Challenges to Transport Theory for Heavy-Ion Collisions, May 20-24, 2019, ECT*

Correlations, fluctuations, and fragment formation in transport models

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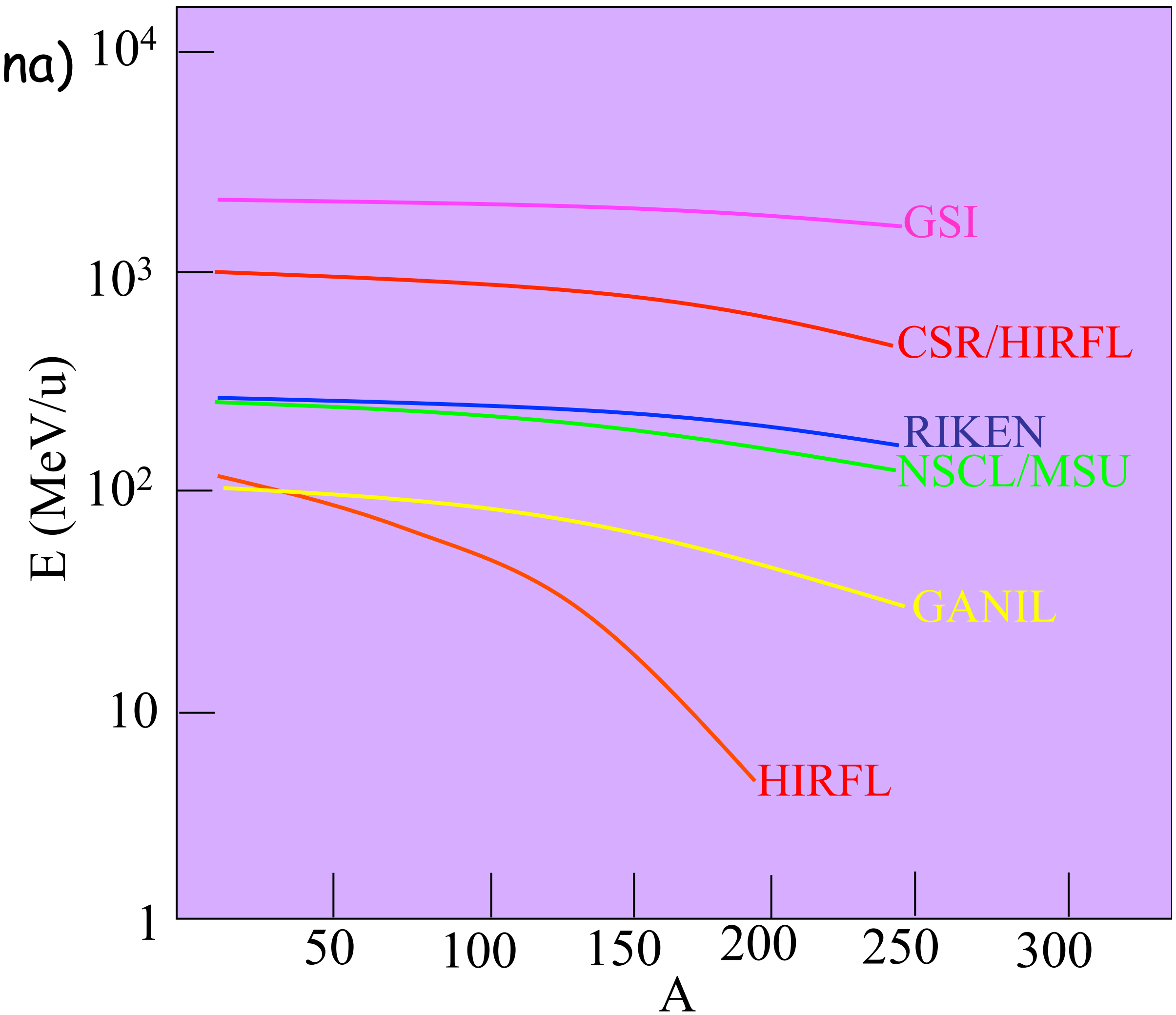
Outline

- Introduction
- Early stage of the transport models (1970-1990)
- Correlations, fluctuations, and fragmentations (1987-2013)
- Recent progress (after 1992)
- Conclusions and suggestions

Heavy-Ion Accelerators at Intermediate Energies in 1990

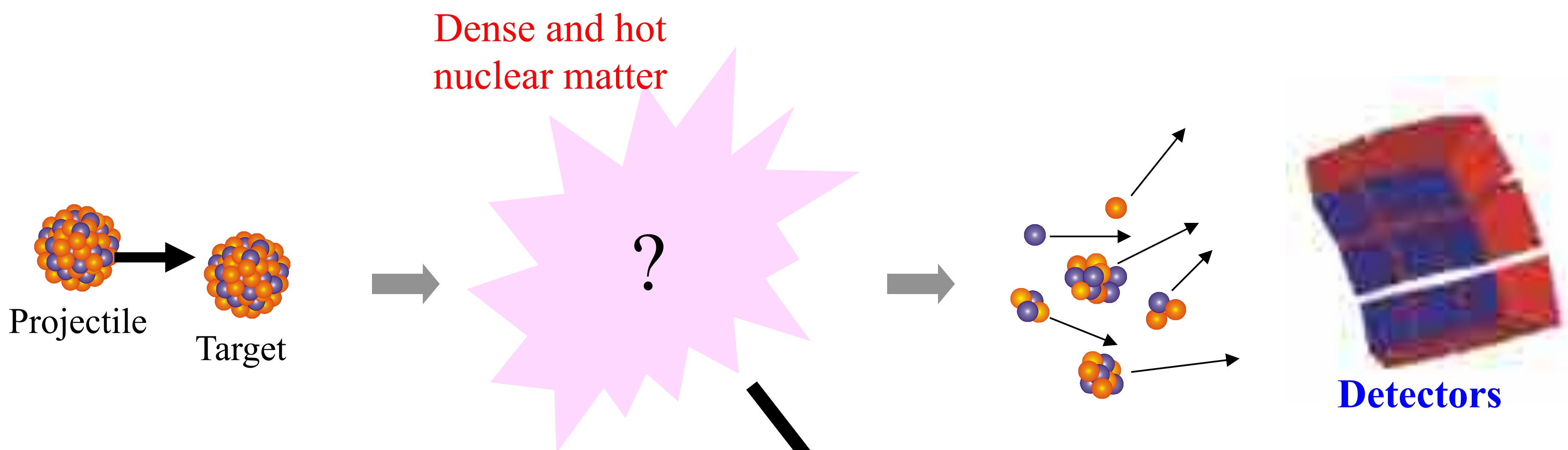
1. HIRFL, CSR/HIRFL (China)
2. GANIL (France)
3. GSI (Germany)
4. NSCL/MSU
5. RIKEN (Japan)



Dubna, LBL, ORNL, TAMU, INFN, KVI,...

BNL, CERN

Nuclear dynamics at intermediate and high energies by tranport models



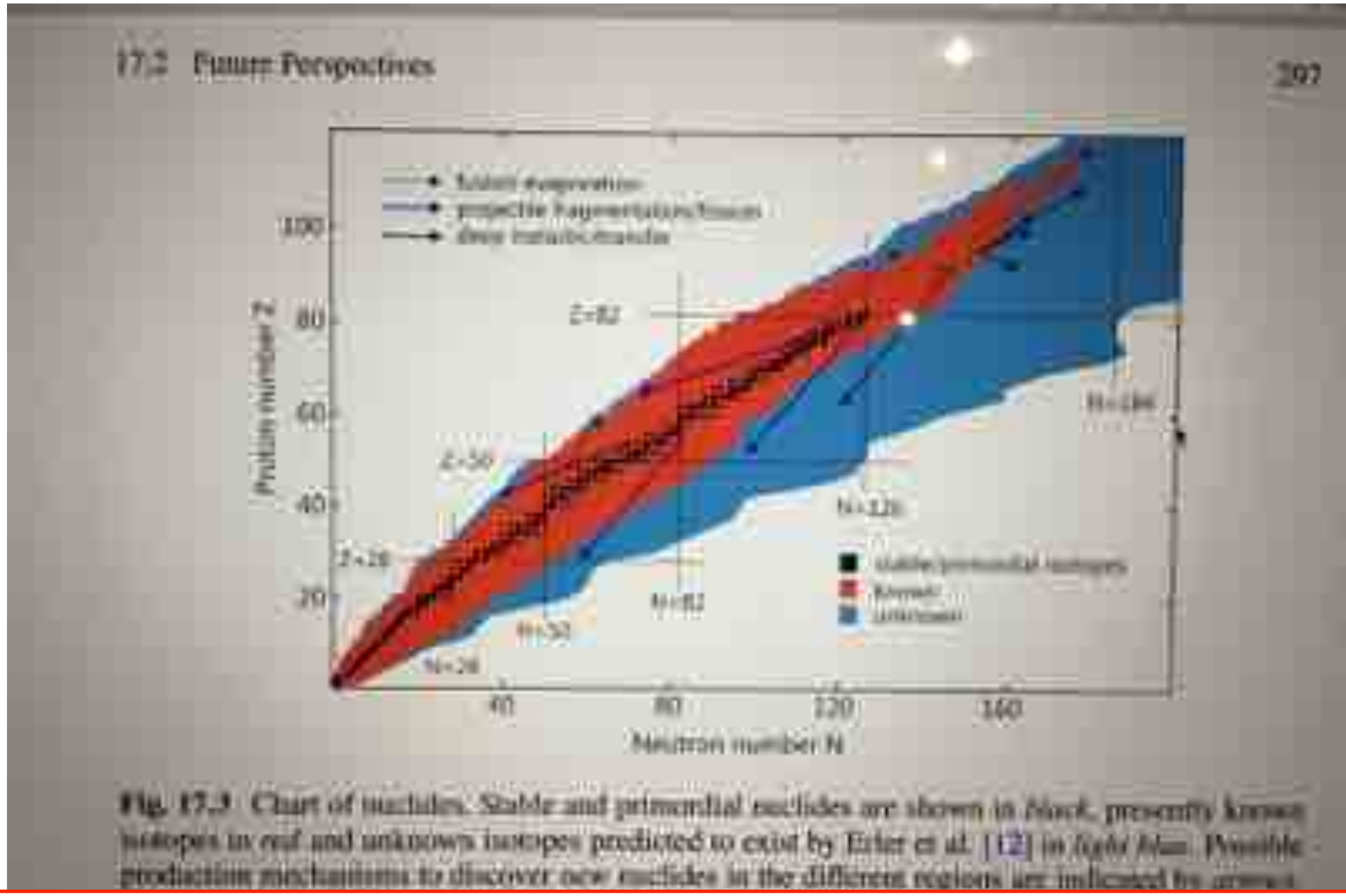
Equation of State Of Nuclear Matter $E(\rho,T,\delta)$?

Liquid-to-Gas transition ?

Transport models ?

