Homework for July 26

Write a code that expresses as a matrix, the lattice Hamiltonian for one free non-relativistic particle:

$$H_{\text{free}} = -\frac{1}{2m} \sum_{\vec{n}} \sum_{l=1,2,3} a^\dagger(\vec{n}) \left[a(\vec{n}+\hat{l}) - 2a(\vec{n}) + a(\vec{n}-\hat{l}) \right]$$

Find the energy spectrum of this matrix for a cubic periodic lattice with mass m equal to 1 (in lattice units) with a periodic lattice of length 8.

Derive analytic expressions for these energies, and check that the first 33 energies agree with the analytic expressions for these energies.