Chiral symmetry and Ward identities

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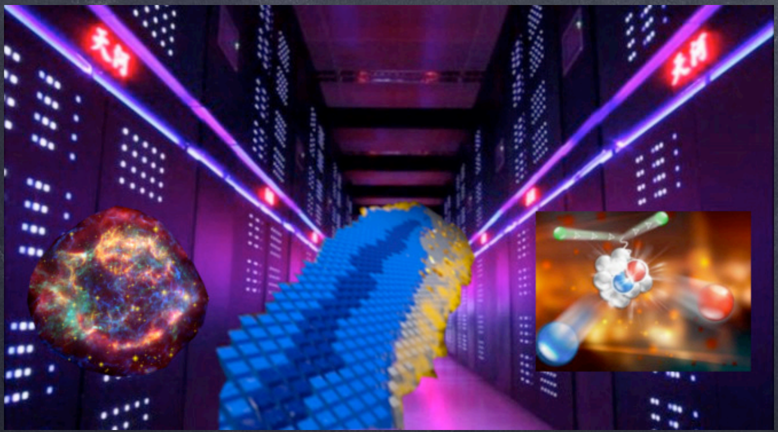
TALENT School - From quarks and gluons to nuclear forces and structures



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One flavor

Chiral symmetry <—> SχSB <—> light pions

Wilson fermions

Ginsparg-Wilson fermions

$$S_F[\psi, \overline{\psi}, A] = \int d^4x \; \mathcal{L}[\psi, \overline{\psi}, A]$$

$$\mathcal{L}[\psi, \overline{\psi}, A] = \overline{\psi}(x) \left[\gamma_\mu D_\mu\right] \psi(x)$$

$$D_\mu = \partial_\mu + i A_\mu$$

$$\begin{cases} \psi(x) \to \mathrm{e}^{i\alpha\gamma_5}\psi(x) \\ \overline{\psi}(x) \to \overline{\psi}(x)\mathrm{e}^{i\alpha\gamma_5} \end{cases}$$

$\alpha \in \mathbb{R}$ constant

$\{\gamma_5, \gamma_\mu\} = 0$

$$\mathcal{L}[\psi', \overline{\psi}', A] = \overline{\psi}(x)\mathrm{e}^{i\alpha\gamma_5} \left[\gamma_\mu D_\mu\right] \mathrm{e}^{i\alpha\gamma_5}\psi(x) = \mathcal{L}[\psi, \overline{\psi}, A]$$

One flavor

$$m\overline{\psi}(x)\psi(x) \to m\overline{\psi}(x)\mathrm{e}^{i\alpha\gamma_5}\mathrm{e}^{i\alpha\gamma_5}\psi(x) = m\overline{\psi}(x)\mathrm{e}^{2i\alpha\gamma_5}\psi(x)$$

Not invariant

Chiral symmetry -> Left- and Right-handed

$$P_R = \frac{1+\gamma_5}{2} \qquad P_L = \frac{1-\gamma_5}{2} \qquad P_{R,L}^2 = P_{R,L} \quad \gamma_5^2 = 1 \quad P_R + P_L = 1$$

$$\psi_{R,L} = P_{R,L}\psi \qquad \overline{\psi}_{R,L} = \overline{\psi}P_{L,R} \qquad P_R P_L = P_L P_R = 0 \qquad \gamma_\mu P_{L,R} = P_{R,L}\gamma_\mu$$

$$\mathcal{L}_{m=0} = \overline{\psi}\left(P_R + P_L\right)\gamma_\mu D_\mu \left(P_R + P_L\right)\psi = \overline{\psi}_L\gamma_\mu D_\mu \psi_L + \overline{\psi}_R\gamma_\mu D_\mu\psi_R$$

$$m\overline{\psi}\psi = m\overline{\psi}(P_L + P_R)(P_L + P_R)\psi = m\left(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R\right)$$

$m \to 0$ **Chiral limit** $\gamma_5\gamma_\mu D_\mu + \gamma_\mu D_\mu \gamma_5 = 0$

Several flavors

N_f flavors

$$m_1 = m_u$$
$$m_2 = m_d$$
$$m_3 = m_s$$
$$\vdots$$

$$\psi(x) = \begin{bmatrix} \psi_1(x) \\ \vdots \\ \psi_{N_f}(x) \end{bmatrix} \qquad M = \begin{bmatrix} m_1 & & \\ & \ddots & \\ & & m_{N_f} \end{bmatrix}_{N_f \times N_f}$$

$$S_F[\psi, \overline{\psi}, A] = \int d^4x \; \overline{\psi}(x) \left[\gamma_\mu D_\mu + M\right] \psi(x)$$

