



# Sequential clustering in transport dynamics

by A. Le Fèvre<sup>1</sup>, Y. Leifels<sup>1</sup>, J. Aichelin<sup>2</sup>, Ch. Hartnack<sup>2</sup>

<sup>1</sup>GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany

<sup>2</sup>SUBATECH, IMT Atlantique - IN2P3/CNRS - Université de Nantes, France



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- The FRIGA clustering approach.
- The achievements of the static approach.
- Spectator versus fireball cluster formation: cold-static versus hot-expanding-sequential clustering.
- Sequential FRIGA: new developments and results.



# FRIGA: a clusterisation approach...

## Fragment Recognition In General Applications



Frigg / Friga, spinning the clouds

Friga (Frigg), goddess of harmonious weddings and alliances, setting order in the chaos, in the old Germanic mythology.



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- \* Prediction of (light and heavy) (hyper)isotope yields and full phase space distribution.



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# Simulated Annealing Clusterization Algorithm (SACA): The principles

If we want to **identify fragments early**, one has to use **momentum space** info as well as **coordinate space** info.

Idea by **Dorso et al.** (Phys.Lett.B301:328,1993) :

- a) Take the **positions** and **momenta** of **all nucleons** at time  $t$ .
- b) **Combine** them in all possible ways into **fragments** or leave them as **single nucleons**.
- c) Neglect the interaction among clusters.
- d) **Choose that configuration which has the highest binding energy.**

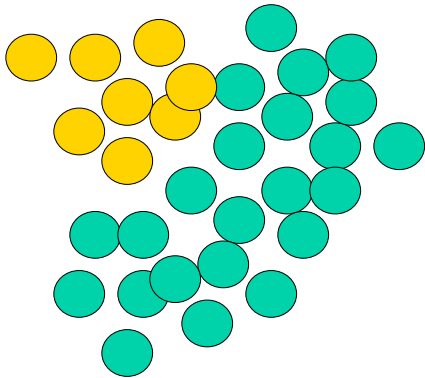
Simulations show: Clusters chosen that way at **early times** are the pre-fragments of the final state clusters, because fragments are not a random collection of nucleons at the end but initial-final state correlations.



# FRIGA: a clusterisation approach...

## Steps:

1) Pre-select good «candidates» for fragments according to proximity criteria: coordinate and momentum space coalescence = Minimum Spanning Tree (MST) procedure.



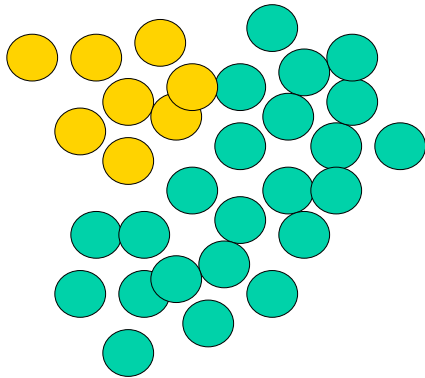
$$E = E_{\text{kin}}^1 + E_{\text{kin}}^2 + V^1 + V^2$$



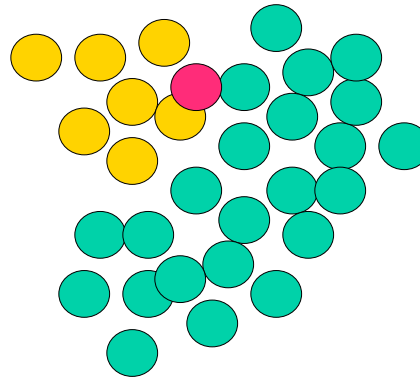
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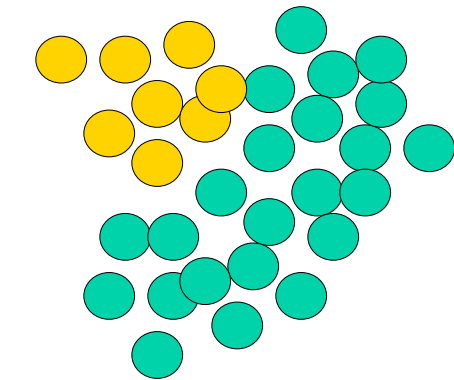




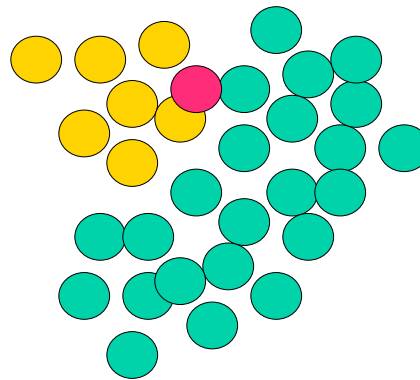
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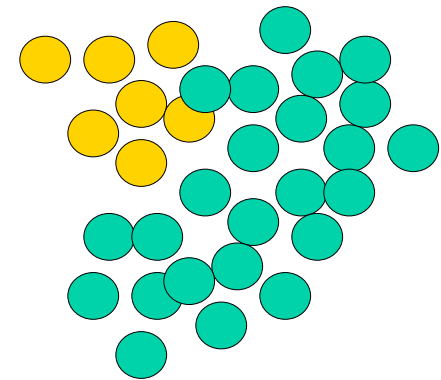
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$$E' = E_{\text{kin}}^{1'} + E_{\text{kin}}^{2'} + V^{1'} + V^{2'}$$

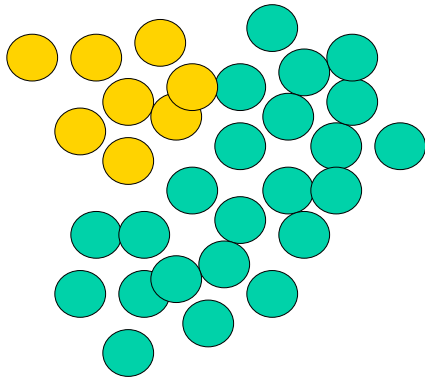




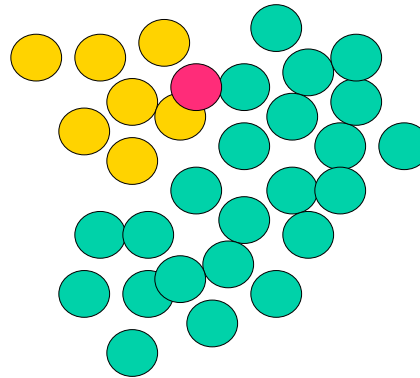
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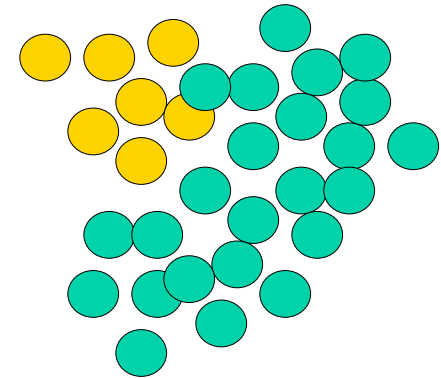
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If  $E' < E$  take the new configuration

If  $E' > E$  take the old with a probability depending on  $E' - E$

Repeat this procedure very many times... (Metropolis procedure)

It leads automatically to **the most bound configuration**.



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**Ingredients of the binding energy of the clusters :**





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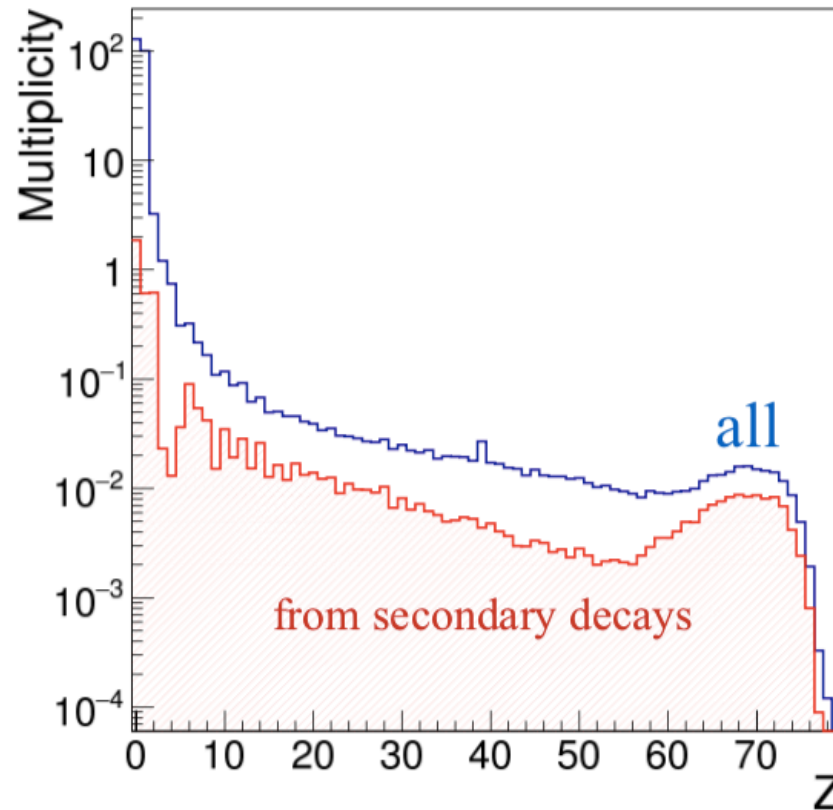
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- ⑧ Rejection of « non-existing » isotopes and hyper-clusters.



# Influence of the secondary decays on light isotope yields

An example: Au+Au @ 600 A.MeV (min. bias),  $b < 6$  fm (passing time =  $2 t_{\text{pass}}$ ) from BQMD\*+FRIGA



\*: J. Aichelin. Phys. Reports 202, 233 (1991).

