Static and Time-Dependent CI in Continuum for Multi-Nucleon Transfer Reactions

The Microscopic Theory behind the Quantum Nuclear Alchemy

The collision $^{48}Ca+^{208}Pb$ near the Coulomb barrier

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Based on: A. Bulgac, arXiv:2024.092173, Phys. Rev. C, in press (2026)
M. Kafker and A. Bulgac, arXiv:2604.218454

Nuclear Chrysopoeia - chēmeia ⟶ Alchemy(al-kīmīā) was originated by Earnest Rutherford with

- the discovery of the size of the nucleus in 1911
- the realization of the first nuclear reaction in 1917

➢ In 2023 the first private company Marathon Fusion was founded, with DOE support and collaborations with universities and other private companies, to create gold from mercury with 14 MeV neutrons from a deuterium-tritium plasma

$$(d+t \longrightarrow n+\alpha) \longrightarrow {}^{198}Hg(n,2n)^{197}Hg(\beta^{-})^{197}Au$$

arXiv:2507.13461, 5000 kg/year ≈ $740M using 1 GW of electricity

^{197}Au is the only stable gold isotope

➢ possibility of creating other critical minerals

- I will explain what and how it was done.
- But not so much why.

- Only 23 slides and 6 figures in total.
- I will provide the minimum necessary number of equations needed to understand the problem we attacked.

We solved numerically all equations on the Frontier supercomputer, OCLF.

We needed 80 TB of memory.

We used 48,000 GPUs for the static problem
and 56,000 GPUs for the time-dependent problem.

Ours is arguably the largest GCM-type problem ever simulated.

One can make a parallel between:

the state of technology in the human society

and

the state of quantum description of strongly interacting many fermion systems.

- Roughly before 1800's people built roads, bridges, pyramids, castles with one or more floors - in a word static structures - and for transportation used animals and wind.

- After 1800 people started building machines and most things were in motion: trains and railways, cars, airplanes, rockets, created a constellation of GPS satellites, people landed on the Moon and soon will likely be landing on Mars, send the probes Voyager 1 and 2 beyond the solar system …

- In quantum mechanics of many-body systems today people construct mostly the equivalent of buildings with one or more floors, and at best the equivalent of skyscrapers with a few hundreds of floors.

- Theorists basically do not have yet at their disposal a sufficiently sophisticated quantum time-dependent technology needed for studying the quantum dynamics of non-equilibrium and dissipative phenomena, where interference and entanglement play a dominant role in the dynamics. On the other hand, there are a plethora of phenomenological models, mostly classical in character, with lots of phenomenological parameters and, some would argue, reasonable assumptions.

As an exercise, compare how many time-dependent problems you solved in high and undergraduate schools and how many real time-dependent problems have you solved until today in quantum mechanics of many-body systems, or even a few bodies.

Quantum mechanics is already more than a century old!

The great majority of all the known 3,300 isotopes known so far were created in Multi-Nucleon Transfer (MNT) reactions between two colliding nuclei in laboratories worldwide.

We expect that the rest of the unknown 3,500-4,000 isotopes expected to exist and be created in the labs will be created in more or less the same manner.

- There is a need a microscopic theory for MNT reactions!

- The MNT reactions are dissipative, non-equilibrium, and time-dependent, and the results are the final quantum thermalized products!

The MNT reactions are described with the following phenomenological formula for the cross section:

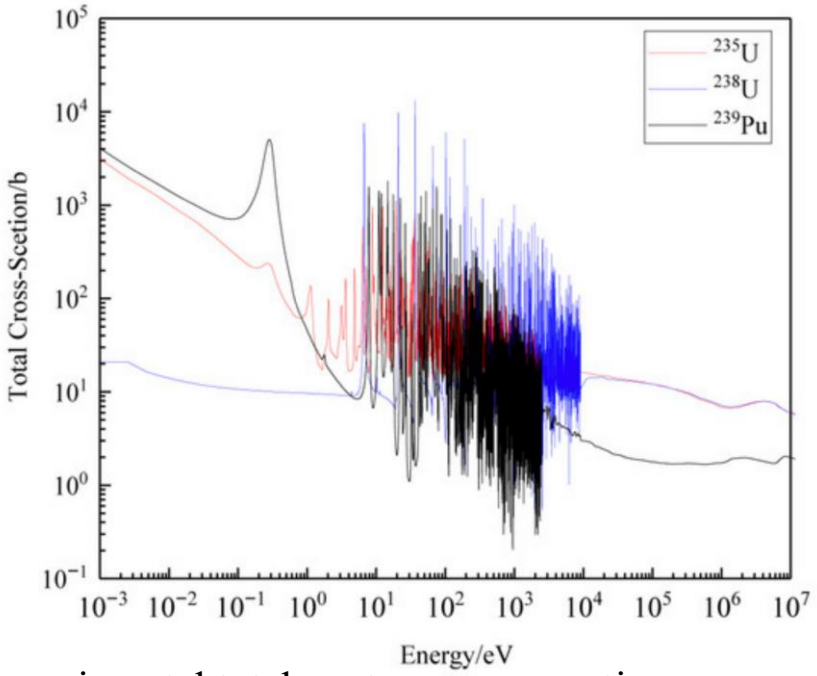
$$\sigma_{EVR} = \sum_{J=0}^{J_{\max}} \sigma_{\mathrm{cap}}(E_{\mathrm{CoM}}, J) P_{\mathrm{CN}}(E^*, J) W_{\mathrm{sur}}(E^*, J)$$

- The first factor in the sum is the capture cross section, which arguably can be evaluated with reasonable accuracy using optical potentials, which typically are phenomenological and but for spherical nuclei mostly.

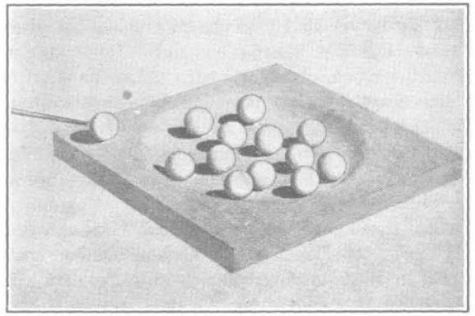
- The second factor is the probability of the formation of the Compound Nucleus (CN). The compound nucleus was predicted in 1936 by Niels Bohr (but never derived) and experimental results (crystal blocking) apparently appear to agree with its presence in nuclear reactions and are supporting its long life-time.

According to Godbey and 18 others, J. Phys. G (2026)
$\boldsymbol{P_{CN}}$ is the weakest link! But MK+AB think that the real situation is but more complicated!

- The third factor is the probability of "evaporation" of the reaction products, for which there are apparently reasonably trustworthy (semi-)microscopic models: Weiskopf (1937), Hauser and Feshbach (1952).



Experimental total neutron cross-section for three nuclei. CN appears at incident neutron energies > 10 eV!



This figure did not appear in Bohr's 1936 paper.
Obviously!
Classical entropy increases!

What are the essential ingredients of a microscopic theory for strongly time evolving interacting many-body systems?

The equations should:

- Describe phenomena in terms of many-body wave functions,
- Various components add up with complex time-dependent coefficients, thus capable of describing time evolution,
- The explicit presence of the Planck's constant.

- The dynamics should describe:

- Interference,
- Entanglement,
- The increasing complexity in time of the many-body wave function, the increasing entropy for an isolated system,
- Dissipation, non-equilibrium quantum dynamics,
- Thermalization of an isolated quantum system (is the Eigenvalue Thermalization Hypothesis really valid?).