The Discovery of Isotopes: A Complete Compilatio

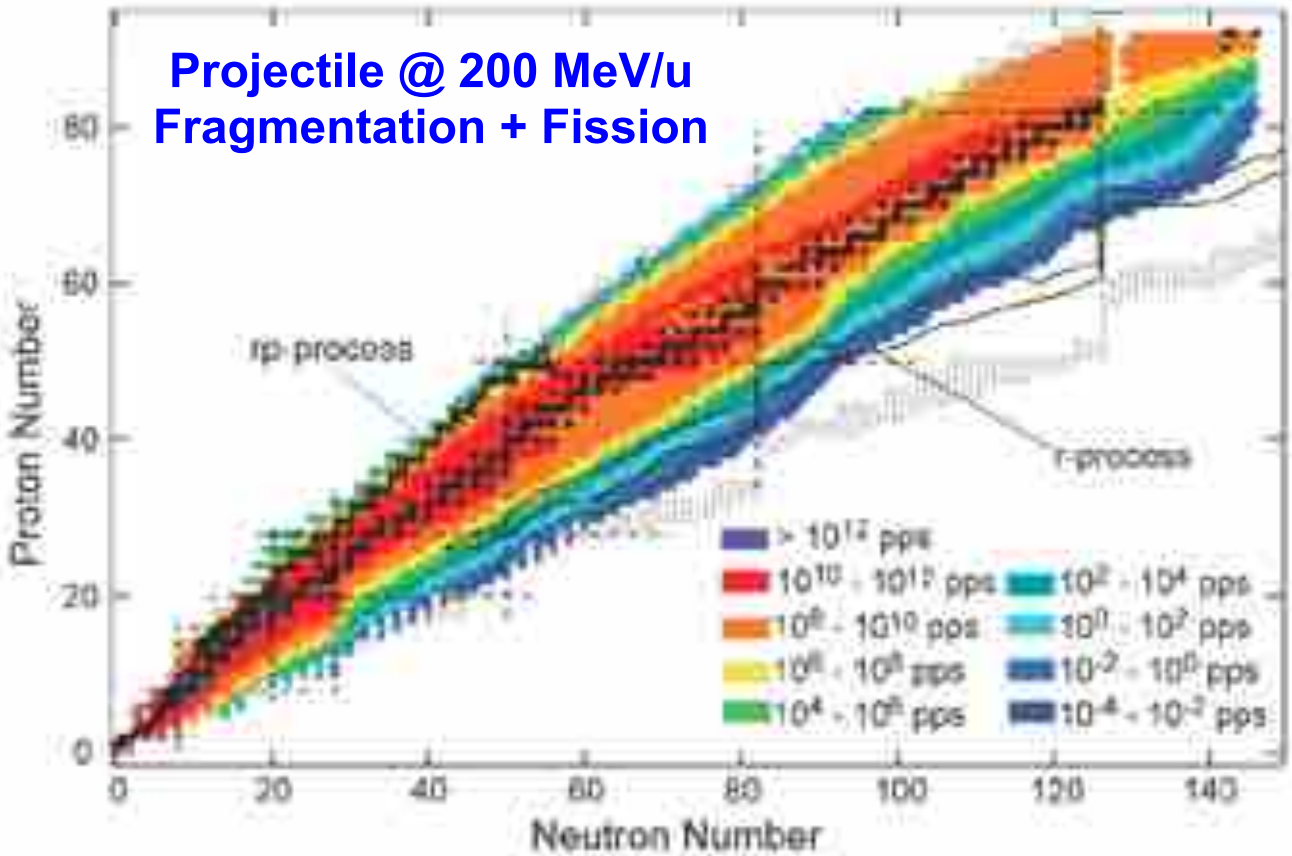
Michael Thoennessen, Springer, 2016



Up to the end of 2018，3386 nuclides have been found。Natural nuclides: 288（Stable: 254, unstable: 34），the others are man made radioactive isotopes。

What Nuclides Will FRIB Produce?

- FRIB will produce more than 1000 NEW isotopes at useful rates (4500 available for study)
- Theory is key to making the right measurements
- Exciting prospects for study of nuclei along the drip line to mass 120 (compared to 24)
- Production of most of the key nuclei for astrophysical modeling
- Harvesting of unusual isotopes for a wide range of applications



Rates are available at http://groups.nscl.msu.edu/frib/rates/

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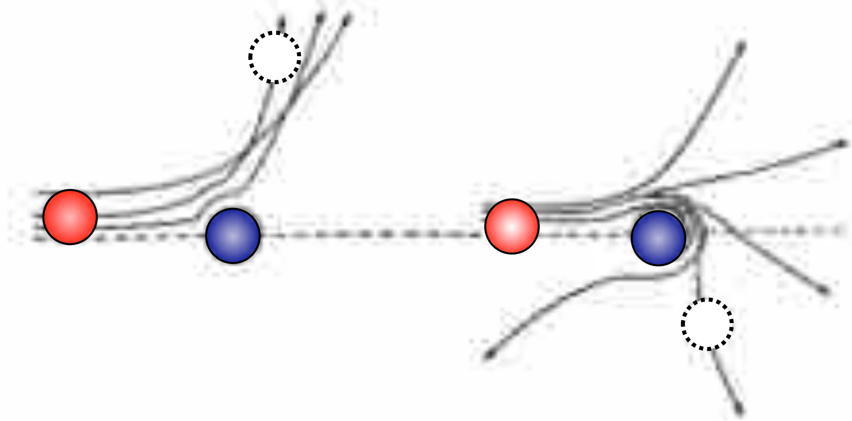
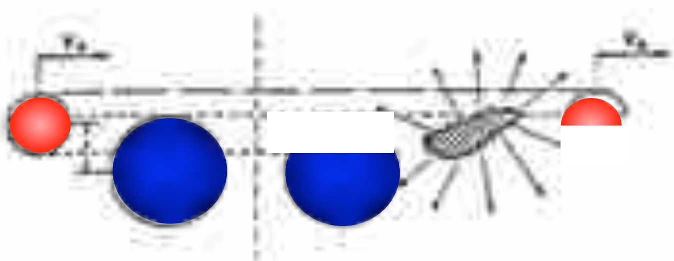
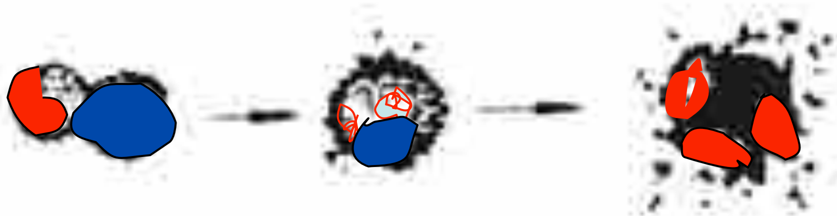


Low Energy

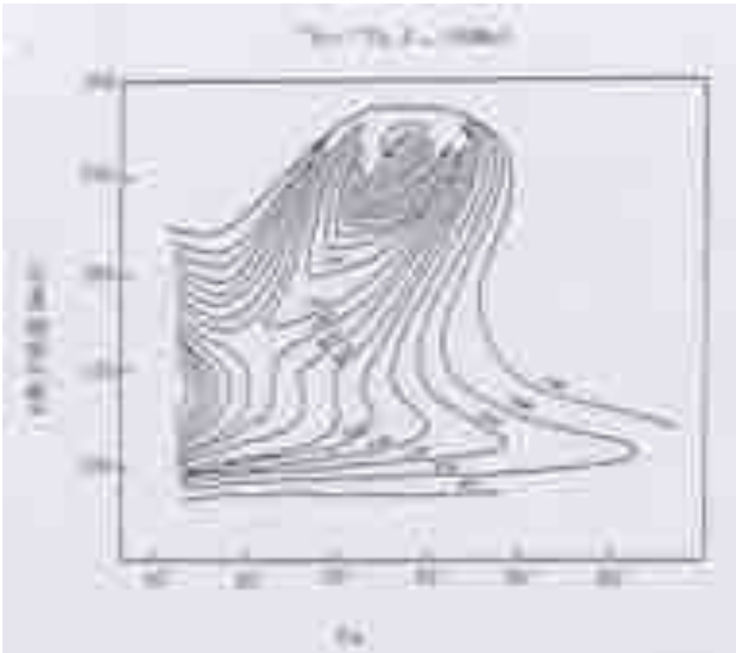


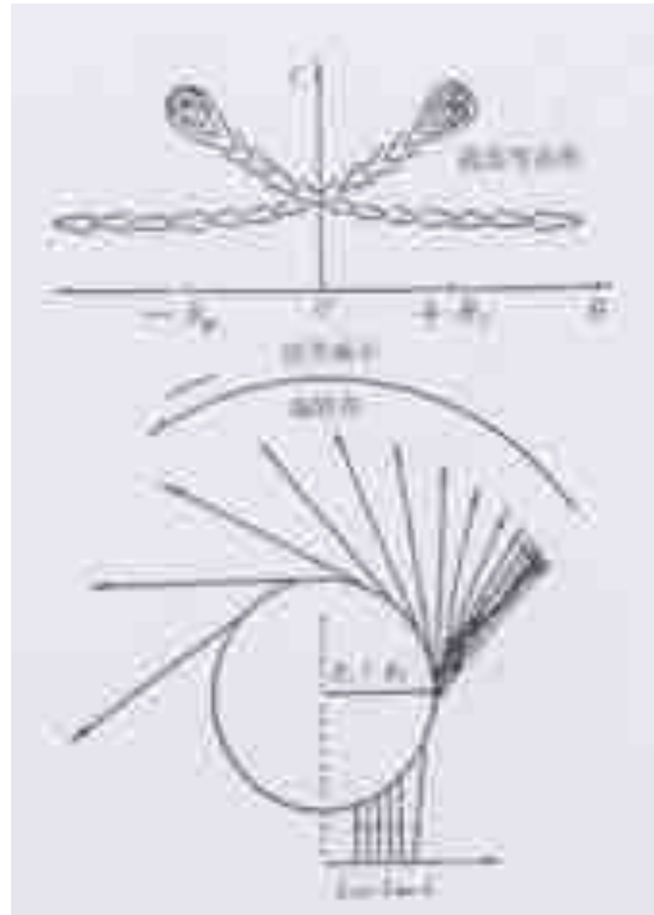
Fermi Energies: Peripheral



Fermi Energies: Central

DIC process



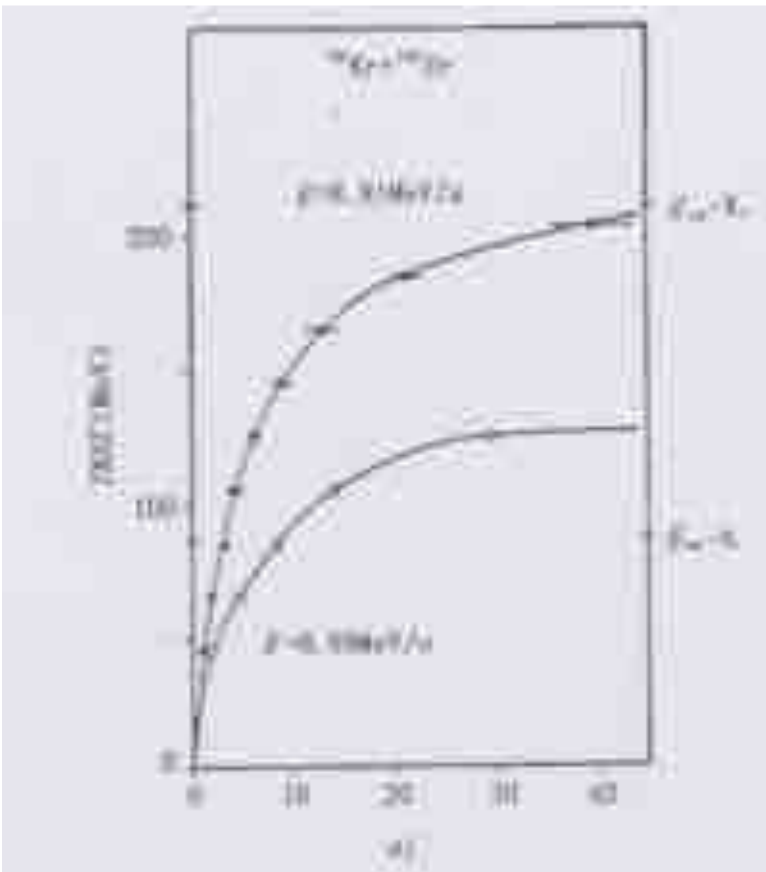


$$l = mr^2\dot{\theta}$$

$$E - V(r) = \frac{l^2}{2mr^2} + \frac{1}{2}m\dot{r}^2$$

$$\theta = \pi - 2\int_{r_{\min}}^{\infty} \frac{b}{r^2}(1 - \frac{V(r)}{E})^{1/2} dr$$

TKEL~ σ_Z^2



$$p(Z,t) = \frac{1}{\sqrt{2\pi\sigma_Z^2}} \exp\{-\frac{(Z-Z_0)^2}{\sigma_Z^2}\}$$

TKEL increasing， σ_Z^2 increasing

The master equation

$$\frac{\partial}{\partial t}P(x,t)=\sum_{x'}\lambda(x,x',t)\rho(x)P(x,t)-\lambda(x',x,t)\rho(x')P(x',t)]$$

