Pauli and cluster correlations

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ECT* Workshop "Challenges to Transport Theory for Heavy-Ion Collisions",

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- Pauli
  - A little about $1 - f$ in AMD
- Clusters
  - Clusters and fragments in transport models
  - Effects of clusters on collision dynamics

Transport theories (BUU equation without fluctuation)

A many-body system is approximately described by using **one-body distribution function** $f(\mathbf{r}, \mathbf{p}, t)$ in phase space or one-body density matrix $\rho(\mathbf{r}, \mathbf{r}')$.

$$\frac{\partial f(\mathbf{r}, \mathbf{p}, t)}{\partial t} = \frac{\partial h}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{p}} - \frac{\partial h}{\partial \mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{r}} + I_{\text{coll}}$$

- Mean field, or single-particle Hamiltonian $h[f](\mathbf{r}, \mathbf{p}, t)$
- Collision term with Pauli blocking

$$I_{\text{coll}}[f](\mathbf{r}, \mathbf{p}, t) = \int \frac{d\mathbf{p}_2}{(2\pi\hbar)^3} \int d\Omega\, |v| \left(\frac{d\sigma}{d\Omega}\right)_v \Big\{ f(\mathbf{r}, \mathbf{p}_3, t) f(\mathbf{r}, \mathbf{p}_4, t) \big[1 - f(\mathbf{r}, \mathbf{p}, t)\big] \big[1 - f(\mathbf{r}, \mathbf{p}_2, t)\big]$$
$$- f(\mathbf{r}, \mathbf{p}, t) f(\mathbf{r}, \mathbf{p}_2, t) \big[1 - f(\mathbf{r}, \mathbf{p}_3, t)\big] \big[1 - f(\mathbf{r}, \mathbf{p}_4, t)\big] \Big\}$$

- How can one describe many-body dynamics with the one-body distribution function?

  — Fluctuation of f, or more generally many-body correlations
- How to guarantee the Pauli principle.
  - Technical/fundamental problems due to the use of test particles.
  - Quantum case beyond local-density approximation.

Pauli principle for $f(\mathbf{r}, \mathbf{p})$

The Wigner transform of the one-body density operator $\hat{\rho}$ for a many-body state $|\Psi\rangle$

$$\hat{\rho} = A \operatorname*{Tr}_{2,\ldots,A} |\Psi\rangle\langle\Psi|$$

$$f(\mathbf{r}, \mathbf{p}) = \int \langle \mathbf{r} - \tfrac{1}{2}\mathbf{s}|\, \hat{\rho}\, |\mathbf{r} + \tfrac{1}{2}\mathbf{s}\rangle e^{i\mathbf{p}\cdot\mathbf{s}/\hbar} d\mathbf{s}$$

Slater determinant

$$\hat{\rho}^2 = \hat{\rho} \quad \Leftrightarrow \quad f(\mathbf{r}, \mathbf{p}) \cos(\tfrac{1}{2}\hbar\Lambda) f(\mathbf{r}, \mathbf{p}) = f(\mathbf{r}, \mathbf{p}) \quad \rightsquigarrow \quad f = 1 \text{ or } 0, \quad \text{for } \hbar \to 0 \text{ or } \frac{\partial f}{\partial \mathbf{r}} = 0$$

with $\Lambda = (\overleftarrow{\partial}/\partial\mathbf{r}) \cdot (\overrightarrow{\partial}/\partial\mathbf{p}) - (\overleftarrow{\partial}/\partial\mathbf{p}) \cdot (\overrightarrow{\partial}/\partial\mathbf{r})$.

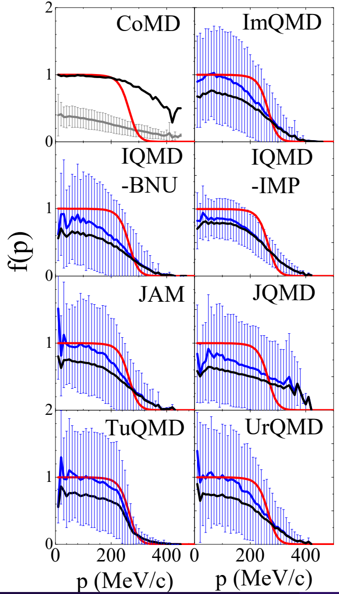
In general, the Pauli principle for the one-body density may be expressed as

$$0 \le \langle\psi|\hat{\rho}|\psi\rangle \le 1, \ {}^\forall\psi \quad \text{i.e.} \quad 0 \le \iint \frac{d\mathbf{r}d\mathbf{p}}{(2\pi\hbar)^3} g(\mathbf{r}, \mathbf{p}) f(\mathbf{r}, \mathbf{p}) \le 1 \quad \rightsquigarrow \quad 0 \le f \le 1, \text{ for } \hbar \to 0$$

where $g(\mathbf{r}, \mathbf{p})$ is the Wigner transform of $|\psi\rangle\langle\psi|$, for any normalized one-body wave function ψ.
So $g(\mathbf{r}, \mathbf{p})$ must satisfy the uncertainty principle.

Box Homework 1: Problem of Pauli blocking probability

Zhang et al., PRC97, 034625 (2018)



QMD models

The nucleon coordinates $(\mathbf{R}_k, \mathbf{P}_k)$ are samples taken from the Fermi-Dirac probability distribution f.

The evaluated f for the collision final states:

$$f(\mathbf{r},\mathbf{p}) \propto \sum_{k=1}^{A} e^{-\alpha(\mathbf{r}-\mathbf{R}_k)^2-\beta(\mathbf{p}-\mathbf{P}_k)^2}$$

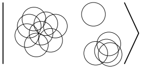
A fundamental problem

The original f cannot be reproduced by the evaluated f.

- The evaluated $\langle f\rangle$ is more broadly distributed than the original f.
- The blocking probability cannot be greater than 1, and therefore $\langle \min(f,1)\rangle \le \langle f\rangle$.

$\Rightarrow$ affects the stability of a nucleus and a Fermi gas.