1939 (Meitner and Frisch), 1967 (Strutinski) and 2016 (BMRS)

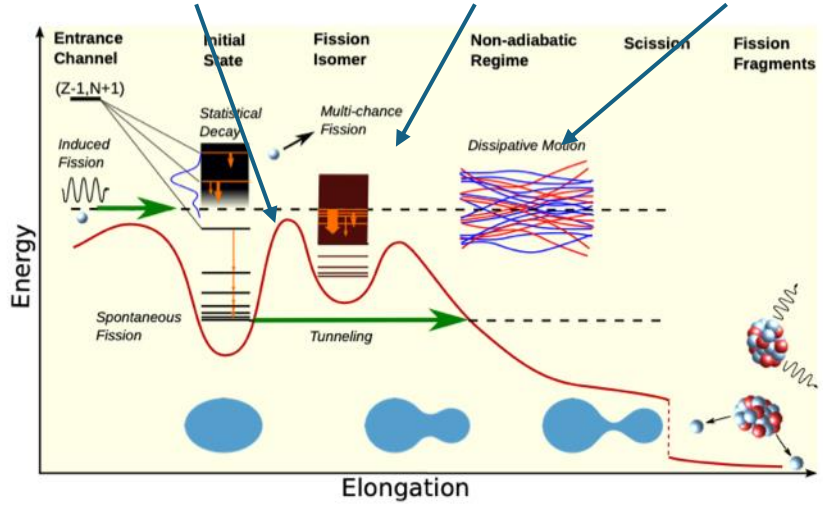


FIG. 1. This open access figure from Refs. [1, 2], courtesy of N. Schunck, vividly illustrates the complexity of the three main stages of induced fission. In the initial state and in the fission isomer stage a fissioning nucleus spends approximately 10^{-15} seconds [3], while in the third stage, the non-adiabatic stage, before scission the nucleus evolves during approximately 10^{-20} seconds [3–6], and the neck rupture and the separation of the fissioning nucleus into FFs happens in about 10^{-21} seconds [7]. Subsequent processes, such as prompt neutron and light charge particle emission [8], γ, and β decay happen at much longer time scales [3].

Figure from Bender et al., (courtesy of N. Schunck)
J. Phys. G: Nucl. Part. Phys. 47, 113002 (2020)

Induced fission is a reaction where the compound nucleus is always expected to be created and eventually fission fragments are produced.

The microscopic theory of the third stage, the descent from the top of the outer fission barrier, to scission and evolution of fission fragments, and the fact that it is in fact a dissipative evolution (not an adiabatic one) was established only in 2015.

<u>There is still no complete microscopic theory encompassing all three stages of the entire reaction:</u>

- the capture,
- the probability of creation and the evolution of a compound nucleus up to the outer fission barrier, and
- then the descent from the top of outer barrier to the scission configurations and formation of fission fragments.

This last stage was until 2015 still believed to be an adiabatic evolution.

The third stage was proven to be a strongly dissipative process, which is also surprisingly much slower than an adiabatic evolution!

- I will present results for the collision $^{48}Ca+^{208}Pb$ in a simulation cubic box $(64\ fm)^3$, with a lattice constant 1 fm.

- 235 MeV kinetic energy in the center of mass reference system (289 MeV in the lab), just above the Coulomb barrier. We solve TDHF equations for 152 impact parameters in the orange ring in the plane of the impact parameters.

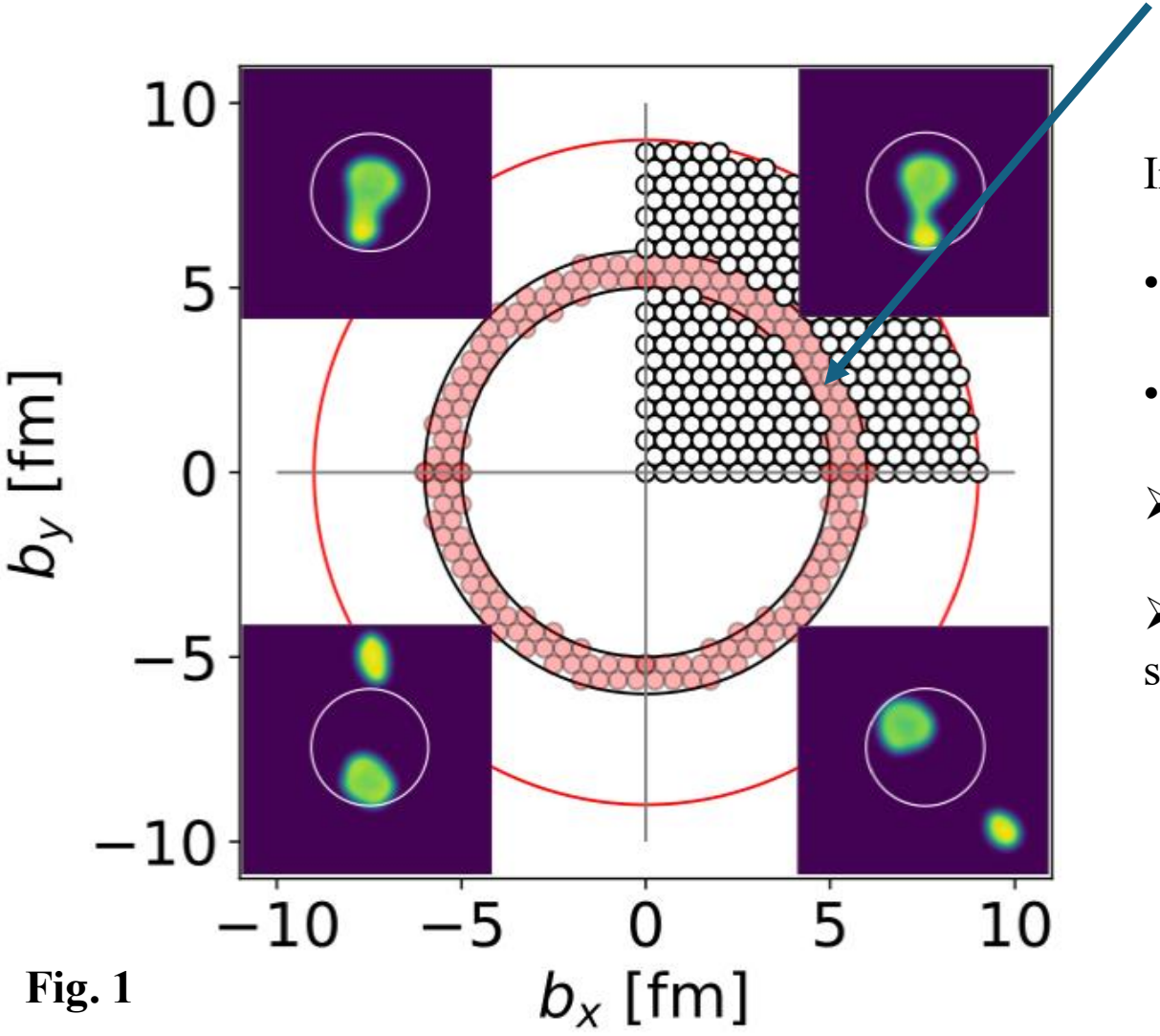


Fig. 1

Impact parameters considered $5 \leq b \leq 6$ fm.

- In the left images TDHF trajectories with b = 5 fm.
- In the right images TDHF trajectories with b = 6 fm.

➢ At the top, the two reaction partners come into contact.

➢ At the bottom, the two reaction fragments are spatially separated

The microscopic formalism I will use is called enhanced GCM (eGCM)

- First part, static eGCM

The static eGCM/CI solution in the continuum for the reaction $^{48}Ca+^{208}Pb$.

This is a typical combination of reaction partners in creating transuranium elements.

There will be 3 figures for the static part.

Let me proceed with outlining our approach.

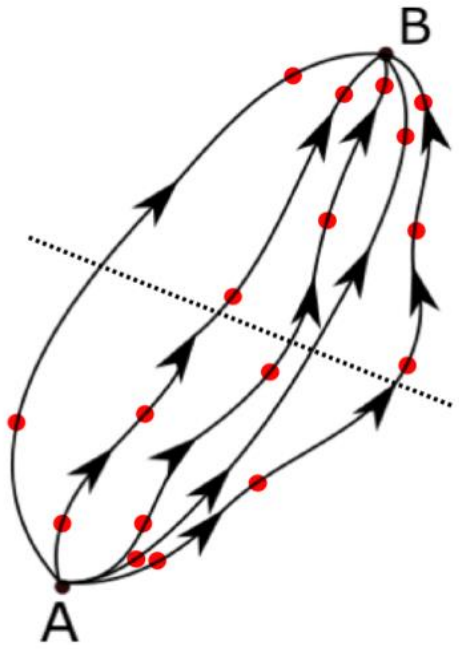


FIG. 2. These are five of the infinitely many paths available in case of neutron induced fission from point A (the time of impinging neutron impact) to point B (detector) at a later time. This figure with added interaction times at random positions (red dots) is reproduced from Wikipedia entry *Path integral formulation*, is a pictorial representation of a many-body quantum propagator $K(\xi_1 \dots \xi_A, \tau | \xi'_1 \dots \xi'_A, \tau')$ from point A at time τ' to point B at time τ.

This artist's depiction of the Feynman's path integral is for an isolated particle.