$P(x,t) is \quad probability$ density of state x at t， $\rho(x) is \quad weight$，

$\lambda(x,x',t)$ transition probability from x’ to x

1st: $\lambda(x,x',t)\rho(x)P(x,t), gain \quad term, from \quad x' to \quad x$

2nd: $\lambda(x',x,t)\rho(x')P(x',t), loss \quad term$， from x to x’ when x approaches x'，one expands this eq around x'=x, up to the 2nd order of (x-x')，one gets Fokker-Plank eq

$$\frac{\partial}{\partial t}P(x,t) = -\frac{\partial}{\partial x}[v(x,t)P(x,t)] + \frac{\partial^2}{\partial x^2}[D(x,t),P(x,t)]$$

$v(x,t): drift \quad coefficient \quad D(x,t): diffusion \quad coefficient$

$$v(x,t) = 2\mu_2(x,t)\frac{\partial}{\partial x}\rho(x,t) + \rho(x,t)\frac{\partial}{\partial x}\mu_2(x,t)]$$

$$D(x,t) = \mu_2(x,t)\rho(x,t)$$

$$\mu_2(x,t) = \int_{-\infty}^{+\infty}\lambda(x,x',t)(x'-x)^2 dx'$$

$In \quad nuclear \quad reaction \quad, during \quad time \quad \tau \quad, v(x,t), D(x,t) \quad are \quad const \quad,$

$P(x,t=0) = \delta(x-x_0),$

$one \quad gets \quad the \quad solution: \quad Gaussian \quad function$

$$p(x,\tau) = \frac{1}{\sqrt{2\pi D\tau}}\exp\{-\frac{(x-x_0-v\tau)^2}{4D\tau}\}$$

For the average value <x> and its mean square deviation σ_x^2

$$< x >= \int_{-\infty}^{+\infty} xP(x,t) = x_0 + v\tau$$

$$\sigma_x^2 =< x - < x >^2 >= \int_{-\infty}^{+\infty} (x - x_0 - vt)P(x,t)dx = 2D\tau$$

That means the <x> is proportional to v, and the mean square deviation σ_x^2 is also proportional to D

TDHF-->ETDHF, Wong, and Tang, PRL40(1978)1070

[(a)]Permanent address.
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[3]A. K. Kerman and A. F. R. de Toledo Piza, Ann. Phys. (N. Y.) 48, 173 (1968).
[4]G. E. Brown, Rev. Mod. Phys. 31, 893 (1959).
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[8]A. M. Lane, J. E. Lynn, and J. D. Moses, Nucl. Phys. A232, 189 (1974).
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[11]W. M. MacDonald, "Strength Functions for the Analysis of Configuration Broadened States" (to be published).

Extended Time-Dependent Hartree-Fock Approximation with Particle Collisions

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(Received 3 October 1977)

We formulate an extended time-dependent Hartree-Fock approximation which includes particle collisions. As the configuration-space analog of the quantum Boltzmann equation, it can be utilized to study the dynamics of nuclear or other fermion systems when irreversible dissipation is present.

INC, J. Cugnon, NPA251(1981)505

Nuclear Physics **A352** (1981) 505–534 © *North-Holland Publishing Co., Amsterdam*

EQUILIBRATION IN RELATIVISTIC NUCLEAR COLLISIONS. A MONTE CARLO CALCULATION*

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Received 4 March 1980
(Revised 2 June 1980)

Abstract: A relativistic Monte Carlo calculation of the nucleus–nucleus interaction in the GeV range is presented. The interaction process is described as a sequence of classical, binary, on-shell baryon-baryon collisions. Pion production is introduced via the formation of Δ-resonances. The latter are given a definite mass and a lifetime against pion emission larger than the collision time. They are, however, assumed to scatter or disappear in collisions with nucleons. At the end of the collision process, they are allowed to decay. The model is used to study the equilibration during a head-on collision between two ^{40}Ca nuclei. The system is found to be compressed up to a time 6–8 fm/c and to decompress very rapidly. The final nucleon and pion momentum distributions are not completely thermalized. They are, however, tentatively described by effective temperatures. The rapidity distributions show larger temperatures than the perpendicular momentum distributions. Also, nucleon temperatures are generally larger than pion temperatures. The theoretical transverse temperatures and the pion multiplicities agree fairly well with the experimental data. The role of the delta particles is investigated. It is shown that the delta production quickens the equilibration process by transforming longitudinal kinetic energy into mass energy. Furthermore, it favours high compression of the system. Non-central collisions are studied. The results are consistent with the concept of geometrical separation between participant and spectator nucleons. However, our model

Psuedoparticle, C.Y. Wong, PRC25(1982)1460

PHYSICAL REVIEW C VOLUME 25, NUMBER 3 MARCH 1982

Dynamics of nuclear fluid. VIII. Time-dependent Hartree-Fock approximation from a classical point of view

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(Received 3 September 1981)

In order to facilitate the comparison of the time-dependent Hartree-Fock approximation with other classical theories and to help guide our intuition in understanding the underlying physics, we study the time-dependent Hartree-Fock approximation from a classical viewpoint. We show that the time-dependent Hartree-Fock approximation is approximately equivalent to a purely classical pseudoparticle simulation. In this simulation, a collection of pseudoparticles is introduced to discretize the phase space of spatial and momentum coordinates. The dynamics is completely determined by following the pseudoparticle trajectories which are the same as the trajectories of real particles moving in the self-consistent field. As an application of the concept of the pseudoparticle simulation, we study the origin of the nonfusion events in nearly-head-on heavy-ion collisions as obtained in the time-dependent Hartree-Fock approximation. It is argued that for these nearly-head-on collisions, the emergence of the most energetic pseudonucleons of one nucleus outside the far surface of the other nucleus initiates a coherent flow-through motion because of self-consistency and leads to the breakup of the composite system. Based on this picture, we obtain quantitative estimates of the threshold energies and the low-*l* fusion window which agree quite well with the time-dependent Hartree-Fock results.

BUU model

PHYSICAL REVIEW C VOLUME 29, NUMBER 2 FEBRUARY 1984

Rapid Communications

The Rapid Communications section is intended for the accelerated publication of important new results. Manuscripts submitted to this section are given priority in handling in the editorial office and in production. A Rapid Communication may be no longer than 3½ printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but, because of the rapid publication schedule, publication is not delayed for receipt of corrections unless requested by the author.

Boltzmann equation for heavy ion collisions

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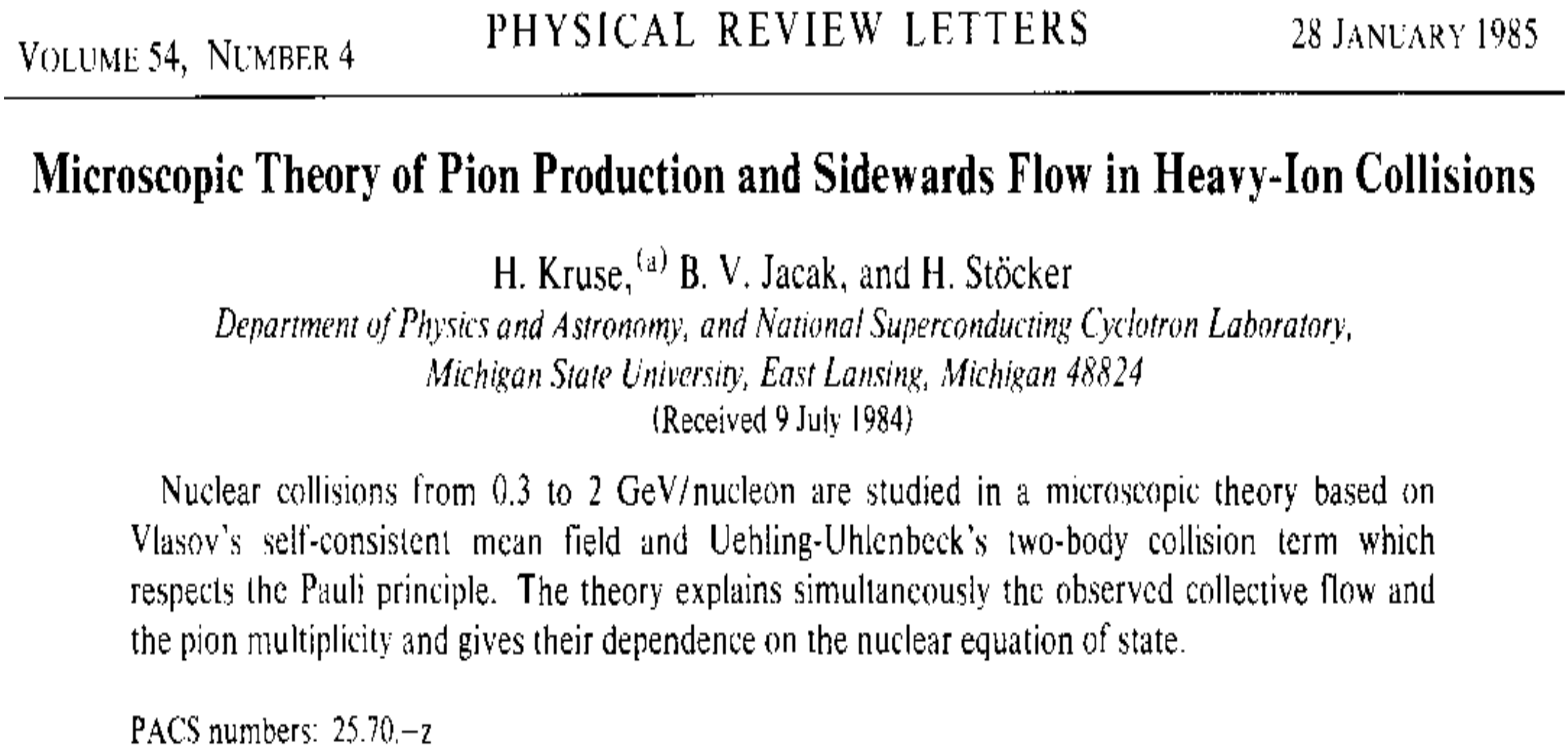
(Received 11 October 1983)

The sensitivity of inclusive observables in heavy ion collisions to the nuclear equation of state can be tested with the Boltzmann equation. We solve the Boltzmann equation, including mean field and Pauli blocking effects, by a method that follows closely the cascade model. We find that the inclusive pion production is insensitive to the nuclear equation of state, contrary to recent claims.

VUU model


VOLUME 54, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JANUARY 1985

Microscopic Theory of Pion Production and Sidewards Flow in Heavy-Ion Collisions

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(Received 9 July 1984)

Nuclear collisions from 0.3 to 2 GeV/nucleon are studied in a microscopic theory based on Vlasov's self-consistent mean field and Uehling-Uhlenbeck's two-body collision term which respects the Pauli principle. The theory explains simultaneously the observed collective flow and the pion multiplicity and gives their dependence on the nuclear equation of state.

PACS numbers: 25.70.−z

LV model

Nuclear Physics A465 (1987) 317-338
North-Holland, Amsterdam

SEMI-CLASSICAL DYNAMICS OF HEAVY-ION REACTIONS

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Received 22 April 1986
(Revised 30 October 1986)

Abstract: We present a semi-classical approach of the heavy-ion collision theory in the intermediate energy domain (10–100 MeV incident kinetic energy per nucleon) based on the Vlasov equation and its extension – the Landau–Vlasov equation – when the residual interaction is accounted for through a collision kernel. We use the coherent state set as an overcomplete basis for the decomposition of the nuclear phase-space distributions. We show that the uniform repartition of coherent states in phase space provides semi-classical descriptions of nuclei at equilibrium which are the correct initial conditions of the Vlasov and Landau–Vlasov dynamical equations.

In the slab geometry, we compare the results of the Vlasov equation with those of the TDHF theory for the crossing of a potential barrier and the collision of two slabs. We present sample results of three-dimensional calculations of heavy-ion collisions with a Skyrme self-consistent interaction and inclusion of the Coulomb interaction; the individual collisions being described by the Uehling Uhlenbeck kernel. These calculations illustrate the incomplete fusion process for central collisions at 27 MeV/u incident energy and the onset of an abrasion-like process for more peripheral collisions at 35 MeV/u.