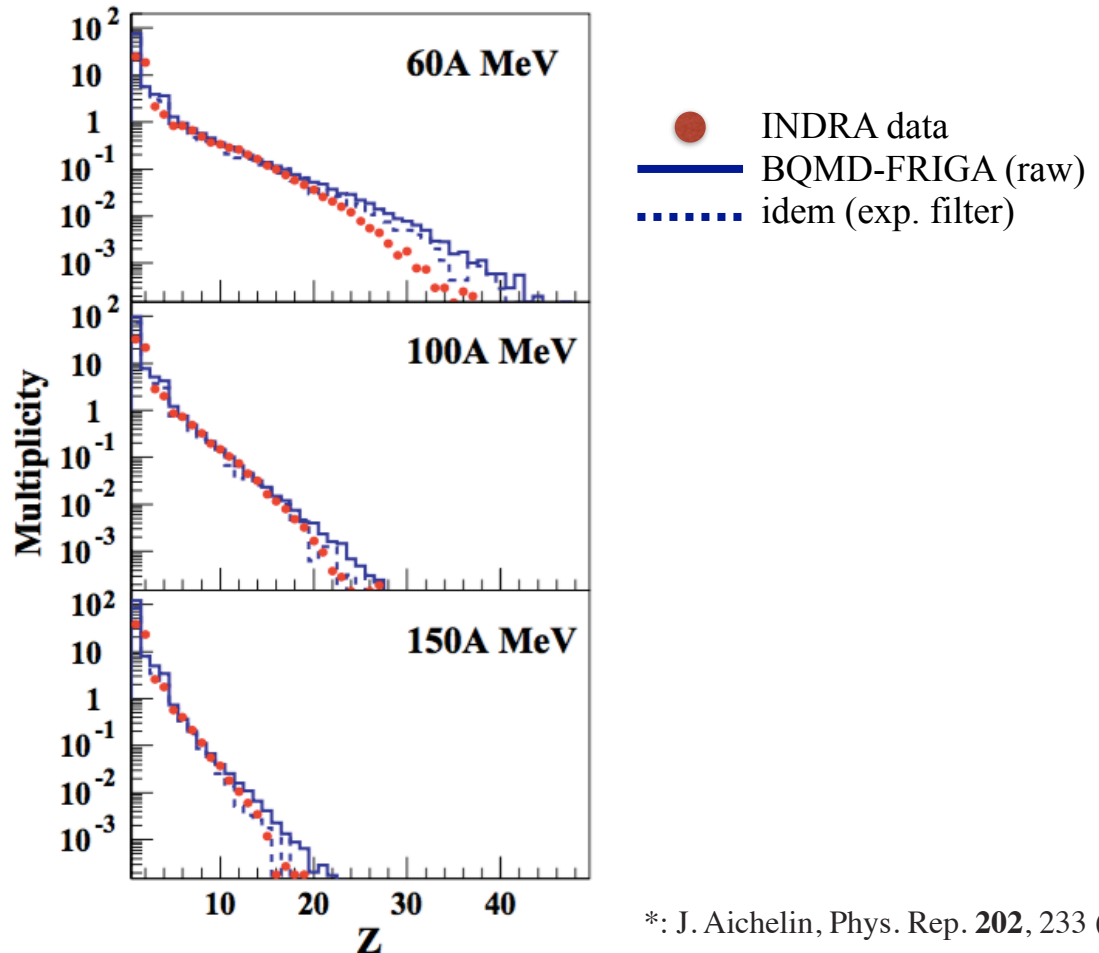
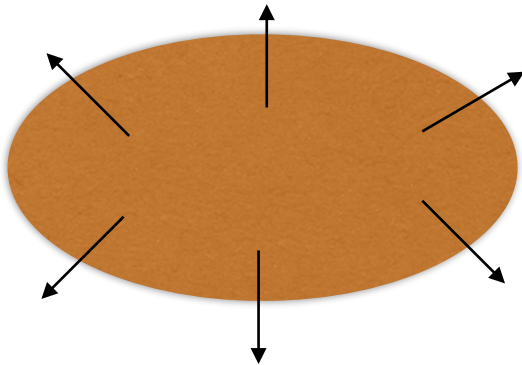


# Some successful applications at intermediate energies

INDRA central Au+Au

K. Zbiri et al., PHYSICAL REVIEW C **75**, 034612 (2007)

BQMD\*+FRIGA  
( $t = 2 t_{\text{pass}}$ )



\*: J. Aichelin, Phys. Rep. **202**, 233 (1991).



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INDRA Au+Au excitation function  
A. Le Fèvre and J. Aichelin - PRL 100, 042701 (2008)

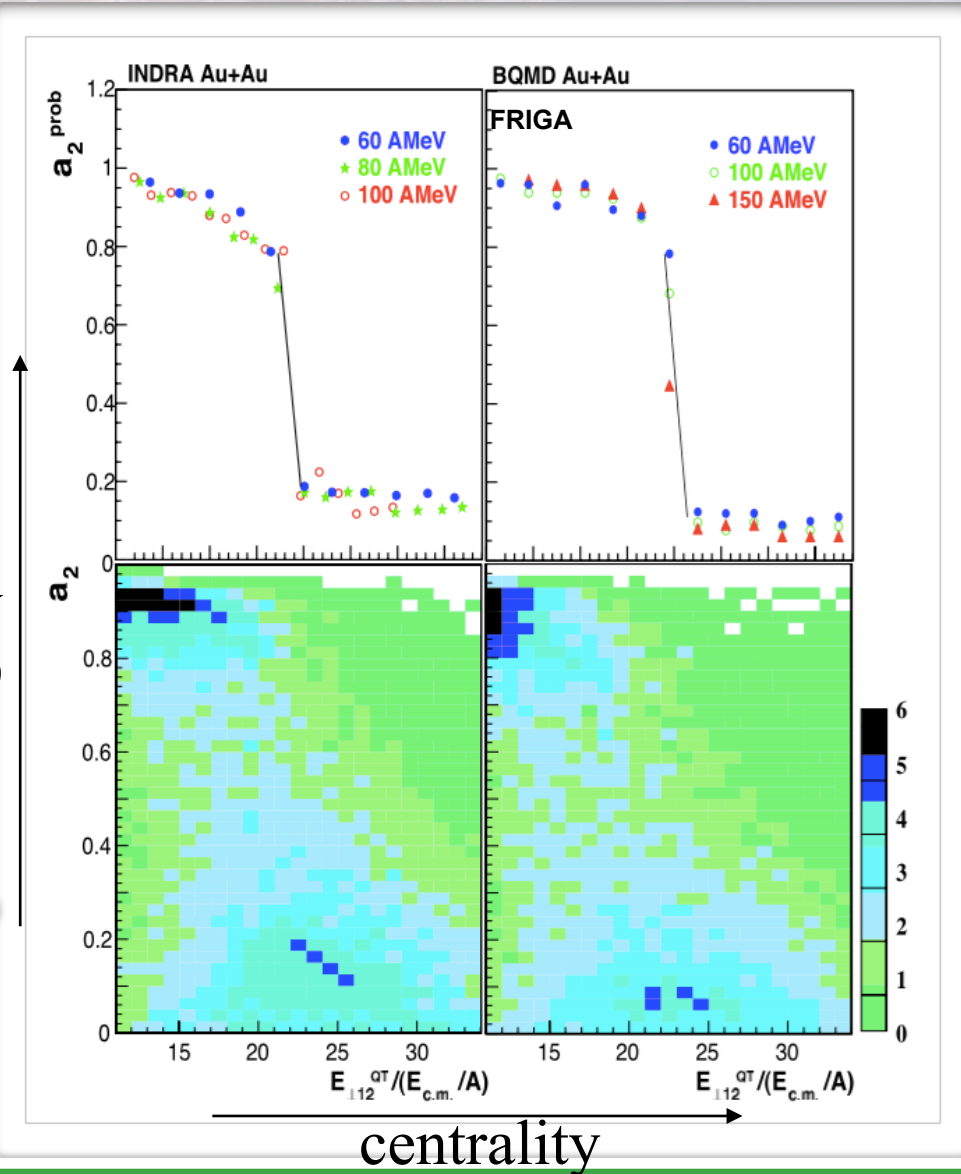
BQMD\*+FRIGA  
( $t = 2 t_{\text{pass}}$ )

=> Bimodality\*\* =  
a mechanical instability  
(critical phenomenon, metastability)

fragment size asymmetry  
 $a_2 = (Z_1 - Z_2) / (Z_1 + Z_2)$

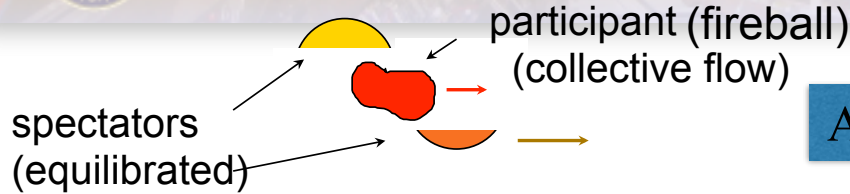
=> sufficient fluctuations  
necessary in transport models

\*: J. Aichelin, Phys. Rep. **202**, 233 (1991).  
\*\*: also found in Xe+Sn system in  
M. Pichon et al, INDRA-ALADIN Coll.,  
Nuclear Physics A 779 (2006) 267–296

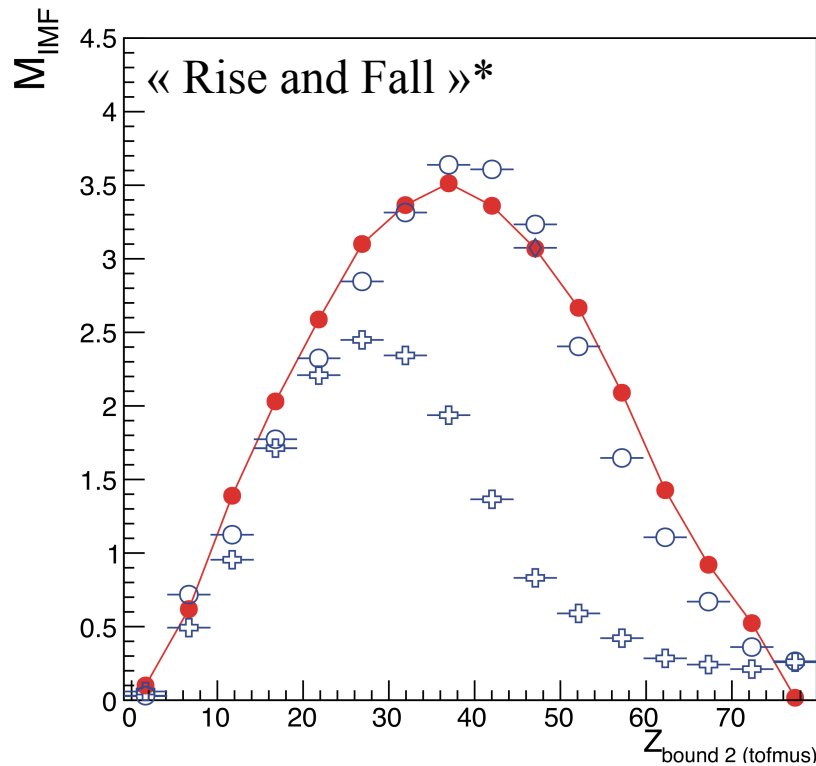




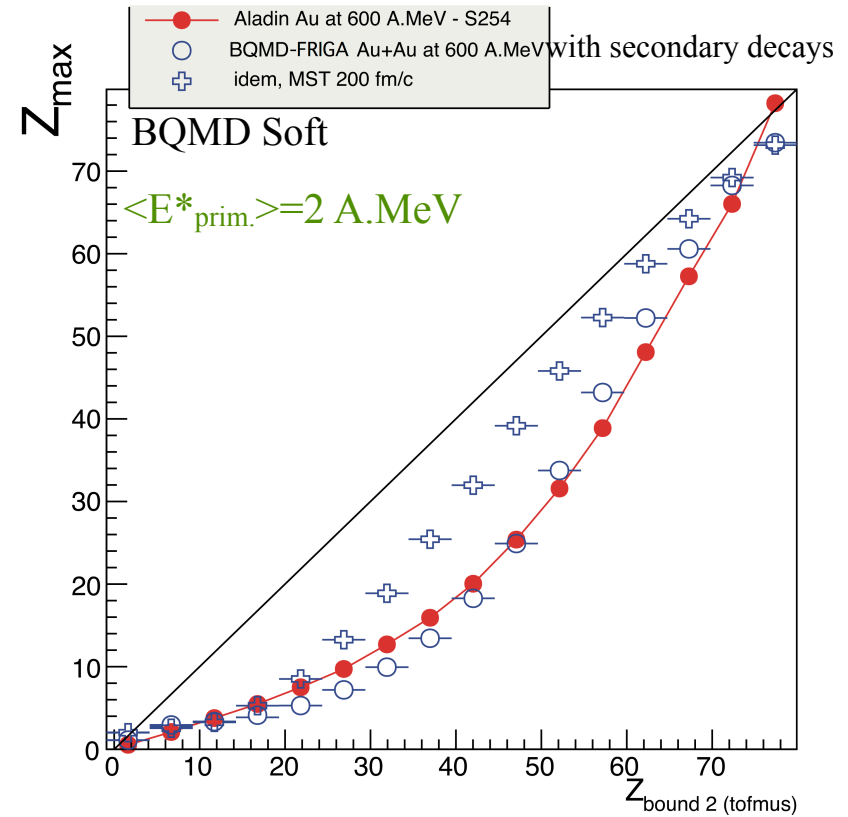
# Some successful applications in the spectator regime



