QGP dileptons and photons: perturbation theory meets the lattice

Jacopo Ghiglieri, CERN

ECT*, November 26, 2018

In this talk

- In this talk:
  - the *thermal* photon and dilepton rates at **NLO** in an infinite, equilibrated medium in different kinematical regimes
    - at zero virtuality JG Hong Kurkela Lu Moore Teaney **JHEP1305** (2013)
    - at small virtuality JG Moore **JHEP1412** (2014)
    - at larger virtuality Ghisoiu Laine **JHEP1410** (2014), JG Moore
  - a comparison with lattice data JG Kaczmarek Laine Meyer **PRD94** (2016)

Basics of e/m production

- $\alpha \ll 1$ implies that real/virtual photon production are rare events and that rescatterings and back-reactions are negligible: medium is transparent to/not cooled by EM radiation

- At leading order in QED and to all orders in QCD the photon and dilepton rates are given by (in eq.)

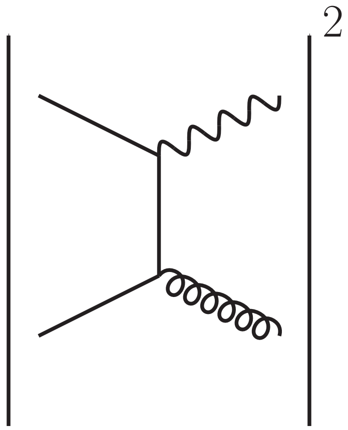
$$\frac{d\Gamma_\gamma(k)}{d^3k} = -\frac{\alpha}{4\pi^2 k} n_B(k^0)\, \rho_{EM}(k)$$

$$\frac{d\Gamma_{l^+l^-}(K)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2} n_B(k^0)\, \rho_{EM}(K)$$

thermal distribution x spectral function of the EM current

Theory approaches

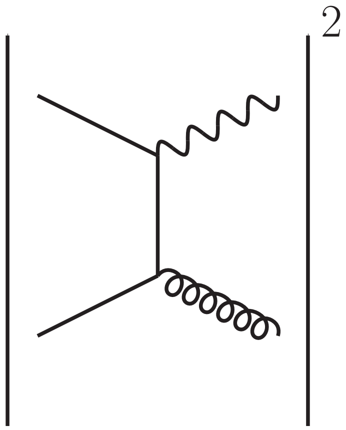
Theory approaches



pQCD: QCD action (and EFTs thereof), thermal average can be generalized to non-equilibrium.

Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$

Theory approaches



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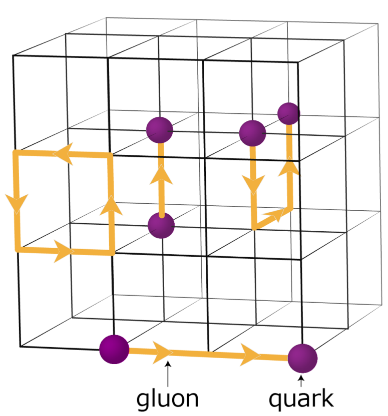

lattice QCD: Euclidean QCD action, pure thermal average. Real world: analytically continue to Minkowskian domain

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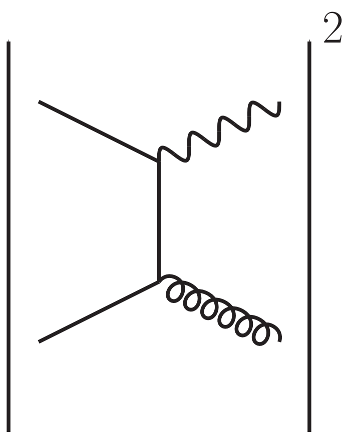

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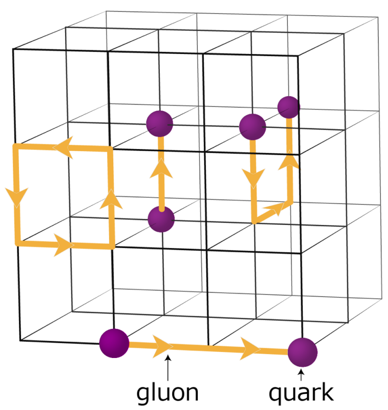

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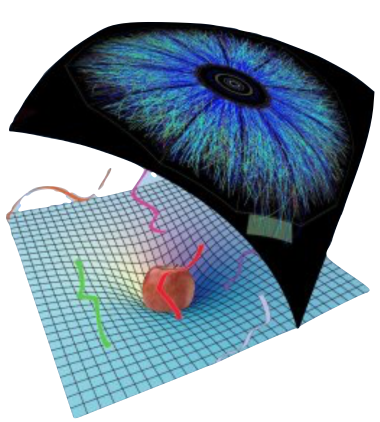

AdS/CFT: $\mathcal{N}=4$ action, in and out of equilibrium, weak and strong coupling. Real world: extrapolate to QCD

Motivation

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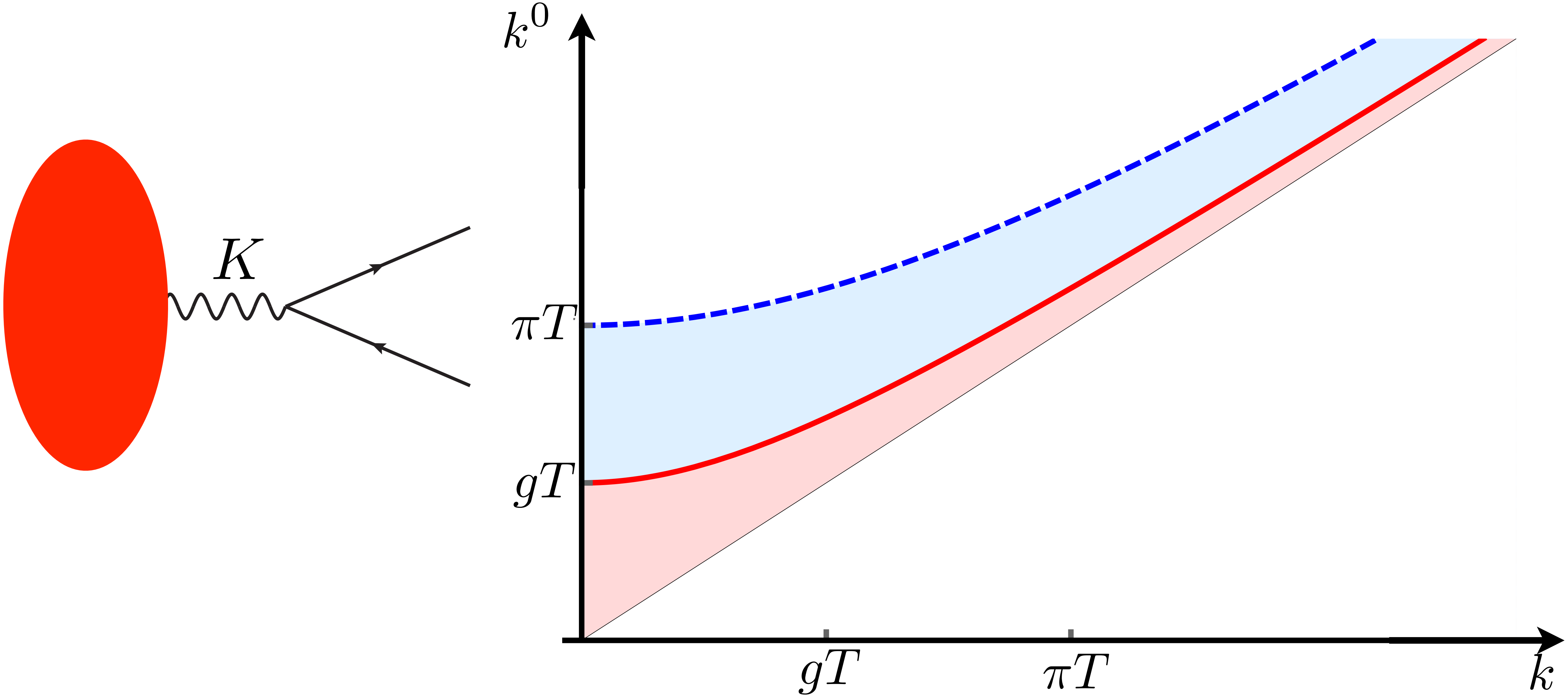
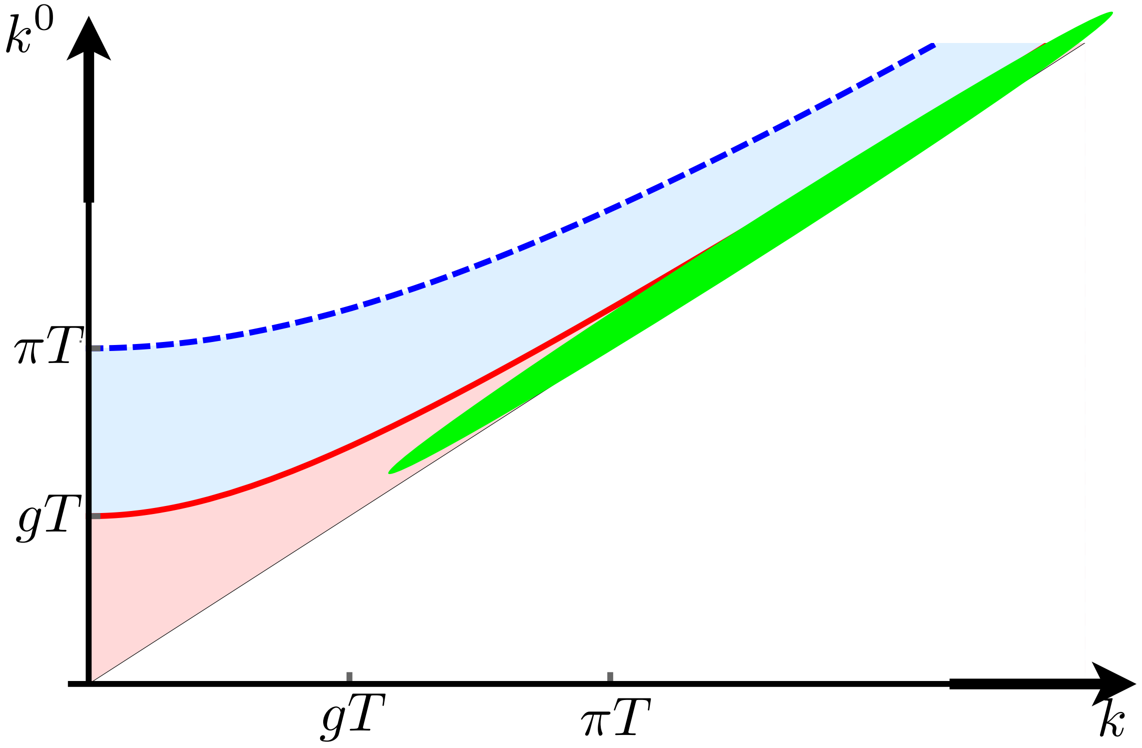
- Phenomenological motivation clear
- More theoretical motivation: lots of knowledge about perturbative thermodynamics to high orders, not so much about dynamical quantities. Is convergence better/worse?

Kinematics of e/m production

$$\frac{d\Gamma_{l^+l^-}(K)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2} n_{\rm B}(k^0)\, \rho_{\rm EM}(K)$$



NLO at small K^2



- Consider $k^0 + k \sim T$ $k^0 - k \sim g^2 T$

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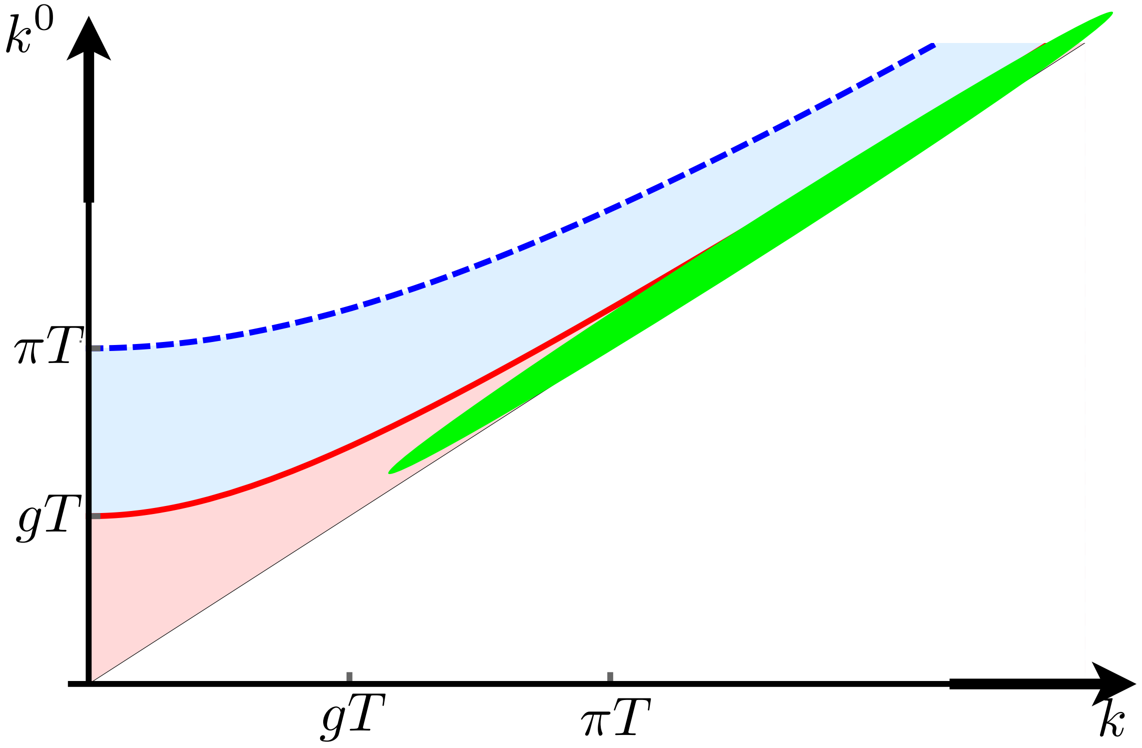
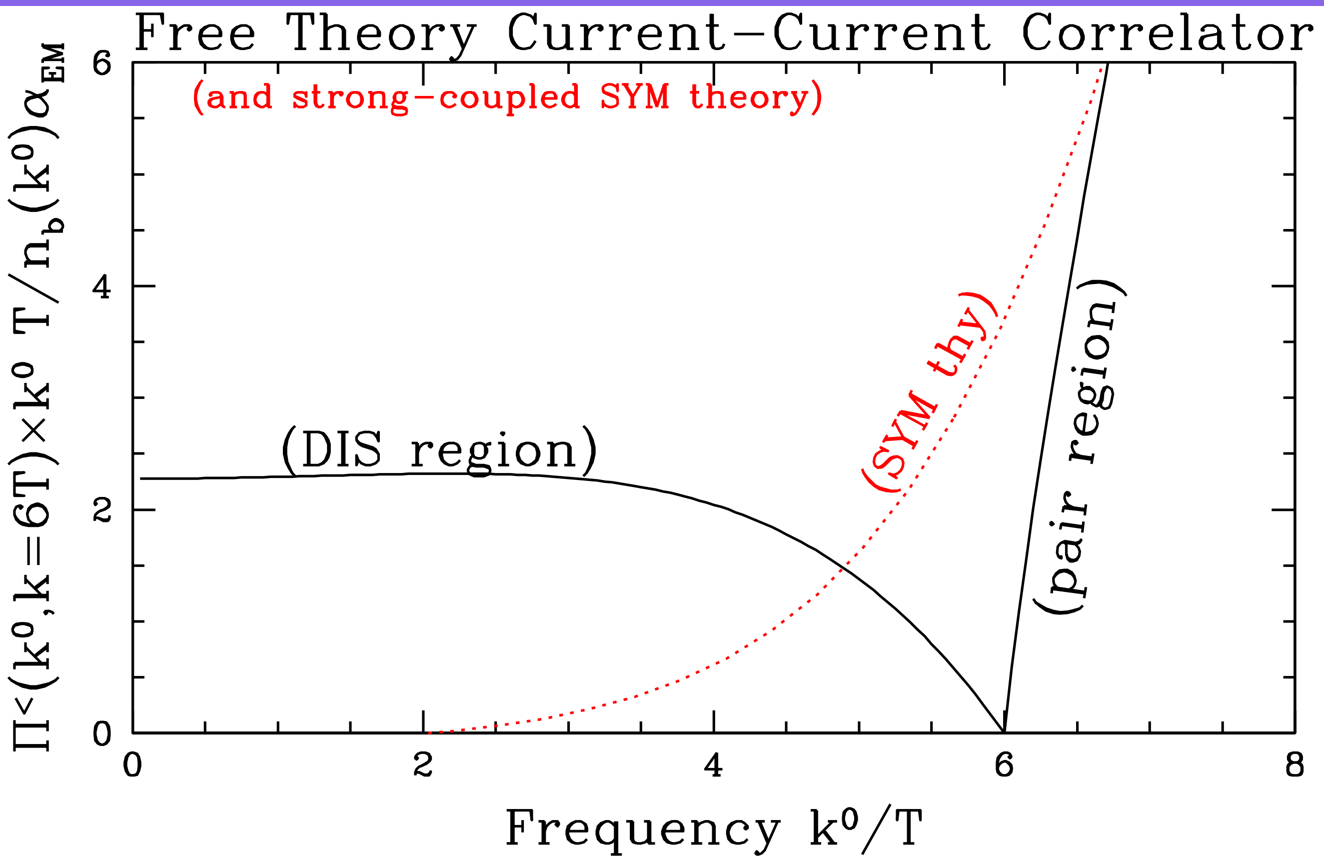
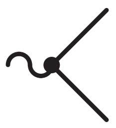