Milestone of Transport models (1987)

PHYSICS REPORTS (Review Section of Physics Letters) 160, No. 4 (1988) 189–233. North-Holland, Amsterdam

A GUIDE TO MICROSCOPIC MODELS FOR INTERMEDIATE ENERGY HEAVY ION COLLISIONS

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Received September 1987

Contents:

QMD J. Aichelin, PR202(1991)233

PHYSICS REPORTS (Review Section of Physics Letters) 202, Nos. 5 & 6 (1991) 233–360. North-Holland

"QUANTUM" MOLECULAR DYNAMICS—A DYNAMICAL MICROSCOPIC *n*-BODY APPROACH TO INVESTIGATE FRAGMENT FORMATION AND THE NUCLEAR EQUATION OF STATE IN HEAVY ION COLLISIONS

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Editor: G.E. Brown Received June 1990

Contents:

Two Kinds of Transport Models

Quantum Molecular Dynamics like: solve N-body equation of motion

$$\Phi_i(\vec{r}) = \frac{1}{(2\pi\sigma_r^2)^{3/4}} \exp\left[-\frac{(\vec{r}-\vec{r}_i)2}{4\sigma_r^2} + \frac{i}{\hbar}\vec{r}\cdot\vec{p}_i \right]$$

$$f(\vec{r},\vec{p}) = \sum_i \frac{1}{(\pi\hbar)^3} \exp\left[-\frac{(\vec{r}-\vec{r}_i)^2}{2\sigma_r^2} - \frac{2\sigma_r^2}{\hbar^2}(\vec{p}-\vec{p}_i)^2 \right]$$

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial r_i}, \quad \dot{\vec{r}}_i = \frac{\partial H}{\partial p_i}$$

Two body collision: occurs between nucleons

Version: QMD, IQMD, ImQMD, LQMD, CoMD, UrQMD , ……
AMD, FMD

Boltzmann-like: f(r,p,t) one body phase space density

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\cdot\nabla_r - \nabla_r U\left(\hat{f}\right)\cdot\nabla_p \right)\hat{f}\left(\vec{r},\vec{p},t\right) = K\left(\hat{f}\right) + \delta K\left(\vec{r},\vec{p},t\right)$$

$$f(\vec{r},\vec{p}) = \frac{1}{\tilde{N}} \sum \delta(\vec{r}-\vec{r}_i)\delta(\vec{p}-\vec{p}_i)$$

Two-body collision: occurs between test part.

Version: IBUU04, BLE,……

QMD-like

wave function

The N-body phase-space distribution function

$$f(\vec{r},\vec{p}) = \sum_i \frac{1}{(\pi\hbar)^3} \exp\left[-\frac{(\vec{r}-\vec{r}_i)^2}{2\sigma_r^2} - \frac{2\sigma_r^2}{\hbar^2}(\vec{p}-\vec{p}_i)^2 \right]$$

Hamiltonian equations

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial r_i}, \qquad \dot{\vec{r}}_i = \frac{\partial H}{\partial p_i}$$

$$H = T + U_{Coul} + U_2 + U_3 + U_{sym} + U_{sur} + U_{MDI}$$

two-body collision

$$N+N \to N+N, \quad N+N \to N+\Delta,$$

$$N+\Delta \to N+\Delta, \quad N+\Delta \to N+N,$$

$$\Delta+\Delta \to \Delta+\Delta,\ \mathrm{L}$$

Fermionic nature

Pauli blocking

phase-space density constraint

Comparisons in 2004 and 2009

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **31** (2005) S741–S757 doi:10.1088/0954-3899/31/6/015

Transport theories for heavy-ion collisions in the 1 *A* GeV regime

E E Kolomeitsev[1,2], C Hartnack[3], H W Barz[4], M Bleicher[5], E Bratkovskaya[5], W Cassing[6], L W Chen[7,8], P Danielewicz[9], C Fuchs[10], T Gaitanos[11], C M Ko[7], A Larionov[6,13], M Reiter[5], Gy Wolf[12] and J Aichelin[3,14]

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Received 8 December 2004
Published 23 May 2005

Comparisons in 2014, 2017, and 2019

PHYSICAL REVIEW C 93, 044609 (2016)

Understanding transport simulations of heavy-ion collisions at 100 A and 400 A MeV: Comparison of heavy-ion transport codes under controlled conditions

Jun Xu,[1,*] Lie-Wen Chen,[2,†] ManYee Betty Tsang,[3,‡] Hermann Wolter,[4,§] Ying-Xun Zhang,[5,‖] Joerg Aichelin,[6] Maria Colonna,[7] Dan Cozma,[8] Pawel Danielewicz,[3] Zhao-Qing Feng,[9] Arnaud Le Fèvre,[10] Theodoros Gaitanos,[11] Christoph Hartnack,[6] Kyungil Kim,[12] Youngman Kim,[12] Che-Ming Ko,[13] Bao-An Li,[14] Qing-Feng Li,[15] Zhu-Xia Li,[5] Paolo Napolitani,[16] Akira Ono,[17] Massimo Papa,[7] Taesoo Song,[19] Jun Su,[20] Jun-Long Tian,[21] Ning Wang,[22] Yong-Jia Wang,[15] Janus Weil,[19] Wen-Jie Xie,[23] Feng-Shou Zhang,[24] and Guo-Qiang Zhang[1]

PHYSICAL REVIEW C 97, 034625 (2018)

Comparison of heavy-ion transport simulations: Collision integral in a box

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Comparison of heavy-ion transport simulations: Collision integral with pions and Δ resonances in a box

submitted to PRC, 2019

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Boltzmann-Langevin equation

BBGKY Hierarchy （Born, Bogoliubov, Green, Kirkwood, and Yvon）

$$\hat{H} = \sum_{i=1}^{N} \left(\frac{1}{2m} p_i^2 + V(q_i)\right) + \sum_{1 \le i < j}^{N} V_{ij}$$

$V(q)$ □ □ □

$\mathbf{V}_{ij}$ □ □ □ □ □ □ □ □

□ □ $1,2,3,\ldots s$ □ □ □ □

$$F_1(q_1, p_1, t), F_2(q_1, q_2, p_1, p_2, t), F_3(q_1, q_2, q_3, p_1, p_2, p_3, t), \ldots$$

$$Fs(q_1, \ldots, q_s, p_1, \ldots, p_s, t)$$

$$\frac{\partial F_s}{\partial t} = (H_s, F_s) + \frac{N_s}{V} \iint dq_{s+1} dp_{s+1} \left(\sum_{i=1}^{s} \phi_{i,s+1}, F_{s+1}\right)$$

$$(A, B) = \sum \left(\frac{\partial A}{\partial q_k}\frac{\partial B}{\partial p_k} - \frac{\partial A}{\partial p_k}\frac{\partial B}{\partial q_k}\right)$$ □ **poisson** □ □

$$H_s = \sum_{i=1}^{s} \left(\frac{1}{2m} p_i^2 + V(q_i)\right) + \sum_{1 \le i < j}^{s} V_{ij}$$

□ □ □ **BBGKY** （*Bogoliubov, Born, Green, Kirkwood, Yvon*） □ □

□ *Liouville* □ □ □ □

$s = 1$

$$\frac{\partial F_1}{\partial t} + \frac{\vec{p}}{m} \bullet \frac{\partial F_1}{\partial \vec{q}_1} - \frac{\partial V(q_1)}{\partial \vec{q}_1} \bullet \frac{\partial F_1}{\partial \vec{p}_1} = n \iint d\vec{q}_2 d\vec{p}_2 \frac{\partial V_{12}}{\partial \vec{q}_1} \bullet \frac{\partial F_2}{\partial \vec{p}_1}$$

□□□□□□□□□□□□□□□□□，□□□□□□。

□□，$\mathbf{F}_1$□$\mathbf{F}_2$, $\mathbf{F}_2$□$\mathbf{F}_3$, $\mathbf{F}_3$□$\mathbf{F}_4$,...,□□□□□□。

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_r - \nabla_r U(\hat{f}) \cdot \nabla_p \right) \hat{f}(\mathbf{r}, \mathbf{p}, t) = K(\hat{f}) + \delta K(\mathbf{r}, \mathbf{p}, t). \quad (1)$$

$$K(\hat{f}_1) = \int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 W(12;34)[\hat{f}_3 \hat{f}_4 (1 - \hat{f}_1)(1 - \hat{f}_2) - \hat{f}_1 \hat{f}_2 (1 - \hat{f}_3)(1 - \hat{f}_4)], \quad (2)$$

$$\langle \delta K(\mathbf{r}_1, \mathbf{p}_1, t_1) \delta K(\mathbf{r}_2, \mathbf{p}_2, t_2) \rangle = C(\mathbf{p}_1, \mathbf{p}_2) \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(t_1 - t_2). \quad (4)$$

Bauer, Bertsch, and Das Gupta, 1987

VOLUME 58, NUMBER 9 PHYSICAL REVIEW LETTERS 2 MARCH 1987

Fluctuations and Clustering in Heavy-Ion Collisions

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and

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(Received 3 December 1986)

We propose a new theory to treat fluctuation phenomena in heavy-ion reactions. In practical terms, the method is an extension of the theories of the one-body density based on mean-field plus collisional dynamics. In an exploratory study of the $^{20}Ne + ^{20}Ne$ reaction, we find considerable fragmentation with a rapidly falling mass spectrum.

PACS numbers: 24.60.Ky, 24.10.−i, 25.70.Np

Ayik and Gregoire, 1988

Volume 212, number 3 PHYSICS LETTERS B 29 September 1988

FLUCTUATIONS OF SINGLE-PARTICLE DENSITY IN NUCLEAR COLLISIONS

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Received 10 June 1988

In order to incorporate fluctuations into the extended TDHF, a new approach is proposed. The evolution of the single-particle density is considered as a "generalized Langevin process" in which the correlated part of the two-body collisions acts as a "random force". In the semi-classical approximation, the correlation function of the random force is calculated. A possible algorithm for the numerical solution is discussed.

Chomaz,Burgio,and Randrup, 1991

me 254, number 3,4 PHYSICS LETTERS B 24 January 1991

Inclusion of fluctuations in nuclear dynamics

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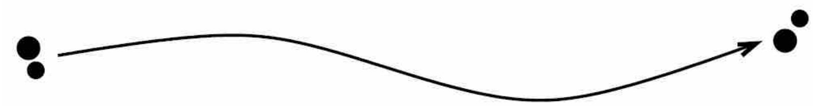
Received 10 September 1990; revised manuscript received 23 October 1990

In this letter we present a new method to include fluctuations into dynamical simulations of the Nordheim type, in which individual nucleons moving in their self-consistent mean field experience Pauli-blocked two-body collisions. The method consists of including a suitably scaled amount of noise in the basic two-body scattering process. The method is illustrated for a gas of fermions on a two-dimensional torus and the results exhibit the desired behavior: the mean phase-space occupancy relaxes towards the appropriate Fermi–Dirac distribution, the associated variance evolves as expected from quantum statistics, and the covariance reflects the various correlations inherent in the two-body collision process.