ALADiN Au+Au @ 600 A.MeV (S254 exp., 2003)

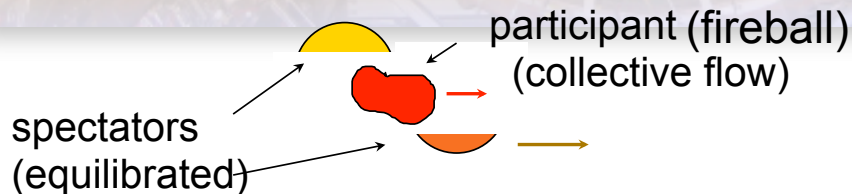
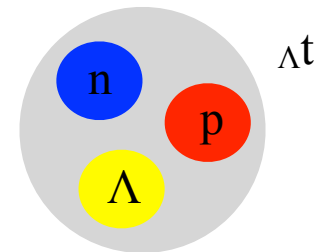


← centrality



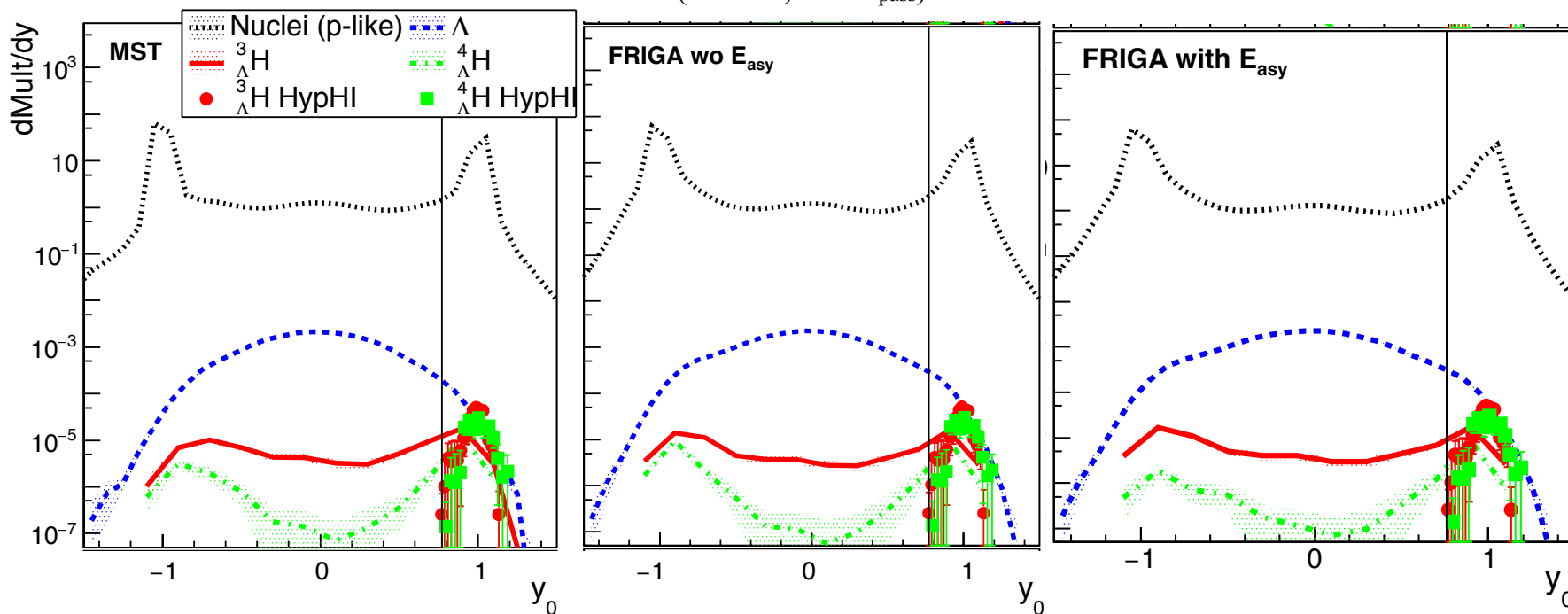
← centrality

# Some successful applications in the spectator regime (hypernuclei)



HyPHI experiment @ GSI  
Ch. Rappold et al.,  
PLB 747 (2015) 129–13

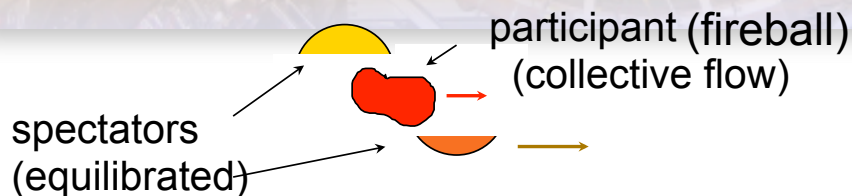
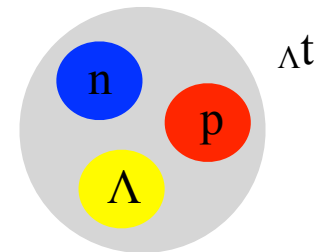
IQMD\*+FRIGA  
 ${}^6\text{Li}+{}^{12}\text{C}$  @ 2A.GeV  
( $b > 3$  fm,  $t = 4 t_{\text{pass}}$ )



\*: Ch.Hartnack et al., Eur. Phys. J. A 1(1998) 151.

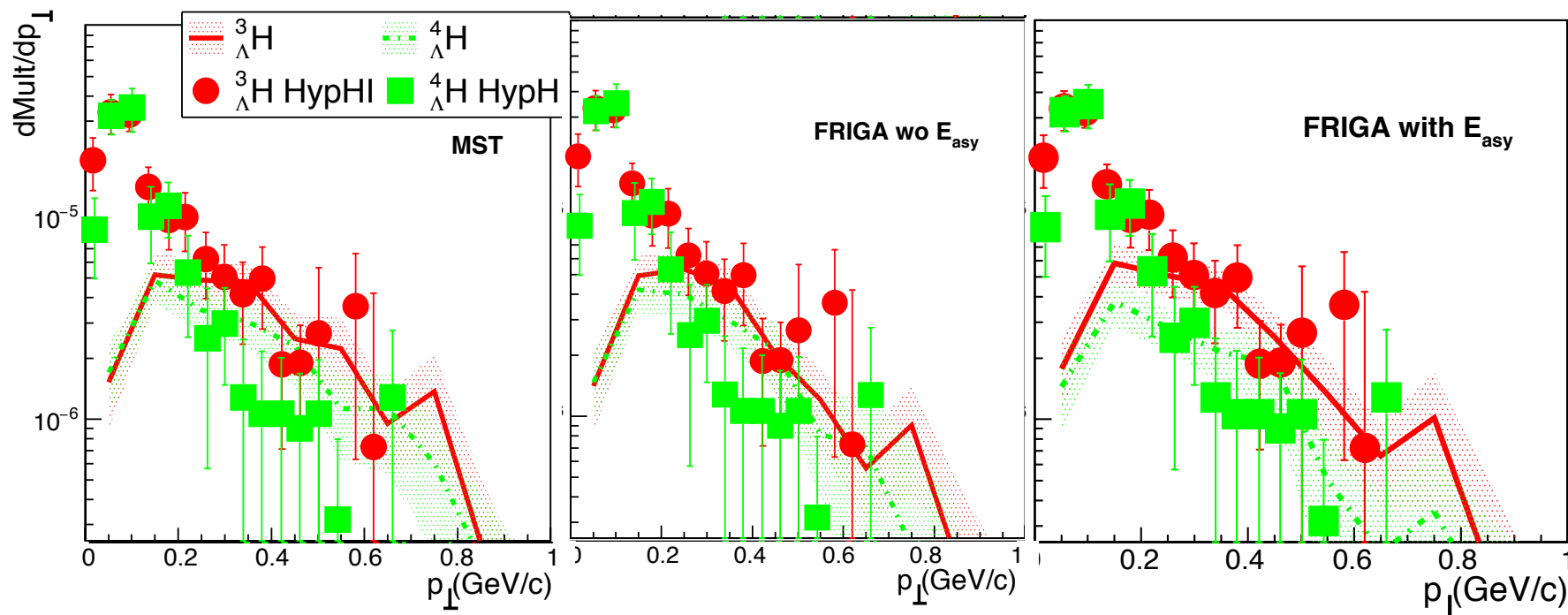


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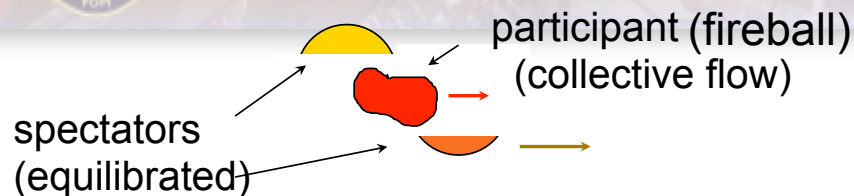
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# The fireball regime with the static approach



Central

Au+Au:

IQMD-FRIGA (dashed lines)

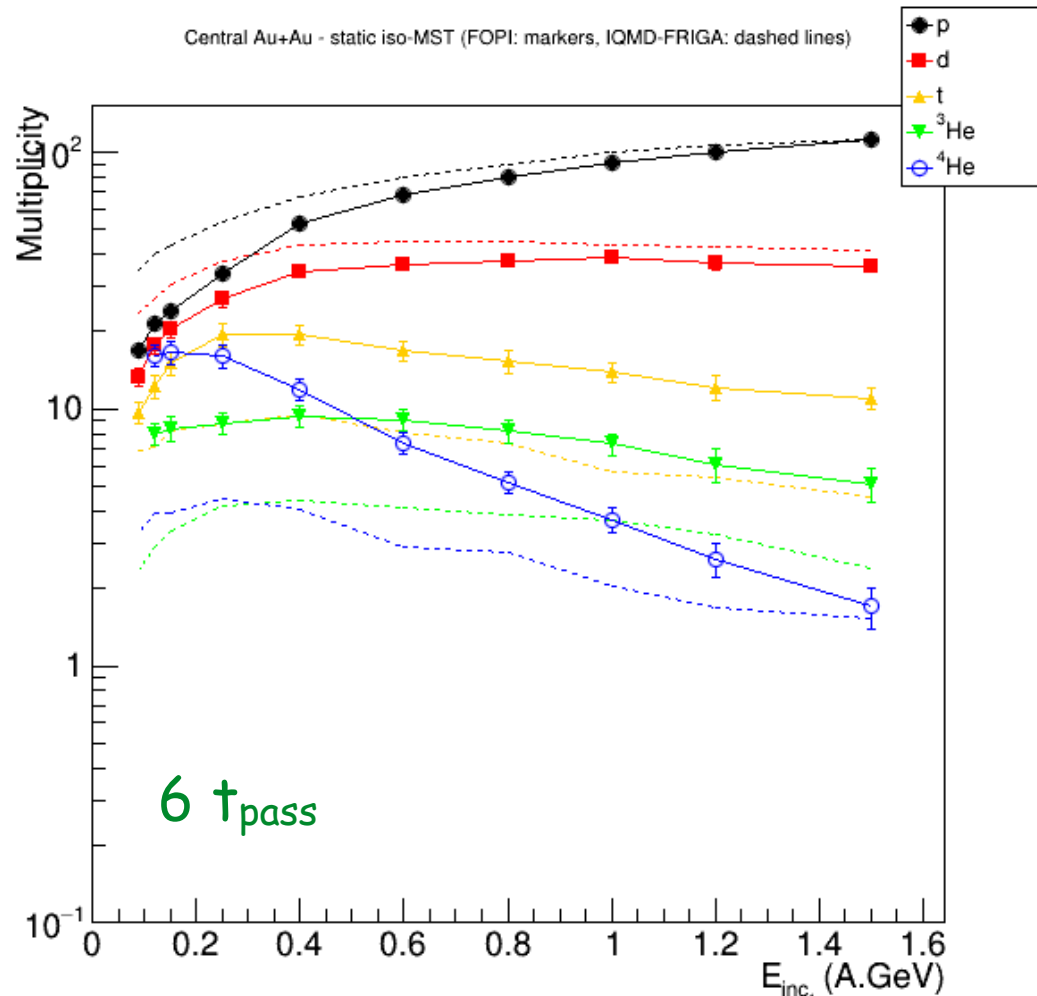
vs

FOPI data (markers)\*

static  
iso-MST\*

$$*\Delta r_{pp} < 2.5 \text{ fm}$$

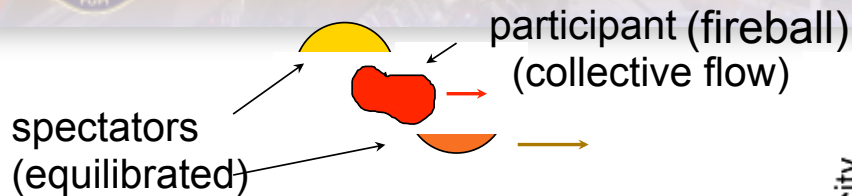
$$\Delta r_{nn,np} < 3.8 \text{ fm}$$



\*: W. Reisdorf et al., FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427



