Generalized Density Functionals with Clusters for Transport Theory

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Workshop on Challenges to Transport Theory for Heavy-Ion Collisions

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Use of Densities

Considerations: qualitative + microscopic + phenomenology

Ability to replace $\mathcal{N}$-particle information with 1-particle or few-particle, $\hat{\rho}_{\mathcal{N}} \longrightarrow \hat{\rho}_1$, relies on the assumption that 2-particle and few-particle correlations relax quickly under specified $\hat{\rho}_1$

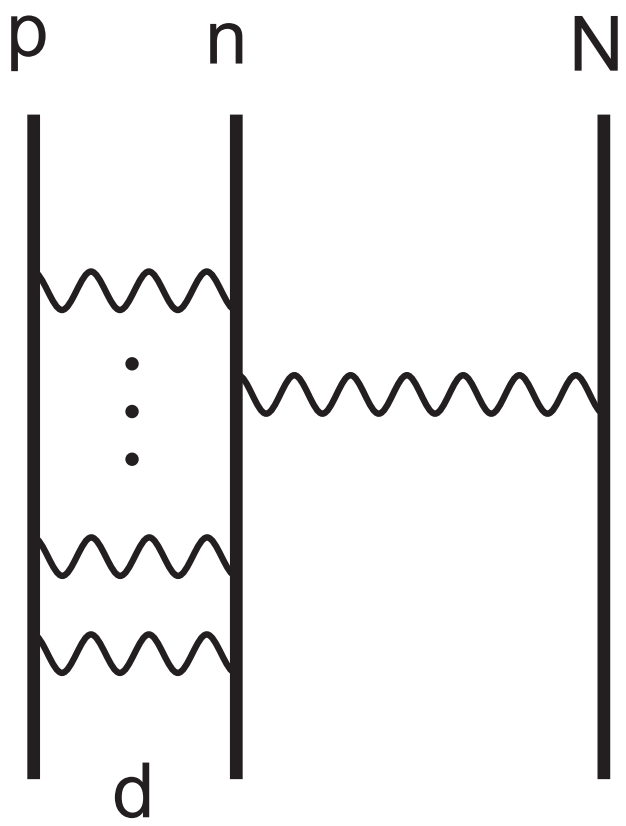
Justified when interactions frequent in dense matter

Not justified when system thins out and correlations begin to freeze in time as fragments form

??Isolate long-lived correlations by separating out fragments X as independent degrees of freedom, in terms of $\hat{\rho}_X$?? Energy:

$$E(\hat{\rho}_{\mathcal{N}}) \rightarrow E(\hat{\rho}_1) \quad \Longrightarrow \quad E(\hat{\rho}_{\mathcal{N}}) \rightarrow E(\hat{\rho}_N, \hat{\rho}_d, \hat{\rho}_t, \ldots)$$

Single-Particle Energies ϵ



$$d \leftrightarrow n + p$$

$$-\delta n_d = \delta n_p = \delta n_n$$

$$\frac{\delta E}{\delta \rho_d} \simeq \frac{\delta E}{\delta \rho_n} + \frac{\delta E}{\delta \rho_p}$$

$$\epsilon_d \simeq \epsilon_p + \epsilon_n - B_d$$

$\rightarrow$There in direct reaction models with folding/superposition potentials

Density Options

Details of density mtx/Wigner function unlikely to matter - use of test ptcles e.g. requires that only integrated Wigner functions are employed

Energy density functional should depend on properly chosen generalized densities $\{n_\alpha\}$. Many choices, e.g.

$$e_{\rm kin}(x,t) = \sum_X \int {\rm d}p \sqrt{m_X^2 + p^2}\, f_X(p,x,t) \qquad \text{(kinetic en density)}$$

$$n_B(x,t) = \sum_X A_X \int {\rm d}p\, f_X(p,x,t) \qquad \text{(baryon density)}$$

$\ldots$

Simon&PD PRC87(13)054619; MSc Thesis, MSU '13
Yuanyuan Zhang, Bachelor's Thesis, XJTU '14

Constructing Energy Functional

Tempting Choice

$$E = E_{\text{kin}} + E_{\text{pot}} = \sum_X \int \mathrm{d}x\,\mathrm{d}p\,\epsilon_{X0}(p)\,f_X(p,x,t) + \int \mathrm{d}x\,e_{\text{pot}}(x,t)$$

$$\stackrel{?}{=} \sum_X \int \mathrm{d}x\,\mathrm{d}p\,\epsilon_X(p,\{n_\alpha\})\,f_X(p,x,t) + \int \mathrm{d}x\,e_c(\{n_\alpha\})$$

Motivation: intuition/microscopic insights

common for single particle energies

$$\epsilon_X(p) = \frac{\delta E}{\delta f_X(p)}$$

Consistency conditions, with $\mathrm{d}e_c = \sum_\alpha n^*_\alpha\,\mathrm{d}n_\alpha$,

$$n^*_\alpha = -\sum_X \int \mathrm{d}p\,\frac{\partial \epsilon_X}{\partial n_\alpha}\,f_X \quad \text{and} \quad \frac{\partial n^*_\alpha}{\partial n_\beta} = \frac{\partial n^*_\beta}{\partial n_\alpha}$$

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Current Context

All current quasiparticle models in use follow this outline. In Walecka/relatistic models, the second (mixed derivative) condition is satisfied automatically with

$$e_c = \sum_\alpha e_{c\alpha}(n_\alpha)$$

yielding vanishing mixed derivatives, and the first condition dictates the form of generalized densities. E.g.

$$\epsilon_X = \sqrt{m_X^2(n_s) + p^2}$$

then first condition

$$n^*(n_s) = -\sum_X \int \mathrm{d}p \, \frac{\partial \epsilon_X}{\partial n_s} f_X = -\sum_X m_X \, m_X' \int \mathrm{d}p \, \frac{f_X}{\epsilon_X}$$

i.e. $n_s \propto \sum_X \ldots \int \mathrm{d}p \, f_X/\epsilon_X$.

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Another Example

$$\epsilon_X = \epsilon_{X0}(p) + U_X(p, n_v) = \epsilon_{X0}(p) + \frac{A_X\, n_v}{v^2 + v_0^2}$$

where the dependence on velocity v & proportionality to mass number A_X introduced for coalescence invariance. Then

$$n_v^*(n_v) = -\sum_X \int \mathrm{d}p \frac{\partial \epsilon_X}{\partial n_v} f_X = -\sum_X A_X \int \mathrm{d}p \frac{f_X}{v^2 + v_0^2}$$

i.e. up to a factor

$$n_v = \sum_X A_X \int \mathrm{d}p \frac{f_X}{v^2 + v_0^2}$$

$\Rightarrow$ Following the scheme QGP phase transitions, mo-dep isovector potentials can be consistently incorporated

$\rightarrow$??Rates for composites?? $\rightarrow$Microscopic Theory

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Many-Body Theory

Transport eq. for nucleons follows from the eq. of motion for the 1-ptcle Green's function (KB eq.). Transport eq. for deuterons ($A = 2$) from the eq. for 2-ptcle Green's function??