Wigner and Husimi functions for AMD wave function



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter $= (2.5\ \mathrm{fm})^{-2}$

χ_{α_i} : Spin-isospin states $= p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Wigner function for the AMD wave function

$$f_\alpha(\mathbf{r},\mathbf{p}) = 8\sum_{i\in\alpha}\sum_{j\in\alpha} e^{-2\nu(\mathbf{r}-\mathbf{R}_{ij})^2} e^{-(\mathbf{p}-\mathbf{P}_{ij})^2/2\hbar^2\nu} B_{ij}B^{-1}_{ji}, \qquad \alpha = p\uparrow, p\downarrow, n\uparrow, n\downarrow$$

$$\mathbf{R}_{ij} = \tfrac{1}{2\sqrt{\nu}}(\mathbf{Z}_i^* + \mathbf{Z}_j), \quad \mathbf{P}_{ij} = i\hbar\sqrt{\nu}(\mathbf{Z}_i^* - \mathbf{Z}_j), \quad B_{ij} = e^{-\frac{1}{2}(\mathbf{Z}_i^* - \mathbf{Z}_j)^2}$$

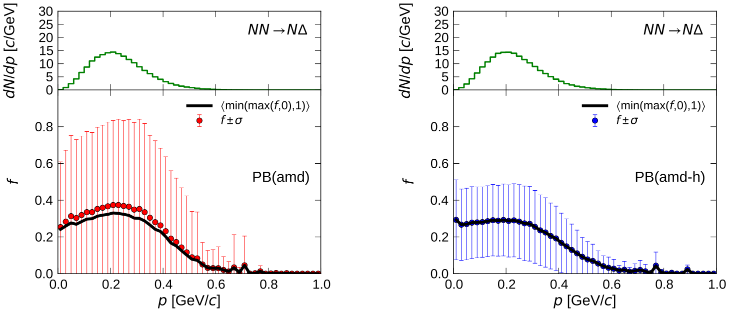
Husimi function for the AMD wave function

$$F_\alpha(\mathbf{R},\mathbf{P}) = \langle\mathbf{RP}|\hat{\rho}|\mathbf{RP}\rangle = \iint \frac{d\mathbf{r}'d\mathbf{p}'}{(\pi\hbar)^3} e^{-2\nu(\mathbf{r}'-\mathbf{R})^2} e^{-(\mathbf{p}'-\mathbf{P})^2/2\hbar^2\nu} f_\alpha(\mathbf{r}',\mathbf{p}')$$

$$= \sum_{j\in\alpha}\sum_{k\in\alpha} e^{-\nu(\mathbf{R}-\mathbf{R}_{jk})^2} e^{-(\mathbf{P}-\mathbf{P}_{jk})^2/4\hbar^2\nu} B_{jk}B^{-1}_{kj}$$

Wigner or Husimi for Pauli blocking

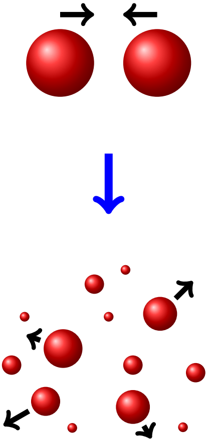
The Wigner and Husimi functions calculated for the AMD states $|\Phi(t, \text{event})\rangle$ for the blocking of the final nucleon in $NN \to N\Delta$, in ^{124}Sn + ^{132}Sn central collisions at 270 MeV/nucleon.

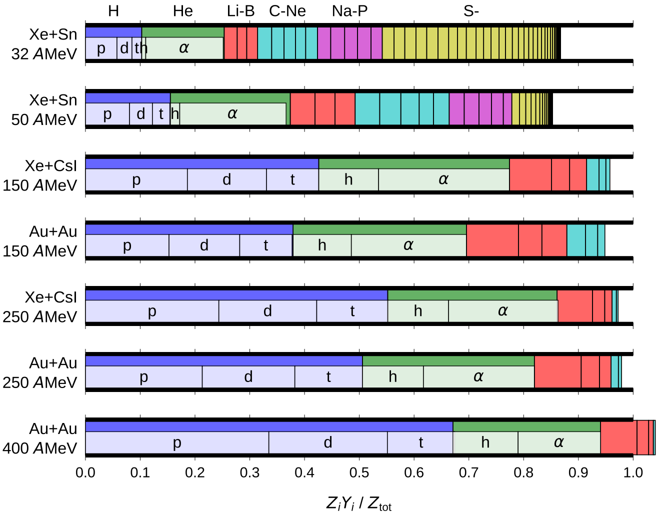


- The Wigner function can be $f > 1$ and $f < 0$ by its nature. $\Rightarrow \langle \min(\max(f, 0), 1)\rangle \leq \langle f \rangle$
- The Husimi function satisfies $0 \leq F(\mathbf{r}, \mathbf{p}) \leq 1$, but it has been smeared.

What is a good way for the Pauli-blocking factor, $(1 - f)$? ⇒ Ikeno's talk

Fraction of protons in clusters and fragments in heavy-ion collisions





INDRA: Hudan et al., PRC67 (2003) 064613.

FOPI: Reisdorf et al., NPA 848 (2010) 366.

Recognition of clusters and fragments

Assume that we know the many-body state $|\Psi\rangle$ at an intermediate time of a heavy-ion collision.

Problem of counting clusters (maybe ill-posed)

What is the probability (or number) of finding clusters and fragments in $|\Psi\rangle$?

To recognize clusters and fragments that will be observed in the final state $e^{-iHt}|\Psi\rangle$ $(t \to \infty)$.

- Find clusters and fragments at a very late time (by MST), in QMD/AMD.
- Coalescence method applied to the one-body distribution $f(\mathbf{r}, \mathbf{p})$ at a late time, in BUU.
- Recognition of clusters and fragments at a relatively early time in QMD (e.g. SACA, FRIGA).

To find cluster correlations in a many-body state $|\Psi\rangle$, which then affects the dynamics.

- AMD with clusters

To represent the state explicitly with clusters $f_p, f_n, f_d, f_t, \ldots$

- pBUU

Can we count the number of clusters in a many-body state $|\Psi\rangle$?

Creation operator of a deuteron ($M = 1$), [e.g. Röpke and Schulz, NPA 477 (1988) 472]

$$a_d^\dagger(\mathbf{P}) = \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} \psi_d(\mathbf{p})\, a_{p\uparrow}^\dagger(\tfrac{1}{2}\mathbf{P} + \mathbf{p})\, a_{n\uparrow}^\dagger(\tfrac{1}{2}\mathbf{P} - \mathbf{p})$$

Caution: These are not boson operators, $[a_d(\mathbf{P}),\ a_d^\dagger(\mathbf{P}')] \neq \delta_{\mathbf{PP}'}$.

In a coalescence method for BUU, [e.g. L.W. Chen, C.M. Ko, B.A. Li, NPA 729 (2003) 809]
for an uncorrelated state $|\Psi\rangle = |f_p f_n\rangle$ at a late time, the deuteron spectrum is calculated as

$$\langle\Psi(t)|a_d^\dagger(\mathbf{P})a_d(\mathbf{P})|\Psi(t)\rangle = \int \frac{d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{p}}{(2\pi\hbar)^3} g_d(\mathbf{r}_1 - \mathbf{r}_2, \mathbf{p})\, f_{p\uparrow}(\mathbf{r}_1, \tfrac{1}{2}\mathbf{P} + \mathbf{p}, t)\, f_{n\uparrow}(\mathbf{r}_2, \tfrac{1}{2}\mathbf{P} - \mathbf{p}, t)$$

where $g_d(\mathbf{r}, \mathbf{p})$ is the Wigner transform of $|\psi_d\rangle$.

