Nucleon Lightfront Wave Functions: Observing diquarks correlations in hadron structure

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- In order to recover a probabilistic interpretation, we need to project "covariant" QFT onto the Lightfront.

LightFront Wave Functions

Hadrons seen as Fock States

- Lightfront quantization allows us to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}} |q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}} |q\bar{q},q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particle Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

LFWFs and Hadron structure

- Since they describe hadron states, LFWFs can be used to compute matrix element of the type :

$$\langle p'|O(z_1,\dots z_n)|p\rangle \to \sum_N \sum_{N'} \psi^*_{N'}\psi_N \langle q_1 \dots q_{N'}|O(z_1,\dots,z_n)|q_1 \dots q_N\rangle$$

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- These matrix elements typically encode hadron structure properties (EFF, PDFs, GPDs, etc)
- Thus, we can compute hadron structure distributions, as a convolution (or overlap) of Lightfront Wave functions

M. Diehl *et al.*, Nucl. Phys B596 (2001)

Three-body Nucleon LFWFs

The different possibilities to combine three quarks with a definite helicity projection into a nucleon of helicity $\uparrow$ can be seen as:

$$\begin{aligned}|P,\uparrow\rangle =& |u_\uparrow, u_\downarrow, d_\uparrow; P,\uparrow\rangle_{\frac{1}{2}} + |u_\uparrow, u_\downarrow, d_\downarrow; P,\uparrow\rangle_{-\frac{1}{2}} \\ &+ |u_\uparrow, u_\uparrow, d_\uparrow; P,\uparrow\rangle_{\frac{3}{2}} + |u_\downarrow, u_\downarrow, d_\downarrow; P,\uparrow\rangle_{-\frac{3}{2}} + \text{permutations}\end{aligned}$$

Fock state decomposition

$$|P;\uparrow\rangle_{1/2} = \int [\mathcal{D}\kappa]\,(\psi^1 + i(k_1^x k_2^y - k_1^y k_2^x)\psi^2)$$
$$\times \frac{\epsilon^{c_1 c_2 c_3}}{\sqrt{6}} b_{u,\uparrow}^{+c_1\dagger}(\kappa_1)(b_{u,\downarrow}^{+c_2\dagger}(\kappa_2) b_{d,\uparrow}^{+c_3\dagger}(\kappa_3) - b_{d,\downarrow}^{+c_2\dagger}(\kappa_2) b_{u,\uparrow}^{+c_3\dagger}(\kappa_3))|0\rangle$$

$$|P;\uparrow\rangle_{-1/2} = \int [\mathcal{D}\kappa]\,((k_1^x + ik_1^y)\psi^3 + (k_2^x + ik_2^y)\psi^4)$$
$$\times \frac{\epsilon^{c_1 c_2 c_3}}{\sqrt{6}} (b_{u,\uparrow}^{+c_1\dagger}(\kappa_1) b_{u,\downarrow}^{+c_2\dagger}(\kappa_2) b_{d,\downarrow}^{+c_3\dagger}(\kappa_3) - b_{d,\uparrow}^{+c_1\dagger}(\kappa_1) b_{u,\downarrow}^{+c_2\dagger}(\kappa_2) b_{u,\downarrow}^{+c_3\dagger}(\kappa_3))|0\rangle$$

$$|P;\uparrow\rangle_{3/2} = \int [\mathcal{D}\kappa]\,(-k_1^x + ik_1^y)\psi^5$$
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X. Ji *et al.*, Nucl.Phys.B 652 (2003) 383-404

Fine but how can one compute consistently these LFWFs?



Definitions and Classification of LFWFs

LFWFs: formal definitions

$$\langle 0|O^{\alpha,\cdots}(\{z_1^-, z_{\perp 1}\}, \ldots, \{z_n^-, z_{\perp n}\})|P,\lambda\rangle\big|_{z_i^+=0}$$

- Lightfront operator O of given number of quark and gluon fields

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Both mesons and baryons can (in principle) have multiple independent leading-twist LFWFs

An example on the pion

$$\langle 0|\bar{q}_\alpha(\gamma_5 q)_\beta|\pi\rangle|_{z^+=0}$$

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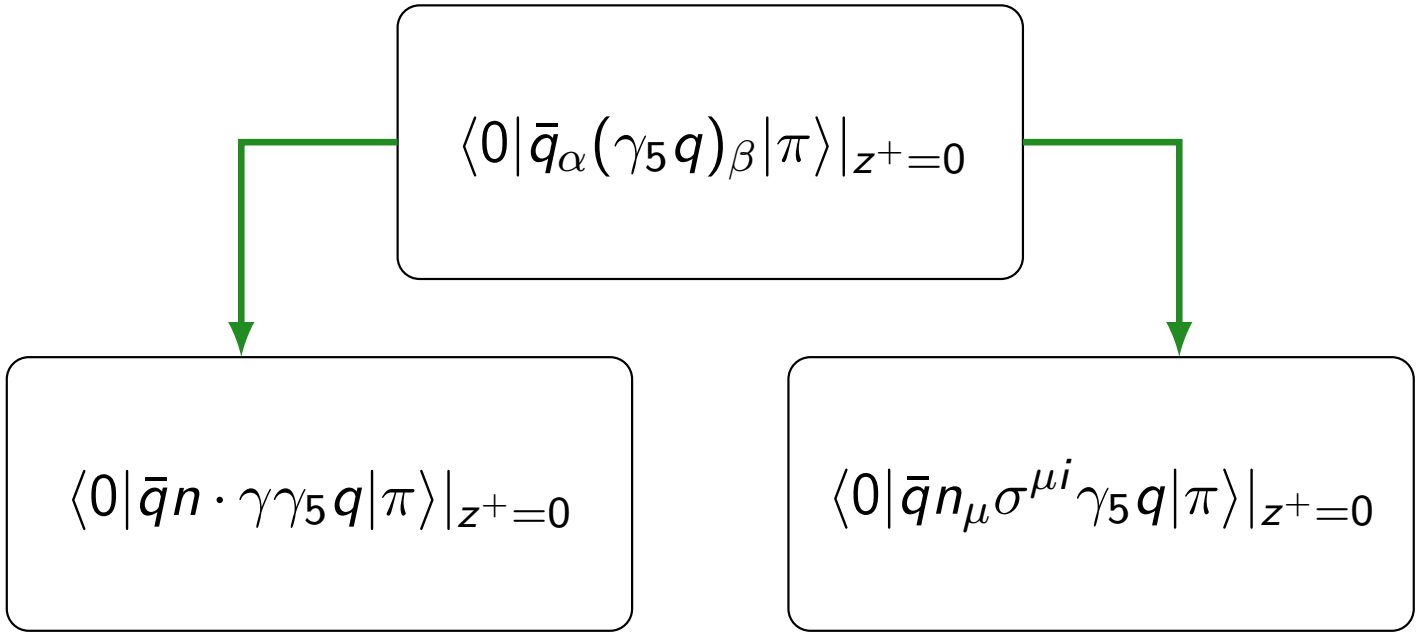

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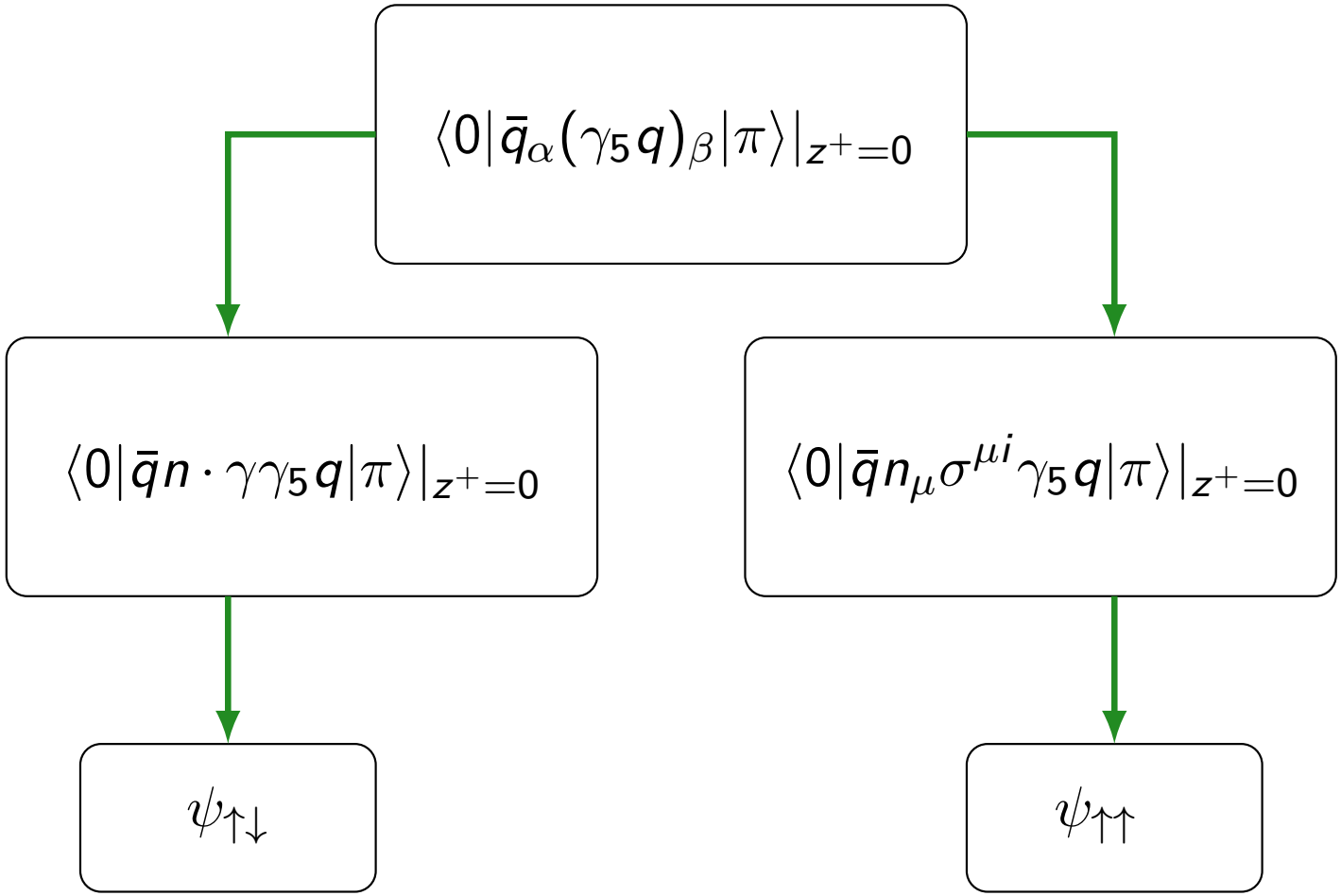

