partial^\mu f = C^{2\leftrightarrow 2} + \ldots$$

Solved discretizing the space in $(\eta, x, y)_\alpha$ cells

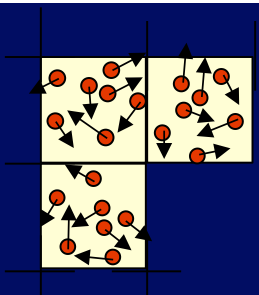
$$\frac{\Delta N_{coll}^{2\to2}}{\Delta t \frac{1}{(2\pi)^3}\Delta^3 x \Delta^3 p_1} = \frac{1}{2E_1}\frac{\Delta^3 p_2}{(2\pi)^3 2E_2} f_1 f_2 \times \frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'}|\mathcal{M}_{12\to1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p_1'-p_2').$$

Rate of collisions per unit of phase space

$$f_i = \frac{\Delta N_i}{\frac{1}{(2\pi)^3}\Delta^3 x\, \Delta^3 p_i},$$

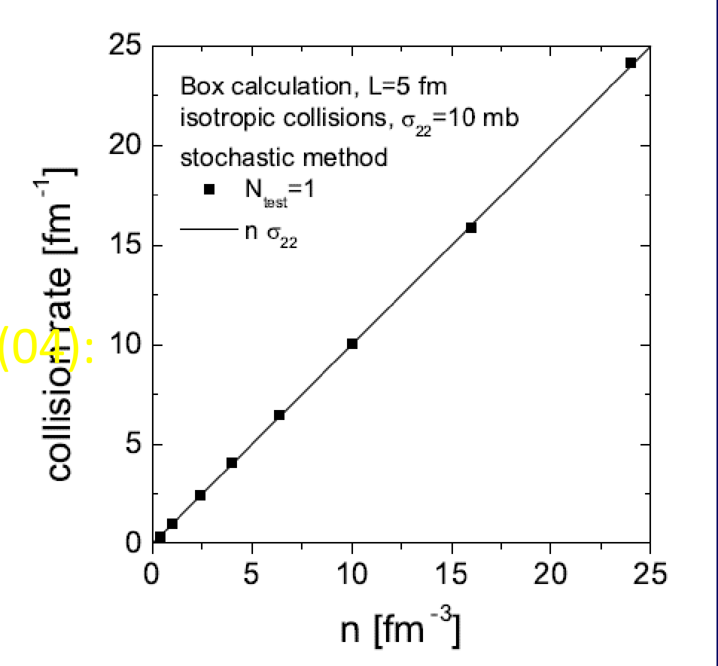
$$P_{22} = \frac{\Delta N_{\rm coll}^{2\to2}}{\Delta N_1 \Delta N_2} = v_{\rm rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