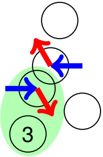
This is valid in a dilute system. In general, however,

$$N_d = \int \frac{d\mathbf{P}}{(2\pi\hbar)^3}(\cdots) = \int \frac{d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi\hbar)^6} g_d\big(\mathbf{r}_1 - \mathbf{r}_2, \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)\big)\ f_{p\uparrow}(\mathbf{r}_1, \mathbf{p}_1, t)\ f_{n\uparrow}(\mathbf{r}_2, \mathbf{p}_2, t)$$

cannot be the number of deuterons, because N_d can be $N_d > N_{p\uparrow}$ or $N_d > N_{n\uparrow}$.

NN collisions in AMD (with cluster correlations)

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



In the usual way of NN collision, only the two wave packets are changed.

$$\Big\{|\Psi_f\rangle\Big\} = \Big\{|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\rangle\Big\}$$

(ignoring antisymmetrization for simplicity of presentation.)

$$\left|\,③②\,\right\rangle \xrightarrow{\text{AMD}} \left|\,③\;\;②\,\right\rangle$$

What is the probability for that ...

1. the scattered nucleon forms a state ψ_d with another nucleon, and
2. they are propagated as a cluster until it collides with something,
3. more than included in the usual AMD.

Extension for cluster correlations

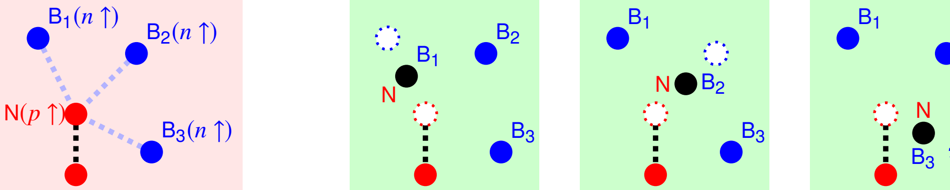
Include correlated states in the set of the final states of each NN collision.

$$\Big\{|\Psi_f\rangle\Big\} \ni |\varphi_{k_1}(1)\psi_d(2,3)\Psi(4,\ldots)\rangle, \ \ldots$$

$$\left|\,③②\,\right\rangle = c\left|\,(\psi_d)\,\right\rangle + \psi_{\text{cont}} \xrightarrow{e^{-iHt}} ce^{-i\omega t}\left|\,(\psi_d)\,\right\rangle + \psi_{\text{cont}}(t)$$

The cluster is not isolated but coupled with the rest of the system. (Danielewicz, Röpke)

Clusters (in the final states) are assumed to have $(0s)^N$ configuration.



$|\Phi^{\mathbf{q}}\rangle$ After $\mathbf{p}^{(0)} \longrightarrow \mathbf{p}^{(0)} + \mathbf{q}$

$|\Phi'_1\rangle$ $\mathrm{N} + \mathrm{B}_1 \longrightarrow \mathrm{C}_1$

$|\Phi'_2\rangle$ $\mathrm{N} + \mathrm{B}_2 \longrightarrow \mathrm{C}_2$

$|\Phi'_3\rangle$ $\mathrm{N} + \mathrm{B}_3 \longrightarrow \mathrm{C}$

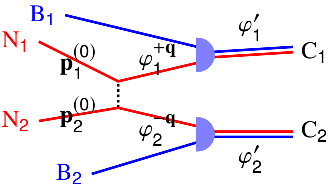
Final states are not orthogonal: $N_{ij} \equiv \langle \Phi'_i | \Phi'_j \rangle \neq \delta_{ij}$

The probability of cluster formation with one of B's:

$$\hat{P} = \sum_{ij} |\Phi'_i\rangle N_{ij}^{-1} \langle \Phi'_j|, \qquad P = \langle \Phi^{\mathbf{q}} | \hat{P} | \Phi^{\mathbf{q}} \rangle \qquad \neq \sum_i |\langle \Phi'_i | \Phi^{\mathbf{q}} \rangle|^2$$

$$\begin{cases} P & \Rightarrow \text{Choose one of the candidates and make a cluster.} \\ 1 - P & \Rightarrow \text{Don't make a cluster (with any n}\uparrow\text{).} \end{cases}$$

$\mathrm{N}_1 + \mathrm{B}_1 + \mathrm{N}_2 + \mathrm{B}_2 \rightarrow \mathrm{C}_1 + \mathrm{C}_2$



- $\mathrm{N}_1, \mathrm{N}_2$: Colliding nucleons
- $\mathrm{B}_1, \mathrm{B}_2$: Spectator nucleons/clusters
- $\mathrm{C}_1, \mathrm{C}_2$: N, $(2N)$, $(3N)$, $(4N)$ (up to α cluster)

Transition probability

$$W(\mathrm{NBNB} \rightarrow \mathrm{CC}) = \frac{2\pi}{\hbar} |\langle \mathrm{CC}|V|\mathrm{NBNB}\rangle|^2 \delta(E_f - E_i)$$

$$v\,d\sigma \propto |\langle \varphi_1' | \varphi_1^{+\mathbf{q}} \rangle|^2 |\langle \varphi_2' | \varphi_2^{-\mathbf{q}} \rangle|^2 |M|^2 \delta(E_f - E_i) p_{\mathrm{rel}}^2 \, dp_{\mathrm{rel}} \, d\Omega$$

$|M|^2 = |\langle \mathrm{NN}|V|\mathrm{NN}\rangle|^2$: Matrix elements of NN scattering

$\Leftarrow (d\sigma/d\Omega)_{\mathrm{NN}}$ in free space (or in medium)

$$\mathbf{p}_{\mathrm{rel}} = \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) = p_{\mathrm{rel}} \hat{\mathbf{\Omega}}$$

$$\mathbf{p}_1 = \mathbf{p}_1^{(0)} + \mathbf{q}$$

$$\mathbf{p}_2 = \mathbf{p}_2^{(0)} - \mathbf{q}$$

$$\varphi_1^{+\mathbf{q}} = \exp(+i\mathbf{q} \cdot \mathbf{r}_{\mathrm{N}_1}) \varphi_1^{(0)}$$

$$\varphi_2^{-\mathbf{q}} = \exp(-i\mathbf{q} \cdot \mathbf{r}_{\mathrm{N}_2}) \varphi_2^{(0)}$$

