In-medium effects on pion production

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- Introduction
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- Summary
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Microscopic Theory of Pion Production and Sidewards Flow in Heavy-Ion Collisions

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(Received 9 July 1984)

Nuclear collisions from 0.3 to 2 GeV/nucleon are studied in a microscopic theory based on Vlasov's self-consistent mean field and Uehling-Uhlenbeck's two-body collision term which respects the Pauli principle. The theory explains simultaneously the observed collective flow and the pion multiplicity and gives their dependence on the nuclear equation of state.

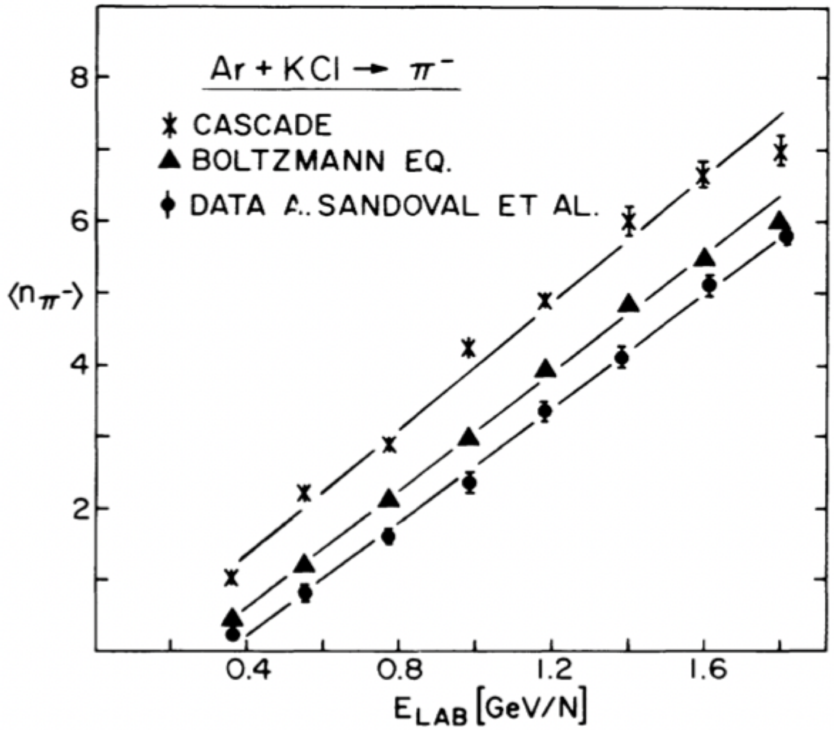
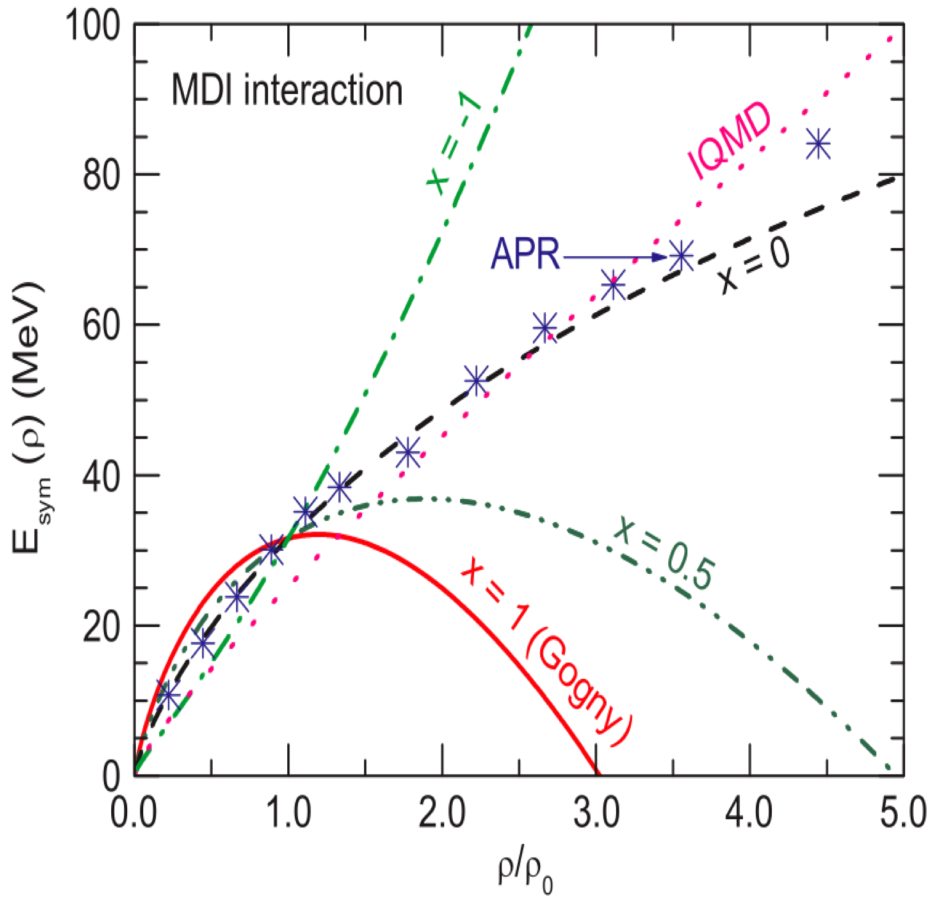


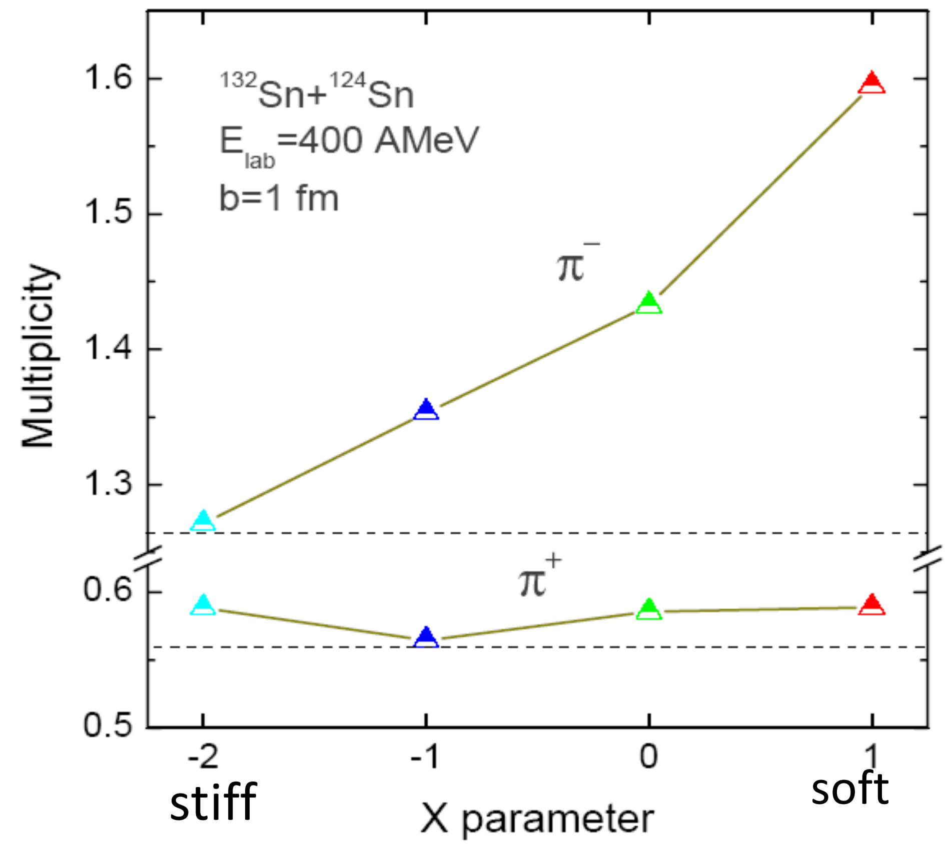
FIG. 2. Pion multiplicity for central collisions ($b < 2.4$ fm) of Ar+KCl. The data (Ref. 6, circles) are compared to the present theory in the "cascade mode" (crosses) and to the same theory with compression energy and phase-space Pauli blocking included (triangles).

- Although difference between cascade and BUU with mean fields can be 40%, difference between stiff and soft EOS is much smaller.
- No definitive conclusion on stiffness of nuclear EOS from pion yield!

Near-threshold pion production with high energy radioactive beams (IBUU)

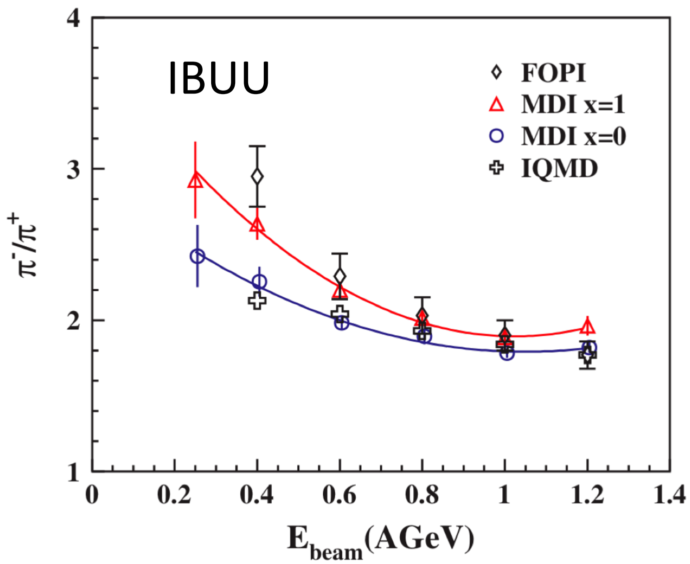
B. A. Li, PRL 88, 192701 (2002)



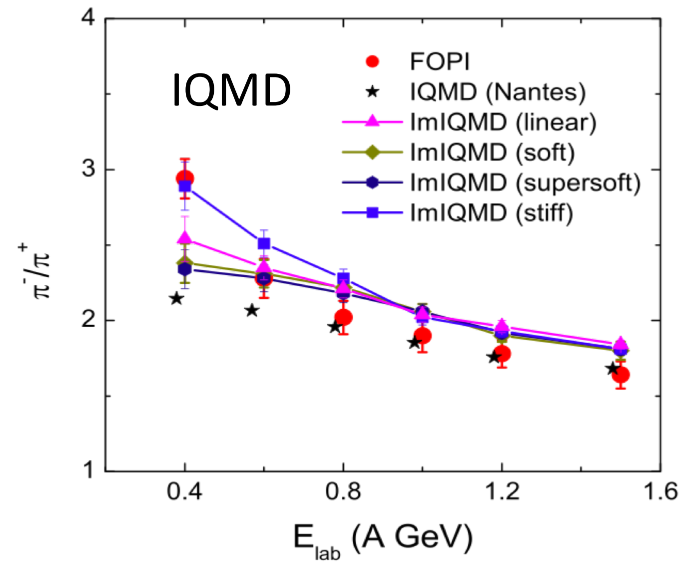


π^- yield is sensitive to the symmetry energy $E_{sym}(\rho)$ since they are mostly produced in the neutron-rich region, with softer one giving more π^- than stiffer one. Difference between $^{\pi^-}/_{\pi^+}$ from super soft (x = 1) and super stiff (x = -1) $E_{sym}(\rho)$ is, however, only about 30%, which makes it very challenging to determine $E_{sym}(\rho)$ from data using transport models.