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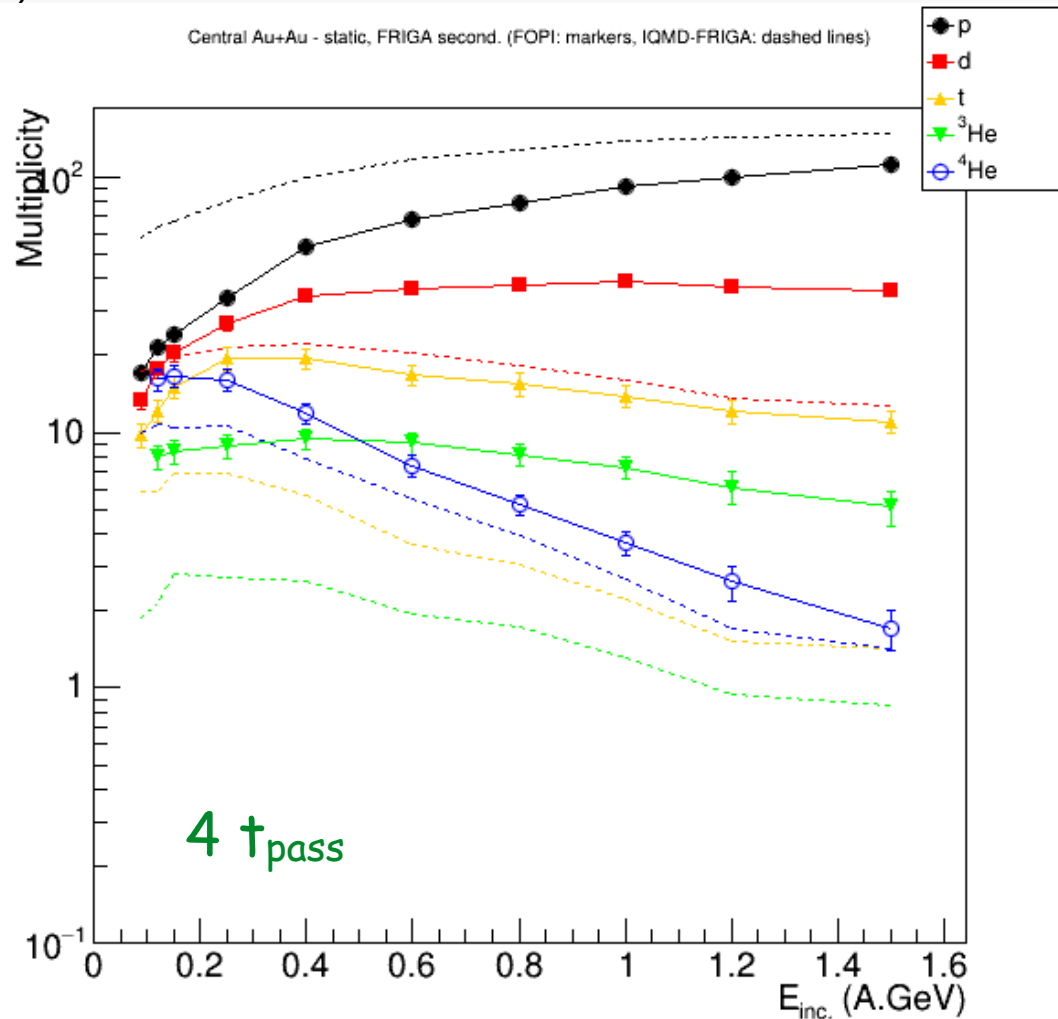
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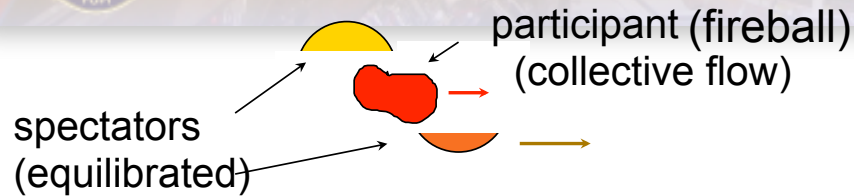
static  
FRIGA  
(with  $B_{asy}$ , second.)



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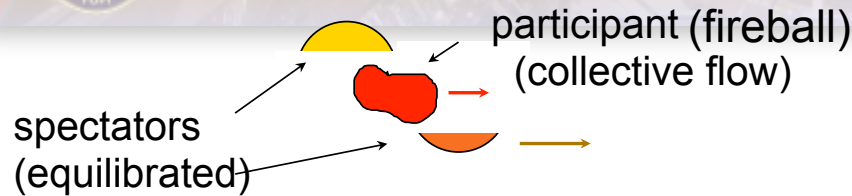


- ⇒ In central collisions, the static/instantaneous FRIGA strategy (including asymmetry and structure binding energies) provides inaccurate light isotope yields
- ⇒ Reversely, the MST static coalescence approach is more reliable at the highest SIS energies.

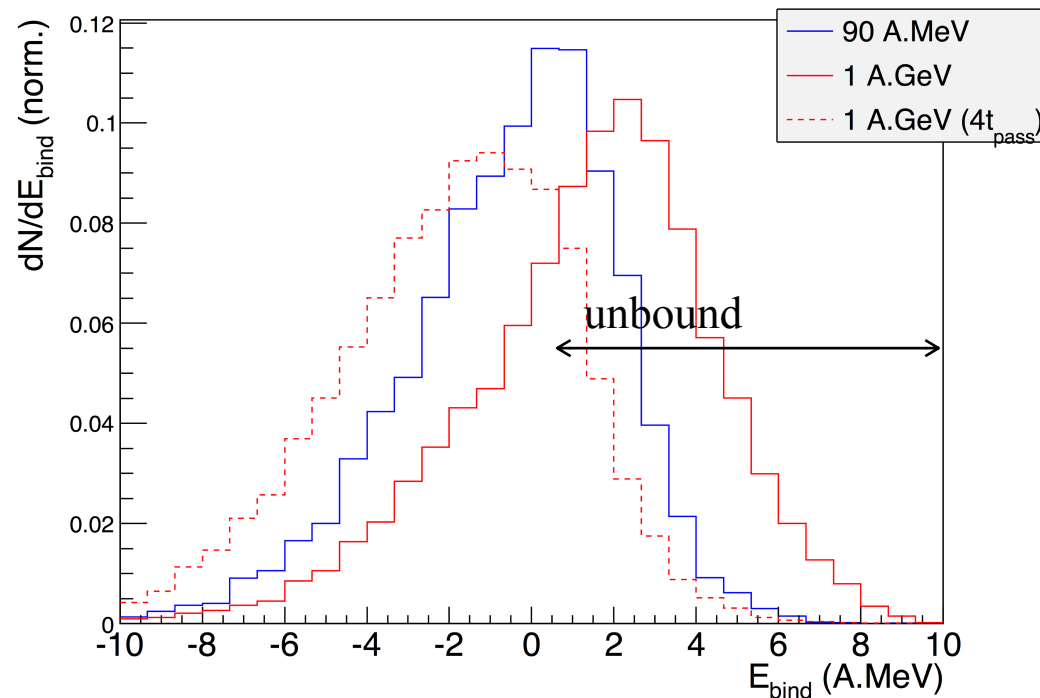
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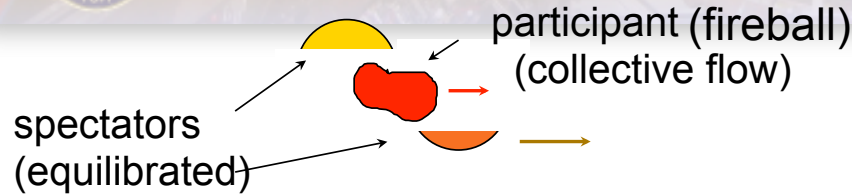


Binding energy of early ( $2 t_{\text{pass}}$ ) tritons  
identified with coalescence (MST)

$B_{\text{asy}}$  on  
 $B_{\text{struct}}$  on



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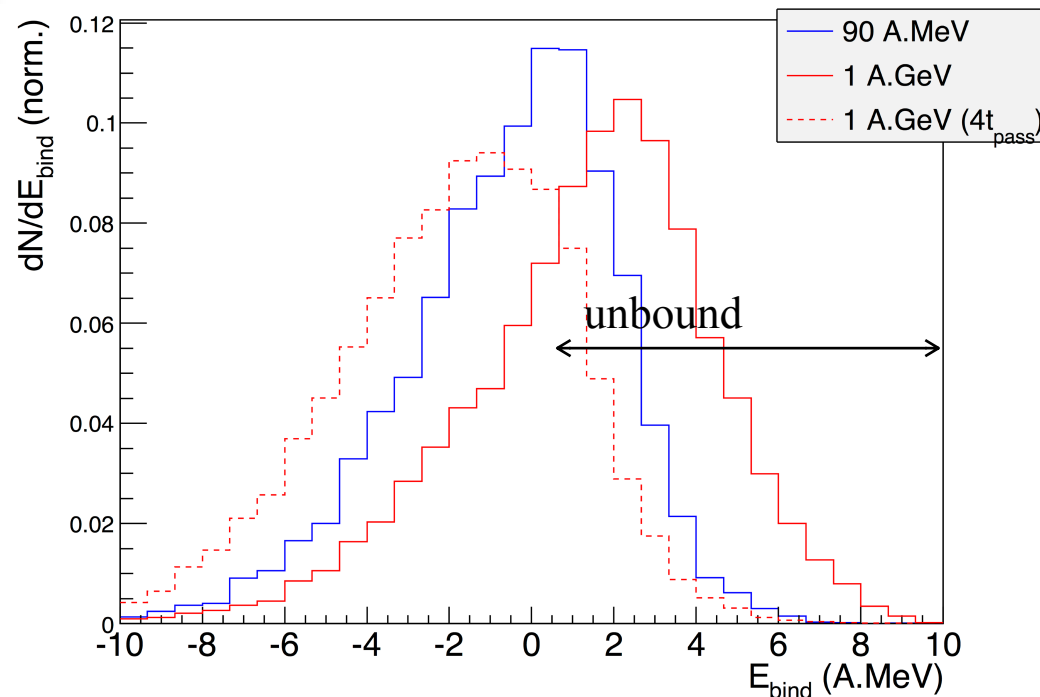


Central  
Au+Au:  
IQMD-FRIGA

1- On the contrary of the rather cool central source of intermediate energies (and spectator), in the hot fireball, early pre-fragments are mostly hot and unbound.