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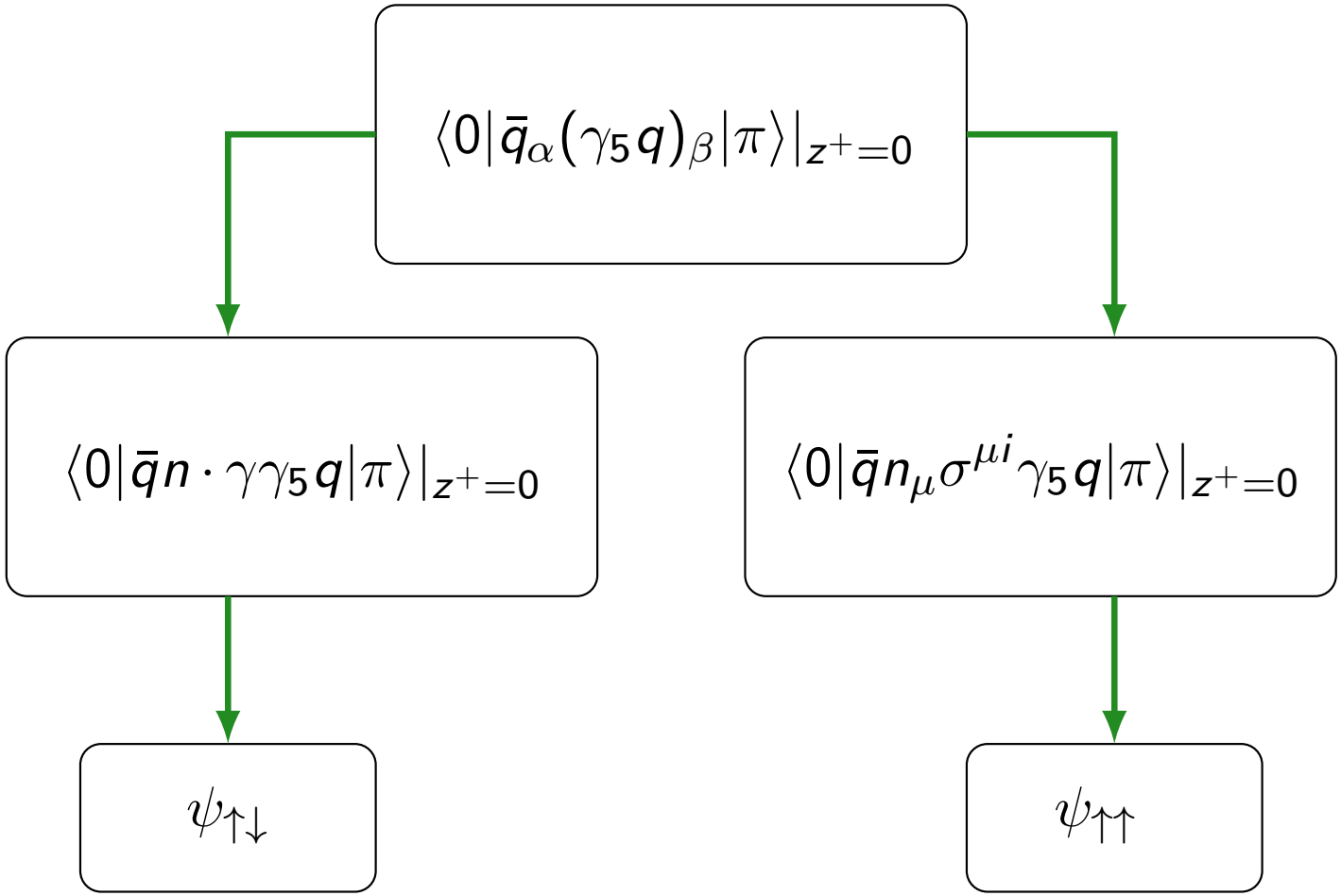

We can build one LFWFs with OAM projection 0, and one with OAM projection 1.

Nucleon LFWFs classification

- In the nucleon case, the procedure applies with three quarks at leading Fock state:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P,\uparrow\rangle$$

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X. Ji, *et al.*, Nucl Phys B652 383 (2003)

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- The LFWFs carry different amount of OAM projections:

states	$\langle\downarrow\downarrow\downarrow\|P,\uparrow\rangle$	$\langle\downarrow\downarrow\uparrow\|P,\uparrow\rangle$	$\langle\uparrow\downarrow\uparrow\|P,\uparrow\rangle$	$\langle\uparrow\uparrow\uparrow\|P,\uparrow\rangle$
OAM	2	1	0	-1
LFWFs	ψ^6	ψ^3, ψ^4	ψ^1, ψ^2	ψ^5

Relation with the Faddeev Wave function

- Since the Faddeev wave function χ is given as:

$$\langle 0|\, T\left\{q_{\alpha_1}(z_1)q_{\alpha_2}(z_2)q_{\alpha_3}(z_3)\right\}|P,\lambda\rangle = \frac{1}{4} f_N N_\sigma(P,\lambda)$$

$$\times \int \prod_{j=1}^{3} \mathrm{d}^{(4)}k_j e^{-ik_j z_j}\delta^{(4)}(P-\sum_j k_j)\chi_{\alpha_1\alpha_2\alpha_3\sigma}(k_1,k_2,k_3),$$

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- one can get the LFWFs schematically through

$$\psi_i \Gamma_{\alpha_3'\sigma'} = \int \prod_{j=1}^{3} [\mathrm{d}k_j^-]\mathcal{P}_{i;\alpha_1\alpha_2\alpha_3\alpha_3'\sigma\sigma'}\chi_{\alpha_1\alpha_2\alpha_3\sigma}$$

where $\mathcal{P}_i$ are the relevant leading-twist and OAM projectors.

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Important

The FWF allows a **consistent** derivation of the 6 leading-Fock states LFWFs of the nucleon

Modelling the Faddeev wave Function

Baryon and Diquarks

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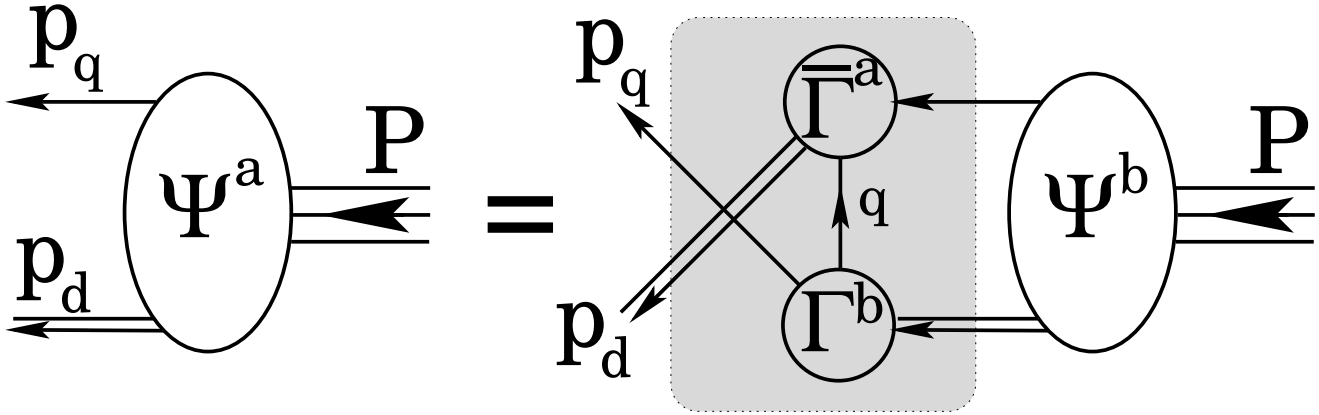
- It predicts the existence of strong diquarks correlations inside the nucleon.



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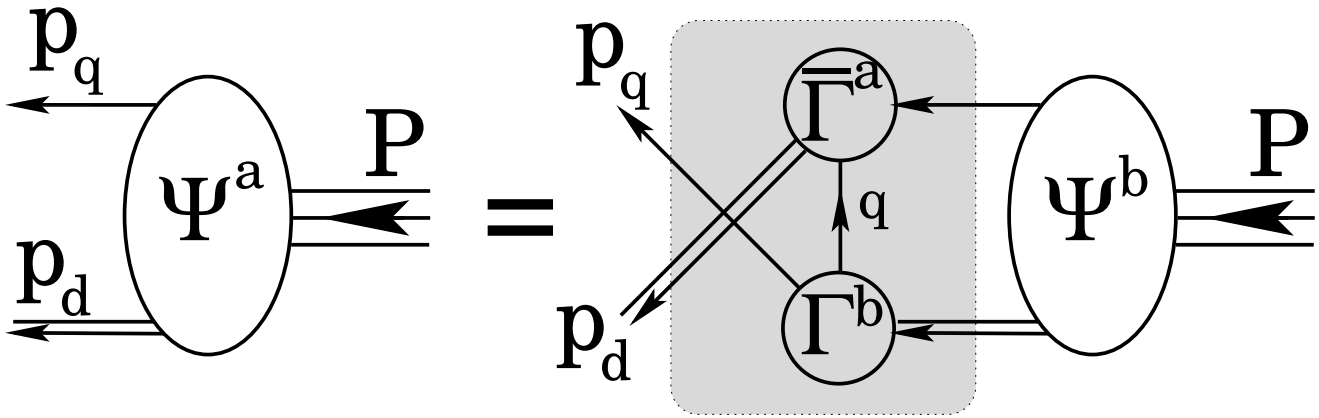