$$R = n < v_{rel}\sigma_{22} >$$



$\Delta t \to 0$

$\Delta^3 x \to 0$ → exact solution

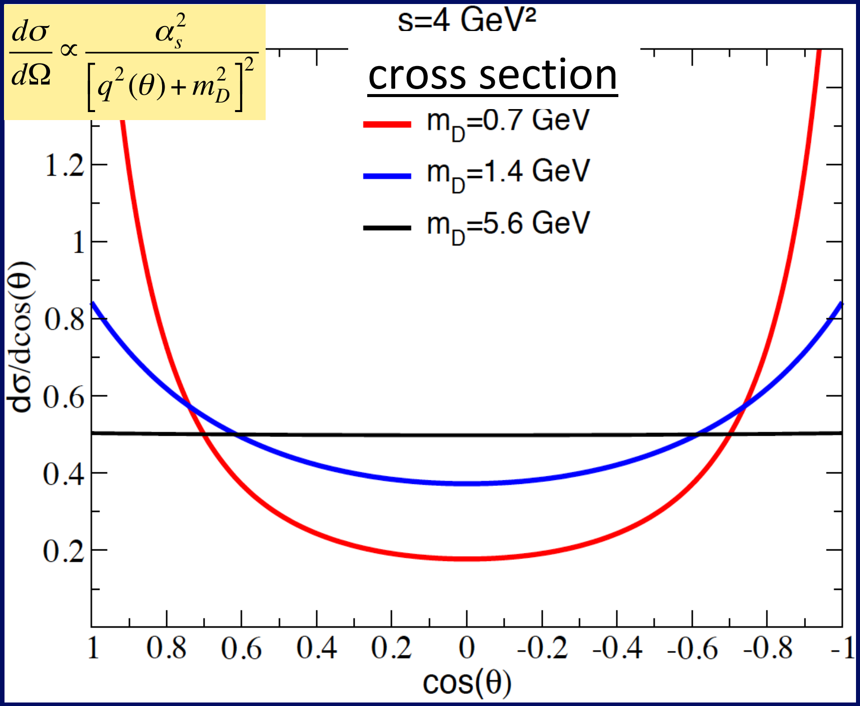


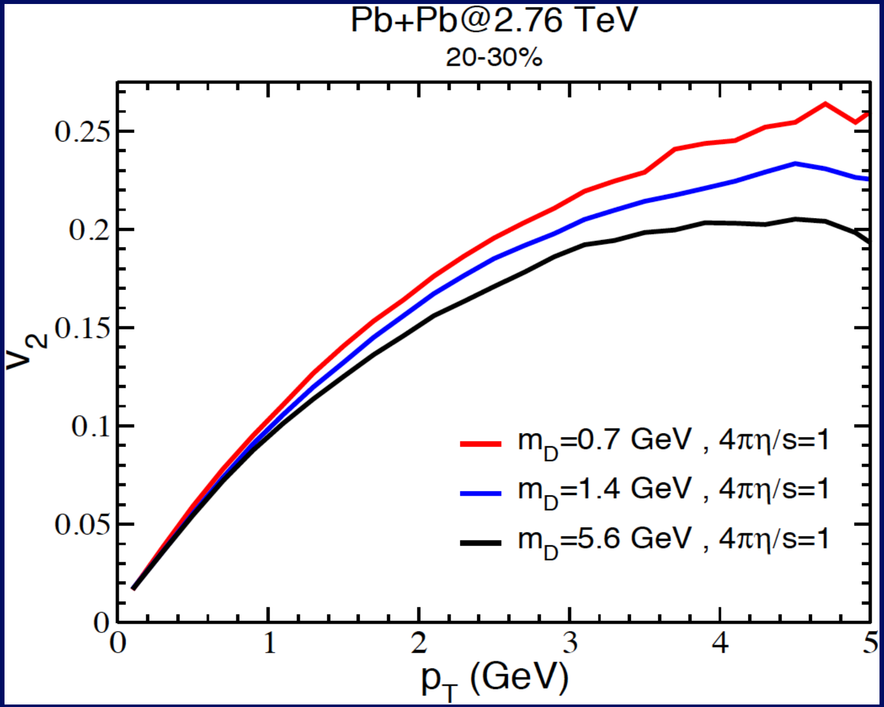
✧ Done all test of convergency in a box re-checking all the test agrees with Z. Xu and C. Greiner PRC71 (04): $\delta\eta, N_{test}, \sigma$, expanding 1D-cylinder

✧ We added a check if convergency vs $N_{test}, \sigma, \delta\eta, \delta x$, of dN/dp_T, $v_2(p_T)$ for the expanding fireball -> convergency needs more N_{test}

η/s or details of the cross section?

$$\frac{d\sigma}{d\Omega} \propto \frac{\alpha_s^2}{\left[q^2(\theta)+m_D^2\right]^2}$$





Keep same η/s means:

$$\frac{\eta}{s} = \frac{1}{15}\langle p\rangle \cdot \tau_\eta$$

$$\tau_\eta^{-1} =$$

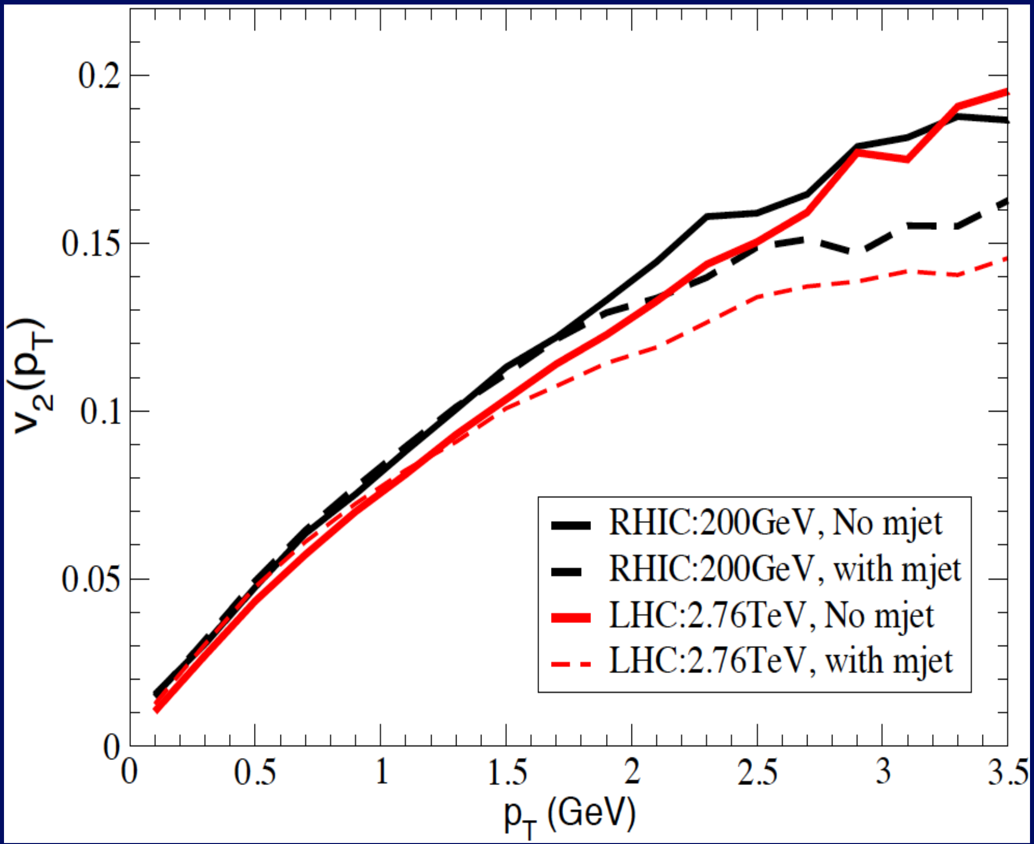
$$\frac{\sigma_{TOT}(m_{1D})}{\sigma_{TOT}(m_{2D})} = \frac{g(m_{2D})}{g(m_{1D})}$$

