NANJING UNIVERSITY

ECT* EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

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https://arxiv.org/abs/2604.06510

Distribution amplitudes and functions of ground-state scalar and pseudoscalar charmonia

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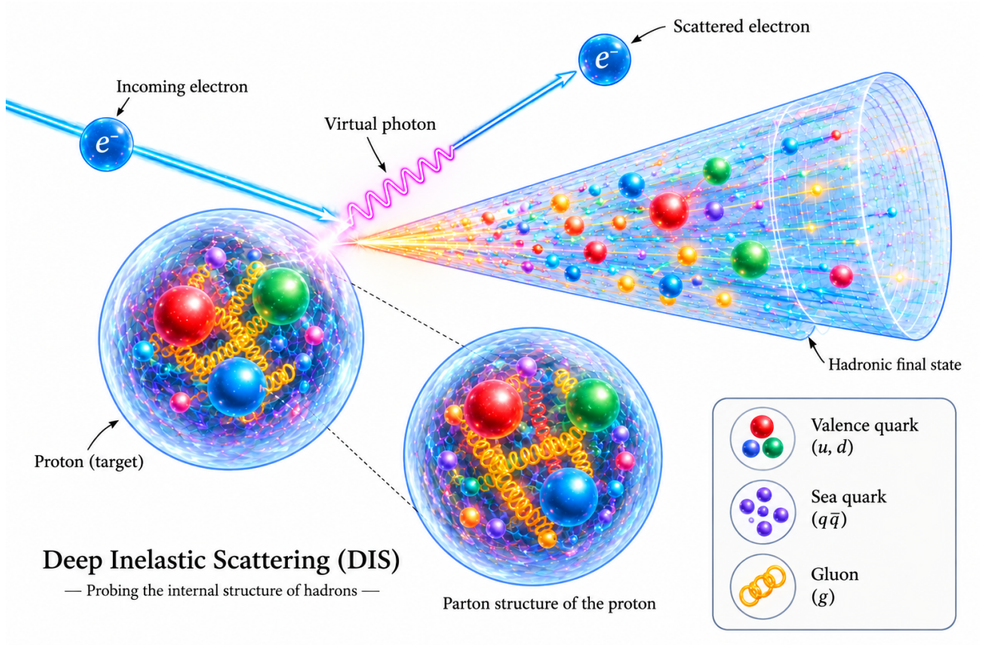
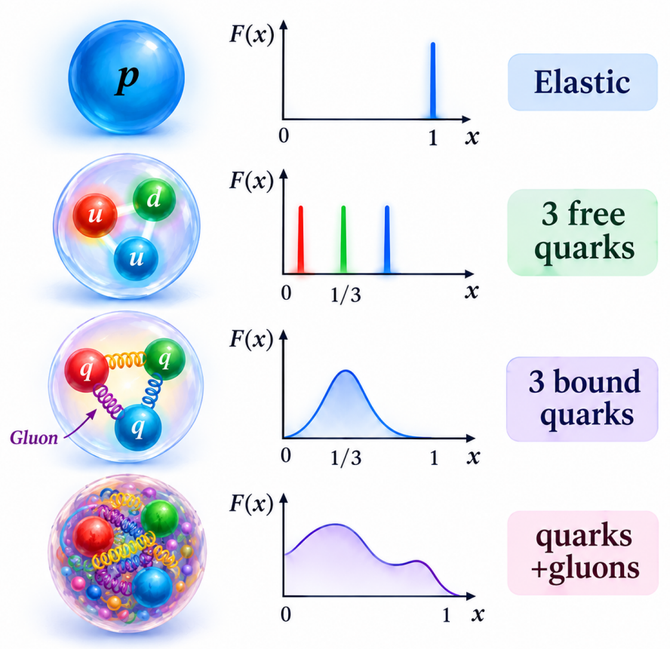
22-10-2026

The Study of Hadrons Structure

- *In 1969, Richard Feynman proposed the Parton Model to explain deep inelastic scattering (DIS), describing the proton as a collection of nearly free, point-like partons that interact with electrons via virtual photon exchange.*
- *Parton distribution functions (PDFs) were introduced to describe how these partons are distributed inside hadrons.*

- *As the energy scale increases, the internal structure of hadrons and the interactions among their constituents become increasingly complex, and all these features are reflected in the PDFs.*

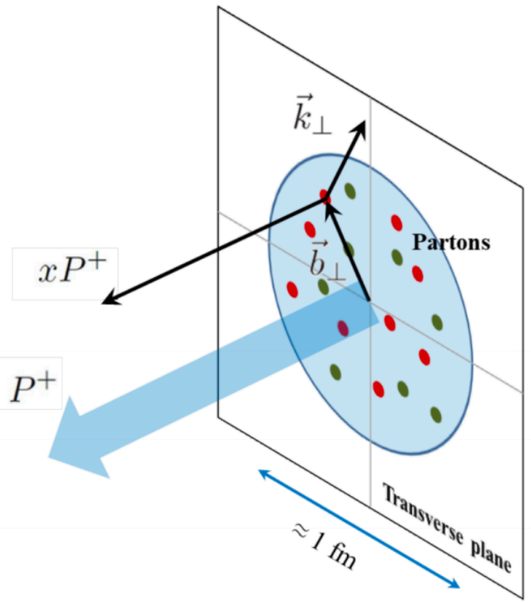




Modern Theoretical Approaches to Hadron Structure

➢ *In the infinite-momentum frame (IMF), a nucleon moving with $P_z \to \infty$ appears as a two-dimensional pancake due to Lorentz contraction. Its constituents (partons) carry momentum fraction x, transverse momentum $\vec{k}_\perp$, and transverse position $\vec{b}_\perp$. Their distributions and polarization dependence reveal the internal structure of hadrons.*