Wigner function in second quantization

$$f(\mathbf{p};\mathbf{R},T) = \int d\mathbf{r}\, e^{-i\mathbf{p}\mathbf{r}} \langle \hat{\psi}^{\dagger}_{H}(\mathbf{R}-\mathbf{r}/2,T)\hat{\psi}_{H}(\mathbf{R}+\mathbf{r}/2,T)\rangle$$

$\hat{\rho}$

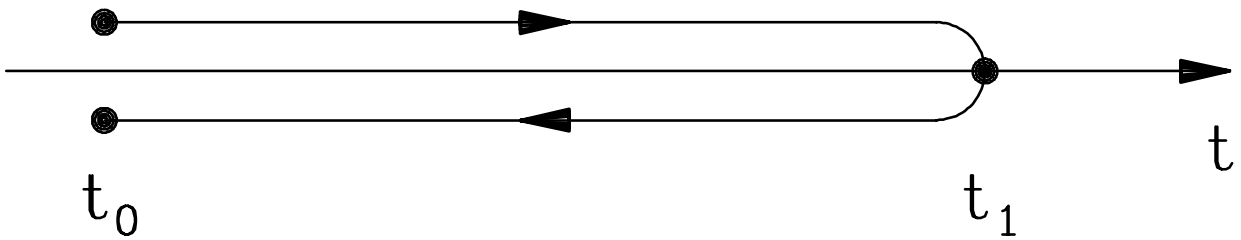
where $\langle\cdot\rangle \equiv \langle\Psi|\cdot|\Psi\rangle$ and $|\Psi\rangle$ describes the initial state.

Evolution driven by a Hamiltonian. Interaction Hamiltonian:

$$\hat{H}^1 = \frac{1}{2}\int d\mathbf{x}\, d\mathbf{y}\, \hat{\psi}^{\dagger}(\mathbf{x})\hat{\psi}^{\dagger}(\mathbf{y})v(\mathbf{x}-\mathbf{y})\hat{\psi}(\mathbf{y})\hat{\psi}(\mathbf{x})\,,$$

Evolution Contour

$$\begin{aligned}\langle \hat{O}_H(t_1)\rangle &= \langle T^a\left[\exp\left(-i\int_{t_1}^{t_0} dt'\,\hat{H}_I^1(t')\right)\right]\hat{O}_I(t_1)\\ &\quad\times T^c\left[\exp\left(-i\int_{t_0}^{t_1} dt'\,\hat{H}_I^1(t')\right)\right]\rangle\\ &= \langle T\left[\exp\left(-i\oint_{t_0}^{t_0} dt'\,\hat{H}_I^1(t')\right)\hat{O}_I(t_1)\right]\rangle,\end{aligned}$$



Expectation value expanded perturbatively in terms of V and noninteracting 1-ptcle Green's functions on the contour

$$iG_0(\mathbf{x},t,\mathbf{x}',t') = \langle T\left[\hat{\psi}_I(\mathbf{x},t)\hat{\psi}_I^\dagger(\mathbf{x}',t')\right]\rangle$$

Single-Particle Evolution

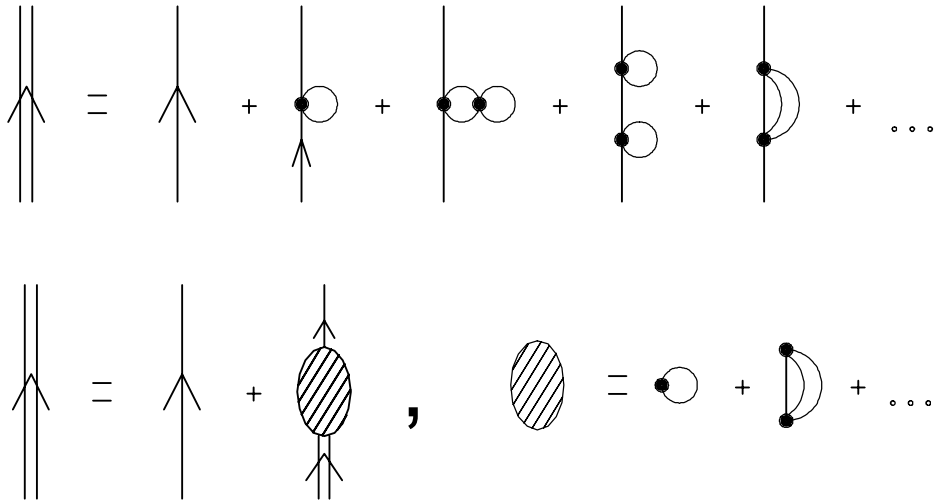
Wigner function corresponds to a particular case of the Green's function on contour:

$$f(\mathbf{p};\mathbf{R},T) = \int d\mathbf{r}\, e^{-i\mathbf{pr}}\,(\mp i)G^{<}(\mathbf{R}+\mathbf{r}/2,T,\mathbf{R}-\mathbf{r}/2,T)$$

If we find an equation for G, this will also be an equation for f.

Dyson eq. from perturbation expansion:

$$G = G_0 + G_0\,\Sigma\,G$$



Outcome of Evolution

Formal solution of the Dyson eq:

$$\mp iG^{<}(x,t;x',t') = \int \mathrm{d}x_1\,\mathrm{d}t_1\,\mathrm{d}x_1'\,\mathrm{d}t_1'\; G^{+}(x,t;x_1,t_1) \times(\mp i)\Sigma^{<}(x_1,t_1;x_1',t_1')\,G^{-}(x,t;x_1,t_1)$$

and

$$\mp i\Sigma^{<}(x,t;x',t') = \langle \hat{j}^{\dagger}(x',t')\,\hat{j}(x,t)\rangle_{irred}$$