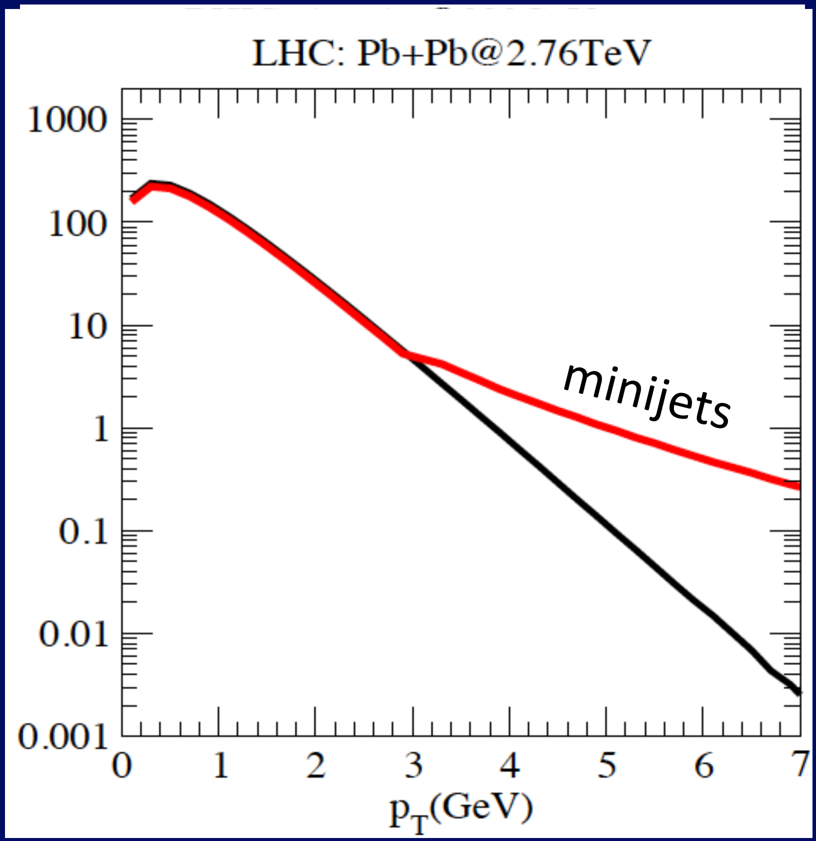
- ✧ η/s is really the physical parameter determining v_2 at least up to 1.5-2 GeV
- ✧ microscopic details become relevant at higher p_T
- ✧ First time η/s<-> v_2 hypothesis is verified!

Differences arises just where in viscous hydro δf becomes relevant

$$\delta f = \frac{\pi^{\mu\nu}}{\varepsilon + P}\frac{p_\mu p_\nu}{T^2} f_{eq}$$