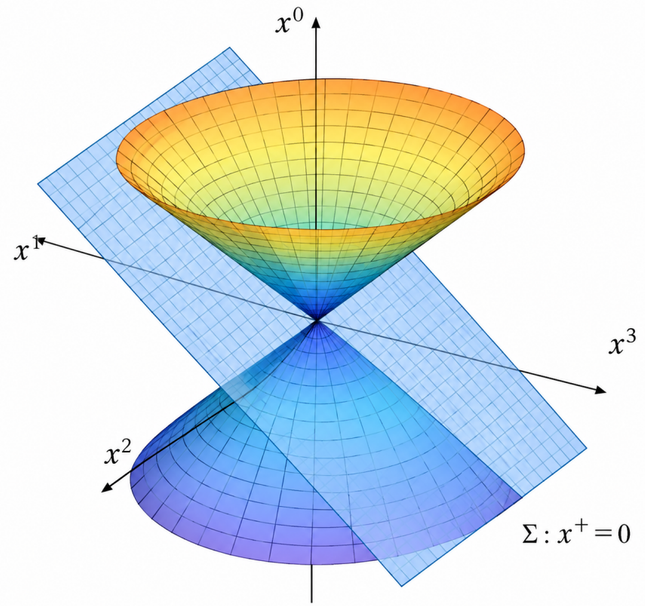
➢ *Hadron structure is described by DAs,PDFs, FFs, TMDs, GPDs, and GTMDs.*



➢ *PDFs and FFs characterize one-dimensional distributions, while TMDs and GPDs probe transverse momentum and spatial structure, respectively. GTMDs combine both, providing a more comprehensive picture of hadron structure.*

Parton Distribution Amplitude and Parton Distribution Functions of χ_{c0}

➢ *Light-Front Formalism:*



✔*quantum mechanics-like wave functions can be defined*

✔*quantum-mechanics-like expectation values can be defined and evaluated*

Why Light-Front Description is effective?

- *In Instant Form quantization, boosts mix Fock states ($[H, K_i] = iP_i$). causing particle-number non-conservation (quantum fluctuation).*

- *The Light-Front approach resolves this by using coordinates where the boost operator acts as a simple dilation, $[K_i, H + iP_i] =+ i(H + iP_i)$. simplifying the vacuum structure.*

- *To the corresponding conjugate coordinates, i.e.:$H + iP_i \rightarrow t + x_i$ (This is exact LF coordinate)*

Why Light-Front Description is effective?

- *Light-Front Quantization (LFQ) ensures boost invariance by transforming the boost into a dilation ($x^+ \to \lambda x^+$), fixing the $x^+ = 0$ surface. Consequently:*

  ✓*Particle Number Conservation : The Fock space expansion retains a clear physical meaning.*

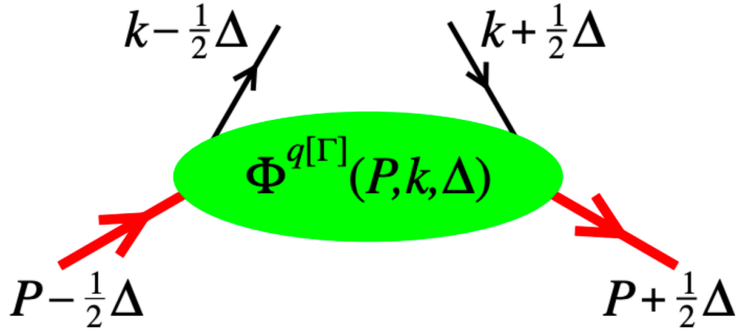
  ✓*Existence of Wave Functions : It allows fo* $\Psi_{x,\vec{k}_\perp}$ *definition of a frame-independent Light-Front Wave Function (LFWF) , which has a probabilistic interpretation in a relativistic system.*

- *This, in turn, introduces the concepts of distribution amplitudes (DAs) and parton distribution functions (DFs), which are analogous to wave functions and probability densities, respectively.*

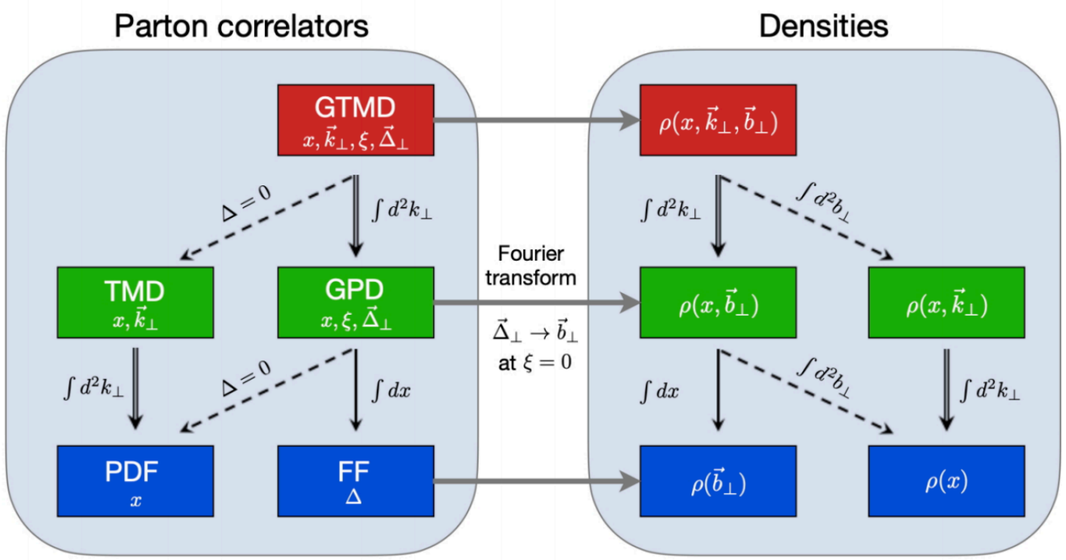
Hadrons Structure in Light Front coordinate