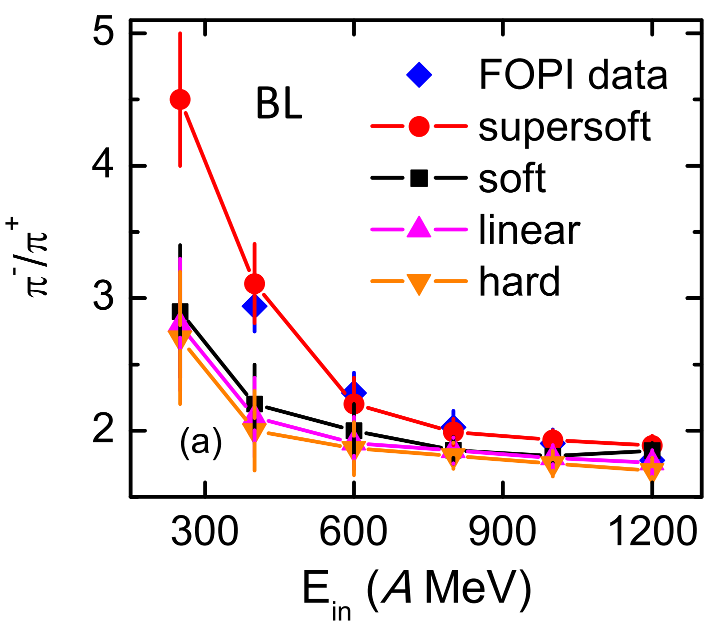
Conflicting results on symmetry energy from charged pion ratio



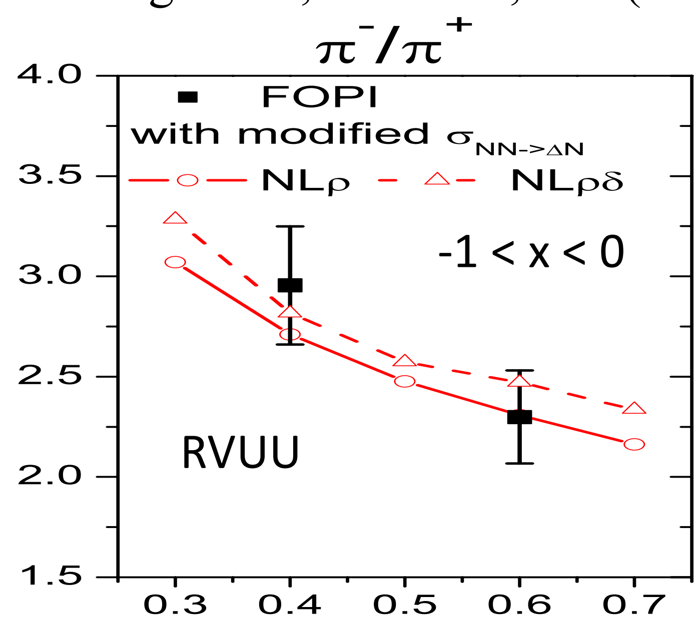
Xiao et al, PRL 102, 062502 (2009)



Feng & Jin, PLB 683, 140 (2010)



Shi et al., PLB 718, 1510 (2013)



Song & Ko, PRC 91, 014901 (2015)

In-medium threshold effect on pion production

$$U_{asy}^{\Delta^{++}} = U_{asy}^{p}, \quad U_{asy}^{\Delta^{+}} = \frac{2}{3}U_{asy}^{p} + \frac{1}{3}U_{asy}^{n}, \quad U_{asy}^{\Delta^{0}} = \frac{1}{3}U_{asy}^{p} + \frac{2}{3}U_{asy}^{n}, \quad U_{asy}^{\Delta^{-}} = U_{asy}^{n}$$

- $pn \rightarrow p\Delta^0$

Initial-state potential: $U_p + U_n$

Final-state potential: $U_p + U_{\Delta^0} = U_p + \frac{U_p}{3} + \frac{2}{3}U_n$

$\rightarrow$ difference in initial and final potentials is $(U_n - U_p)/3 > 0$ in neutron-rich matter, which reduces the production threshold [Ferini, Colonna, Gaitanos and Di Toro (NPA 762, 147 (2005)]

Also, momenta $\mathbf{k}_3$ and $\mathbf{k}_4$ of final particles are determined from

$$E_3 + E_4 = E_1 + E_2 + \frac{1}{3}U_n - \frac{1}{3}U_p$$

So potentials affect both propagation and collisions. The latter is not included in essentially all transport models.

Relativistic Vlasov-Uehling-Uhlenbeck model

Ko, NPA 495, 321 (1989)

$$\boxed{\frac{\partial}{\partial t} f + \vec{v} \cdot \nabla_r f - \nabla_r H \cdot \nabla_p f = \mathcal{C}[f]}$$

Mean-field potential $\boxed{H = \sqrt{m^{*2} + p^{*2}} + g_\omega \omega^0 \pm g_\rho (\rho_3)_0}$

Collisional integral $C[f]$ includes nucleon-nucleon elastic scattering NN → NN based on empirical cross sections as well as inelastic scattering NN → NΔ and its inverse reaction NΔ → NN using cross sections from the one-boson exchange model of Huber and Aichelin [NPA 573, 587 (1994)]

$$\begin{aligned}
\mathcal{C}[f] \quad = \quad & -\frac{1}{(2\pi)^3} \int \int d\mathbf{k}_2 d\mathbf{k}_3 \int d\Omega |v_{12}| \frac{d\sigma}{d\Omega} \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) \\
& \times \{ f(\mathbf{x}, \mathbf{k}_1, t) f(\mathbf{x}, \mathbf{k}_2, t)[1 - f(\mathbf{x}, \mathbf{k}_3, t)][1 - f(\mathbf{x}, \mathbf{k}_4, t)] \\
& - f(\mathbf{x}, \mathbf{k}_3, t) f(\mathbf{x}, \mathbf{k}_4, t)[1 - f(\mathbf{x}, \mathbf{k}_1, t)][1 - f(\mathbf{x}, \mathbf{k}_2, t)] \}
\end{aligned}$$

Delta resonances satisfy a similar RVUU equation with mean-field potentials related to those of nucleons via their isospin structures in terms of those of nucleons and pions.

Equilibrium requires $f_1 f_2(1-f_3)(1-f_4) = f_1 f_2(1-f_3)(1-f_4)$ or $f_1 f_2 = f_3 f_4$ if fermi statistics is neglected. Then

$$\ln f_1 + \ln f_2 = \ln f_3 + \ln f_4$$

Energy conservation

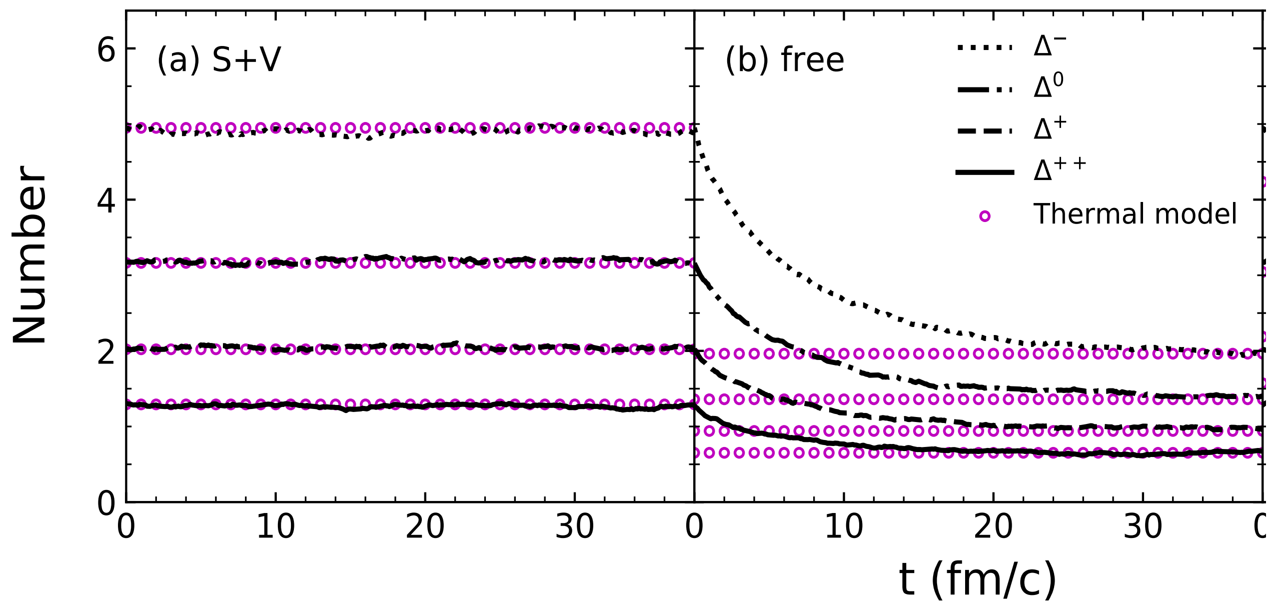
$$E_1 + U_1 + E_2 + U_2 = E_3 + U_3 + E_4 + U_4$$

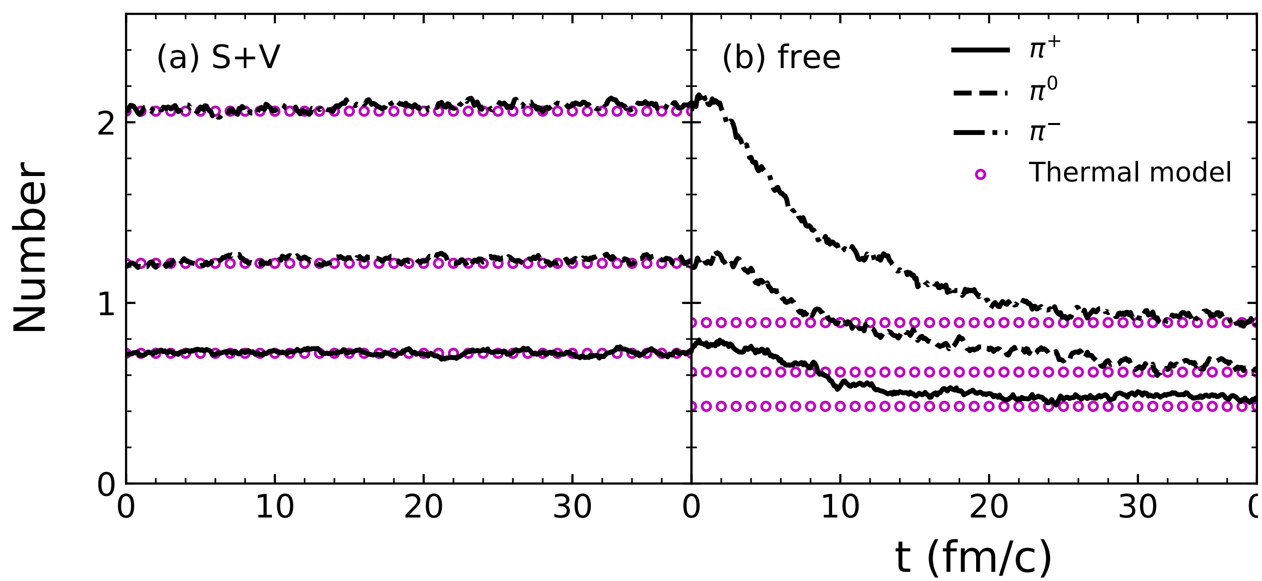
with $E_i = \sqrt{m_i^2 + k_i^2}$ and U_i being the potential, implies that

$$\ln f_i \propto E_i + U_i \to f_i = e^{-\frac{\sqrt{m_i^2+k_i^2}+U_i}{T}}$$

- For elastic scatterings $nn \to nn, np \to np, pp \to pp$, $U_1 + U_2 = U_3 + U_4$. So $\mathbf{k}_3$ and $\mathbf{k}_4$ are determined from $E_1 + E_2 = E_3 + E_4$, and potentials have no effect on collisions.