Non equilibrium at larger p_T: impact of minijets on $v_2(p_T)$

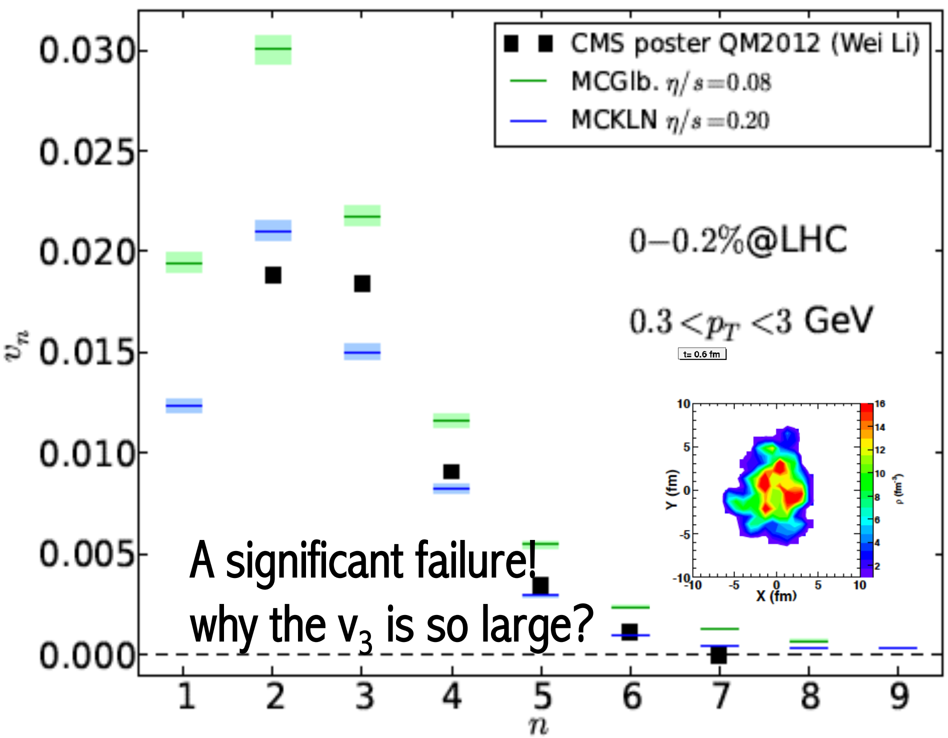


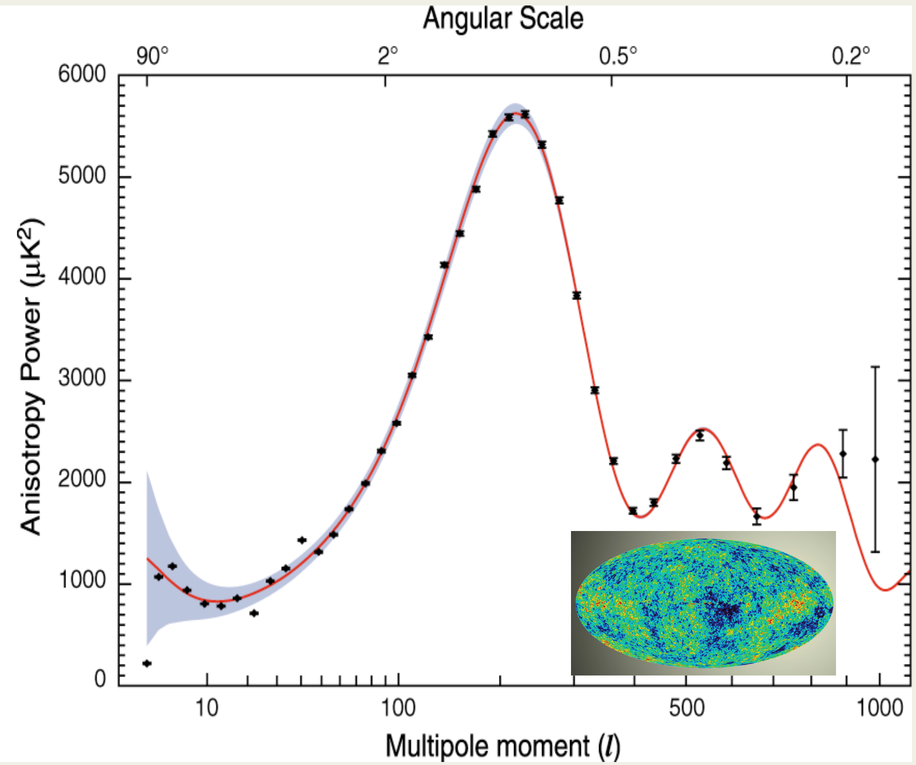


Mini-jets starts to affect $v_2(p_T)$ for p_T>1.5 GeV

Effect non-negligible. Again a flatter spectrum leads to smaller v2

Going deeply into Hot QCD matter





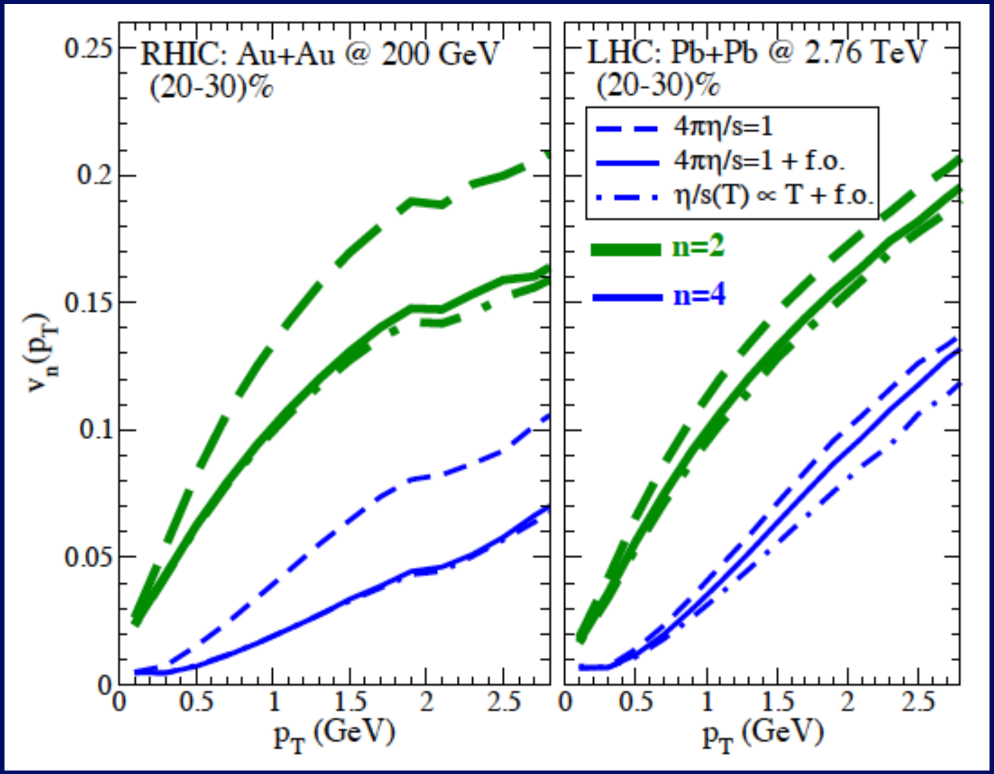
- Initial QCD quantum fluctuations
- T dependence of η/s
- Equation of State
- Freeze-out dynamics