More about cluster production in NN collisions

$$\mathrm{N}_1 + \mathrm{B}_1 \; + \; \mathrm{N}_2 + \mathrm{B}_2 \; \to \mathrm{C}_1 + \mathrm{C}_2$$

$$v d\sigma \propto |\langle \varphi_1' | \varphi_1^{+\mathbf{q}} \rangle|^2 |\langle \varphi_2' | \varphi_2^{-\mathbf{q}} \rangle|^2 \; |M|^2 \; \delta(E_f - E_i) \; p_{\mathrm{rel}}^2 dp_{\mathrm{rel}} d\Omega \qquad E_i, \; E_f = \frac{\langle \Phi(Z)|H|\Phi(Z)\rangle}{\langle \Phi(Z)|\Phi(Z)\rangle}$$

$$\Rightarrow P(\mathrm{C}_1, \mathrm{C}_2, p_{\mathrm{rel}}, \Omega) \times \left| M(p_{\mathrm{rel}}^{(0)}, p_{\mathrm{rel}}, \Omega) \right|^2 \times \frac{{p_{\mathrm{rel}}}^2 d\Omega}{\partial E_f / \partial p_{\mathrm{rel}}}$$

- Gaussian width $\nu_{\mathrm{cl}} = 0.24\ \mathrm{fm}^{-2}$ for the overlap factors.
- There are a huge number of final cluster configurations $(\mathrm{C}_1, \mathrm{C}_2)$.

$$\sum_{\mathrm{C}_1 \mathrm{C}_2} P(\mathrm{C}_1, \mathrm{C}_2, p_{\mathrm{rel}}, \Omega) = 1 \quad \text{for any fixed } (p_{\mathrm{rel}}, \Omega)$$

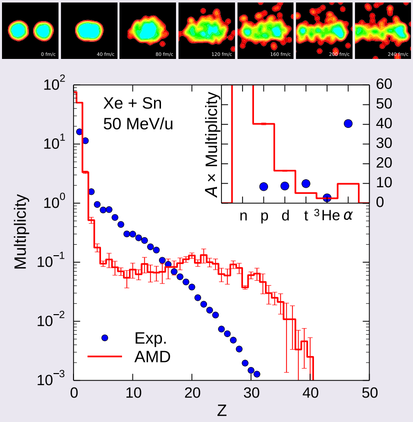
- The energy-conserving final momentum depends on the cluster configuration

$$p_{\mathrm{rel}} = p_{\mathrm{rel}}(\mathrm{C}_1, \mathrm{C}_2, \Omega)$$

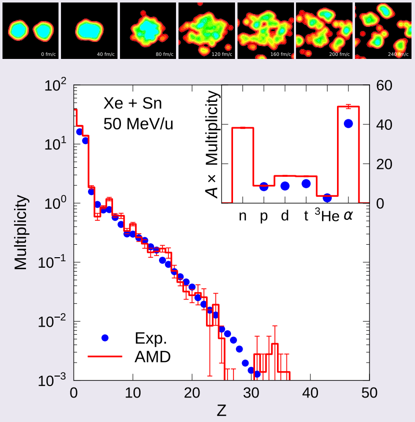
  When clusters are formed, p_{rel} tends to be large, and the effect of collisions will increase.
  - the phase space factor ↑
  - Pauli blocking ↓ (collision probability ↑)
  - momentum transfer ↑

Effect of cluster correlations: central Xe + Sn at 50 MeV/u

Without clusters

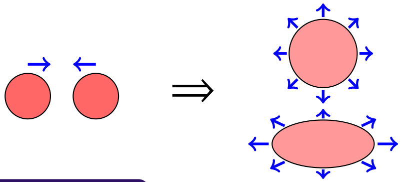


With clusters



Stopping

Initial state



A quantity to represent stopping

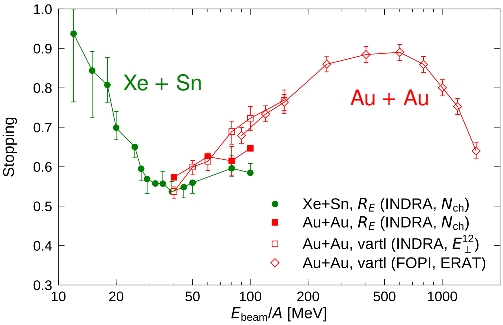
$$R_E = \frac{\sum(E_x + E_y)}{2\sum E_z}$$

$\sum$: for all charged products ($Z \geq 1$)

Stopping should depend on

- Inmedium NN cross sections
- Treatment of Pauli blocking
- Effective interaction (EOS)
- How to select central events

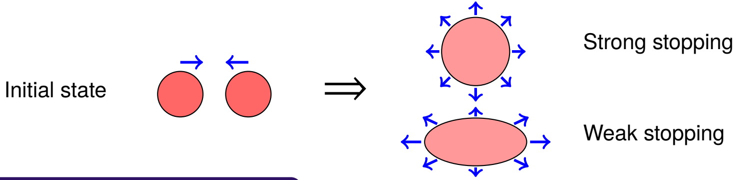
It is a many-body quantity.



INDRA: Lehaut et al., PRL104 (2010) 232701.

FOPI: Reisdorf et al., NPA 848 (2010) 366.

Stopping



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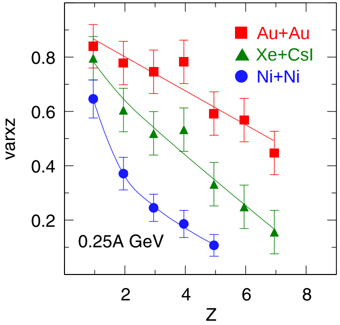
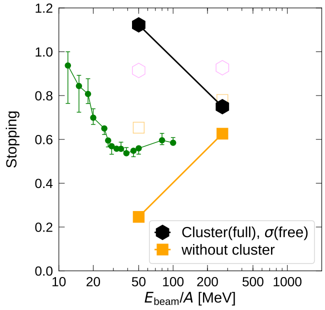


FOPI: Reisdorf et al., NPA 848 (2010) 366.

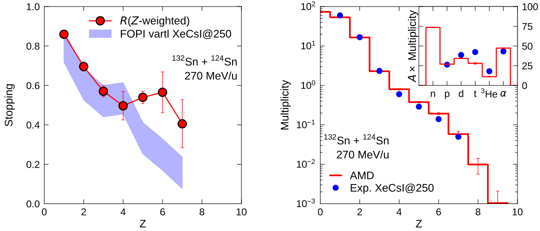
Results with full clusters



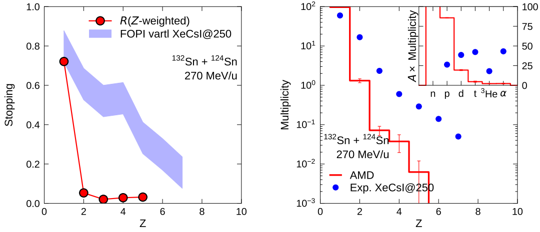
Central collisions (b < 1–2 fm)

- Xe + Sn for $E \leq 50A$ MeV
- ^{132}Sn + ^{124}Sn at 270A MeV

Cluster(full) and σ_{NN}(free)

