Energy conservation in pion production

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Challanges to Transport Theory for Heavy-Ion Collisions
ECT*, Trento, 22 May 2019

Elliptic Flow vs. Pion ratios

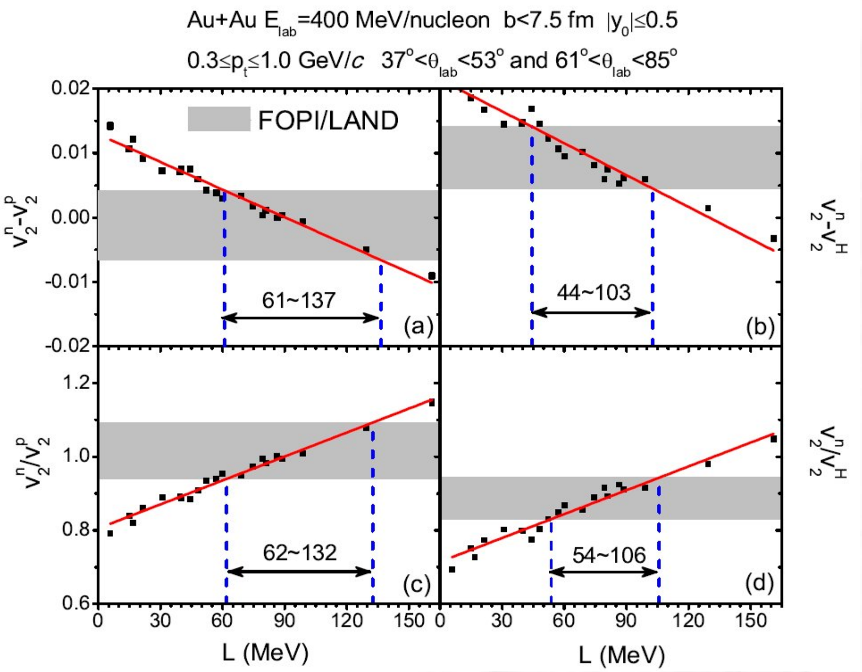
$$\frac{dN}{d\phi} \sim 1+2v_1\cos\phi+2v_2\cos 2\phi$$

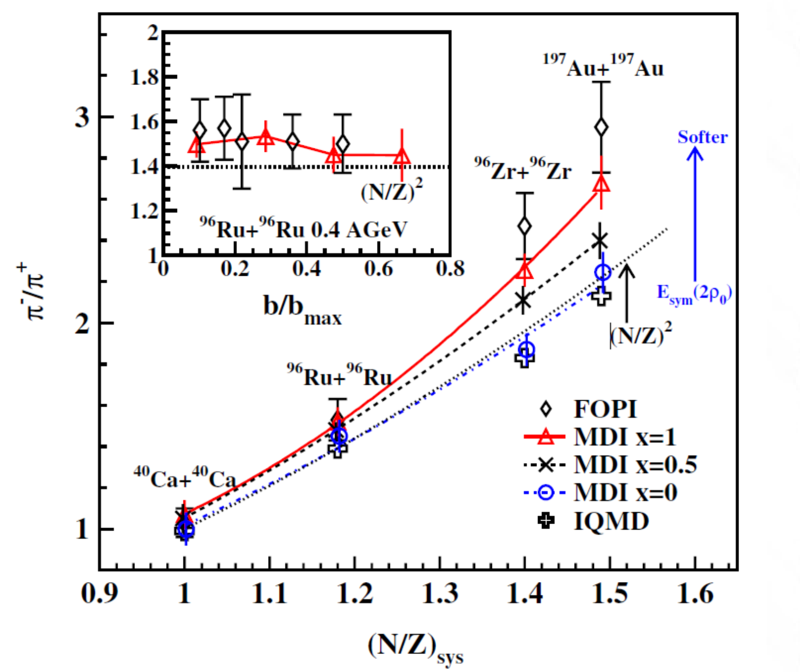
UrQMD - Y. Wang et al. PRC 89, 044603 (2014)

Isobar model (no symmetry potential)

$\pi^-/\pi^+=(5N^2+NZ)/(5Z^2+NZ)$

IBUU - Z.Xiao et al. PRL 102,062502 (2009)





TuQMD - linear/moderately stiff
M.D. Cozma et al. PRC 88, 044912 (2013)
UrQMD – linear
P. Russotto et.al PLB 697, 491 (2011)
IBUU - linear/moderately stiff
G.-C. Yong private communication

ImQMD – stiff
Z.-Q. Feng et al., PLB 683, 140 (2010)
Boltzmann-Langevin – super-soft
W.-J. Xie et al., PLB 718, 1510 (2013)
TuQMD (VEC) – super-soft
M.D. Cozma, PLB 753, 166 (2016)
pBUU – no sensitivity to SE
J. Hong et al. PRC 90, 024605 (2014)

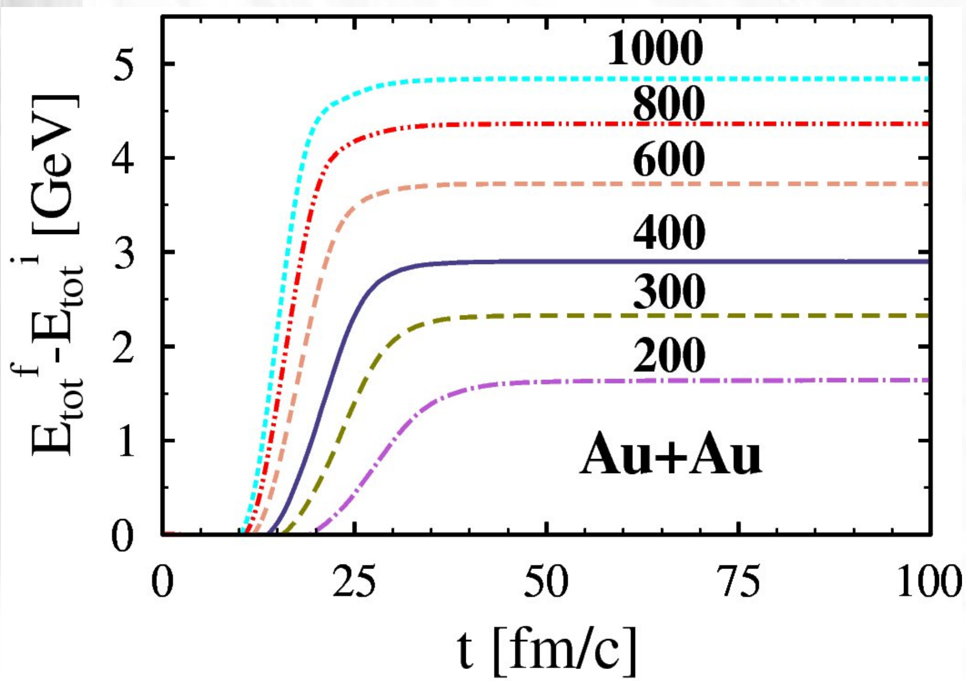
Energy Conservation (QMD type models)

80's transport models – total energy conserved on average (potentials dependent <u>only</u> on density)