where the source j is

$$\hat{j}(x,t) = \left[\hat{\psi}(\mathbf{x},t), \hat{H}^{1}\right]$$

Quasiparticle Limit

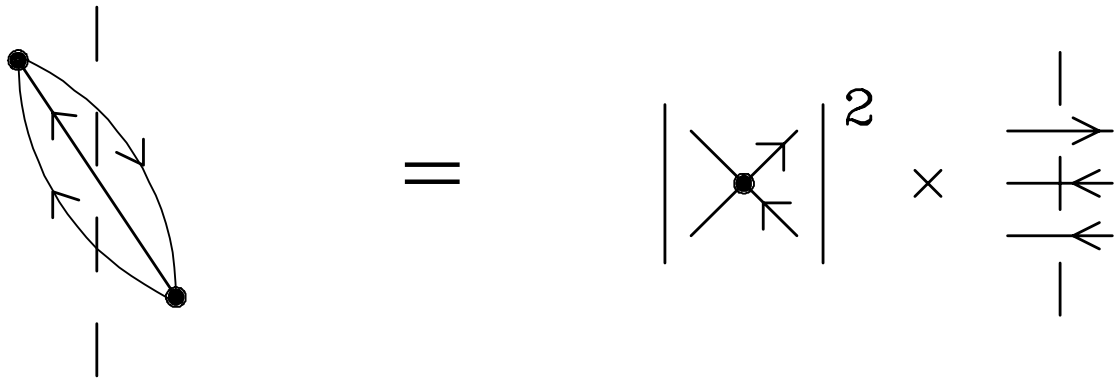
Under slow spatial and temporal changes in the system, the Green's function expressible in terms of the Wigner function f and 1-ptcle energy ϵ_p

$$\mp iG^<(x,t;x',t') \approx \int \mathrm{d}p\, f\left(p; \frac{x+x'}{2}, \frac{t+t'}{2}\right) \mathrm{e}^{i(p(x-x')-\epsilon_p(t-t'))}$$

Then also Boltzmann eq:

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = -i\Sigma^<(1-f) - i\Sigma^> f$$

$\mp i\Sigma^<$:



2-Particle Green's Function

Transport eq. for deuterons ($A = 2$) from the eq. for 2-ptcle Green's function??

$$iG_2^< = \langle \hat{\psi}^\dagger(\mathbf{x}_1' \, t') \, \hat{\psi}^\dagger(\mathbf{x}_2' \, t') \hat{\psi}(\mathbf{x}_2 \, t) \, \hat{\psi}(\mathbf{x}_1 \, t) \rangle$$

For the contour function:

$$G_2 = \mathcal{G} + \mathcal{G} \, v \, G_2$$

where $\mathcal{G}$ – irreducible part of G_2 (w/o two 1-ptcle lines connected by the potential v; anything else OK)

In terms of retarded Green's function $G_2^<$:

$$iG_2^< = (1 + G_2^+ \, v) \; i\mathcal{G}^< \; (1 + v \, G_2^-)$$

Deuteron Quasiparticle Limit

In the limit of slow spatial and temporal changes, deuteron contribution to the 2-ptcle Green's function:

$$
\begin{aligned}
iG_2^< &= \langle \hat{\psi}^\dagger(\mathbf{x}_1' \, t') \, \hat{\psi}^\dagger(\mathbf{x}_2' \, t') \hat{\psi}(\mathbf{x}_2 \, t) \, \hat{\psi}(\mathbf{x}_1 \, t) \rangle \\
&\simeq \int d\mathbf{p} \, f_d(\mathbf{p} \, \mathbf{R} \, T) \, \phi_d^*(r') \, \phi_d(r) \, \mathrm{e}^{i\mathbf{p}\left(\frac{\mathbf{x}_1+\mathbf{x}_2}{2} - \frac{\mathbf{x}_1'+\mathbf{x}_2'}{2}\right)} \, \mathrm{e}^{-i\epsilon_d \, (t-t')} \\
&\quad + \cdots ,
\end{aligned}
$$

where $\mathbf{R} = \frac{1}{4}(\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_1' + \mathbf{x}_2')$, $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$
ϕ_d and f_d – internal wave function and cm Wigner function
$\cdots \equiv$ continuum

Transport eq from integral quantum eq of motion:

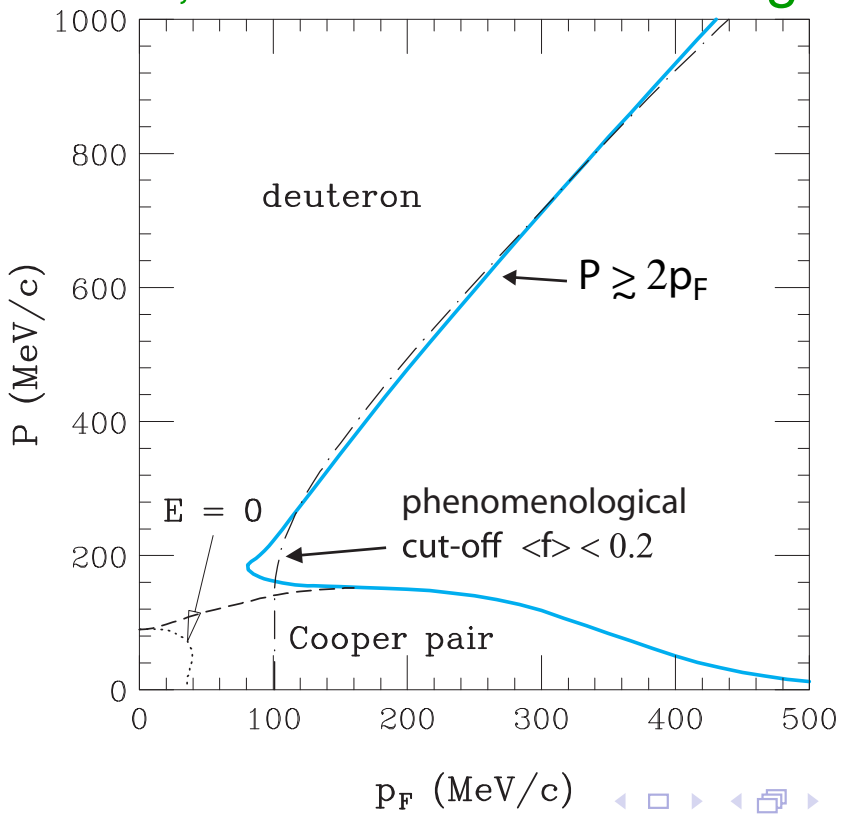
$$
\frac{\partial f_d}{\partial T} + \frac{\partial \epsilon_d}{\partial \mathbf{p}} \frac{\partial f_d}{\partial \mathbf{R}} - \frac{\partial \epsilon_d}{\partial \mathbf{R}} \frac{\partial f_d}{\partial \mathbf{p}} = \mathcal{K}^< \, (1 + f_d) - \mathcal{K}^> \, f_d
$$