The differences among the Vlasov, BUU and BL models

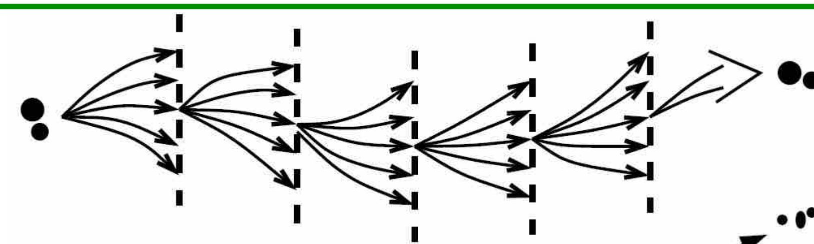
Vlasov

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\cdot\nabla_r - \nabla_r U\left(\hat{f}\right)\cdot\nabla_p\right)\hat{f}\left(\vec{r},\vec{p},t\right) = 0$$



BUU

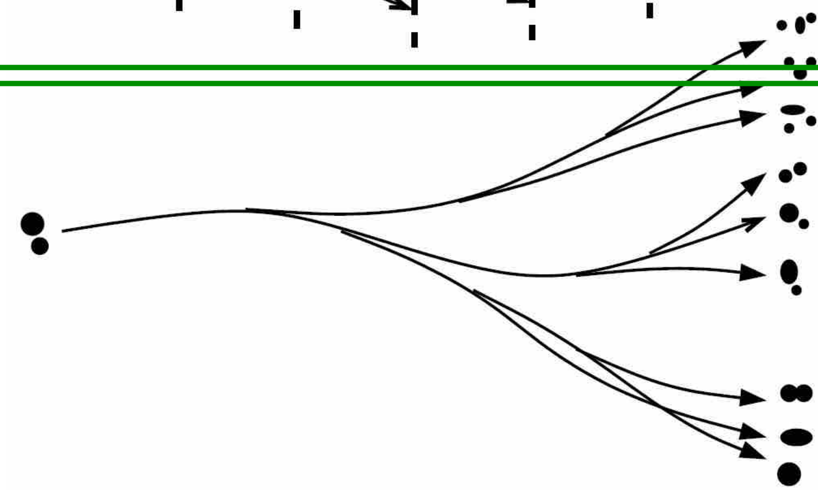
$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\cdot\nabla_r - \nabla_r U\left(\hat{f}\right)\cdot\nabla_p\right)\hat{f}\left(\vec{r},\vec{p},t\right) = K\left(\hat{f}\right)$$



BL

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\cdot\nabla_r - \nabla_r U\left(\hat{f}\right)\cdot\nabla_p\right)\hat{f}\left(\vec{r},\vec{p},t\right) = K\left(\hat{f}\right)$$

$$+\delta K\left(\vec{r},\vec{p},t\right)$$



A. Ono, J. Randrup, Eur. Phys. J. A 30 (2006) 109

The 1st comparision of the fluctuations in the transport models, 1992

Nuclear Physics A540 (1992) 227–260
North-Holland

NUCLEAR
PHYSICS A

Fluctuations in nuclear dynamics: Comparison of different methods *

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Received 4 December 1991
(Revised 20 January 1992)

Abstract: We compare several methods for incorporating the effect of the fluctuating collision integral in Boltzmann–Uehling–Uhlenbeck simulations of nuclear dynamics: the method of correlated pseudo-particle collisions proposed by Bauer, Bertsch and Das Gupta, the method of projection onto collective variables (specifically the local quadrupole moment of the momentum distribution) proposed by Ayik and Gregoire, and the method of direct simulation on a lattice in phase space recently proposed by Burgio, Chomaz and Randrup. Considering a periodic two-dimensional nucleon gas in a constant one-body field, we examine the evolution of the fluctuations of the phase-space occupancy and their correlations.

Conclusion 1

6. Concluding remarks

In the present work, we have investigated several methods proposed for incorporating the fluctuating part of the BUU collision term into numerical simulations of one-body nuclear dynamics. First we briefly reviewed the lattice simulation method recently developed by Chomaz, Burgio and Randrup [7,8]) and illustrated its ability to produce the desired behavior of the fluctuations in the phase-space occupancy $f(\boldsymbol{r},\boldsymbol{p})$ and their associated correlations. This method presents a well-founded numerical solution of the Boltzmann–Langevin type transport equation for the reduced one-body phase-space density. However, as of yet, its practical applicability is severely limited by the

computing speed and storage capacity of equipment readily available and it would be highly desirable to develop further approximations.

However, because of its satisfactory behavior under well understood idealized conditions, the method offers a valuable tool for testing more approximate methods that may be more easily applicable to realistic situations. Therefore, we have examined two approximate methods that have been suggested for the incorporation of the fluctuations into the BUU-type dynamics.

Conclusions 2

The method proposed by Ayik *et al.* reduces the microscopic Boltzmann–Langevin equation to equations for a set of collective variables whose stochastic changes are then used to make an approximate reconstruction of the microscopic density. The choice of collective variables is of course somewhat arbitrary, as is the associated reconstruction procedure, and the proponents of this method have in fact explored several alternative prescriptions. However, our investigations indicate that none of these yields a very satisfactory result, even though some improvement might be achieved by careful adjustments of the various numerical parameters involved, such as the size of the time step, and the inclusion of more complicated collective variables (though this is far from certain). However, it would appear that the resulting correlations associated with the fluctuating one-body density will tend to reflect the symmetries and other characteristics of the employed reconstruction procedure rather than that of the underlying physical fluctuations. Therefore, at this stage of development, the method appears unsuitable for calculating particle production which depends sensitively on the details of the momentum distribution.

Conclusion 3

When applying the method proposed by Bauer *et al.* to our idealized system, we found that it is able to produce fluctuations of the correct general magnitude, provided that a suitable coarse graining is performed and that these display some features of the correlations expected from the basic characteristics of the two-body collision processes. However, the detailed momentum dependence of the variance in phase-space occupancy deviates significantly from what is dictated by quantum statistics. The method requires the definition of a distance in phase-space, which is of course associated with some arbitrariness. Since the particular distance definition influences the repartition of fluctuations, it should probably be adapted to each different physical case, so as to mimic the extension of the nucleonic wave functions in phase space, although this may not always be an easy task.

General conclusion

Finally, we wish to stress that as the complexity of the studied phenomena grows it becomes increasingly important to subject the models to theoretical tests in idealized scenarios where the physical behavior is well understood – only then is confrontation with actual data likely to be informative.

Zhang and Suraud, 1993

Physics Letters B 319 (1993) 35–40
North-Holland

PHYSICS LETTERS B

Boltzmann–Langevin equation, dynamical instability, and multifragmentation

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Editor: G.F. Bertsch

By using simulations of the Boltzmann–Langevin equation which incorporates dynamical fluctuations beyond usual transport theories and by coupling it with a coalescence model, we obtain information on multifragmentation in heavy-ion collisions. From a calculation of the $^{40}Ca+^{40}Ca$ system, we show that we can compute with confidence physical observables related to recent multifragmentation data.

Suraud, Ayik, Belkacem, Zhang, 1994

ELSEVIER

Nuclear Physics A580 (1994) 323–334

NUCLEAR PHYSICS A

On transient effects in violent nuclear collisions *

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Received 16 july 1993; revised 11 January 1994

Abstract

It is shown that the numerical simulations of the recently developed Boltzmann–Langevin model exhibit large dynamical fluctuations in momentum space during the early stages of heavy-ion collisions, which arise from an interplay between the nuclear mean-field and binary collisions. It is pointed out that this transient behaviour provides an initial seed for the development of density fluctuations, and could strongly influence the particle production cross-sections at subthreshold energies.

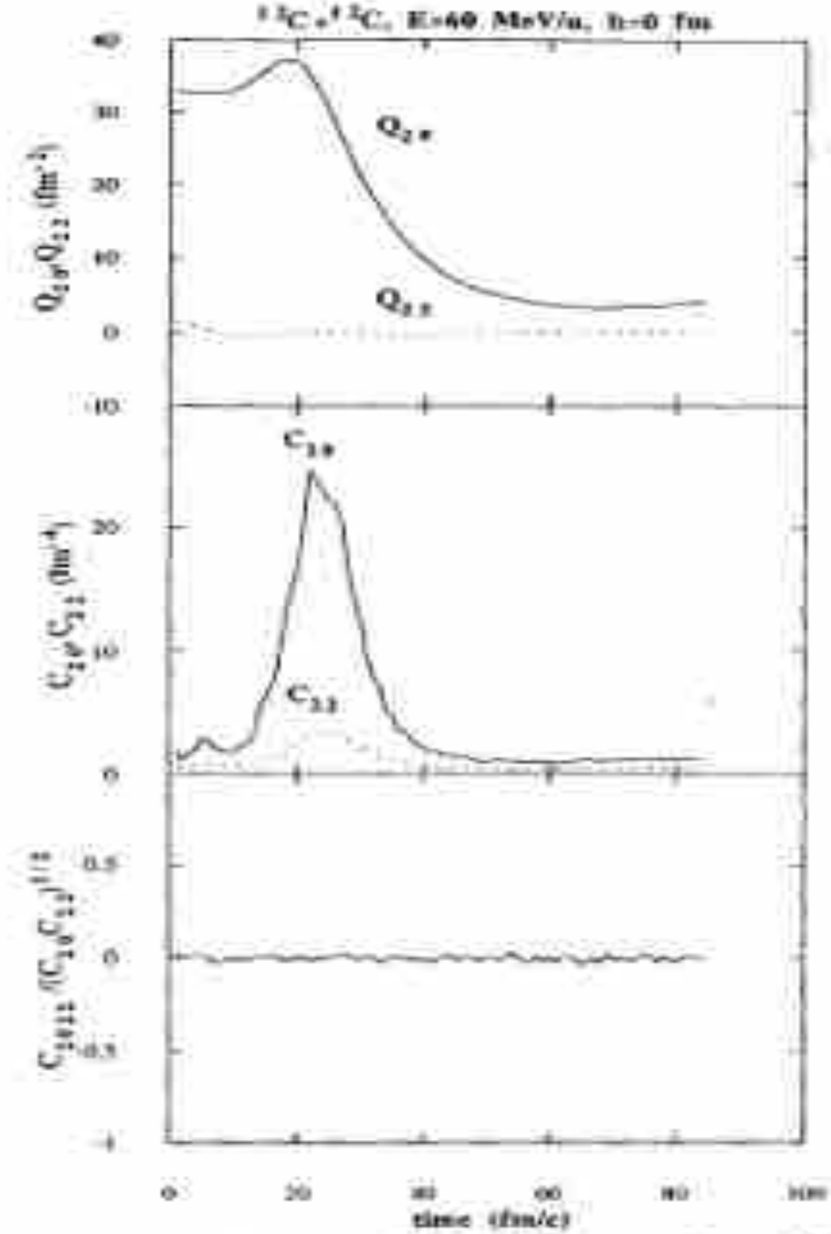

Fig. 1. Time evolution of the ensemble-averaged quadrupole moment Q_{20} and Q_{22} (with the magnetic quantum number $M=0$ and 2) (a) of the momentum distribution and their diffusion coefficient C_{20} and C_{22} (b), and the cross correlation ratio $C_{2022}/\sqrt{C_{20}C_{22}}$ (c) for central $^{12}C+^{12}C$ collisions at a bombarding energy of 60 MeV/u.