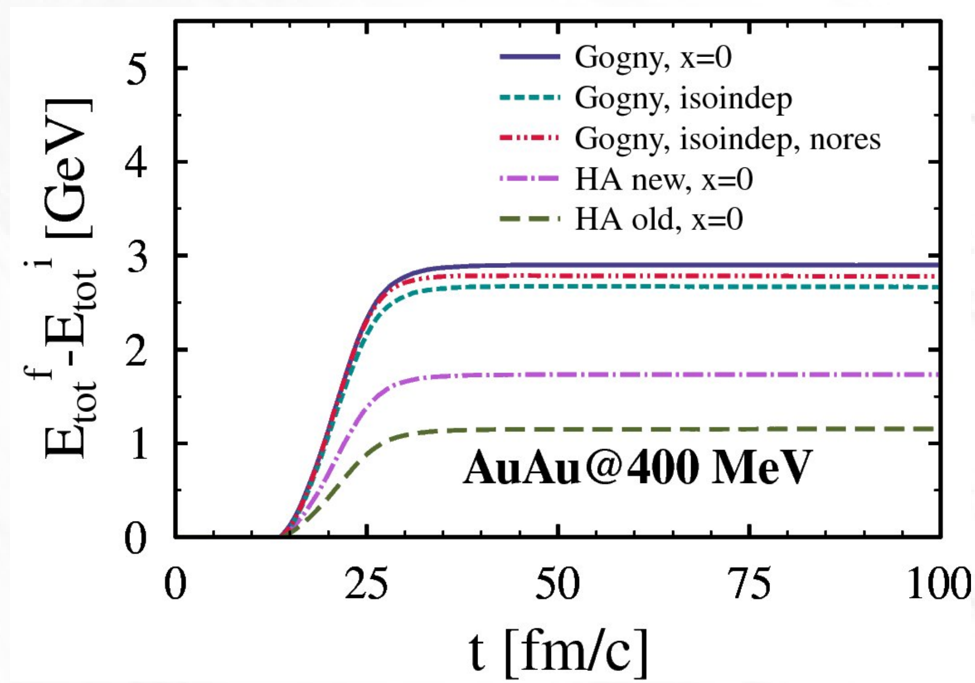
Collective phenomena – momentum dependence

Isospin effects – isospin asymmetry dependence → **Violation of total energy conservation**

Determination of final state kinematics of 2-body collisions <u>neglects</u> medium effects



M.D. Cozma, EPJ Web Conf. 117 (2016) 07016



Gogny: Das, Das Gupta, Gale, Li PRC67, 034611 (2003)
HA: Hartnack, Aichelin, PRC 49, 2901 (1994)

Energy conservation (in-medium)

$$\sqrt{p_1^2+m_1^2}+U(p_1)+\sqrt{p_2^2+m_2^2}+U(p_2)=\sqrt{p'^2_1+m'^2_1}+U(p'_1)+\sqrt{p'^2_2+m'^2_2}+U(p'_2)$$

- rarely considered in transport models below 1 AGeV, with a few exceptions:
  G. Ferini et al. PRL 97, 202301 (2006), C.Fuchs et al. PRC 55, 411 (1997),
  T. Song, C.M. Ko PRC 91, 014901 (2015)

- Ansatz for the isospin 3/2 resonance potential motivated by decay channel
  – see also S.A. Bass et al., PRC 51, 3343 (1995)

- imposed in the CM of the colliding nuclei (not in the Eckart frame)

- reactions: NN↔NR, R↔Nπ (R↔Nππ not corrected)

$$U(\Delta^{++})=U^p$$
$$U(\Delta^{+})=\frac{2}{3}U^p+\frac{1}{3}U^n$$
$$U(\Delta^{0})=\frac{1}{3}U^p+\frac{2}{3}U^n$$
$$U(\Delta^{-})=U^n$$

B.-A. Li, NPA 708 (365) 2002

Reaction	$\Delta U=U^f-U^i$	Effect
nn → $p\Delta^-$	$U^p-U^n<0$	more π^-
nn → $n\Delta^0$	$1/3(U^p-U^n)<0$	more π^-, π^0
np → $p\Delta^0$	$1/3(U^p-U^n)<0$	more π^-, π^0
np → $n\Delta^+$	$1/3(U^n-U^p)>0$	less π^+, π^0
pp → $p\Delta^+$	$1/3(U^n-U^p)>0$	less π^+, π^0
pp → $n\Delta^{++}$	$U^n-U^p>0$	less π^+

Transport Model

Quantum Molecular Dynamics (TuQMD): HIC 0.1-2.0 GeV/A

previously applied to study:

- dilepton emission in HIC: K.Shekter, PRC 68, 014904 (2003); D. Cozma, PLB640,170 (2006); E.Santini PRC78,03410 (2008)
- EoS of symmetric nuclear matter: C. Fuchs, PRL 86, 1974 (2001); Z.Wang NPA 645, 177 (1999)
- In-medium effects and HIC dynamics: C. Fuchs, NPA 626,987 (1997); U. Maheswari NPA 628,669 (1998)

upgrades implemented at IFIN-HH (Bucharest):

- various parametrizations for the EoS: optical potential, symmetry energy PRC 88, 044912 (2013)
- various parametrizations for elastic cross-sections (also in medium ones) PLB 700, 139 (2011)
- threshold effects for baryon resonance & π meson emission/absorption PLB 753, 166 (2016)
- pion optical potential PRC 95, 014601 (2017)
- **planned**: threshold effects for reactions involving strangeness degrees of freedom

Pion production: two step process
- resonance excitation in baryon-baryon collisions
  parametrization of the OBE model of
  S.Huber et al., NPA 573, 587 (1994)
- resonance decay:
  Breit-Wigner shape of the resonance spectral function;
  parameters -> K. Shekhter, PRC 68, 014904 (2003)
  decay channels:

$$R \to N\pi, \quad R \to N\pi\pi$$
$$R \to \Delta(1232)\pi, \quad R \to N(1440)\pi$$

Pion absorption:
-resonance model (all 4* resonances below 2 GeV)
K. Shekhter, PRC 68, 014904 (2003)

Isospin dependence of EoS

momentum dependent – generalization of the Gogny interaction:

MDI potential

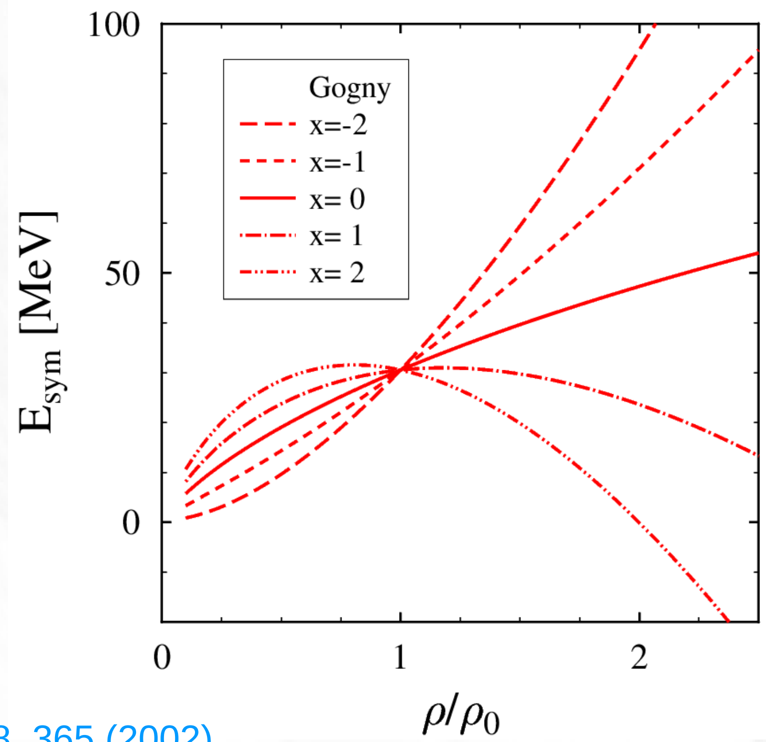
Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

$$\frac{E}{N}(\rho,\beta,x)=\frac{1}{2}A_1 u+\frac{1}{2}A_2(x)u\beta^2+\frac{B}{\sigma+1}(1-x\beta^2)+\frac{1}{u\rho_0^2}\sum_{\tau,\tau'}C_{\tau\tau'}\int\int d^3p\,d^3p'\frac{f_\tau(p,p')f_{\tau'}(p,p')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2}$$

$$A_2(x)=A_2^0+\frac{2xB}{\sigma+1}\bar{u}^{\sigma-1} \qquad u=\frac{\rho}{\rho_0}$$

x	L_{sym} [MeV]	K_{sym} [MeV]
-2	152	418
-1	106	127
0	61	-163
1	15	-454
2	-301	-745

$$S(\rho)=J+\frac{L}{3}\frac{\rho-\rho_0}{\rho_0}+\frac{K_{sym}}{18}\frac{(\rho-\rho_0)^2}{\rho_0^2}$$



resonance potential: isovector component unknown

$$V(\Delta^{++})=V_n=V_s+\frac{3}{2}V_v$$

$$V(\Delta^{+})=\frac{2}{3}V_n+\frac{1}{3}V_p=V_s+\frac{1}{2}V_v$$

$$V(\Delta^{0})=\frac{1}{3}V_n+\frac{2}{3}V_p=V_s-\frac{1}{2}V_v$$

$$V(\Delta^{-})=V_p=V_s-\frac{3}{2}V_v$$

$$V_s\equiv\frac{1}{2}(V_n+V_p)$$

$$\delta\equiv\frac{1}{3}(V_n-V_p)$$

$$V_v=\delta$$

B.-A. Li, NPA 708, 365 (2002)