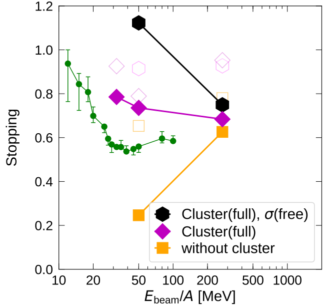


Without Cluster



FOPI data: Xe + CsI at 250A MeV

Results with full clusters



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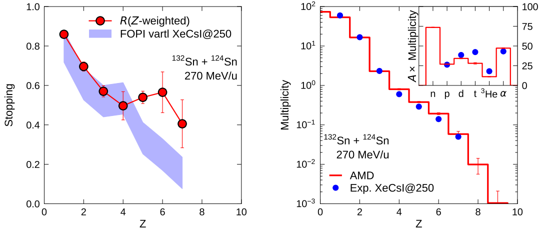
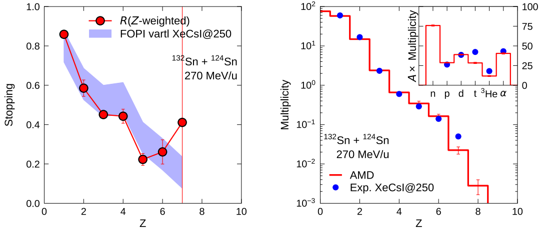


Cluster(full) and σ_{NN}(in-medium)



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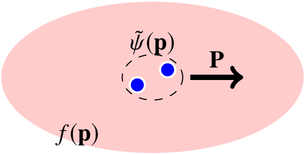
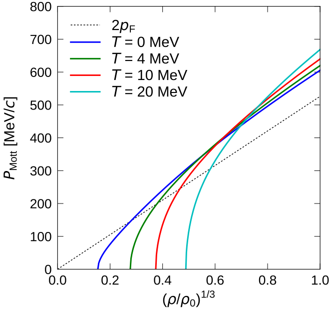


Equation for a deuteron in uncorrelated medium

$$\Big[e(\tfrac{1}{2}\mathbf{P}+\mathbf{p})+e(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\Big]\tilde{\psi}(\mathbf{p})$$
$$+\Big[1-f(\tfrac{1}{2}\mathbf{P}+\mathbf{p})-f(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\Big]\int\frac{d\mathbf{p}'}{(2\pi)^3}\langle\mathbf{p}|v|\mathbf{p}'\rangle\tilde{\psi}(\mathbf{p}')$$
$$=E\tilde{\psi}(\mathbf{p})$$



Formula from Röpke, NPA867 (2011) 66.

- A bound deuteron cannot exist inside the Fermi sphere, except at very low densities.
- A deuteron can exist if its momentum is high enough.
- In AMD, Pauli blocking has already been considered for NN collisions. More suppression of clusters may be introduced. c.f. $\langle f\rangle_d < 0.2$ by Danielewicz et al., NPA533 (1991) 712.

Condition to switch on/off clusters

With or without clusters

$$N_1 + N_2 + B_1 + B_2 \to C_1 + C_2 \qquad \text{or} \qquad N_1 + N_2 \to N_1 + N_2$$

The condition to switch on clusters

$$\rho' < \rho_c, \qquad \rho_c = 0.125\ \text{fm}^{-3} \text{ or } 0.060\ \text{fm}^{-3} \text{ etc.}$$

Density with a momentum cut for the nucleon N_i $(i = 1, 2)$

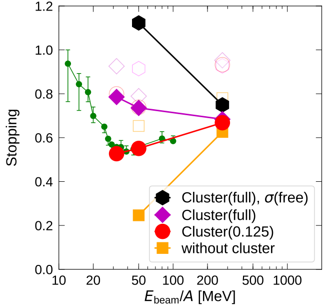
$$\rho_i'^{(\text{ini})} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{k(\neq i)} \theta\big(p_{\text{cut}} > |\mathbf{P}_i - \mathbf{P}_k|\big)\ e^{-2\nu(\mathbf{R}_i - \mathbf{R}_k)^2}$$

$$\rho_i'^{(\text{fin})} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{k(\neq i)} \theta\big(p_{\text{cut}} > |\mathbf{P}_i^{(\text{fin})} - \mathbf{P}_k|\big)\ e^{-2\nu(\mathbf{R}_i - \mathbf{R}_k)^2}$$

$$\rho' = \big(\rho_1'^{(\text{ini})} \rho_1'^{(\text{fin})} \rho_2'^{(\text{ini})} \rho_2'^{(\text{fin})}\big)^{\frac{1}{4}}$$

An energy-dependent momentum cut was chosen, $p_{\text{cut}} = (375\ \text{MeV}/c)\ e^{-\epsilon/(225\ \text{MeV})}$, where ϵ is the collision energy (i.e. the sum of the kinetic energies of N_1 and N_2 in their c.m. frame).

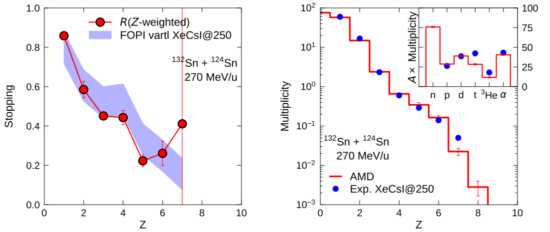
Effects of in-medium cluster suppression, with σ_{NN}(in-medium)



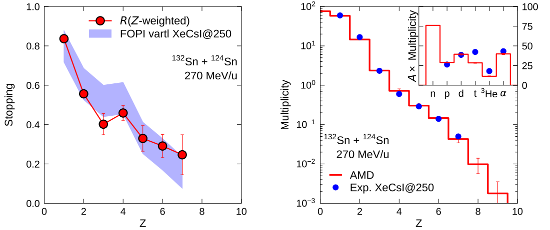
Central collisions (b < 1–2 fm)

- Xe + Sn for $E \leq 50A$ MeV
- ^{132}Sn + ^{124}Sn at 270A MeV

Cluster(full) and σ_{NN}(in-medium)

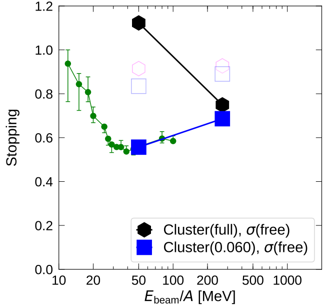


Cluster(0.125), and σ_{NN}(in-medium)



FOPI data: Xe + CsI at 250A MeV

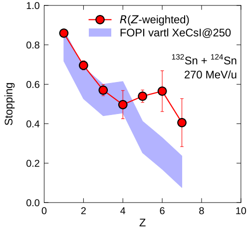
Effects of in-medium cluster suppression, with σ_{NN}(free)