2- In a fireball, the hot expanding medium needs more time to generate clusters than the spectator regime (fast clustering from a rather cold non expanding medium)

⇒ Invalidity of static/instantaneous FRIGA as an early « afterburner » in the fireball regime. Better alternative: follow the process of cluster formation up to a relatively longer time.



Binding energy of early ( $2 t_{\text{pass}}$ ) tritons  
identified with coalescence (MST)

$B_{\text{asy}}$  on  
 $B_{\text{struct}}$  on

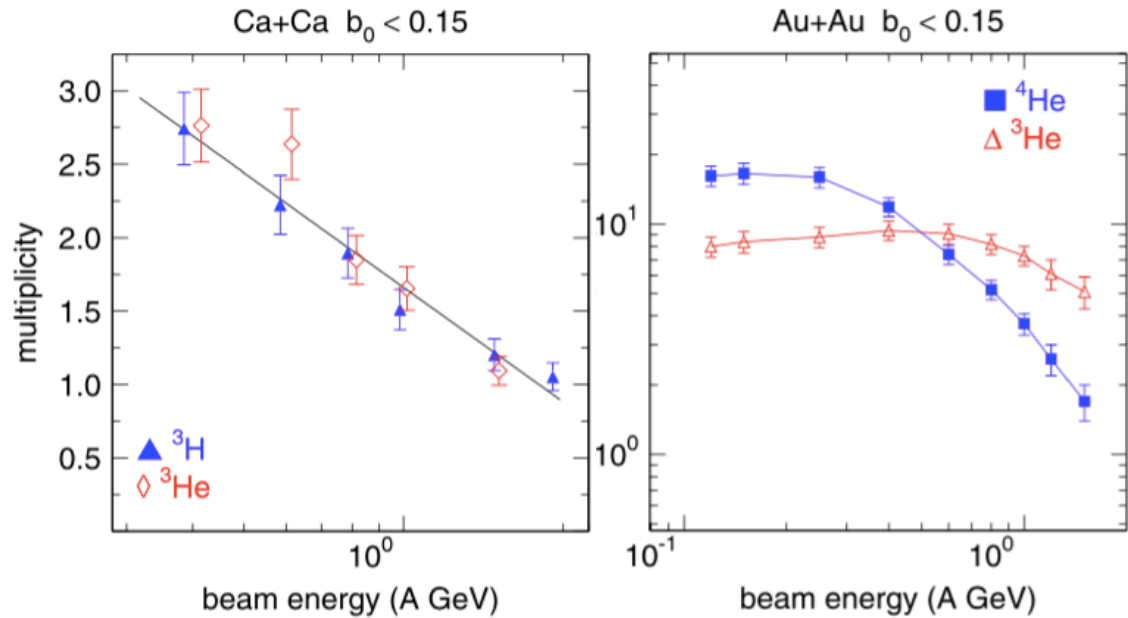




# Clustering in the fireball regime: what says the experiment?

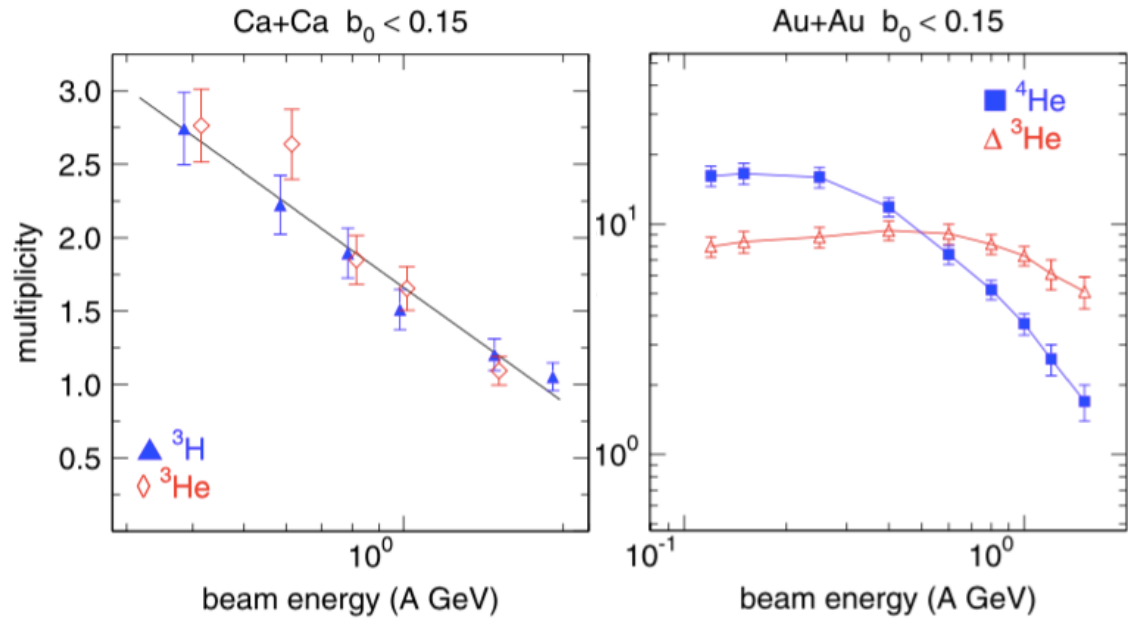


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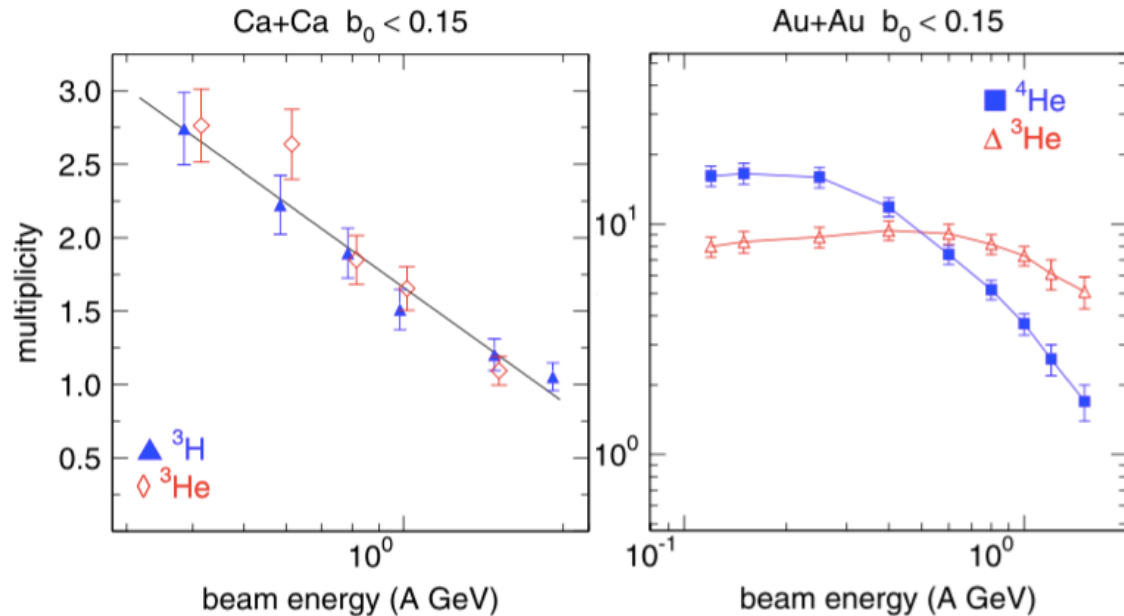
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FOPI Collaboration / Nuclear  
physics A 848 (2010) 366-427

A high degree of clusterisation  
even at high energies:  
extrapolation of FOPI@GSI trend  
→ clustered fraction >10% up to  
4A GeV. Persistence of a  
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⇒ Signal of **local cooling**  
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⇒ Strong constraint on the  
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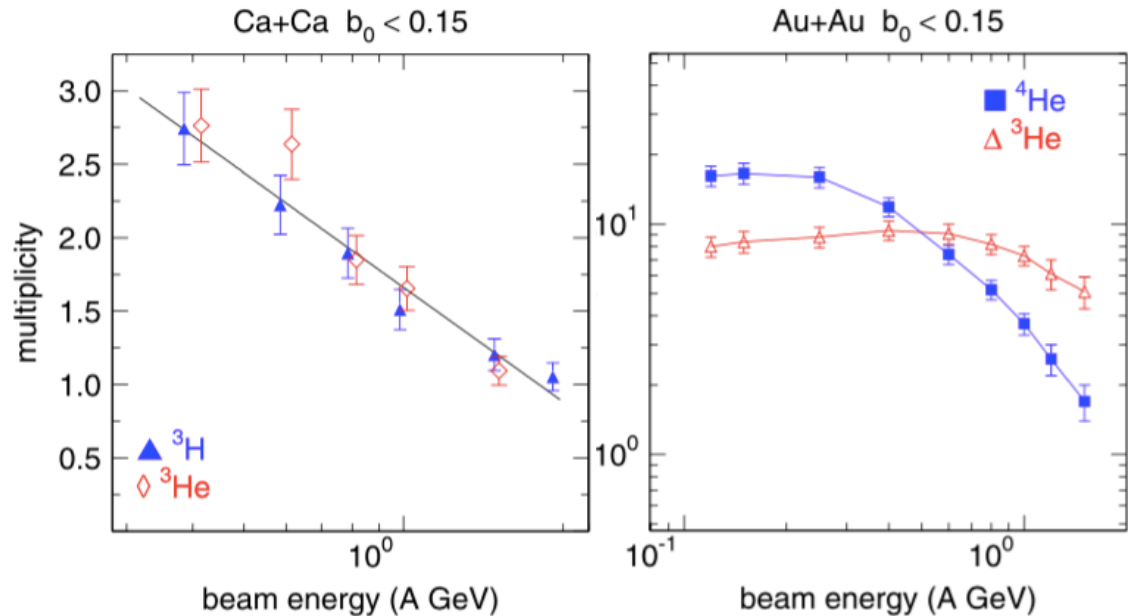
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An interpretation: Increased **stopping** (right panels) ↔  
increased **compression** ↔ increasing **radial flow** developed  
thereafter in the expansion phase coupled to increased  
**cooling** ('**droplet formation**')



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  - 3) Process with MST/FRIGA **free** hadrons only.