Including

Q_{22}, C_{22}

and the cross term

Zhang and Suraud, 1995

YSICAL REVIEW C VOLUME 51, NUMBER 6 JUNE 1

Analysis of multifragmentation in a Boltzmann-Langevin approach

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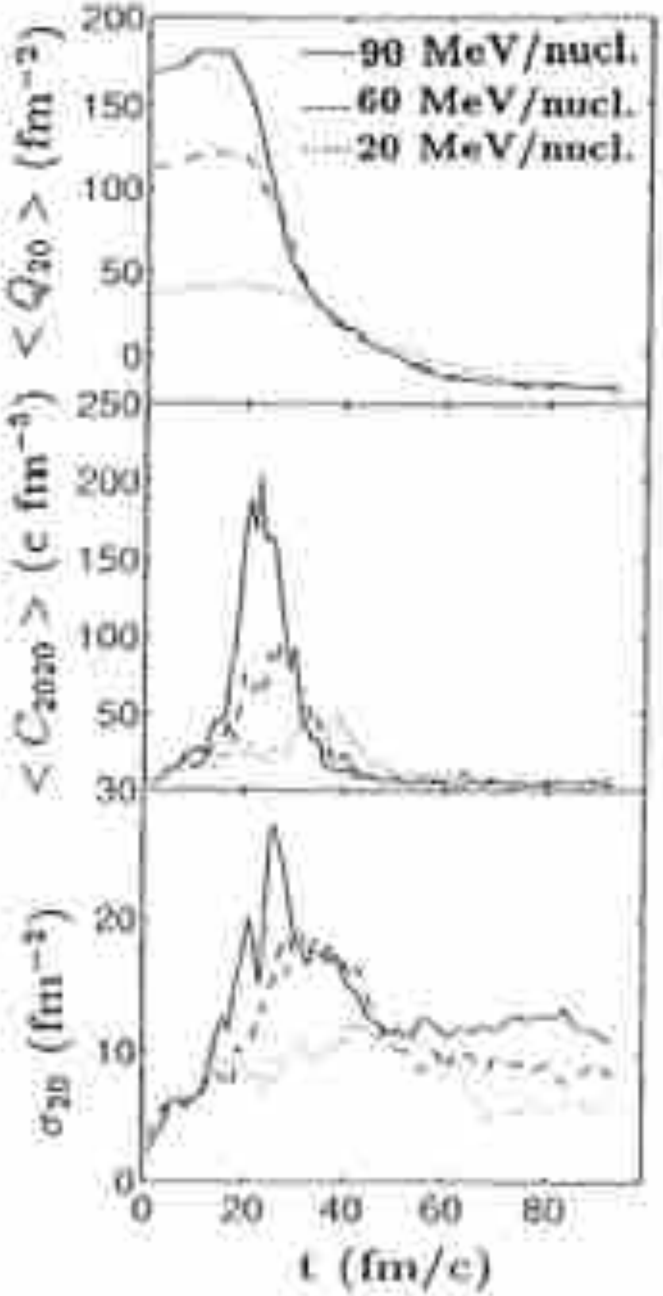
(Received 21 October 1994)

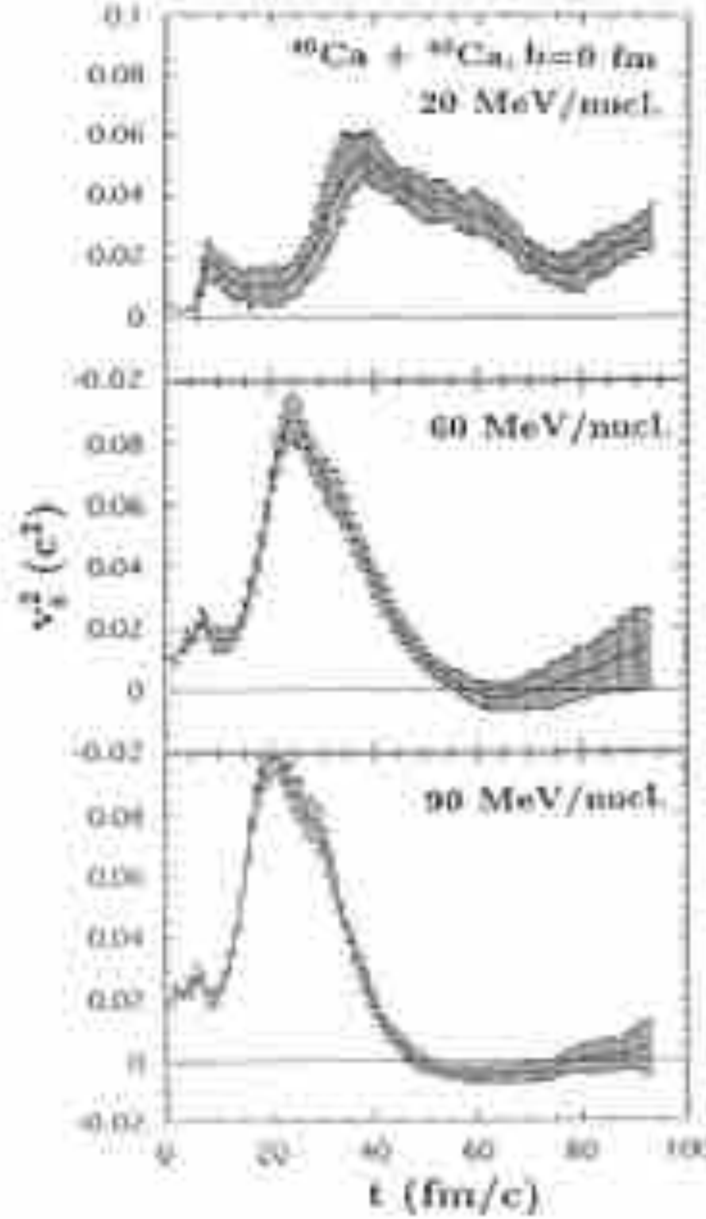
By using the Boltzmann-Langevin equation, which incorporates dynamical fluctuations beyond usual transport theories, we simulate the ^{40}Ca+^{40} Ca reaction system at different beam energies 20, 60, and 90 MeV/nucleon for different impact parameters. Dynamical fluctuations become larger and larger with increasing bombarding energy and the system can reach densities corresponding to the unstable region of the nuclear matter equation of state at energies above 60 MeV/nucleon. By coupling the Boltzmann-Langevin equation with a coalescence model in the late stages of the reaction, we obtain the distribution of the intermediate mass fragments in each event. From the correlation analysis of these fragments, we recover some trends of recent multifragmentation data. A critical behavior analysis is also provided.

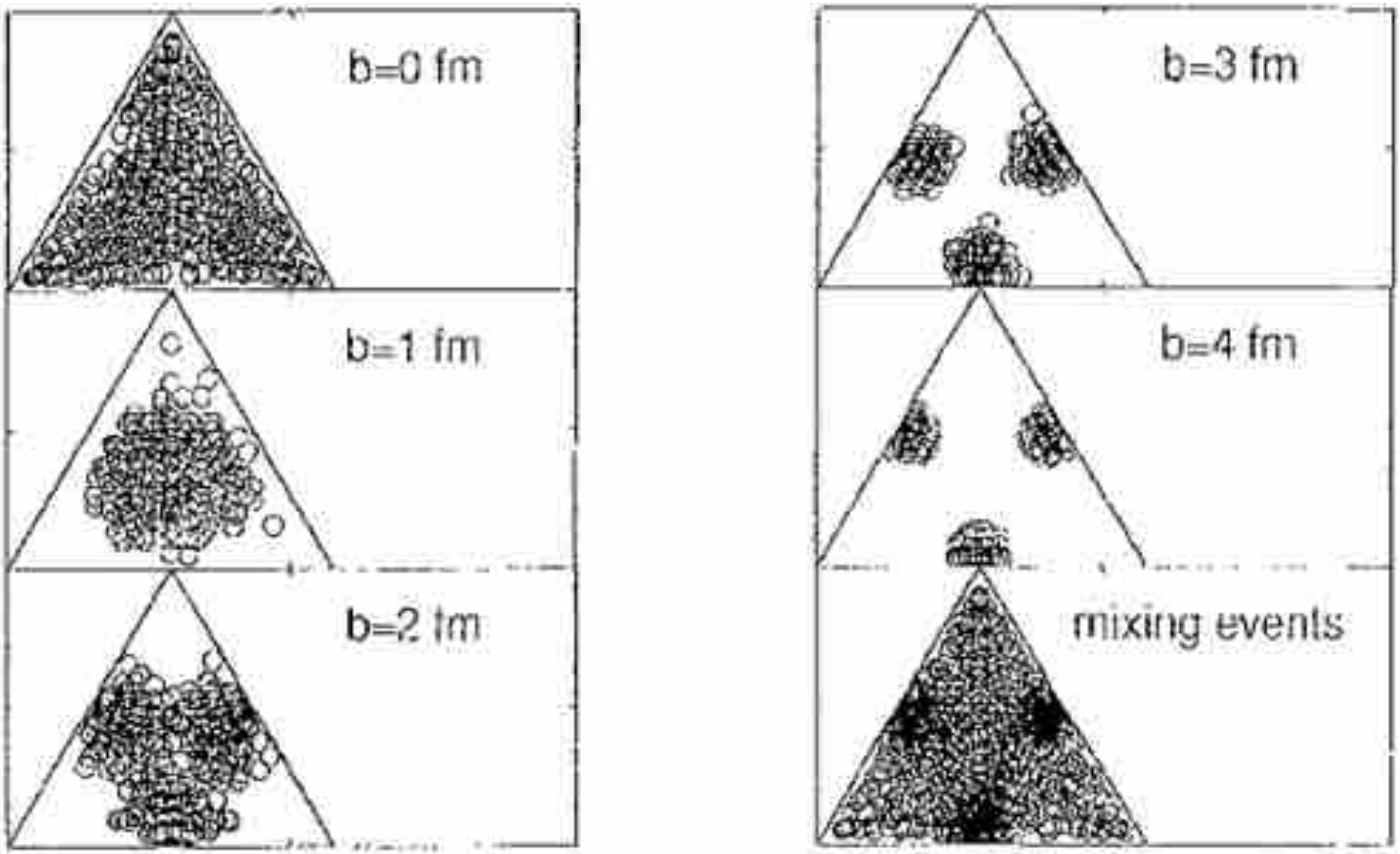
PACS number(s): 25.70.Pq

^{40}Ca + ^{40}Ca, b=0 fm



$$v_s^2 = \frac{1}{m}\left[\frac{10}{9}\langle E_k\rangle + A\left(\frac{\rho}{\rho_0}\right) + B\sigma\left(\frac{\rho}{\rho_0}\right)^{\sigma}\right]$$





F. S. Zhang and E. Suraud,
Physical Review C51, 1995, 2301

BLE with projection method, 1996

ELSEVIER

Physics Reports 275 (1996) 49–196

PHYSICS REPORTS

On stochastic approaches of nuclear dynamics

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Received December 1995; editor: G.E. Brown

Chomaz, Colonna, Randrup, 2004

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Physics Reports 389 (2004) 263–440
PHYSICS REPORTS
www.elsevier.com/locate/physrep

Nuclear spinodal fragmentation

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Accepted 22 September 2003
editor: G.E. Brown

Abstract

Spinodal multifragmentation in nuclear physics is reviewed. Considering first spinodal instability within the general framework of thermodynamics, we discuss the intimate relationship between first-order phase-transitions and convexity anomalies in the thermodynamic potentials, clarify the relationship between mechanical and chemical instability in two-component systems, and also address finite systems. Then we analyze the onset of spinodal fragmentation by various linear-response methods. Using the Landau theory of collective modes in bulk matter as a starting point, we first review the application of mean-field methods for the identification of the unstable collective modes and the determination of their structure and the associated dispersion relations

Baran,Colonna, Greco, Di Toro, 2005

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Physics Reports 410 (2005) 335–466

PHYSICS REPORTS

www.elsevier.com/locate/physrep

Reaction dynamics with exotic nuclei

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Available online 17 March 2005

editor: G.E. Brown

Abstract

We review the new possibilities offered by the reaction dynamics of asymmetric heavy-ion collisions, using stable and unstable beams. We show that it represents a rather unique tool to probe regions of highly asymmetric nuclear matter (ANM) in compressed as well as dilute phases, and to test the in-medium isovector interaction for high-momentum nucleons. The focus is on a detailed study of the symmetry term of the nuclear equation of state (EOS) in regions far away from saturation conditions but always under laboratory controlled conditions.

Thermodynamic properties of ANM are surveyed starting from non-relativistic and relativistic effective interactions. In the relativistic case, the role of the isovector–scalar δ-meson is stressed. The qualitative new features of the liquid–gas phase transition, "diffusive" instability and isospin distillation, are discussed. The results of ab initio simulations of n-rich, n-poor, heavy-ion collisions, using stochastic isospin-dependent transport equations, are analyzed as a function of beam energy and centrality. The isospin dynamics plays an important role in all steps of the reaction, from prompt nucleon emissions to the final fragments. The isospin diffusion is also of large interest, due to the interplay of asymmetry and density gradients. In relativistic collisions, the possibility of a direct study of the covariant structure of the effective nucleon interaction is shown. Results are discussed for particle production, collective flows and isotransparency.