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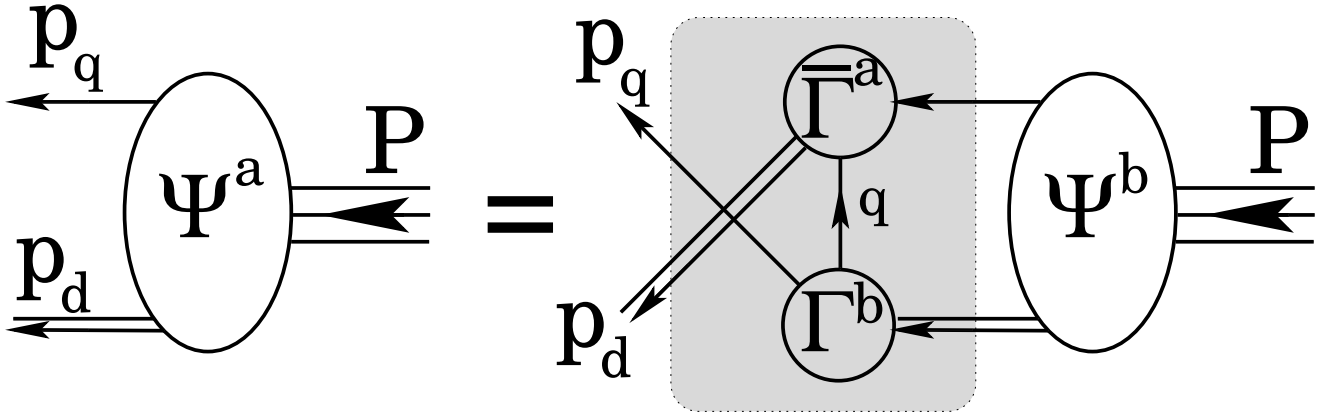

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
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- In the following we build a model inspired by numerical solutions of the Faddeev equations

Example with ψ_1 and ψ_2

- A single projector allows us to compute both ψ_1 and ψ_2:

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1^-, z_{1\perp})C\not{n}u^j_\downarrow(z_2^-, z_{2\perp})\right)\not{n}d^k_\uparrow(z_3^-, z_{3\perp})|P,\uparrow\rangle$$
$$\rightarrow\psi_1(x_1, k_{1\perp}, x_2, k_{2\perp}) + \epsilon^{ij}k_i^1 k_j^2\psi_2(x_1, k_{1\perp}, x_2, k_{2\perp})$$

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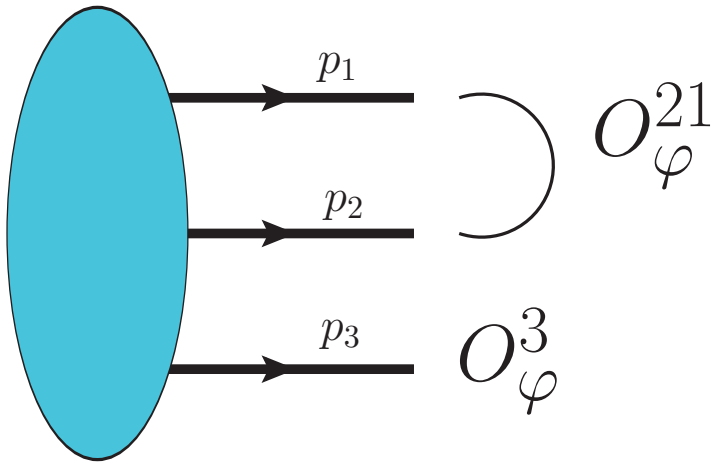

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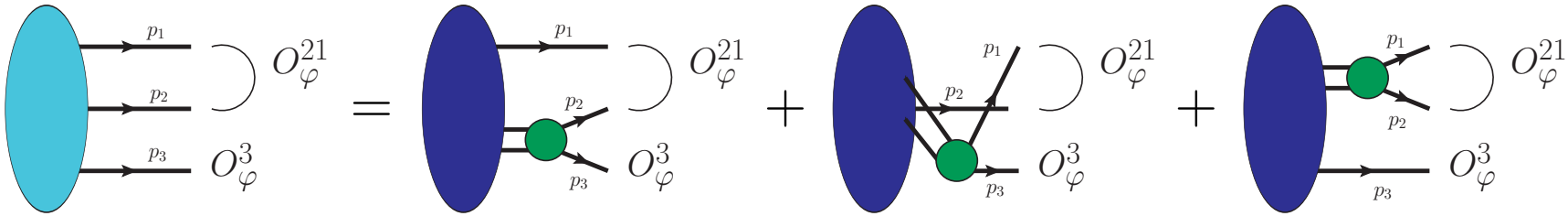

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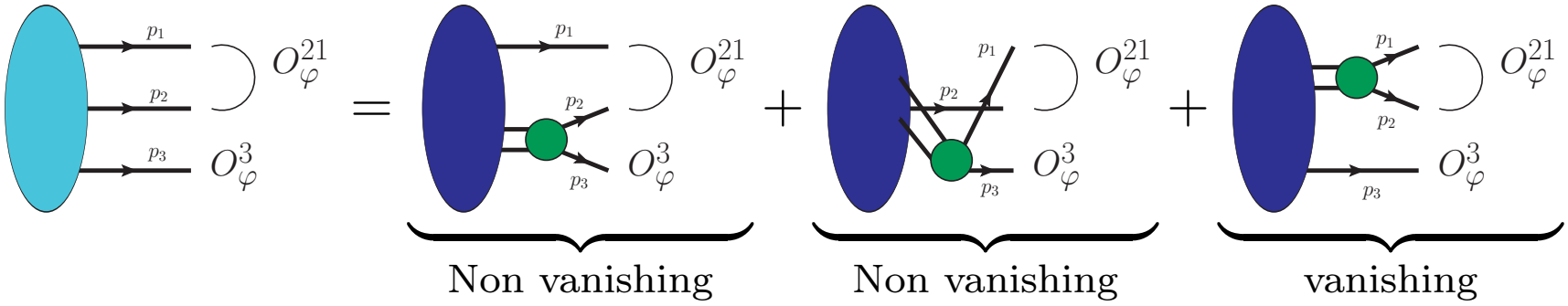
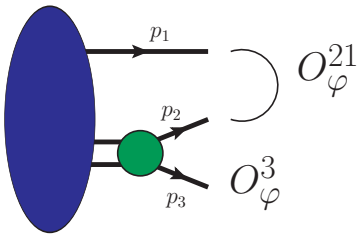


- The operator then selects the relevant component of the wave function.

Dirac Structure and Factorisation I

Considering the diquark amplitude Γ_0 and the Quark-diquark amplitude $\mathcal{S}$, we choose the following tensorial structure:

$$\Gamma_0 \propto i\gamma_5 C, \quad \mathcal{S} \propto I$$



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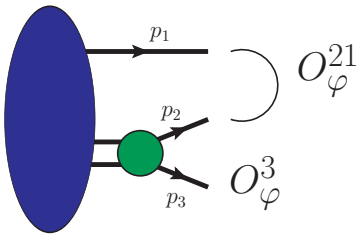

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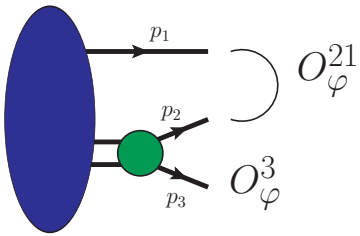

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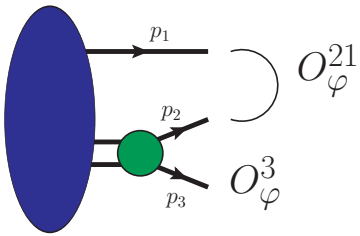


$$\propto \frac{\gamma_\nu}{4} \mathrm{Tr}\left[\gamma^\nu \not{n} L^\uparrow S(k_3) \Gamma^{0T} S^T(k_2) L^\downarrow \not{n}^T (C^\dagger)^T L^\uparrow S(k_1) \mathcal{S}\right] \Delta(K)$$

$$\propto \frac{\gamma_\nu}{4} \underbrace{\mathrm{Tr}\left[S(k_3)\Gamma^{0T} S^T(k_2) L^\downarrow C^\dagger \not{n} L^\uparrow\right]}_{\text{Diquark LFWF } \psi_{\uparrow\downarrow}} \underbrace{\mathrm{Tr}\left[\gamma^\nu \not{n} L^\uparrow S(k_1)\mathcal{S}\right]}_{\substack{\text{Projection of the} \\ \text{Faddeev WF}}} \Delta(K)$$

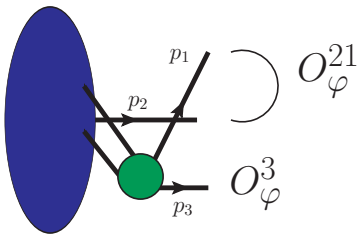
$$\propto \psi_1(x_1, k_{1\perp}, x_2, k_{2\perp})$$

Note that $\int \mathrm{d}^{(2)} k_{1\perp} \mathrm{d}^{(2)} k_{2\perp} \psi_1 = \varphi$, the nucleon DA.