! In the following: V_v expressed in units of [δ]

Approximations

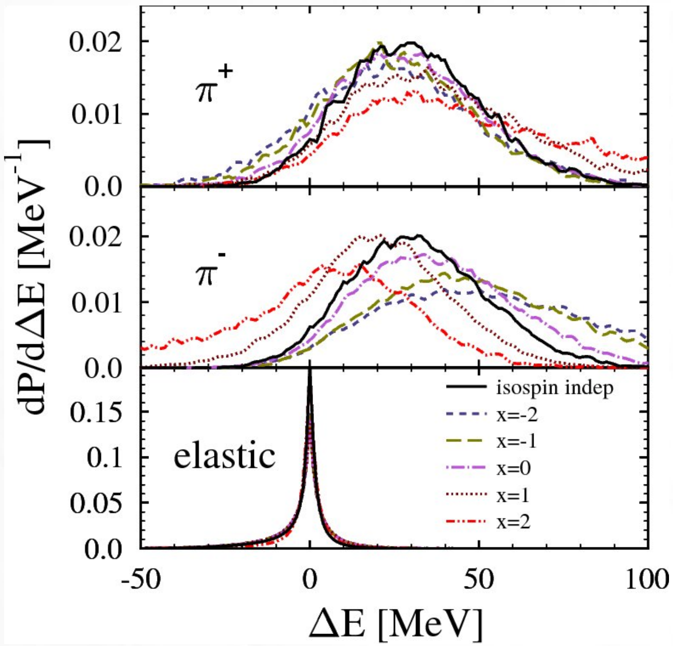
PLB 753, 166 (2016)

Elastic scattering:

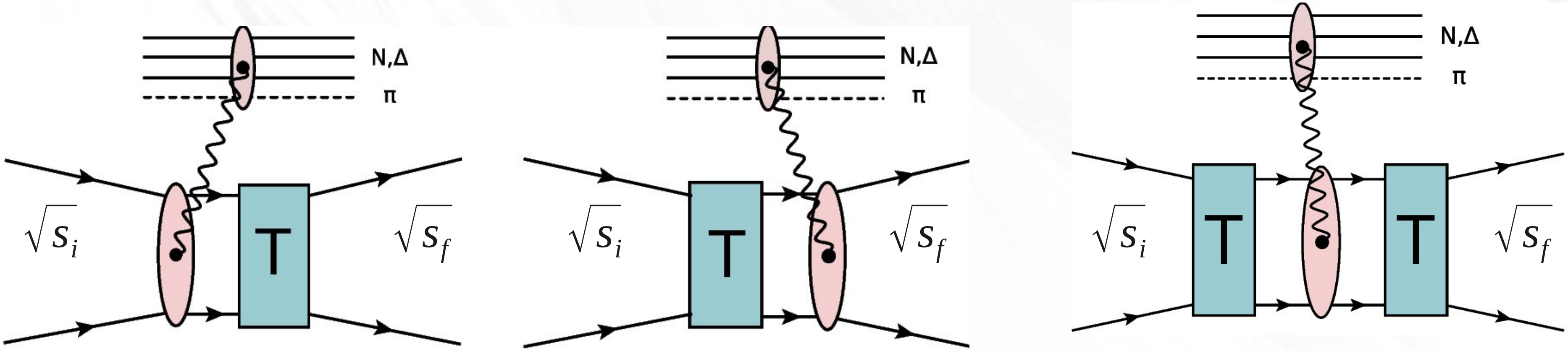
$$\sqrt{s_f} \approx \sqrt{s_i}$$
$$\sqrt{s^*} = 0.5\left(\sqrt{s_f} + \sqrt{s_i}\right)$$

Resonance excitation:

$$\sqrt{s_f} - \sqrt{s_i} \approx 25\,\mathrm{MeV}$$
$$\sqrt{s^*} = \sqrt{s_f}$$



$$\Delta E = \sqrt{s_f} - \sqrt{s_i}$$



initial **final state** **rescattering**

Different "scenarios"

VEC – vacuum energy conservation constraint

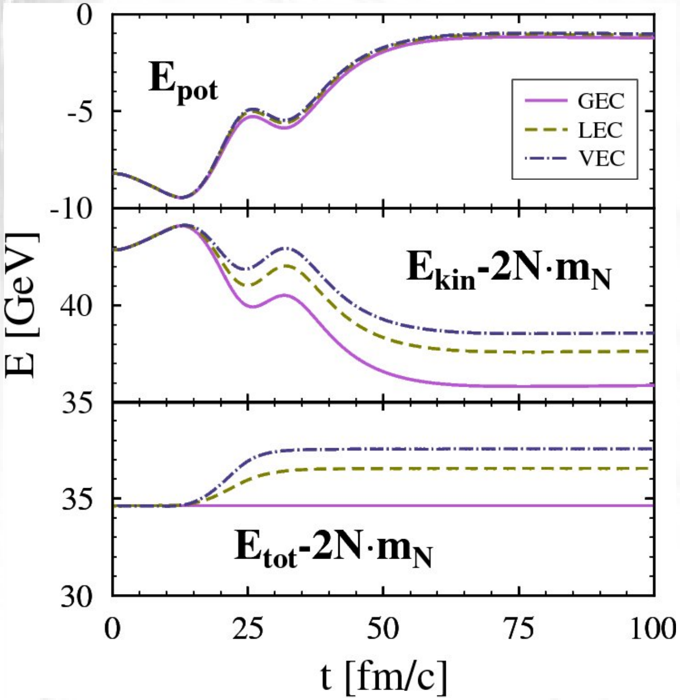
LEC - "local" energy conservation – limited impact on multiplicities and ratios

GEC- "global" energy conservation – conserve energy of the entire system

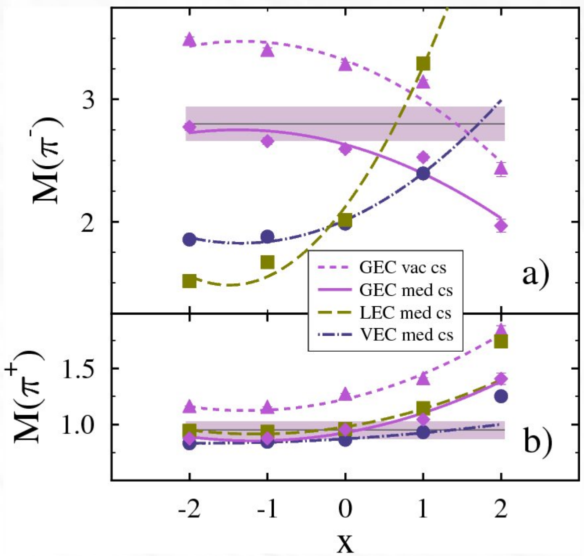
-in-medium cross-sections for the inelastic channels

$$\sigma^{*(12\rightarrow 23)} = \left[\frac{m_1^* m_2^* m_3^* m_4^*}{m_1 m_2 m_3 m_4}\right]^{1/2} \sigma^{(12\rightarrow 34)}$$

