Quantum entanglement in hadron scattering





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Contents

Overview

- Entanglement suppression for NN scattering

S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019);
I. Low, T. Mehen, PRD104, 074014 (2021)

(- Entanglement for diquark correlation)

C. Qian, S. Xu, Y.-G. Yang, X. Zhao, arXiv:2603.16681 [hep-ph]

Application of entanglement suppression

- SU(3) baryon decuplet ($\Omega\Omega$) scattering

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, Phys. Rev. Res. 7, 043306 (2025);
K. Sone, T.R. Hu, F.K. Guo, T. Hyodo, I. Low, MPLA 41, 2642007 (2026)

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Summary

Basic concept of entanglement suppression

Entanglement power (EP) of S-matrix $E(\hat{S})$

- ability of generating entanglement



$$|\psi_{\text{out}}\rangle = \hat{S}\,|\psi_{\text{in}}\rangle$$

$E(\hat{S})$

entangle?



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Entanglement suppression

S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)

$E(\hat{S}) = 0$ —> emergent symmetries of NN scattering

Generalization to other hadrons and nuclei, …

Q. Liu, I. Low, T. Mehen, PRC107, 025204 (2023);
T. Kirchner, W. Elkamhawy, H.-W. Hammer, Few-Body Syst. 65, 29 (2024);
T.R. Hu, S. Chen, F.K. Guo, PRD110, 014001 (2024); …
C.E.P. Robin, M.J. Savage, arXiv:2604.26376 [quat-ph]

Entanglement measures

Entanglement measure for bipartite state $|\psi\rangle \in \mathscr{H}_A \otimes \mathscr{H}_B$

- von Neumann entropy

$$\mathscr{E}_{\mathrm{vN}}(|\psi\rangle) = -\,\mathrm{Tr}_A[\rho_A \log \rho_A], \quad \rho_A = \mathrm{Tr}_B\,|\psi\rangle\langle\psi|$$

- linear entropy (expansion $\log \rho_A \sim (\rho_A - 1)$ **)**

$$\mathscr{E}(|\psi\rangle) = 1 - \mathrm{Tr}_A[\rho_A^2]$$

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- **linear entropy (expansion** $\log\rho_A \sim (\rho_A - 1)$ **)**

$$\mathcal{E}(|\psi\rangle) = 1 - {\rm Tr}_A[\rho_A^2]$$



- **product state: minimum (=0)**

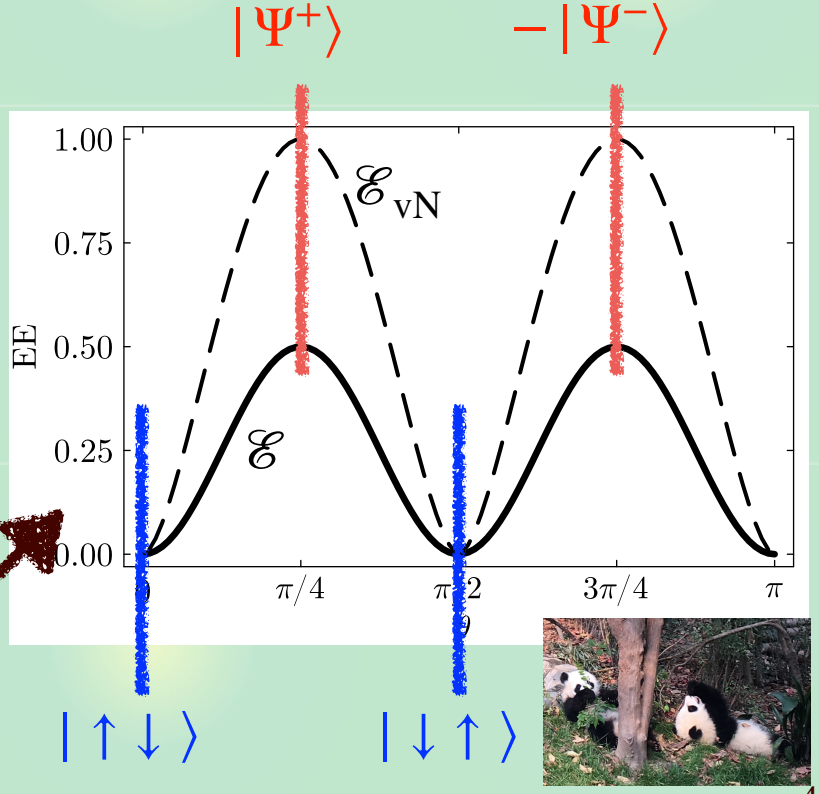
$$|\uparrow\downarrow\rangle = |\uparrow_A\rangle \otimes |\downarrow_B\rangle$$