Dirac Structure and Factorisation II

Considering the diquark amplitude Γ_0 and the Quark-diquark amplitude $\mathcal{S}$, we choose the following tensorial structure:

$$\Gamma_0 \propto i\gamma_5 C, \quad \mathcal{S} \propto I$$



$$\propto \frac{\gamma_\nu}{4}\mathrm{Tr}\left[\gamma^\nu \not{n} L^\uparrow S(k_3)\Gamma^{0T} S^T(k_1) L^\uparrow \not{n}^T (C^\dagger)^T L^\downarrow S(k_2)\mathcal{S}\right]\Delta(K)$$

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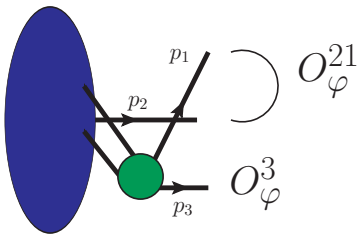

$$\propto \frac{\gamma_\nu}{4} \mathrm{Tr}\left[\gamma^\nu \not{p} L^\uparrow S(k_3) \Gamma^{0T} S^T(k_1) L^\uparrow \not{n}^T (C^\dagger)^T L^\downarrow S(k_2) \mathcal{S}\right] \Delta(K)$$

$$\propto \frac{\gamma_\nu}{4} \underbrace{\mathrm{Tr}\left[S(k_3)\Gamma^{0T} S^T(k_1) L^\uparrow C^\dagger \not{n} \gamma_\alpha\right]}_{\text{Diquark LFWF } \psi_{\uparrow\uparrow}} \underbrace{\mathrm{Tr}\left[\gamma^\nu \not{p} \gamma^\alpha L^\downarrow S(k_2)\mathcal{S}\right]}_{\substack{\text{Projection of the} \\ \text{Faddeev WF}}} \Delta(K)$$

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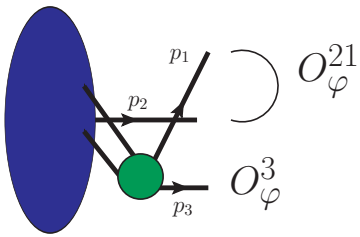

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$$\propto \epsilon^{\mu\nu\rho\sigma} n_\mu p_\nu k_{1\perp\rho} k_{2\perp\sigma} \psi_2(x_1, k_{1\perp}, x_2, k_{2\perp})$$

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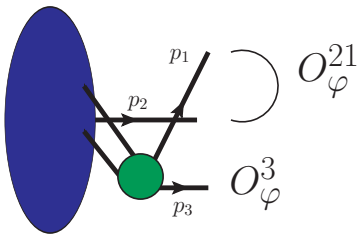

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$$\propto \epsilon^{\mu\nu\rho\sigma} n_\mu p_\nu k_{1\perp\rho} k_{2\perp\sigma} \psi_2(x_1, k_{1\perp}, x_2, k_{2\perp})$$

Note that the antisymmetric structure guarantees that the contribution vanishes when integrated over the transverse momenta.

Example with ψ_3 and ψ_4

- ψ_3 and ψ_4 are given by:

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1^-,z_{1\perp})C\not{n}u^j_\downarrow(z_2^-,z_{2\perp})\right)\not{n}d^k_\uparrow(z_3^-,z_{3\perp})|P,\downarrow\rangle$$
$$\to\left(k_{1,\perp}\psi_3(x_1,k_{1\perp},x_2,k_{2\perp})+k_{2,\perp}\psi_4(x_1,k_{1\perp},x_2,k_{2\perp})\right)\gamma^\perp$$

Braun *et al.*, Nucl.Phys. B589 (2000)
X. Ji *et al.*, Nucl.Phys. B652 (2003)

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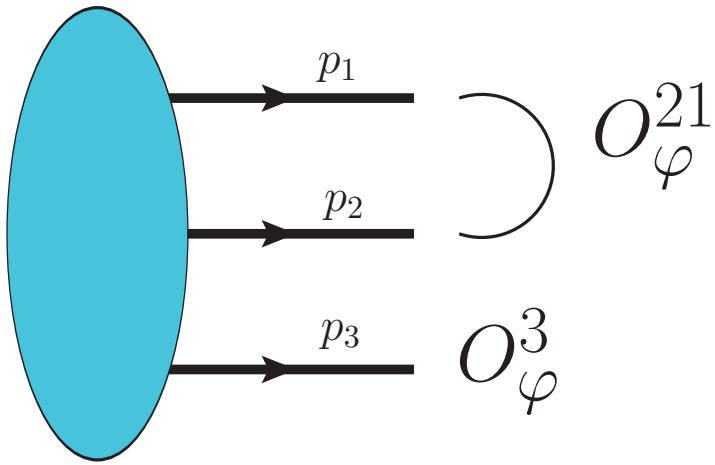
- We can apply it on the Faddeev wave function:



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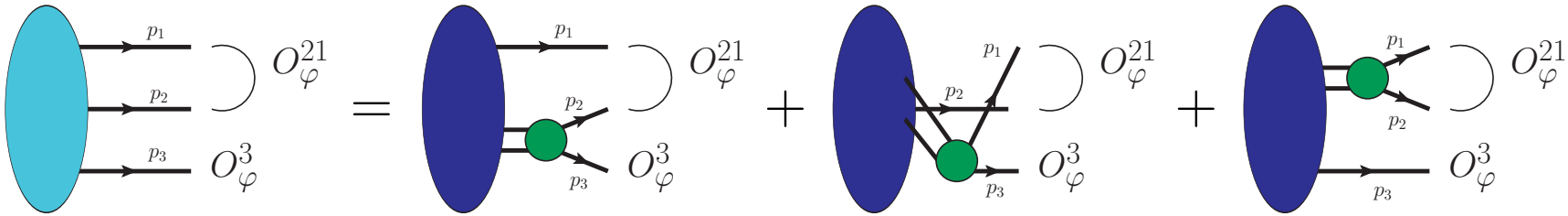

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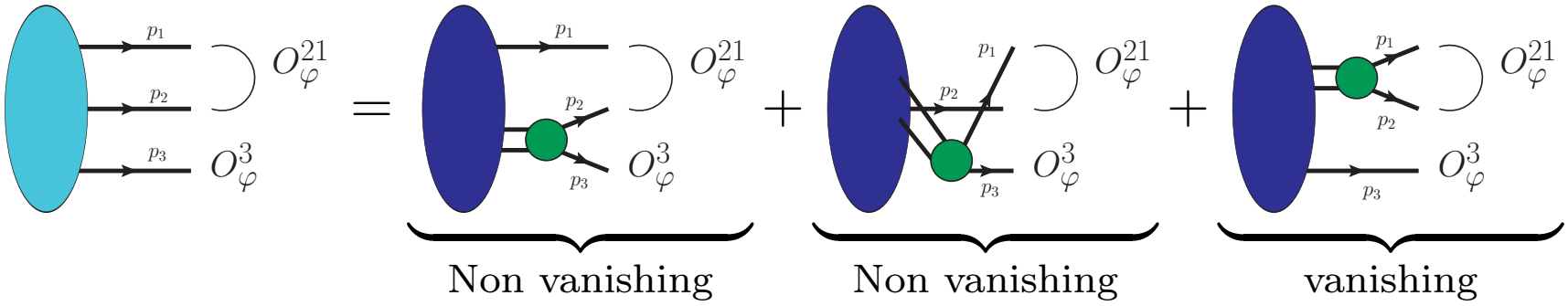
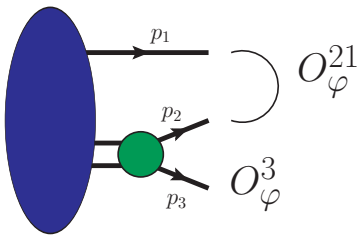


- The operator then selects the relevant component of the wave function.

Dirac Structure and Factorisation I

Considering the diquark amplitude Γ_0 and the Quark-diquark amplitude $\mathcal{S}$, we choose the following tensorial structure:

$$\Gamma_0 \propto i\gamma_5 C, \quad \mathcal{S} \propto I$$



$$\propto \frac{\sigma_{\mu\nu}}{48} \mathrm{Tr}\left[\sigma^{\mu\nu} \not{n} L^{\uparrow} S(k_3) \Gamma^{0T} S^T(k_2) L^{\downarrow} \not{n}^T (C^\dagger)^T L^{\uparrow} S(k_1) \mathcal{S} \Lambda^+\right] \Delta(K)$$

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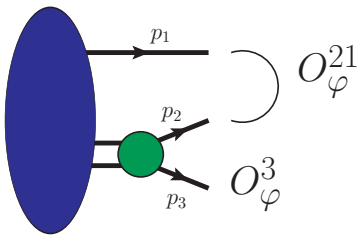

$$\propto \frac{\sigma_{\mu\nu}}{48} \mathrm{Tr}\left[\sigma^{\mu\nu} \not{p} L^{\uparrow} S(k_3) \Gamma^{0T} S^T(k_2) L^{\downarrow} \not{p}^T (C^{\dagger})^T L^{\uparrow} S(k_1) \mathcal{S} \Lambda^+\right] \Delta(K)$$

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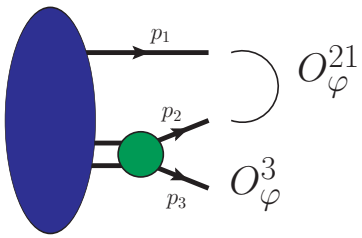

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$$\propto k_{1\perp} \psi_3(x_1, k_{1\perp}, x_2, k_{2\perp})$$

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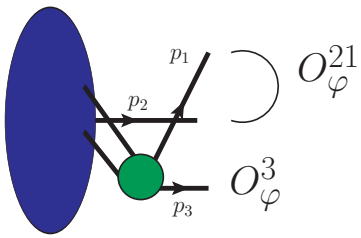

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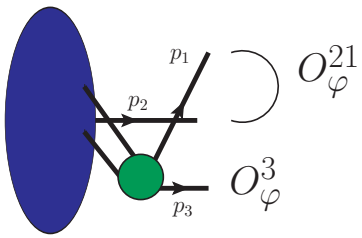

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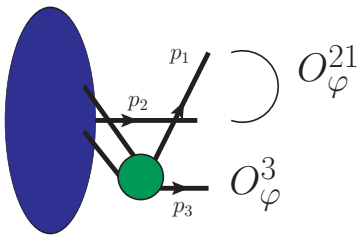

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$$\propto k_{1\perp\rho} \psi_3(x_1, k_{1\perp}, x_2, k_{2\perp}) + k_{2\perp\sigma} \psi_4(x_1, k_{1\perp}, x_2, k_{2\perp})$$

Case of ψ_5 and ψ_6

- ψ_5 is connected to the matrix element:

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1^-, z_{1\perp})Ci\sigma^{\nu\perp}n_\nu u^j_\uparrow(z_2^-, z_{2\perp})\right)\not{n}d^k_\uparrow(z_3^-, z_{3\perp})|P,\uparrow\rangle$$
$$\to k_{1,\perp}(\psi_5(x_1, k_{1\perp}, x_2, k_{2\perp}) - \psi_5(x_1, k_{1\perp}, x_3, k_{3\perp})) + \dots$$