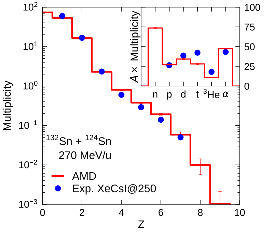


Central collisions ($b <$ 1–2 fm)

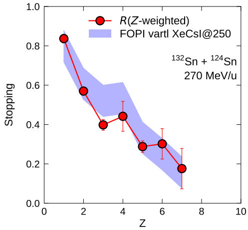
- Xe + Sn for $E \leq 50A$ MeV
- ^{132}Sn + ^{124}Sn at $270A$ MeV

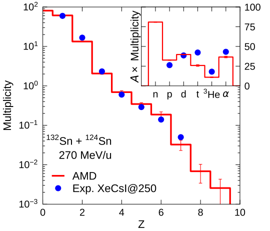
Cluster(full) and σ_{NN}(free)





Cluster(0.060) and σ_{NN}(free)





FOPI data: Xe + CsI at $250A$ MeV

Summary on clusters and stopping

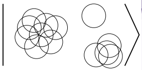
- Cluster correlations can have strong impacts, not only on the cluster emission, but also on the collision dynamics (e.g. stopping) in central heavy-ion collisions.
- Suppression of clusters at high (phase-space) densities was considered, in the cluster production process in AMD.
- Information on stopping can give a constraint on a combination of the in-medium suppression of clusters and the in-medium NN cross section (and ...).
  - Too strong suppression (e.g. without clusters ■) cannot be compatible with the experimental data of the cluster yield and the cluster-size dependence of stopping.
  - In some range of the degree of suppression ($\sim$ ● $\sim$ ■ $\sim$), the fragment observables can be roughly consistent with data.

- What can fix the degree of suppression of clusters (and in-medium NN cross section)?
- How is the isospin dynamics, i.e. the difference of neutrons and protons?

Discussions on “clustering and correlations in transport”

1. What are clusters?
   - Different aims and concepts of production/recognition of clusters
   - How to count the number of clusters in a dense system
2. Interplay of one-body dynamics ⇔ cluster correlations
   - Thermodynamics, EOS
   - Collision dynamics
3. Clusters in the existence of many other nucleons
   - Change of binding energy and wave function. Spectral function.
   - At low and high densities
   - In general many-body states? (correlated and/or time-dependent)
4. Handling clusters in transport models
   - $f_p, f_n, f_d, f_t, f_h, f_\alpha, \ldots$
   - Have a many-nucleon state $|\Psi\rangle$ and try to find clusters in it.
5. Fluctuation / branching / bifurcation
6. Clusters in usual QMD
   - Is it really the problem of binding energies?
7. Off-shell transport?

Antisymmetrized Molecular Dynamics (very basic version)



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = (2.5 fm)$^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} \quad + \quad \text{(NN collisions)}$$

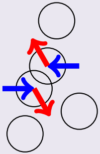
$\{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$: Motion in the mean field

$$\mathcal{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)}$$

H: Effective interaction (e.g. Skyrme force)

NN collisions

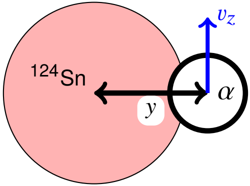
$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



- $|V|^2$ or σ_{NN} (in medium)
- Pauli blocking

Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.

A cluster put into a nucleus in AMD



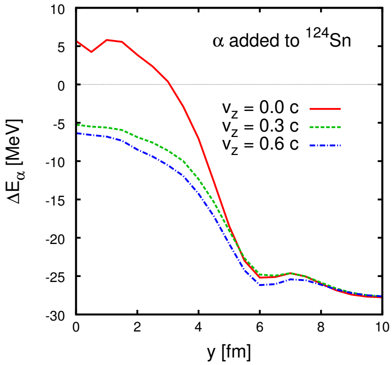
α cluster $|\alpha, \mathbf{Z}\rangle$: Four wave packets with different spins and isospins at the same phase space point $\mathbf{Z}$.

$$E_\alpha : \quad \mathcal{A}\,|\alpha, \mathbf{Z}\rangle|^{124}\text{Sn}\rangle$$

$$E_\text{N} : \quad \mathcal{A}\,|\mathbf{Z}\rangle|^{124}\text{Sn}\rangle \quad (\text{N} = p\uparrow, p\downarrow, n\uparrow, n\downarrow)$$

$$-B_\alpha = \Delta E_\alpha = E_\alpha - (E_{p\uparrow} + E_{p\downarrow} + E_{n\uparrow} + E_{n\downarrow})$$

(Energies are defined relative to $|^{124}\text{Sn}\rangle$.)



$$\frac{\text{Re}\,\mathbf{Z}}{\sqrt{\nu}} = (0, y, 0),$$

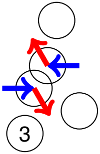
$$\frac{2\hbar\sqrt{\nu}\,\text{Im}\,\mathbf{Z}}{M} = (0, 0, v_z)$$

- Distance from the center: y
  $\approx$ Dependence on density
- Dendence on $P_\alpha = M_\alpha v_z$
- Due to the density dependence of the Skyrme force, the interaction between nucleons in the α cluster is weakened in the nucleus.

Energy is OK, but dynamics is …

NN collisions without or with cluster correlations

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

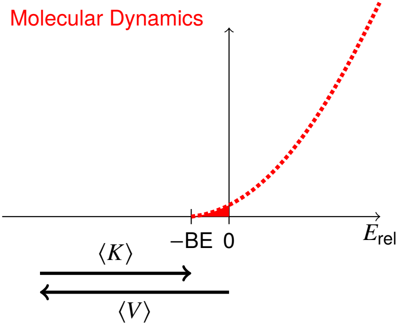


In the usual way of NN collision, only the two wave packets are changed.

$$\Big\{|\Psi_f\rangle\Big\} = \Big\{|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\rangle\Big\}$$

(ignoring antisymmetrization for simplicity of presentation.)

Phase space or the density of states for two nucleon system



NN collisions without or with cluster correlations

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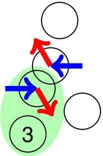

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$$\Big\{|\Psi_f\rangle\Big\} = \Big\{|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\rangle\Big\}$$

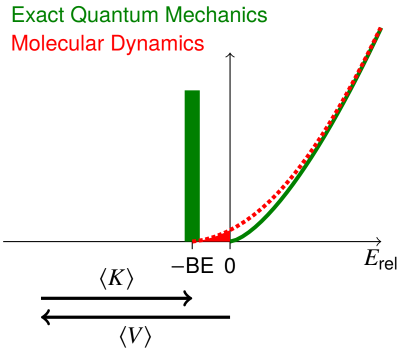
(ignoring antisymmetrization for simplicity of presentation.)