➢ *Quark-Quark correlators:*

$$\Phi^{q[\Gamma]}(P,k,\Delta)=\frac{1}{2}\int\frac{d^4z}{(2\pi)^4}e^{ik\cdot z}\langle p',S'|\overline{\psi}_q\left(-\frac{z}{2}\right)\Gamma\mathcal{W}\left[-\frac{z}{2},\frac{z}{2}\right]\psi_q\left(\frac{z}{2}\right)|p,S\rangle.$$



➢ *All the functions about hadron structure can be defined in terms of this:*



➢ *FF:* $\Phi^{q[\Gamma]}_{\mathrm{FF}}(P,\Delta)=\int d^4k\Phi^{q[\Gamma]}(P,k,\Delta)$

➢ *PDF:* $\Phi^{q[\Gamma]}_{\mathrm{PDF}}(P,x)=\int d^4k\Phi^{q[\Gamma]}(P,k,0)\delta(k^+-xP^+)$

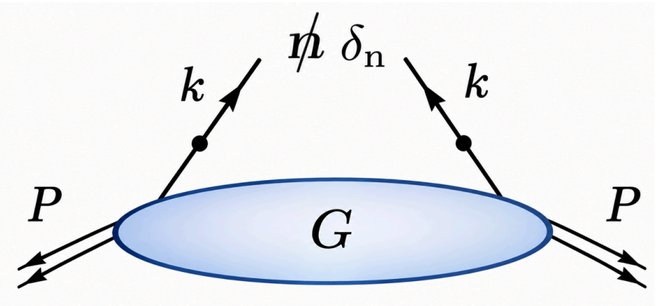
➢ *TMD:* $\Phi^{q[\Gamma]}_{\mathrm{TMD}}(P,x,\vec{k}_\perp)=\int dk^+dk^-\Phi^{q[\Gamma]}(P,k,0)\delta(k^+-xP^+)$

➢ *GPD:* $\Phi^{q[\Gamma]}_{\mathrm{GPD}}(P,x,\xi,\vec{\Delta}_\perp)=\int d^4k\Phi^{q[\Gamma]}(P,k,\Delta)\delta(k^+-xP^+)$

➢ *GTMD:* $\Phi^{q[\Gamma]}_{\mathrm{GTMD}}(P,x,\vec{k}_\perp,\xi,\vec{\Delta}_\perp)=\int dk^+dk^-\Phi^{q[\Gamma]}(P,k,\Delta)\delta(k^+-xP^+)$

Parton Distribution Functions in CSMs framework

- *So, Leading twist unpolarized PDFs means:*
- *1.Forwrad limit:* $\Delta \equiv P' - P = 0;$
- *2. Leading twist:* $\Gamma \to \gamma^+$



[Pion and kaon valence-quark parton distribution functions Trang Nguyen,Craig D. Roberts et al. Published in: Phys.Rev.C 83 (2011) 062201 • e-Print: 1102.2448 [nucl-th]]

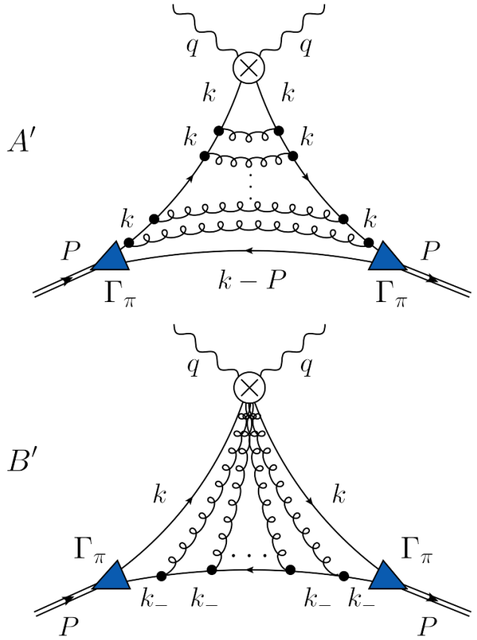
[Basic features of the pion valence-quark distribution function Lei Chang et al. (Jun 20, 2014) Published in: Phys.Lett.B 737 (2014) 23-29 • e-Print: 1406.5450 [nucl-th]]

- *In rainbow ladder truncation:*
  - *BS wave function:* $= \chi$

$$q_A(x) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta) \Gamma(k_\eta, -P) \partial_{k_n} S(k_n) \Gamma(k_{\bar{\eta}}, P) S(k_{\bar{\eta}})$$