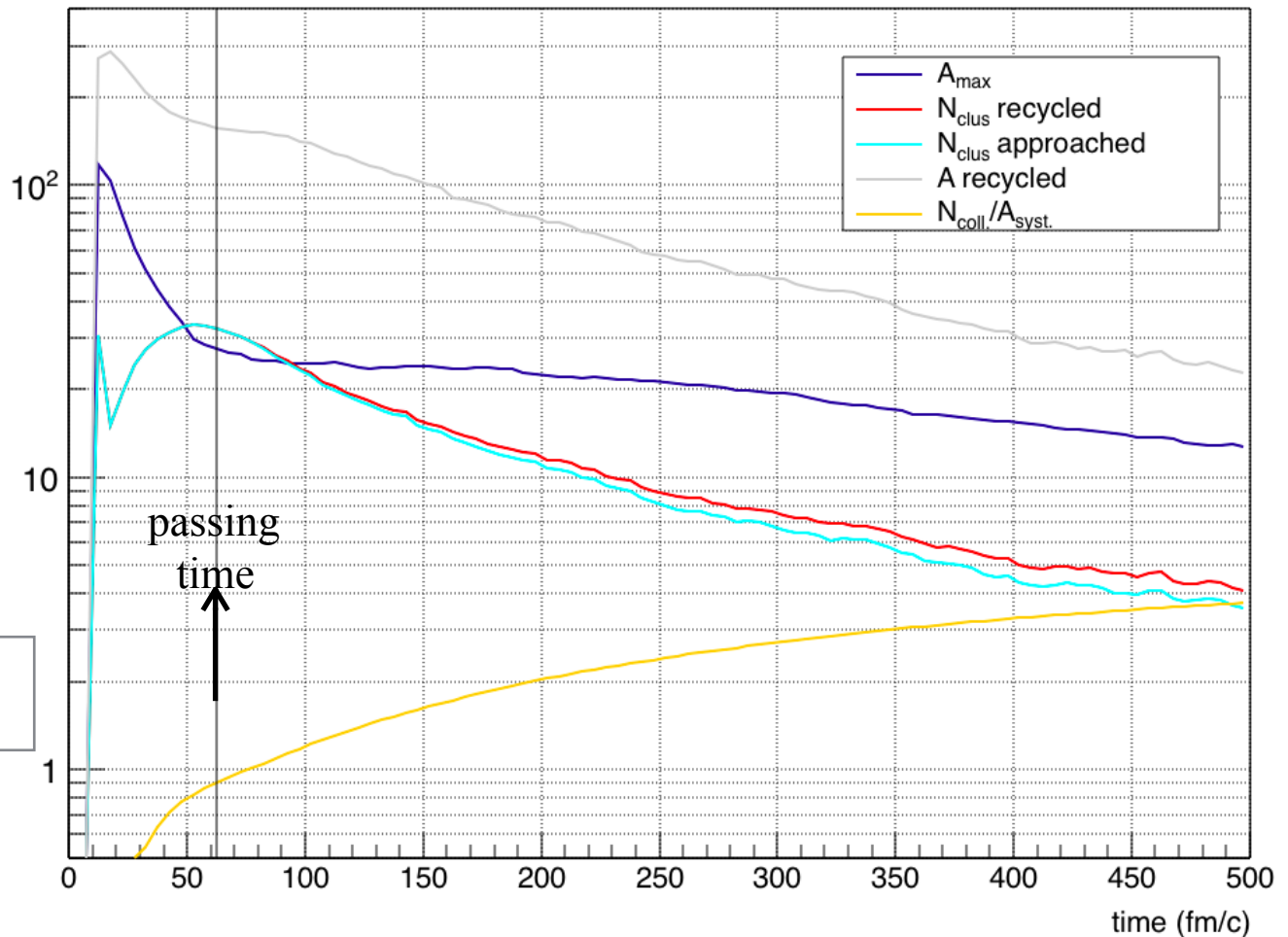


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Central  
Au+Au:  
IQMD-FRIGA

90A MeV

IQMD+seqFRIGA2.9.6(Epair1Easy0+IsoMST) central Au+Au@90A.MeV,  $\Delta t_{\text{FRIGA}} = 1 \text{ fm/c}$ ,  $a_l = 4.33 \text{ fm}^2$ ,  $P_{\text{ini}} < 0.9 P_{\text{Fermi}}$



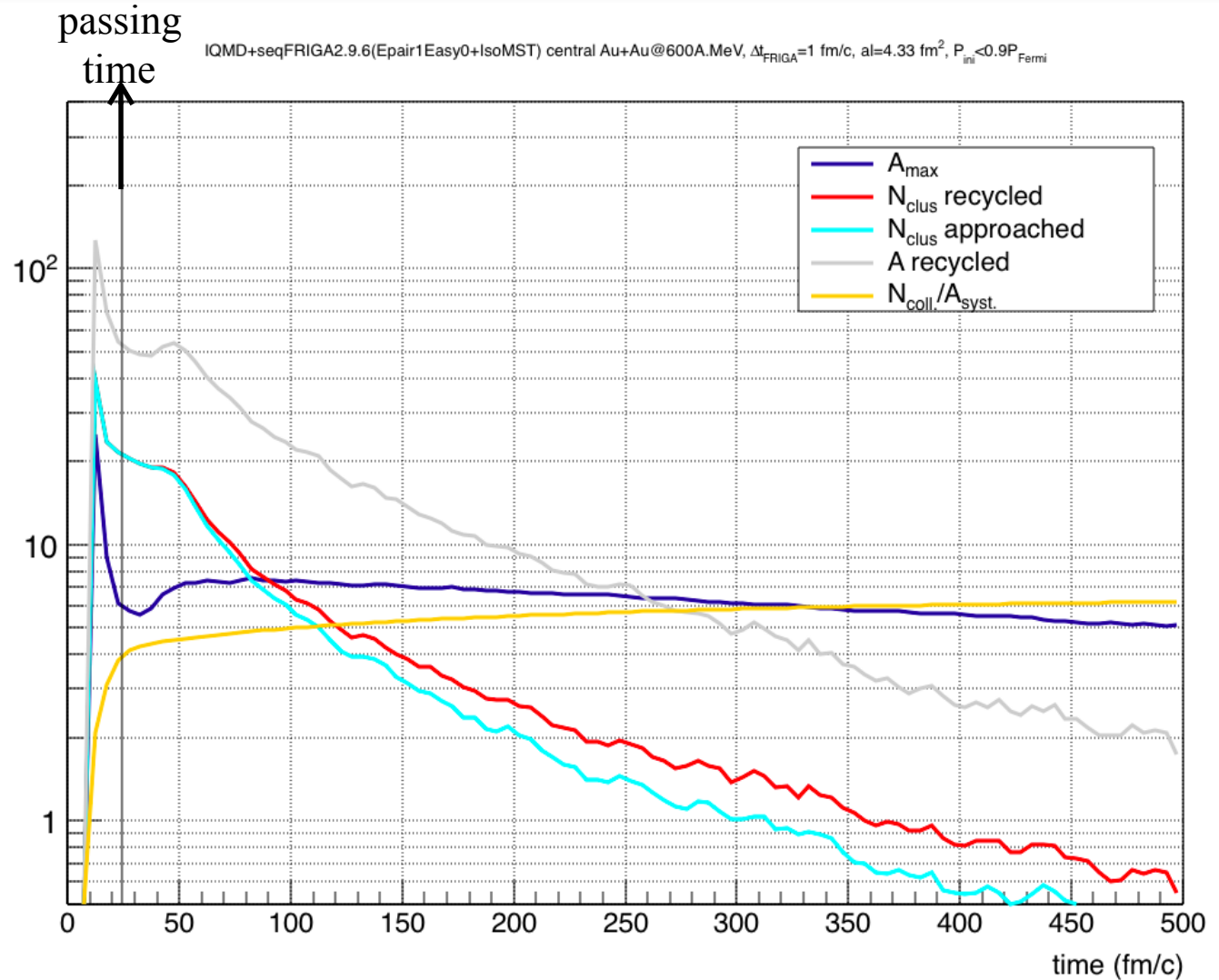
General trend with IQMD:  
 $A_{\text{max}}$  leak after 250 fm/c



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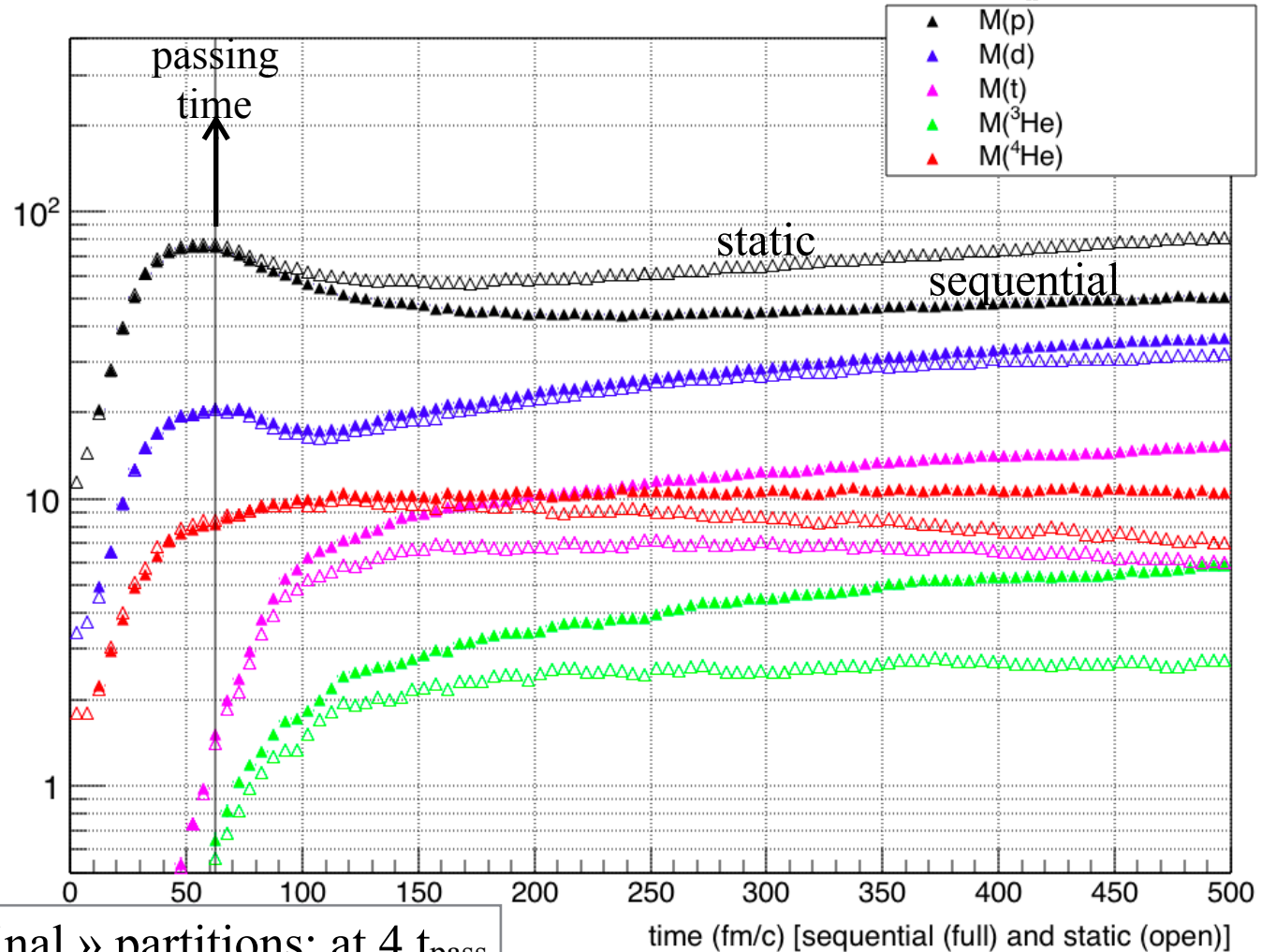