Effects of energy conservation on chemical equilibrium in hot dense symmetric nuclear matter (RVUU with $NL\rho$)





Zhang & Ko, PRC 97, 014910 (2018)

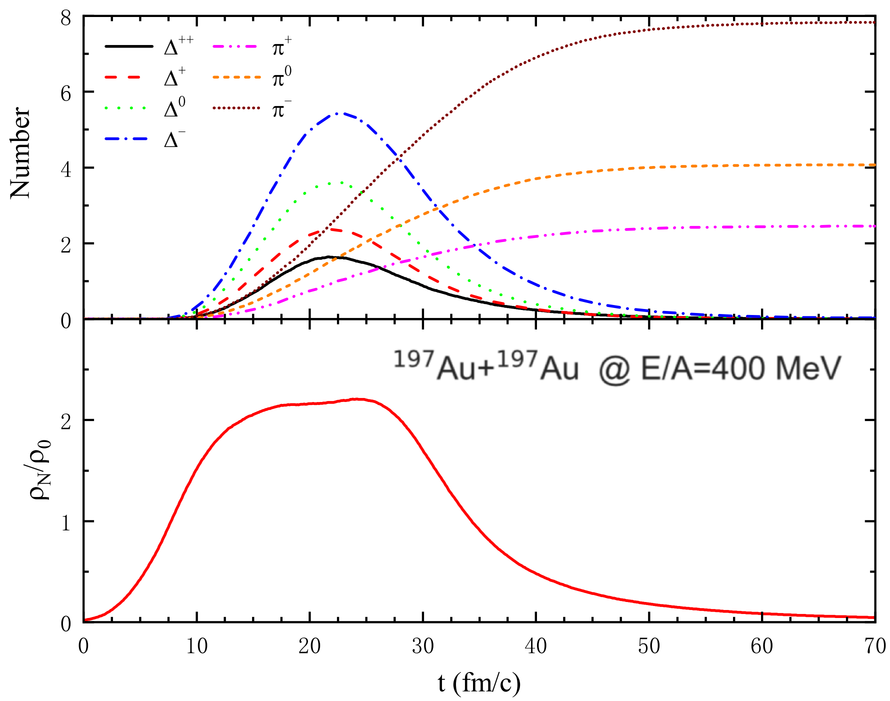
Nucleons, Deltas and pions in a box at T= 60 MeV,

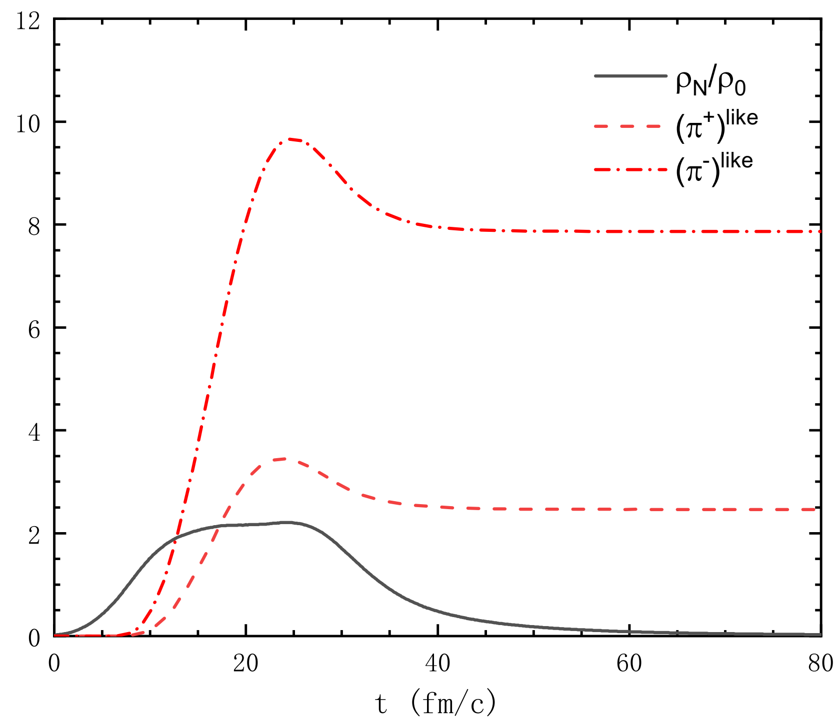
$$\rho = 0.24\ \mathrm{fm}^{-3},$$
$$\rho_I = 0.096\ \mathrm{fm}^{-3}$$

- Including potentials in the energy conservation during collisions keeps correct equilibrium distributions.
- Treating collisions as in free space leads to equilibrium distributions without potential effects.

Pion production in Au+Au collisions at E = 400 AMeV and b= 1fm (RVUU)

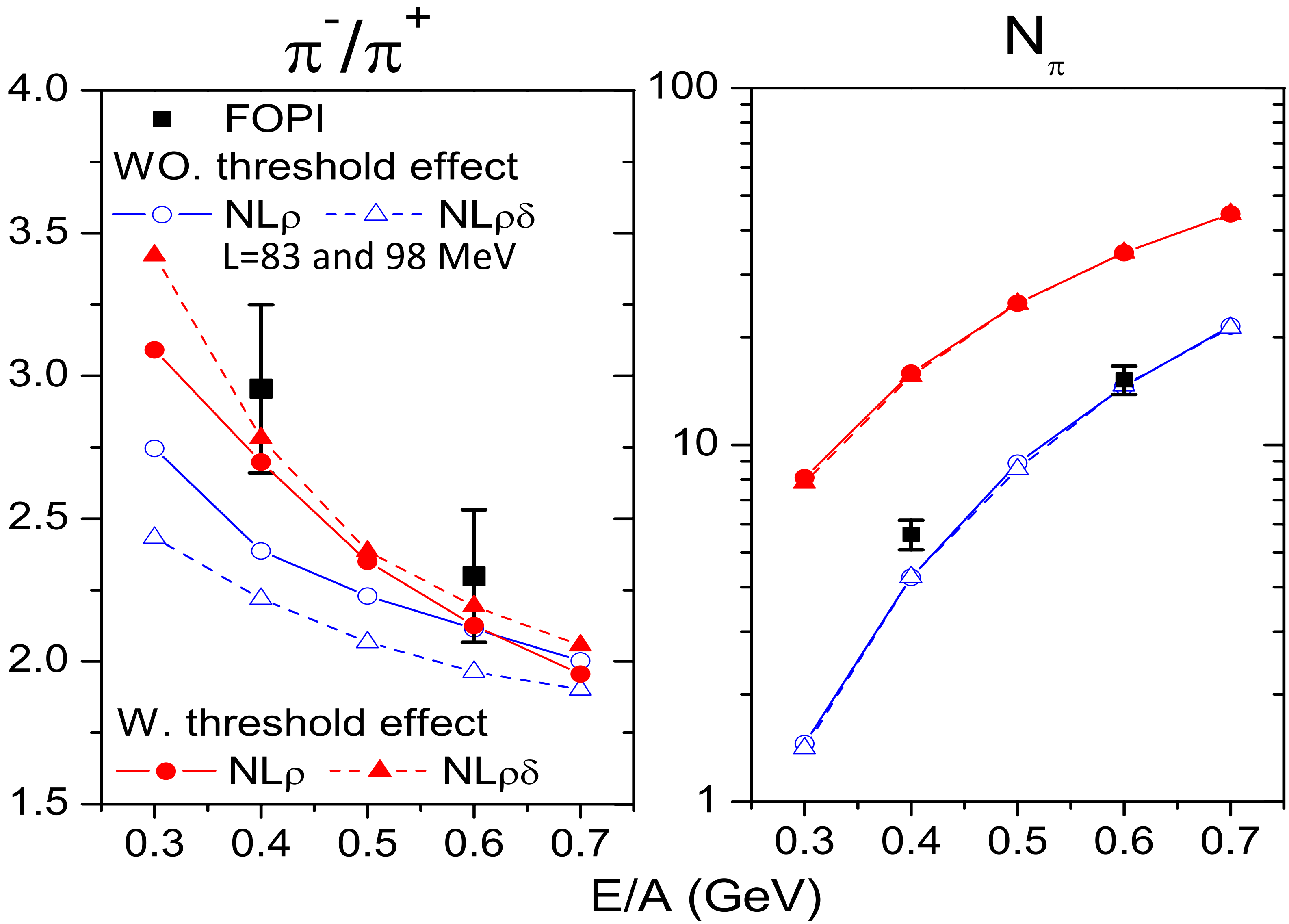
Song & Ko, PRC 91, 014901 (2015)





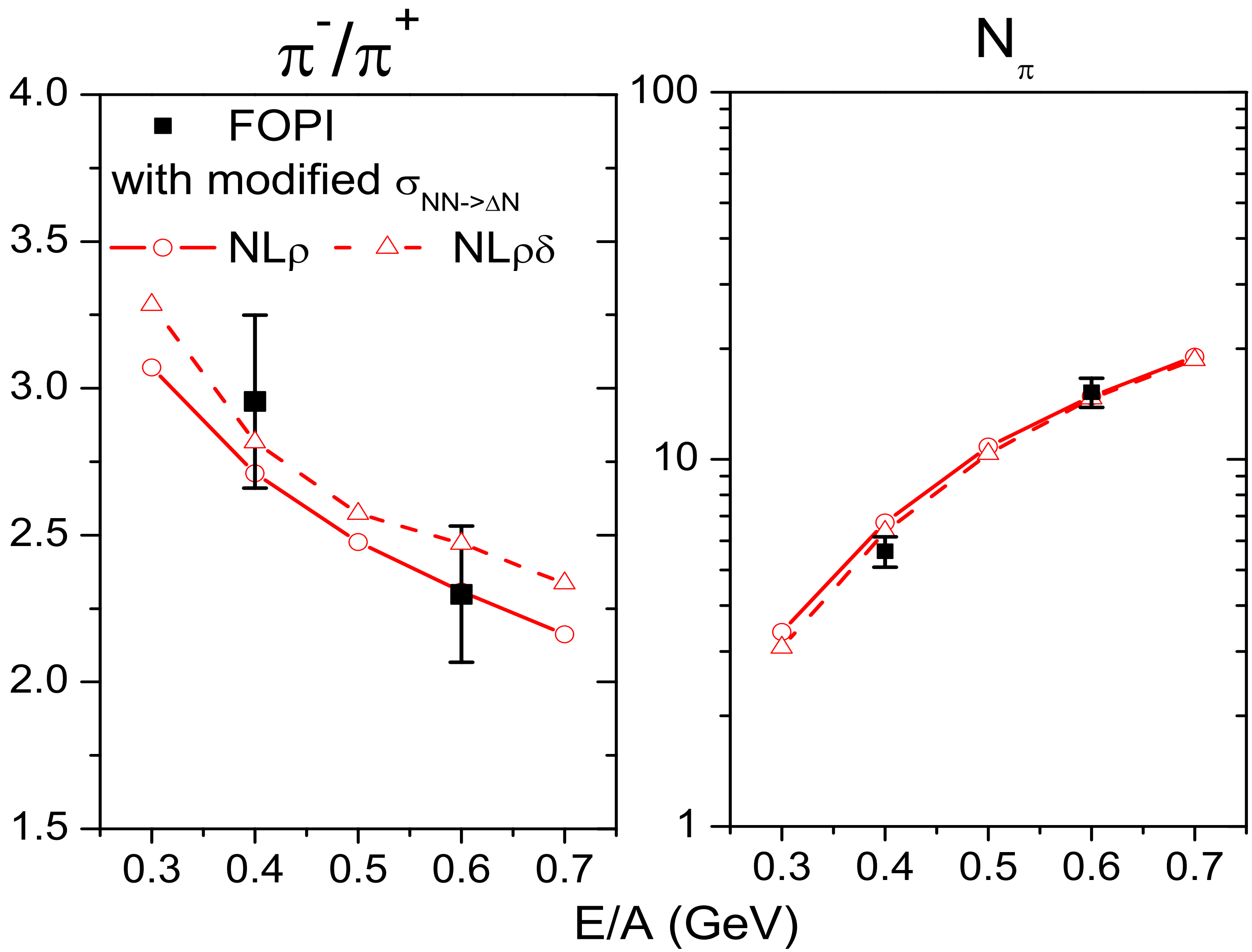
- Deltas are produced during high density stage and decay to pions as the matter expands.
- Both effective π^- and π^+ numbers (including those in Delta resonances change only slightly after maximum compression (chemical freeze out), due to constancy of entropy per particle [Xu & Ko, PLB 772, 290 (2017)].

In-medium threshold effects on π^-/π^+ ratio



- In-medium threshold effects increase the total pion yield, the π^-/π^+ ratio, and reverse the effect of symmetry energy.

Effects of in-medium Delta production cross sections

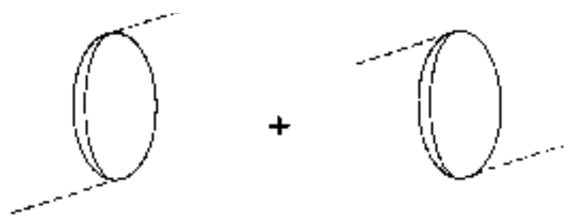


- Reproducing the total pion yield requires density-dependent Delta production cross section $\sigma_{NN\rightarrow N\Delta}(\rho) = \sigma_{NN\rightarrow N\Delta}(0)\exp(-1.65\rho/\rho_0)$, similar to those by Larionov and Mosel, NPA 728, 135 (2003) and Prassa et al., NPA 789, 311 (2007).