- **Bell state: maximum**

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle \pm \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$

Linear combination:

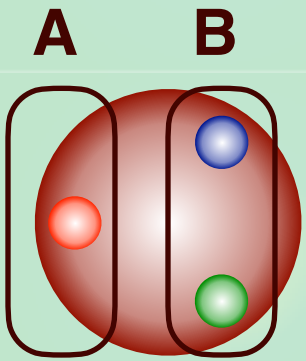
$$|\psi\rangle = \cos\theta\,|\uparrow\downarrow\rangle + \sin\theta\,|\downarrow\uparrow\rangle$$



Detour: Entanglement for diquark?

Color entanglement of quarks in a proton

A. Dumitru, E. Kolbusz, PRD105, 074030 (2022)



- q and (qq) are maximally entangled in color

$$\rho_A = \mathrm{Tr}_B^{\mathrm{color}} |qqq\rangle\langle qqq| = \frac{1}{3} I_{3\times 3} \quad \Rightarrow \quad \mathscr{E}_{\mathrm{vN}} = \log 3$$

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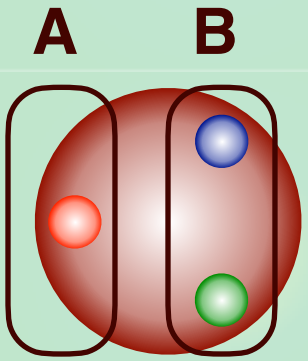
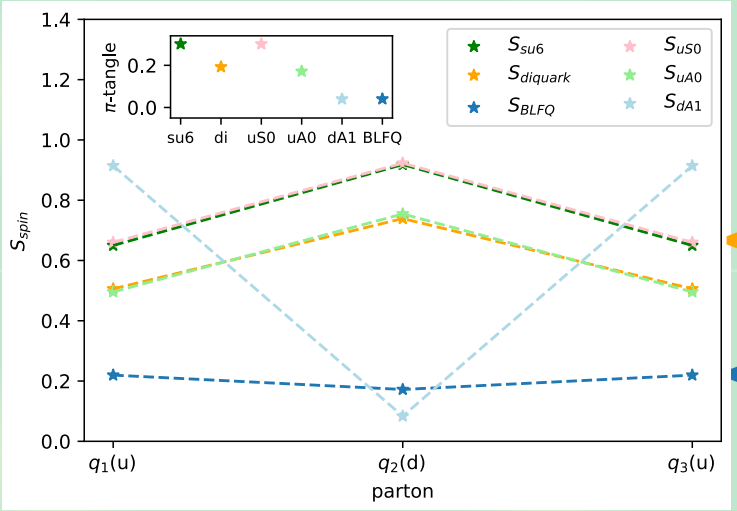


Spin entanglement of quarks in a proton

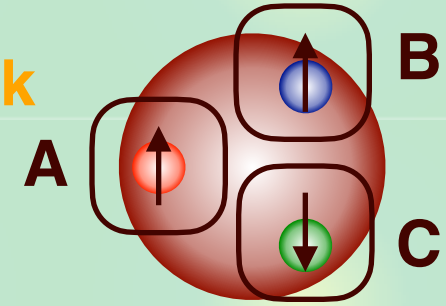
C. Qian, S. Xu, Y.-G. Yang, X. Zhao, arXiv:2603.16681 [hep-ph]

Entanglement



<— quark-diquark

<— BLFQ



Entanglement ~ diagnostic of emergent diquark correlations

Entanglement power

Entanglement power (EP) of operator $\hat{U}$ acting on $|\psi\rangle$

P. Zanardi, C. Zalka, L. Faoro, PRA62, 030301 (2000)

- average over entanglement measure of $\hat{U}|\psi_A\rangle \otimes |\psi_B\rangle$

$$E(\hat{U}) = \overline{\mathscr{E}\left(\hat{U}|\psi_A\rangle \otimes |\psi_B\rangle\right)}$$

$$= \int d\omega_A d\omega_B \; \mathscr{E}(\hat{U}|\psi_A(\omega_A)\rangle \otimes |\psi_B(\omega_B)\rangle)$$

$$\mathscr{E}(|\psi_A\rangle \otimes |\psi_B\rangle) = 0$$

—> ability of $\hat{U}$ to generate entanglement (state independent)