- Each of these oriented lines are the equivalent of TDHF trajectories in MNT reactions or TDDFT in induced fission, which are stored in time at the position of the red dots.

- At each time during the evolution the true many-body wave function of the many-body system is a quantum superposition of many time-dependent Hartree-Fock (TDHF) trajectories.

- <u>In eGCM the final many-body wave function is a coherent superposition of all red dots (TDHF) above the dotted line.</u>

- This last statement might appear incorrect to some.

  This is what actually Reinhard, Cusson and Goeke suggested in 1983, as a TDGCM extension.

- eGCM is however the correct extension of both static and time-dependent GCM

- For each impact parameter we follow in time the two reaction partners in TDHF,
- We store the full Slater determinants for the system ^{48}Ca + ^{208}Pb with a time step $\Delta\tau = 6.4$ fm/c, in total approximately 385 instances in time for each impact parameter (more for b = 5 fm and less for b = 6 fm),
- In total we end up with non-orthogonal 39,630 Slater determinants, each denoted by $\Phi(\alpha)$, where $\alpha = (b_x, b_y, \tau)$, but linearly independent Slater determinants for all 256 nucleons.

$$\sum\!\!\!\!\!\!\int_{\alpha'} \langle\Phi(\alpha)|\Phi(\alpha')\rangle g_k(\alpha') = \nu_k g_k(\alpha), \quad \nu_k > 0.00112,$$

$$\sum\!\!\!\!\!\!\int_{\alpha} g_k^*(\alpha) g_l(\alpha) = \delta_{kl}, \quad \sum\!\!\!\!\!\!\int_{k} g_k(\alpha) g_k^*(\beta) = \delta(\alpha - \beta),$$

$$|\overline{\Phi}_k\rangle = \sum\!\!\!\!\!\!\int_{\alpha} \nu_k^{-1/2} g_k(\alpha)|\Phi(\alpha)\rangle, \; \nu_k > 0, \; \langle\overline{\Phi}_n|\overline{\Phi}_m\rangle = \delta_{nm}$$

CI → $$\langle\overline{\Phi}_k|\hat{H}|\Psi_n\rangle = \sum\!\!\!\!\!\!\int_{l} \langle\overline{\Phi}_k|\hat{H}|\overline{\Phi}_l\rangle\, h_{ln} = h_{kn} E_n,$$

$$|\overline{\Phi}(\alpha)\rangle = \sum\!\!\!\!\!\!\int_{l} g_l^*(\alpha)|\overline{\Phi}_l\rangle, \quad \langle\overline{\Phi}(\alpha)|\overline{\Phi}(\beta)\rangle = \delta_{\alpha\beta},$$

CI → $$\langle\overline{\Phi}(\alpha)|\hat{H}|\Psi_n\rangle = \sum\!\!\!\!\!\!\int_{\beta} \langle\overline{\Phi}(\alpha)|\hat{H}|\overline{\Phi}(\beta)\rangle\, h_{\beta n} = h_{\alpha n} E_n.,$$

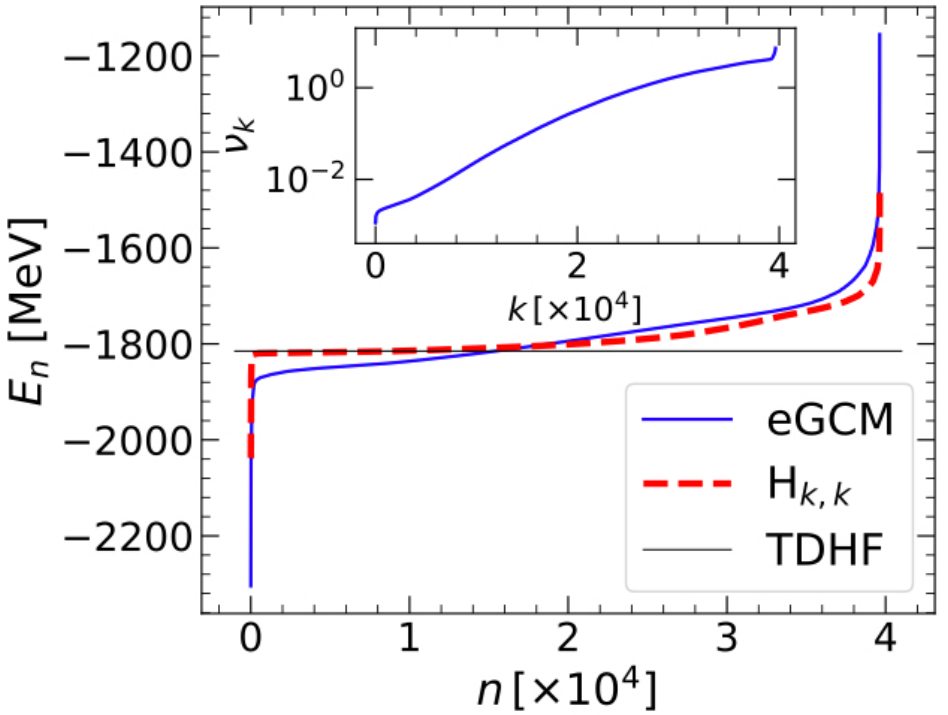


Fig. 2

$|\overline{\Phi}_k\rangle$ and $|\overline{\Phi}(\alpha)\rangle$ They are orthogonal to each other and the equivalent of Bloch (running waves) and Wanier (localized states) in crystals. These are linear combinations of 39,630 Slater determinants.

The E_n energy spectrum is consistent with a Gaussian Orthogonal Ensemble spectrum.

Inverse participation ratios quantify the delocalization magnitude of a many-body wave function, or the number of major components of a many-body wave function

$$\mathrm{IPR_N}(k) = \left[\sum\!\!\!\!\!\!\int_{\alpha} |g_k(\alpha)|^4\right]^{-1}, \mathrm{IPR_N}(\alpha) = \left[\sum\!\!\!\!\!\!\int_{k} |g_k(\alpha)|^4\right]^{-1},$$

$$\mathrm{IPR_H}(n) = \left[\sum\!\!\!\!\!\!\int_{k} |h_{k,n}|^4\right]^{-1}, \quad \mathrm{IPR_H}(\alpha) = \left[\sum\!\!\!\!\!\!\int_{k} |h_{\alpha,n}|^4\right]^{-1}.$$

$|\overline{\Phi}_k\rangle$ and $|\overline{\Phi}(\alpha)\rangle$

$$\sum\!\!\!\!\!\!\int_{\alpha'} \langle\Phi(\alpha)|\Phi(\alpha')\rangle g_k(\alpha') = \nu_k g_k(\alpha), \quad \nu_k > 0.00112,$$

$$\sum\!\!\!\!\!\!\int_{\alpha} g_k^*(\alpha) g_l(\alpha) = \delta_{kl}, \quad \sum\!\!\!\!\!\!\int_{k} g_k(\alpha) g_k^*(\beta) = \delta(\alpha-\beta),$$

$$|\overline{\Phi}_k\rangle = \sum\!\!\!\!\!\!\int_{\alpha} \nu_k^{-1/2} g_k(\alpha)|\Phi(\alpha)\rangle, \ \nu_k > 0, \ \langle\overline{\Phi}_n|\overline{\Phi}_m\rangle = \delta_{nm}$$

CI → $\langle\overline{\Phi}_k|\hat{\mathrm{H}}|\Psi_n\rangle = \sum\!\!\!\!\!\!\int_{l} \langle\overline{\Phi}_k|\hat{\mathrm{H}}|\overline{\Phi}_l\rangle h_{ln} = h_{kn} E_n,$

$$|\overline{\Phi}(\alpha)\rangle = \sum\!\!\!\!\!\!\int_{l} g_l^*(\alpha)|\overline{\Phi}_l\rangle, \quad \langle\overline{\Phi}(\alpha)|\overline{\Phi}(\beta)\rangle = \delta_{\alpha\beta},$$

CI → $\langle\overline{\Phi}(\alpha)|\hat{\mathrm{H}}|\Psi_n\rangle = \sum\!\!\!\!\!\!\int_{\beta} \langle\overline{\Phi}(\alpha)|\hat{\mathrm{H}}|\overline{\Phi}(\beta)\rangle h_{\beta n} = h_{\alpha n} E_n.,$