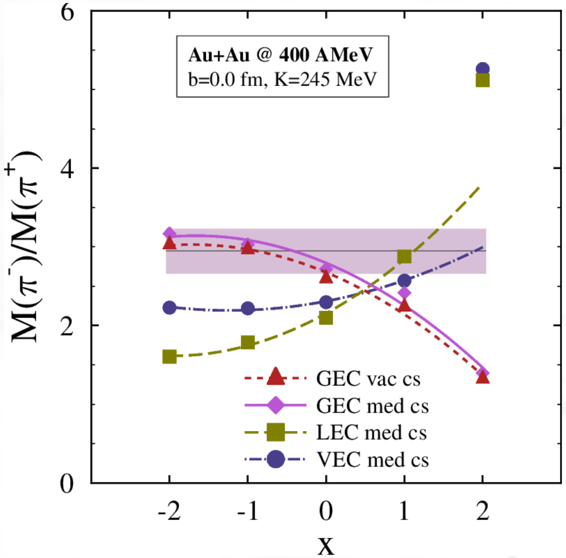
AuAu@400 MeV, b=0 fm



multiplicities



ratios



Experimental data: W. Reisdorf et al. (FOPI) NPA 848, 366 (2010)

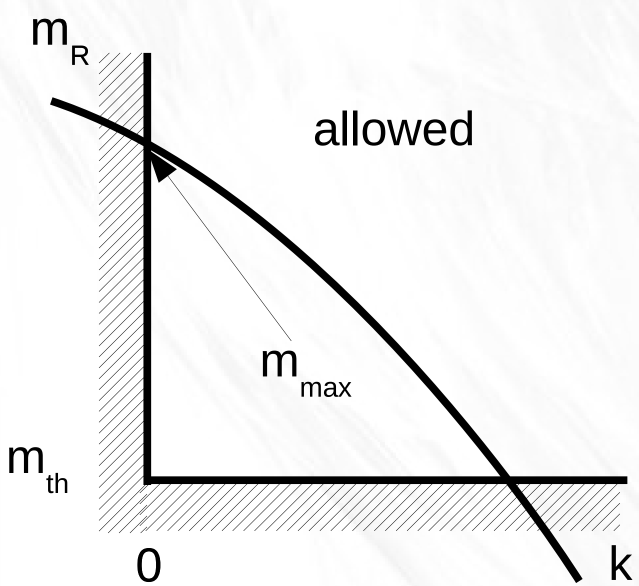
softer

Final state kinematics

final state phase space in NN->NR

VEC

$$\sqrt{s}=\sqrt{m_N^2+k^2}+\sqrt{m_R^2+k^2}$$

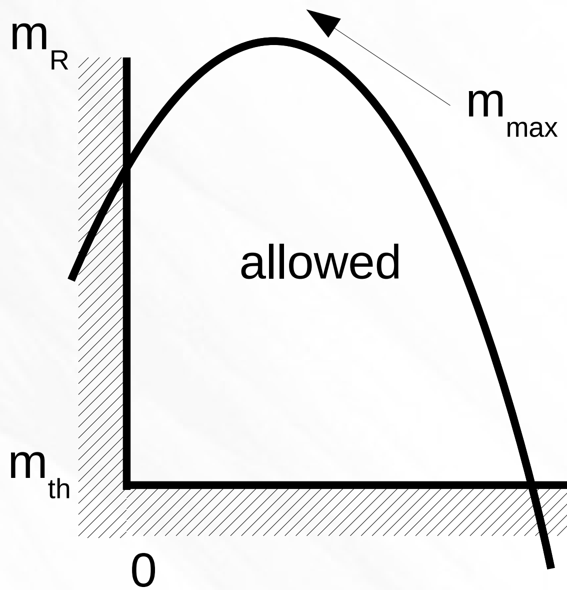


$$m_{th}\leqslant m_R\leqslant m_{max}$$
$$0\leqslant\theta\leqslant\pi$$
$$0\leqslant\Phi\leqslant 2\pi$$

consistent with $\frac{d\sigma}{d\Omega}$

LEC, GEC

$$\sqrt{s_{fin}}=\sqrt{m_N^2+k^2}+\sqrt{m_R^2+k^2}$$
$$\sqrt{s_{fin}}=\sqrt{s_{ini}}+\delta\sqrt{s}$$



$$m_{th}\leqslant m_R\leqslant m_{max} \longrightarrow m_{th}\leqslant m_R\leqslant m_R(k=0)$$

$$\theta_{min}\leqslant\theta\leqslant\theta_{max}$$
$$\Phi_{min}\leqslant\Phi\leqslant\Phi_{max}$$

} approximate to vacuum case

Detailed Balance (GEC)

Resonance absorption: detailed balance

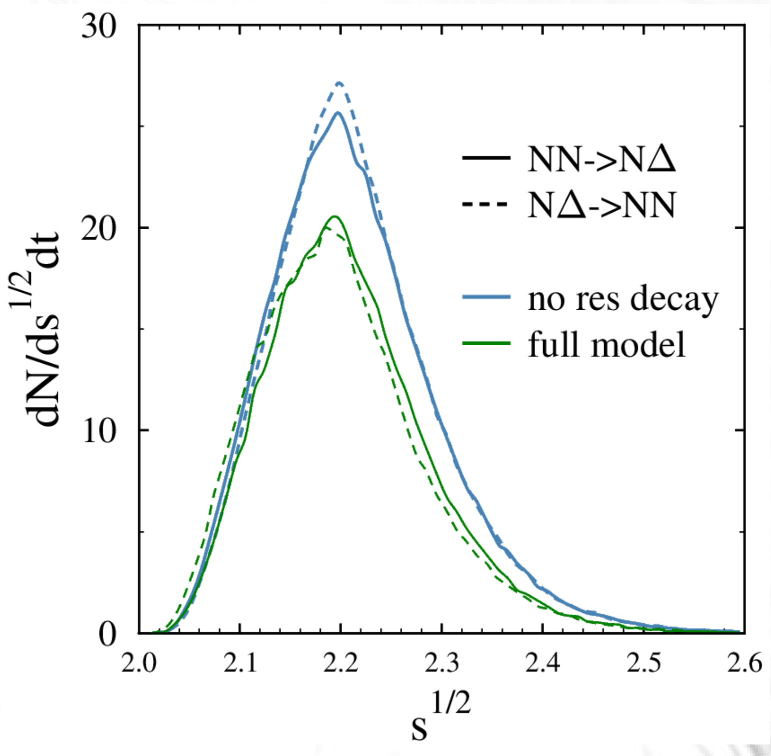
P. Danielewicz et al., NPA 533, 712 (1992)

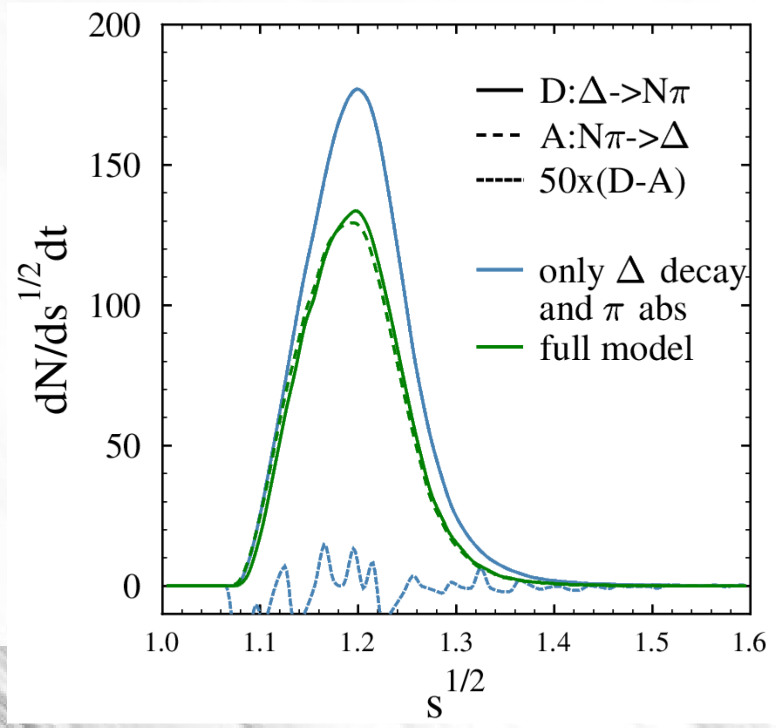
$$\sigma^{NR\to NN} = \frac{1}{4}\frac{m_R\, p_{NN}^2}{p_{NR}}\sigma^{NN\to NR} \times \left[\frac{1}{4\pi}\int d\Omega \frac{1}{2\pi}\int_{m_N+m_\pi}^{M_{max}(\theta,\phi)} dM\, M\, p'_{NR}\, A_R(M)\right]^{-1}$$

$$\sqrt{s_i} \to p_{NR}, \qquad \sqrt{s_f} \to p_{NN}$$

average over solid angle not considered in PLB 753, 166 (2016):