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—> ability of $\hat{U}$ to generate entanglement (state independent)

- spin 1/2 (qubit): rays in $\mathbb{C}^2 \simeq \mathbb{CP}^1$ **—> Fubini-Study measure**

$$|\psi(\omega_2)\rangle = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 e^{i\nu_1} \end{pmatrix}, \quad d\omega_2 = \frac{1}{\pi} d\theta_1 \cos\theta_1 \sin\theta_1 d\nu_1 \qquad \begin{matrix} \theta_1 \in [0,\pi/2) \\ \nu_1 \in [0,2\pi) \end{matrix}$$

(equivalent to Bloch sphere with $\theta_1 = \theta/2$, $\nu_1 = \phi$, $d\omega_2 = d\Omega/4\pi$)

S-matrix for NN scattering

S-matrix for pn scattering: 2 phase shifts δ_0, δ_1

$$\hat{S} = \underbrace{\exp\{2i\delta_0\}}_{J=0}\frac{1-\boldsymbol{\sigma}_A\cdot\boldsymbol{\sigma}_B}{4} + \underbrace{\exp\{2i\delta_1\}}_{J=1}\frac{3+\boldsymbol{\sigma}_A\cdot\boldsymbol{\sigma}_B}{4}$$

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Entanglement power of NN scattering

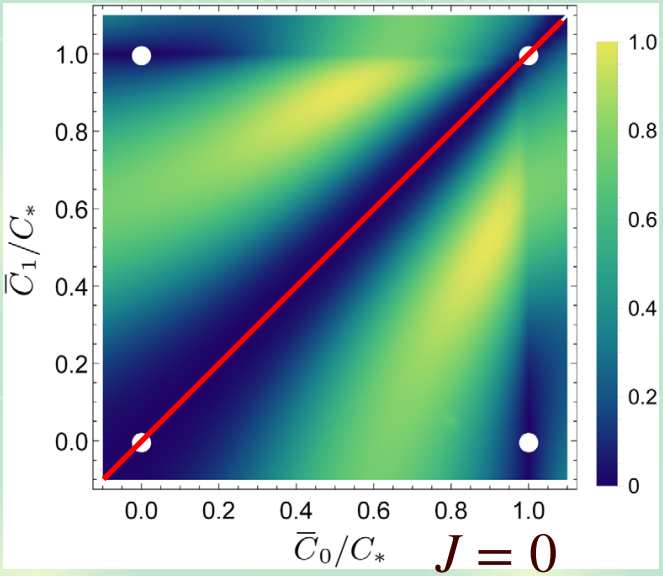
S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)

$$E(\hat{S}) = \frac{1}{6}\sin^2[2(\delta_0-\delta_1)] \geq 0$$

$E(\hat{S})=0$ **is achieved if**

$$\begin{cases} \delta_0 = \delta_1 & \textbf{spin-flavor SU(4)} \\ (\delta_0,\delta_1) = (0,0),(0,\pi/2),(\pi/2,0),(\pi/2,\pi/2) \end{cases}$$

$|a_0| \to \infty$: **NR conformal symmetry**



$\delta_0=\delta_1 : \hat{S} \sim$ **Identity gate,** $|\delta_0-\delta_1| = \pi/2 : \hat{S} \sim$ **SWAP gate**

I. Low, T. Mehen, PRD104, 074014 (2021)

Further evidences of entanglement supression

Spin-flavor symmetries with three flavors

S.R. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)

- $\mathrm{SU}(6) \sim (u, d, s) \otimes (\uparrow, \downarrow)$ <— large Nc
- $\mathrm{SU}(16) \sim (n, p, \Lambda, \ldots) \otimes (\uparrow, \downarrow)$ <— Entanglement suppression
- Lattice QCD evaluation of LEC favors $\mathrm{SU}(16)$