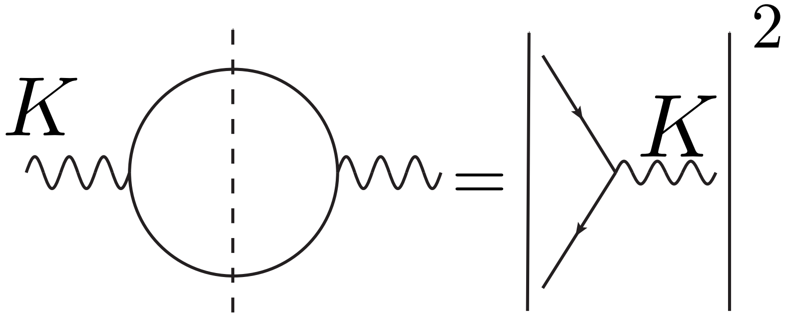
- Consider k^0+k~T, k^0-k~g^2T. Includes real photons
- A phenomenological motivation: low-mass dileptons as an ersatz real photon measurement (see for instance PHENIX). Is the spectral function smooth approaching the light cone?

Small K^2 dileptons

$$\frac{d\Gamma_{l^+l^-}(K)}{dk^0 d^3k} = -\frac{\alpha^2}{6\pi^3 K^2}\, n_{\rm B}(k^0)\, \rho_{\rm EM}(K) \qquad J^\mu = \sum_{q=uds} e_q \bar{q}\gamma^\mu q :$$

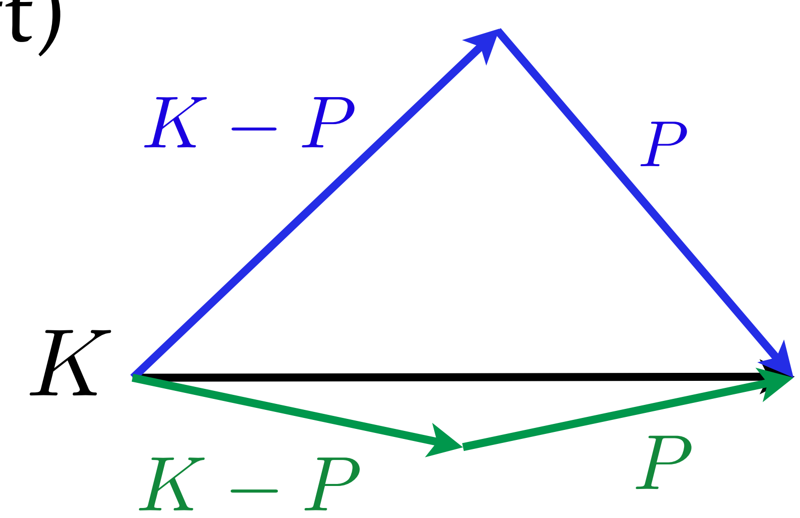


- At zeroth order ($\alpha_{\rm EM}\, g^0$):



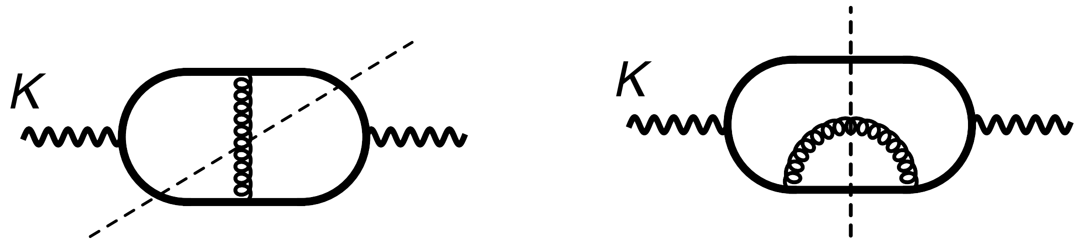
Apparently LO, but very small phase space, proportional to $K^2 \sim g^2T^2$. This is a collinear process.

- As in the real photon case, the calculation is split in the distinct 2↔2 processes (hard+soft) and collinear processes. Only collinear processes are modified wrt the photon case

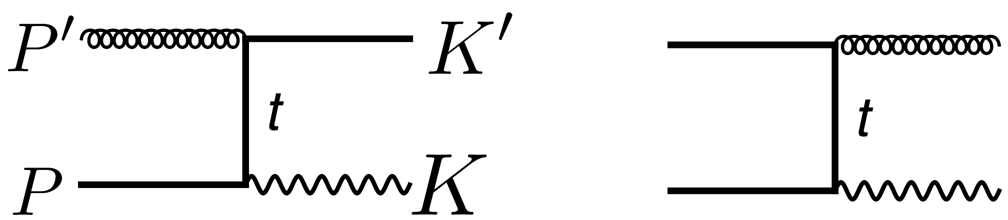


2↔2 processes

- Cut two-loop diagrams ($\alpha_{\mathrm{EM}}\, g^2$)



2↔2 processes (with crossings and interferences):

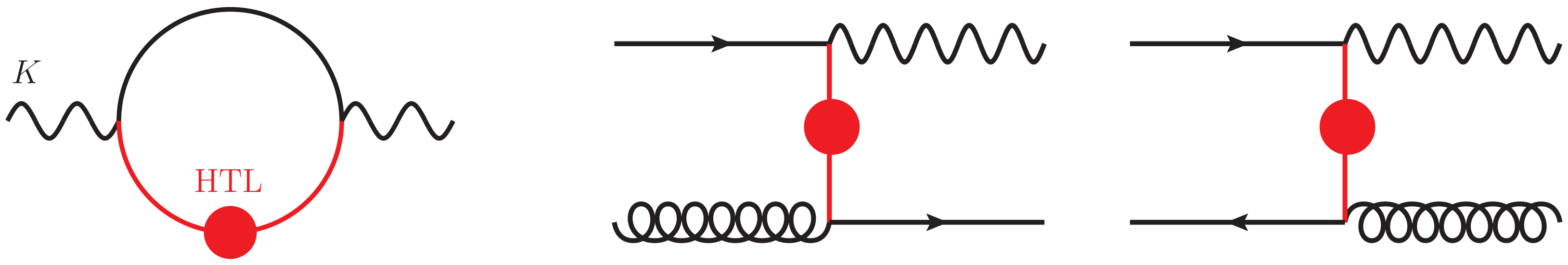


$$\int_{\mathrm{ph.\,space}} f(p) f(p')(1 \pm f(k'))|\mathcal{M}|^2 \delta^4(P + P' - K - K')$$

- 1↔3 processes suppressed by small K^2
- Equivalence with kinetic theory: distributions x matrix elements
- IR divergence (Compton) when t goes to zero

2↔2 processes

- The IR divergence disappears when **Hard Thermal Loop** resummation is performed Braaten Pisarski **NPB337** (1990)



2↔2 processes

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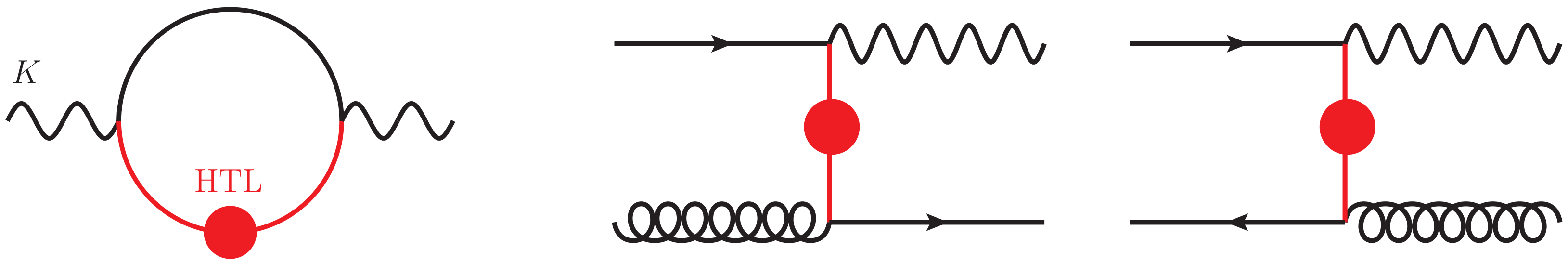
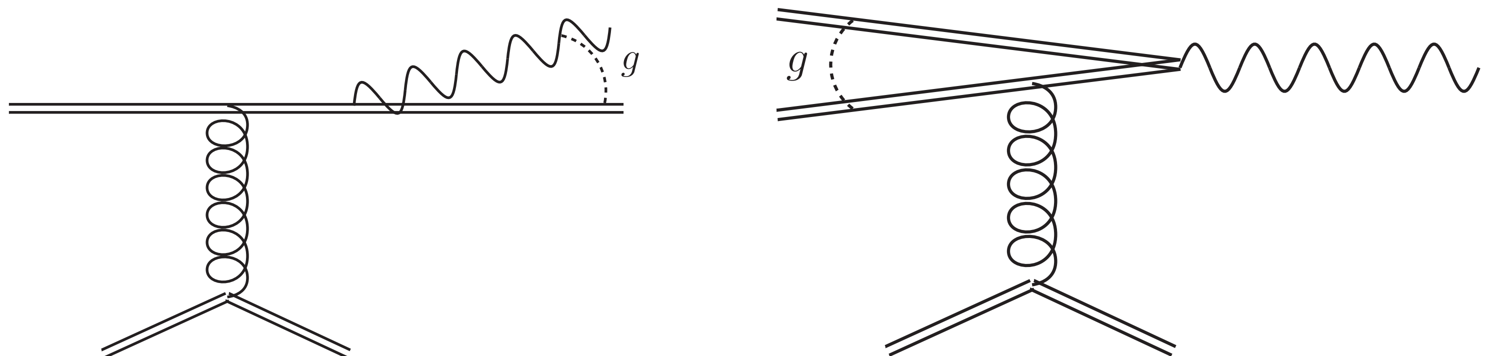


- In the end one obtains the result

$$\left.\frac{d\Gamma_{l^+l^-}(K)}{dk^0 d^3k}\right|_{2\leftrightarrow 2} \propto e^2 g^2 \left[\log\frac{T}{m_\infty} + C_{2\leftrightarrow 2}\left(\frac{k}{T}\right)\right]$$

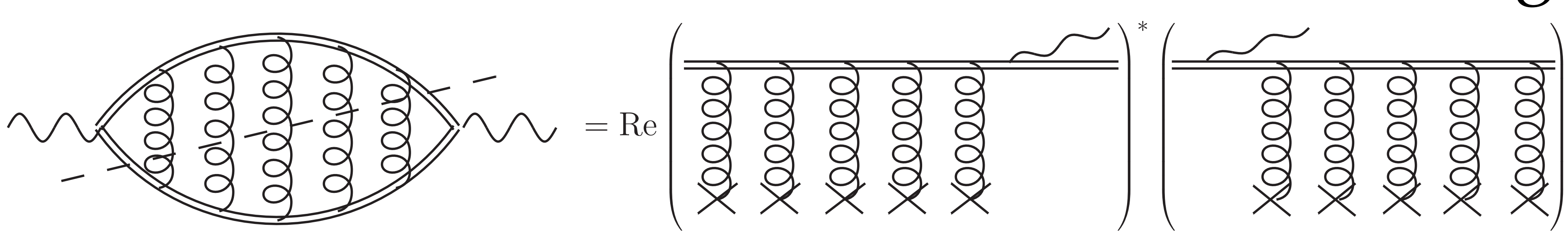
Kapusta Lichard Siebert **PRD44** (1991) Baier Nakkagawa Niegawa Redlich **ZPC53** (1992)

Collinear processes



- These diagrams contribute to LO if small (g) angle radiation/ annihilation Aurenche Gelis Kobes Petitgirard Zaraket 1998-2000
- Virtual photon formation times is then of the same order of the soft scattering rate ⇒ interference: *LPM effect*
- Requires resummation of infinite number of ladder diagrams

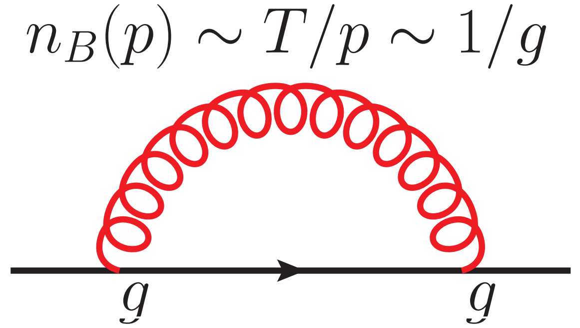
$$\left.\frac{d\Gamma_{l^+l^-}}{dk^0 d^3k}\right|_{\mathrm{coll}} =$$