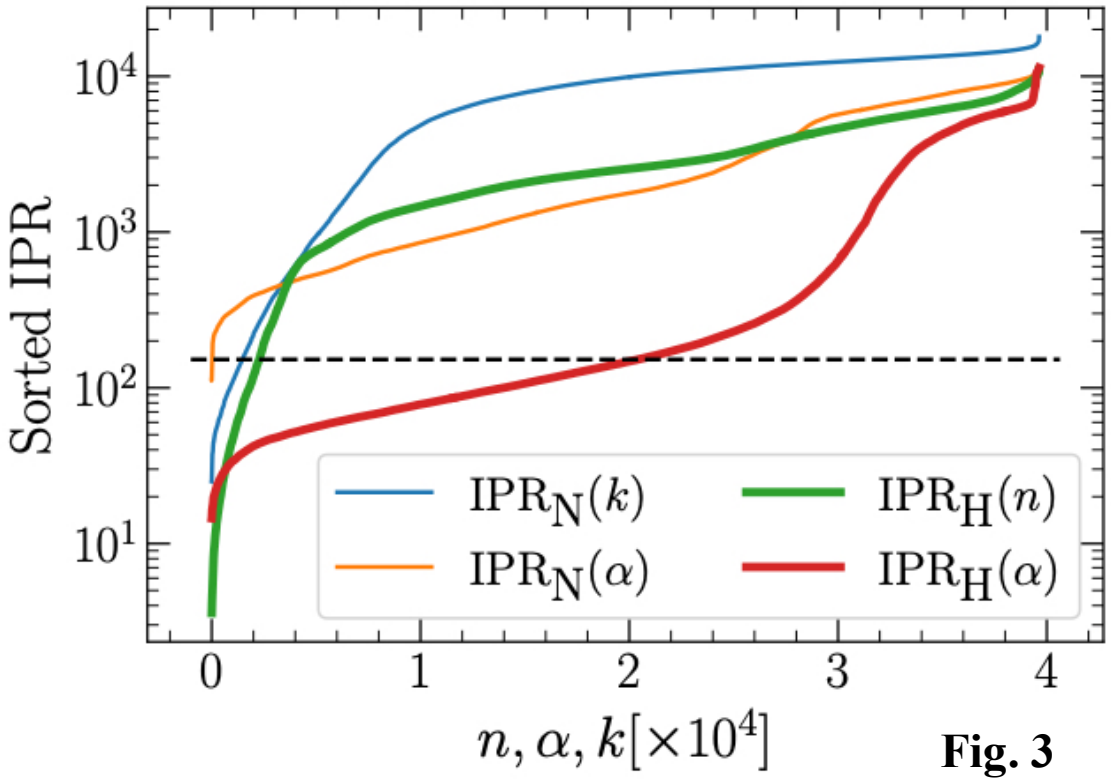
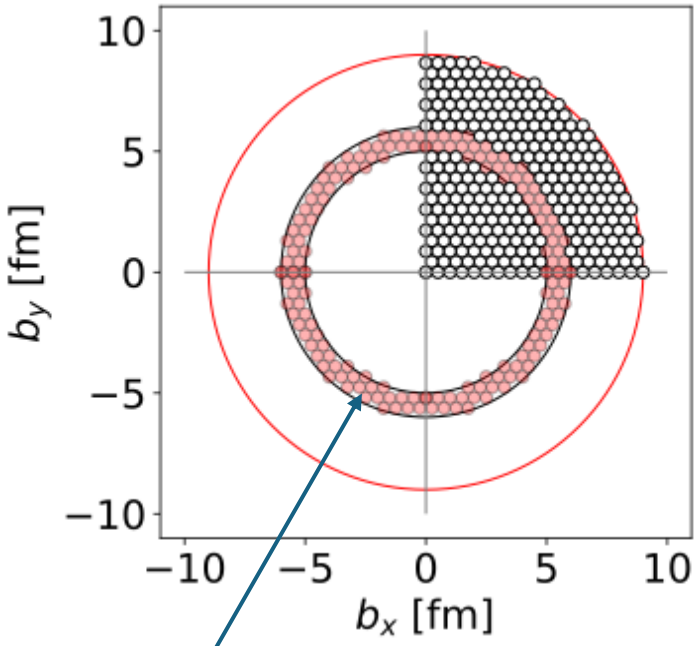


Fig. 3

➢ The second part, the time-dependent eGCM/CI solution in the continuum for the reaction ^{48}Ca + ^{208}Pb, three figures.



1)

$$i\hbar\partial_t|\Psi(t)\rangle = \hat{\mathrm{H}}|\Psi(t)\rangle, \quad \langle\Psi(t)|\Psi(t)\rangle = 1,$$

$$|\Psi(t)\rangle = \sum\!\!\!\!\!\!\int_n c_n(t)|\Psi_n\rangle,\ c_n(t) = c_n(0)\exp\left(-\frac{iE_nt}{\hbar}\right),$$

$$c_n(0) = \langle\Psi_n|\Psi(0)\rangle = \mathcal{N}\sum\!\!\!\!\!\!\int_{\mathbf{b}}\sum\!\!\!\!\!\!\int_k g_k^*(\mathbf{b},0)\nu_k^{1/2}h_{kn}^*,$$

Initial many-body state is a superposition of all impact parameters for ^{48}Ca+^{208}Pb in the ring and moving towards each other in the center of mass system.

$$|\Psi(t)\rangle = \sum\!\!\!\!\!\!\int_k a_k(t)|\overline{\Phi}_k\rangle = \sum\!\!\!\!\!\!\int_\alpha d(\alpha|t)|\overline{\Phi}(\alpha)\rangle,$$

2) “Bloch waves”

3) “Wannier states”

NB Three different representations of the total time-dependent many-body wave function.

Fig. 4

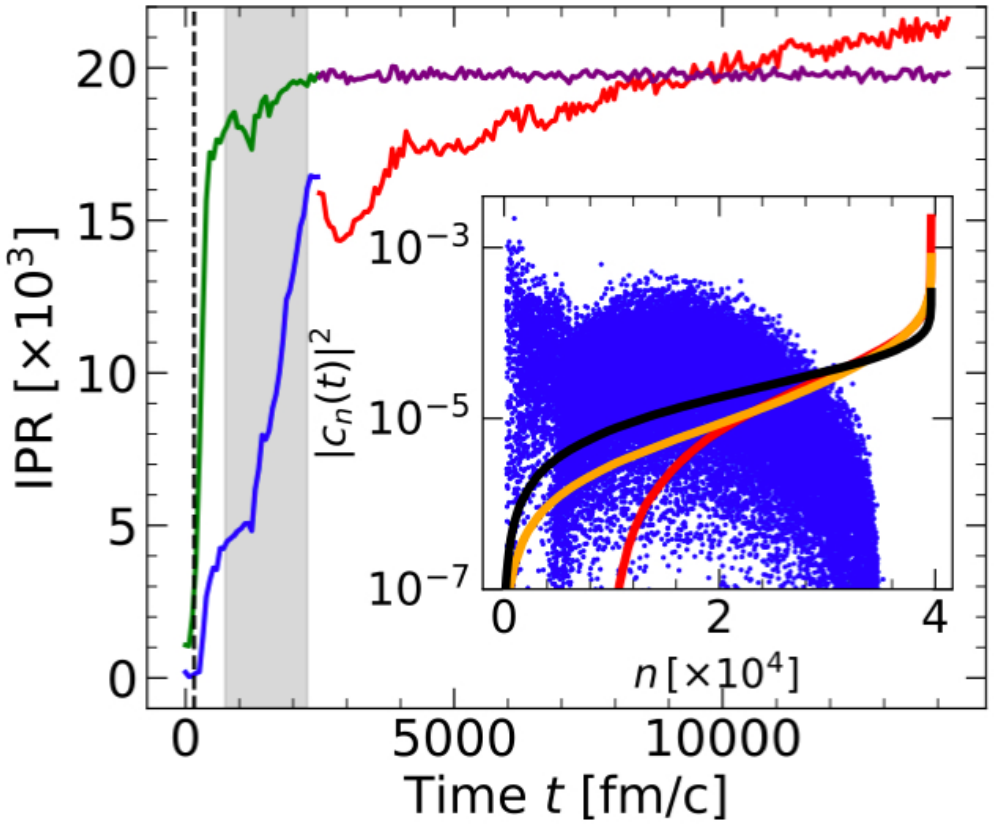


FIG. 4. The IPR is evaluated for the wave function $|\Psi(t)\rangle$, see Eq. (14), using either the expansion coefficients $a_k(t)$ (blue line, continued red after $t_{fin} = 2,482$ fm/c), and coefficients $d(\alpha|t)$ (green line, continued purple after t_{fin}). The vertical black dashed line indicates the time when all the nuclei first make contact in TDHF. The gray banded region indicates the range of times during which the nuclei separate in TDHF. The inset displays the coefficients $|c_n(t)|^2$, as well as the sorted coefficients $|d(\alpha|t_{fin})|^2$, $|a_k(t_{fin})|^2 \times 1.75$, and $|c_n(t_{fin})|^2$ with black, orange, and red lines respectively.