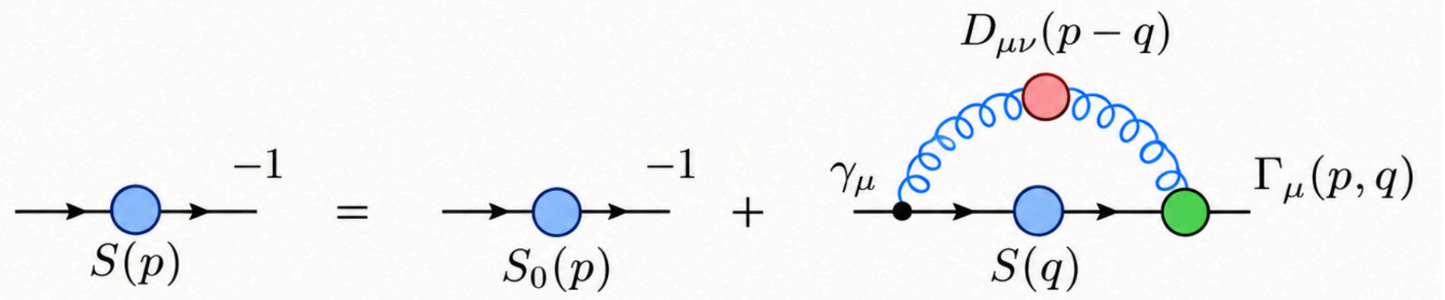
[Symmetry, symmetry breaking, and pion parton distributions Minghui Ding et al. (May 13, 2019) Published in: Phys.Rev.D 101 (2020) 5, 054014 • e-Print: 1905.05208 [nucl-th]]

- *This means that we can define and directly calculate PDFs in a nonperturbative manner!*



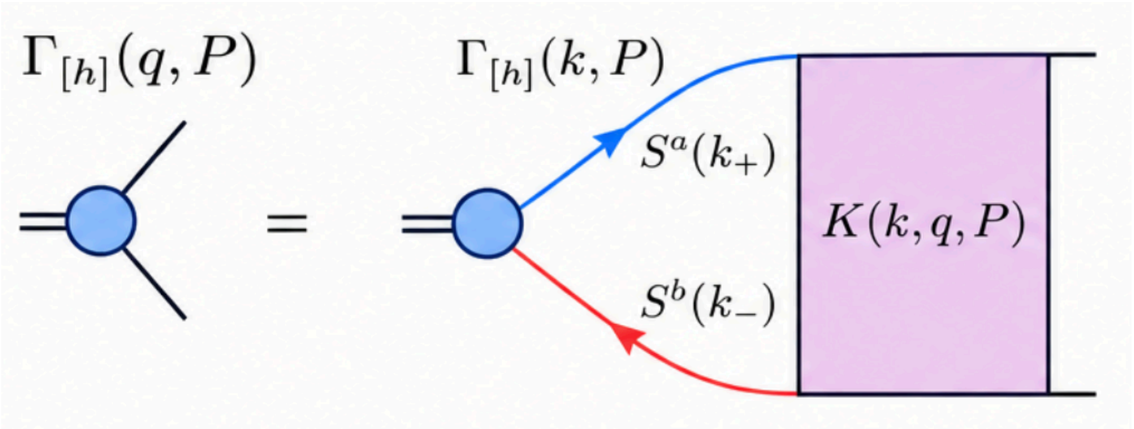
Non-perturbative methods in QCD: Continuous Schwinger Methods(CSM)

➢ *Dyson-Schwinger Equation (Gap equation) :* $S(p)^{-1} = Z_2 i\gamma\cdot p + Z_4 m(\mu) + Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p)$



➢ *Bethe-Salpeter Equation:*

$$\Gamma^{ab}_M(p;P) = \int^\Lambda \frac{d^4q}{(2\pi)^4} K(p,q;P)\times S^a(q+\eta P)\Gamma^{ab}_M(q;P)S^b(q-\bar{\eta}P)$$



Non-perturbative methods in QCD: Continuous Schwinger Methods(CSM)

➢ *Model and Approximation:*

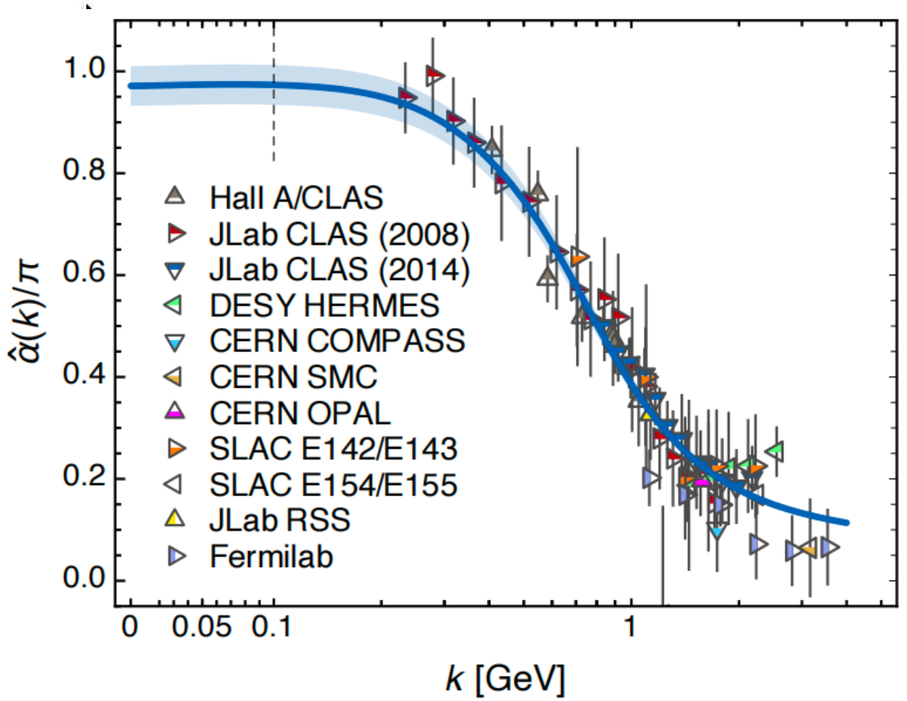
•Rainbow-Ladder Truncation:

$$Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p) \to \int_q^\Lambda \mathcal{G}\left[(p-q)^2\right] D^{\text{free}}_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\gamma_\nu$$