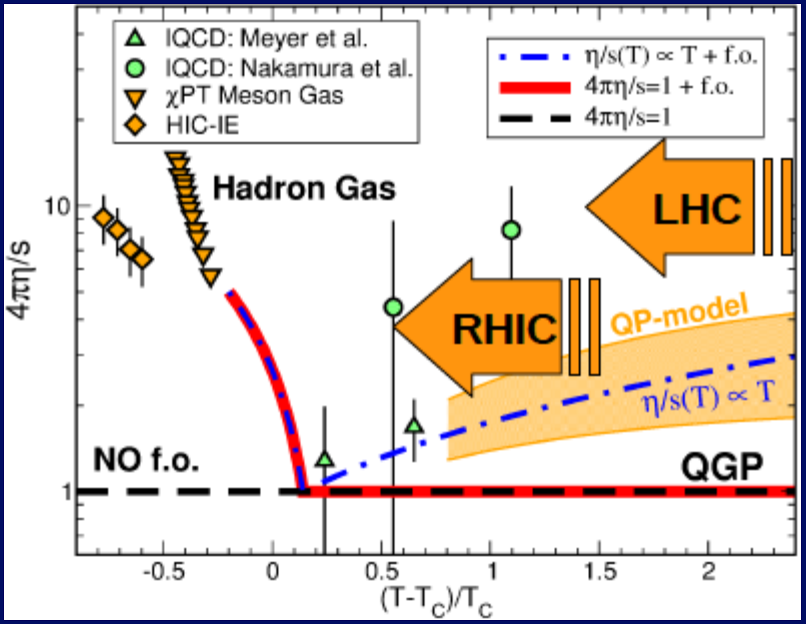
Keeping size and time of QGP (p_T spectra)

- Standard Model Matter
- Cold Dark Matter
- Dark Energy
- Hubble Constant

Keeping Age and Flatness of the Universe

Include Initial State Fluctuations : $v_n(p_T)$ & $\eta/s(T)$





- ✓ $v_{2,3}$ at RHIC affected by freeze-out dynamics
- ✓ $v_{2,3}$ at LHC determined essentially by the QGP η/s

Another sector where Boltzmann transport is playing a role in the QGP physics:

Heavy Flavor

HQ diffusion in the expanding QGP

c,b quarks

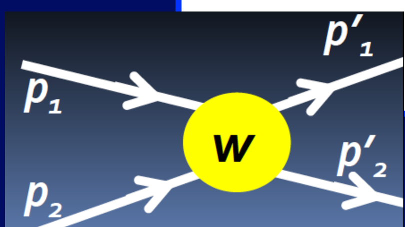
Two main approaches:

1) **Langevin approach** ($T \ll m_q$ soft scattering)
[TAMU, Duke, Nantes, Torino, Catania, …]

2) **Boltzman kinetic transport** (…Kadanoff-Baym-PHSD)
[Catania, Nantes, Frankfurt, LBL, CCNU,…]

Boltzmann (BM)

$$\frac{Df_Q(p)}{Dt} = C_{22} = \frac{1}{2E_p}\int \frac{d^3q}{(2\pi)^3 2E_q}\int \frac{d^3p'}{(2\pi)^3 2E'_p}\frac{d^3q'}{(2\pi)^3 2E'_q}\Big[f'_g(q')f'_Q(p')\left|M_{gQ\to gQ}(p'q'\to pq)\right|^2 - f_g(q)f_Q(p)\left|M_{gQ\to gQ}(pq\to p'q')\right|^2\Big](2\pi)^4\delta^4(p+q-p'-q')$$



Small $q^2 \ll M$, $M \ll gT$
Brownian motion

Langevin/Fokker Planck (LV)

Fluct.-Dissip. Th.
$D = ET\gamma$

$$\frac{\partial f_Q}{\partial t} = \gamma\frac{\partial (pf_Q)}{\partial p} + D\frac{\partial^2 f_Q}{\partial p^2}$$