Braun *et al.*, Nucl.Phys. B589 (2000)
X. Ji *et al.*, Nucl.Phys. B652 (2003)

- ψ_6 is obtained projecting:

$$\langle 0|\epsilon^{ijk}\left(u^i_\downarrow(z_1^-, z_{1\perp})Ci\sigma^{\nu\perp}n_\nu u^j_\downarrow(z_2^-, z_{2\perp})\right)\not{n}d^k_\downarrow(z_3^-, z_{3\perp})|P,\uparrow\rangle$$
$$\to k_{1,\perp}k_{1,\perp'}\psi_6(x_1, k_{1\perp}, x_2, k_{2\perp}) + \dots$$

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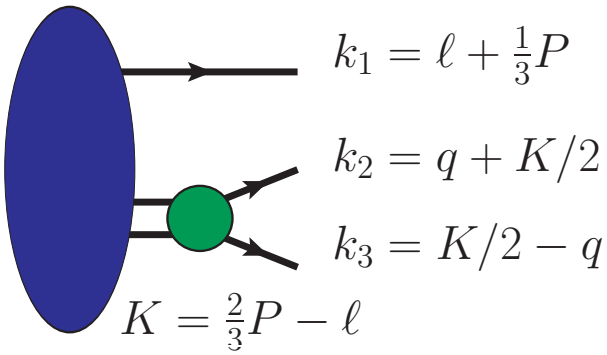
Scalar Diquark part of the nucleon

Computation Strategy

The diquark approach allows us to simplify a three-body system into two convoluted two-body systems. The strategy is thus:



1. Compute the virtuality (K) dependent LFWFs of the diquark by integrating over q
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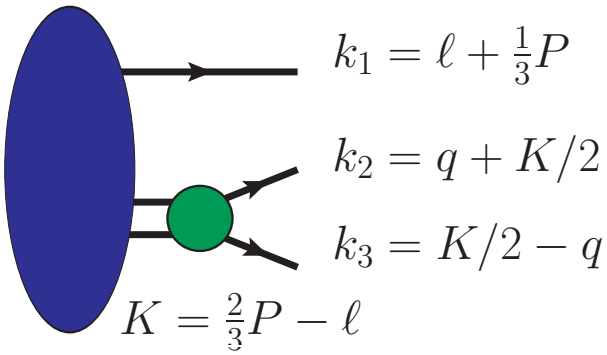


1. Compute the virtuality (K) dependent LFWFs of the diquark by integrating over q
2. Convolute with the quark-diquark amplitude by integrating over ℓ.

- For LFWFs, only the q^- and ℓ^- momenta should be integrated
- For the Distribution Amplitude, we have to integrate also on $q_\perp$ and $\ell_\perp$.

Mellin Moments

- Reminder : we use a model, not a solution of the Faddeev equation
- We do not integrate over the q^- and ℓ^- directly because we work in Euclidean space. Instead, we work with Mellin moments of it:

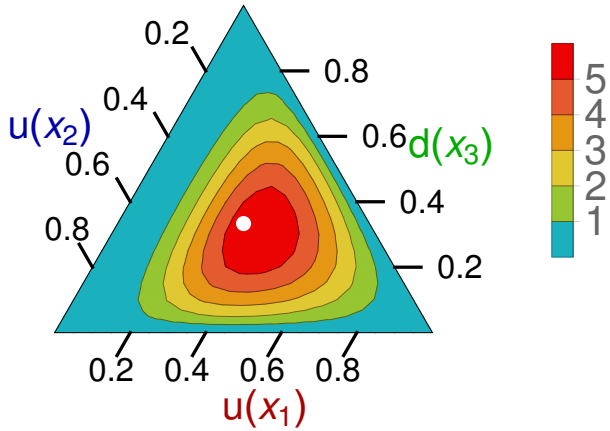
$$\langle x_1^m x_2^n \rangle (k_{1\perp}, k_{2\perp}) = \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \; x_1^m x_2^n \psi(x_1, x_2, k_{1\perp}, k_{2\perp})$$

- For a general moment $\langle x_1^m x_2^n \rangle$, we change the variable in such a way to write down our moments as:

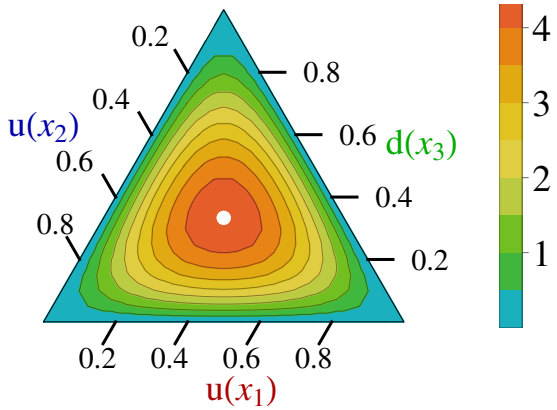
$$\langle x_1^m x_2^n \rangle (k_{1\perp}, k_{2\perp}) = \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \; \alpha^m \beta^n f(\alpha, \beta, k_{1\perp}, k_{2\perp})$$

- f is a complicated function involving the integration on 6 parameters
- Uniqueness of the Mellin moments of continuous functions allows us to identify f and ψ

Preliminary Results at DA level



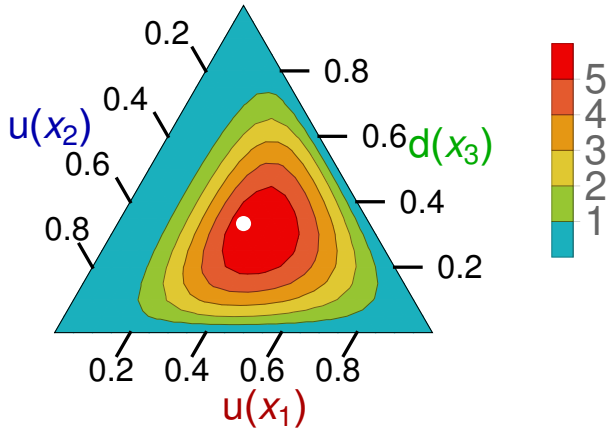
Scalar diquark Only



Asymptotic DA

- Typical symmetry in the pure scalar case

Preliminary Results at DA level



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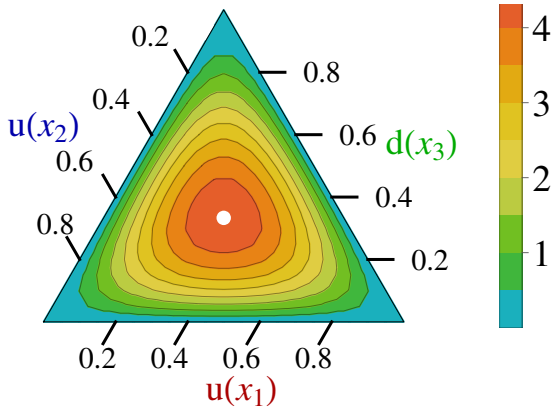

Asymptotic DA

- Typical symmetry in the pure scalar case
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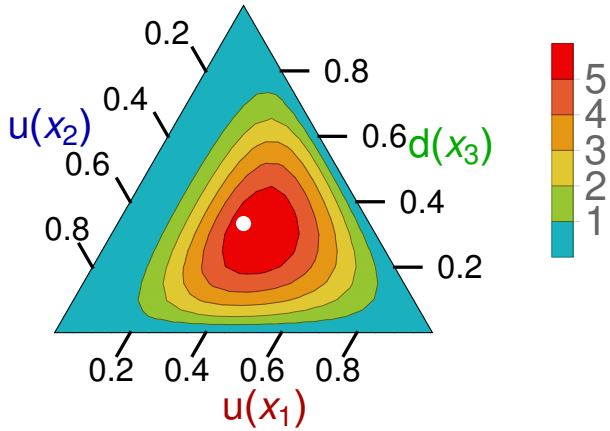
Preliminary Results at DA level



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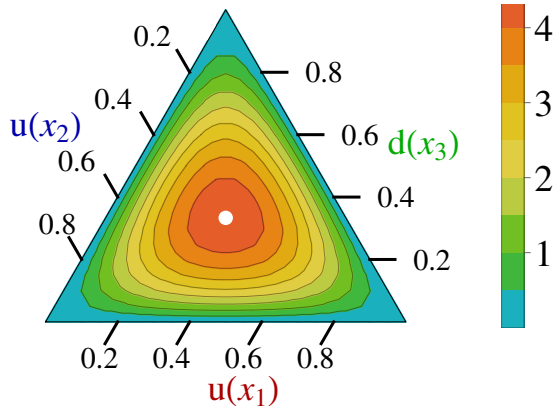
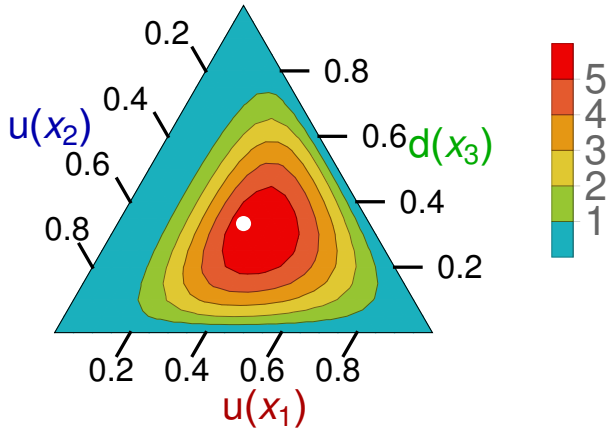


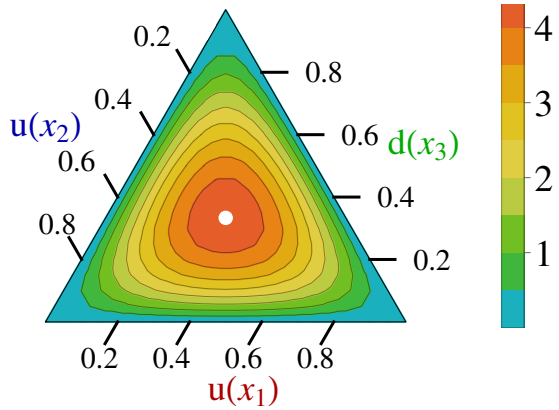
Asymptotic DA

- Typical symmetry in the pure scalar case
- Nucleon DA is skewed compared to the asymptotic one
- Deformation along the symmetry axis and orthogonally to it
  - Impact of the virtuality dependence of the diquark WF