$SU(N_f)$ Not to be confused with $SU(N_c)$

$$SU(N_f)_V : \begin{cases} \psi(x) \to \mathrm{e}^{i\alpha^a \lambda^a} \psi(x) \\ \overline{\psi}(x) \to \overline{\psi}(x) \mathrm{e}^{-i\alpha^a \lambda^a} \end{cases} \qquad U(1)_V : \begin{cases} \psi(x) \to \mathrm{e}^{i\alpha} \psi(x) \\ \overline{\psi}(x) \to \overline{\psi}(x) \mathrm{e}^{-i\alpha} \end{cases}$$

S_F is invariant for $M = 0$ and $M = \mathrm{diag}(m, \cdots, m)$

Several flavors

$$SU(N_f)_V : \begin{cases} \psi(x) \to e^{i\alpha^a \lambda^a} \psi(x) \\ \overline{\psi}(x) \to \overline{\psi}(x) e^{-i\alpha^a \lambda^a} \end{cases} \qquad U(1)_V : \begin{cases} \psi(x) \to e^{i\alpha} \psi(x) \\ \overline{\psi}(x) \to \overline{\psi}(x) e^{-i\alpha} \end{cases}$$

S_F is invariant for $M = 0$ and $M = \mathrm{diag}(m, \cdots, m)$

$$\begin{cases} \psi(x) \to e^{i\gamma_5 \alpha^a \lambda^a} \psi(x) \\ \overline{\psi}(x) \to \overline{\psi}(x) e^{i\gamma_5 \alpha^a \lambda^a} \end{cases} \qquad U(1)_A : \begin{cases} \psi(x) \to e^{i\gamma_5 \alpha} \psi(x) \\ \overline{\psi}(x) \to \overline{\psi}(x) e^{i\gamma_5 \alpha} \end{cases}$$

$$M = 0 \qquad \boxed{SU(N_f)_L \times SU(N_f)_R} \times U(1)_V \times \boxed{U(1)_A}$$

Anomalous

SχSB

$$Z_F = \int \mathcal{D}[\psi, \overline{\psi}] e^{-S_F[\psi, \overline{\psi}, U]}$$

Measure not invariant

Fujikawa

Chiral symmetry breaking

$M = \mathrm{diag}(m, \cdots, m)$ → Explicit breaking

$SU(N_f)_L \times SU(N_f)_R \times U(1)_V$ → $SU(N_f)_V \times U(1)_V$

$M = \mathrm{diag}(m_1, \cdots, m_{N_f})$ → $\underbrace{U(1)_V \times \cdots \times U(1)_V}_{N_f \text{ factors}}$

$m_u,\ m_d \sim 5$ MeV $m_s \sim 100$ MeV $m_P \sim 1$ GeV $\Lambda_{\mathrm{QCD}} \sim 300$ MeV

$N_f = 2$ Good $N_f = 3$ Good?

If $m_u = m_d = 0$ $SU(2)_L \times SU(2)_R \times U(1)_V$ Good symmetry

→ $m_N = m_{N^*}$

$$\left.\begin{aligned} m_N &\simeq 940 \text{ MeV} \\ m_{N^*} &\simeq 1535 \text{ MeV} \end{aligned}\right\} \text{S}\chi\text{SB}$$

$m_{u,d} \neq 0$ too small

Spontaneous chiral symmetry breaking

Spin systems -> action invariant | ground state is not

$\langle \overline{\psi}(x)\psi(x)\rangle$ Not invariant in the chiral limit

$$\lim_{m\to 0}\lim_{V\to\infty}\langle \overline{\psi}(x)\psi(x)\rangle = 0$$

$$\lim_{m\to 0}\lim_{V\to\infty}\langle \overline{\psi}(x)\psi(x)\rangle \neq 0$$

Goldstone theorem $\mathcal{G}\to\mathcal{H} \quad \Rightarrow N = N_{\mathcal{G}} - N_{\mathcal{H}}$ massless particles

$SU(2)_L \times SU(2)_R \to SU(2)_V \Rightarrow 3$ Goldstone bosons (3 pions)

- Pions are not massless because
- Chiral symmetry is useful if $\mathcal{M} \neq 0$ is small

$$m_q \ll \Lambda_{QCD} \simeq 250 \text{ MeV} \qquad (m_q \simeq 4 \text{ MeV})$$

$$M_\pi \ll \Lambda_\chi = 4\pi F_\pi \simeq 1.2 \text{ GeV} \qquad (M_\pi \simeq 140 \text{ MeV})$$