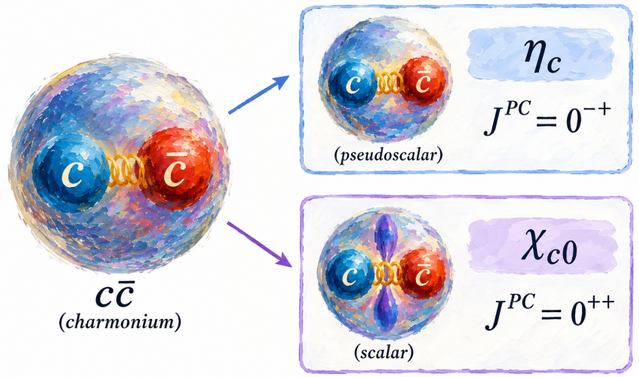
•Gauge Sectors:

$$\mathscr{G}\left(q^2\right) \simeq \frac{4\pi\hat{\alpha}\left(q^2\right)}{q^2 + m_g^2\left(q^2\right)}$$

[M. Ding, C. D. Roberts and S. M. Schmidt, Emergence of Hadron Mass and Structure, Particles no.1, 57-120]



Why do we study Charmonium?



➢ *The "Positronium" of QCD: An Analogy for Precision Physics*

Charmonium ($c\bar{c}$) , the QCD analogue of positronium, is a bound state formed by the strong force. Its spectrum and internal structure serve as a critical probe of non-perturbative QCD, including confinement, and test the completeness of QCD as the theory of the strong interaction.

➢ *Unique Mass Scale*

The charm quark mass ($m_c \approx 1.27$ GeV) sits in the transition region between perturbative and non-perturbative QCD, bridging two distinct physical regimes.

➢ *Probing the Extension of DCSB*

Although the charm quark mass is dominated by the Higgs mechanism, studying its internal structure (e.g., PDA) reveals whether DCSB effects extend into the heavy quark sector.

Parton Distribution Amplitude and Parton Distribution Functions of η_c

- *Parton Distribution Amplitude(PDA):light-front projection of BS wave function*

$$f_5\varphi_5(x) = N_c Z_2 \mathrm{Tr}_D \int_{dk}^{\Lambda} \delta\left(n\cdot k_+ - xn\cdot P\right)\gamma_5 \gamma\cdot n\chi$$

Bethe-Salpeter wave function

- *Parton Distribution Functions(PDF):the quantum mechanical analogue of* $|\psi|^2$

Bethe-Salpeter amplitude

$$q^{\eta_c}(x;\zeta_{\mathcal{H}}) = N_c \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\delta_n^x(k_\eta)\Gamma^P_{\eta_c}(k_{\bar{\eta}\eta};\zeta_{\mathcal{H}})S(k_{\bar{\eta}};\zeta_{\mathcal{H}})$$

$$\times\{n\cdot\frac{\partial}{\partial k_\eta}\left[\Gamma^{-P}_{\eta_c}(k_{\eta\bar{\eta}};\zeta_{\mathcal{H}})S(k_\eta;\zeta_{\mathcal{H}})\right\}$$

Dressed quark propagator

Parton Distribution Amplitude and Parton Distribution Functions of χ_{c0}

➢ *Parton Distribution Amplitude(PDA):light-front projection of BS wave function*

$$\tilde{f}_0\varphi_0(x) = N_c Z_2 \mathrm{Tr}_D \int_{dk}^{\Lambda} \delta\left(n\cdot k_+ - xn\cdot P\right)\gamma\cdot n\chi$$

Bethe-Salpeter wave function

➢ *Parton Distribution Functions(PDF):the quantum mechanical analogue of* $|\psi|^2$

Bethe-Salpeter amplitude

$$q^{\chi_{c0}}(x;\zeta_{\mathcal{H}}) = N_c \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\delta_n^x(k_\eta)\Gamma^P_{\chi_{c0}}(k_{\bar{\eta}\eta};\zeta_{\mathcal{H}})S(k_{\bar{\eta}};\zeta_{\mathcal{H}})$$

$$\times\{n\cdot\frac{\partial}{\partial k_\eta}[\Gamma^{-P}_{\chi_{c0}}(k_{\eta\bar{\eta}};\zeta_{\mathcal{H}})S(k_\eta;\zeta_{\mathcal{H}})\}$$

Dressed quark propagator

Results

https://arxiv.org/abs/2604.06510

Distribution Amplitude of χ_{c0} and η_c :