Preliminary Results at DA level



Scalar diquark Only



Asymptotic DA

- Typical symmetry in the pure scalar case
- Nucleon DA is skewed compared to the asymptotic one
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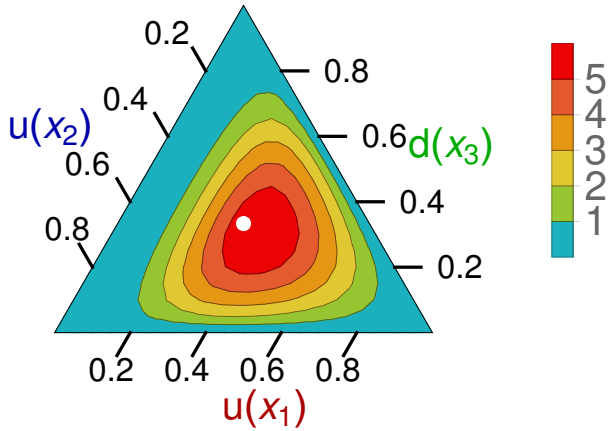
- These properties are consequences of our quark-diquark picture

Preliminary Results at DA level



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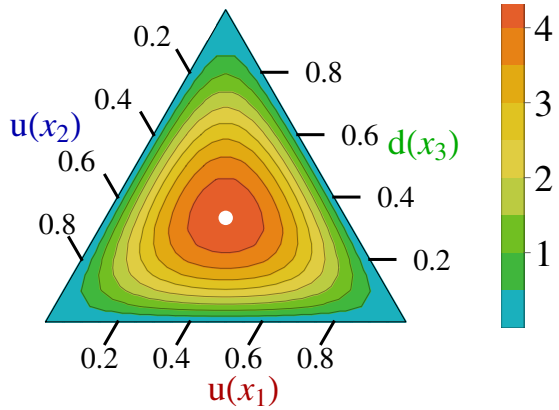

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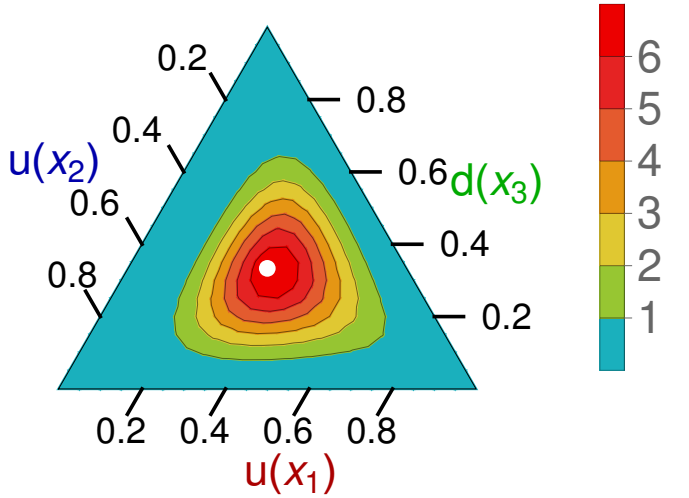
- Improvement in the modelling with respect to our previous work

see B. Almeida-Zamora talks this morning

Preliminary results for ψ_1

Momentum dependence



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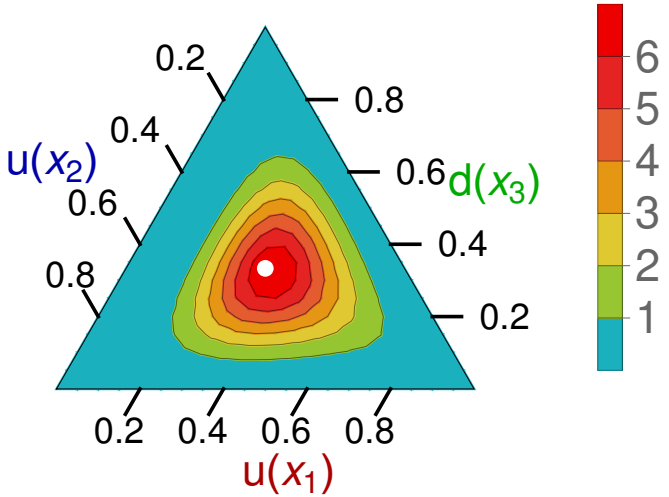

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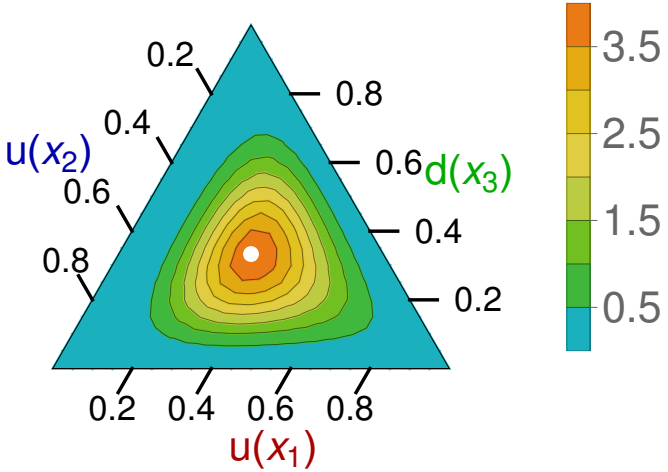

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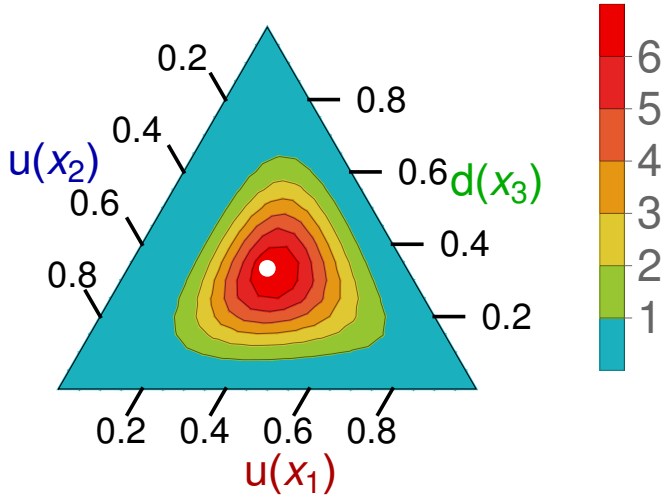

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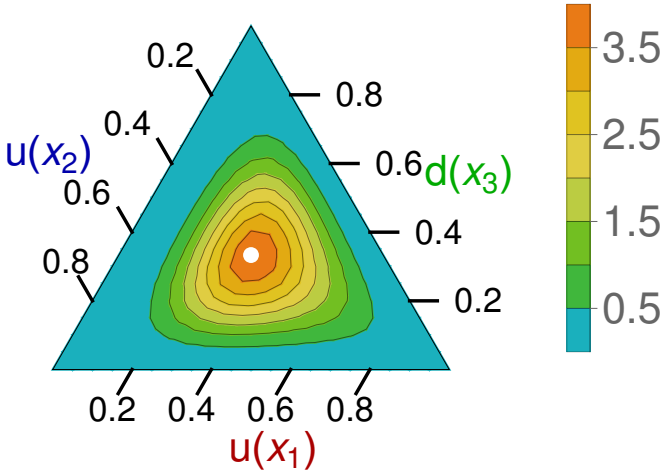

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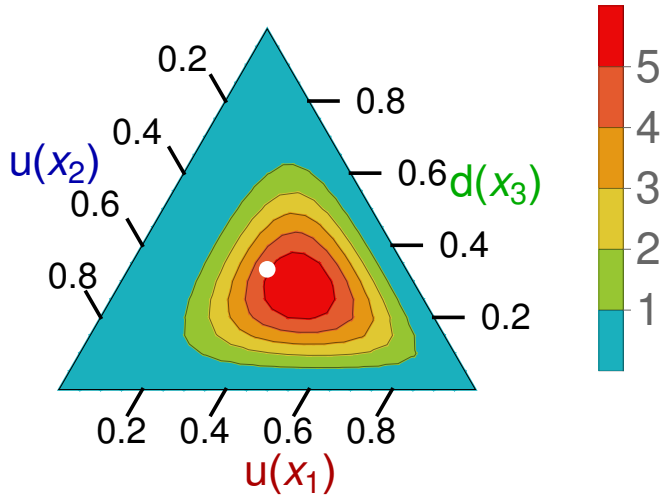

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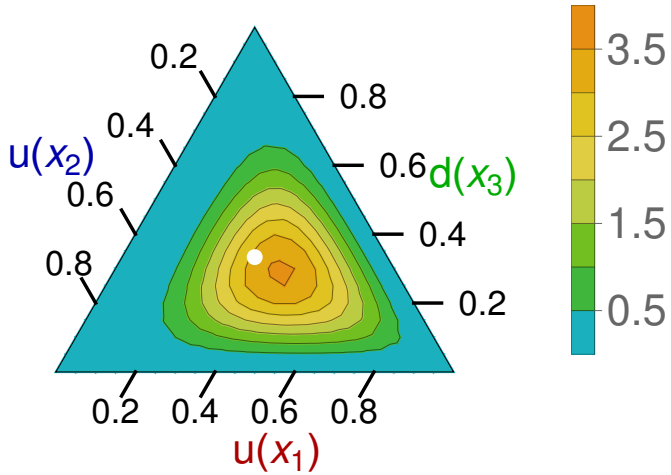