Wave Equation

From Green's function eq, the equation for wavefunction:

$$\left(\epsilon_d(\mathbf{P}) - \epsilon_N(\mathbf{P}/2+\mathbf{p}) - \epsilon_N(\mathbf{P}/2-\mathbf{p})\right)\phi_d(\mathbf{p})$$
$$-\left(1 - f_N(\mathbf{P}/2+\mathbf{p}) - f_N(\mathbf{P}/2-\mathbf{p})\right)\int d\mathbf{p}'\, v(\mathbf{p}-\mathbf{p}')\,\phi_d(\mathbf{p}') = 0$$

In zero-temperature matter, discrete states lacking over a vast range of momenta



Cluster Production & Absorption

?? Production & absorption rates: $i\mathcal{K}^{><} = \phi^* \, v \, i\mathcal{G}^{><} \, v \, \phi$

Leading contribution

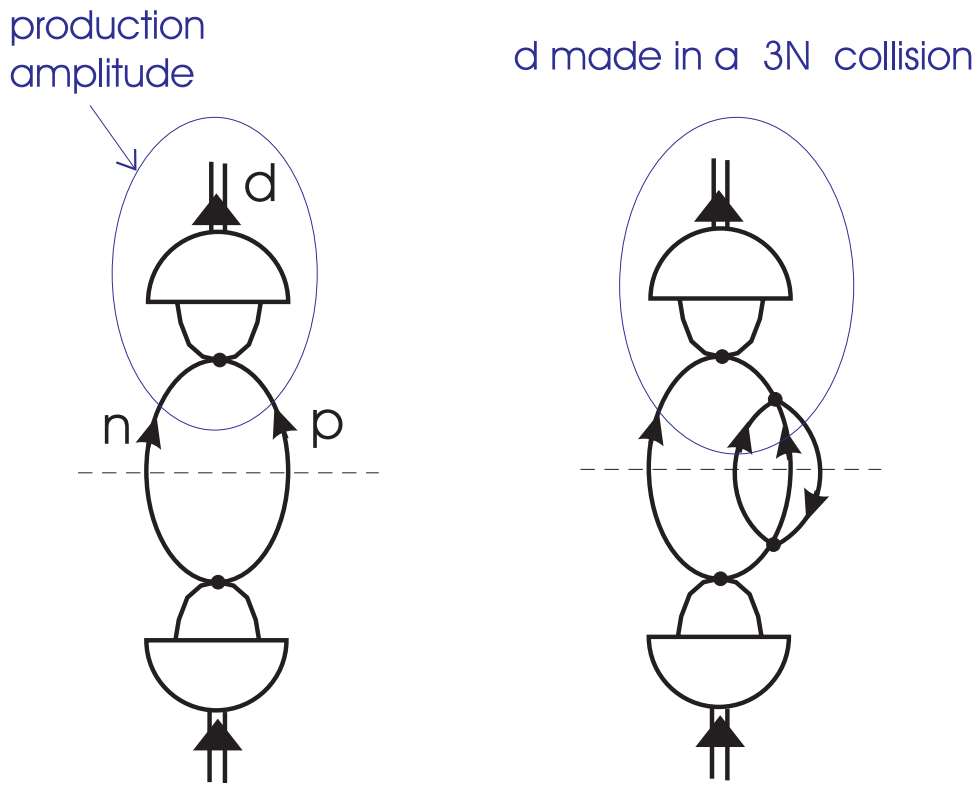
$$\mathcal{K}^{<} = \int d\mathbf{r}\, d\mathbf{r}' \, \phi_d^* \, v \, \langle \hat{\psi}^\dagger(\mathbf{x}_1' \, t') \, \hat{\psi}(\mathbf{x}_1 \, t) \rangle \, \langle \hat{\psi}^\dagger(\mathbf{x}_2' \, t') \, \hat{\psi}(\mathbf{x}_2 \, t) \rangle \, v \, \phi_d$$

Leading-order in the quasiparticle expansion: neutron & proton come together and make a deuteron.

If system approximately uniform and stationary, the process not allowed by energy-momentum conservation.

Process possible in a mean field varying in space, but, in nuclear case, the high-energy production rate low – tested in Glauber model.

3-Nucleon Collisions



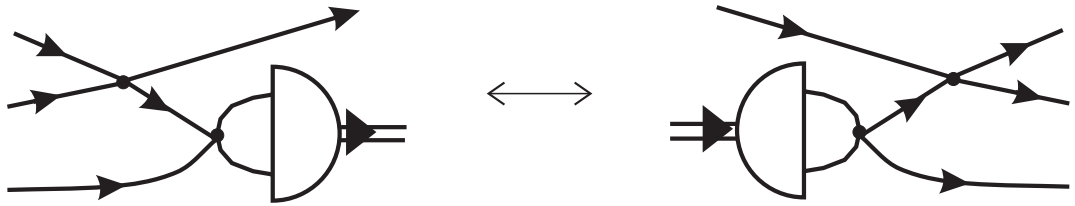
First correction to the pure 1-ptcle state, from a coupling to p-h excitations, yields a contribution to the d-production due to 3-nucleon collisions.

Still more nucleons involved in production of heavier clusters.

Deuteron Production

Detailed balance:

$$\overline{|\mathcal{M}^{npN\to Nd}|^2} = \overline{|\mathcal{M}^{Nd\to Nnp}|^2} \propto \mathrm{d}\sigma^{Nd\to Nnp}$$



Thus, production can be described in terms of breakup.

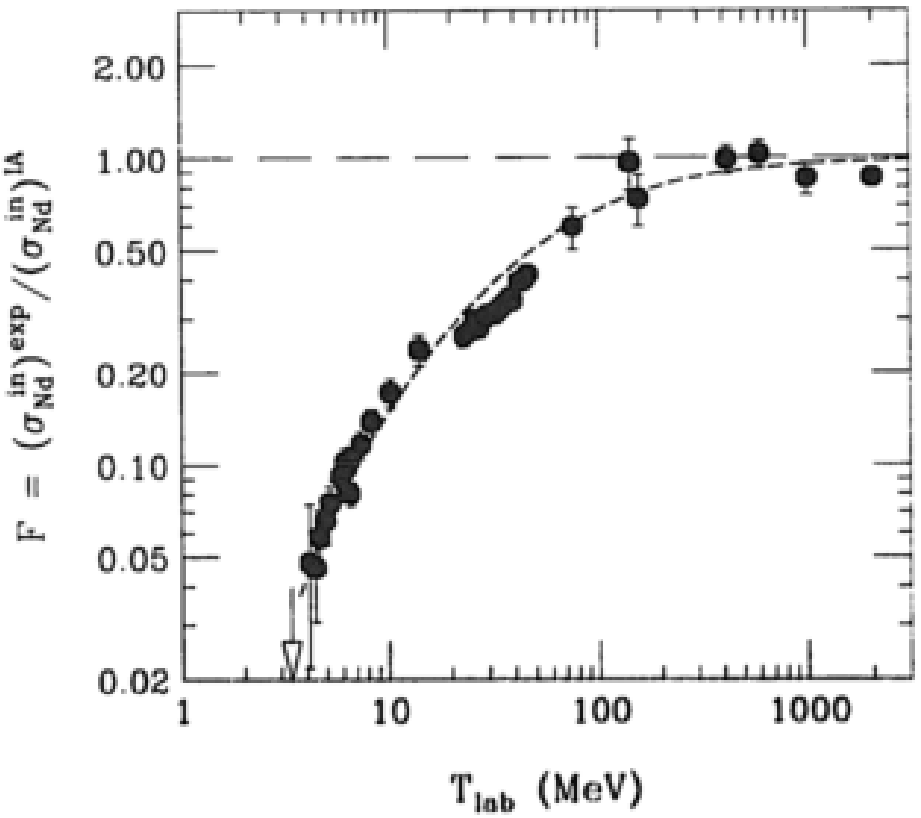
Problem: Breakup cross section only known over limited range of final states - Interpolation/extrapolation needed