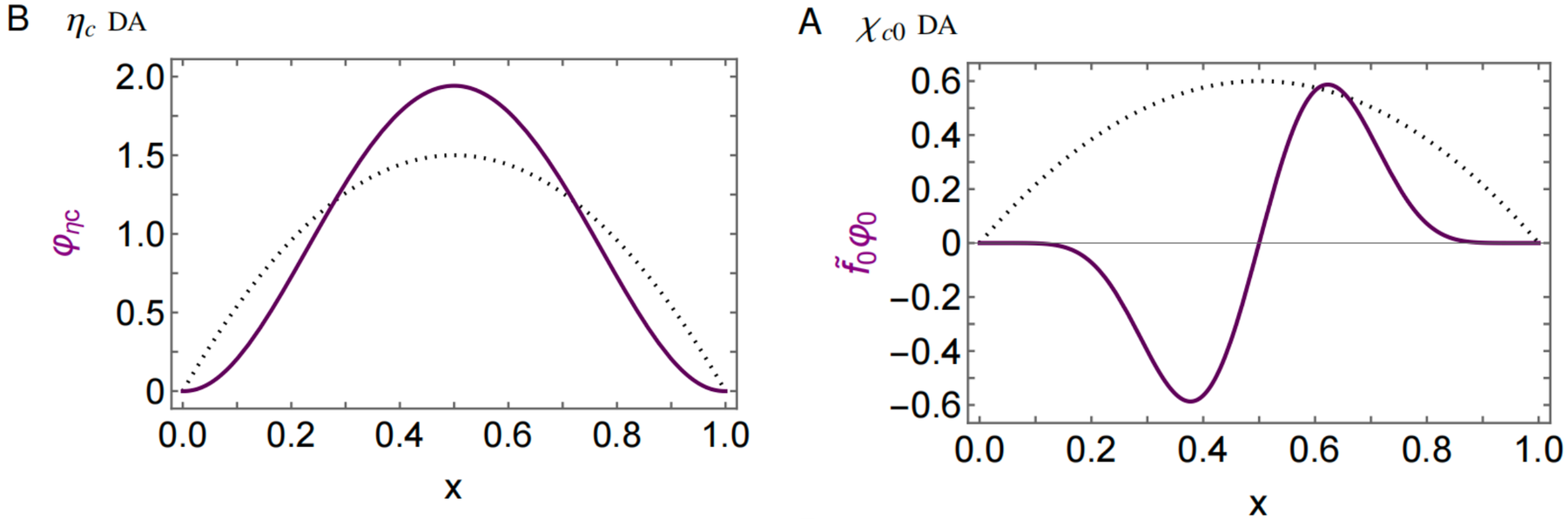


Figure 2: Leading-twist valence dof DAs: Panel A – χ_{c0} (solid purple) and $(2/5)\varphi_{\rm as}$ (dotted black); Panel B – η_c (solid purple) and $\varphi_{\rm as}$ (dotted black).

1. Owing to QCD symmetries, scalar meson DA contains positive and negative regions whose integrals cancel.

2. (Anti)Symmetric about $x = \frac{1}{2}$ and more concentrated than the asymptotic DA.

3. Both DAs are suppressed as $x = 0$ or $x = 1$, disfavoring extreme momentum-sharing configurations.

Results

https://arxiv.org/abs/2604.06510

Distribution Functions in hadron scale ζ_H:

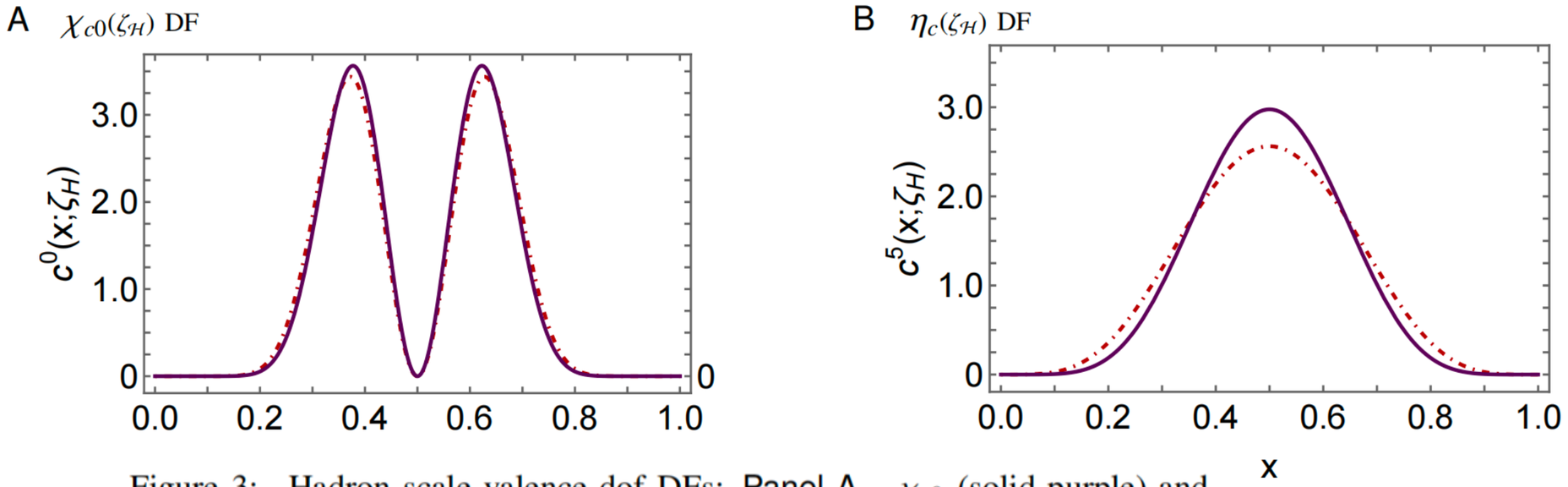


Figure 3: Hadron scale valence dof DFs: Panel A – χ_{c0} (solid purple) and zeroth-moment unit-normalised φ_0^2 (dot-dashed red); Panel B – η_c (solid purple) and zeroth-moment unit-normalised φ_5^2 (dot-dashed red)

1. Valence-quark PDFs are suppressed at $x = 0$ and $x = 1$, reflecting the characteristic momentum distributions of heavy charmonia, which differ from light mesons' PDF, like pion.