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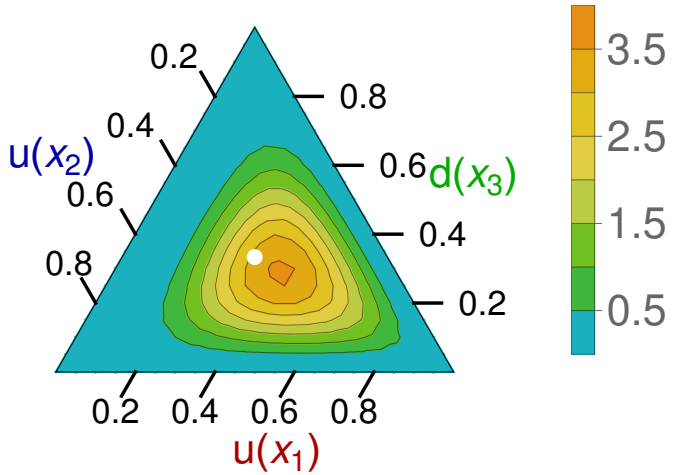

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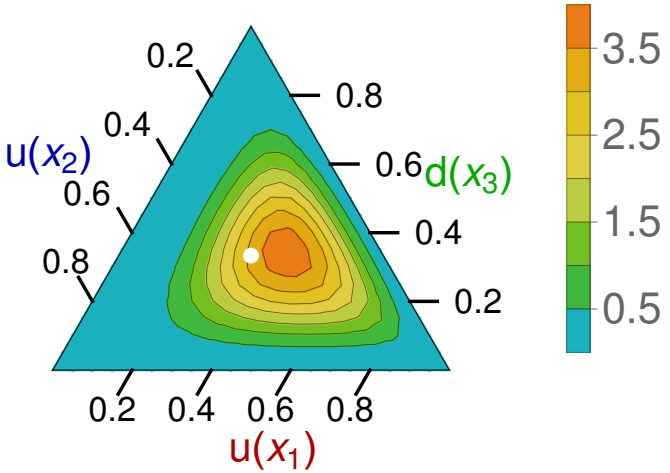

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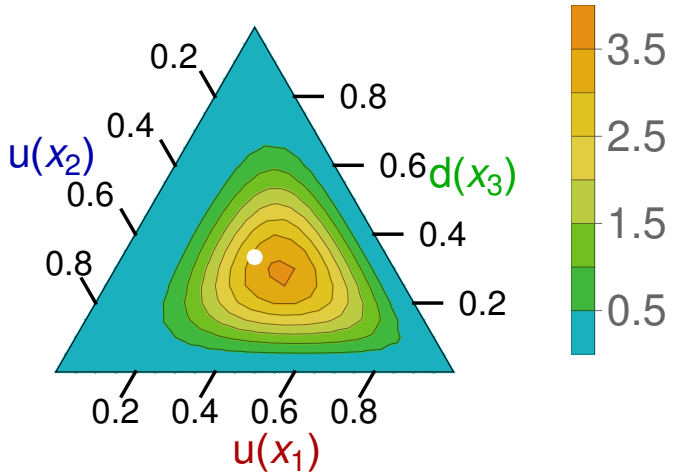

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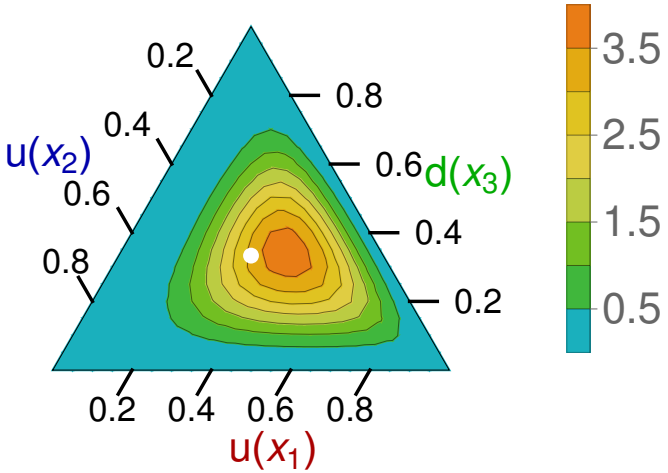

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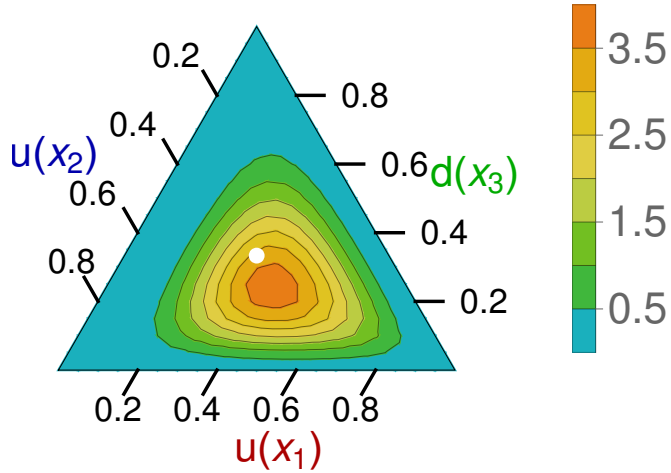

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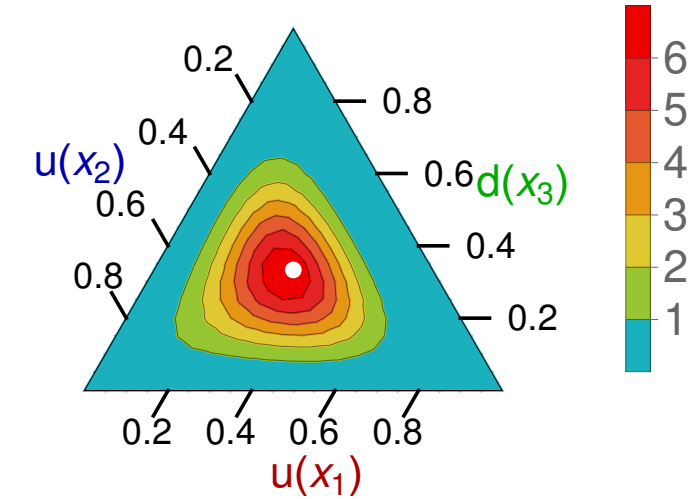
Adding tensorial structures or modifying the propagators will break this symmetry between ψ_1 and ψ_2.

The cases of $\psi_3 \ldots \psi_6$

- Pretty much the same analyses can be applied to the other LFWFs...
- ... but at the exception of small differences coming from kinematics
  $\Rightarrow$ one can get an additional power of x_1 or x_2
- ... and contributions of multiple diquark configurations.

Some illustrations



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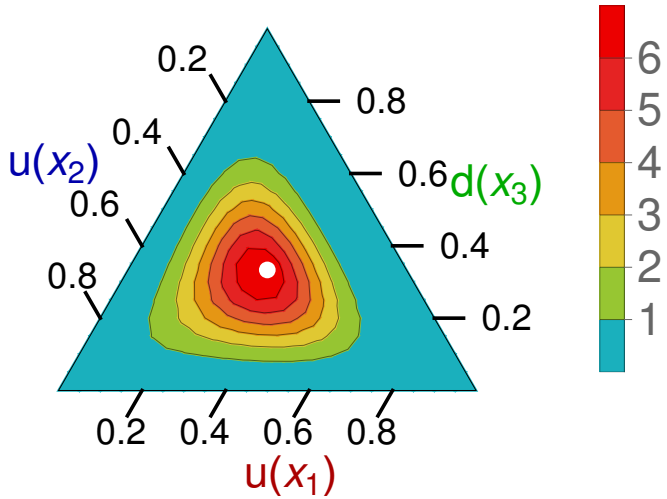

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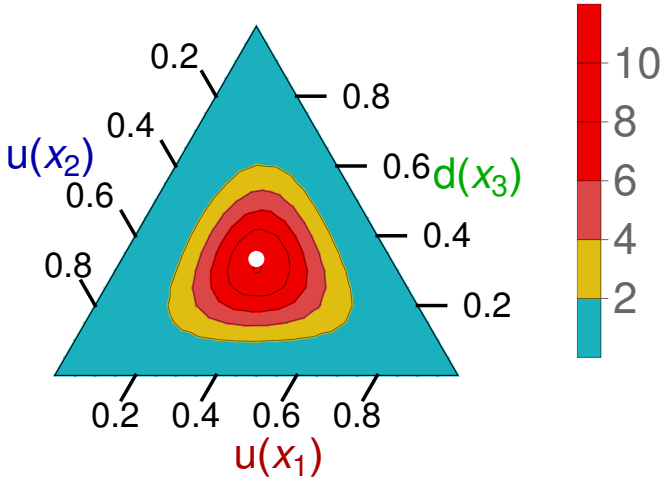

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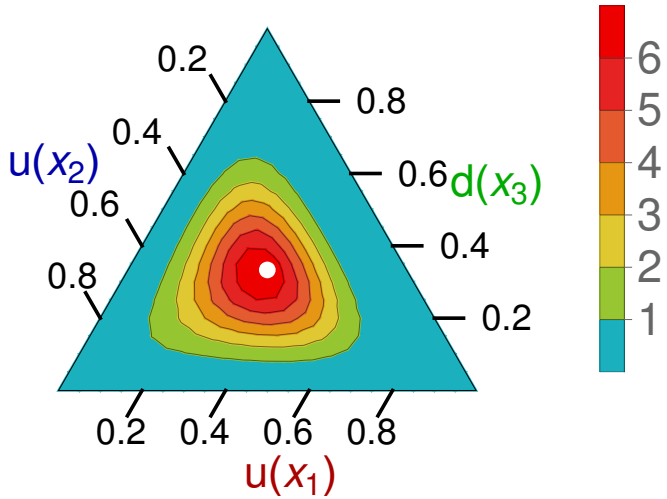

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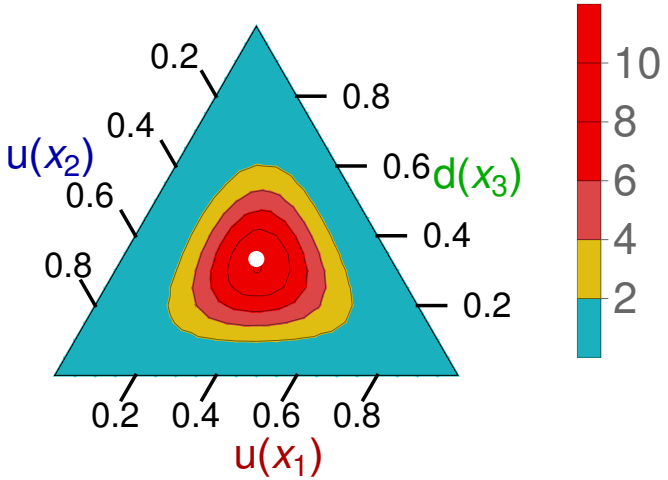