$$i\hbar\partial_t|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle, \quad \langle\Psi(t)|\Psi(t)\rangle = 1,$$

$$|\Psi(t)\rangle = \sum\!\!\!\!\!\!\int_n c_n(t)|\Psi_n\rangle,\ c_n(t) = c_n(0)\exp\left(-\frac{iE_nt}{\hbar}\right),$$

$$c_n(0) = \langle\Psi_n|\Psi(0)\rangle = \mathcal{N}\sum\!\!\!\!\!\!\int_{\mathbf{b}}\sum\!\!\!\!\!\!\int_k g_k^*(\mathbf{b},0)\nu_k^{1/2}h_{kn}^*,$$

$$\sum\!\!\!\!\!\!\int_{\alpha'}\langle\Phi(\alpha)|\Phi(\alpha')\rangle g_k(\alpha') = \nu_k g_k(\alpha), \quad \nu_k > 0.00112,$$

$$\sum\!\!\!\!\!\!\int_\alpha g_k^*(\alpha)g_l(\alpha) = \delta_{kl}, \quad \sum\!\!\!\!\!\!\int_k g_k(\alpha)g_k^*(\beta) = \delta(\alpha-\beta),$$

$$|\overline{\Phi}_k\rangle = \sum\!\!\!\!\!\!\int_\alpha \nu_k^{-1/2}g_k(\alpha)|\Phi(\alpha)\rangle,\ \nu_k > 0,\ \langle\overline{\Phi}_n|\overline{\Phi}_m\rangle = \delta_{nm}$$

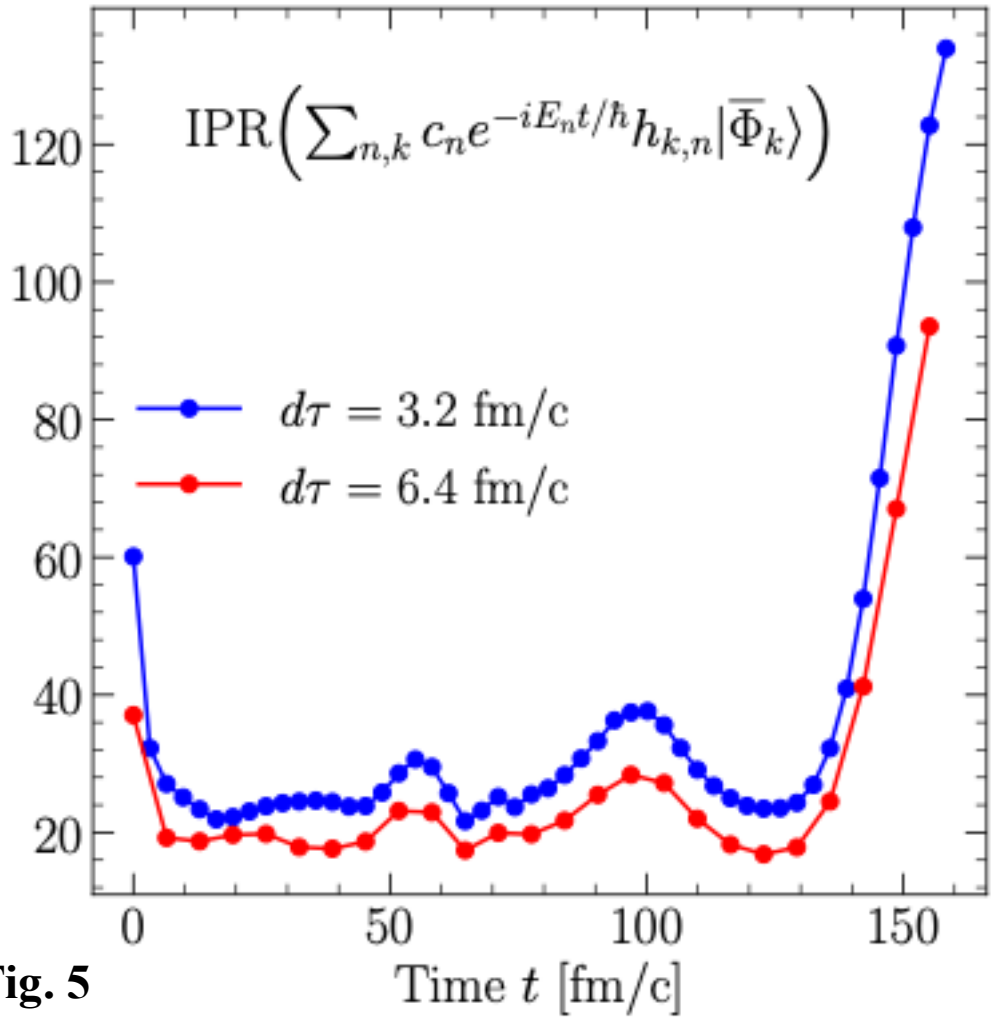
$$\langle\overline{\Phi}_k|\hat{H}|\Psi_n\rangle = \sum\!\!\!\!\!\!\int_l\langle\overline{\Phi}_k|\hat{H}|\overline{\Phi}_l\rangle\, h_{ln} = h_{kn}E_n,$$

$$|\overline{\Phi}(\alpha)\rangle = \sum\!\!\!\!\!\!\int_l g_l^*(\alpha)|\overline{\Phi}_l\rangle, \quad \langle\overline{\Phi}(\alpha)|\overline{\Phi}(\beta)\rangle = \delta_{\alpha\beta},$$

$$\langle\overline{\Phi}(\alpha)|\hat{H}|\Psi_n\rangle = \sum\!\!\!\!\!\!\int_\beta\langle\overline{\Phi}(\alpha)|\hat{H}|\overline{\Phi}(\beta)\rangle\, h_{\beta n} = h_{\alpha n}E_n.,$$

BTW **ETH** would predict in this case $\frac{\hbar}{\Delta E} \approx 7\frac{fm}{c}$.

The "thermalization" appears to be significantly slower, similar to a result we obtained for the UFG
Phys. Rev. Res. Letter, 6, L042003 (2024).