2. The χ_{c0} PDF has two peaks with a central minimum, unlike the η_c PDF.

3. The valence-quark PDF is approximately proportional to the squared DA, with better accuracy for χ_{c0} than for η_c.

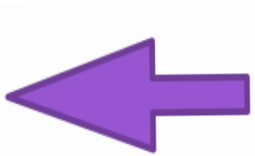
Evolution of PDF

- *At the hadron scale ζ_H (~0.3GeV) , all degrees of freedom are carried by the valence quarks; however, this scale is not experimentally accessible. To compare our results with experimental measurements, we must evolve them to a higher scale using the DGLAP equations.*

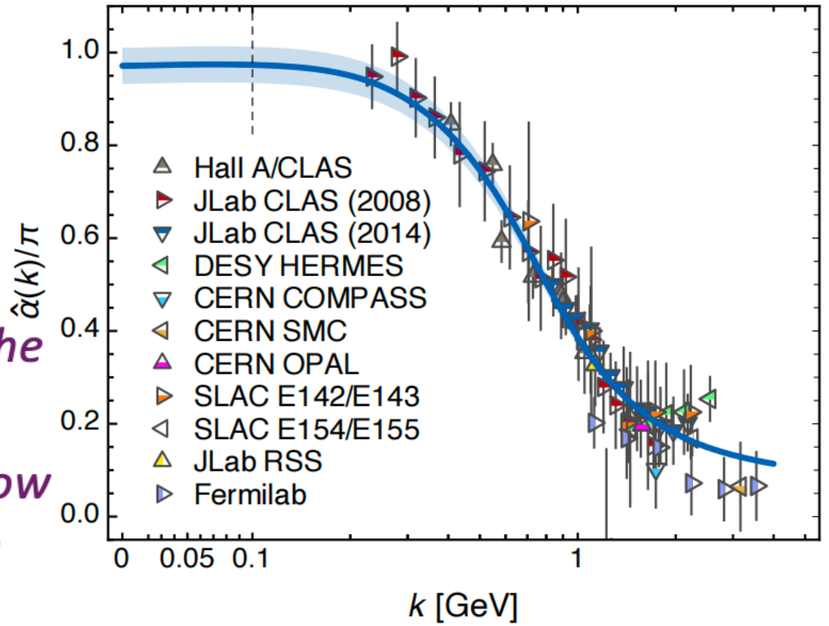
- *All Orders Evolution Scheme of Parton Distribution:*

[All-Orders Evolution of Parton Distributions: Principle, Practice, and Predictions
Pei-Lin Yin et al. (Jun 5, 2023) Published in: Chin.Phys.Lett. 40 (2023) 9, 091201 • e-Print: 2306.03274 [hep-ph]]

$$\frac{\partial f_i\left(x,\zeta^2\right)}{\partial \ln \zeta^2} = \frac{\widehat{\alpha_{1\ell}}\left(\zeta^2\right)}{2\pi} \sum_j \int_x^1 \frac{dy}{y} P_{ij}\left(\frac{x}{y}\right) f_j\left(y,\zeta^2\right)$$



- *Use the process-independent (PI) effective coupling $\hat{\alpha}(k^2)$ to replace the perturbative running coupling in the standard DGLAP equations.*
- *This effective coupling describes the strength of QCD interactions from low to high momenta and incorporates nonperturbative dynamical effects in the low-momentum region.*



Evolution of PDF

Introducing mass-dependent splitting functions :

- the c-quark is more massive than the u-quark.
- Hence, physically: valence $\bar{c}$-quarks must produce fewer gluons than valence u-quarks; and gluon splitting must produce fewer $\bar{c}c$-pairs than light-quark pairs.
- Such effects can be expressed in the splitting functions, and we now illustrate their impact.

Splitting functions:

[Kaon and pion parton distributions Zhu-Fang Cui et al. (Jun 24, 2020) Published in: Eur.Phys.J.C 80 (2020) 11, 1064]

$$\begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix}$$

$$P_{c\leftarrow c}(z) \to P_{q\leftarrow q}(z) - \Delta_{c\leftarrow c}(z,\zeta),$$
$$P_{c\leftarrow g}(z) \to P_{c\leftarrow g}(z) + \Delta_{c\leftarrow g}(z,\zeta),$$

where

$$\Delta_{c\leftarrow c}(z,\zeta) = \sqrt{3}(1-2z)\frac{\sigma_s\delta^2}{\delta^2+(\zeta-\zeta_H)^2}$$

$$\Delta_{c\leftarrow g}(z,\zeta) = \sqrt{5}\left(1-6z+6z^2\right)\frac{\sigma_s\delta^2}{\delta^2+(\zeta-\zeta_H)^2}$$

- the $\sigma_s = 1$ is a strength parameter and $\delta_{u/d} = 0$, $\delta_s = 0.1$, $\delta_c = 0.9$ to distinguish different flavors of quarks.

Evolution of PDF

https://arxiv.org/abs/2604.06510

Evolved: $\zeta_H \to \zeta_3$ =3.2 GeV :