Impulse approximation works at high incident energy

$$\overline{|\mathcal{M}_{Nd\to Npn}|^2_{IA}} = \left|\begin{array}{l}1\\2\\3\end{array}\right|^2 + \left|\begin{array}{l}2\\1\\3\end{array}\right|^2 + \left|\begin{array}{l}3\\1\\2\end{array}\right|^2$$

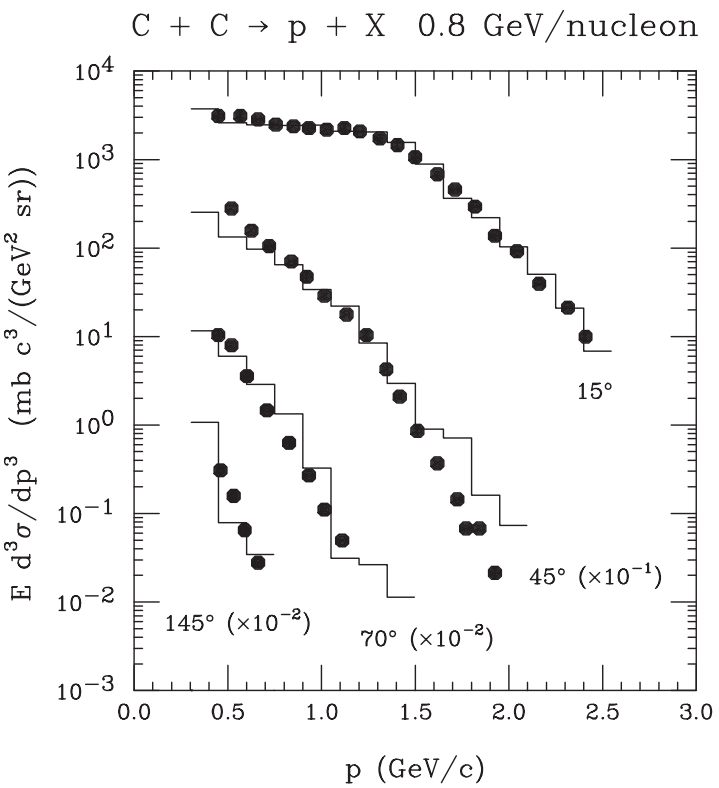
Renormalized Impulse Approximation

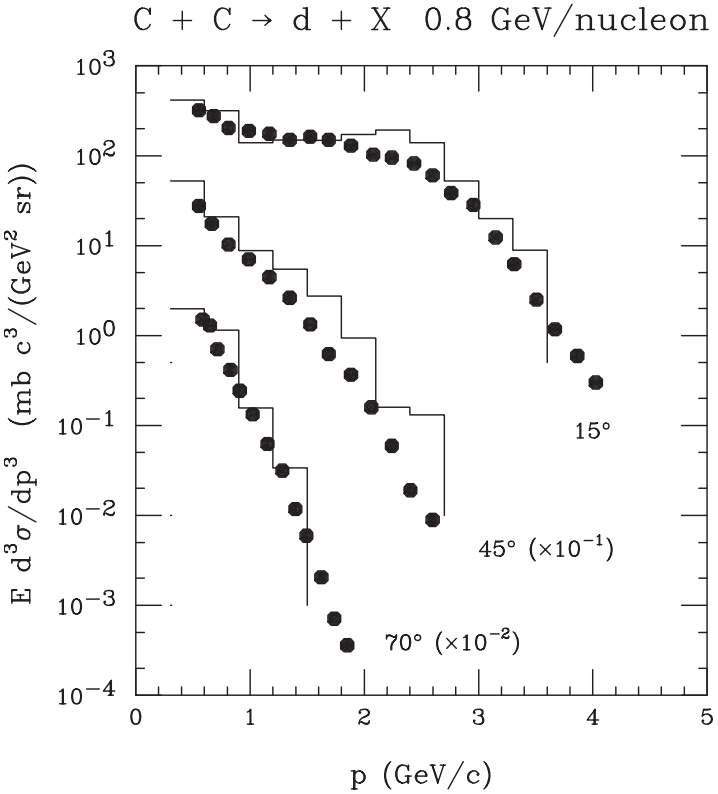
Renormalization factor for squared matrix element to get breakup cross section right as a function of energy



$$d\sigma^{Nd\to Nnp} \propto F\,\sigma_{NN}\,|\phi_d(p)|^2$$

Single-Particle Spectra





proton & deuteron inclusive spectra

histograms: calculations using

$|\mathcal{M}^{npN\to Nd}|^2 = |\mathcal{M}^{Nd\to npN}|^2 \propto \mathrm{d}\sigma_{Nd\to npN}$