Pion in nuclear matter

Brown & Weise, PR 22, 279 (1975)

- Pion p-wave selfenergy

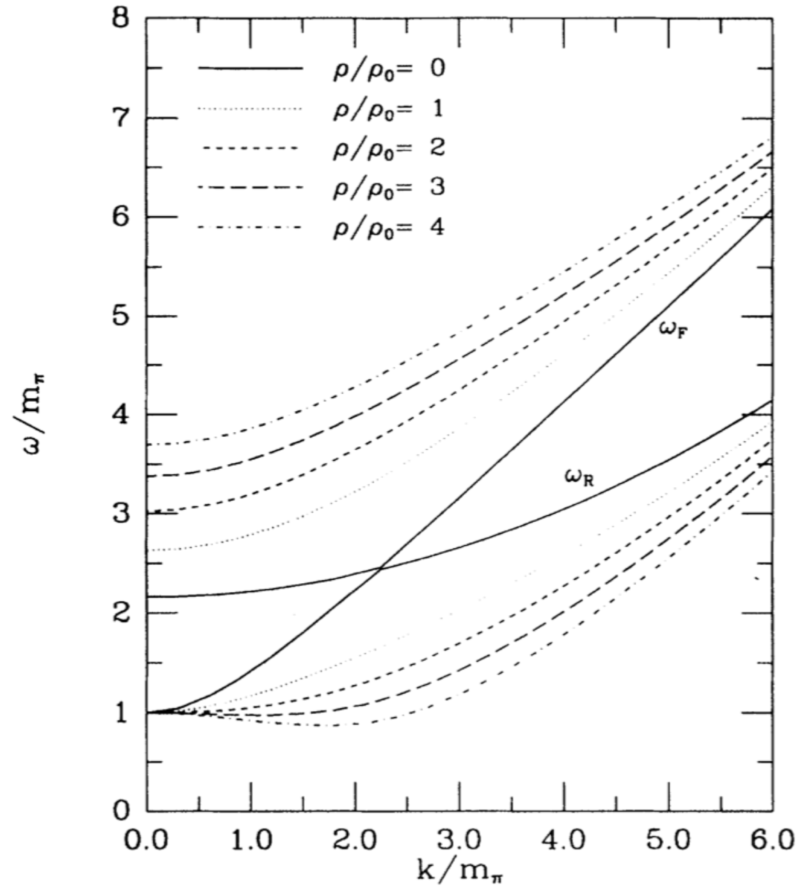


$$\Pi_0(\omega,k) \approx \frac{4}{3}\left(\frac{f_\Delta}{m_\pi}\right)^2 k^2 F^2(k)\rho \frac{\omega_0}{\omega^2-\omega_0^2}$$

$$\omega_0 \approx \frac{k^2}{2m_\Delta} + m_\Delta - m_N$$

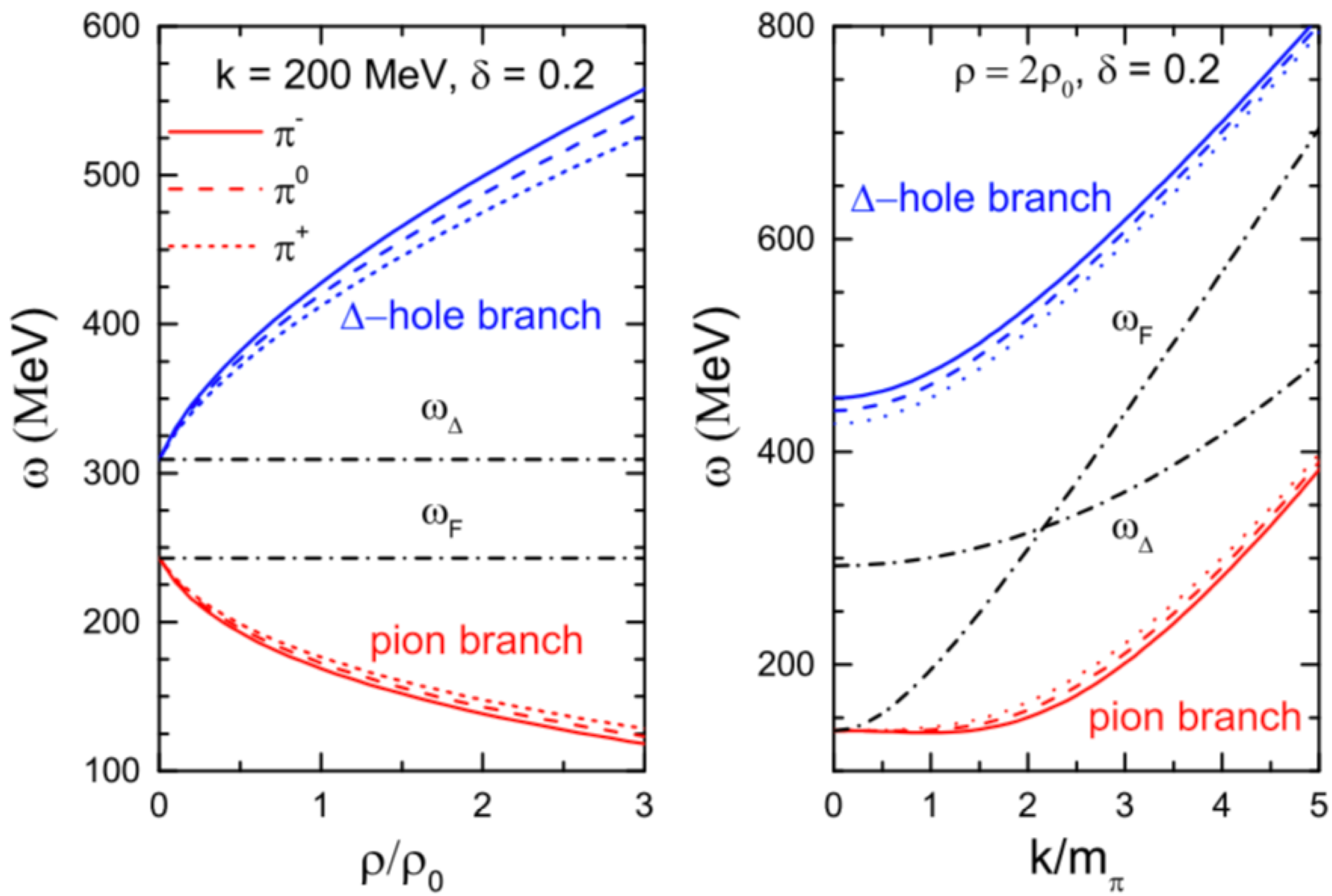
Including short-range repulsion through the Migdal parameter G’ ~ 0.3

$$\Pi^{m_t}(\omega,k) = \frac{\Pi_0^{m_t}}{1-g'\Pi_0^{m_t}/k^2}$$



- Leading to a softening of the pion dispersion relation

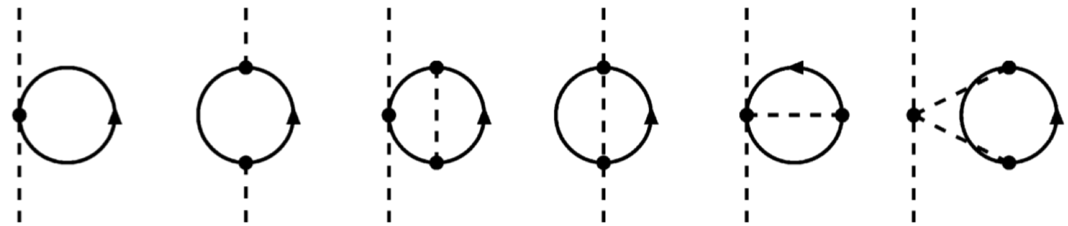
Pion energy in asymmetric nuclear matter



- Pion branch is lower in energy and thus more important.
- π^- is lower than π^+ and thus expect to enhance the π^-/π^+ ratio as for a softer symmetry energy.

Pion s-wave interaction in asymmetric nuclear matter

- Chiral perturbation theory   Kaiser & Weise, PLB 512, 283 (2001)



$$\Pi^{-}(\rho_n,\rho_p) = \rho_n[T^{-}_{\pi N} - T^{+}_{\pi N}] - \rho_p[T^{-}_{\pi N} + T^{+}_{\pi N}] + \Pi^{-}_{\mathrm{rel}}(\rho_n,\rho_p) + \Pi^{-}_{\mathrm{cor}}(\rho_n,\rho_p)$$

$$\Pi^{+}(\rho_p,\rho_n) = \Pi^{-}(\rho_n,\rho_p)$$

$$\Pi^{0}(\rho_n,\rho_p) = -(\rho_p + \rho_n)T^{+}_{\pi N} + \Pi^{0}_{\mathrm{cor}}(\rho_n,\rho_p)$$

Isospin even and odd πN-scattering matrices extracted from energy shift and width of 1s level in pionic hydrogen atom

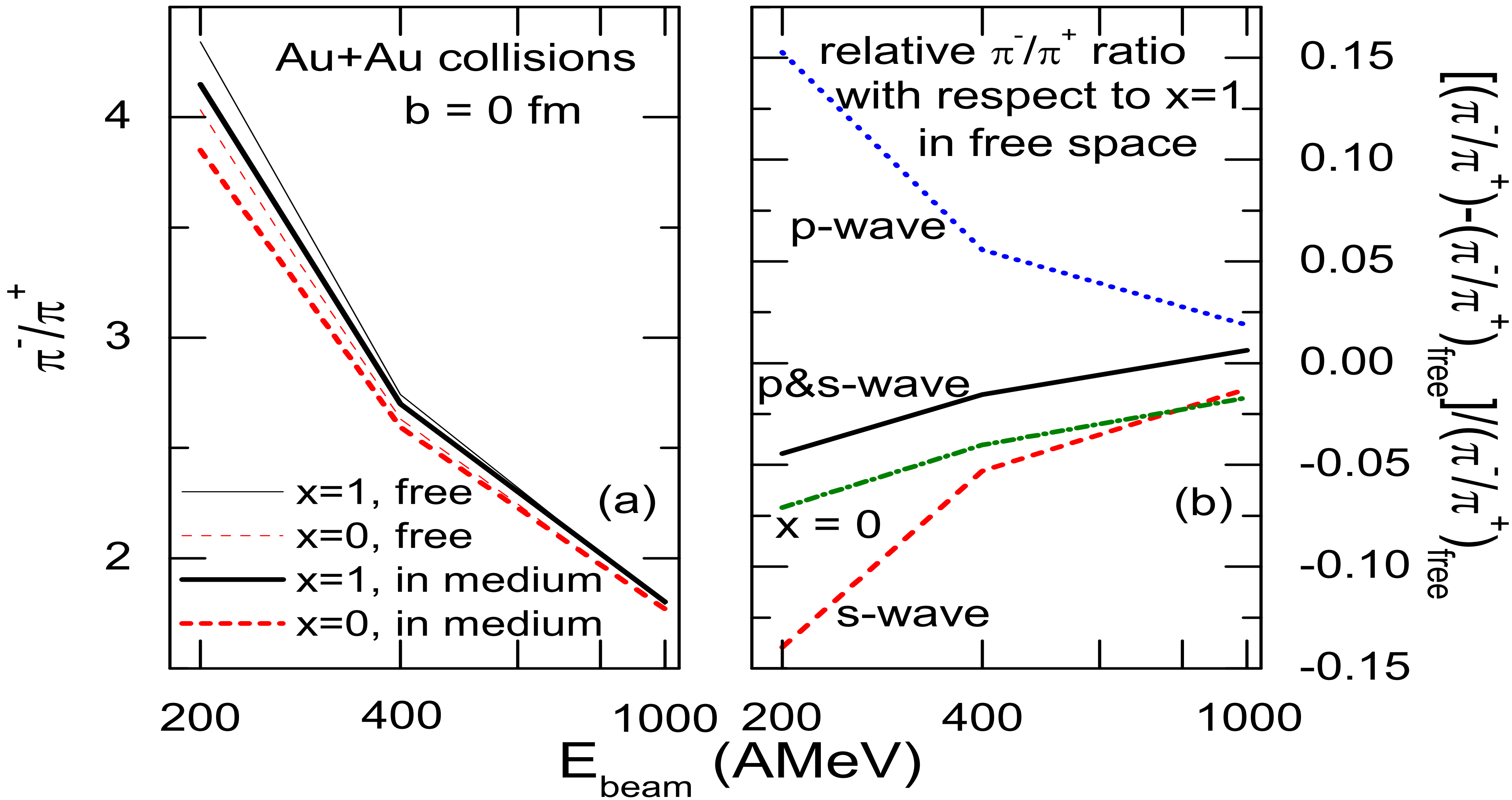
$$T^{+}_{\pi N} \approx 1.847\,\mathrm{fm} \text{ and } T^{-}_{\pi N} \approx -0.045\,\mathrm{fm}$$