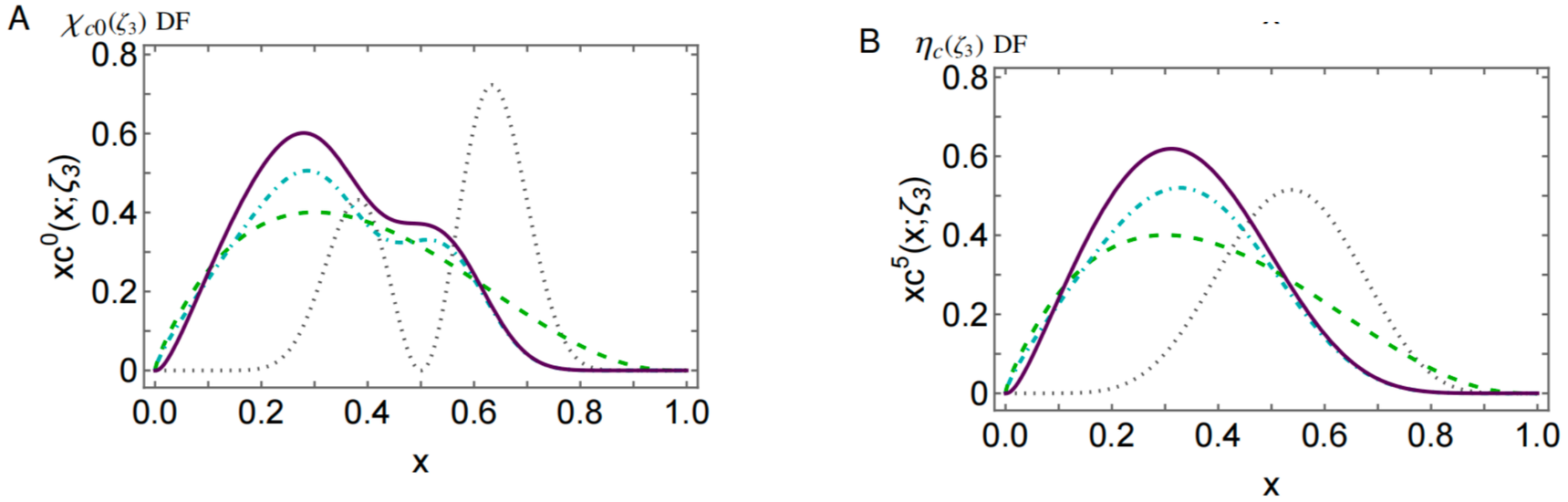


Figure 4: Valence dof DFs in Fig. 3 evolved $\zeta_{\mathcal{H}} \to \zeta_3$: Panel A – χ_{c0}. Legend. Solid purple curve – $xc^0(x;\zeta_3)$ obtained using mass-dependent splitting functions; dot-dashed cyan – $xc^0(x;\zeta_3)$ with mass-independent valence dof splitting; dashed green – $xu^\pi(x;\zeta_3)$; dotted grey – $(1/3)xc^0(x;\zeta_{\mathcal{H}})$. Panel B – η_c. Legend. Analogous to that in A.

1. The χ_{c0} PDF retains two peaks, with a less pronounced central minimum.

2.QCD evolution makes the valence PDFs of χ_{c0} and η_c more alike at higher energy scales.

Evolution of PDF

https://arxiv.org/abs/2604.06510

Evolved: $\zeta_H \to \zeta_3$ =3.2 GeV :

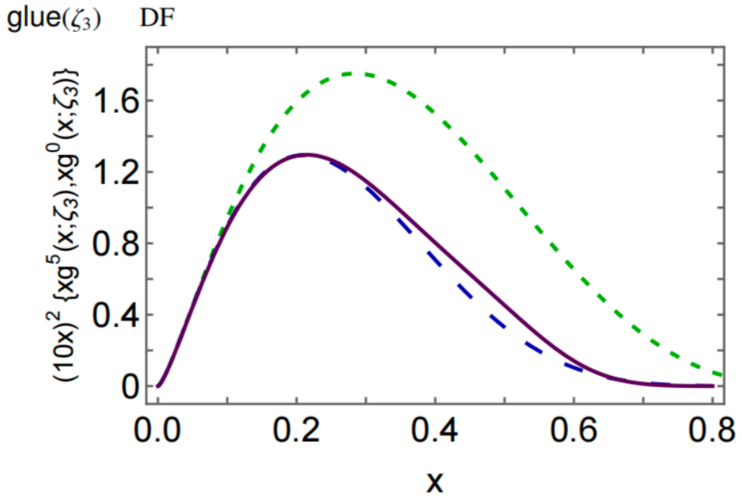


Figure 5: Glue DFs at ζ_3 in the χ_{c0} (solid purple), η_c (dashed blue), and π (short-dashed green). *N.B.* The image depicts $(10x)^2 \times xg(x;\zeta_3)$ so as to highlight differences that appear on the valence domain. On $x \lesssim 0.1$, the glue DFs of all three systems are substantively identical.

Table 6: Species light-front momentum fractions at $\zeta = \zeta_3 := 3.2\,\text{GeV}$. The third row provides a comparison with pion values [24]. Column 1 is $(c+\bar{c})_{\text{valence}}$ for χ_{c0} and η_c; and $(u+\bar{d})_{\text{valence}}$ for π. The final row records the relative difference between the momentum fractions in the light (L) and heavy (H) systems.

	valence	glue	$(u+\bar{u})_{\text{sea}}$	$(d+\bar{d})_{\text{sea}}$	$(s+\bar{s})_{\text{sea}}$	$(c+\bar{c})_{\text{sea}}$
χ_{c0}	0.49	0.40	0.033	0.033	0.027	0.017
η_c	0.49	0.40	0.033	0.033	0.027	0.017
π	0.44	0.44	0.038	0.038	0.031	0.019
$\%^{L/H}_{\text{diff.}}$	−10	10	15	15	15	12

Gluon distributions become nearly indistinguishable, reflecting similar parton evolution patterns.

Summary

- ***Theoretical framework:*** *CSM, guided by symmetry-preserving RL truncation, provides a nonperturbative framework for calculating hadron DAs and PDFs.*

- ***Key finding:****The approximate squared relation between DAs and PDFs suggests a factorized structure of charmonium light-front wave functions.*

- ***Physical implications:****At high energy scales, the PDFs of χ_{c0} and η_c become increasingly similar, yet remain distinct from those of light mesons. These results provide benchmarks for future studies of heavy-quark hadron structure.*

Overall, our results provide new insights into the nonperturbative structure of charmonia.

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