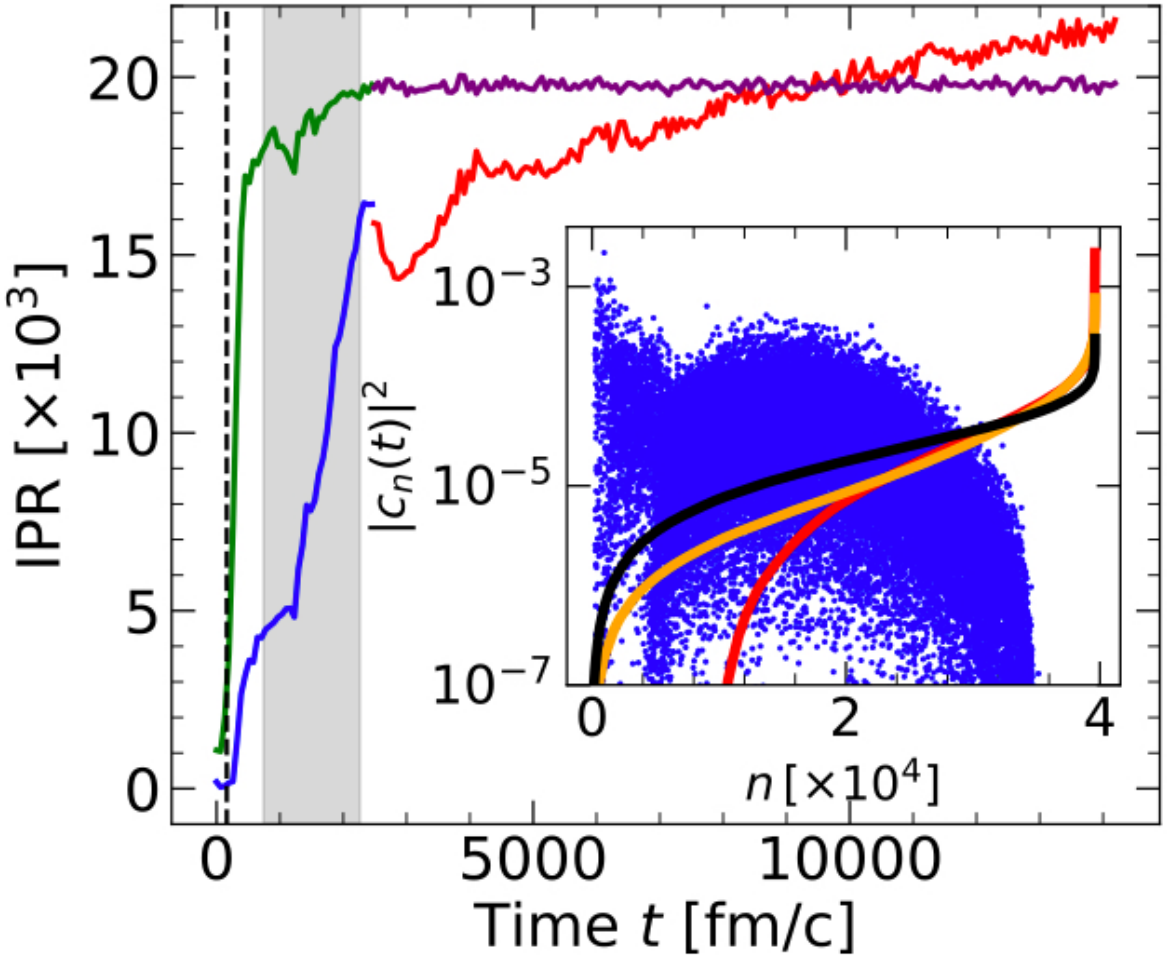
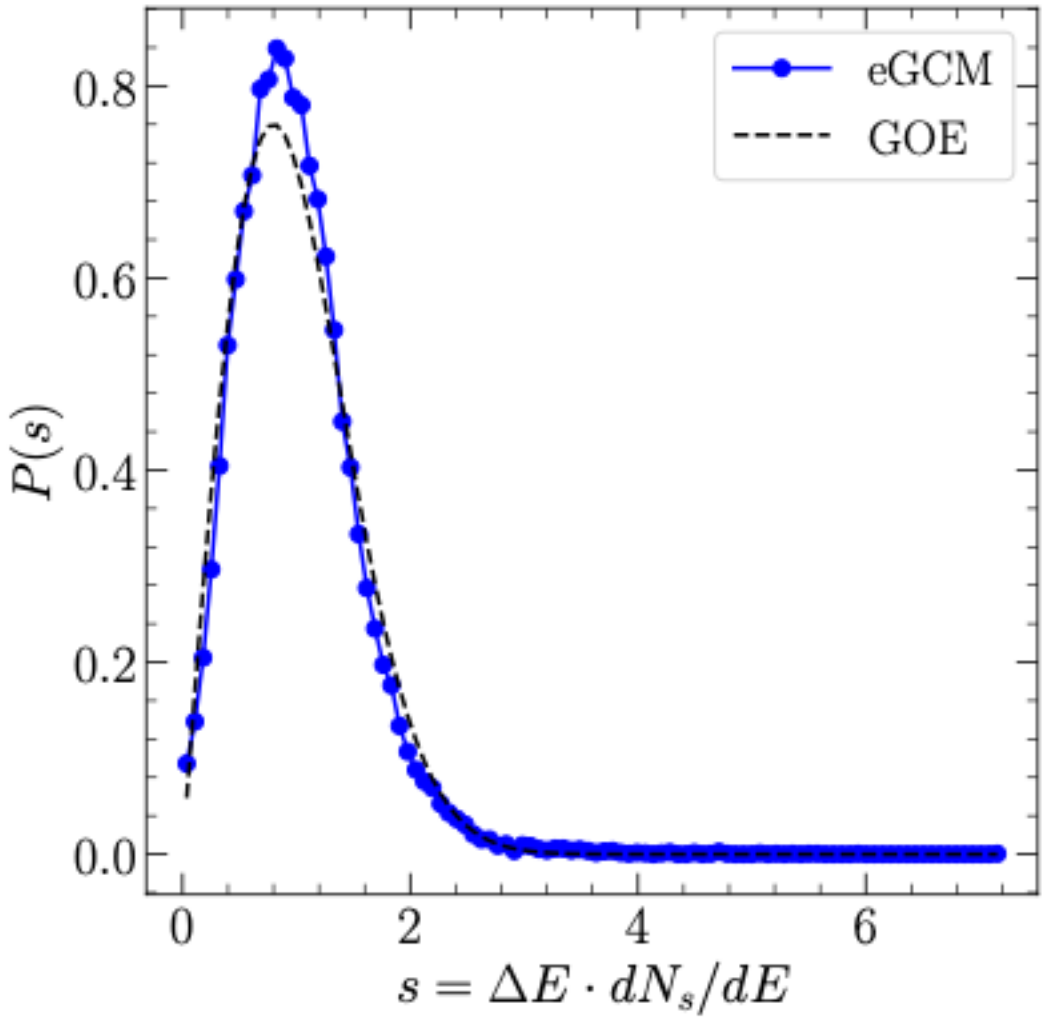


Fig. 5

Even before the two reaction partners come into contact there is a significant mixing present in the eGCM time-dependent many-body wave function. The complexity of the final eGCM Many-body wave functions increases from initially about 20 to finally 15,000-20,000 “simple configurations.”



eGCM level spacing distribution is very similar to GOE spectrum,
It is consistent with Wigner-Dyson surmise in Random Matrix Theory

Fig. 6

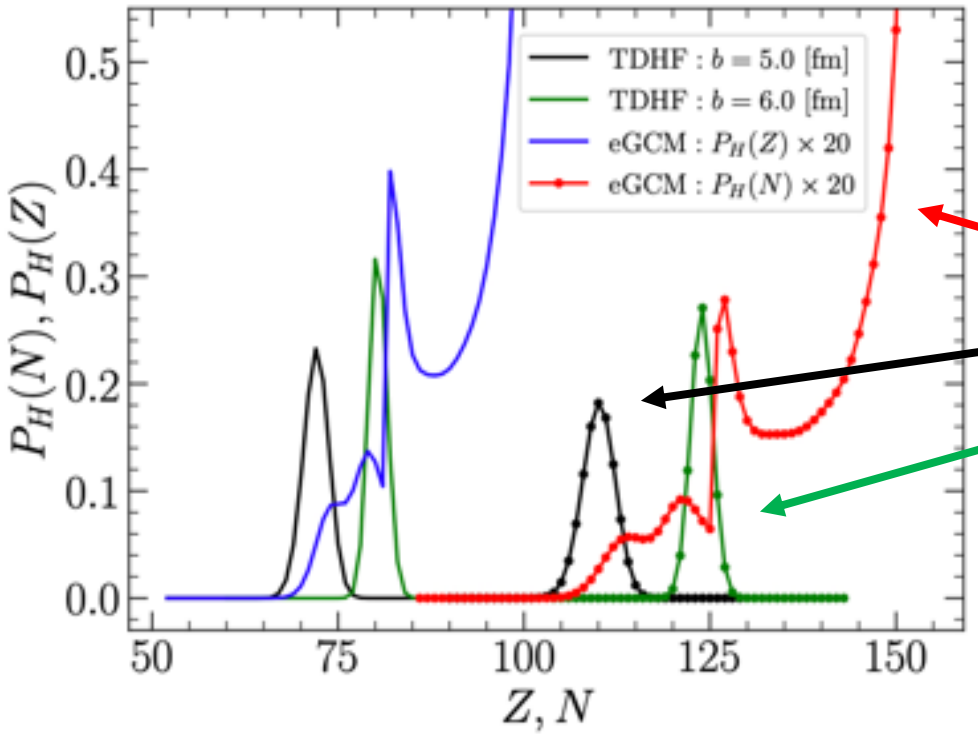
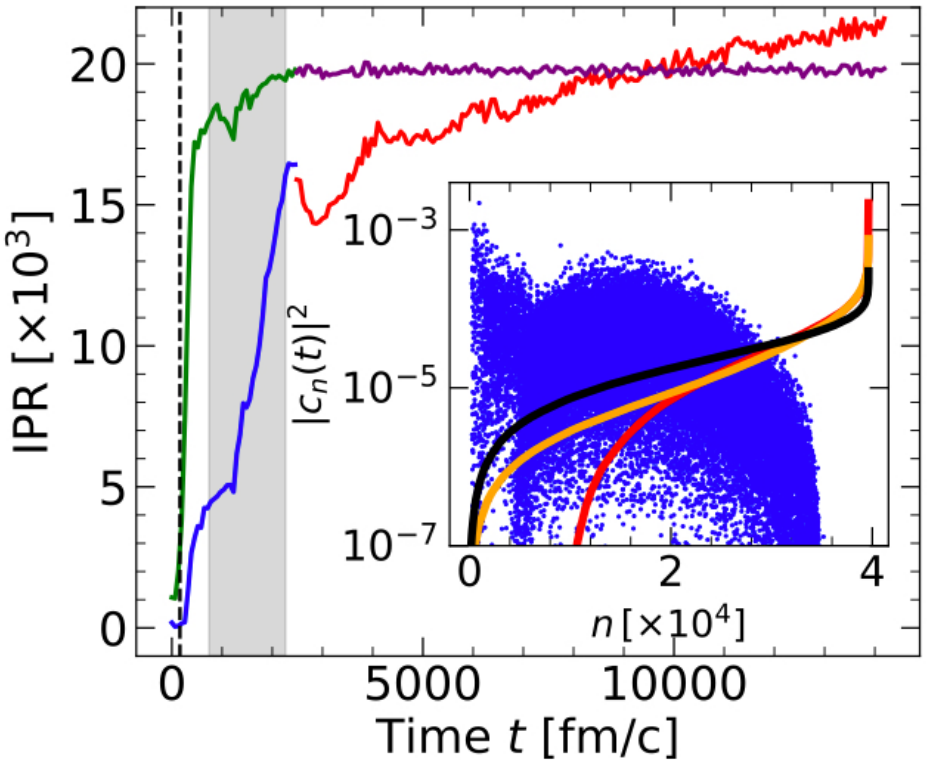


