Systematic study of Superdeformation in ^{36}Ar using mean field formalism

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ECT* Workshop

Evolution of Nuclear Shapes



independent particle motion
(single particle motion)
↓
weak gamma transition
spherical shape



correlated particle motion
(collective rotation, vibration)
↓
strong gamma transition
rugby football or pancake shape

Key points

- Shape evolution
- Collective motion
- $R_{4/2} = E(4_1^+)/E(2_1^+)$
- Transition to superdeformation

[P. Ring and P. Schuck, The Nuclear Many-Body Problem (Springer-Verlag, Berlin, 1980]

Independent Particle Approximation

Exact Many-Body Hamiltonian

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} V_{ij}, \tag{1}$$

where the first term represents the kinetic energy of the nucleons and the second term accounts for the nucleon–nucleon interaction.

Mean-Field Approximation

The complicated many-body interaction is replaced by an average potential, allowing each nucleon to move independently:

$$H_{\mathrm{MF}} = \sum_{i=1}^{A} \left[-\frac{\hbar^2}{2m} \nabla_i^2 + U(\mathbf{r}_i) \right]. \tag{2}$$

[A. Bohr and B. R. Mottelson, Nuclear Structure, vol. 1, (W. A. Benjamin, Inc., 1969).]

Cranked Mean-Field Approximation

Cranked Mean-Field Hamiltonian

$$H^{\omega} = H_{\mathrm{MF}} - \omega J_x, \tag{3}$$

where

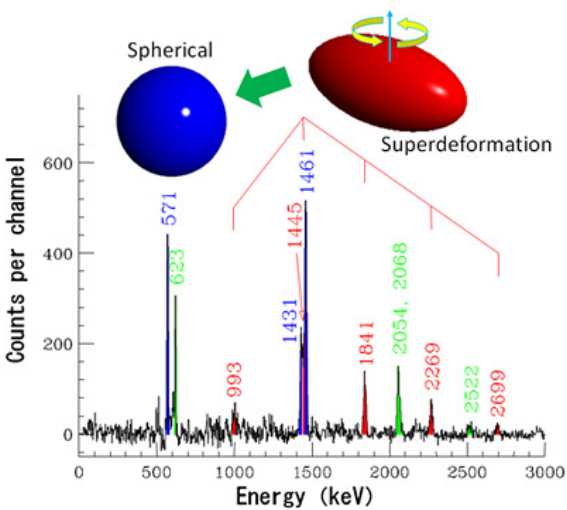
- H_{MF} : Mean-field Hamiltonian,
- ω : Rotational (cranking) frequency,
- J_x : Angular momentum about the cranking axis.

Why cranking?

- Describes nuclei in a rotating reference frame. Incorporates the Coriolis interaction through the term $-\omega J_x$. Enables a microscopic description of high-spin rotational bands.
- Provides equilibrium nuclear shapes and rotational observables through the Total Routhian Surface (TRS) calculations.

[D. R. Inglis, Phys. Rev. 96, 1059 (1954), Phys. Rev. 103, 1786 (1956).]

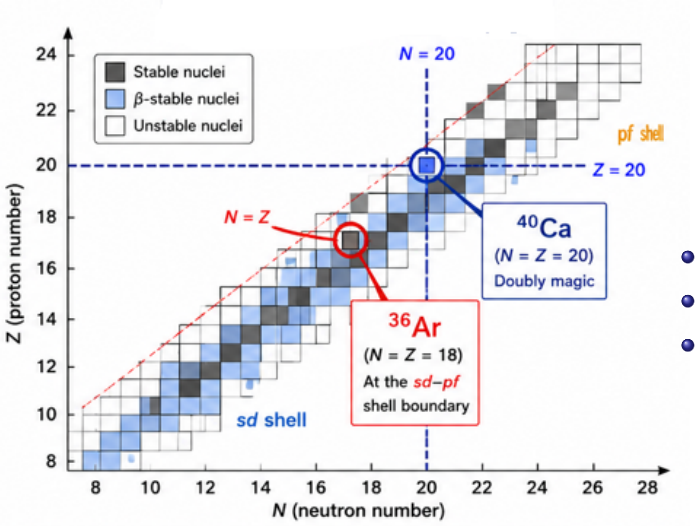
What is Superdeformation?