BUU including correlated binary collisions with many-body effects from Bogoliubov approach B.J. Yang, PRC36(1987)667

PHYSICAL REVIEW C VOLUME 36, NUMBER 2 AUGUST 1987

Kinetic equation of nuclear gas

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(Received 27 January 1987)

The kinetic equation on nuclear gas is derived by means of the Bogoliubov approach. It is an improved Boltzmann-Uehling-Uhlenbeck equation including correctional binary collisions with many-body effects.

$$\frac{\partial f_1(\mathbf{x}_1)}{\partial t}+\frac{\mathbf{p}_1}{m}\cdot\frac{\partial f_1(\mathbf{x}_1)}{\partial \mathbf{q}_1}+\mathcal{F}\cdot\frac{\partial f_1(\mathbf{x}_1)}{\partial \mathbf{p}_1}=\frac{\pi}{(2\pi)^3\hbar}\int d\mathbf{k}\left(e^{(\hbar\mathbf{k}/2)\cdot\partial/\partial\mathbf{p}_1}-e^{-(\hbar\mathbf{k}/2)\cdot\partial/\partial\mathbf{p}_1}\right)\times\int d\mathbf{p}_2\,\delta\left[\mathbf{k}\cdot\left(\frac{\mathbf{p}_1}{m}-\frac{\mathbf{p}_2}{m}\right)\right]\frac{\tilde{V}_{12}^2(\mathbf{k})}{|1+(1/\hbar)\tilde{V}_{23}\Psi|^2}\times[f_1^+(\mathbf{x}_1)f_1^-(\mathbf{x}_2)-f_1^-(\mathbf{x}_1)f_1^+(\mathbf{x}_2)]\ .$$

Equation (28) is the kinetic equation of a nuclear gas in the quasihomogeneous case. This is an improved BUU equation. It is reduced to the usual BUU equation provided to neglect many-body effects and to take the first approximation of the term $\mathcal{F}$:

$$\frac{\partial f_1(\mathbf{x}_1)}{\partial t}+\frac{\mathbf{p}_1}{m}\cdot\frac{\partial f_1(\mathbf{x}_1)}{\partial \mathbf{q}_1}-\nabla U_1\cdot\frac{\partial f_1(\mathbf{x}_1)}{\partial \mathbf{p}_1}=\frac{\pi}{\hbar(2\pi)^9}\int d\mathbf{p}_2 d\mathbf{p}_1' d\mathbf{p}_2'\,|\langle\mathbf{p}_1\mathbf{p}_2|V_{12}|\mathbf{p}_1'\mathbf{p}_2'\rangle|^2\times\{f_1(\mathbf{x}_1')f_1(\mathbf{x}_2')[1-f_1(\mathbf{x}_1)][1-f_1(\mathbf{x}_2)]-f_1(\mathbf{x}_1)f_1(\mathbf{x}_2)[1-f_1(\mathbf{x}_1')][1-f_1(\mathbf{x}_2')]\}\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1'-\mathbf{p}_2')\ .$$

Lingxiao Ge，Yi-Zhong Zhuo，Non-Relativistic extended BUU equation，1989

第 13 卷 第 7 期　　高能物理与核物理　　Vol. 13, No. 7
1989 年 7 月　　HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS　　July, 1989

非相对论 BUU 方程*

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摘　　要

基于闭合时间格林函数技术，在 H-F 和 Born 近似下，完成了一级和二级微扰计算，得到了不同微扰下的自能和格林函数运动方程，并分别在局域和非局域近似下，得到了单粒子格林函数的 Wigner 函数随时间变化的 Boltzmann-Uehling-Uhlenbeck（BUU）方程和它的扩展形式。并对导出 BUU 方程的近似作了总结和讨论

三、Vlasov 方程

一般，两体相互作用可以写为下面形式，

$$V(\vec{x}_3, t_3, \vec{x}_4, t_4) = V(\vec{x}_3, \vec{x}_4)\delta(t_3 - t_4), \tag{3.1}$$

因此，(2.17)和(2.18)式中的正时序和反时序格林函数做类似于

$$iG^{(0)++}(\vec{x}_3, t_3, \vec{x}_4, t_4)|_{t_3=t_4+0^+} \Longrightarrow iG^{(0)-+}(\vec{x}_3, t_3, \vec{x}_4, t_4), \tag{3.2}$$

近似，(2.17)式变为，

$$\Sigma^{(1)++}(\vec{x}_3, t_3, \vec{x}_4, t_4) = \delta(t_3 - t_4)\left[\delta(\vec{x}_3 - \vec{x}_4)\int d\vec{x}_4' V(\vec{x}_3, \vec{x}_4') \times G^{(0)-+}(\vec{x}_4', \vec{x}_4') - V(\vec{x}_3, \vec{x}_4)G^{(0)-+}(\vec{x}_3, \vec{x}_4)\right], \tag{3.3}$$

这正是 Hatree-Fork 自能形式[12]，(3.3)式中第一项是直接项，第二项是交换项，即

$$\Sigma^{HF}(\vec{x}_3, \vec{x}_4, t_3) = \Sigma^{(1)++}(\vec{x}_3, \vec{x}_4, t_3), \tag{3.4}$$

二、闭合时间格林函数和微扰

非平衡格林函数允许我们去研究多粒子量子系统的时间发展。我们知道，基于 Gell-mann 和 Low 理论所得到的算符的期望值不能用于非静止状态的期望值[12]，如果我们讨论相对于 t_0 规定状态的算符期望值，会得到：

$$\langle \hat{O}_H(t)\rangle = \langle U(t_0, t)\hat{O}_I(t)U(t, t_0)\rangle, \tag{2.1}$$

算符的下标 H 和 I 分别表示海森堡和相互作用表象。其中：

$$U(t, t_0) = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}T^c\left[\int_{t_0}^{t}dt_1\cdots\int_{t_0}^{t}dt_n[H_I'(t_1)\cdots\cdots H_I'(t_n)]\right], \tag{2.2}$$

$$U(t_0, t) = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}T^a\left[\int_{t}^{t_0}dt_1\cdots\int_{t}^{t_0}dt_n[H_I'(t_1)\cdots\cdots H_I'(t_n)]\right]. \tag{2.2'}$$

$H_I'(t)$ 是互作用表象中的相互作用哈密顿量，故

$$\langle \hat{O}_H(t)\rangle = \left\langle T^a\left[\exp\left(-i\int_t^{t_0}dt' H_I(t')\right)\right]\hat{O}_I(t)T^c\left[\exp\left(-i\int_{t_0}^{t}dt' H_I'(t')\right)\right]\right\rangle \tag{2.3}$$

T^c 和 T^a 分别为正时序和反时序次序算符，当算符的左边和右边都加入了指数函数，又引进了时间的次序 T，就可分辨场算符属于正时序还是属于反时序部分，因此，引入闭合回路，在时间上朝前从 $t_0 \to t$，又返回从 $t \to t_0$，如图 1 所示。由此，我们沿着闭合回路来定义格林函数，其时间变量沿着回路，这称为闭合的时间格林函数（CTGF），根据场算符在回路中的不同位置，我们定义下面四种格林函数。

四、碰撞项

Kadanoff-Baym 方程在一级和二级微扰近似下，可从 Dyson 方程直接得到，

$$\begin{aligned} G_{01}^{0-1}G^{-+}(\vec{x}_1, t_1, \vec{x}_2, t_2) &= \int d\vec{x}_3 dt_3 \Sigma^{HF}(\vec{x}_1, \vec{x}_3, t_1)G^{-+}(\vec{x}_3, t_1, \vec{x}_2, t_2) \\ &- \int_{t_0}^{t_1}[\Sigma^{(2)+-}(\vec{x}_1, \vec{x}_3, t_1) - \Sigma^{(2)-+}(\vec{x}_1, \vec{x}_3, t_1)]G^{-+}(\vec{x}_3, t_1, \vec{x}_2, t_2)d\vec{x}_3 dt_3 \\ &+ \int_{t_0}^{t_2}\Sigma^{(2)-+}(\vec{x}_1, \vec{x}_3, t_1)[G^{+-}(\vec{x}_3, t_1, \vec{x}_2, t_2) - G^{-+}(\vec{x}_3, t_1, \vec{x}_2, t_2)]d\vec{x}_3 dt_3, \end{aligned} \tag{4.1}$$

Outline

- Introduction
- Early stage of the transport models (1970-1990)
- Correlations, fluctuations, and fragmentations (1987-2013)
- Recent progress (after 1992)
- Conclusions and suggestions

Chomaz, Colonna, Guarnera, and Randrup, 1994

VOLUME 73, NUMBER 26 PHYSICAL REVIEW LETTERS 26 DECEMBER 1994

Brownian One-Body Dynamics in Nuclei

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(Received 15 August 1994)

A novel method is presented for introducing fluctuations in one-body dynamics. It consists of employing a Brownian force in the kinetic equations. For nuclear matter within the spinodal zone, the magnitude of the Brownian force can be determined by demanding correspondence with the growth of the most unstable mode, as given by Boltzmann-Langevin simulations. The method is illustrated and tested for idealized two-dimensional matter and promises to provide a practical means for addressing catastrophic nuclear processes.

PACS numbers: 24.60.Ky, 21.65.+f

Guarnera, Colonna, Chomaz, 1996

25 April 1996

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 373 (1996) 267–274

3D stochastic mean-field simulations of the spinodal fragmentation of dilute nuclei ★

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Editor: G.F. Bertsch

Abstract

We study the spinodal decomposition of hot and dilute nuclear systems using a stochastic one-body approach in 3D. The early clusterization process appears dominated by unstable modes with well defined multipolarity and radial structure, which can be related to infinite nuclear matter properties. These instabilities favour primary partitions of the system in nearly equal mass fragments, in association with a lack of small clusters. Finally, we discuss how these features are affected by the final decay of the formed fragments.

PACS: 24.60.Ky; 21.65.+f

Bernard Borderie et al., 2001

VOLUME 86, NUMBER 15 PHYSICAL REVIEW LETTERS 9 APRIL 2001

Evidence for Spinodal Decomposition in Nuclear Multifragmentation

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(Received 3 October 2000)

Multifragmentation of a "fused system" was observed for central collisions between 32 MeV/nucleon ^{129}Xe and ^{nat}Sn. Most of the resulting charged products were well identified due to the high performances of the INDRA 4π array. Experimental higher-order charge correlations for fragments show a weak but nonambiguous enhancement of events with nearly equal-sized fragments. Supported by dynamical calculations in which spinodal decomposition is simulated, this observed enhancement is interpreted as a "fossil" signal of spinodal instabilities in finite nuclear systems.

DOI: 10.1103/PhysRevLett.86.3252 PACS numbers: 25.70.Pq, 24.60.Ky

Maria Colonna, 2013

PRL 110, 042701 (2013) PHYSICAL REVIEW LETTERS week ending 25 JANUARY 2013

Fluctuations and Symmetry Energy in Nuclear Fragmentation Dynamics

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Within a dynamical description of nuclear fragmentation, based on the liquid-gas phase transition scenario, we explore the relation between neutron-proton density fluctuations and nuclear symmetry energy. We show that, along the fragmentation path, isovector fluctuations follow the evolution of the local density and approach an equilibrium value connected to the local symmetry energy. Higher-density regions are characterized by smaller average asymmetry and narrower isotopic distributions. This dynamical analysis points out that fragment final state isospin fluctuations can probe the symmetry energy of the density domains from which fragments originate.

DOI: 10.1103/PhysRevLett.110.042701 PACS numbers: 25.70.Pq, 05.10.Gg, 21.30.Fe, 24.60.−k

Xie, Su, Zhu, and Zhang, 2013

Physics Letters B 718 (2013) 1510–1514

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Physics Letters B

www.elsevier.com/locate/physletb

Symmetry energy and pion production in the Boltzmann–Langevin approach

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Symmetry energy

ABSTRACT

Based on the improved isospin-dependent Boltzmann–Langevin model which incorporates the dynamical fluctuations, we study the π production in central heavy ion collisions at different incident energies from 250 to 1200A MeV. It is found that the π multiplicity is sensitive to the nuclear equation of state. At π subthreshold energy, the fluctuations have a larger effect on the π multiplicity. The π^-/π^+ ratios as a probe of nuclear symmetry energy are calculated with different stiffness of symmetry energy. The results favor a supersoft symmetry energy of the potential term in comparison with the FOPI data, which supports the one obtained by the usual Boltzmann–Uehling–Uhlenbeck model.