Extension for cluster correlations

Include correlated states in the set of the final states of each NN collision.

$$\Big\{|\Psi_f\rangle\Big\} \ni |\varphi_{k_1}(1)\psi_d(2,3)\Psi(4,\ldots)\rangle,\ \ldots$$

Phase space or the density of states for two nucleon system

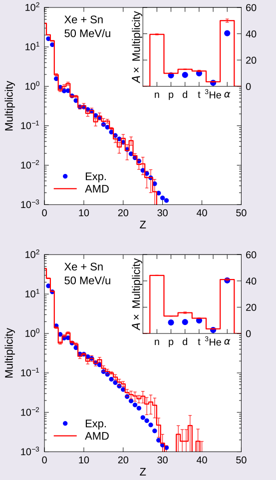


Combinations of σ_{NN} and the in-medium cluster suppression

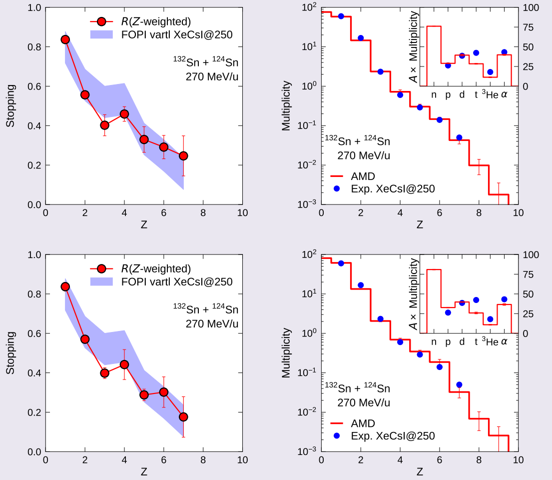
σ_{NN}(in-medium) Cluster(0.125)

σ_{NN}(free) Cluster(0.060)

Xe + Sn at 50A MeV

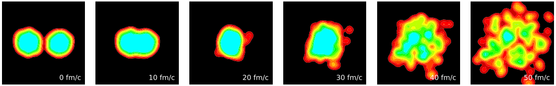


^{124}Sn + ^{132}Sn at 270A MeV



FOPI data: Xe + CsI at 250A MeV

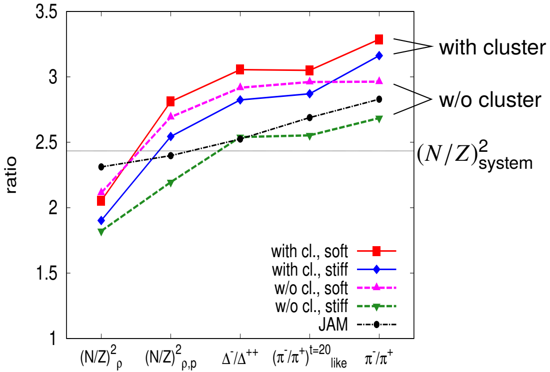
π^-/π^+ ratio in central ^{132}Sn + ^{124}Sn collisions at 300 MeV/nucleon



Ikeno, Ono, Nara, Ohnishi,

PRC 93 (2016) 044612,

Erratum PRC 97 (2018) 069902.



Final π^-/π^+

⇕

$(\pi^-/\pi^+)_{\text{like}}$ at $t = 20$ fm/c

⇕

$(N/Z)^2$ at high density & high momentum

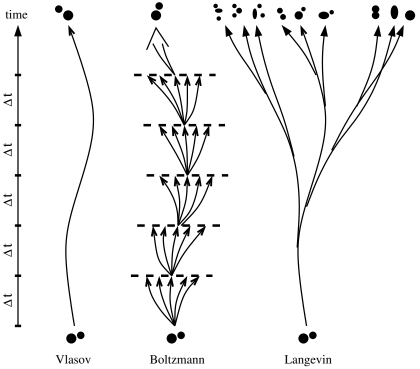
⇕

- Cluster correlations
- Symmetry energy

Motivation and Question for this talk

Where and when do clusters start to appear? How strong should cluster correlations be?

Branching / bifurcation



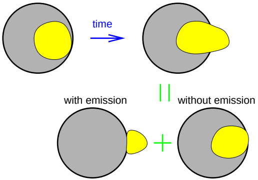
How does randomness appear while the original Schrödinger equation is deterministic?

$$i\hbar \frac{\partial}{\partial t}\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A, t) = H\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A, t)$$

Coupled two subsystems. (A nucleon + the rest, for example)

We can decompose the product state as we like.

$$|\psi\rangle|\tilde{\Phi}\rangle = c_1|\psi_1\rangle|\tilde{\Phi}\rangle + c_2|\psi_2\rangle|\tilde{\Phi}\rangle + \cdots$$



Time evolutions under mean field approximation

$$|\psi\rangle|\tilde{\Phi}\rangle \xrightarrow{\text{mean field } U} |\psi(t)\rangle|\tilde{\Phi}(t)\rangle$$

$$|\psi_1\rangle|\tilde{\Phi}\rangle \xrightarrow{\text{mean field } U_1} |\psi_1(t)\rangle|\tilde{\Phi}_1(t)\rangle$$

$$|\psi_2\rangle|\tilde{\Phi}\rangle \xrightarrow{\text{mean field } U_2} |\psi_2(t)\rangle|\tilde{\Phi}_2(t)\rangle$$

Different results for different ways of decomposition, because of the non-linearity introduced by the mean field approximation.

$$|\psi(t)\rangle|\tilde{\Phi}(t)\rangle \neq c_1|\psi_1(t)\rangle|\tilde{\Phi}_1(t)\rangle + c_2|\psi_2(t)\rangle|\tilde{\Phi}_2(t)\rangle + \cdots$$

What is a good way of decomposition? $\Leftrightarrow$ Choice of a model

We also want to ignore interference between components (treating as fluctuation).

Density matrix

Many-body density operator $\hat{\rho}^{(A)} = |\Psi\rangle\langle\Psi|$, for a normalized pure state Ψ.