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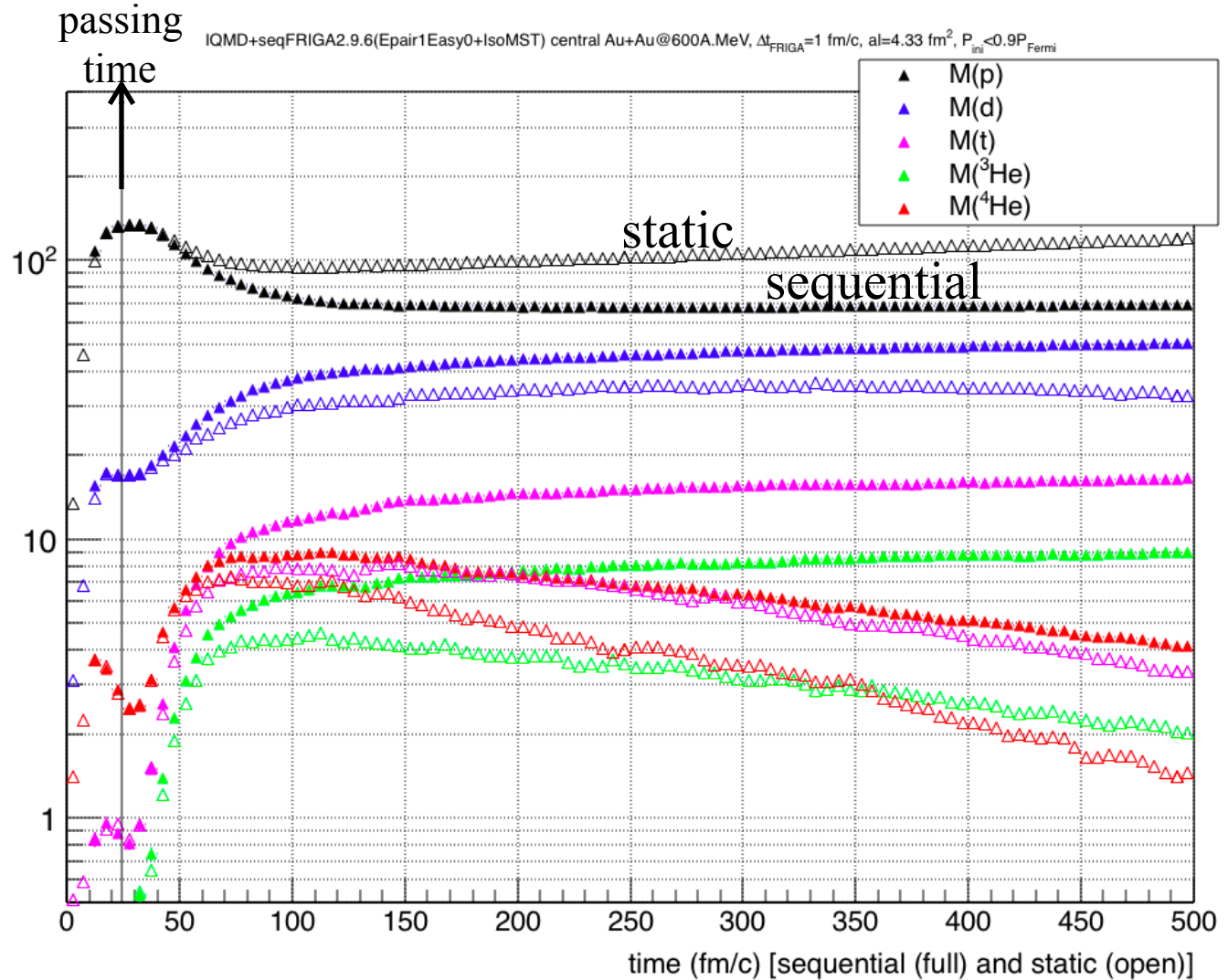
Best compromise for « final » partitions: at  $4 t_{\text{pass}}$



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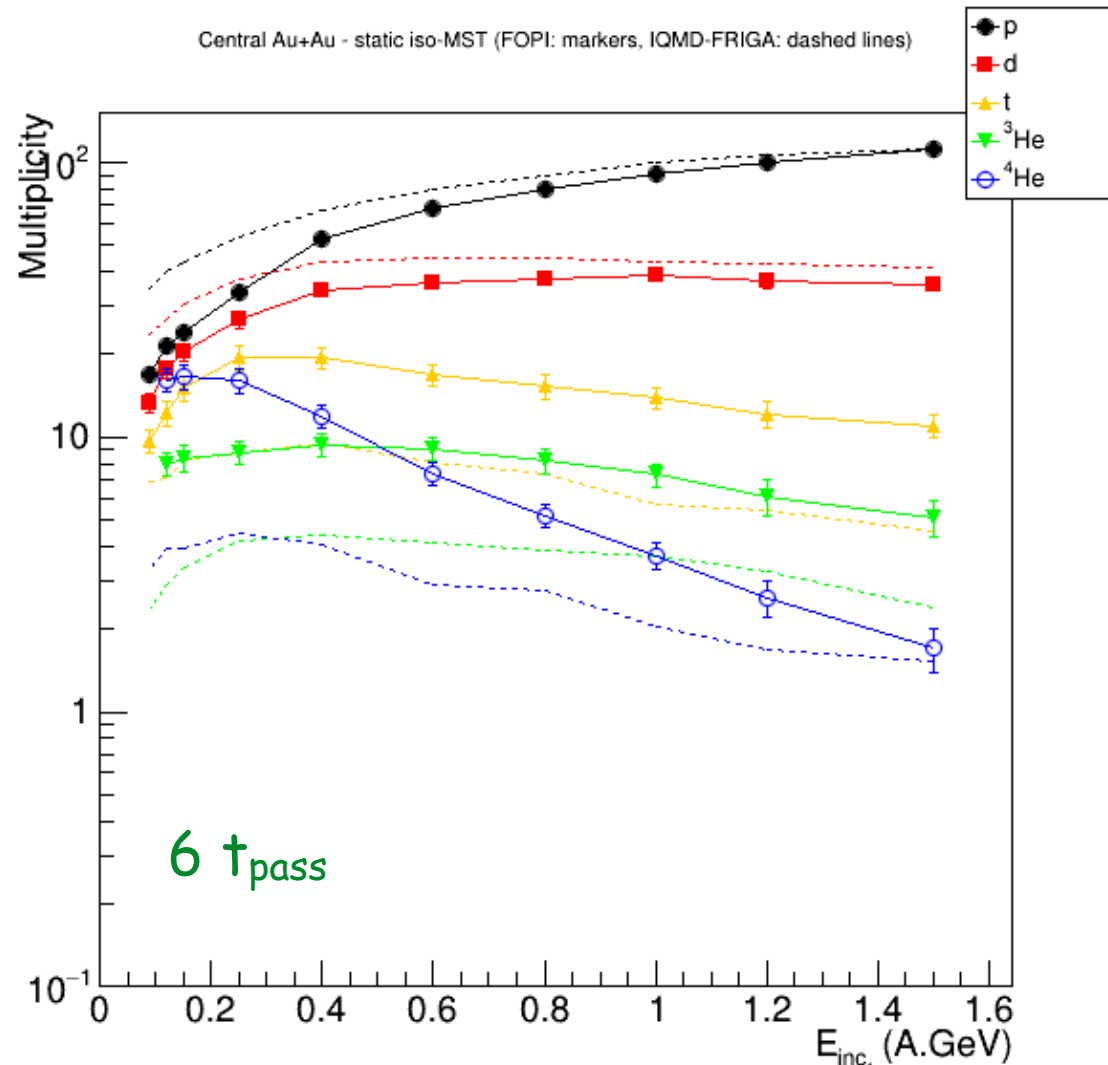
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vs

FOPI data (markers)\*

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\*: W. Reisdorf et al., FOPI Collaboration / Nuclear Physics A 848 (2010) 366–427





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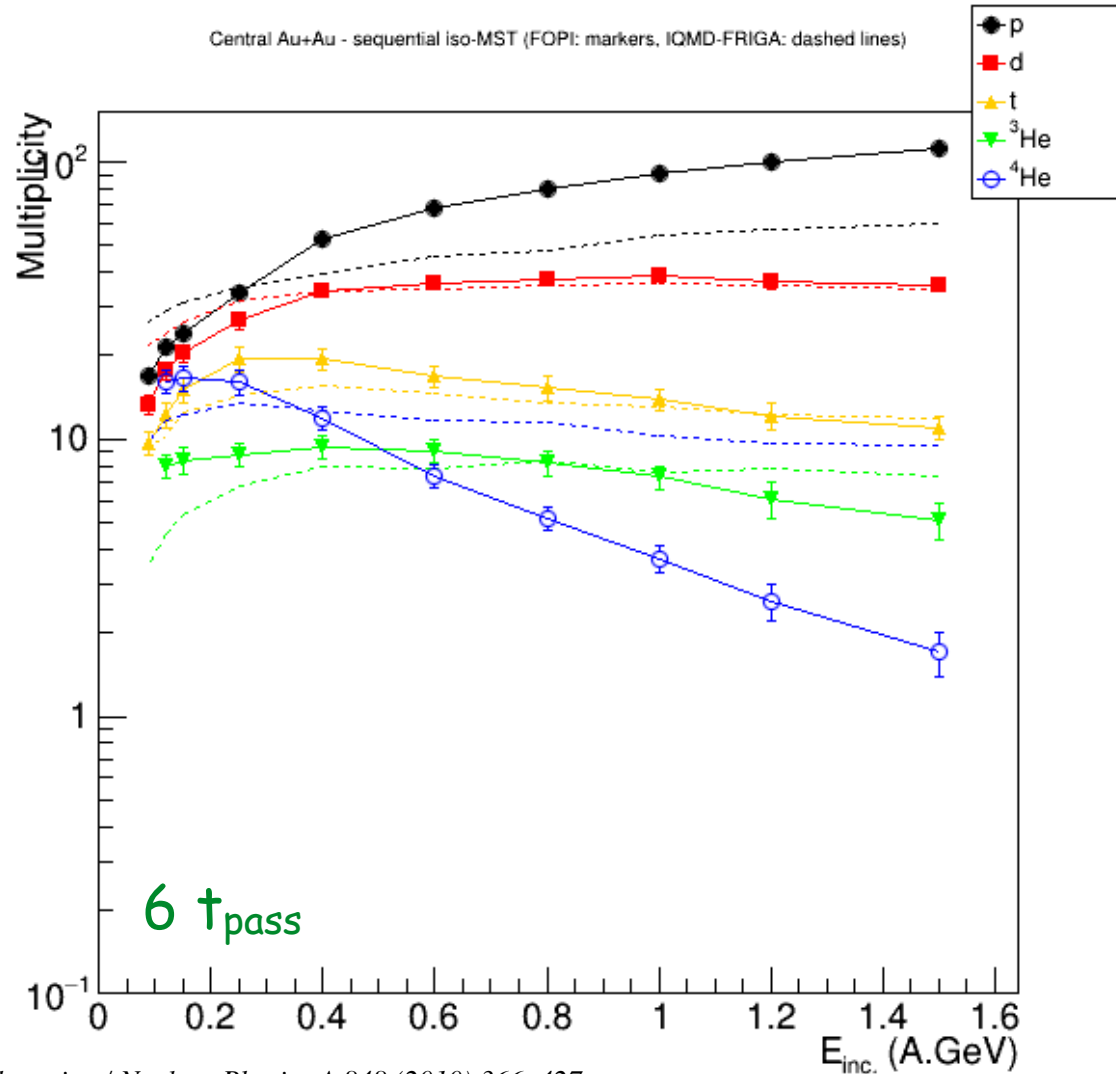
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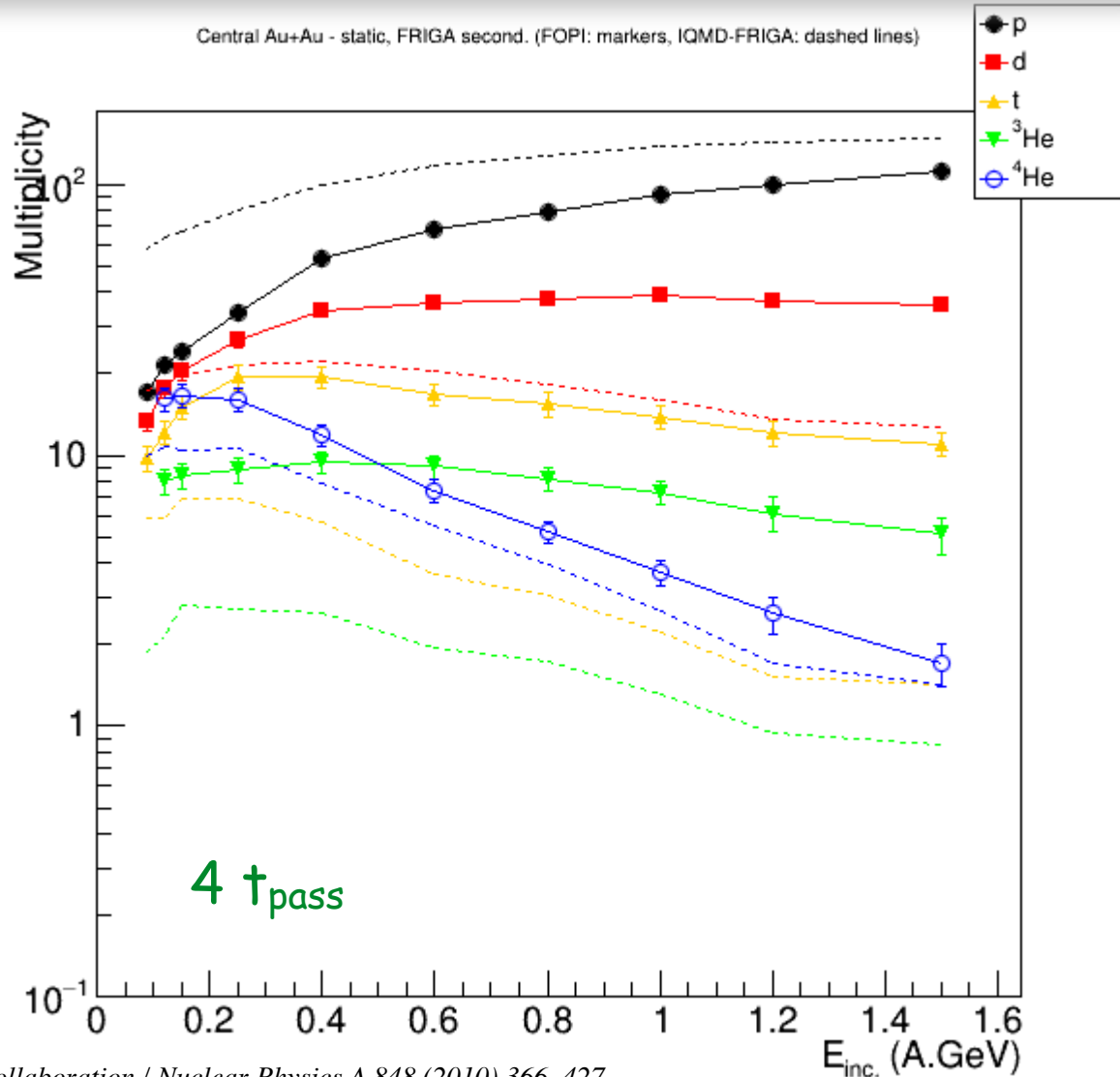