[P.J. Twin, *et al.*, Phys. Rev. Lett. 57, 811 (1986).]

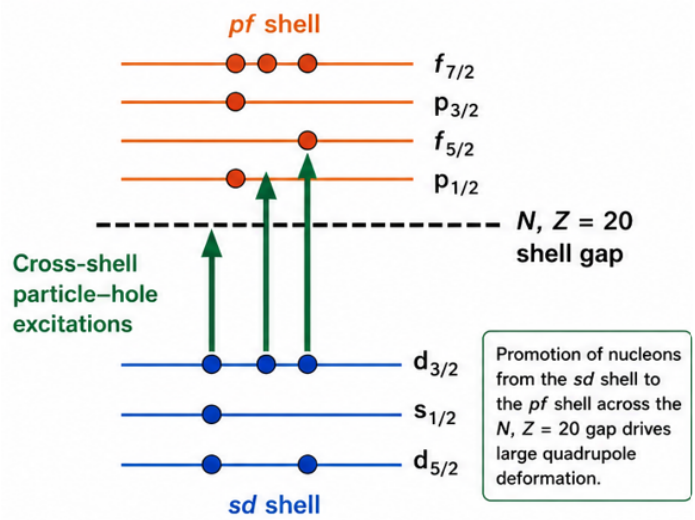
- Large quadrupole deformation. Axis ratio $\sim 2:1$. Rotational bands. Enhanced collectivity

Why Light Mass Region?



- Rapid shell evolution
- sd-pf interaction
- Benchmark for microscopic theories

Why ^{36}Ar?

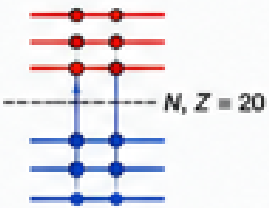


[C. E. Svensson, et al., Phys. Rev. Lett (2000), C. E. Svensson, et al., Phys. Rev. C, 63(6):

Scientific Question

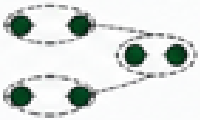
Microscopic ingredients that drive superdeformation

Shell Structure



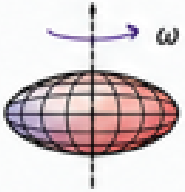
Deformed shell gaps from cross-shell particle–hole excitations

Pairing Correlations



Pairing weakens with rotation; essential for the stability of the SD minimum

Collective Rotation



Sustained rotation leads to the superdeformed rotational band

Shell effect: Formation of superdeformation

The Cranked Woods-Saxon mean-field formalism, based on the Stockholm potential, is employed. [W. Nazarewicz, et al., Nucl. Phys. A (1989).]

$$V_{\mathrm{WS}}(r) = -\frac{V_0}{1 + \exp\left(\frac{r-R}{a}\right)}, \tag{4}$$

Parameter	Symbol	Value
Central potential depth (MeV)	V_0	53.186
Central potential isospin strength	κ	0.791
Spin-orbit isospin strength	κ_{so}	0.162
Spin-orbit potential strength	λ	29.061
Central-potential radius (fm)	r_0	1.136
Spin-orbit radius (fm)	$r_{0,\mathrm{so}}$	0.816
Central-potential diffuseness (fm)	a_0	0.605
Spin-orbit diffuseness (fm)	$a_{0,\mathrm{so}}$	1.000

Shell effect : Formation of superdeformation

Why Shell effects matter ?

- Superdeformation is a shell phenomenon.
- When a nucleus deforms, the degeneracy of the energy levels is lifted.
- The single-particle energy orbitals rearrange themself and generate a gap at large deformation.
- The deformed gaps provide extra binding energy and stabilise the superdeformed minimum.