FIG. 3. Final probabilities $P_H(N) = \sum_Z P_H(N,Z)$ (red lines) and $P_H(Z) = \sum_N P_H(N,Z)$ (blue lines) showcasing the dramatic differences between TDHF and eGCM. The solid dotted black and green lines are the TDHF probabilities $P_H(N)$ or $P_H(Z)$ for the heavy fragments and solid without dots for $P_L(N)$ or $P_L(Z)$. The difference between the TDHF and eGCM particle probabilities are as stark as Isaac Newton's and Thomas Young's predictions would be for the two-slit experiment.

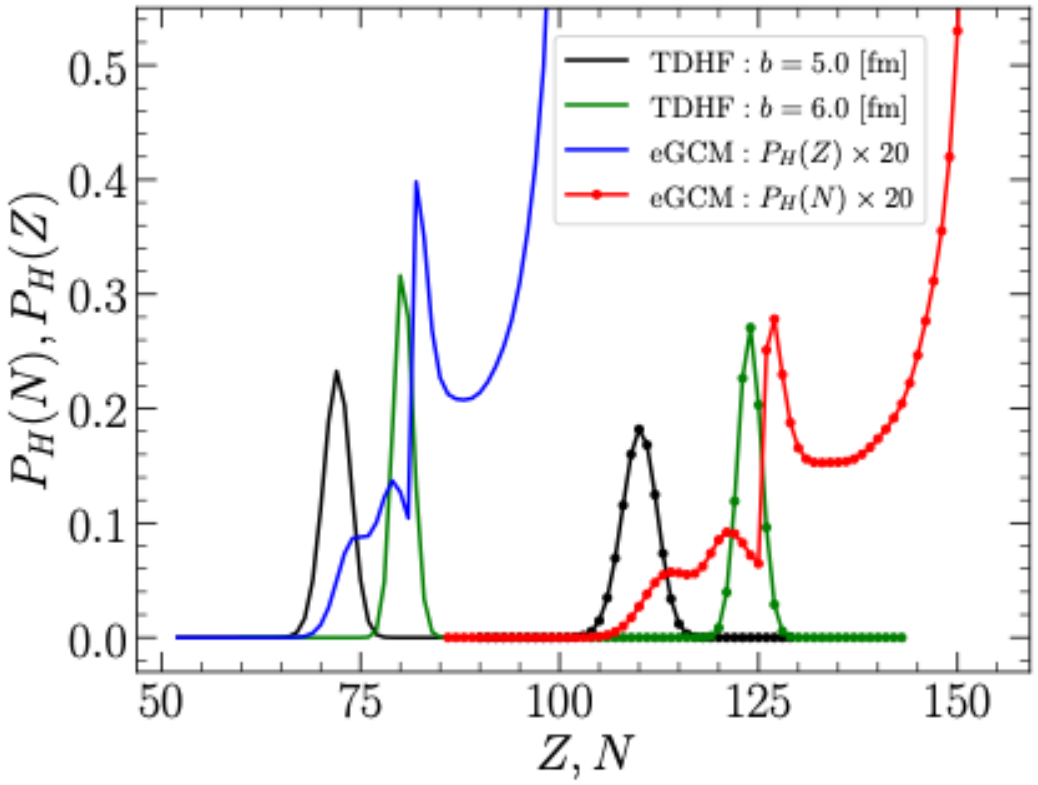
"Newton's" prediction

"Young's" prediction

Notice the amplification factor of 20 needed to make visible the two red little bumps!



Figs. 4 and 6

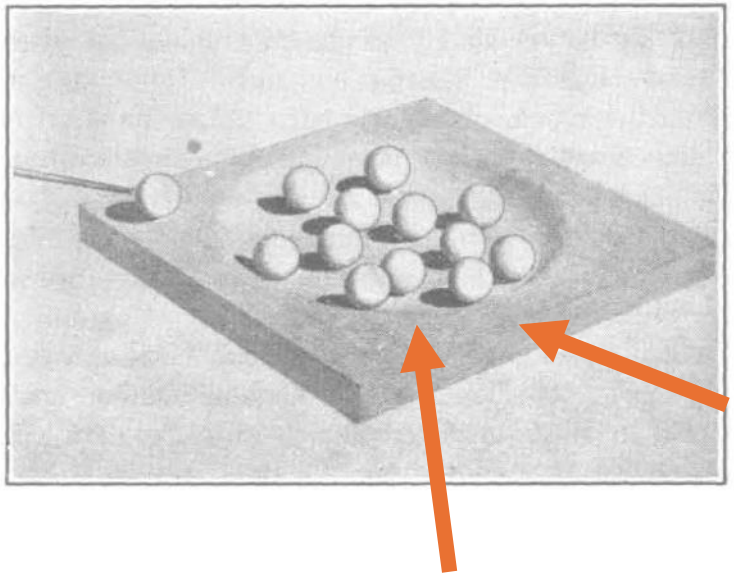


Notice how fast the complexity of **the many-body wave function evolved from about 20 components to about 15,000-20,000 components in a relatively short time.** A nucleon at the Fermi level in ^{208}Pb needs approximately 100 fm/c to get from one end to the other end and back of the nucleus.

Basically in 10-12 nucleon periods the complexity of the many-body wave function increased by a factor of about 150 - 200!

NB! The complexity of the many-body wave function of the many-body wave functions keeps increasing even faster after separating in TDHF, but not in eGCM.

From Editorial to Bohr's paper in Nature (1936).



Does this picture has any relation to the nuclear Compound Nucleus?

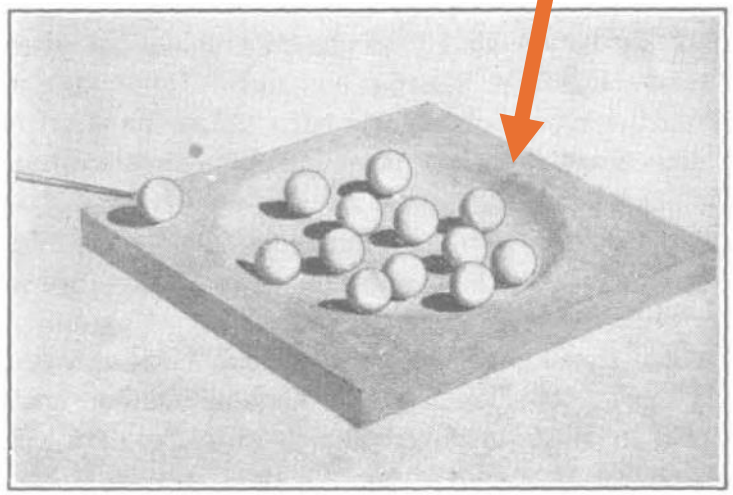
In the collision $^{48}Ca+^{208}Pb$ near the Coulomb barrier the kinetic energy per particle of the partners in their center of mass system is $\frac{235}{256}$= 0.92 MeV and the excitation of formed CN nucleus ^{256}No is 78 MeV or 0.3 MeV/nucleon.