and $\langle f \rangle < 0.2$ cut-off for deuterons

$A = 3$ Particles + Tests

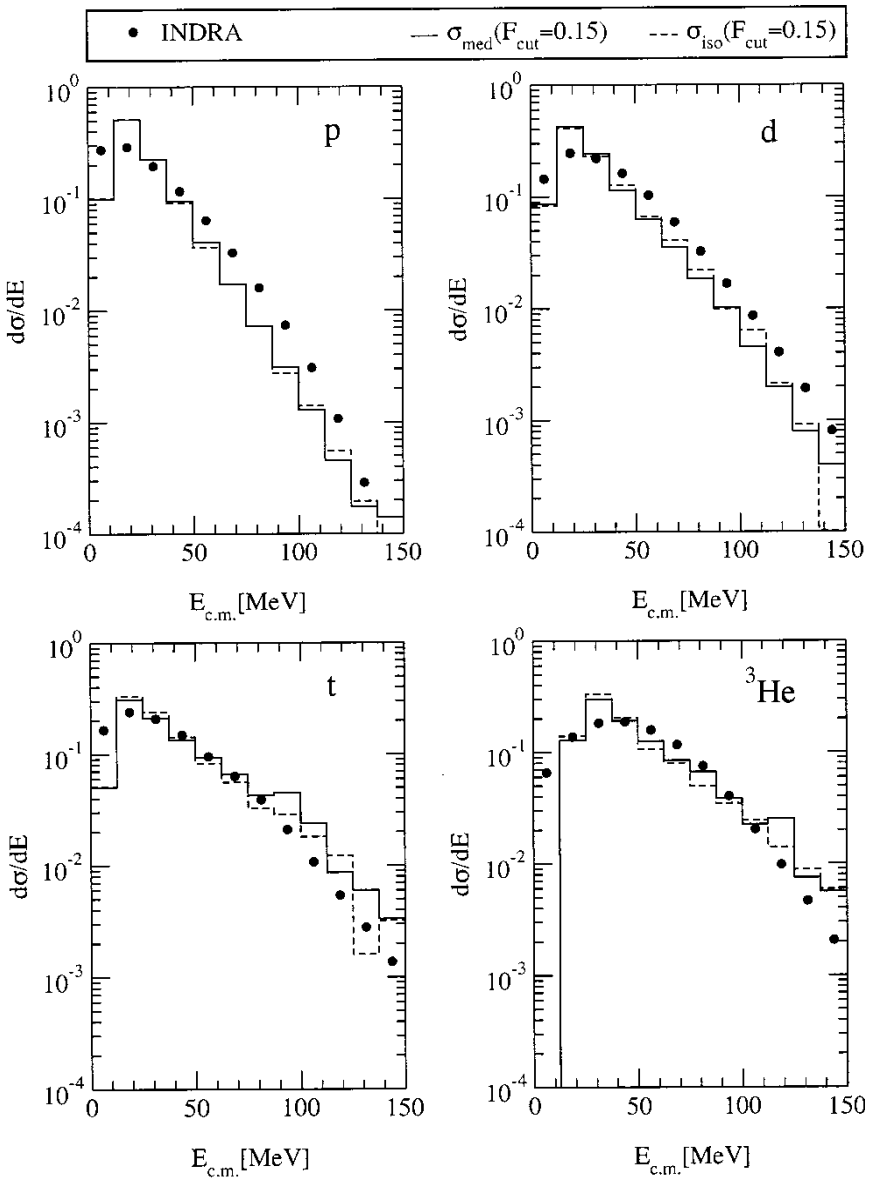
A=3-ptcles from 4N collisions

Christiane Kuhrts: solving finite-T Galitski-Feynman (GF) and modified (in-medium) Alt-Grassberger-Sandhas eqs

solid lines: finite-T GF for cross-sections and existence

dashed lines: free cross sections + $\langle f \rangle$ cut-off

symbols: INDRA data ^{129}Xe + ^{119}Sn at 50 MeV/nucleon



Collective Expansion

Is expansion viscous or isentropic?? Is pressure carrying out work producing a collective expansion of matter?

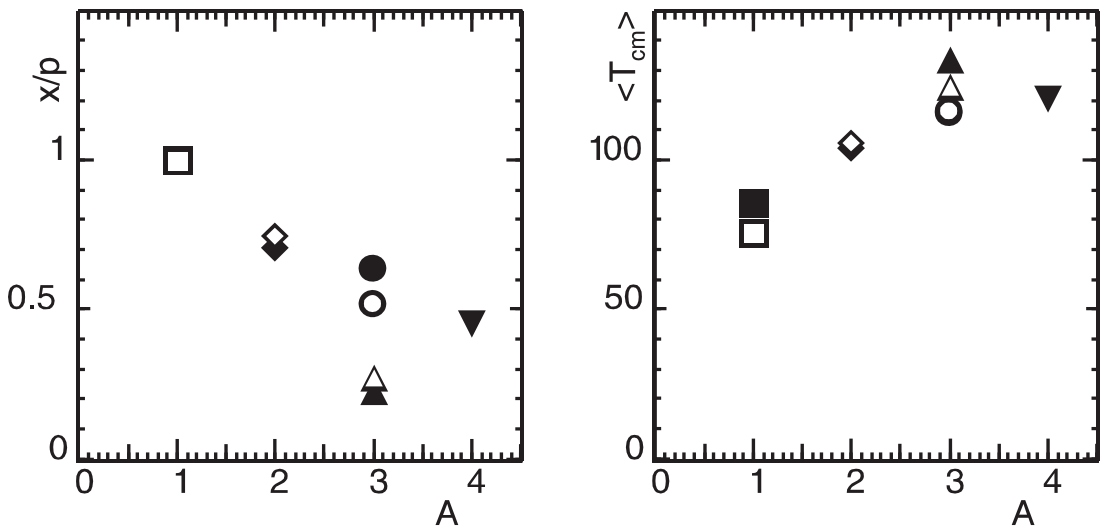
$$\langle E_x \rangle = \frac{3}{2} T + \frac{m_x \langle v^2 \rangle}{2}$$
$$= \frac{3}{2} T + A_x \frac{m_N \langle v^2 \rangle}{2}$$

In isentropic expansion, average kinetic energy should increase with fragment mass.

Au+Au 250AMeV $60^\circ < \theta_{cm} < 90^\circ$

yield ratio

av kin energy



EXP(erat5) Tr.Mod.(b<3.5*fm*)