One-body density operator

$$\hat{\rho} = A \operatorname*{Tr}_{2,\ldots,A} \hat{\rho}^{(A)} \qquad \text{i.e.} \quad \langle \mathbf{r}|\hat{\rho}|\mathbf{r}'\rangle = A \int \cdots \int \Psi(\mathbf{r}, \mathbf{r}_2, \ldots, \mathbf{r}_A)\Psi^*(\mathbf{r}', \mathbf{r}_2, \ldots, \mathbf{r}_A) d\mathbf{r}_2 \cdots d\mathbf{r}_A$$

Slater determinant

$$\text{Slater determinant} \quad \Leftrightarrow \quad \hat{\rho}^2 = \hat{\rho} \quad \text{with } \operatorname{Tr}\hat{\rho} = A$$

Anything can be written by $\hat{\rho}$, such as the two-body density operator

$$\hat{\rho}^{(2)} = A(A-1) \operatorname*{Tr}_{3,\ldots,A} \hat{\rho}^{(A)} = \mathcal{A}_{12}\hat{\rho}_1\hat{\rho}_2, \qquad \mathcal{A}_{12} = 1 - \mathcal{P}_{12}$$

In general, the many-body state Ψ does not continue to be a Slater determinant, and then the equation for $\hat{\rho}$ cannot be closed.

$$i\hbar \frac{d}{dt}\hat{\rho} = [\frac{\hat{\mathbf{p}}^2}{2M},\ \hat{\rho}] + \operatorname*{Tr}_{2}[\hat{v}_{12},\ \hat{\rho}^{(2)}]$$

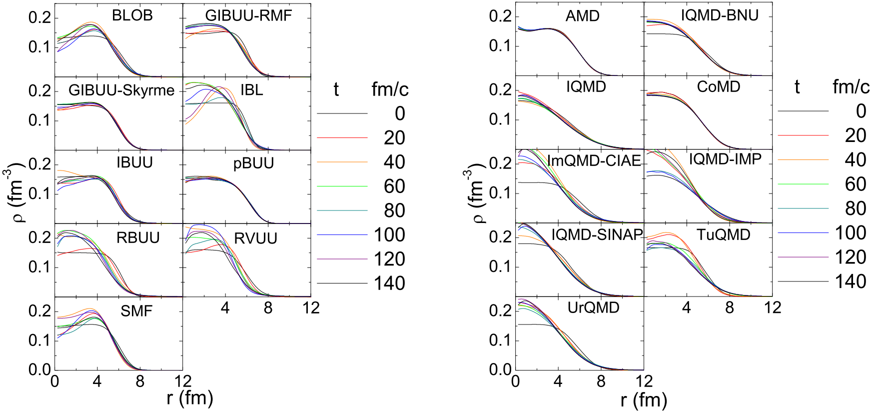
However, usual transport theories assume e.g. $\hat{\rho}^{(2)} = \mathcal{A}_{12}\hat{\rho}_1\hat{\rho}_2$ (i.e. the absence of correlations), to close the equation.

Transport code comparison: Initial density and stability

An example of the importance of Pauli principle

the saturation property of nuclear matter and the ground-state nuclei

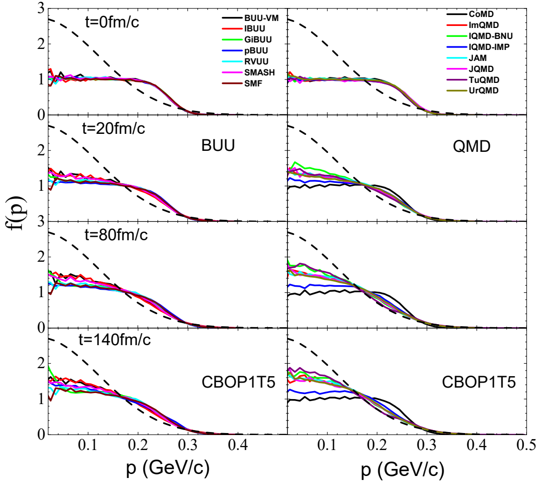
Time evolution of the ground-state Au nucleus



J. Xu et al., PRC93 (2016) 044609.

Box Homework 1: Violation of Pauli principle

As the time progresses, the Fermi-Dirac distribution is gradually destroyed, because of imperfect Pauli blocking.



Parametrization of in-medium NN cross section

The matrix element $|M|^2$ is obtained from the NN cross section.

- Free cross section $\sigma_{\text{free}}(\epsilon)$, taken from the JAM code.
- In-medium cross section which depends on ρ' (with momentum cut).

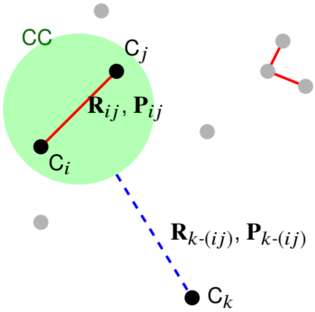
$$\sigma(\rho', \epsilon) = \sigma_0 \tanh\Big(\sigma_{\text{free}}(\epsilon)/\sigma_0\Big), \qquad \sigma_0 = 0.5 \times (\rho')^{-2/3}$$

parametrization by Danielewicz

Several clusters may form a loosely bound state.

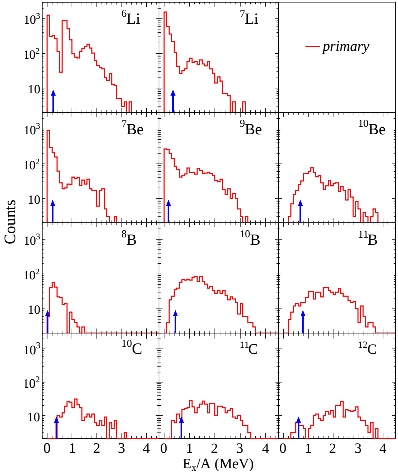
e.g., $^7\text{Li} = \alpha + t - 2.5$ MeV

Need more probability of $|\alpha + t\rangle \longrightarrow |^7\text{Li}\rangle$ etc.



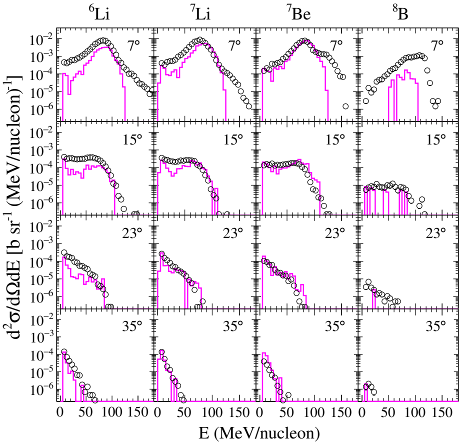
$^{12}C + {}^{12}C$ at 95 MeV/nucleon

Tian et al., PRC 97 (2018) 034610.



Excitation energy per nucleon

Some light nuclei are emitted at large angles ($\theta_{\text{lab}} > 20^\circ$) almost in their ground states, at $t = 300$ fm/c.



Does the probability fluctuate?

$f(\mathbf{r},\mathbf{p})$ is a probability distribution. Does it make sense to say that the probability fluctuates?

Even if the probability f fluctuates in such a way that $f = f_i$ with a probability w_i, the eventual probability is

$$f(\mathbf{r},\mathbf{p}) = \sum_i w_i f_i(\mathbf{r},\mathbf{p})$$

$$\left[\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6}\right] = 0.5\times\left[\frac{1}{3},0,\frac{1}{3},0,\frac{1}{3},0\right] + 0.5\times\left[0,\frac{1}{3},0,\frac{1}{3},0,\frac{1}{3}\right]$$

This is equivalent to the original idea that the probability is $f(\mathbf{r},\mathbf{p})$.

これに対する反論.