$\langle p\rangle \approx e^{-\gamma T}$
Drag

$\langle \Delta p^2\rangle$
Diffusion

$$\gamma = \int d^3k\left|M(k,p)\right|^2 p$$

$$D = \frac{1}{2}\int d^3k\left|M(k,p)\right|^2 p^2$$

$|M|^2$ scatt. matrix from some theory

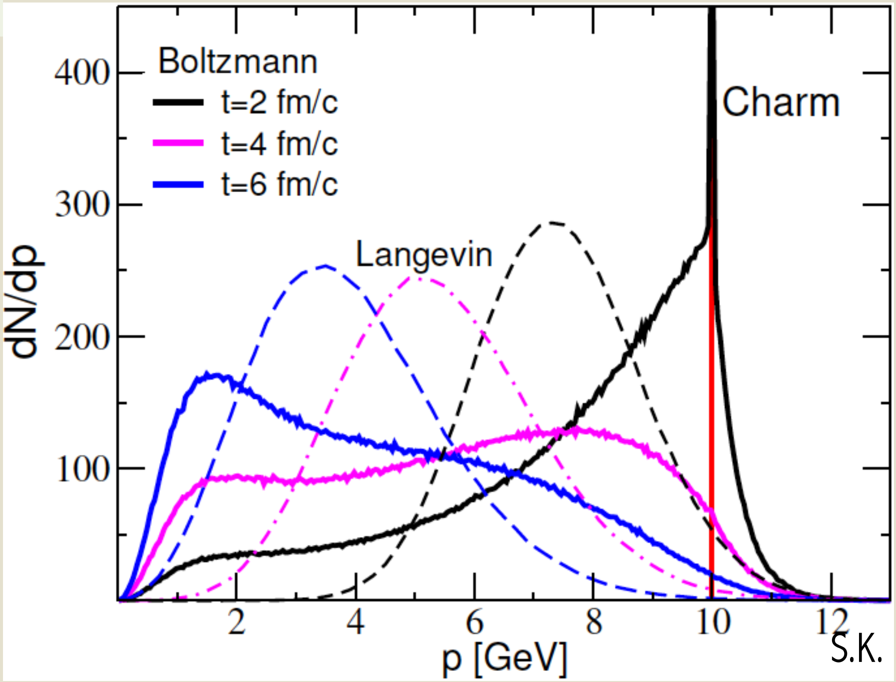
Boltzmann vs Langevin for Heavy Quarks

$$\frac{dN}{d^3p}_{initial} = \delta(p - 10GeV)$$

Brick problem

$m_D = gT = 0.83$ GeV

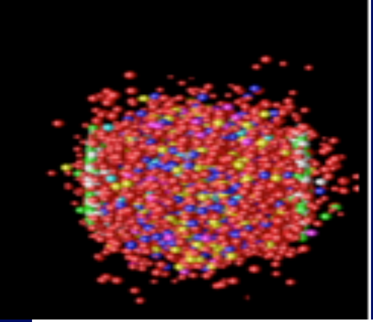
$$\frac{d\sigma}{d\Omega} \propto \frac{1}{\left(q^2(\theta) + m_D^2\right)^2}$$



S.K. Das et al., PRC90 (14) 044901

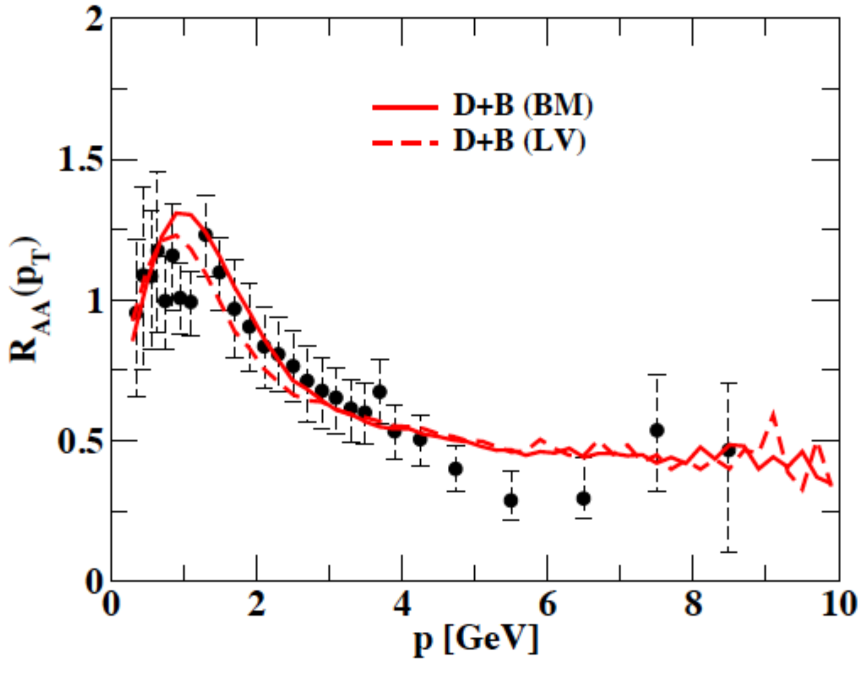
- ✧ Kinematics of collisions (Boltzmann) can throw particles at very low p soon.
- ✧ The motion of single HQ does not appear to be of Brownian type, on the other hand $M_c/T \approx 3$ -> $M_c/\langle p_{bulk}\rangle \approx 1$ & $p \gg m_Q$
- ✧ Evolution of <p> is nearly identical in BM & LV