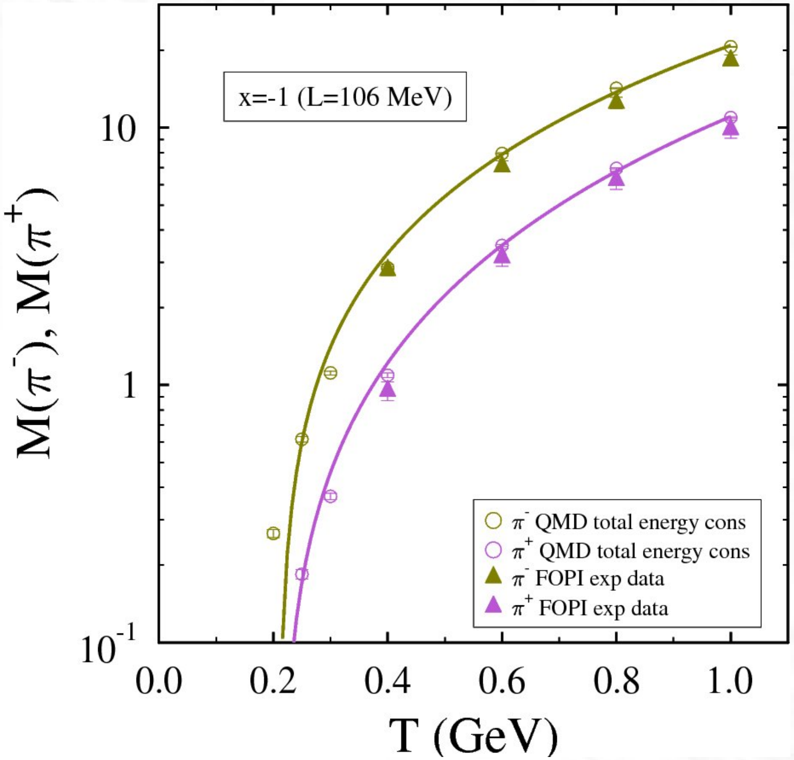
Box calculation: β=0.0, T=60 MeV

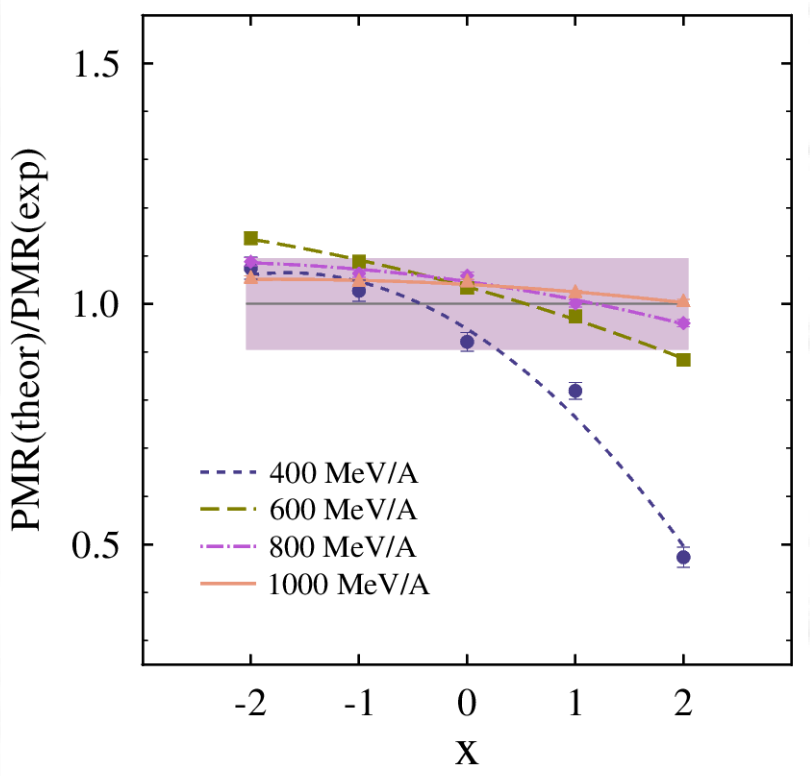




Energy dependence

FOPI data: W.Reisdorf et al., NPA 781, 459 (2007)





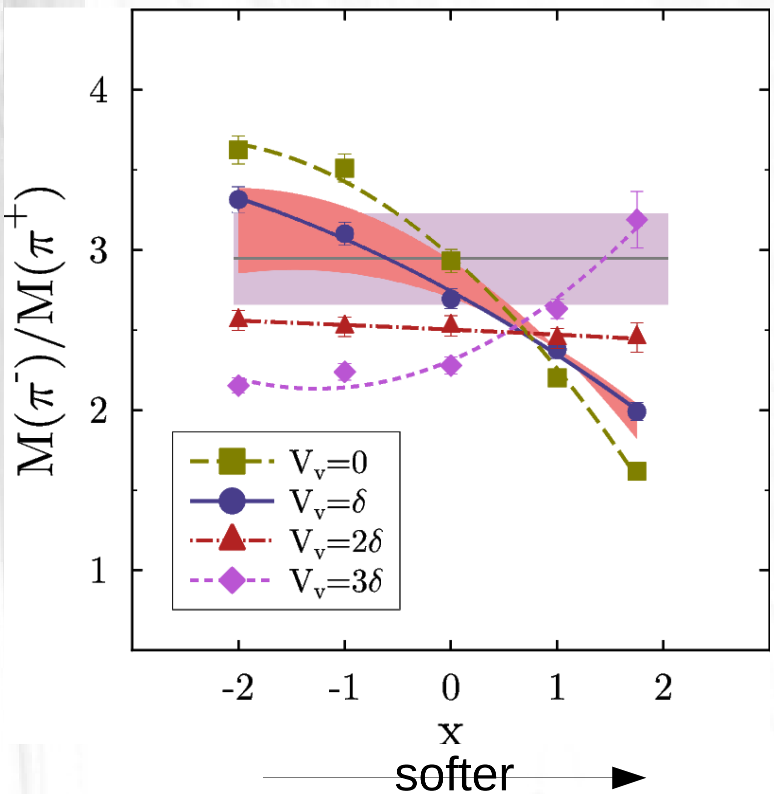
Impact of the Δ(1232) potential

Phenomenology – inclusive electron nucleus scattering (He,C,Fe) attractive
- Δ-nucleus potential deeper than the nucleon-nucleus potential

O'Connel, Sealock PRC 42, 2290 (1990)