M.L. Wagman, *et al.*, (NPLQCD), PRD96, 114510 (2017)

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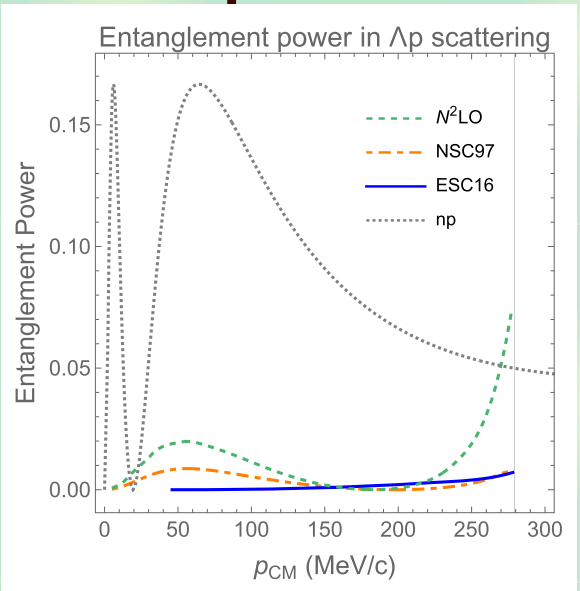
Momentum dependence of Entangelement power

Q. Liu, I. Low, PLB, 856, 138899 (2024)

- $np, \Lambda p$ scatterings

$$E(p_{\mathrm{CM}}) = \frac{1}{6}\sin^2[2(\delta_0(p_{\mathrm{CM}}) - \delta_1(p_{\mathrm{CM}}))]$$

- EP suppressed at low energies



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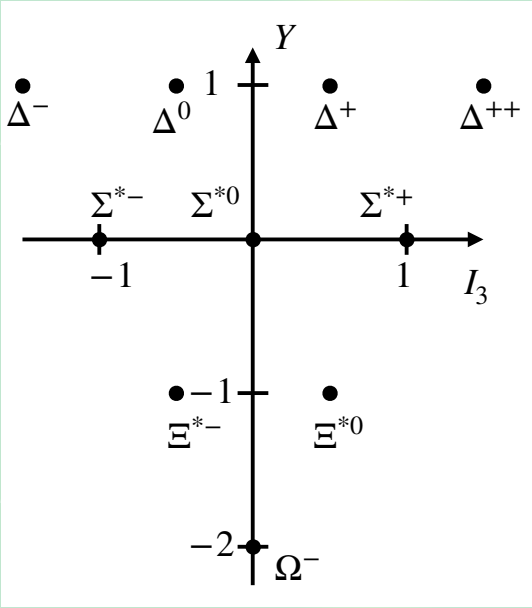
Motivation

Decuplet baryons

- spin 3/2
- ground states with Octet in quark model



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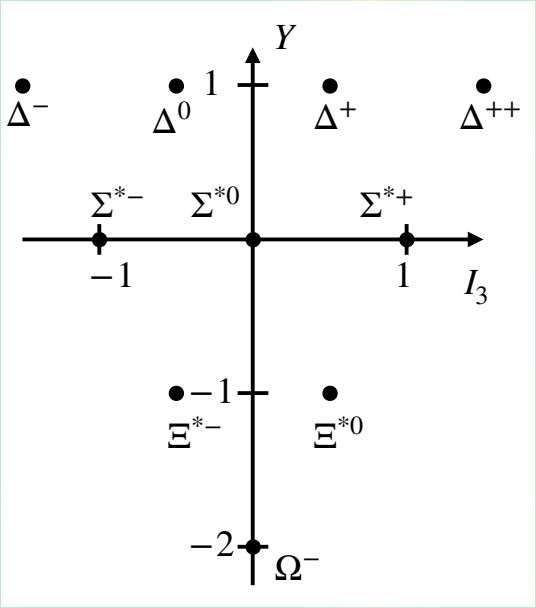

Large scattering length a_0 ($\delta \sim \pi/2$)?

- $\Delta\Delta$ dibaryon, $d^*(2380)$?