Nucleon velocity at the Fermi level is approximately c/4.
Average nucleon-nucleon separation is approximately 1.9 fm.
Proton diameter is 1.7 fm.

ETH thermalization time 7 fm/c is the time it takes two sitting next-to-other "classical nucleons" to collide, therefore is the mean free path of "classical billiard balls."

A quantum nucleon at the Fermi level, which is basically an independent particle in a nucleus, needs about 50 fm/c to cross a heavy nucleus.

Does this picture has any relation to the nuclear Compound Nucleus?



From Editorial to Bohr's paper in Nature (1936).

What experiment has to say?

Estimate of mean free path from neutron total cross sections (*Fig. 2-3*)

Information on the mean free path of nucleons in a nucleus can be obtained from a study of cross sections as a function of energy. The measured total cross section for the interaction of neutrons with Cu, Cd, and Pb are shown in Fig. 2-3.

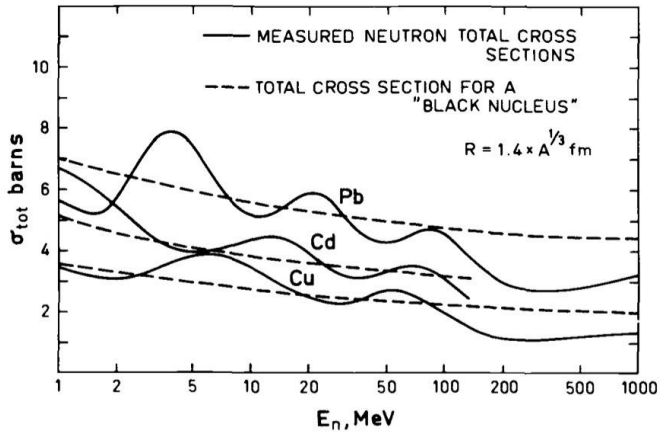
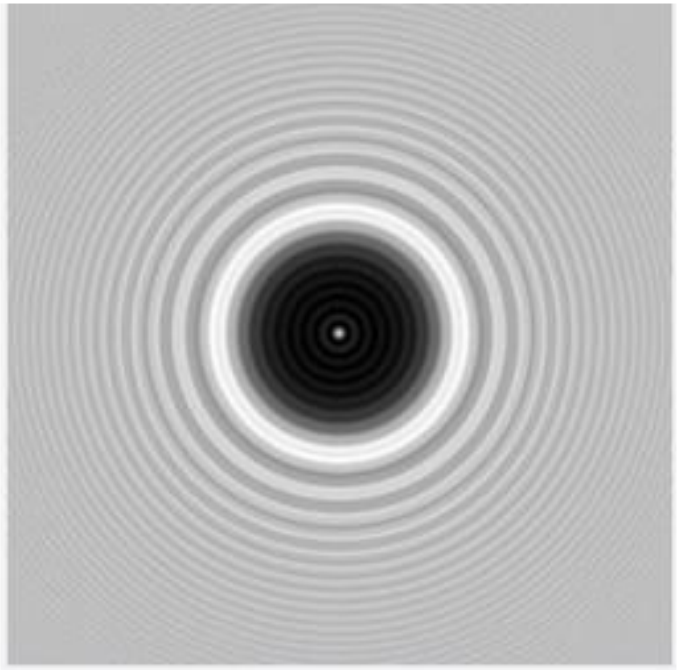


Figure 2-3 The experimental data for $E_n < 100$ MeV are taken from the review by J. M. Peterson, *Phys. Rev.* **125**, 955 (1962) and from the compilation *Neutron Cross Sections*, *BNL* **325**, Sigma Center, Brookhaven National Laboratory (1964). For energies above $E_n = 100$ MeV, the data are taken from V. A. Nedzel, *Phys. Rev.* **94**, 174 (1954) and T. Coor, D. A. Hill, W. F. Hornyak, L. W. Smith, and G. Snow, *Phys. Rev.* **98**, 1369 (1955). We wish to thank J. M. Peterson for help in the preparation of the figure.



Arago's spot

Does semiclassical approximation (M. V. Berry) or ETH (M. Srednicki) apply to quantum strongly interacting fermion systems such as nuclei or unitary Fermi gas???

ETH would predict in this case $\frac{\hbar}{\Delta E} \approx 7\frac{fm}{c}$.

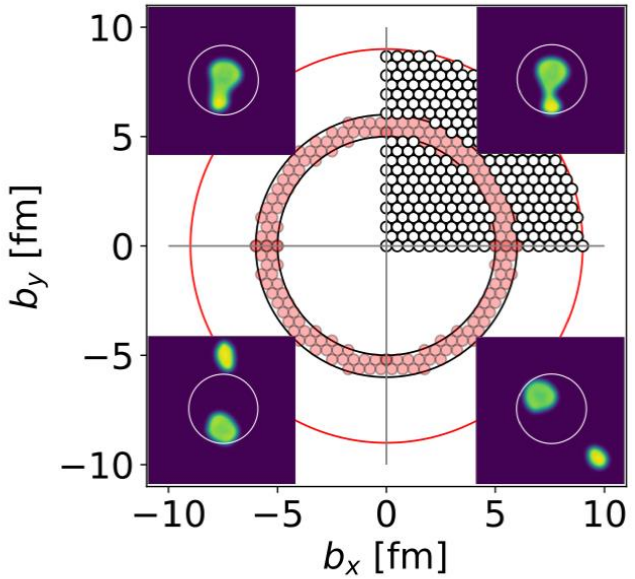
Our CI simulation is what was introduced almost two years ago as the enhanced GCM (eGCM).

Our static and time-dependent CI simulations are de facto the largest type ever of static and time-dependent GCM calculations.

- This is (as far as we are aware of) the first fully microscopic simulation of the formation and evolution of a compound nucleus, conjectured 90 years ago by Niels Bohr (1936).

- **There is a striking qualitative difference between the TDHF and CI time-dependence, as in Newton vs Young.** The images are TDHF simulations!

- In TDHF the two reaction partners are separated at the end of the simulation with probability 1. There is no complexity in the TDHF many-body wave functions. TDDFT with pairing correlations has some complexity, but not enough!



- In the TDCI simulation (which emerged in eGCM framework) the CN wave function is strongly correlated and the CN refused to leave the interaction region and **the probability to form a heavy system with more proton and neutrons than ^{208}Pb is $\Sigma_{Z=83}^{102}\Sigma_{N=127}^{154} P(Z,N) = 0.85$.**

- **The probability to form a stable $_{102}$No$_{154}$ with all A=256 nucleons is 0.34.**

- The "thermalization" of the systems formed in this collision is more than two orders of magnitude slower than what Eigenstate Thermalization Hypothesis (ETH) predicts.

  Since there are obviously no classical periodic orbits in such a reaction this slowing down of thermalization is not due to quantum scarring, see recent Kaplan, Heller, and Keski-Rahkonene, Physics Today, April, 2026.

- **These results are an example of the most coherent structures ever shown to exist in nuclear systems.** The time-dependent many-body wave function we constructed has about 20,000 components of similar magnitude, evolving from an initial state with only 152 components. Perhaps the most coherent phenomenon identified in nuclear theory is the case of pairing correlations in ^{120}Sn (the cleanest condensate in nuclei) where only about 10 Cooper pairs/components (20 neutrons) form a condensate.

- The eGCM many-body wave functions are as important for the nuclear quantum evolution as the Cooper's pair wave function was for superconductivity.

- Size of our Direct Numerical Simulation (DNS) = $((2 \times 256) \times 64^3 \times 2) \times 39630 = 10.6 \times 10^{12}$ real DP numbers
  4 GB per matrix element, one must evaluate **1.57× 109 real DP overlap matrix elements = 5.85 exabytes**

  vs 9.2 petabytes on Frontier memory and 700 petabytes on file system on Frontier

  vs Largest DNS so far on Frontier
  Yeung et al., Classical Turbulence, J. Fluid Mech. 1019, R2 (2025), $32{,}768^3 = 35.18 \times 10^{12}$ real DP numbers

Thank you!