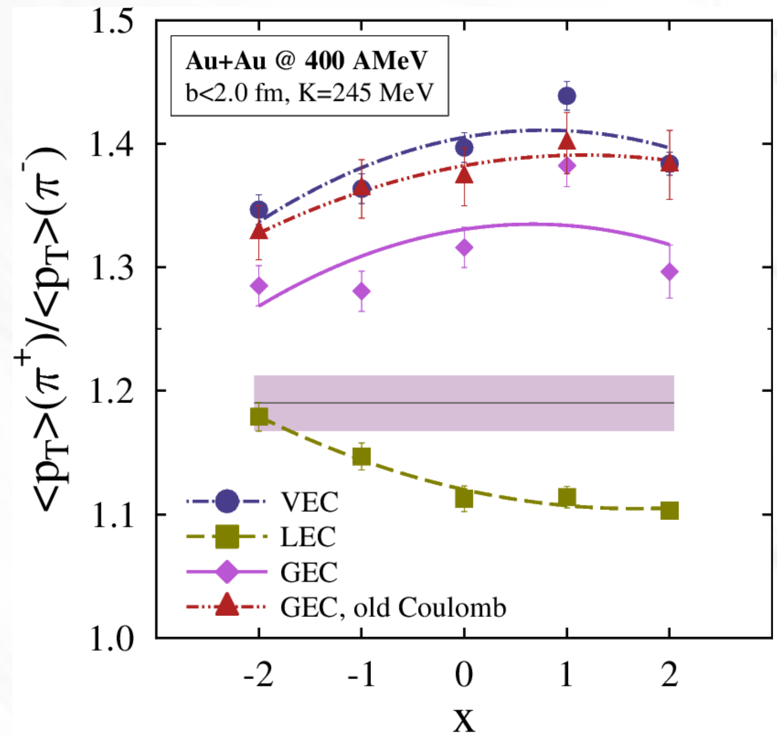
Ab initio calculations – Argonne v_{28} interaction (BBG)=> repulsive Δ potential
- 3D reduction of Bethe-Salpeter equation similar (DB)
- strong dominant repulsive contribution from T=2 sector

Baldo, Ferreira NPA 569, 645 (1994)
Malfliet, de Jong PRC 46, 2567 (1992)



$$V(\Delta^{++})=V_s+\frac{3}{2}V_v$$
$$V(\Delta^{+})=V_s+\frac{1}{2}V_v$$
$$V(\Delta^{0})=V_s-\frac{1}{2}V_v$$
$$V(\Delta^{-})=V_s-\frac{3}{2}V_v$$

$$V_s=\frac{1}{2}(V_n+V_p)$$
$$\delta=\frac{1}{3}(V_n-V_p)$$



M.D. Cozma, PLB 753, 166 (2016)

DC, PRC 95, 14601 (2017)

Exp. data: W. Reisdorf et al., NPA 781, 459 (2007)

Pion optical potential

Sources:

Theoretical models: Effective Hadronic Models

J. Nieves et al., NPA 554, 509; 554 (1993)
M. Doring et al., PRC 77, 024602 (2008)

pion self-energy starting from basic interaction terms πNN, πNΔ, πNN*(1440)

Chiral Perturbation Theory

N. Kaiser et al., PLB 512, 283 (2001)
C. Baru et al., NPA 872, 69 (2011)
W.Weise, Acta. Phys. Pol, B31, 2715 (2000)

Experiment: Pionic Atoms

T. Yamazaki et al., Phys. Rep. 514, 1 (2012)
E. Friedman, A. Gal, Phys. Rep. 452, 89 (2007)

Pion-Nucles Scattering

R. Seki et al, PRC 27, 2799; 2817 (1983)

Ericson-Ericson parametrization (M. Ericson et al. Ann. Phys. 36, 323 (1966))

$$V_{opt}(r)=\frac{2\pi}{\mu}\left[-q(r)+\vec{\nabla}\frac{\alpha(r)}{1+4/3\pi\lambda\alpha(r)}\vec{\nabla}\right]$$

$$q(r)=\epsilon_1(\bar{b}_0\rho+\bar{b}_1\beta\rho)+\epsilon_2 B_0\rho^2$$

$$\alpha(r)=\epsilon_1^{-1}(c_0\rho+c_1\beta\rho)+\epsilon_2^{-1}(C_0\rho^2+C_1\beta\rho^2)$$

$$\epsilon_1=1+m_\pi/m_N \qquad \epsilon_2=1+2m_\pi/m_N$$

	$\bar{b}_0[m_\pi^{-1}]$	$\bar{b}_1[m_\pi^{-1}]$	Re B_0 $[m_\pi^{-4}]$	Im B_0 $[m_\pi^{-4}]$	λ	c_0 $[m_\pi^{-3}]$	c_0 $[m_\pi^{-3}]$	Re C_0 $[m_\pi^{-6}]$	Im C_0 $[m_\pi^{-6}]$
SM-1	-0.0283	-0.120	0.0	0.042	1	0.223	0.250	0.0	0.10
SM-2	0.030	-0.143	-0.150	0.046	1	0.210	0.180	0.11	0.09
Batty-1	-0.017	-0.130	-0.048	0.0475	1	0.255	0.170	0.0	0.09
Batty-2	-0.023	-0.085	-0.021	0.049	1	0.210	0.089	0.118	0.058
Konijn-2	0.025	-0.094	-0.265	0.0546	1	0.273	0.184	-0.140	0.105

subset from K. Itahashi et al., PRC 62, 025202 (2000)

Symmetry Energy Constraints

Au+Au@400 MeV/A, b<2.0 fm

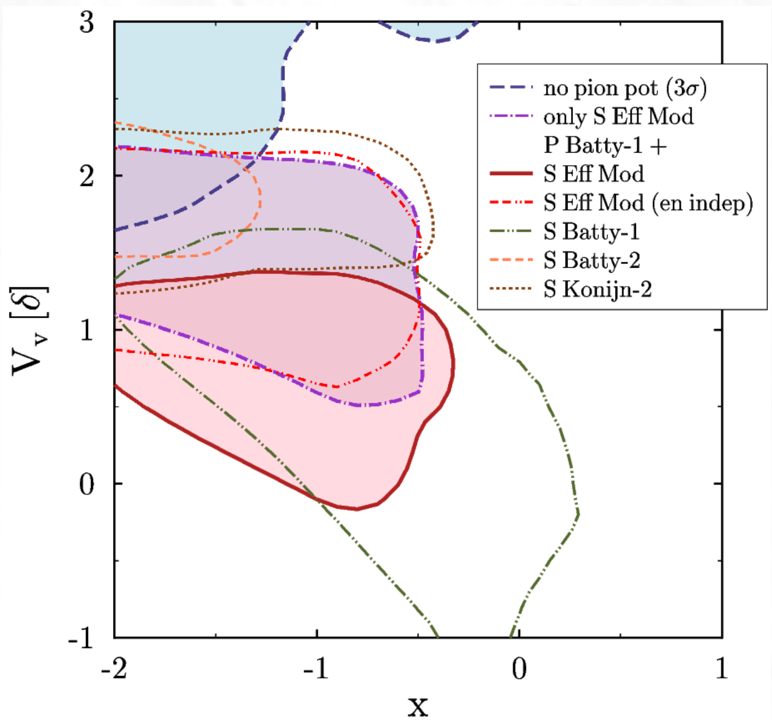
S-wave impact

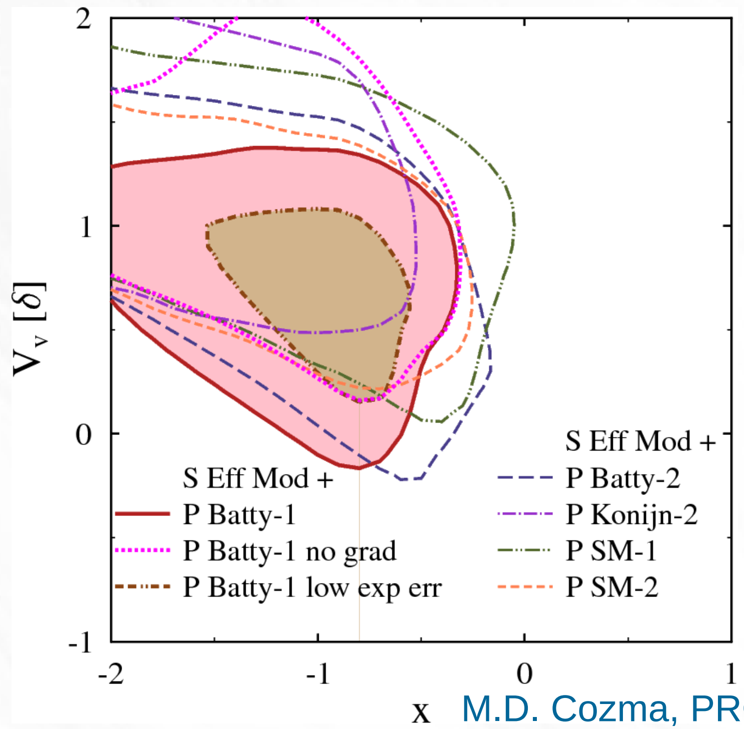
P-wave: Batty-1

1σ CL contour plots

P-wave impact

S-wave: effective model





M.D. Cozma, PRC 95, 14601 (2017)