F. Dyson, N.H. Xuong, PRL13, 815 (1964);
P. Adlarson *et al.*, (WASA-at-COSY), PRL102, 052301 (2009)

- lattice QCD for $\Omega\Omega$ scattering: $a_0(J=0) = 4.6 \text{ fm} \gg 1 \text{ fm}$

S. Gongyo, *et al.*, (HAL QCD), PRL120, 212001 (2018)

Quantum information viewpoint

- spin 3/2 scattering = 4d-qudit quantum gates

Spin 3/2 state

Spin 3/2 (4d-qudit): rays in $\mathbb{C}^4 \simeq \mathbb{CP}^3$

$$|\psi_{\mathbf{4}}\rangle = \begin{pmatrix} \cos\theta_1 \sin\theta_2 \sin\theta_3 \\ \sin\theta_1 \sin\theta_2 \sin\theta_3 e^{i\nu_1} \\ \cos\theta_2 \sin\theta_3 e^{i\nu_2} \\ \cos\theta_3 e^{i\nu_3} \end{pmatrix}, \quad d\omega_{\mathbf{4}} = \frac{3!}{\pi^3} \prod_{i=1}^{3} d\theta_i d\nu_i \cos\theta_i \sin^{2i-1}\theta_i,$$

$$\theta_i \in [0,\pi/2), \quad \nu_i \in [0,2\pi)$$

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$$\theta_i \in [0,\pi/2), \quad \nu_i \in [0,2\pi)$$

Spin decomposition (2 symmetric, 2 anti-symmetric)

$$\frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_{\text{A}} \oplus \underbrace{1 \oplus 3}_{\text{S}}$$

S-matrix (distinguishable): 4 components with $\delta_0, \delta_1, \delta_2, \delta_3$

$$\hat{S} = \mathcal{J}_0 \exp\{2i\delta_0\} + \mathcal{J}_1 \exp\{2i\delta_1\} + \mathcal{J}_2 \exp\{2i\delta_2\} + \mathcal{J}_3 \exp\{2i\delta_3\}$$

$$\mathcal{J}_0 = \frac{33}{128} + \frac{31}{96}(\boldsymbol{t}_{3/2} \cdot \boldsymbol{t}_{3/2}) - \frac{5}{72}(\boldsymbol{t}_{3/2} \cdot \boldsymbol{t}_{3/2})^2 - \frac{1}{18}(\boldsymbol{t}_{3/2} \cdot \boldsymbol{t}_{3/2})^3, \cdots$$

Entanglement power

Entanglement power (~3 days on mathematica)

$$
\begin{aligned}
E(\hat{S}) = \frac{1}{200000}\Big\{ & 77482 - 2100\cos\left[2\left(\delta_0+\delta_1-\delta_2-\delta_3\right)\right] \\
& -2100\cos\left[2\left(\delta_0-\delta_1+\delta_2-\delta_3\right)\right] - 2100\cos\left[2\left(\delta_0-\delta_1-\delta_2+\delta_3\right)\right] \\
& -1200\cos\left[2\left(\delta_0-2\delta_1+\delta_2\right)\right] - 4200\cos\left[2\left(\delta_0+\delta_2-2\delta_3\right)\right] \\
& -8400\cos\left[2\left(\delta_1-2\delta_2+\delta_3\right)\right] \\
& -375\cos\left[4\left(\delta_0-\delta_1\right)\right] - 10800\cos\left[2\left(\delta_0-\delta_2\right)\right] - 625\cos\left[4\left(\delta_0-\delta_2\right)\right] \\
& -875\cos\left[4\left(\delta_0-\delta_3\right)\right] - 2175\cos\left[4\left(\delta_1-\delta_2\right)\right] - 26376\cos\left[2\left(\delta_1-\delta_3\right)\right] \\
& -5481\cos\left[4\left(\delta_1-\delta_3\right)\right] - 10675\cos\left[2\left(\delta_2-\delta_3\right)\right]\Big\}
\end{aligned}
$$

$E(\hat{S}) = 0$ **is achieved if (all cos = +1)**

$$
\delta_0 = \delta_2 = \delta_{\text{even}}, \quad \delta_1 = \delta_3 = \delta_{\text{odd}},
\qquad
\begin{cases} |\delta_{\text{even}} - \delta_{\text{odd}}| = 0 & \text{: Identity gate} \\ |\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2 & \text{: SWAP gate} \end{cases}
$$

