


Homework for July 31

Consider the system of two-component fermions with zero-range attractive interactions.

$$M =: \exp \left[-H_{\text{free}} \alpha_t - C \alpha_t \sum_{n_x} \rho_{\uparrow}(n_x) \rho_{\downarrow}(n_x) \right] :$$

$$\begin{aligned} H_{\text{free}} &= H_{\text{free}}^{\uparrow} + H_{\text{free}}^{\downarrow} = \\ &= -\frac{1}{2m} \sum_{n_x, i=\uparrow, \downarrow} a_i^{\dagger}(n_x) \left[a_i(n_x + 1) - 2a_i(n_x) + a_i(n_x - 1) \right] \end{aligned}$$

We consider one up spin and one down spin. Take the size of the periodic box to be $L = 6$ and choose $L_t = 50$. Do an exact (i.e., not Monte Carlo) calculation of the amplitude

$$Z(L_t) = \langle \psi_{\text{init}} | \overbrace{\text{MMM} \dots \text{MM}}^{L_t} | \psi_{\text{init}} \rangle$$


for the initial state with one up spin and down spin, each with zero momentum in the periodic box. For the parameters take

$$C = -0.200, m = 938.92 \text{ MeV}, a = a_t = (100 \text{ MeV})^{-1}$$

Use the ratio of amplitudes with L_t and $L_t - 1$ time steps to determine an estimate for the energy using the relation

$$e^{-E(L_t)\alpha_t} = Z(L_t)/Z(L_t - 1)$$