No pion potential – unable to describe exp. PMR and PAPTR simultaneously

S-wave - energy dependence impacts V_v

- density dependence – impact both x (60 MeV on L) and V_v

P-wave - moderate impact on both x (25 MeV on L) and V_v

Summary / Conclusions

QMD transport models: - momentum/isospin dependent interactions lead to total energy conservation violation (scattering term)

- solution: account for potential energies when determining final state kinematics in 2-body collisions

- threshold shifts induced by imposing energy conservation

- local energy conservation (roughly 40% reduction of the magnitude of energy conservation violation)

- global energy conservation: does the job by definition

Open issues: - no momentum transfer

- only applicable in regions where cross-sections depend strongly on invariant mass

- not a Lorentz covariant approach (+retardation effects may not be negligible)

Accomplishments: - description of experimental pion multiplicities in HIC in the 0.4-1.0 AGeV range

- compatible constraints for symmetry energy stiffness from pionic and elliptic flow observables

S-wave pion potential

	$b_0[m_\pi^{-1}]$	$b_1[m_\pi^{-1}]$
Exp	-0.0001±0.0021	-0.0885±0.0021
ChPT	0.0075±0.0031	-0.0861±0.0009
WT	0.0	-0.0790

M. Krell et al. NPB 11, 521 (1969)

double scattering

$$\overline{b}_0 = b_0 - \frac{3}{2\pi}(b_0^2 + 2b_1^2)\left(\frac{3\pi^2}{2}\rho\right)^{\frac{1}{3}}$$

ChPT

$$b_0 = 0.0 \qquad b_1 = \frac{-m_\pi}{8\pi(1+m_\pi/m_N)f_\pi^2}$$

W. Weise, NPA 690, 98 (2001)

$$f_\pi^2(\rho) = f_\pi^2(0) - \frac{\sigma\rho}{m_\pi^2}$$

$$b_1(\rho) = \frac{b_1}{1 - \frac{\sigma\rho}{m_\pi^2 f_\pi^2}} \simeq \frac{b_1}{1 - 2.3\rho}$$

Energy dependence
inferred from exp. pion-nucleus scattering

$$\overline{b}_0^{eff} = \overline{b}_0 + \rho^{eff} \, Re \, B_0$$

R. Seki et al., PRC 27, 2799 (1983)
C. Garcia-Recio et al., PRC 40, 1308 (1989)

Free-space & ChPT

R.A. Arndt et al., PRC 74, 045205 (2006)

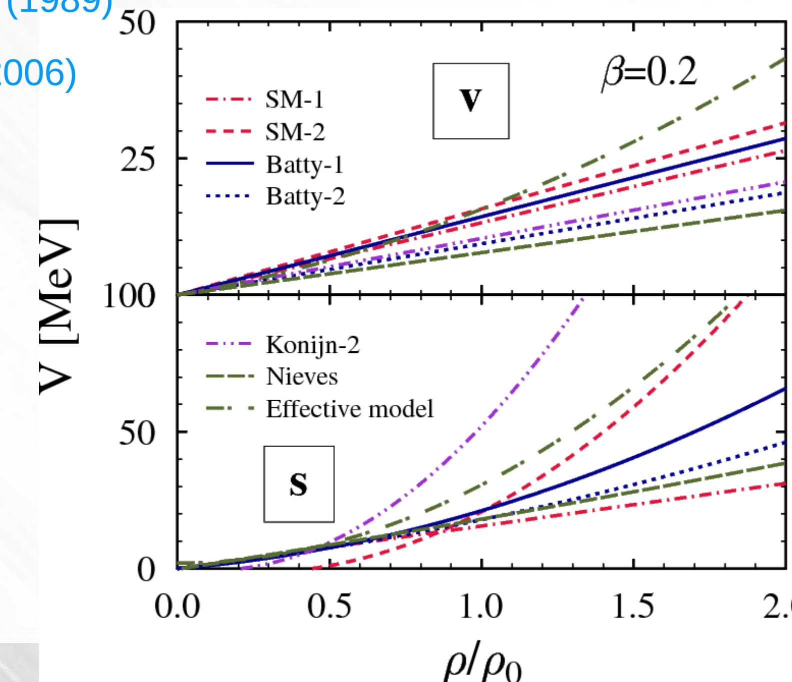
$$\frac{db_0}{d\omega} = -0.00053 \, m_\pi^{-1}/\mathrm{MeV} \qquad \frac{db_1}{d\omega} = 0$$

Effective Model

$$b_0(\omega) = -0.010 - 0.00016\,\omega$$

$$\overline{b}_1(\rho) = -0.088\left(1 + \frac{0.6116}{b_1}\frac{\rho}{\rho_0}\right)$$

$$\overline{b}_0(\rho,\omega) = b_0 - \frac{3}{2\pi}(b_0^2 + 2b_1^2)\left(\frac{3\pi^2}{2}\rho\right)^{\frac{1}{3}}$$



S-wave parametrizations: subset from K. Itahashi et al., PRC 62, 025202 (2000)

P-wave pion potential

Energy dependence:

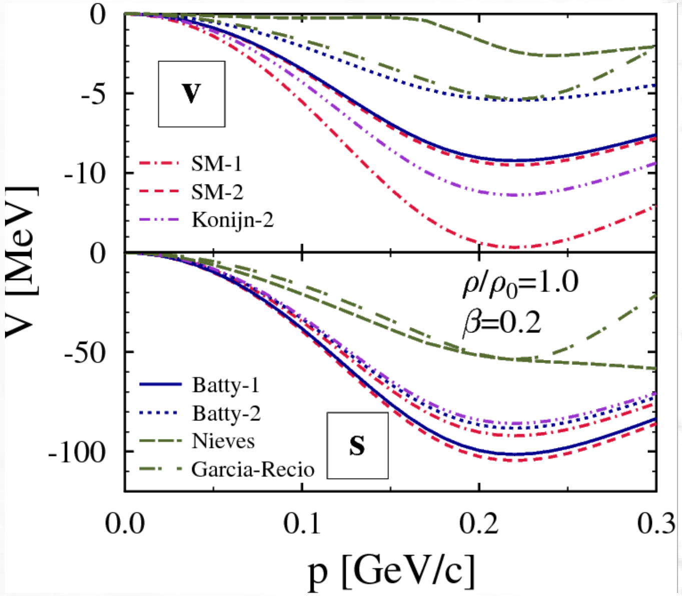
extrapolation of pionic atoms results using a local approximation of the delta-hole model that describes pion-nucleus scattering up to ω=300 MeV

C. Garcia-Recio et al., NPA 526, 685 (1991)

$$f(p^2)=\frac{1-p_{eff}^2/\Lambda_1^2+p_{eff}^4/\Lambda_2^4}{1-p^2/\Lambda_1^2+p^4/\Lambda_2^4}$$