AMY (2001-02), Aurenche Gelis Moore Zaraket (2002), Aurenche Carrington Gynther (2007)

Beyond leading order

- The soft scale gT introduces $O(g)$ corrections



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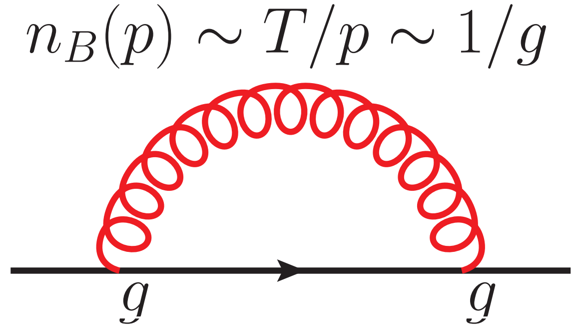
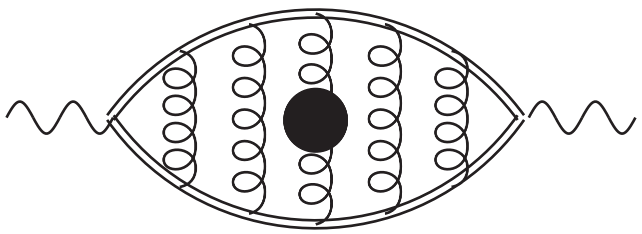


- In the collinear sector: account for 1-loop rungs (related to NLO qhat). Euclidean (EQCD) evaluation
  Caron-Huot **PRD79**



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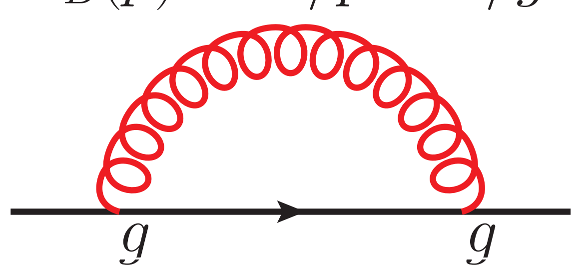

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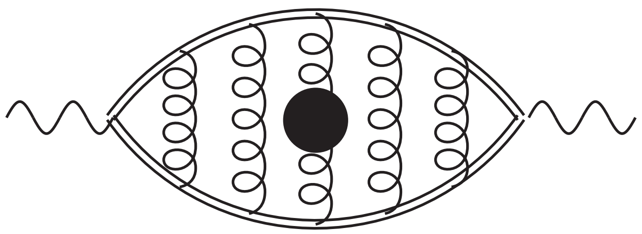
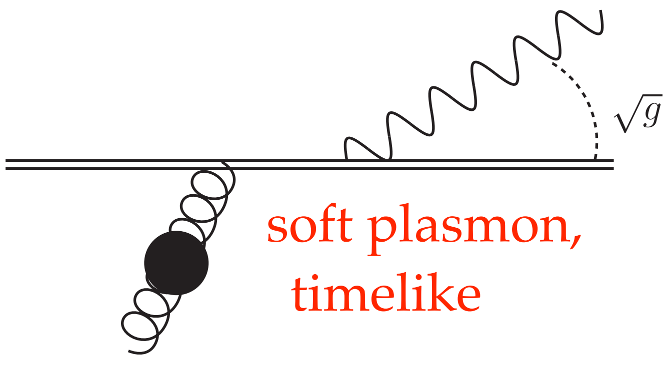
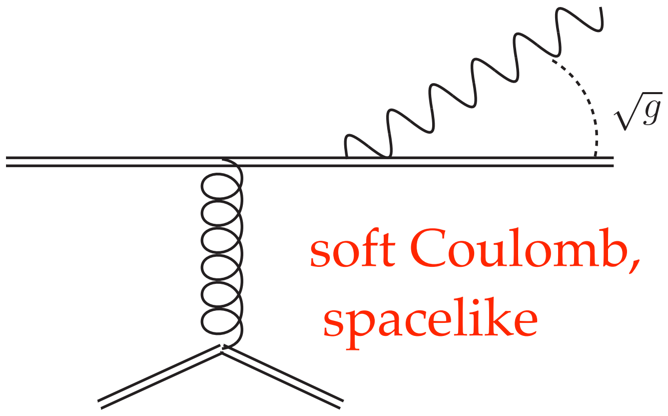


- New semi-collinear processes: larger angle radiation, NLO in collinear radiation approx. Requires a *"modified qhat"*, relevance for jets too





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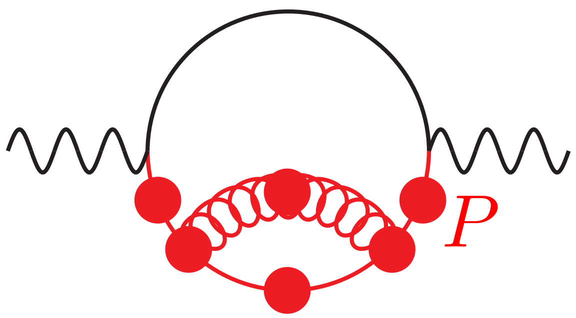

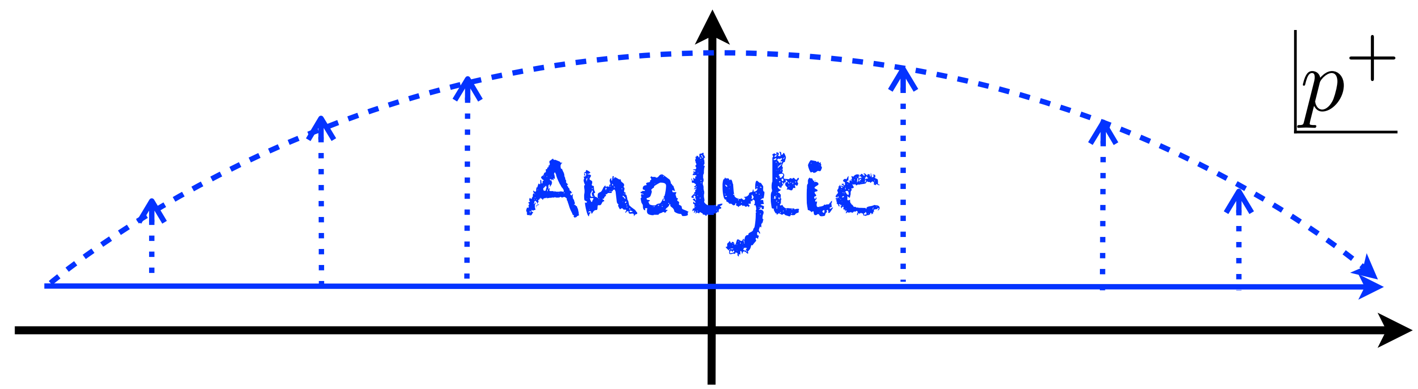

- Add soft gluons to soft quarks: nasty all-HTL region
- Analyticity allows us to take a detour in the complex plane away from the nasty region $\Rightarrow$ compact expression

$$\int \frac{d^2p_\perp}{(2\pi)^2}\frac{m_\infty^2}{p_\perp^2+m_\infty^2} \stackrel{\text{NLO}}{\rightarrow} \int \frac{d^2p_\perp}{(2\pi)^2}\frac{m_\infty^2+\delta m_\infty^2}{p_\perp^2+m_\infty^2+\delta m_\infty^2}$$

JG Hong Kurkela Lu Moore Teaney (2013) for photons
JG Moore (2014) for dileptons

NLO at large K^2

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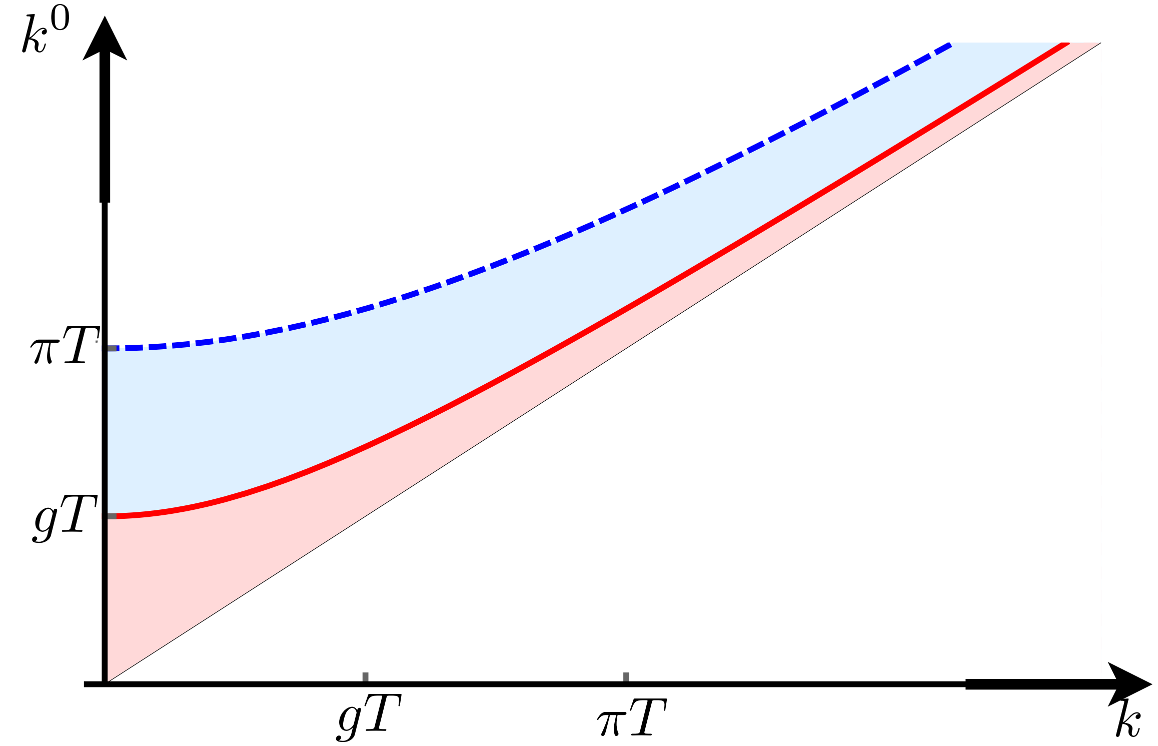
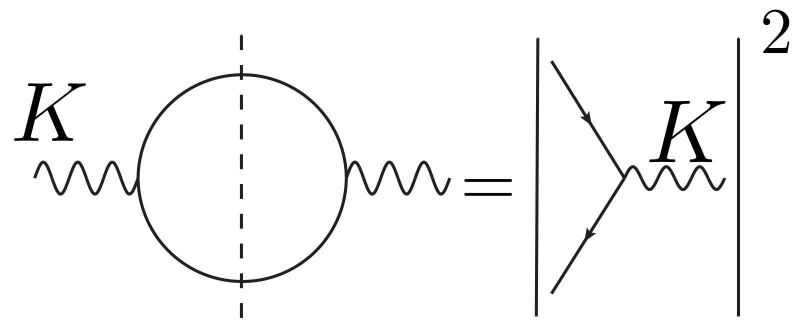
- Before showing any results, let us look at the large-*M* region



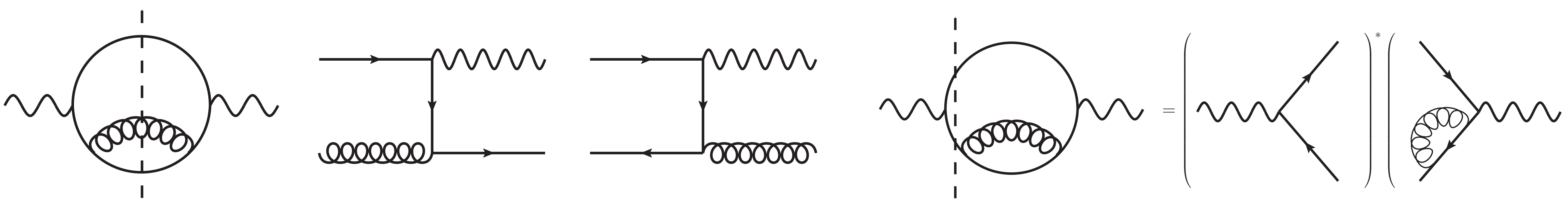
$$k^0 + k \sim T \qquad k^0 - k \sim T$$

- As we have seen, the Born term is proportional to K^2, which is now large ($\sim T^2$), so that the Born term is a well defined LO ($\alpha_{\mathrm{EM}} g^0$)



NLO at large K^2

- At NLO, HTL and LPM resummations are no longer necessary
- Very complicated two-loop integrals with intricate kinematics. Interplay of real and virtual corrections with cancellations of IR divergences



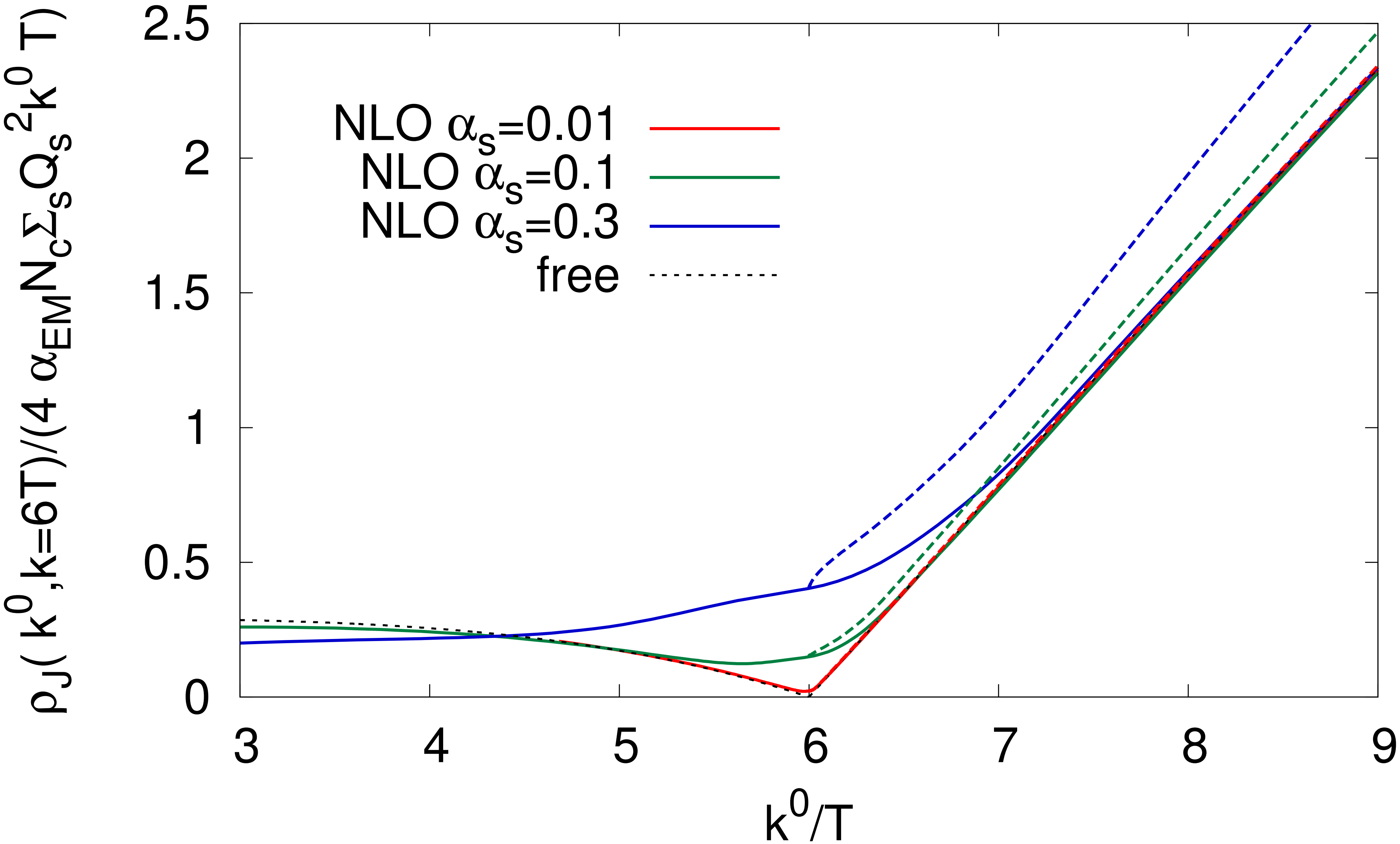
Laine **JHEP1305, JHEP1311** (2013)