At density $\rho=0.165$ fm^{-3} and isospin asymmetry $\delta=0.2$,

$$\Delta m_{\pi^-} = 13.8\,\mathrm{MeV},\ \ \Delta m_{\pi^+} = -1.2\,\mathrm{MeV},\ \ \Delta m_{\pi^0} = 6.1\,\mathrm{MeV},$$

Beam energy dependence of pion in-medium effect

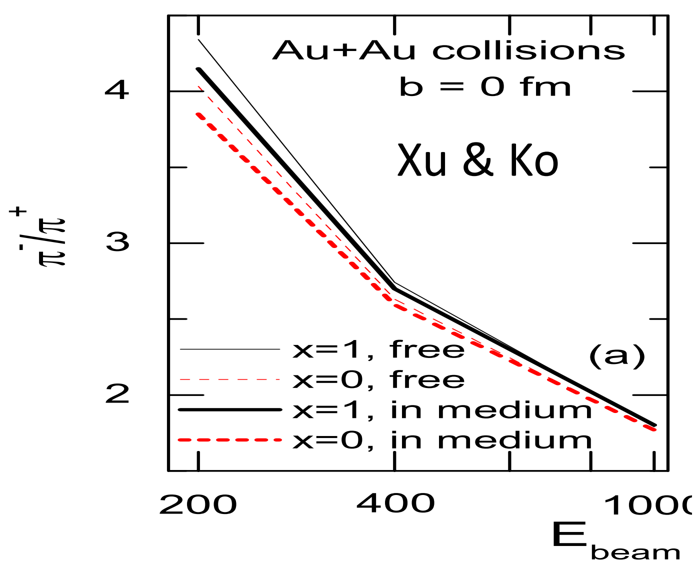
Xu, Chen, Ko, Li & Ma, PRC 87, 067601 (2013)

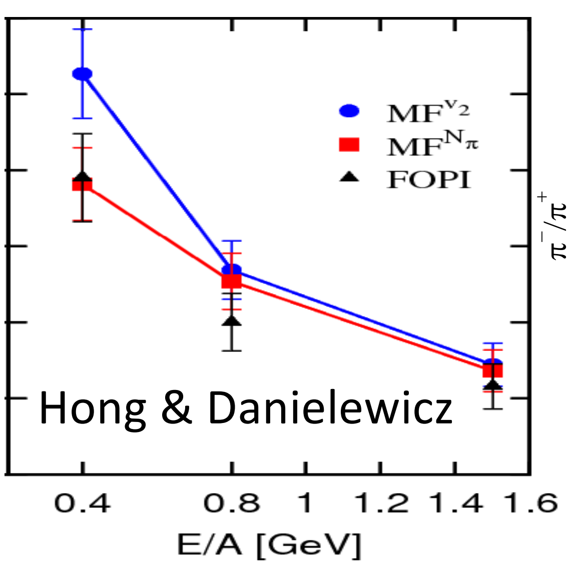


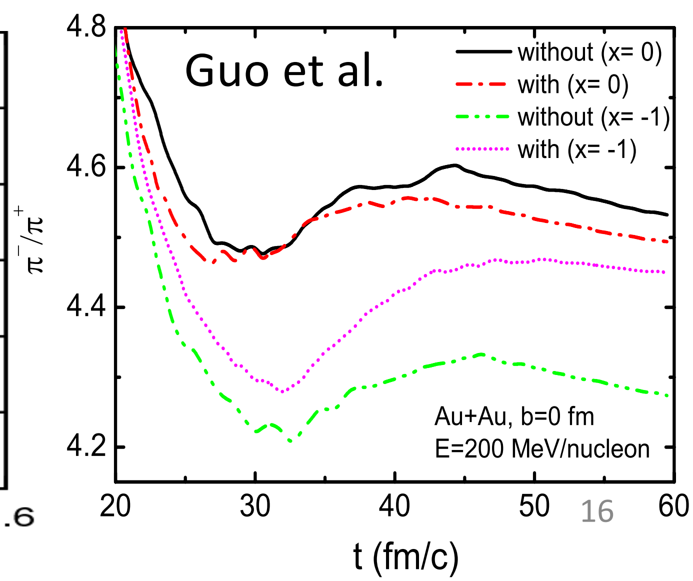
- Pion in-medium effect decreases the π^-/π^+ ratio, and the effect is larger at lower collision energies.

Pion potential effects on charged pion ratio

- Xu & Ko, PRC 81, 024910 (2010); Xu, Chen, Ko, Li & Ma, PRC 87, 067601 (2013): Thermal model → Including both pion s- and p-wave interactions, which have opposite effects, decreases the π^-/π^+ ratio.
- Hong and Danielewicz, PRC 90, 024605 (2014): pBUU → π^-/π^+ ratio is insensitive to stiffness of symmetry energy after including pion s-wave potential.
- Guo, Yong, Liu & Zuo, PRC 91, 054616 (2015): IBUU → pion s- and p-wave potentials and symmetry potential have opposite effects. (p-wave potential essentially vanishes in this study because of average over the pion and Delta-hole branches.)
- Feng, arXiv:1606.01083 [nucl-th]: LQMD → similar to Guo et al.

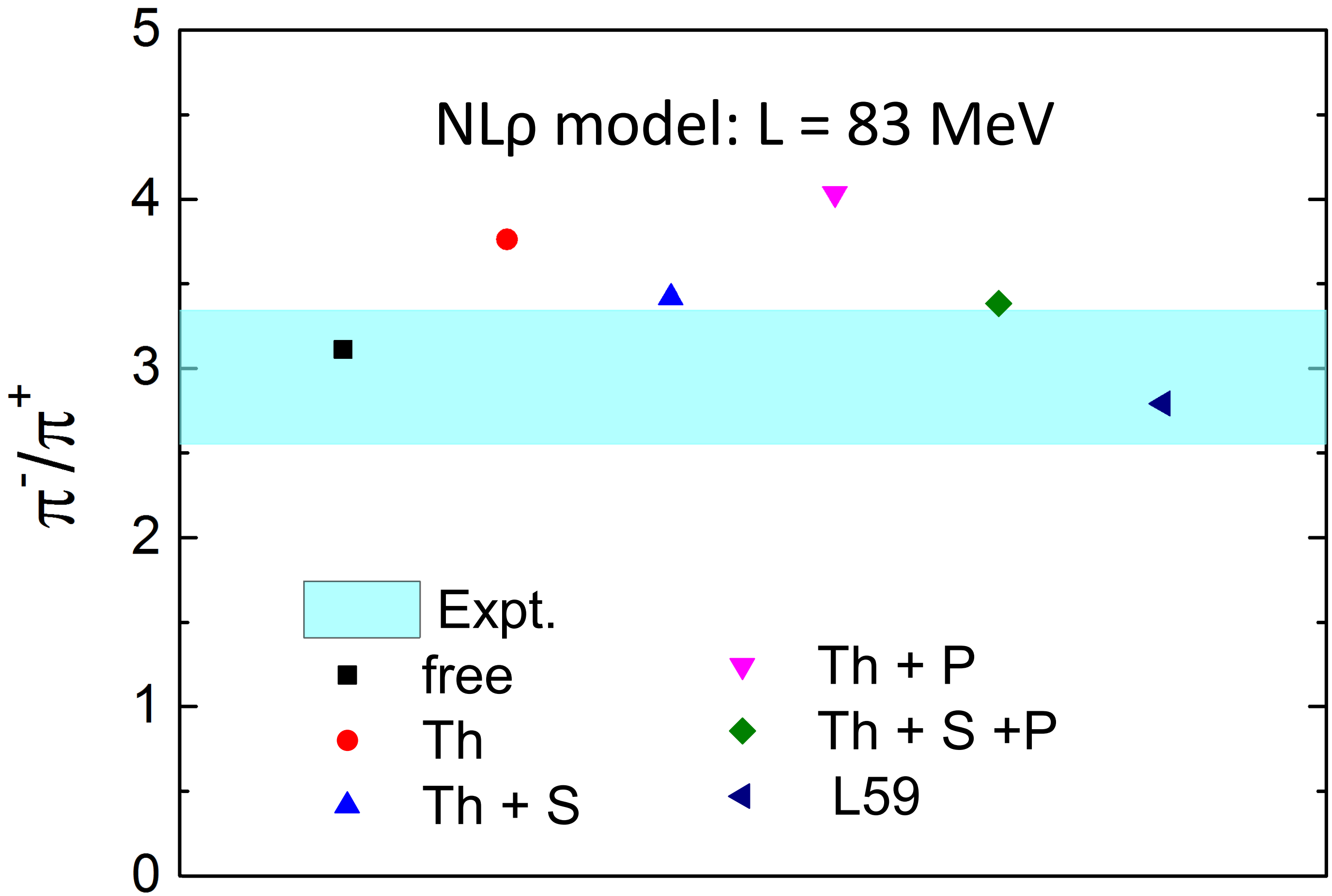






Charged pion ratio in Au+Au @ 400A MeV

Zhang & Ko, PRC 95, 064904 (2017)



- Charged pion ratio is increased by threshold effect, reduced by s-wave potential, increased by p-wave potential, leading to a somewhat lager ratio compared to that without any medium effects.
- Reproducing FOPI data requires a small symmetry energy slope parameter L comparable with the constraints from nuclear structure and reactions as well as neutron star properties.

Summary

- Nuclear symmetry energy affects the π^-/π^+ ratio in HIC, but effect is only 30% between the supersoft and the superstiff, which is larger than current uncertainty among transport models.
- Effect of potentials needs to be included in scattering to ensure correct equilibrium pion abundance.
- In-medium threshold effects increase the total pion yield, the π^-/π^+ ratio, and reverse the effect of symmetry energy.
- Charged pion ratio is reduced by pion s-wave potential and increased by pion p-wave potential. The net effect is a reduction of the ratio if keeping the total pion number unchanged.

→ Require better theoretical modeling of pion production in HIC to extract information on the stiffness of nuclear symmetry energy at high density from the ratio of charged pions.

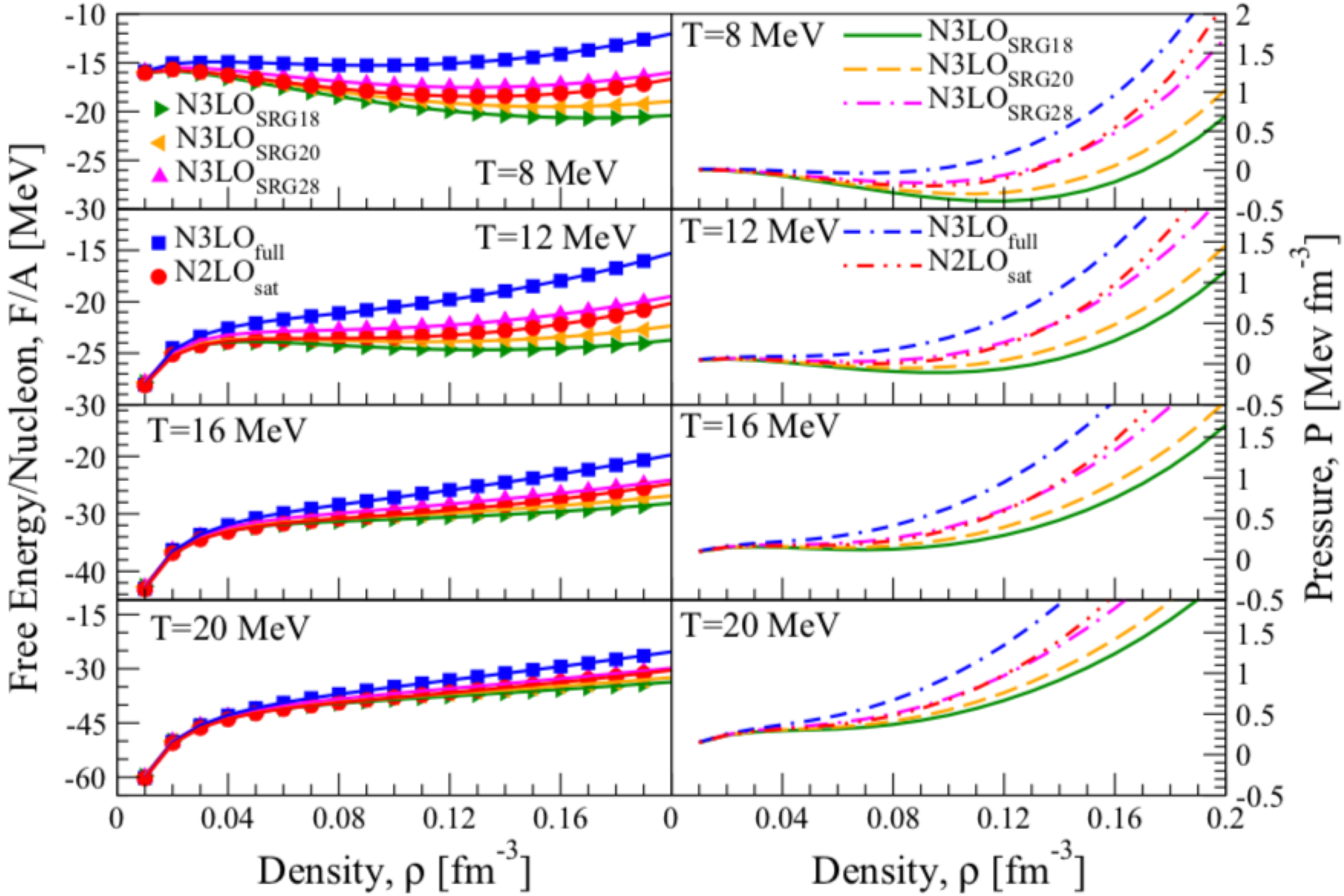
Appendix

How does the nuclear equation of state determined from heavy ion collisions differ from that for a cold nuclear matter that is needed for studying the properties of neutron stars?