$$\Lambda_1=0.55\,\mathrm{GeV} \qquad \Lambda_2=0.22\,\mathrm{GeV}$$

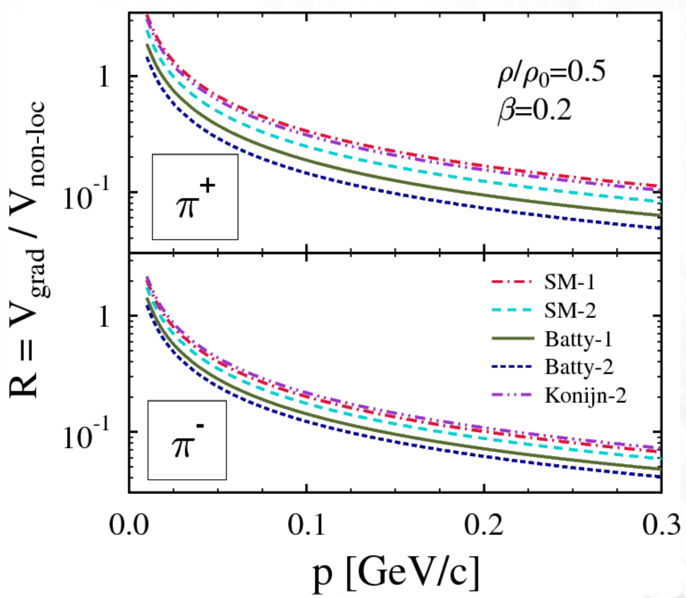
$$p_{eff}=0.05\,\mathrm{GeV}$$



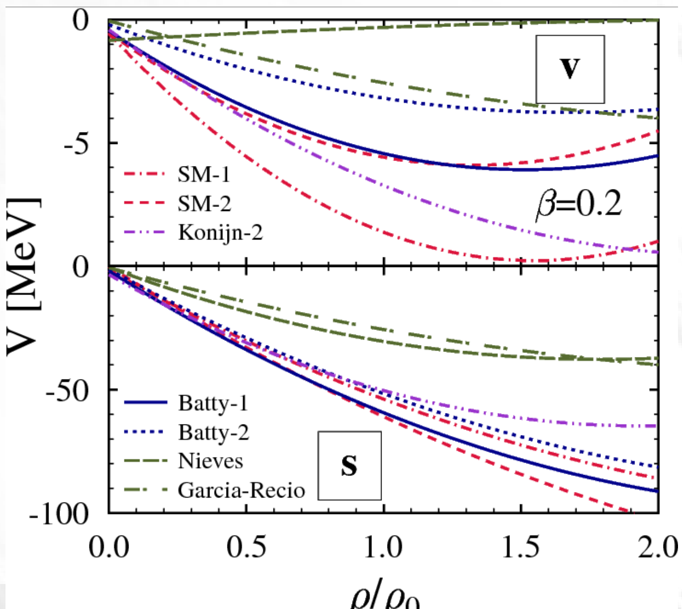
Gradient terms:

$$V_{opt}^{P}(r)=\frac{2\pi}{\mu}\vec{\nabla}\frac{\alpha(r)}{1+4/3\pi\lambda\alpha(r)}\vec{\nabla}$$

$$\Rightarrow \text{ terms } \sim \vec{p}\cdot\vec{\nabla}\rho \qquad \vec{p}\cdot\vec{\nabla}\beta$$



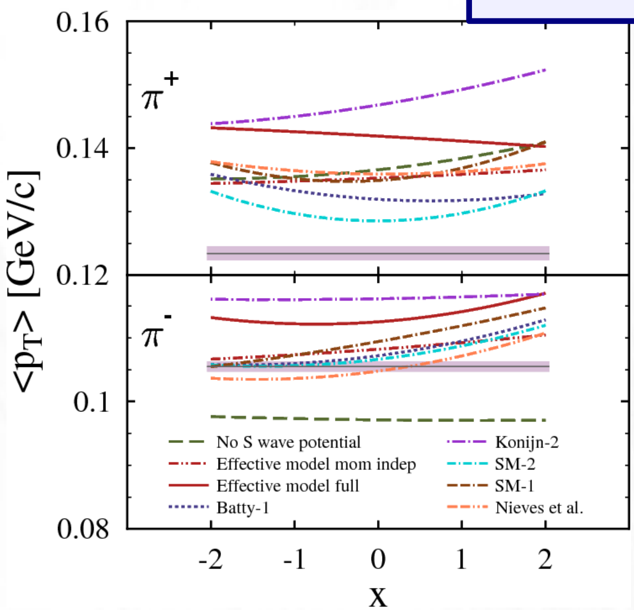
Density dependence



P-wave parametrizations: subset from K. Itahashi et al., PRC 62, 025202 (2000)

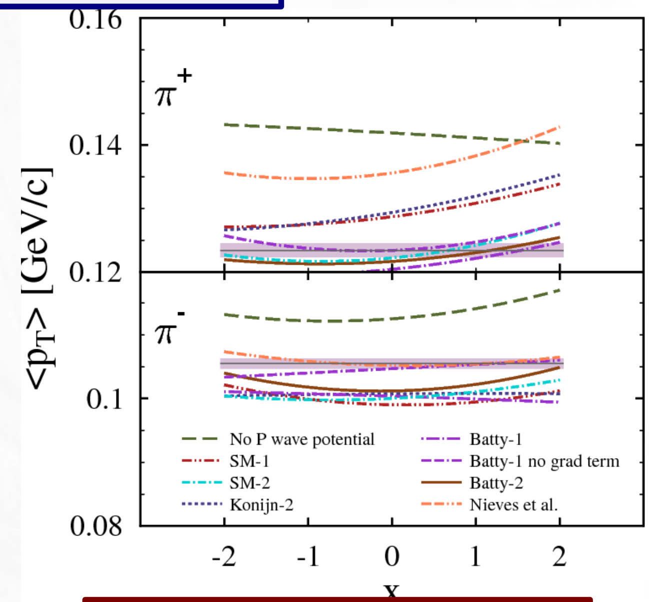
Impact on pion observables

Au+Au@400 MeV/A, 3.35 fm <b<6.0 fm



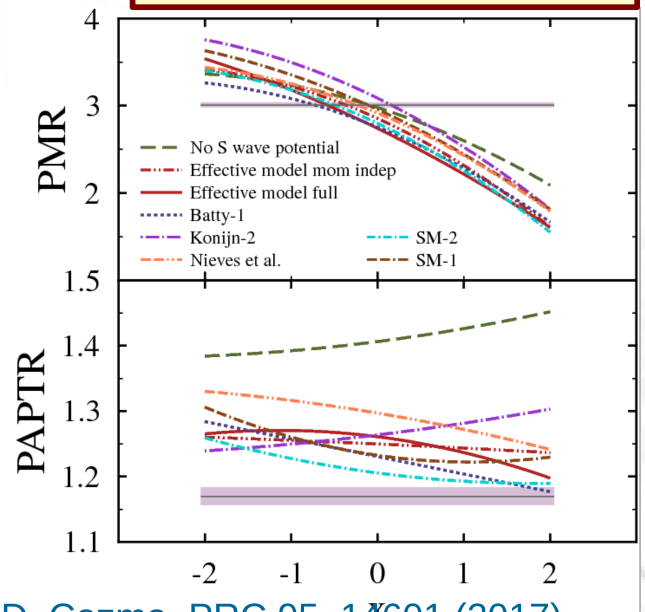
$|y/y_P|<1.75$
$0<p_T<0.33$ GeV/c

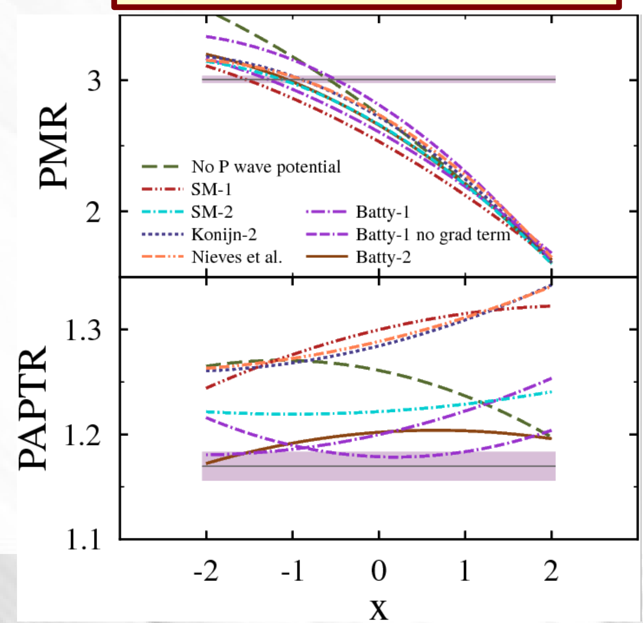
exp data:
W.Reisdorf et al. (FOPI), private communication



S-wave pion potential

P-wave pion potential





M.D. Cozma, PRC 95, 14601 (2017)