Matching small and large K^2

- The large-M calculation diverges logarithmically for $M \to 0$
- The small-M calculation extrapolates for large M to $\rho \propto K^2 + T^2$, in violation of OPE results forbidding a T^2 term Caron-Huot **PRD79** (2009)
- A procedure has been devised to combine the two calculations. In a nutshell,

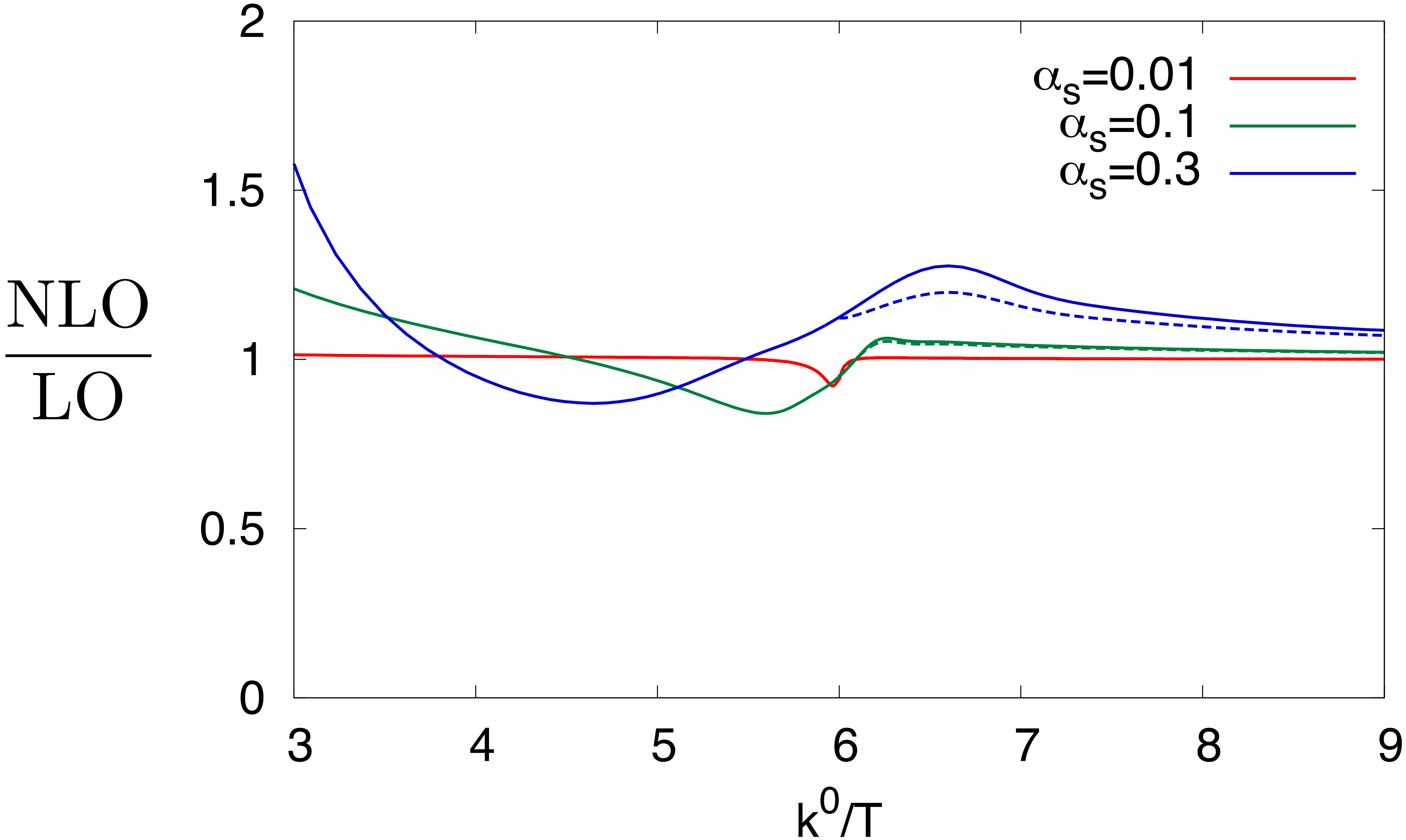
$$\rho_{\rm merge} = \rho_{{\rm large}\, M} + \rho_{\rm LPM} - \rho_{{\rm LPM}\, K^2 \gg T^2}$$

where $\rho_{\rm LPM}$ is the LO collinear part. NLO can be added easily.
Ghisoiu Laine **JHEP1410** (2014), JG Moore **JHEP1412** (2014)



- Full lines: JG Moore, valid at small K^2, does not include Laine (large M)
  Dashed lines: Ghisoiu Laine, valid at large K^2

- At α_s=0.3 the transition at the light cone is smooth
  Ghisoiu Laine **JHEP1410** (2014), JG Moore **JHEP1412** (2014)



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Come visit our website

- http://www.laine.itp.unibe.ch/dilepton-lpm/ Ghisoiu Laine and JG Moore results on a k^0,k mesh, ready for pheno. Used by the McGill group and by Burnier Gastaldi **PRC93** (2015)

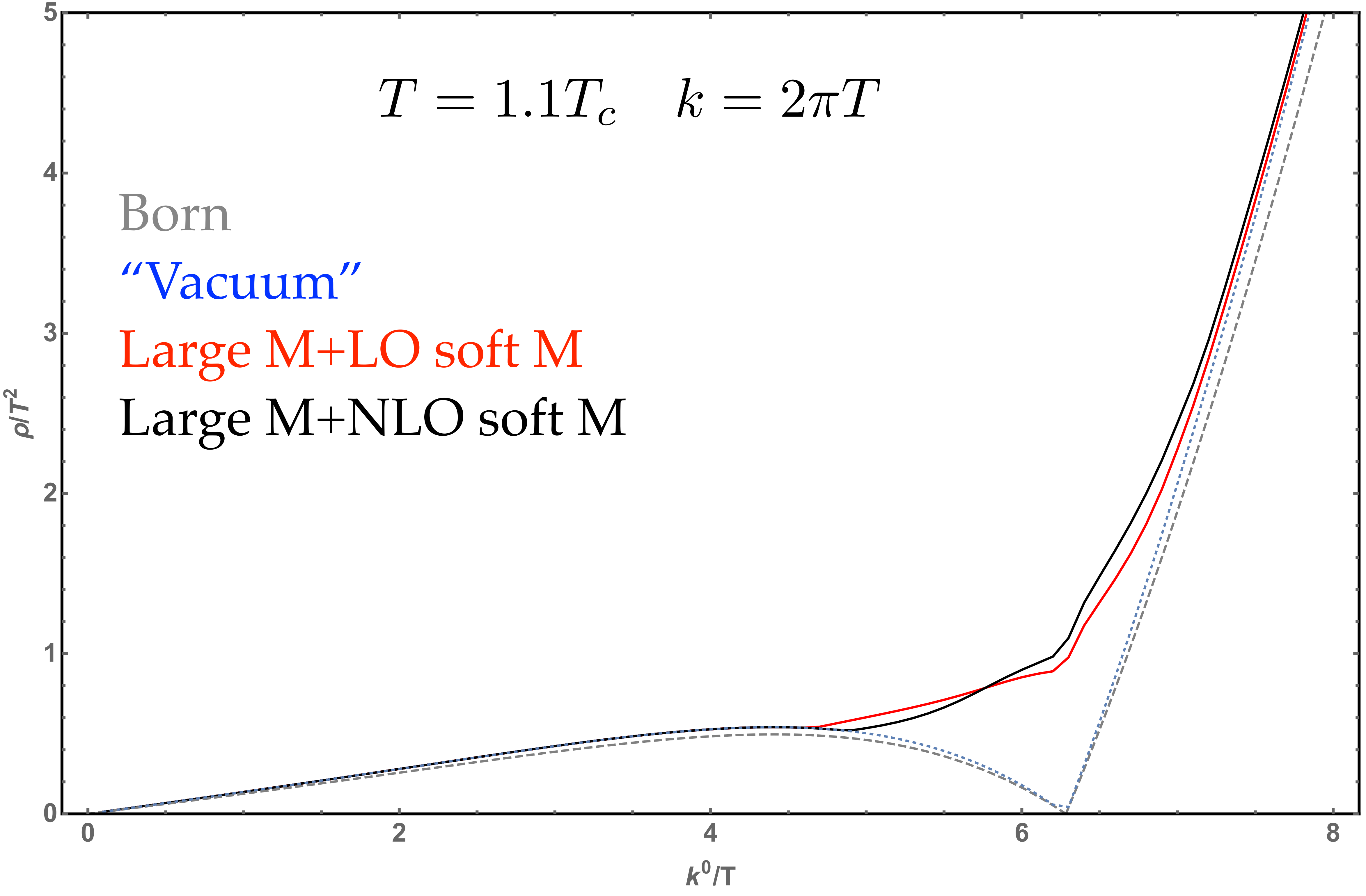
- http://www.laine.itp.unibe.ch/dilepton-lattice/ best available pQCD data for the spectral function

  - at finite k: Ghisoiu Laine plus JG Moore plus vacuum corrections to the Born term

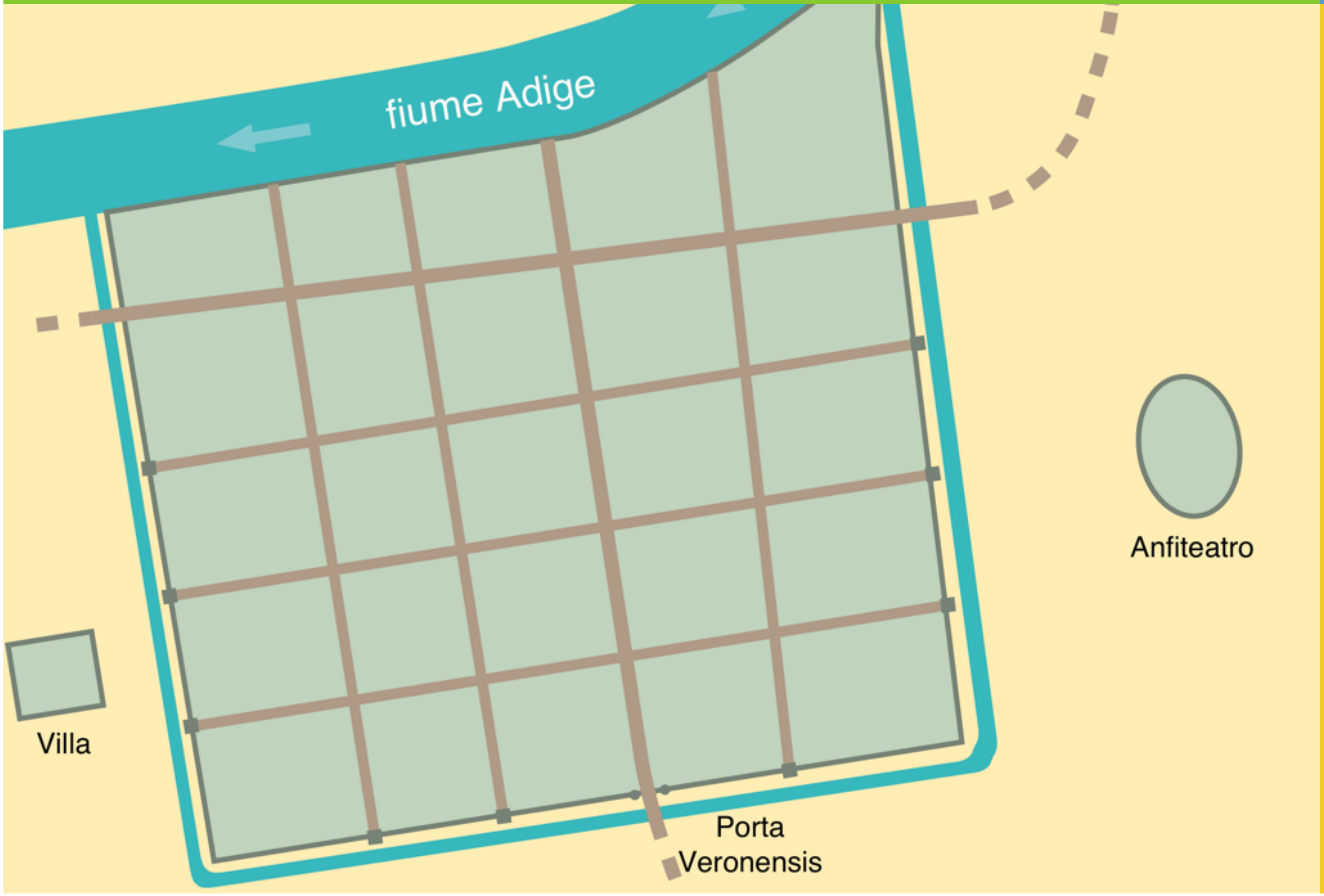
  - at zero k: transport peak from Moore Robert (2006), $k^0>\pi T$, NLO thermal from Aurenche Altherr (1989), vacuum corrections to the Born term. Missing reliable pQCD input in the intermediate region

JG Laine

Come visit our website



A lattice determination



EM probes and the lattice

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- What is measured directly is the Euclidean correlator

$$G_E(\tau, k) = \int d^3x J_\mu(\tau, \mathbf{x}) J_\mu(0,0) e^{i\mathbf{k}\cdot\mathbf{x}}$$

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- Analytical continuation $G_E(\tau, k) = G^<(i\tau, k)$

$$G_E(\tau, k) = \int_0^\infty \frac{dk^0}{2\pi} \rho_V(k^0, k) \frac{\cosh\left(k^0(\tau - 1/2T)\right)}{\sinh\left(\frac{k^0}{2T}\right)}$$