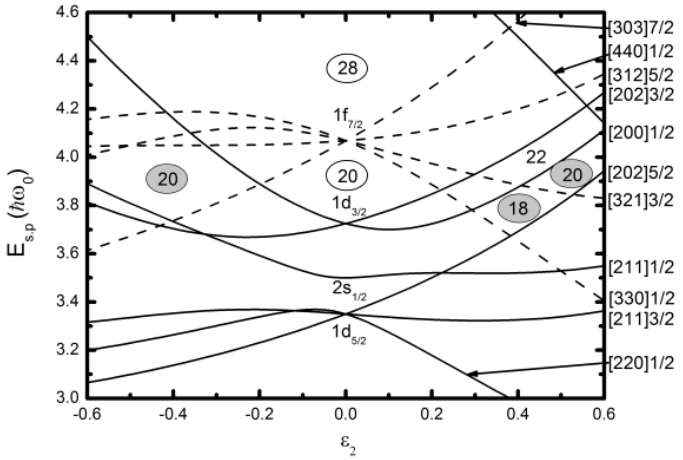
Objective

Investigating whether the mean field framework reproduces the microscopic shell effects.

[V.M. Strutinsky, Nucl. Phys. A 95, 420.]

Shell effect : Formation of superdeformation

Single-energy particle plots are obtained to investigate the role of shell effects.



Pairing Correlations: Shaping the Superdeformed Energy Landscape

How do pairing correlations matter?

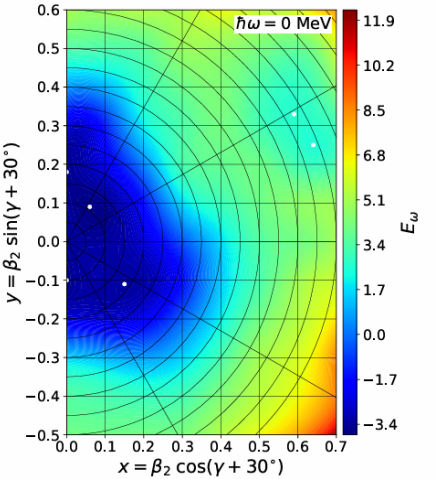
- Nucleons near the Fermi surface participate most actively in particle-hole excitations across shell gaps.
- Pairing correlations couple these valence nucleons, lowering the total energy of the system.
- The competition between shell effects and pairing correlations determines the location and depth of energy minima.
- Consequently, pairing modifies the Total Routhian Surface (TRS) and influences the stability of the superdeformed minimum.

Objective

Investigating the role of pairing correlation through a mean-field framework.

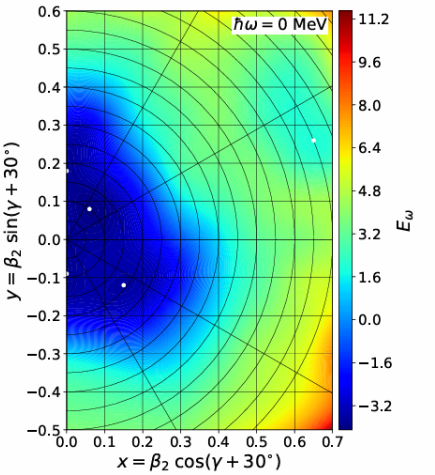
[W. Satula and R. Wyss, Phys. Scr. T56, 159-166 (1995).]

Pairing Correlations: Shaping the Superdeformed Energy Landscape



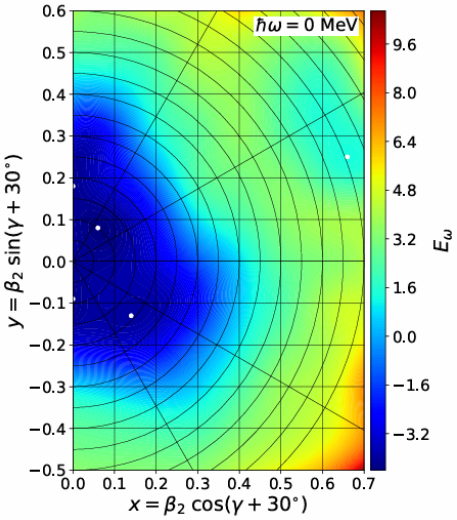
- Pairing Strength is 0.8G (G is the pairing strength).
- Shell effects dominate
- Reduced pairing correlations
- Stronger sensitivity to single-particle level structure
- Deformation-dependent shell correction remains pronounced