Xie, Su, Zhu, and Zhang, 2013

RAPID COMMUNICATIONS

PHYSICAL REVIEW C 88, 061601(R) (2013)

Neutron-proton effective mass splitting in a Boltzmann-Langevin approach

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(Received 19 July 2013; published 19 December 2013)

Based on the Boltzmann-Langevin transport model, in which the isospin and momentum-dependent potential is incorporated, we investigate the neutron-proton effective mass splitting in central reactions using Sn isotopes at 50 MeV/nucleon. It is found that the transverse momentum, rapidity, and kinetic energy distributions of free neutron over proton ratio are sensitive to the neutron-proton effective mass splitting, especially at higher transverse momenta, at higher kinetic energies, and at larger rapidities. The calculated results favor the effective mass of neutrons less than the one of protons in comparison with the experimental data of Famiano *et al.* [Phys. Rev. Lett. 97, 052701 (2006)].

DOI: 10.1103/PhysRevC.88.061601 PACS number(s): 25.70.−z, 24.10.Nz, 14.20.Dh, 21.65.Ef

Gavin, Moschelli, and Zin, 2013

PHYSICAL REVIEW C 95, 064901 (2017)

Boltzmann-Langevin approach to pre-equilibrium correlations in nuclear collisions

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(Received 21 December 2016; published 12 June 2017)

Correlations born before the onset of hydrodynamic flow can leave observable traces on the final-state particles. Measurement of these correlations yield important information on the isotropization and thermalization processes. Starting from a Boltzmann-like kinetic theory in the presence of dynamic Langevin noise, we derive a new partial differential equation for the two-particle correlation function that respects the microscopic conservation laws. To illustrate how these equations can be used, we study the effect of thermalization on long-range correlations. We show quite generally that two-particle correlations at early times depend on S, the average probability that a parton suffers no interactions. We extract S from transverse momentum fluctuations measured in nucleus-nucleus collisions and predict the degree of partial thermalization in proton-nucleus experiments.

DOI: 10.1103/PhysRevC.95.064901

Ayik, Yilmaz 2, Umar, and Turan, 2017

PHYSICAL REVIEW C 96, 024611 (2017)

Multinucleon transfer in central collisions of $^{238}U + ^{238}U$

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(Received 12 June 2017; published 14 August 2017)

Quantal diffusion mechanism of nucleon exchange is studied in the central collisions of $^{238}U + ^{238}U$ in the framework of the stochastic mean-field (SMF) approach. For bombarding energies considered in this work, the dinuclear structure is maintained during the collision. Hence, it is possible to describe nucleon exchange as a diffusion process for mass and charge asymmetry. Quantal neutron and proton diffusion coefficients, including memory effects, are extracted from the SMF approach and the primary fragment distributions are calculated.

DOI: 10.1103/PhysRevC.96.024611

SMF for MNT

S. AYIK, B. YILMAZ, O. YILMAZ, AND A. S. UMAR

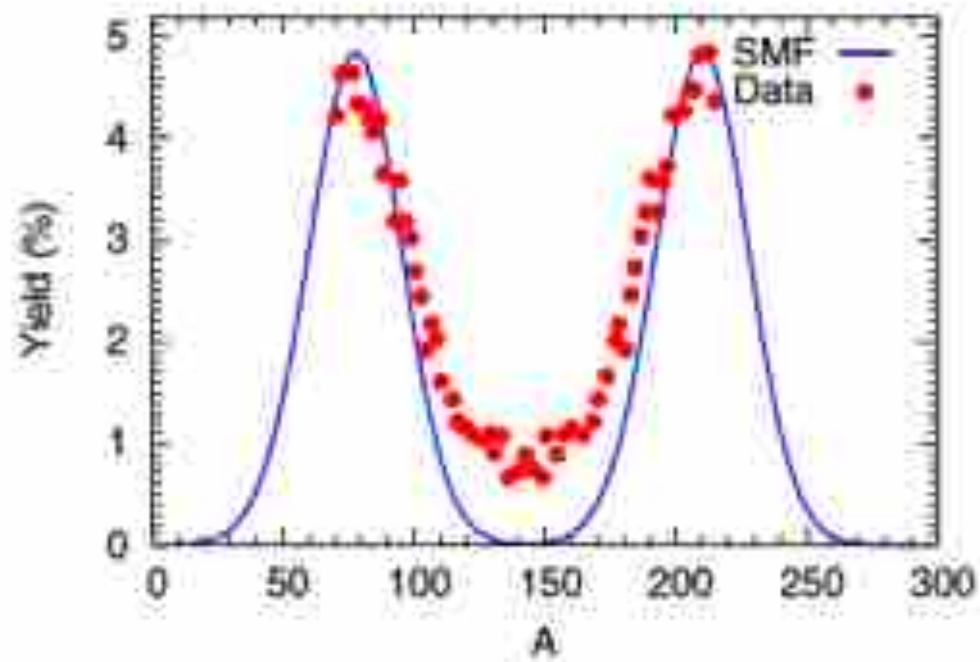

FIG. 7. Primary fragment yield in $^{48}Ca + ^{238}U$ collisions at $E_{c.m.} = 193$ MeV and comparison with data. The solid line is the result of Eq. (48).

Akira Ono, 2019

Progress in Particle and Nuclear Physics 105 (2019) 139–179

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Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

Dynamics of clusters and fragments in heavy-ion collisions

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ARTICLE INFO

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ABSTRACT

A review is given on the studies of formation of light clusters and heavier fragments in heavy-ion collisions at incident energies from several tens of MeV/nucleon to several hundred MeV/nucleon, focusing on dynamical aspects and on microscopic theoretical descriptions. Existing experimental data already clarify basic characteristics of expanding and fragmenting systems typically in central collisions, where cluster correlations cannot be ignored. Cluster correlations appear almost everywhere in excited low-density nuclear many-body systems and nuclear matter in statistical equilibrium where the properties of a cluster may be influenced by the medium. On the other hand, transport models to solve the time evolution have been developed based on the single-nucleon distribution function. Different types of transport models are reviewed putting emphasis both on theoretical features and practical performances in the description of fragmentation. A key concept to distinguish different models is how to consistently handle single-nucleon motions in the mean field, fluctuation or branching induced by two-nucleon collisions, and localization of nucleons to form fragments and clusters. Some transport codes have been extended to treat light clusters explicitly. Results indicate that cluster correlations can have strong impacts on global collision dynamics and correlations between light clusters should also be taken into account.

Outline

- Introduction
- Early stage of the transport models (1970-1990)
- Correlations, fluctuations, and fragmentations (1987-2013)
- Recent progress (2013-update)
- Conclusions and suggestions

The 2st comparision of the fluctuations in the transport models, 2019

MD-like

	Code	Who did?
1	TuQMD	Dan Cozma
2	AMD	Akira Ono
3	SINAP-QMD	Guo-Qiang Zhang
4	UrQMD (L=1)	Yong-Jia Wang
5	UrQMD (L=2)	Yong-Jia Wang
6	BNU-QMD	Jun Su
7	LQMD	Zhao-Qing Feng
8	IQMD	Ch. Hartnack
9	CoMD	M. Papa
10	ImQMD	Ying-Xun Zhang
11	ImQMD-drdp	Ying-Xun Zhang
12	GXNU-QMD	Ning Wang

Boltzmann-like

	Code	Who did?
1	BLOB	P. Napolitani
2	SMF	P. Napolitani
3	GiBUU(Sky)	J. Weil
4	GiBUU(RMF)	J. Weil
5	RVUU	Taesoo Song
6	IBUU(04)	Jun Xu
7	IBL	Wen-Jie Xie
8	RBUU	Kyungil Kim
9	pBUU	P. Danielewicz

How to select the same simple input quantities: physical, numerical

1. Molecular dynamics: N-body approaches
QMD, CoMD,
IQMD, ImQMD,…

AMD, FMD…

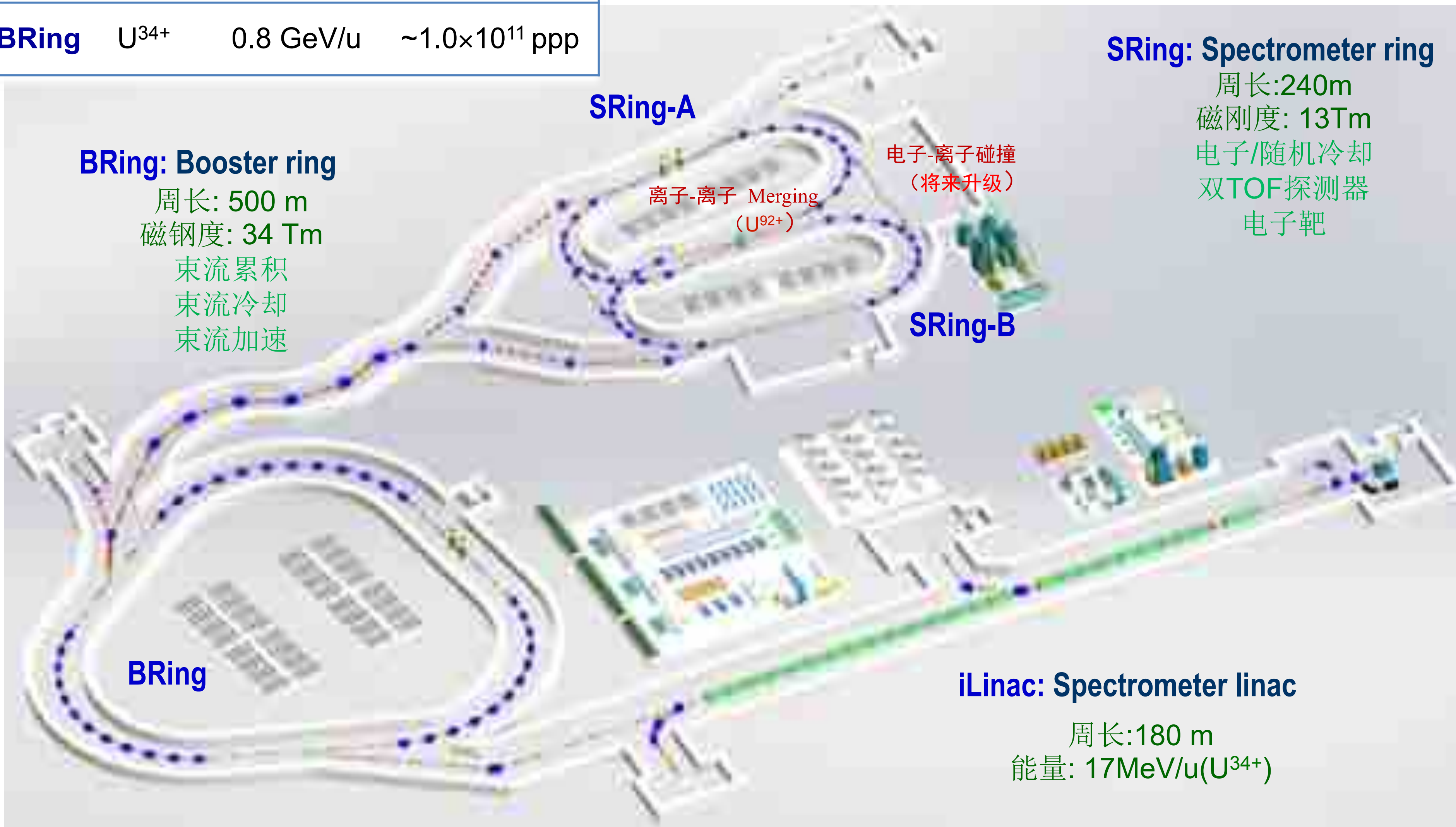
2. Boltzmann-like: 1-body approaches
IBUU (BNV, LV)+Fluc, IBL, SMF, BOB…

Important: Input quantities, Numerical treatments,…

Applications: Fragmentation, MNT, …

	Ions	Energy	Intensity
SECR	U^{34+}	14 keV/u	0.05 pmA
iLinac	U^{34+}	17 MeV/u	0.028 pmA
BRing	U^{34+}	0.8 GeV/u	~1.0×10^{11} ppp

HIAF in Huizhou



Provided by Jiansong Wang in RIBLL1 worskhop


Thank you for your attentions !