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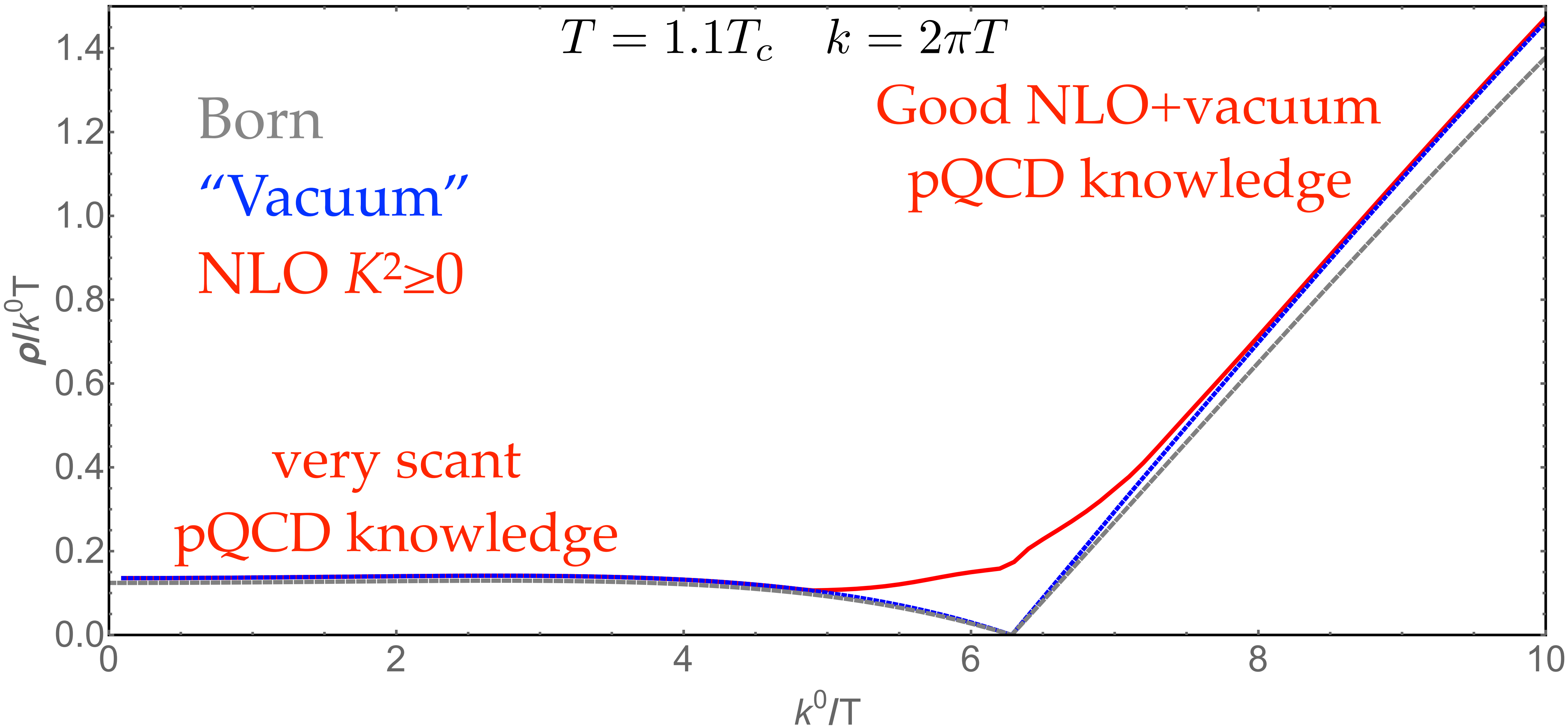
- It contains a lot of info (full spectral function), but hidden in the convolution. Inversion tricky, discrete dataset with errors

At finite momentum

- If k>0 spf describes DIS ($k^0<k$), photons ($k^0=k$) and dileptons ($k^0>k$).



www.laine.itp.unibe.ch/dilepton-lattice/

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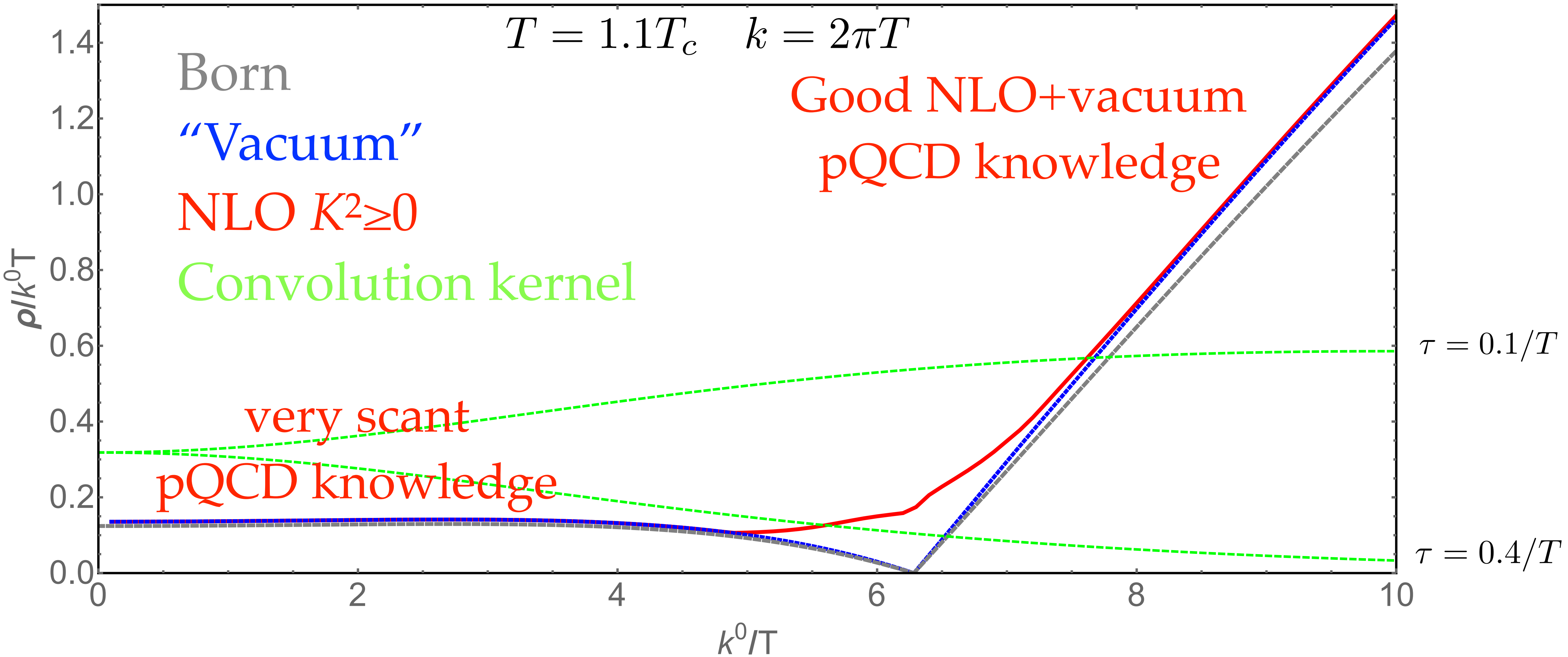


www.laine.itp.unibe.ch/dilepton-lattice/

Fitting to the lattice

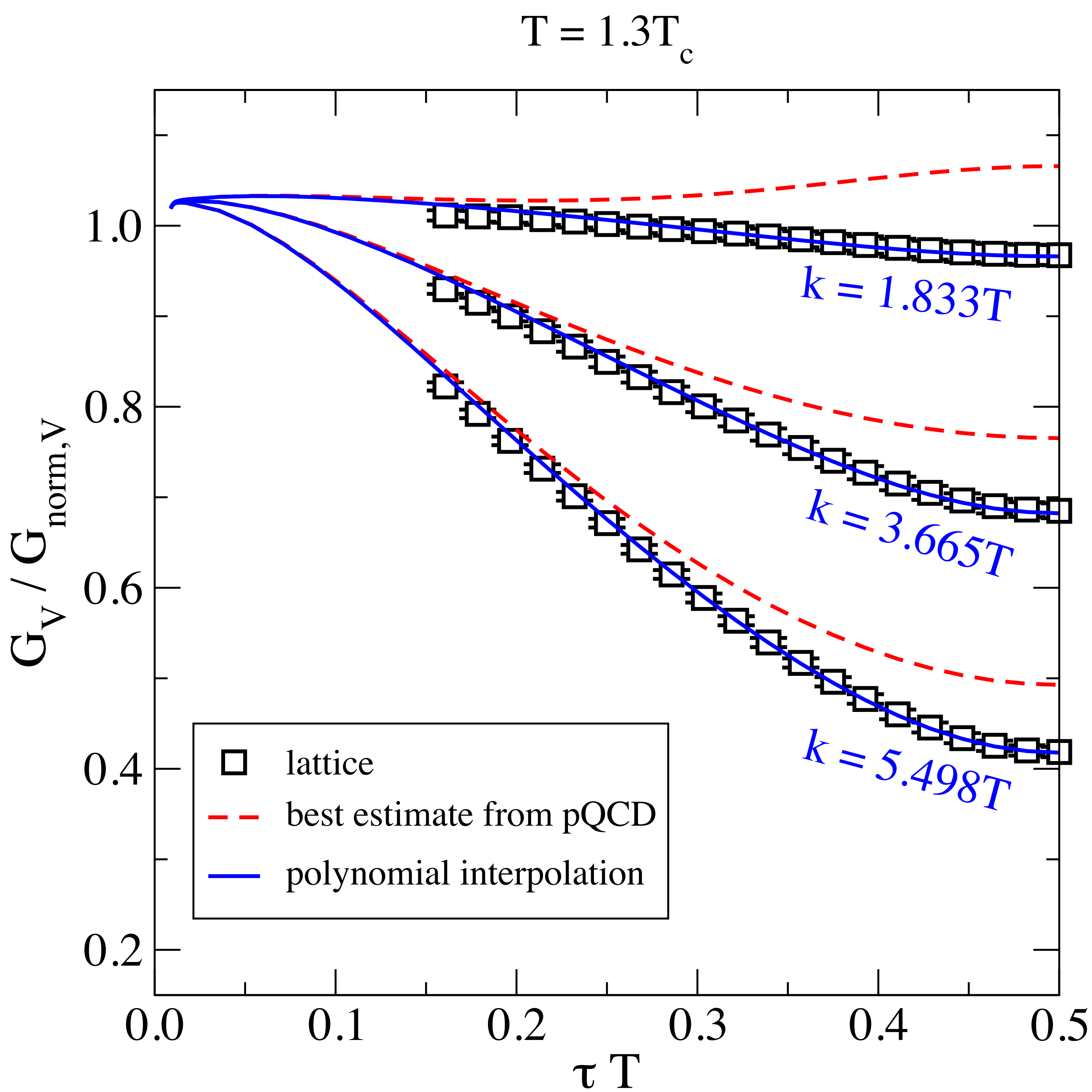
- Getting the Euclidean correlator from the pQCD results is straightforward. It overshoots the lattice data (hold on). Too much spf in the ill-constrained spacelike region?

- Try a fitting Ansatz: pQCD thermal spf above $M \sim \pi T$. 5th degree odd polynomial in ω below $M \sim \pi T$ (3 *coefficients*):

$$\rho_{\rm fit} \equiv \frac{\beta\,\omega^3}{2\omega_0^3}\left(5-\frac{3\omega^2}{\omega_0^2}\right)-\frac{\gamma\,\omega^3}{2\omega_0^2}\left(1-\frac{\omega^2}{\omega_0^2}\right)+\sum_{n\geq 0}^{n_{\rm max}}\frac{\delta_n\omega^{1+2n}}{\omega_0^{1+2n}}\left(1-\frac{\omega^2}{\omega_0^2}\right)^2$$

- Fix two coefficients by requiring smoothness in spf and first derivative at the matching point ω_0. Fit the remaining coefficient to lattice data. Higher order odd polynomials also examined
  JG Kaczmarek Laine Meyer **PRD94** (2016)

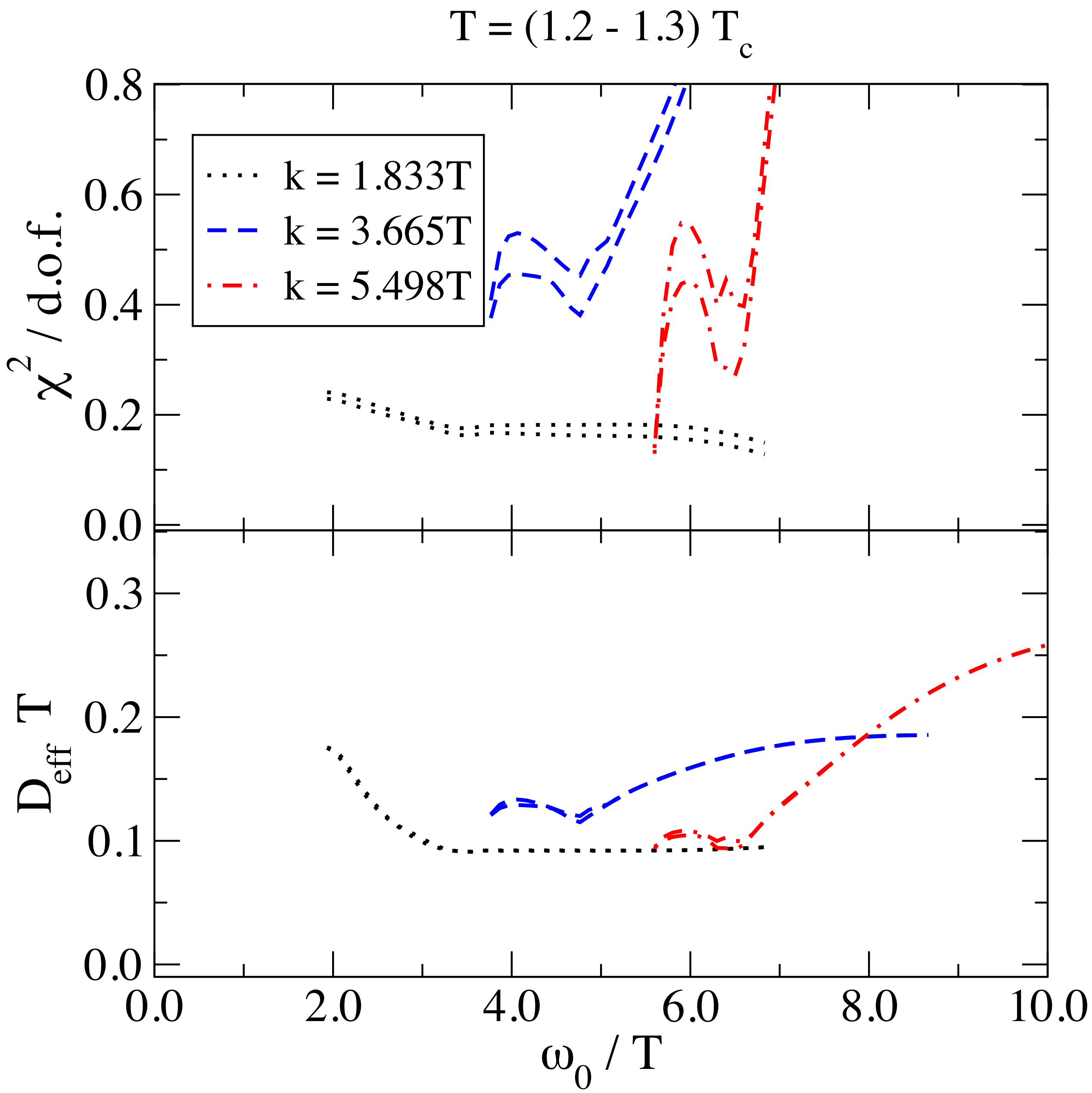
Fitting to the lattice



- Local discretization of J, with non-perturbatively clover-improved Wilson fermions
- Results qualitatively similar at T=1.1T_c
- Lattice continuum extrapolation reliable only from τT>0.22
- Matching point at k^0=k+1.5T