Xu, Carbone, Zhang &ko, arXiv:1904:09669 [nucl-th]

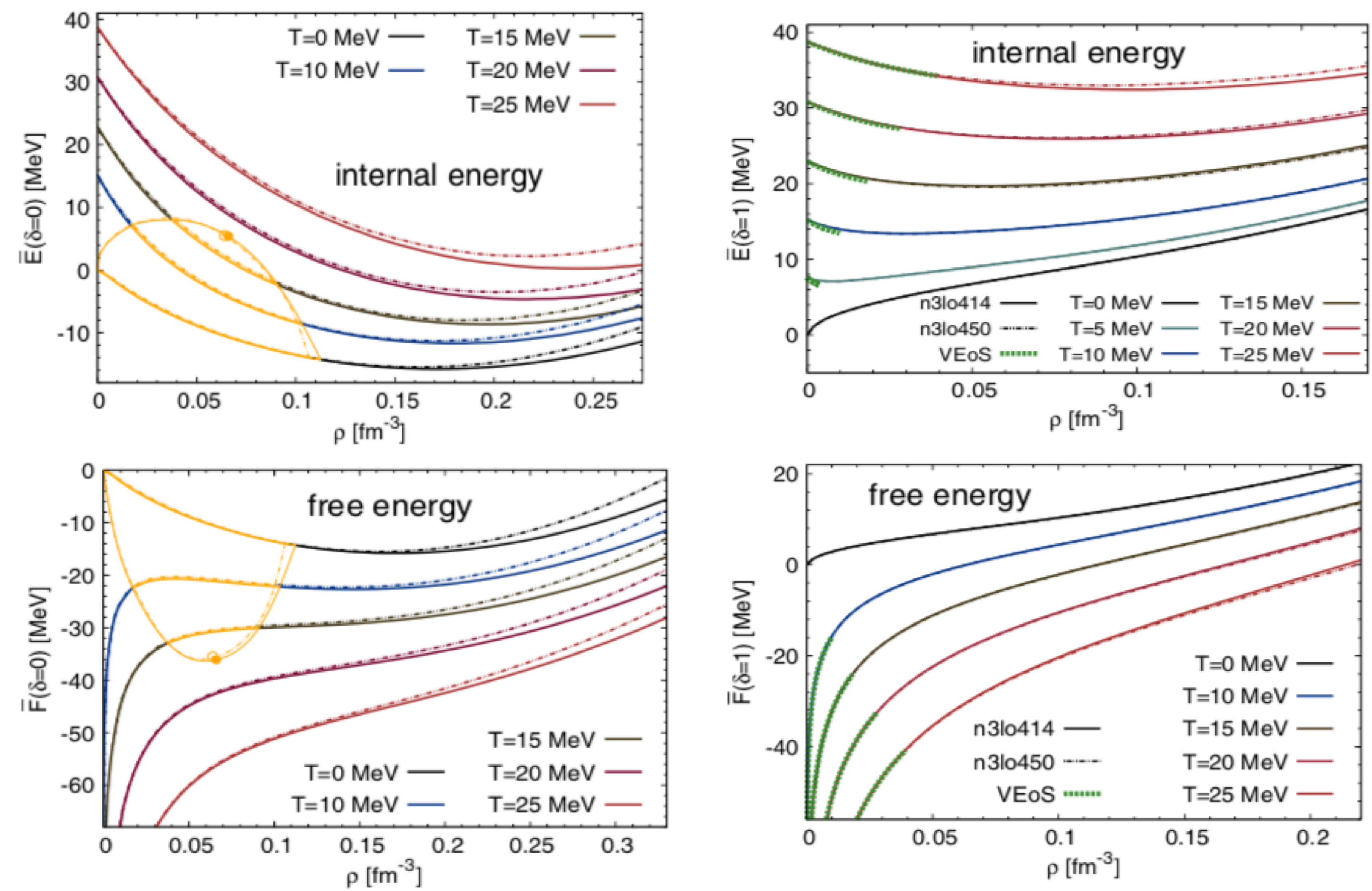
Nuclear equation of state at finite temperature

Carbone, Polls & Rios, PRC 98, 025804 (2018), Green function approach
with chiral NN interactions



Nuclear equation of state at finite temperature

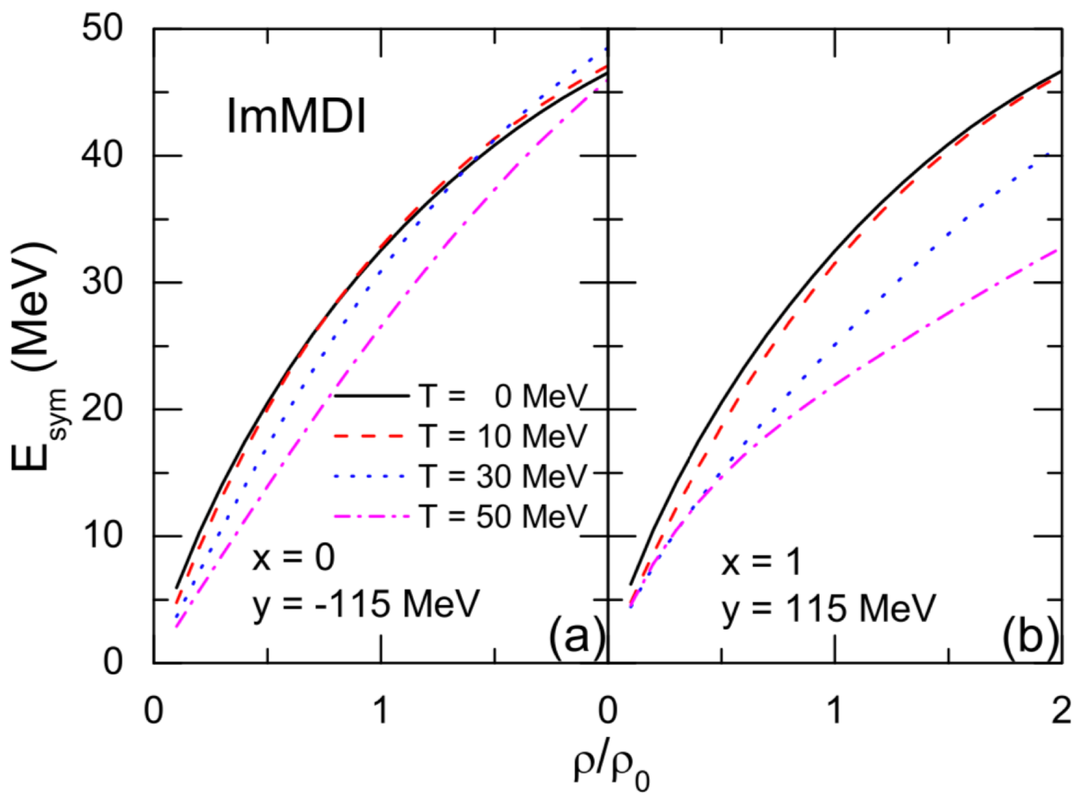
Wellenhofer, Holt &Kaiser, PRC 92, 015801 (2015), Perturbative approach with chiral NN interactions



Improved isospin- and momentum dependent interaction

Xu, Chen & Li, PRC 91, 014611 (2015)

$$V_{\mathrm{ImMDI}} = \frac{A_u \rho_n \rho_p}{\rho_0} + \frac{A_l}{2\rho_0}(\rho_n^2 + \rho_p^2) + \frac{B}{\sigma+1}\frac{\rho^{\sigma+1}}{\rho_0^\sigma} \times (1 - x\delta^2) + \frac{1}{\rho_0}\sum_{q,q'} C_{q,q'} \int\int d^3p d^3p' \frac{f_q(\vec{r},\vec{p}) f_{q'}(\vec{r},\vec{p}')}{1 + (\vec{p}-\vec{p}')^2/\Lambda^2}$$



Chiral effective theory inspired transport model χBUU

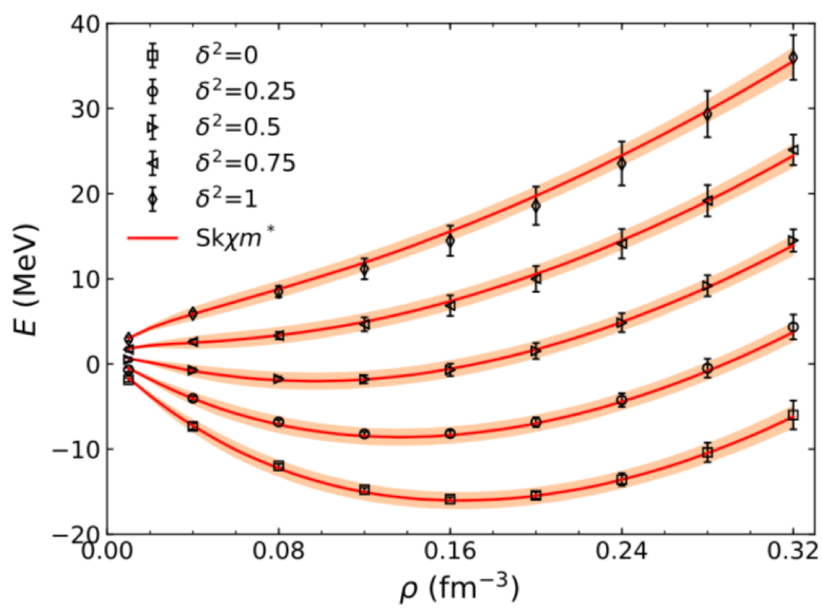
- Skχm* interaction (K_0=230 MeV, E_{sym}=31 MeV, L=45.6 MeV)

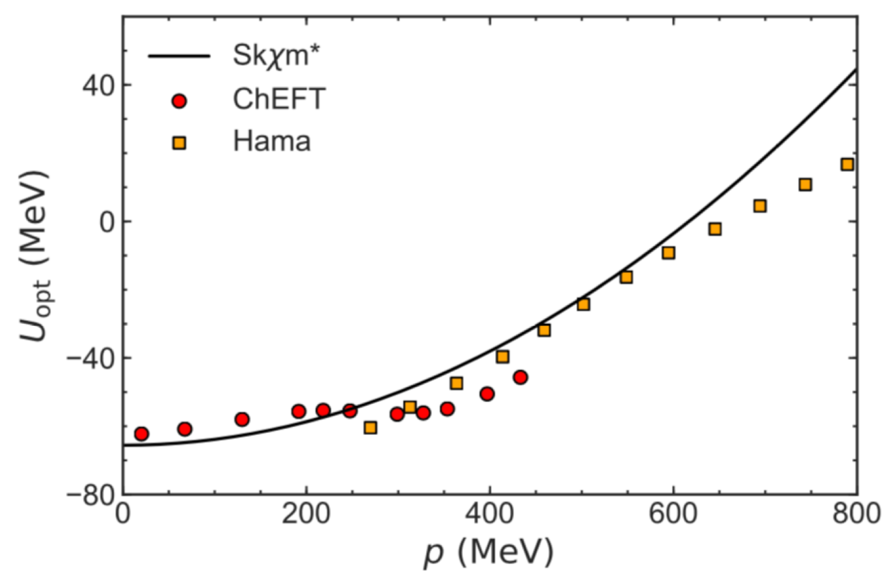
Zhang, Liam, Holt & Ko, PLB 777, 73 (2018), Zhang & Ko, submitted to PRC

$$
\begin{aligned}
v(\mathbf{r_1},\mathbf{r_2}) &= t_0(1+x_0P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2)+\frac{1}{2}t_1(1+x_1P_\sigma)[\mathbf{k}'^2\delta(\mathbf{r}_1-\mathbf{r}_2)+\mathrm{c.c.}] \\
&+ t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k}+\frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)\delta(\mathbf{r}_1-\mathbf{r}_2) \\
&+ iW_0(\sigma_1+\sigma_2)\cdot[\mathbf{k}'\times\delta(\mathbf{r_1}-\mathbf{r_2})\mathbf{k}]
\end{aligned}
$$