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(with  $B_{asy}$ , second.)



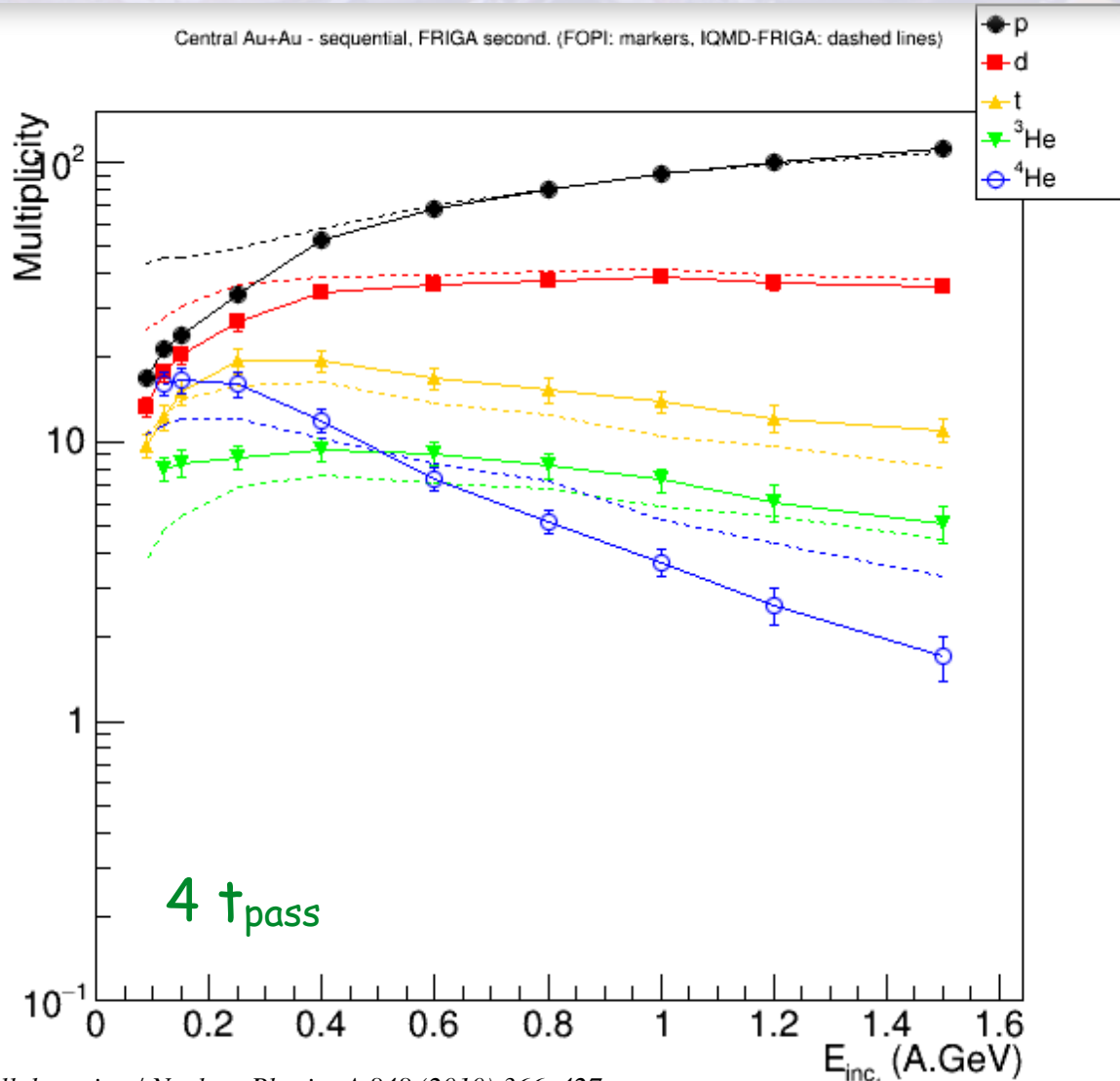
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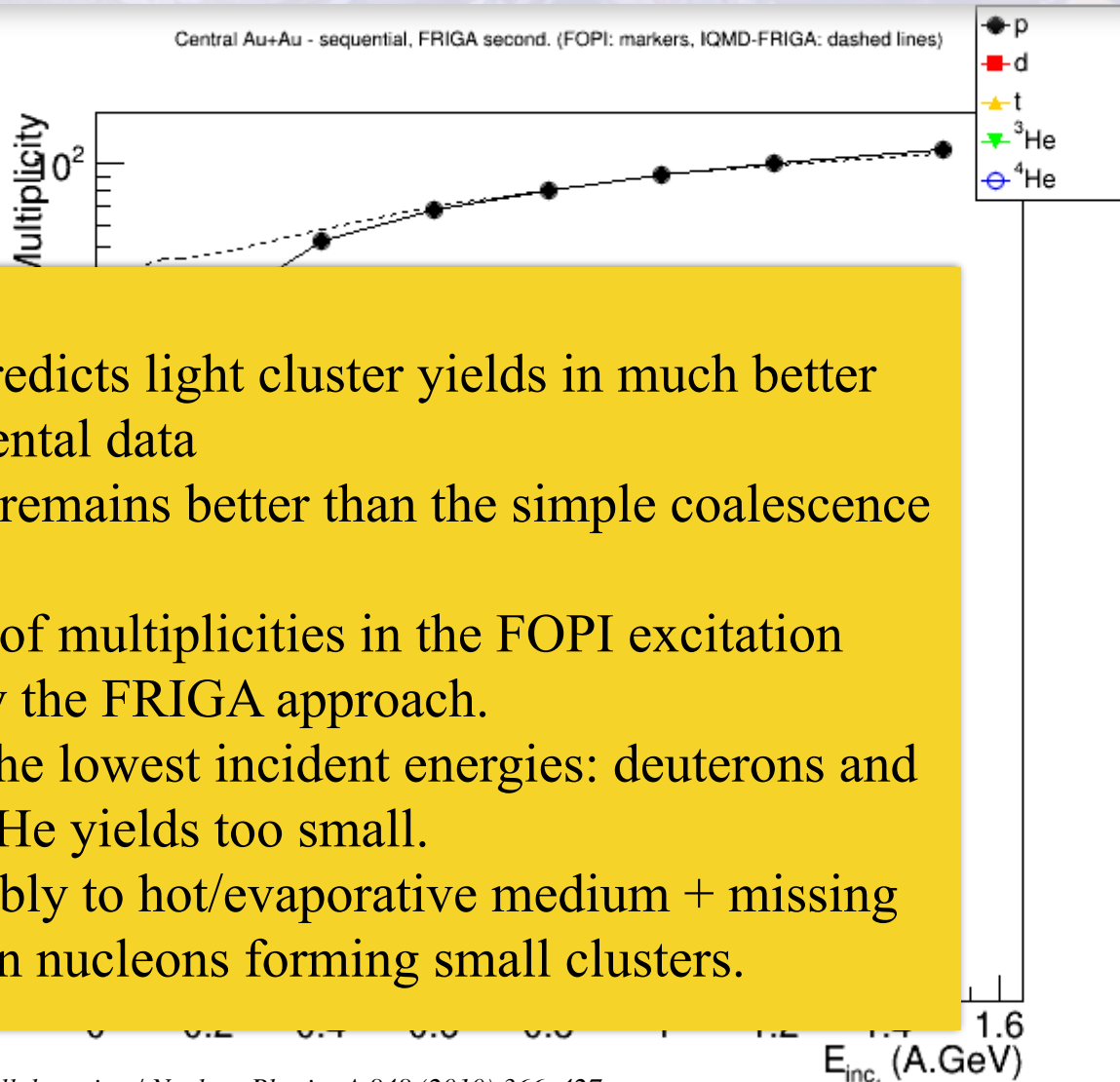


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IQMD-FRIGA  
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- Sequential strategy: predicts light cluster yields in much better agreement with experimental data
- The FRIGA approach remains better than the simple coalescence method.
- The  $^3\text{He}/^4\text{He}$  crossing of multiplicities in the FOPI excitation function is only found by the FRIGA approach.
- Still discrepancies at the lowest incident energies: deuterons and proton yields too large,  $^4\text{He}$  yields too small.
- IQMD creates a probably too hot/evaporative medium + missing quantum binding between nucleons forming small clusters.

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