JG Kaczmarek Laine Meyer **PRD94** (2016)

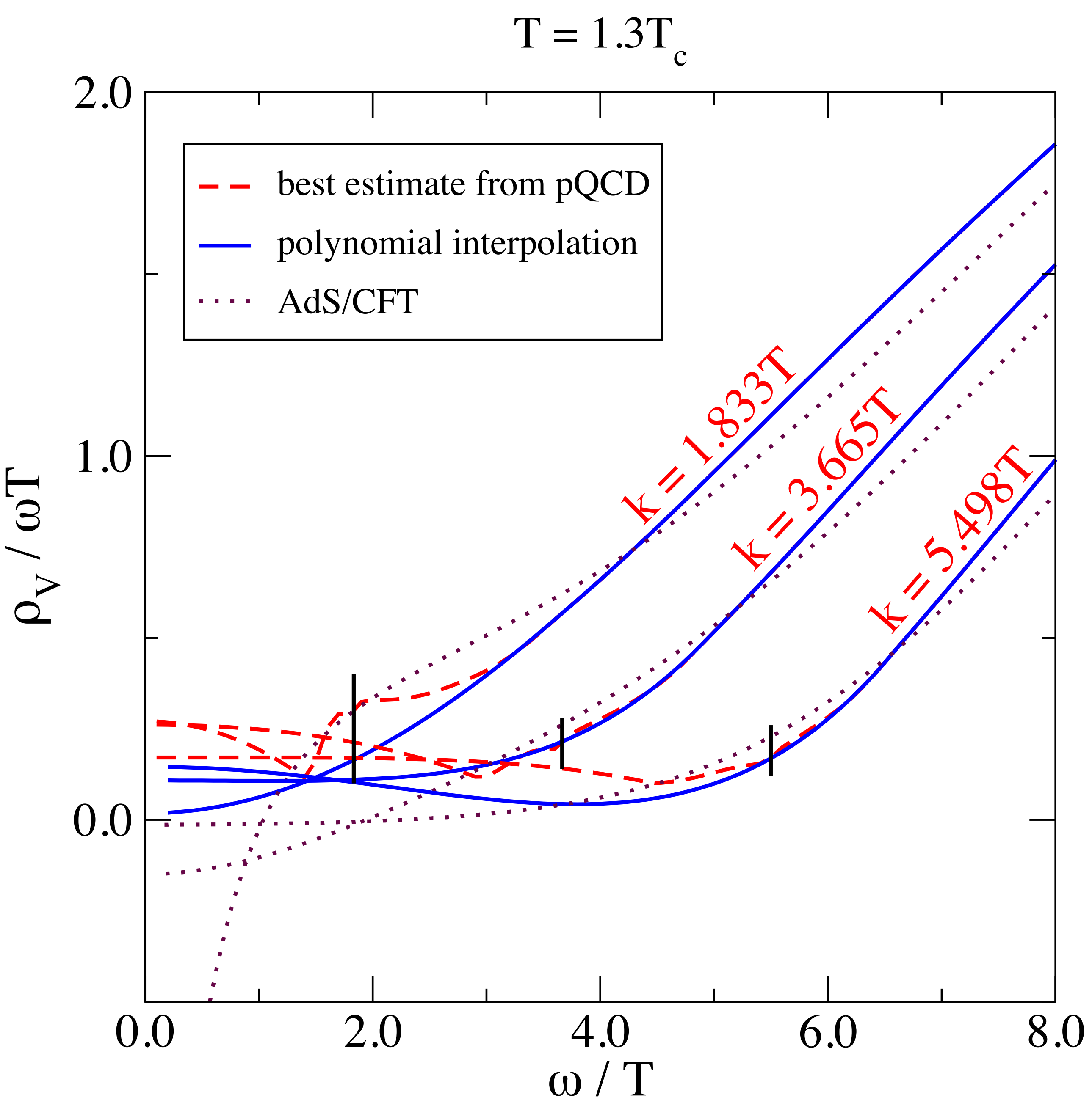
Fitting to the lattice



- Results qualitatively similar at T=1.1T_c
- The fit has a good χ^2, which also has a local minimum for $M \sim \pi T$ and the spf at the photon point is stable against varying the matching point
- D_{eff} proportional to spf at photon point (hold on), quite stable too

JG Kaczmarek Laine Meyer **PRD94** (2016)

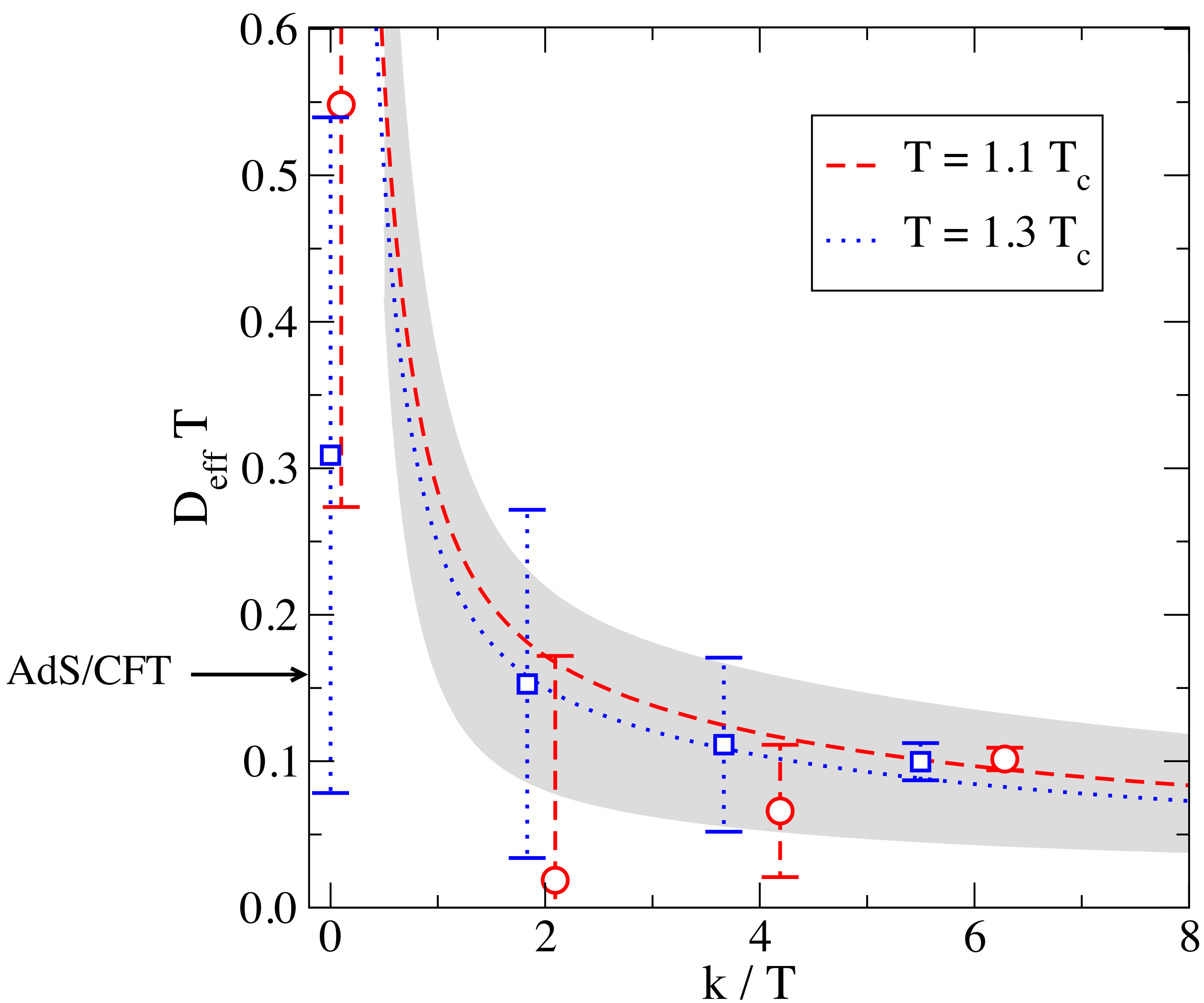
Fitting to the lattice



- At the photon point modest changes from pQCD expectations (below 20% except perhaps at the smallest *k*s, also at 1.1 T_c). Good for pheno!
- AdS/CFT curve adjusted to asymptote to the bare QCD result (extra symmetries make T=0 curve coupling-independent)

JG Kaczmarek Laine Meyer **PRD94** (2016). AdS/CFT: Caron-Huot Kovtun Moore Starinets Yaffe **JHEP0612** (2006)

- Define $D_{\text{eff}}(k) \equiv \begin{cases} \dfrac{\rho_V(k,\mathbf{k})}{2\chi_q k} & , \quad k>0 \\ \lim\limits_{\omega\to 0^+} \dfrac{\rho^{ii}(\omega,\mathbf{0})}{3\chi_q \omega} & , \quad k=0 \end{cases}$



- In the hydro limit $k \ll T$ $D_{\text{eff}} \to D$

$$\sigma = e^2 \sum_{f=1}^{N_f} Q_f^2 \chi_q D$$

- Lattice errors from bootstrap samples
- At large momentum excellent agreement with NLO pQCD from before. At finite $k>0$ this method could be a more controlled approach to the extraction of σ, w/o the large uncertainties associated with the transport peak at k=0.
- Try this for shear?

JG Kaczmarek Laine Meyer **PRD94** (2016). NLO pQCD: JG Hong Kurkela Lu Moore Teaney **JHEP05** (2013)

Recent developments

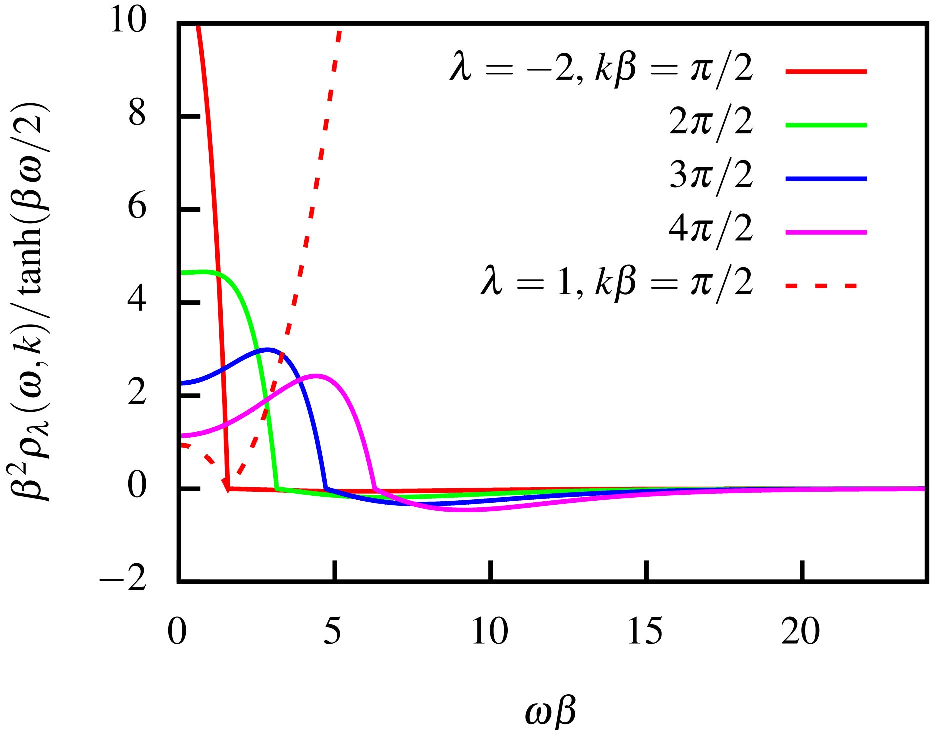
- Main idea: introduce this spf

$$\rho_{\rm Mainz}=2\rho_T-2\rho_L$$

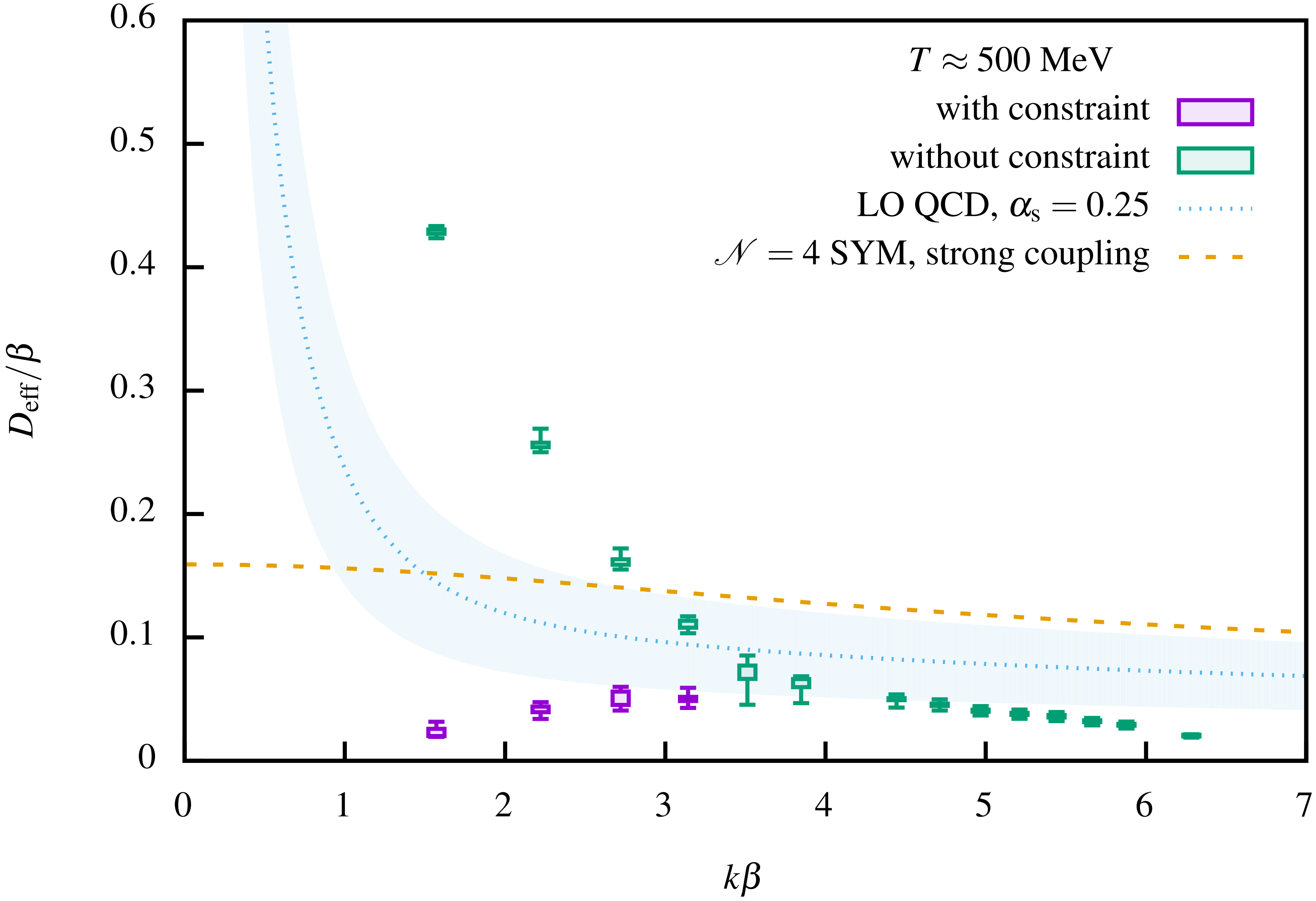
- Through the Ward identity it agrees with the standard $\rho^{\mu}{}_{\mu}$ at the photon point $\omega=k$

$$\rho_V=2\rho_T+\rho_L \qquad \rho_L(\omega=k)=0$$

- The vacuum contribution vanishes identically in this new spf (Lorentz invariance), much better control in the analytical continuation
  Brandt Francis Harris Meyer Steinberg
  1710.07050