EXP(erat5)	Tr.Mod.(b<3.5*fm*)
p	p
d	d
t	t
^{3}He	^{3}He
^{4}He	

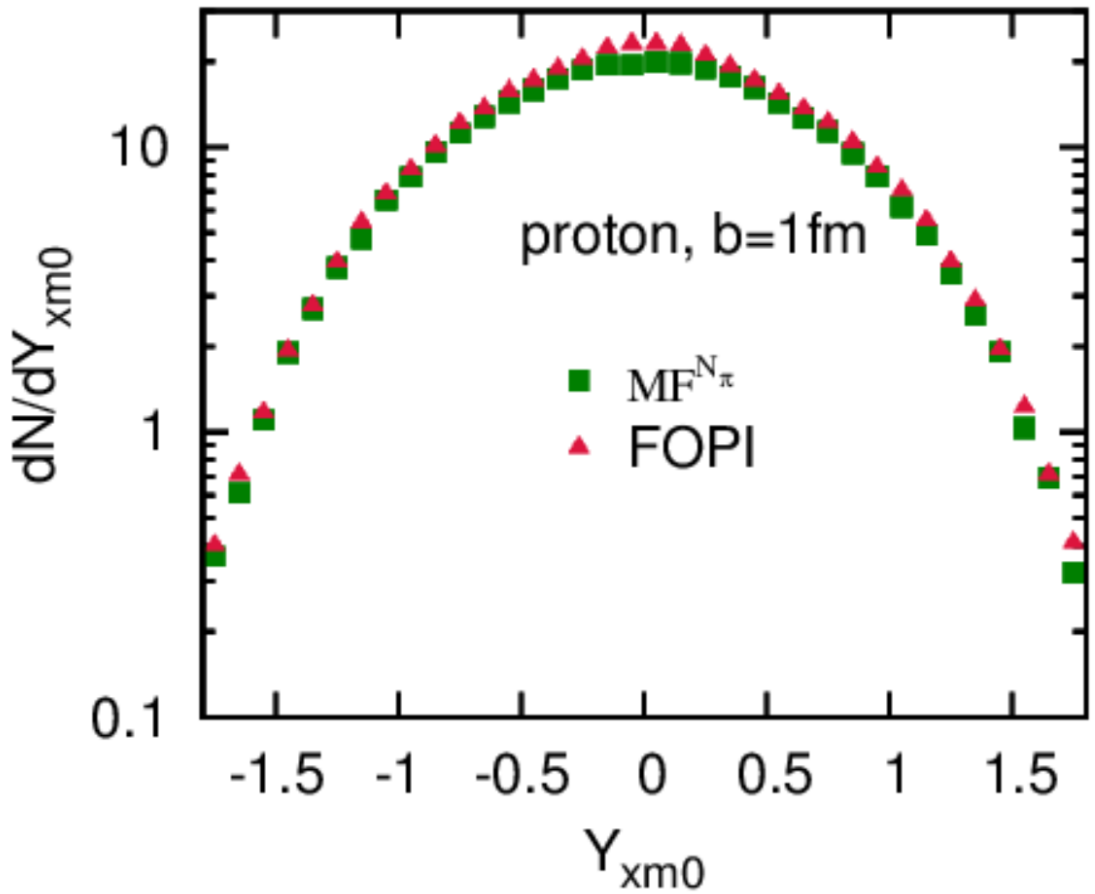
filled symbols -- data, Poggi et al

open symbols -- calculation

Energy increases linearly!

Head-On Au + Au (FOPI)

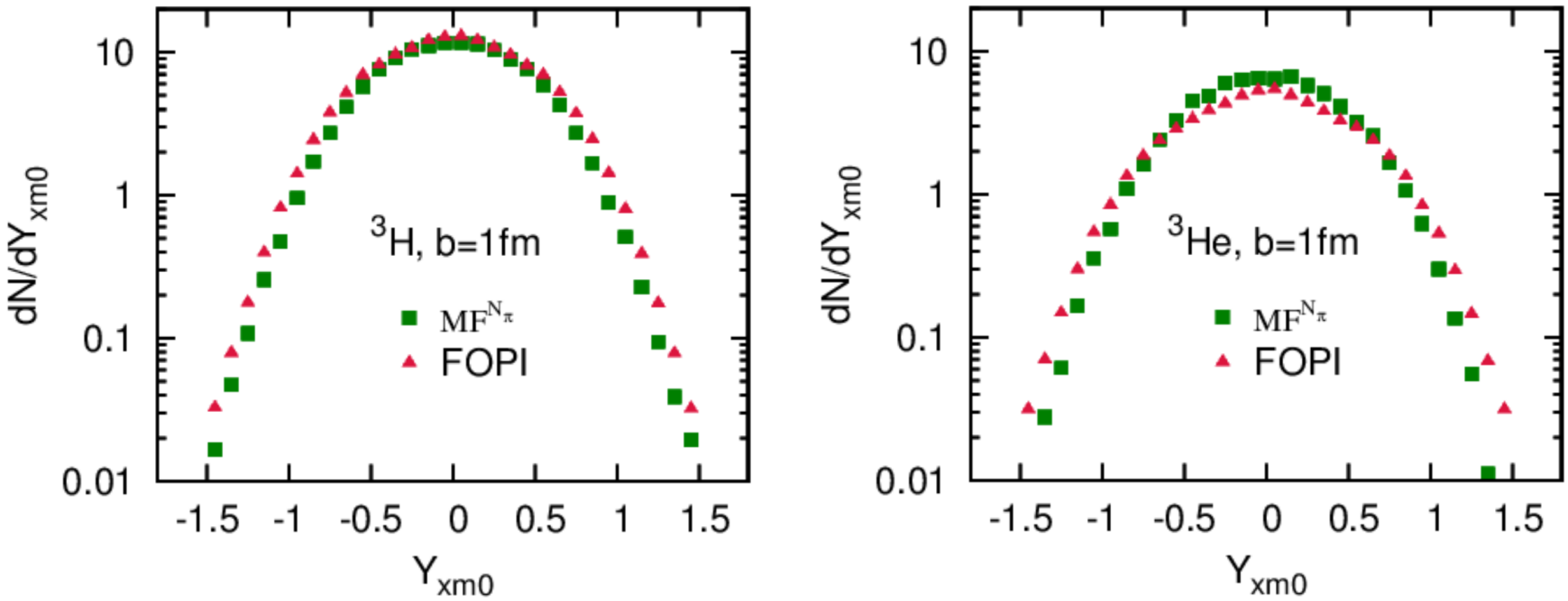
Rapidity Distribution



400 MeV/nucleon

$A = 3$ in Head-On Au + Au (FOPI)