Idntical particles and sub-unitary S-matrix

Identical particles: symmetric spin states are forbidden

- nn scattering/ΩΩ scattering: only antisymmetric component

$$\hat{S}_{nn} = \mathcal{J}_0 \exp\{2i\delta_0\}$$ **spin 1 absent**

$$\hat{S}_{\Omega\Omega} = \mathcal{J}_0 \exp\{2i\delta_{0,\mathbf{28}}\} + \mathcal{J}_2 \exp\{2i\delta_{2,\mathbf{28}}\}$$ **spin 1 and 3 absent**

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S-matrix is not unitary in full space of $|\psi_{\text{in}}\rangle$: $\hat{S}_{nn}\hat{S}^{\dagger}_{nn} = \mathcal{J}_0 \neq 1$

$$\hat{S} = \begin{pmatrix} (\text{Antisymmetric}) & 0 \\ 0 & (\text{Symmetric}) \end{pmatrix}, \quad \mathcal{J}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

EP is obtained by averaging over initial product states

$$E(\hat{U}) = \overline{\mathcal{E}\left(\hat{U}\,|\psi_A\rangle \otimes |\psi_B\rangle\right)}$$

neither antisymmetric nor symmetric

Normalized entanglement power

Naive calculation for nn scattering: larger than 1/2?

$$E(\hat{S}_{nn}) = \frac{23}{24}$$

Density matrix of out state is not properly normalized

$$\rho = |\psi_{\rm out}\rangle\langle\psi_{\rm out}| = \hat{S}_{nn}|\psi_{\rm in}\rangle\langle\psi_{\rm in}|\hat{S}_{nn}^{\dagger}, \quad {\rm Tr}[\rho] = \langle\psi_{\rm in}|\mathcal{J}_0|\psi_{\rm in}\rangle < 1$$

—> Properly normalized Entanglement power

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—> Properly normalized Entanglement power

Normalized density matrix

$$\tilde{\rho} = \frac{|\psi_{\rm out}\rangle\langle\psi_{\rm out}|}{\langle\psi_{\rm out}|\psi_{\rm out}\rangle} \quad \Rightarrow \quad E(\hat{S}_{nn}, \tilde{\rho}) = \frac{1}{2}$$

- consistent with J=0 final state (Bell state)

ΩΩ scattering

ΩΩ S-matrix

$$\hat{S}_{\Omega\Omega} = \mathcal{J}_0 \exp\{2i\delta_{0,\mathbf{28}}\} + \mathcal{J}_2 \exp\{2i\delta_{2,\mathbf{28}}\}$$

Properly normalized EP of ΩΩ S-matrix

$$E_2(\hat{S}_{\Omega\Omega}) = \frac{1}{48}\left\{25 - \cos\left[4\left(\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}\right)\right]\right\}$$

$E_2(\hat{S}_{\Omega\Omega})$ is minimized (1/2, not 0) if

$$\begin{cases} |\,\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}\,| = 0 & \hat{S} \propto \mathcal{J}_0 + \mathcal{J}_2 \equiv \mathcal{P}_{\mathrm{A}} \\ |\,\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}\,| = \pi/2 & \hat{S} \propto -\mathcal{J}_0 + \mathcal{J}_2 \equiv \mathrm{SWAP}_{\mathrm{A}} \end{cases}$$

spin SU(4)

NR conformal

ΩΩ scattering

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spin SU(4)

NR conformal

$a_0(J=0) \gg 1$ fm by lattice QCD suggests either

- $|a_0(J=2)| \gg 1$ fm (unitary limit, $|\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = 0$) or
- $a_0(J=2) \sim 0$ (noninteracting, $|\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = \pi/2$)

Summary

- Entanglement suppression for spin 3/2
- Final state should be properly normalized for identical particle (e.g. $\Omega\Omega$) scattering
- $\Omega\Omega$ scattering emergent symmetries
  - spin SU(4) symmetry ($\hat{S} \propto \mathscr{P}_{\mathrm{A}}$)
  - NR conformal ($\hat{S} \propto \mathrm{SWAP}_{\mathrm{A}}$)
- Application
  - Spin 2 scattering is constrained from spin 0

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, Phys. Rev. Res. 7, 043306 (2025);
K. Sone, T.R. Hu, F.K. Guo, T. Hyodo, I. Low, MPLA 41, 2642007 (2026)