Recent developments



- Backus-Gilbert inversion, N_f=2
  Brandt Francis Harris Meyer Steinberg **1710.07050**

Recent developments

- What is measured directly is the Euclidean correlator $G_E(\tau, k) = \int d^3x J_\mu(\tau, \mathbf{x}) J_\mu(0,0) e^{i\mathbf{k}\cdot\mathbf{x}}$
- Analytical continuation $G_E(\tau, k) = G^<(i\tau, k)$

$$G_E(\tau, k) = \int_0^\infty \frac{dk^0}{2\pi} \rho_V(k^0, k) \frac{\cosh\left(k^0(\tau - 1/2T)\right)}{\sinh\left(\frac{k^0}{2T}\right)}$$

- New ideas go to beyond this

$$G_E(\omega_n, i\omega_n) = \int_0^\beta d\tau \int d^3x J_\mu(\tau, \mathbf{x}) J_\mu(0,0) e^{i\omega_n \tau} e^{\omega_n z}$$

$$G_E(\omega_n, i\omega_n) - G_E(\omega_r, i\omega_r) = \int \frac{d\omega}{2\pi} \omega\, \rho_V(\omega, \omega) \left[\frac{1}{\omega^2 + \omega_n^2} - \frac{1}{\omega^2 + \omega_r^2}\right]$$

H. Meyer **EPJA54** (2018)

Conclusions

- NLO calculations for dileptons are now available over a wide range of invariant masses (at finite k)
- In both cases convergence seems reasonable. At small K^2 transition to photon is smooth
- A collection of the best available data has been prepared and is ready for use by pheno/lattice practitioners
- Comparison with lattice with a simple, motivated Ansatz gives a good fit and seems to further suggest stability (at the tens of $\%$ level) of the pQCD rates

Backup

LPM resummation

- Quark statistical functions × DGLAP splitting × transverse evolution

$$\frac{d\Gamma}{d^3k} = \frac{\alpha}{\pi^2 k}\int \frac{dp^+}{2\pi} n_{\rm F}(k+p^+)[1-n_{\rm F}(p^+)] \frac{(p^+)^2+(p^++k)^2}{2(p^+(p^++k))^2} \lim_{\mathbf{x}_\perp\to 0} 2\mathrm{Re}\boldsymbol{\nabla}_{\mathbf{x}_\perp}\mathbf{f}(x_\perp)$$

$$\begin{array}{c} x^+ \gg \quad x_\perp \quad \gg x^- \\ 1/g^2T \gg 1/gT \gg 1/T \end{array}$$

LPM resummation

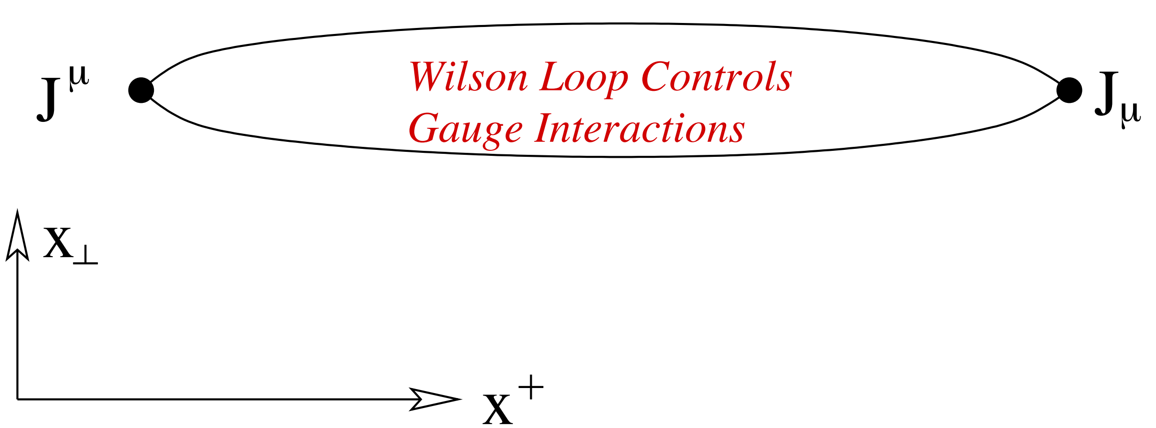
- Quark statistical functions × DGLAP splitting × transverse evolution

$$\frac{d\Gamma}{d^3k} = \frac{\alpha}{\pi^2 k}\int \frac{dp^+}{2\pi} n_F(k+p^+)[1-n_F(p^+)]\frac{(p^+)^2+(p^++k)^2}{2(p^+(p^++k))^2}\lim_{\mathbf{x}_\perp\to 0} 2\mathrm{Re}\boldsymbol{\nabla}_{\mathbf{x}_\perp}\mathbf{f}(x_\perp)$$

- Transverse diffusion and Wilson-loop correlators evolve the transverse density **f** *along the spacetime light-cone*

$$-2i\nabla\delta^2(\mathbf{x}_\perp) = \left[\frac{ik}{2p^+(k+p^+)}\left(m_\infty^2 - \nabla^2_{\mathbf{x}_\perp}\right) + \mathcal{C}(x_\perp)\right]\mathbf{f}(\mathbf{x}_\perp)$$

$$\begin{array}{ccccc} x^+ & \gg & x_\perp & \gg & x^- \\ 1/g^2T & \gg & 1/gT & \gg & 1/T \end{array}$$

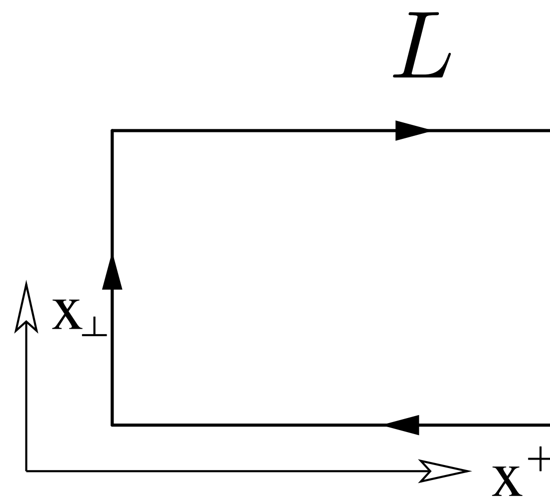


Zakharov 1996-98 AMY 2001-02

LPM resummation: two inputs

- Asymptotic mass $m_\infty^2 = 2g^2 C_R \left(\int \frac{d^3p}{(2\pi)^3} \frac{n_{\rm B}(p)}{p} + \int \frac{d^3p}{(2\pi)^3} \frac{n_{\rm F}(p)}{p} \right)$
- Light-cone Wilson loop, related to $\hat{q}$

$$\hat{q} \equiv \int_0^{q_{\rm max}} \frac{d^2 q_\perp}{(2\pi)^2} q_\perp^2 C(q_\perp)$$



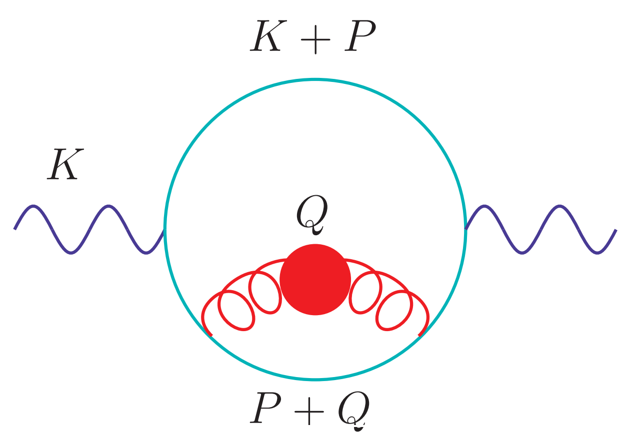
$$\propto e^{\mathcal{C}(x_\perp)L}$$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

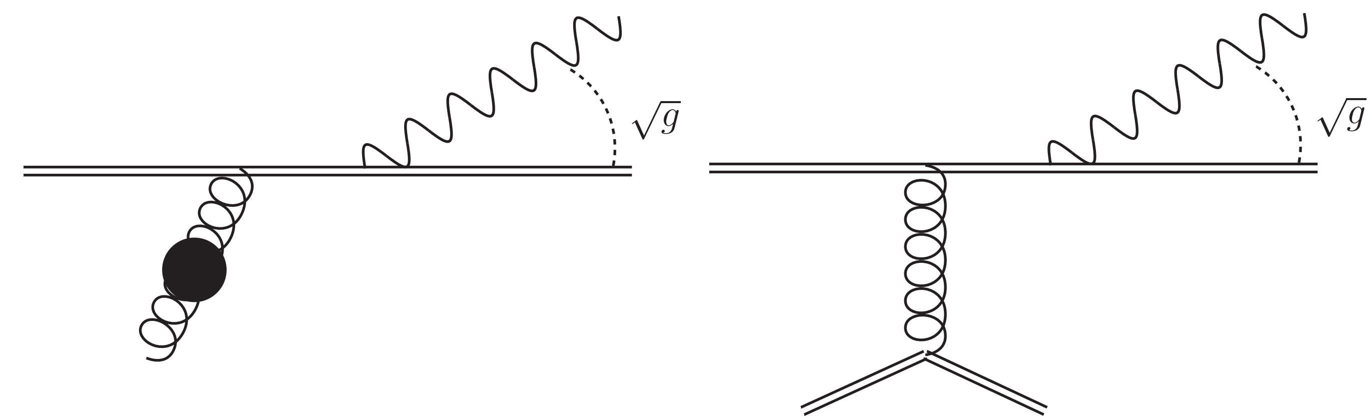
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008), can be "easily" computed in perturbation theory Possible lattice measurements Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **1307.5850** talk by Panero

The semi-collinear region

- Seemingly different processes boiling down to wider-angle radiation



P semi-collinear
Q soft



Q soft plasmon, timelike

Q soft cut, spacelike

- Evaluation: introduce "*modified* $\hat{q}$" that keep tracks of the changes in the small light-cone component p^- of the quarks

"standard" $$\frac{\hat{q}}{g^2C_R} \equiv \frac{1}{g^2C_R}\int\frac{d^2q_\perp}{(2\pi)^2}q_\perp^2\mathcal{C}(q_\perp) \propto \int d^4Q\langle F^{+\mu}(Q)F^{+}{}_{\mu}(-Q)\rangle_{q^-=0}$$

"modified" $$\frac{\hat{q}(\delta E)}{g^2C_R} \propto \int d^4Q\langle F^{+\mu}(Q)F^{+}{}_{\mu}(-Q)\rangle_{q^-=\delta E}$$

- The "*modified* $\hat{q}$" can also be evaluated in EQCD

Euclideanization of light-cone soft physics

For $v=x_z/t=\infty$ correlators (such as propagators) are the equal time Euclidean correlators.

$$G^{>}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n,p)\,e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Causality: retarded functions analytic for positive imaginary parts of all *timelike* and *lightlike* variables: the above result can be extended to the lightcone

$$G^{>}(t=x_z,\mathbf{x}_\perp) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n,p_\perp,p_z+i\omega_n)\,e^{i(\mathbf{p}_\perp\cdot\mathbf{x}_\perp+p_z x_z)}$$

- The sums are dominated by the zero mode for soft physics=>EQCD!