- Parameters fitted to EOS from chiral effective theory and binding energies of 7 doubly magic nuclei

- Good description of momentum dependence of optical potential below 600 MeV, dipole polarizability, and neutron skin thickness





Nucleon momentum distribution at different temperatures

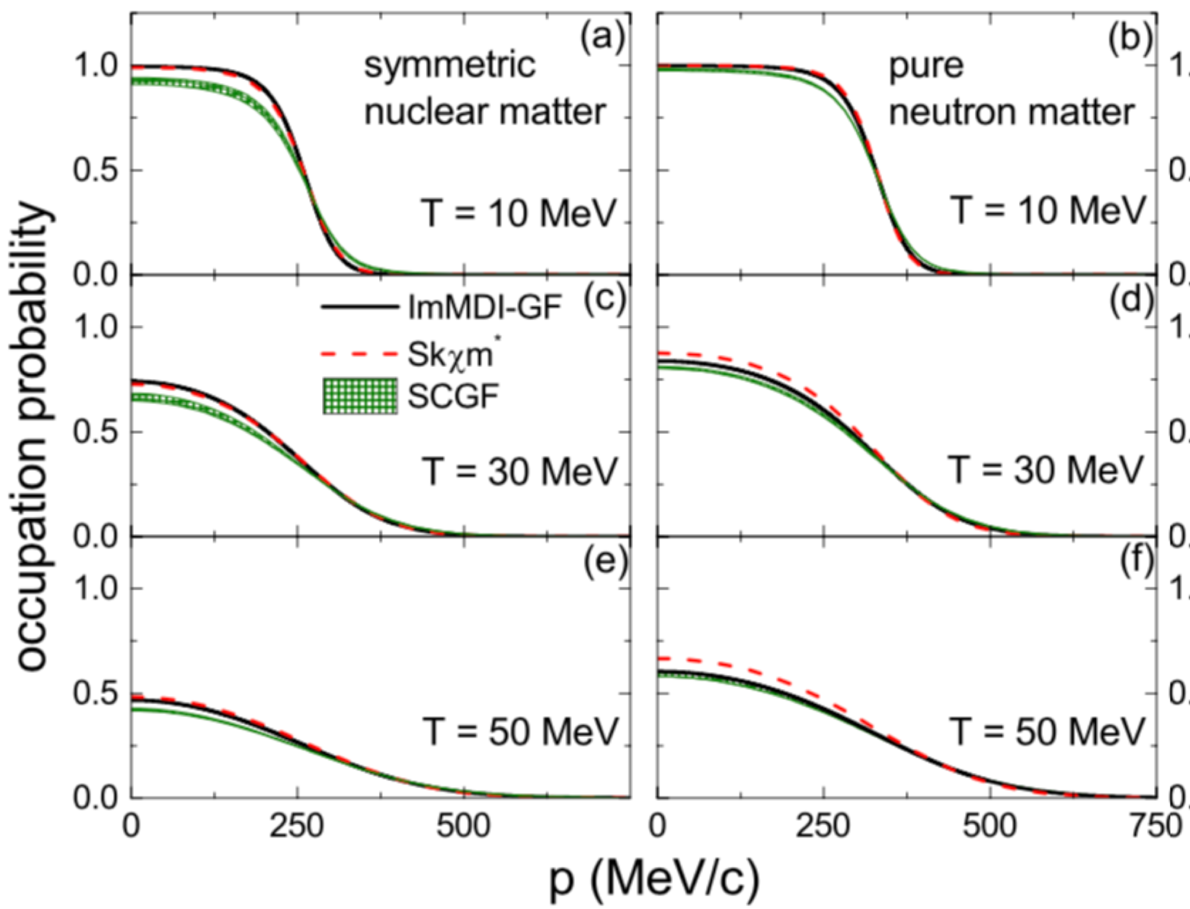


FIG. 1: (Color online) Nucleon occupation probability at $\rho_0 = 0.16\ \mathrm{fm}^{-3}$ as a function of nucleon momentum in symmetric nuclear matter (left) and pure neutron matter (right) at various temperatures from the ImMDI-GF, Skχm*, and SCGF calculations.

Nuclear matter EOS at different temperatures

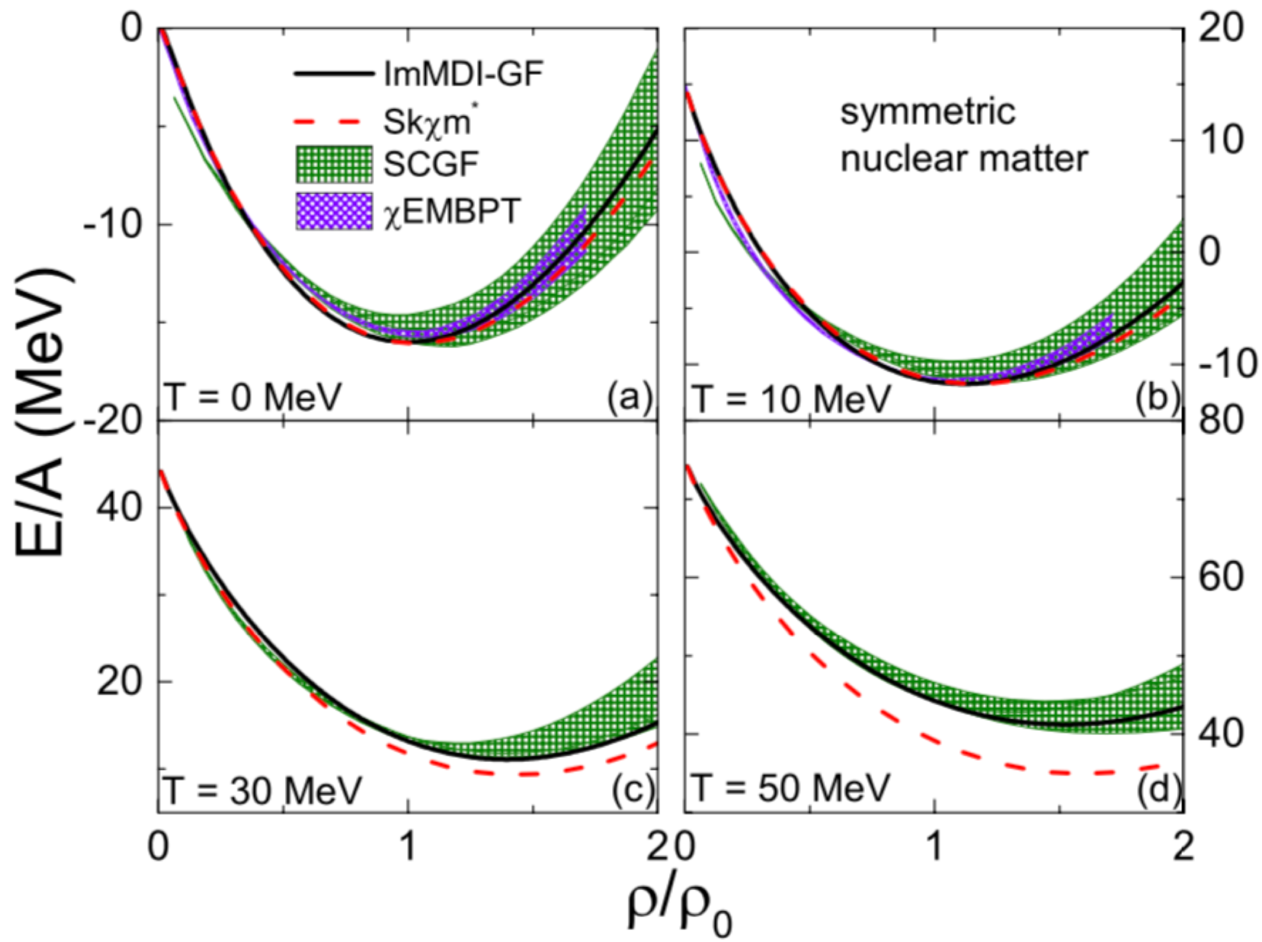
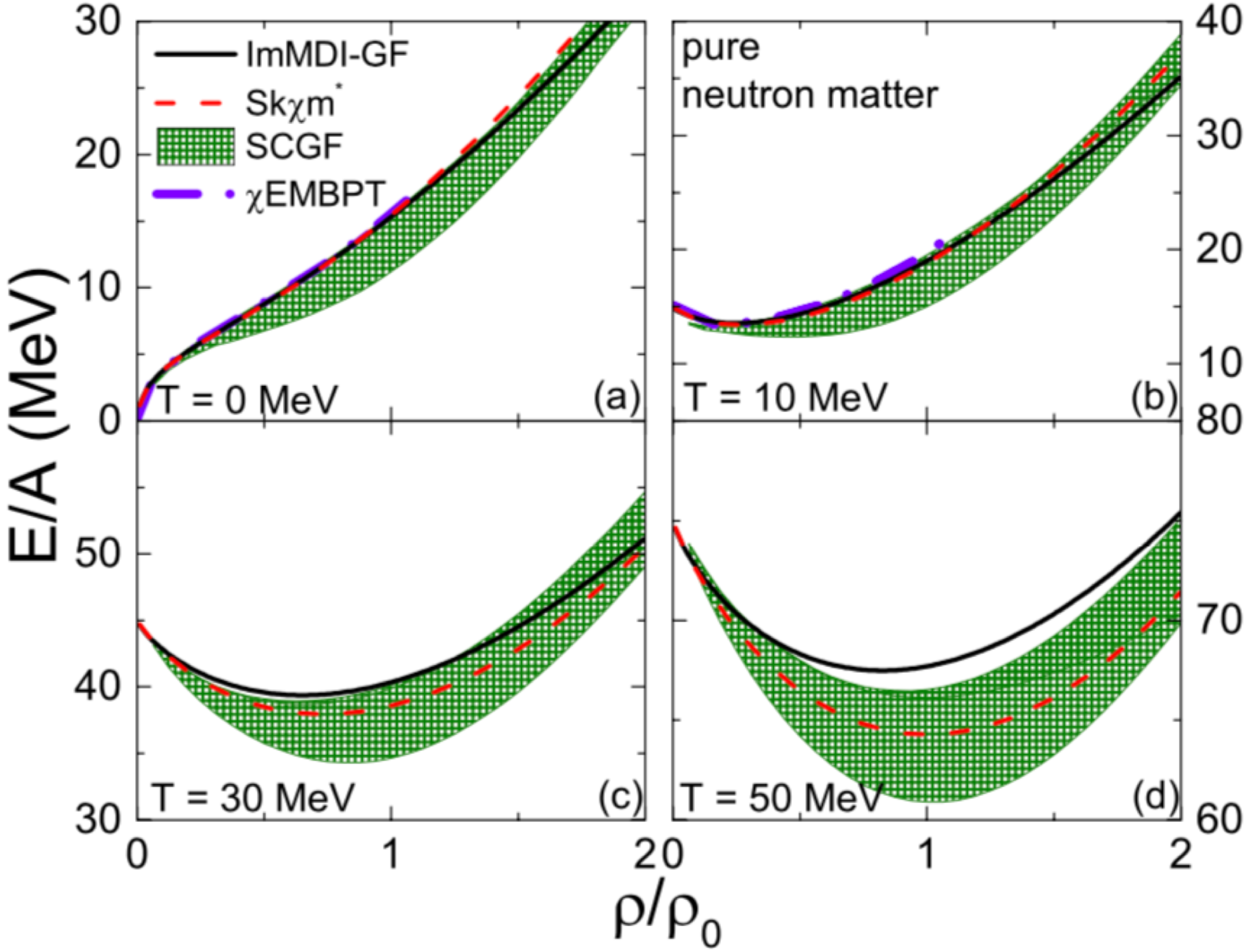


FIG. 3: (Color online) Total energy per nucleon as a function of reduced nucleon density for symmetric nuclear matter at various temperatures from ImMDI-GF and Skχm* compared with results from the SCGF approach. The results with uncertainty bands in panels (a) and (b) are from the χEMBPT using the n3lo414 and n3lo450 forces [15].