Pairing correlations: Shaping the Superdeformed Energy Landscape



- Pairing Strength is 0.9G.
- Competition between shell effects and pairing
- Energy surface becomes smoother
- Equilibrium deformation reflects both effects

Pairing correlations: Shaping the Superdeformed Energy Landscape



Strong pairing: 1G

- Enhanced smoothing of the single-particle occupations
- Shell-induced deformation effects are weakened
- Relative stability of competing minima is modified

Pairing correlations: Shaping the Superdeformed Energy Landscape

$0.8G$	$0.9G$	$1.0G$
Weaker pairing	Intermediate pairing	Stronger pairing
Stronger shell effects	Shell–pairing competition	Stronger pairing smoothing

Purpose

To examine how changing the pairing correlations modifies the competition with shell effects and consequently the deformation minima.

Rotation: Sustains the collective motion

How rotation sustains the collective motion at large spin

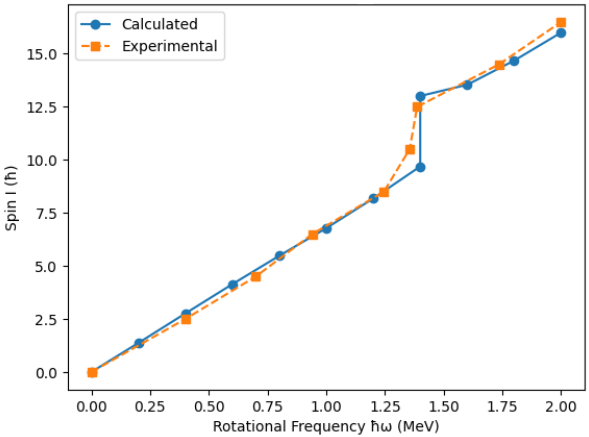
- When a nucleus rotates at very high rotational frequency, the Coriolis force begins to act on particles.
- The Corilos force breaks the Cooper pairs, allowing them to occupy high j-orbitals.
- After occupying high j-orbitals, they align their angular momentum along the rotational axis.
- The rotation favours a large moment of inertia; thus, it sustains the SD bands since they have a large moment of inertia.

Objective

Investigating the role of rotation through a mean-field framework.

[D. R. Inglis, Phys. Rev. 96, 1059 (1954), Phys. Rev. 103, 1786 (1956).]

Rotation: Evolution of Angular Momentum with Frequency



[X. H. Xiang, Chin. Phys. (2018).]

Key observations:

- I increases systematically with $\hbar\omega$. The calculated trend reproduces the experimental behaviour. A change in the slope of $I(\omega)$ indicates a change in the rotational response.
- At high frequency, quasiparticle alignment becomes increasingly important.

$$\hbar\omega \uparrow \Longrightarrow I \uparrow \Longrightarrow \text{alignment} \Longrightarrow \text{enhanced rotational response}$$

Future Perspectives: Theory-Experiment Connection

- **Measure electromagnetic observables** Determine lifetimes, $B(E2)$ values and transition quadrupole moments to constrain the predicted deformation.
- **Map the rotational response** Extend measurements of E_γ and extract $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ over a wider frequency range.
- **Identify quasiparticle alignment** Combine spectroscopy with microscopic configuration assignments to test the predicted alignment mechanism.
- **Extend toward exotic nuclei** Apply the framework to nuclei away from stability where shell evolution and pairing may change substantially.

Goal

Use complementary spectroscopic observables to distinguish the roles of shell structure, pairing, and rotation in shape evolution.

From Microscopic Structure to Precision Electromagnetic Probes

Shell structure
Pairing correlations
Deformation
Rotation

→

Nuclear electromagnetic properties

→

Low-energy electromagnetic probes

Theory

β_2, γ

$E(I)$

$\mathcal{J}^{(1,2)}$

Nuclear observables

Q_t

$B(E2)$

$\langle r^2 \rangle$

Experimental probes γ-ray spectroscopy Laser spectroscopy High-intensity electromagnetic fields

Thanks to My Collaborator

Special Thanks to Prof. Ramon Wyss

Thank You