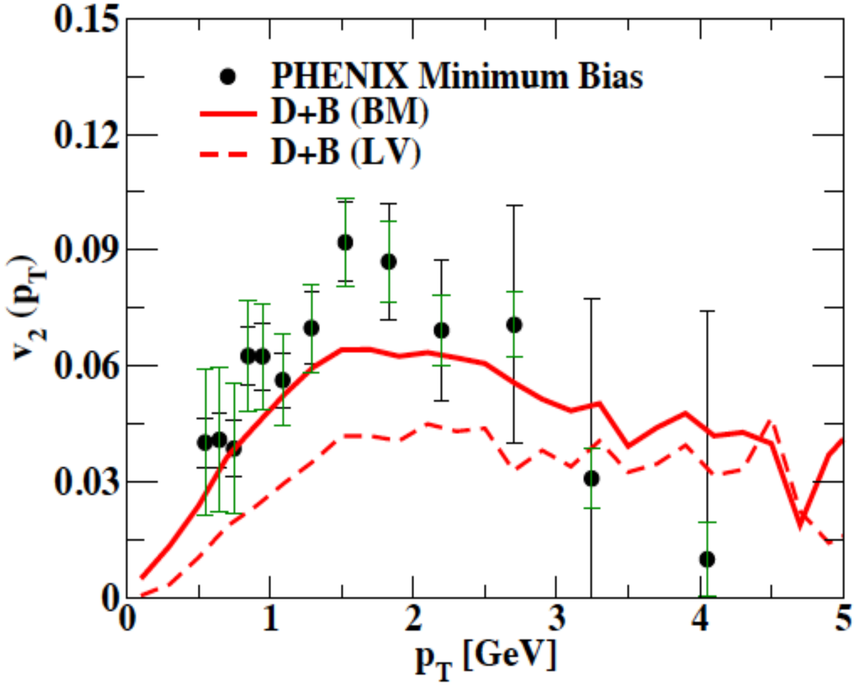
X. Dong & VG, Prog.Part.Nucl.Phys.(2019)



R_{AA} & v_2 Boltzmann vs Langevin

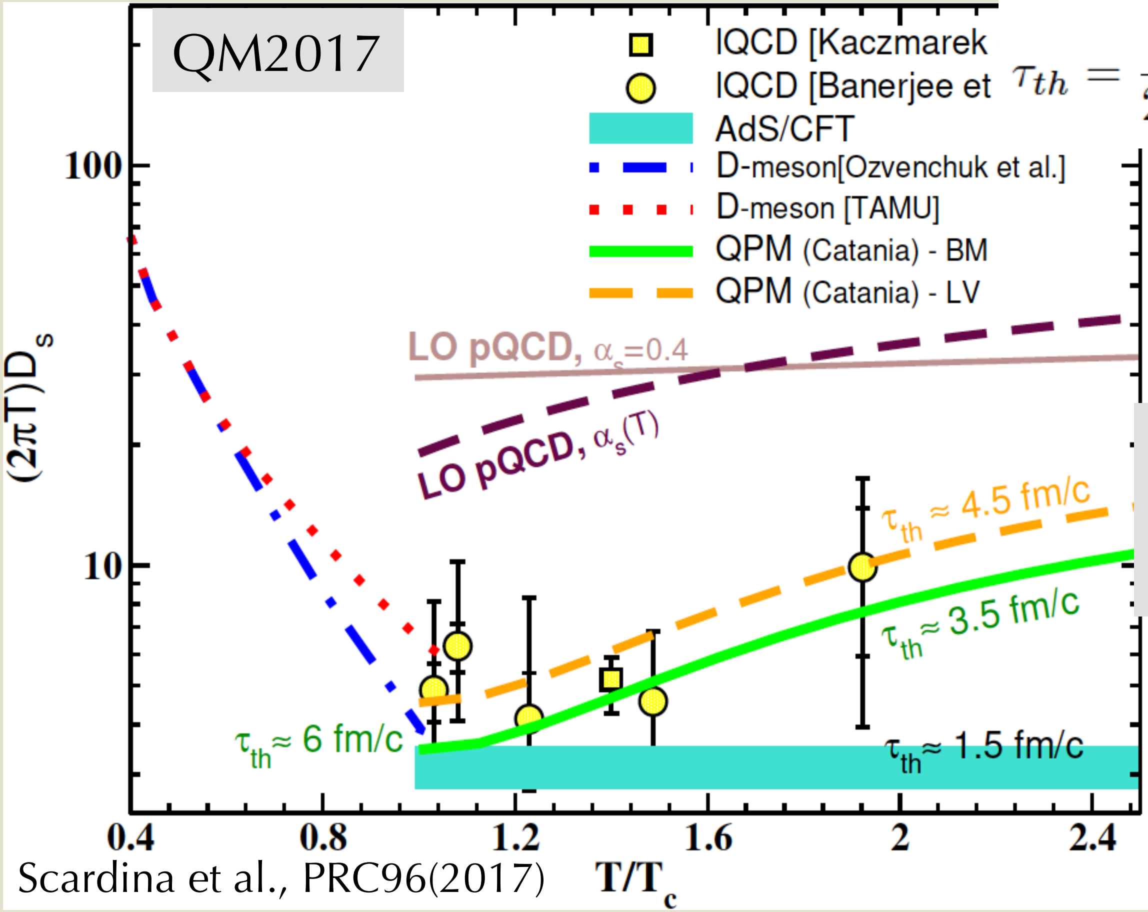
One Preliminary result: Au+Au@200AGeV, b=8 fm





- ✓ Fixed same $R_{AA}(p_T) \rightarrow v_2(p_T)$ about 25% higher
  - dependence on the specfic scattering matrix (isotropic case -> larger effect)
- ✓ This may be the reason of the large v_2 in BAMPS
- ✓ Angular DD correlation? Work under progress

What is the underlying D_s?