Pure neutron matter EOS at different temperatures



Momentum dependence of nucleon potential in nuclear matter at different temperatures

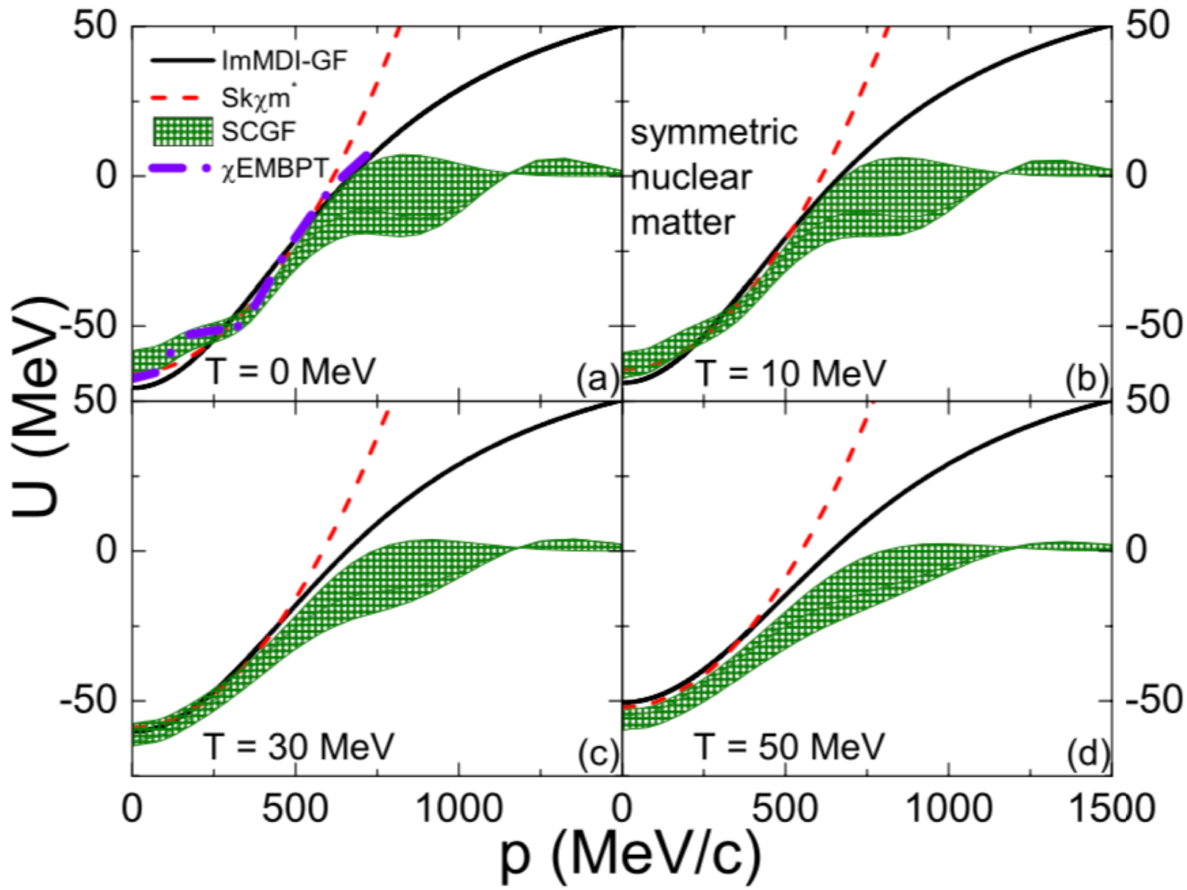
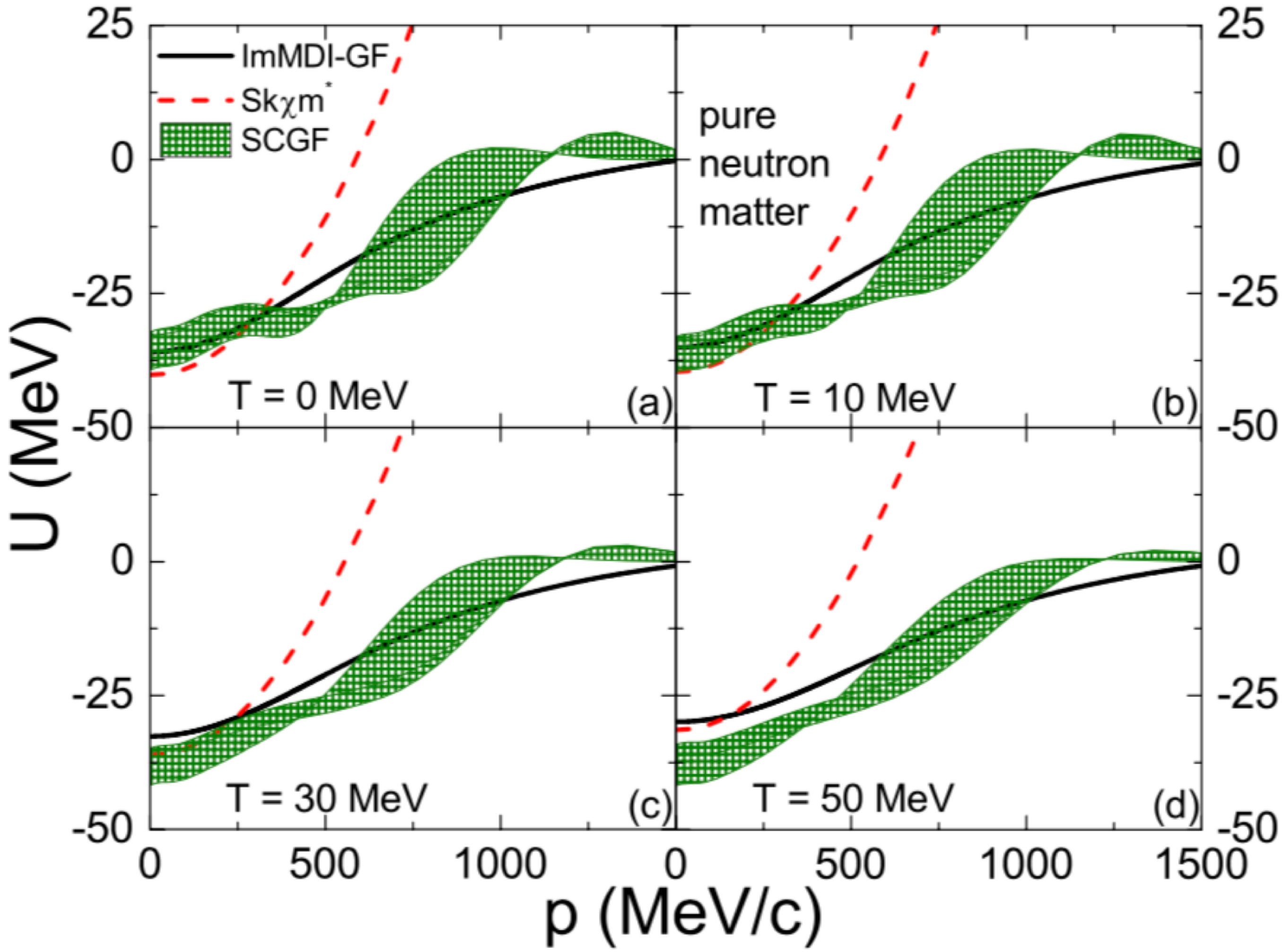


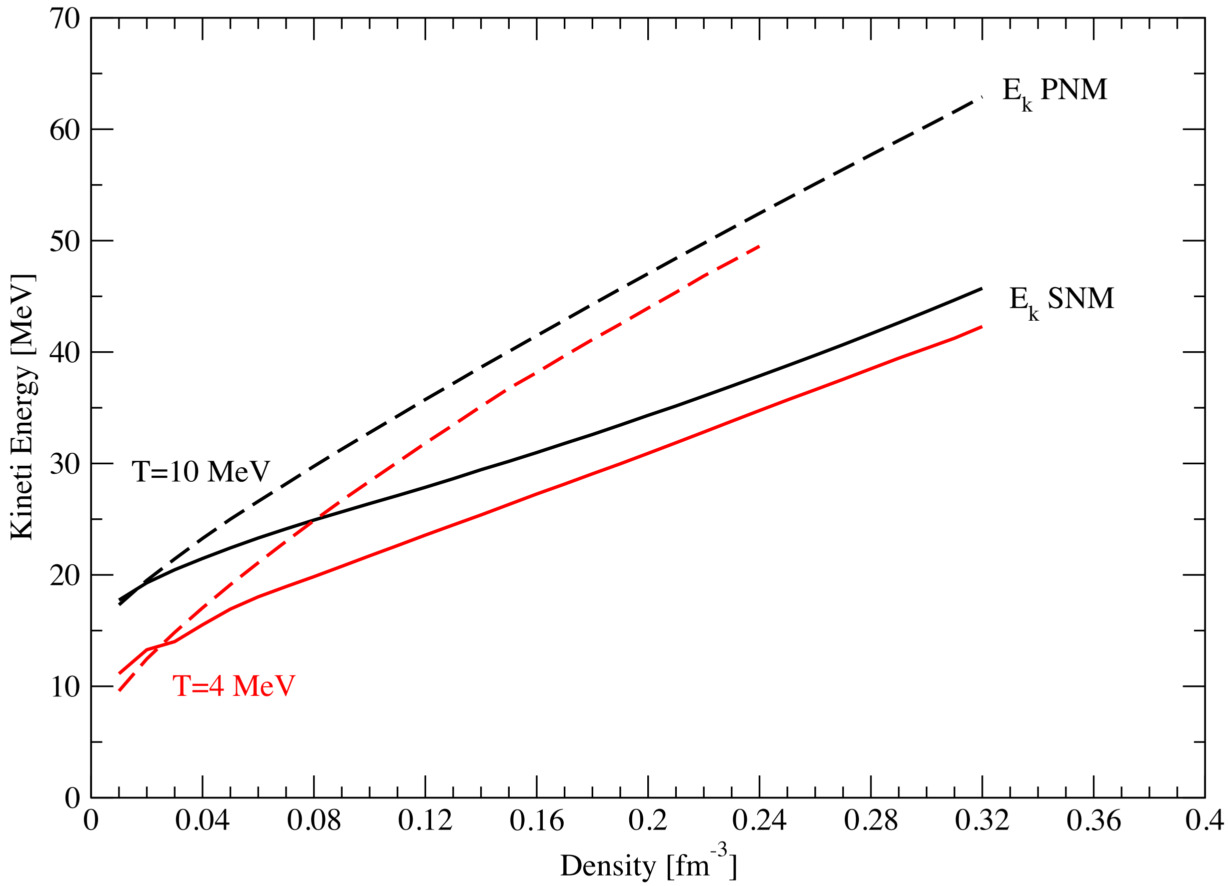
FIG. 5: (Color online) Mean-field potential at $\rho_0 = 0.16\mathrm{fm}^{-3}$ as a function of nucleon momentum in symmetric nuclear matter at various temperatures from ImMDI-GF and Skχm* compared with results from the SCGF approach. Result at $T = 0$ MeV from the χEMBPT using the n3lo450 force [40] is also plotted for comparison.

Momentum dependence of nucleon potential in Pure neutron matter at different temperatures



Symmetry energy from kinetic contribution at finite temperature from self-consistent Green's function approach

Arianna Carbone



At $\rho = 0.16/\mathrm{fm}^3$

- T=4 MeV, $\mathrm{E}_{\mathrm{sym}}^{\mathrm{kin}} \approx 40 - 25 \approx 15$ MeV
- T=10 MeV, $\mathrm{E}_{\mathrm{sym}}^{\mathrm{kin}} \approx 42.5 - 30 \approx 12.5$ MeV

Conclusions

- Effective interactions with the correct momentum dependence of the mean-filed potential can properly describe the properties of hot dense nuclear matter and are thus suitable for use in transport models to study heavy-ion collisions at intermediate energies.