Rapidity Distributions



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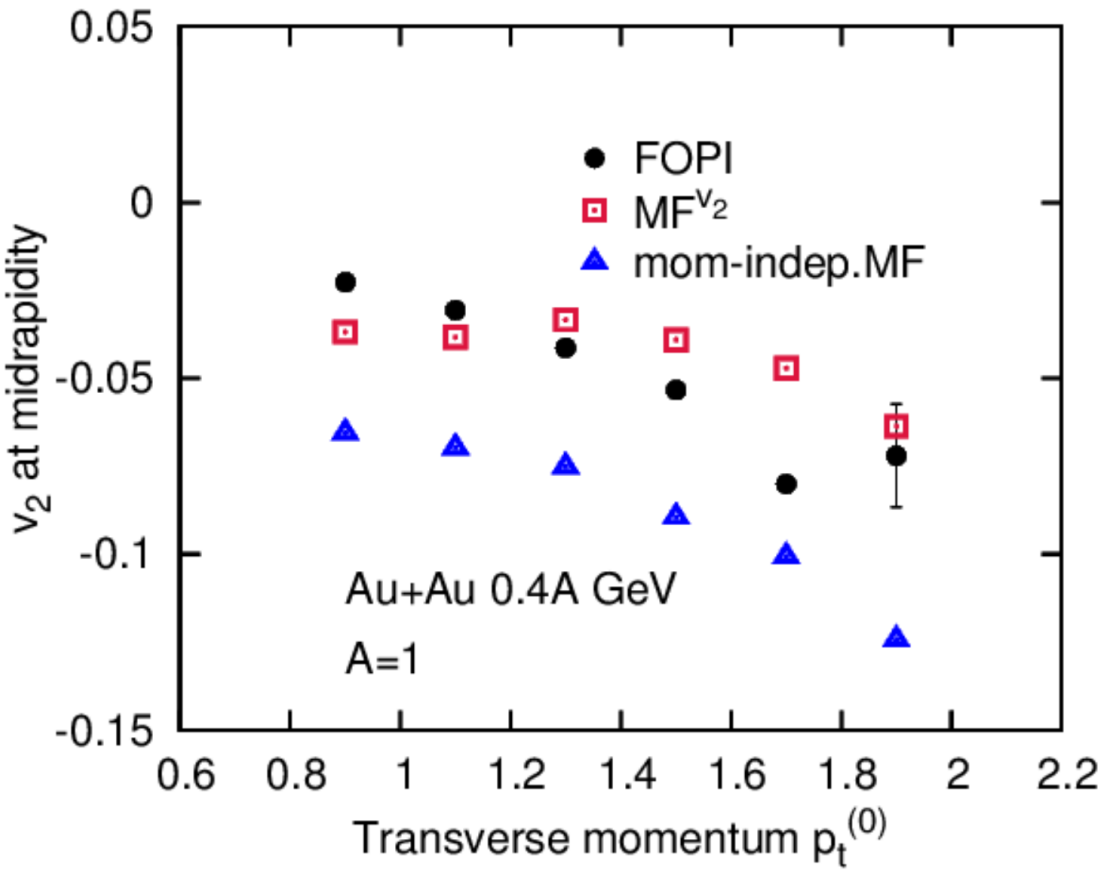
Semicentral Au + Au (FOPI)

Elliptic Flow



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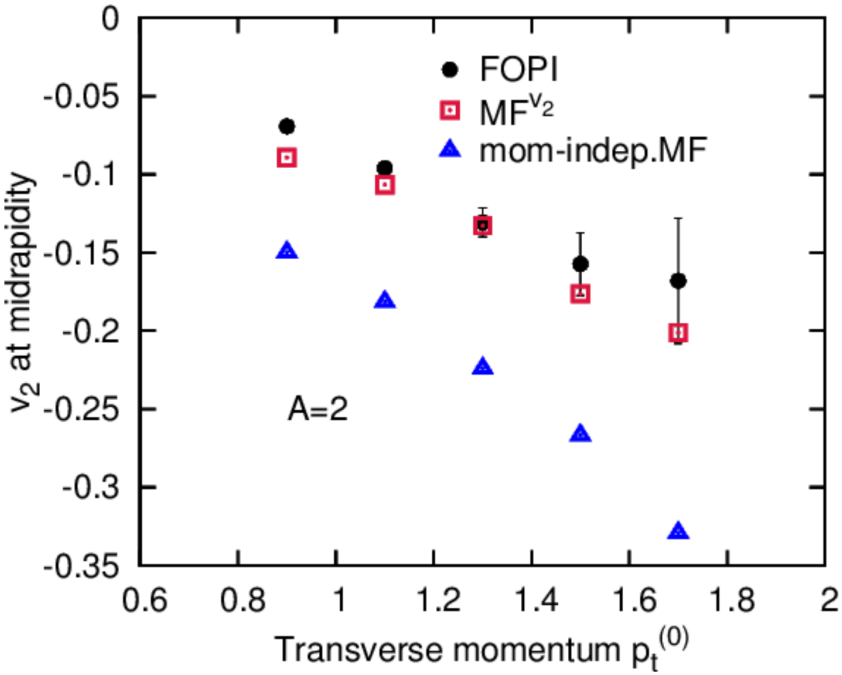
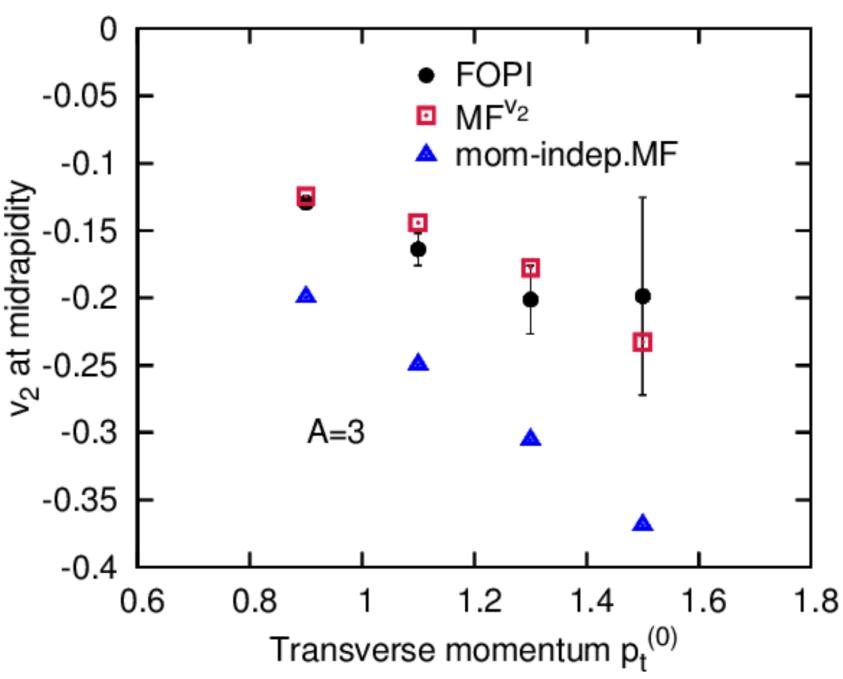
Elliptic Flow





400 MeV/nucleon

Conclusions

- Variety of single-particle energies/mean fields can be incorporated into transport, in conserving manner, provided suitable generalized densities are used
- Real-time many-body theory provides microscopic fundamentals for production of clusters in transport theory
- Few-body collisions or rapidly changing mean-field conditions are needed to spur cluster production
- Detailed balance must be obeyed for thermodynamic consistency
- Breakup data yield production rates in collisions
- Clusters emphasize collective motion

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