Scardina et al., PRC96(2017)

$$\tau_{th} = \frac{M}{2\pi T^2}(2\pi T D_s) \cong 1.8\,\frac{2\pi T D_s}{(T/T_c)^2}\ \text{fm/c}$$

$$D_s = \frac{T}{M\gamma} = \frac{T^2}{D_p}$$

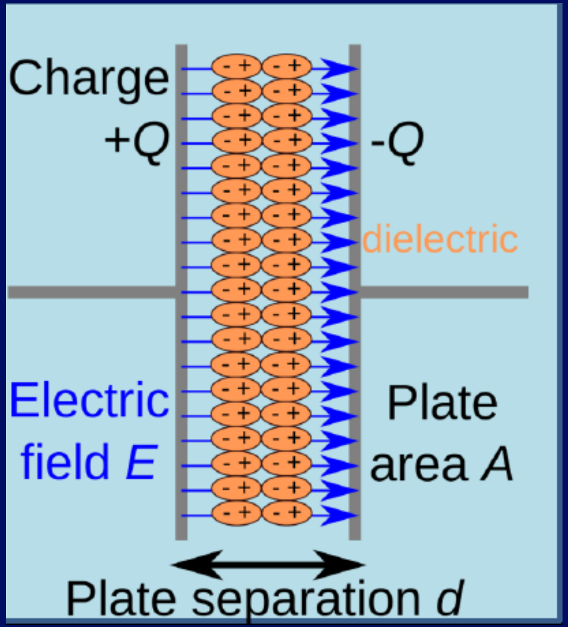
Not a model fit to lQCD data! but the result from the predictions of $R_{AA}(p_T)$ & $v_2(p_T)$

- Other *more differential observables* are *more sensitive* to the difference between BM and LV

This will come after the ALICE upgrade

Schwinger Mechanism in Electrodynamics

Vacuum with and E-field
unstable under pair creation



Quantum Effective Action of a pure electric field, has an imaginary part responsible for field instability

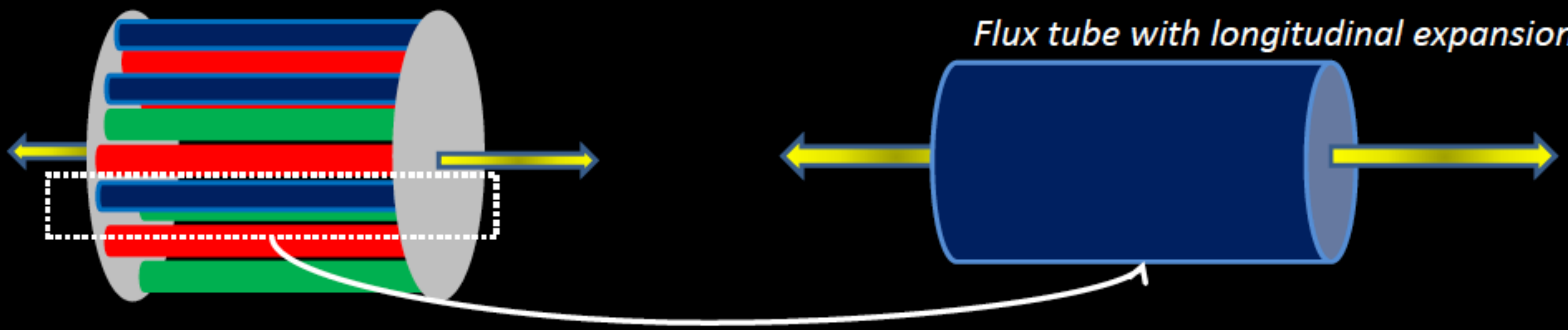
Vacuum Decay Probability
Per unit space-time to create electron-proton

$$\mathcal{W}(x) = \frac{e^2 E^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{|eE|}\right)$$

Quantum tunneling interpretation - Casher et al. , PRD20 (1979) describe Schwinger effect as a dipole formation , $p = 2g\frac{E_T}{|g\vec{E}|}$

Once the pair pop-up charged particles propagate in real time and produce an electric current **J = σ E** – dieletric breakdown

Boost invariant 1+1D expansion



$$(p_\mu \partial^\mu + g Q_{jc} F^{\mu\nu} p_\mu \partial_\nu^p) f_{jc} = p_0 \frac{\partial}{\partial t} \frac{dN_{jc}}{d^3x\, d^3p} + \mathcal{C}[f]$$

We assume field dynamics is ***boost invariant.*** This means E=E(τ), hence independent on η:

$$\left.\begin{aligned} \frac{\partial E}{\partial z} &= \rho \\ \frac{\partial E}{\partial t} &= -j \end{aligned}\right\}$$

Time derivative of dipole moment

$$\frac{dE}{dt} = \rho \tanh\eta - j_M - \frac{j_D}{\cosh\eta}$$

depend on distribution functions

Link Maxwell equation to kinetic equation