- Equivalent to sum rules Caron-Huot **PRD79** (2009)

Summary

- LO rate

$$(2\pi)^3 \frac{d\Gamma}{d^3k}\bigg|_{\rm LO} = \mathcal{A}(k) \overbrace{\left[\log\frac{T}{m_\infty} + C_{2\to 2}(k) + C_{\rm coll}(k)\right]}^{C_{\rm LO}(k)}$$

$$\mathcal{A}(k) = \alpha_{\rm EM}\, g^2\ C_F T^2 \frac{n_{\rm F}(k)}{2k} \sum_f Q_f^2 d_f$$

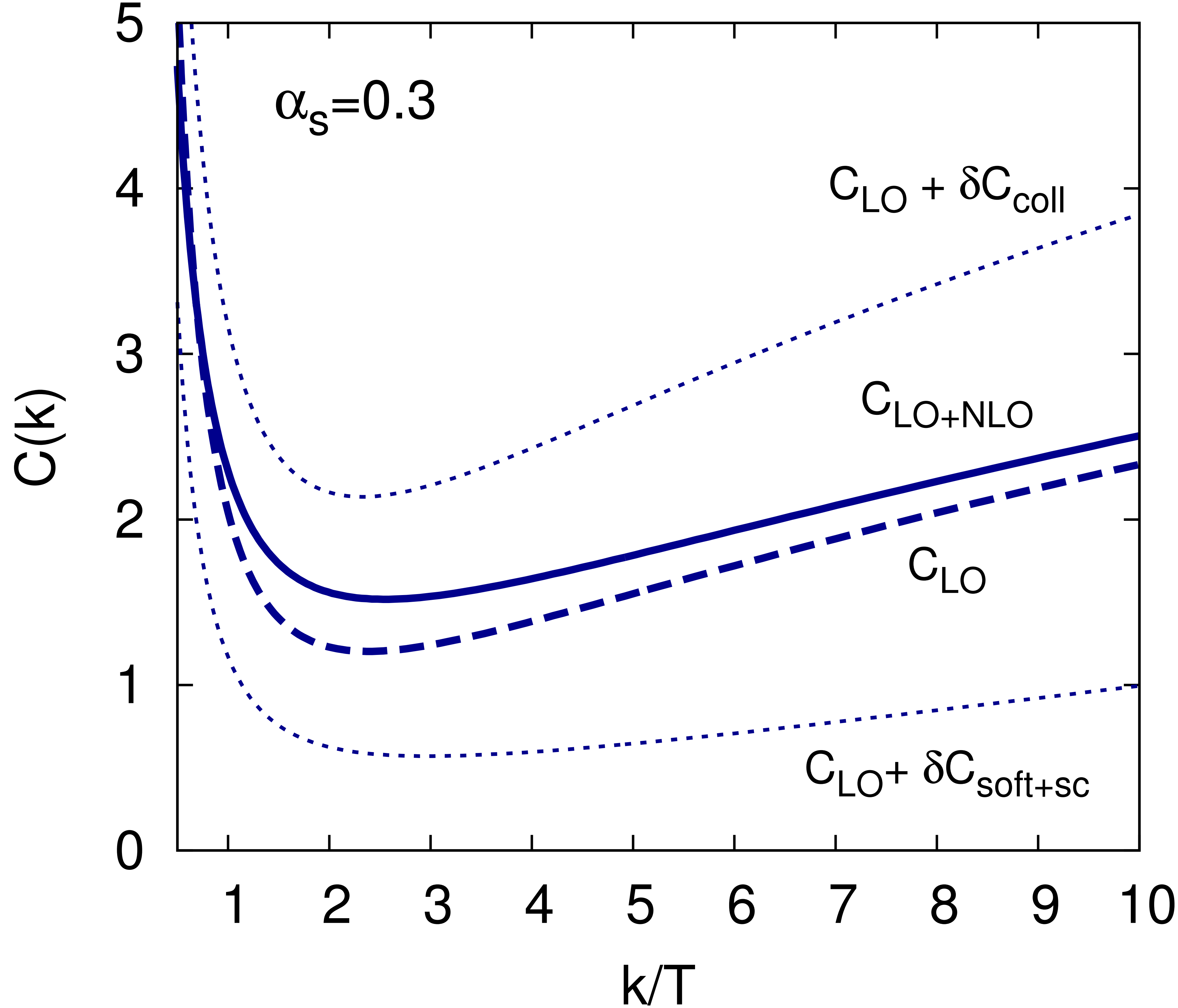
- NLO correction

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k) \overbrace{\left[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2} C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2} C_{\rm coll}^{\delta m}(k) + \frac{g^2 C_A T}{m_D} C_{\rm coll}^{\delta \mathcal{C}}(k)}_{\delta C_{\rm coll}(k)}\right]}^{\delta C_{\rm NLO}(k)}$$

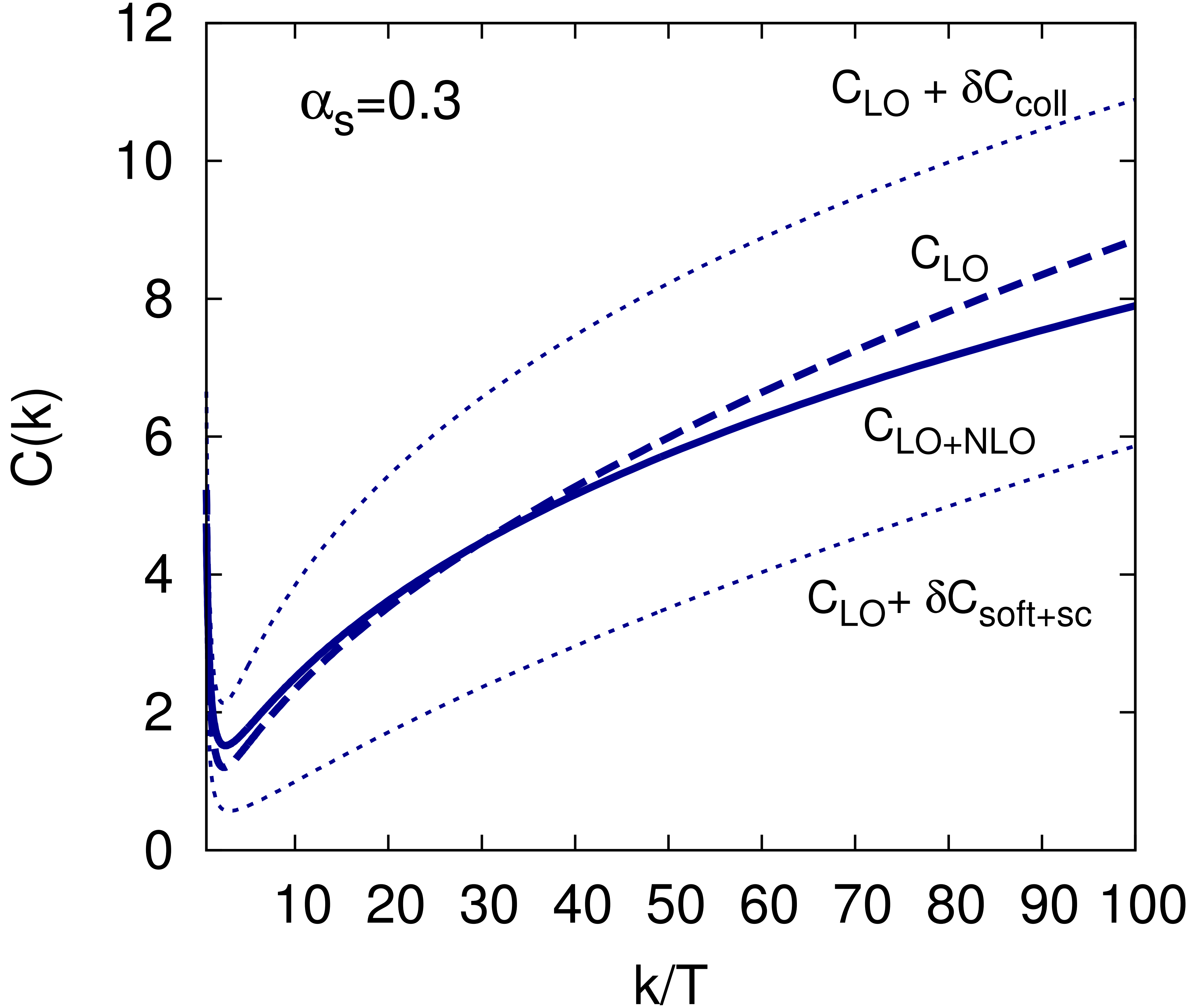
- Fits available in the paper

JG Hong Kurkela Lu Moore Teaney **JHEP0513 (2013)**

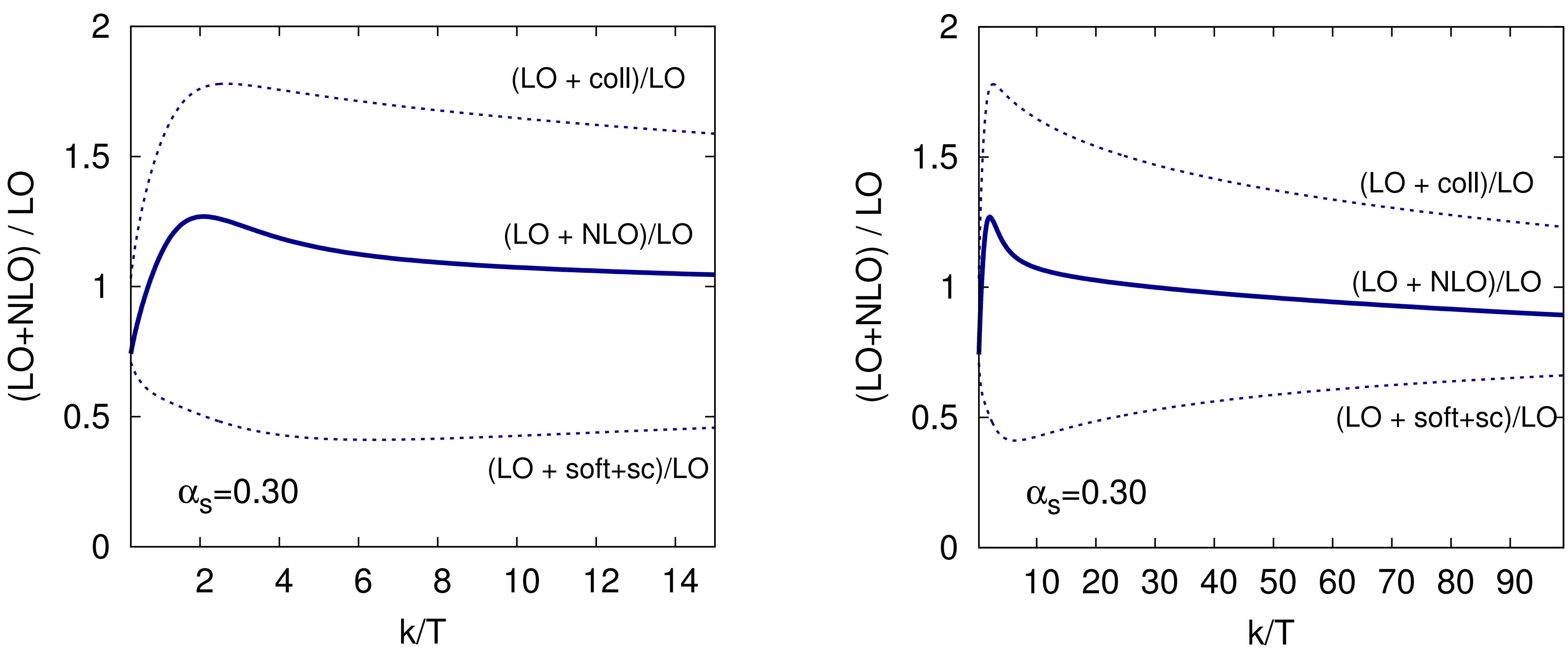
$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\Big|_{\rm NLO} = \mathcal{A}(k)\overbrace{\Bigg[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2} C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2} C_{\rm coll}^{\delta m}(k) + \frac{g^2 C_A T}{m_D} C_{\rm coll}^{\delta \mathcal{C}}(k)}_{\delta C_{\rm coll}(k)}\Bigg]}^{\delta C_{\rm NLO}(k)}$$

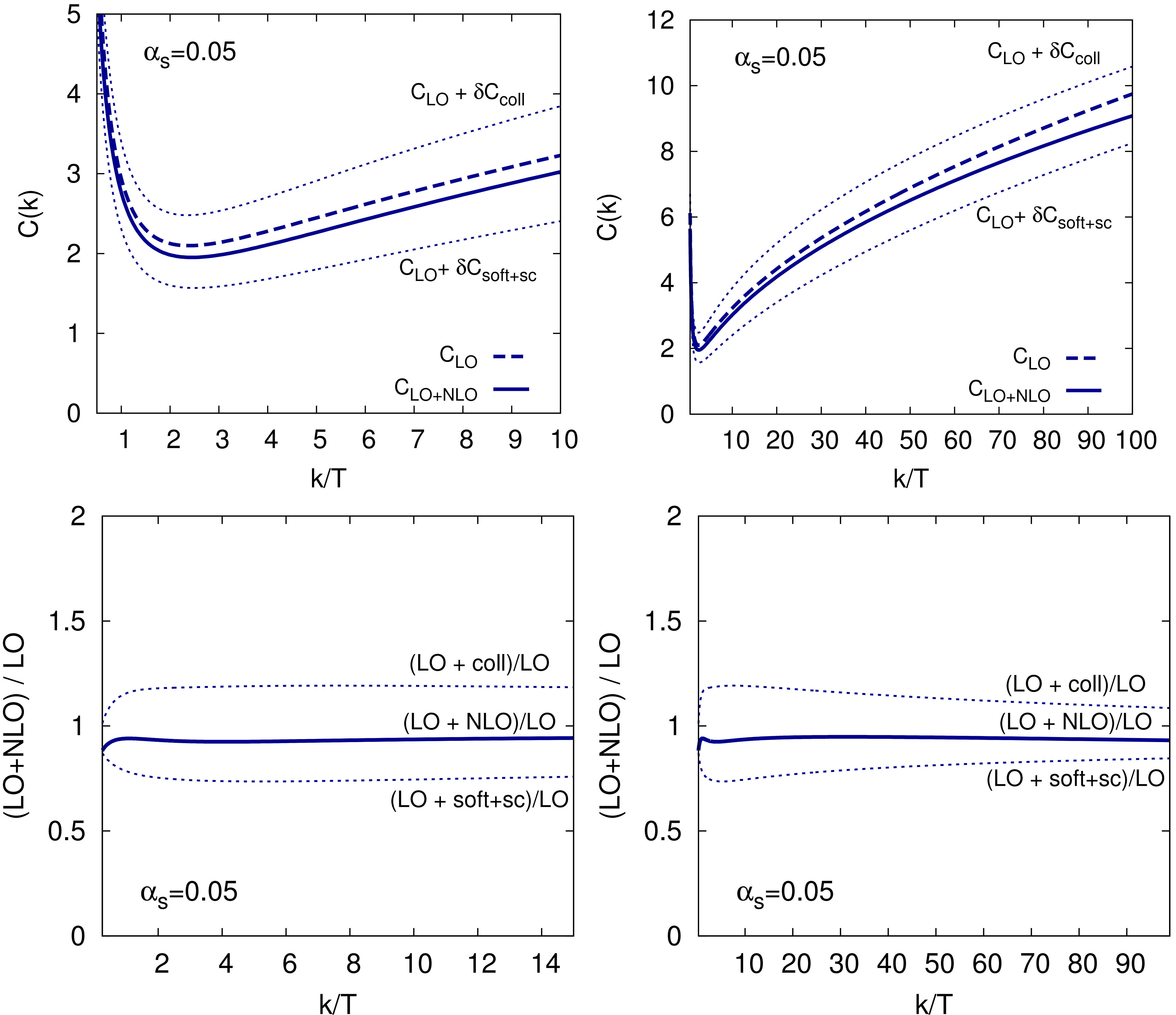


$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\mathrm{NLO}} = \mathcal{A}(k) \overbrace{\Bigg[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2} \log \frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2} C_{\mathrm{soft+sc}}(k)}_{\delta C_{\mathrm{soft+sc}}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2} C_{\mathrm{coll}}^{\delta m}(k) + \frac{g^2 C_A T}{m_D} C_{\mathrm{coll}}^{\delta \mathcal{C}}(k)}_{\delta C_{\mathrm{coll}}(k)} \Bigg]}^{\delta C_{\mathrm{NLO}}(k)}$$



$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k)\overbrace{\left[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2} C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2} C^{\delta m}_{\rm coll}(k) + \frac{g^2 C_A T}{m_D} C^{\delta\mathcal{C}}_{\rm coll}(k)}_{\delta C_{\rm coll}(k)}\right]}^{\delta C_{\rm NLO}(k)}$$





αs=0.05
C(k)
k/T
CLO + δCcoll
CLO+ δCsoft+sc
CLO
CLO+NLO
(LO+NLO) / LO
(LO + coll)/LO
(LO + NLO)/LO
(LO + soft+sc)/LO