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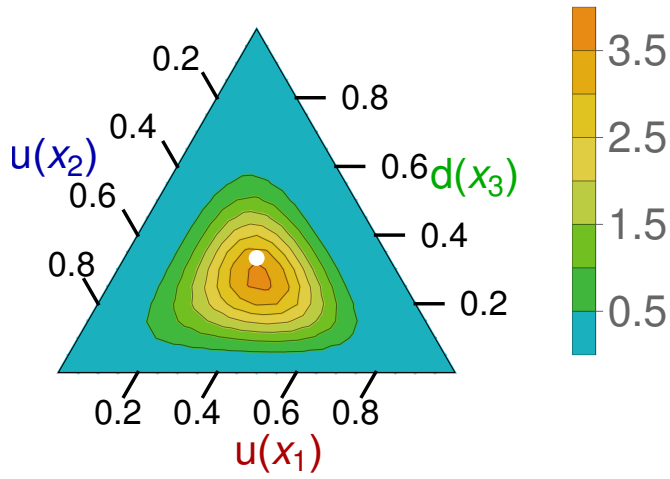

$$\psi_4(x_1, x_2, q_\perp^2 = \ell_\perp^2 = 0)/20$$

Toward baryons tomography

3D hadron structure

- Goal: access the 2D+1D number density of quarks within the nucleon

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- Tool: we use Generalised Parton Distributions (GPDs)
  - They are defined as non-local matrix elements

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma\cdot n\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$=\frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma\cdot nu+E^q(x,\xi,t)\bar{u}\frac{i\sigma^{\mu\alpha}n_\mu\Delta_\alpha}{2M}u\right].$$

  - In the limit $\Delta\to 0$, one recovers Parton Distribution Functions (PDF).
  - When integrated over x, they yields the Dirac and Pauli Electromagnetic Form Factors.

Overlap Representation of GPDs

As mentioned before, the GPDs can be computed as an overlap of LFWFs in the DGLAP region:

$$\begin{aligned}
&\langle P + \frac{\Delta}{2}|\bar{\psi}(-z/2)\gamma \cdot n\psi(z/2)|P - \Delta/2\rangle \\
&= \sum \int \mathcal{D}(x_i, k_{i,\perp})\Psi^*_{out}\Psi_{in}\langle qqq, out|\bar{\psi}(-z/2)\gamma \cdot n\psi(z/2)|qqq, in\rangle \\
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ψ_1 result

We look at the simple possible case, considering only ψ^1, *i.e.*:

$$\Psi_{1,\frac{1}{2}} = \psi^1 + \cancel{i(k_1^x k_2^y - k_1^y k_2^x)\psi^2}$$

An example for $\Psi_{1,1/2}$ in DGLAP

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Similar relations can be found for the d quark and higher OAM LFWFs.

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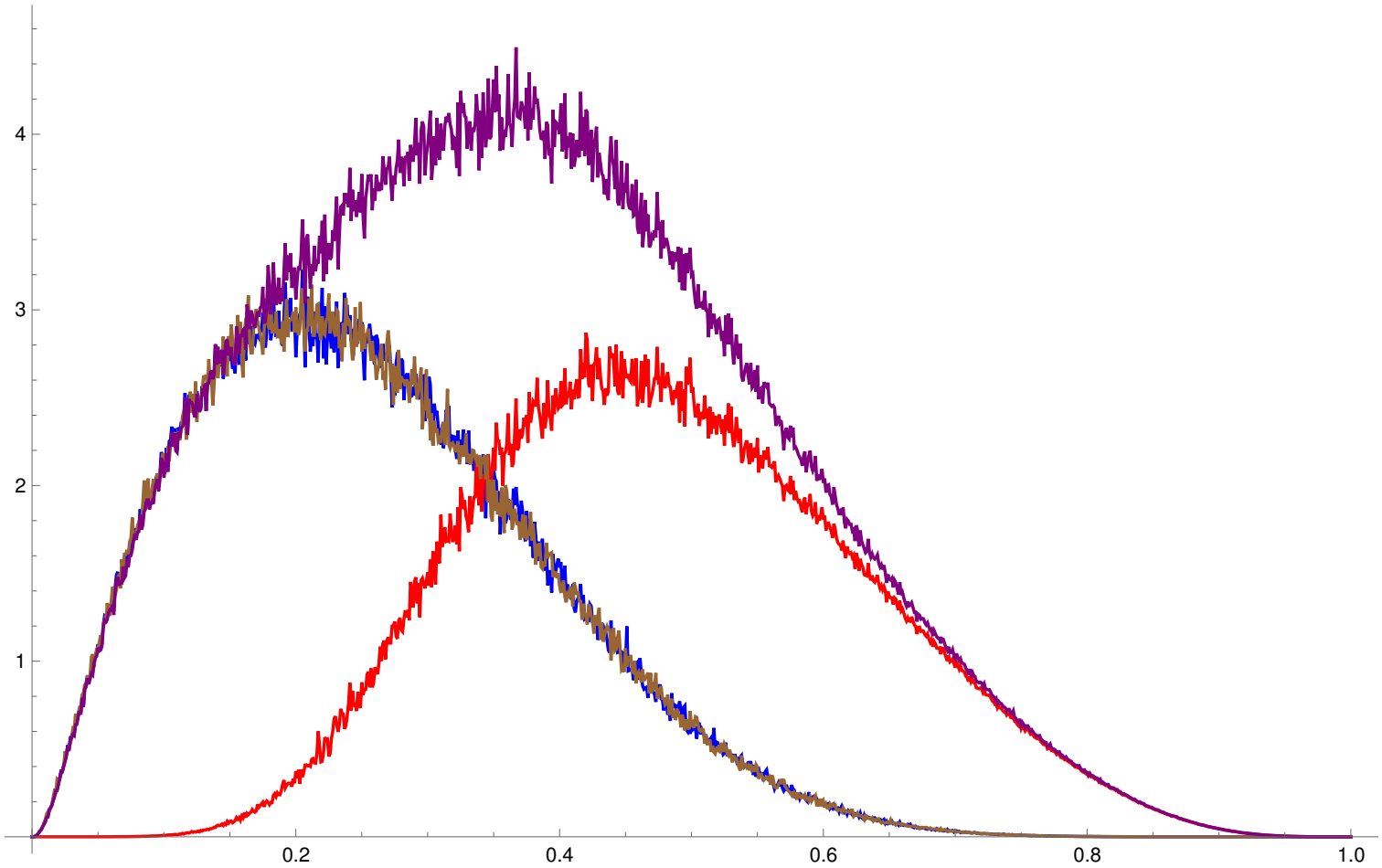
Results

We have obtained preliminary results in the case of the PDF using ψ_1 only.

- Good news: there is no show stopper and the computation can be performed
- However the results are very preliminary

ψ^1 contribution to the Nucleon PDF

Very Preliminary !!!



Summary

Summary and conclusion

Achievements

- **DSE compatible** framework for Nucleon LFWFs computations
- Based on the Nakanishi representation
- Improved models from the first exploratory work on PDA
- Relation between LFWFs and GPDs has been worked out
- Proof of concept up to PDF computations / no show stopper identified

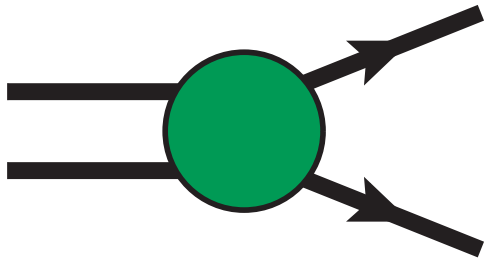
Work in progress/future work

- Finishing the computation for the 6 LFWFs
- Computations of GPDs in the Nakanishi picture with the scalar diquark
- Move to the numerical solutions of the Faddeev WF.
- Map deformations due to orbital angular momentum.

Thank you for your attention

Back up slides

Nakanishi Representation



At all order of perturbation theory, one can write (Euclidean space):

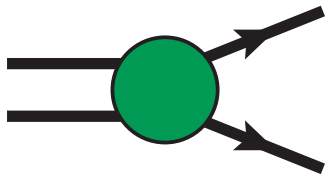
$$\Gamma(k,P) = \mathcal{N}\int_0^\infty \mathrm{d}\gamma \int_{-1}^{1} \mathrm{d}z \frac{\rho_n(\gamma,z)}{(\gamma+(k+\frac{z}{2}P)^2)^n}$$

We use a "simpler" version of the latter as follow:

$$\tilde{\Gamma}(q,P) = \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{\rho_n(z)}{(\Lambda^2+(q+\frac{z}{2}P)^2)^n}$$

Modelling the Scalar Diquark DA

- We need to obtain the structure of the scalar diquark itself



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{(\Lambda_q^2 + (q + \frac{z}{2}K)^2)}$$

  - q is the relative momentum between the quarks and K the total diquark momentum
  - Λ_q is a free parameter to be fit on DSE computations
  - $\rho(z,\gamma) = \rho(z) = 1 - z^2 \rightarrow$ we keep only the 0th degree coefficient in a Gegenbauer expansion of the Nakanishi weight

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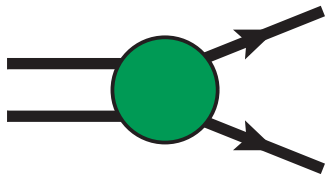

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  - $\rho(z,\gamma) = \rho(z) = 1 - z^2 \rightarrow$ we keep only the 0th degree coefficient in a Gegenbauer expansion of the Nakanishi weight
- We couple this with a simple massive fermion propagator:

$$S(p) = \frac{-ip \cdot \gamma + M}{p^2 + M^2}$$

Adjusting the parameters

- Mass of the quarks: $M = 2/5 M_N$
  - Sum of the frozen mass bigger than the nucleon mass for stability (binding energy)
  - Avoid singularities in the complex plane

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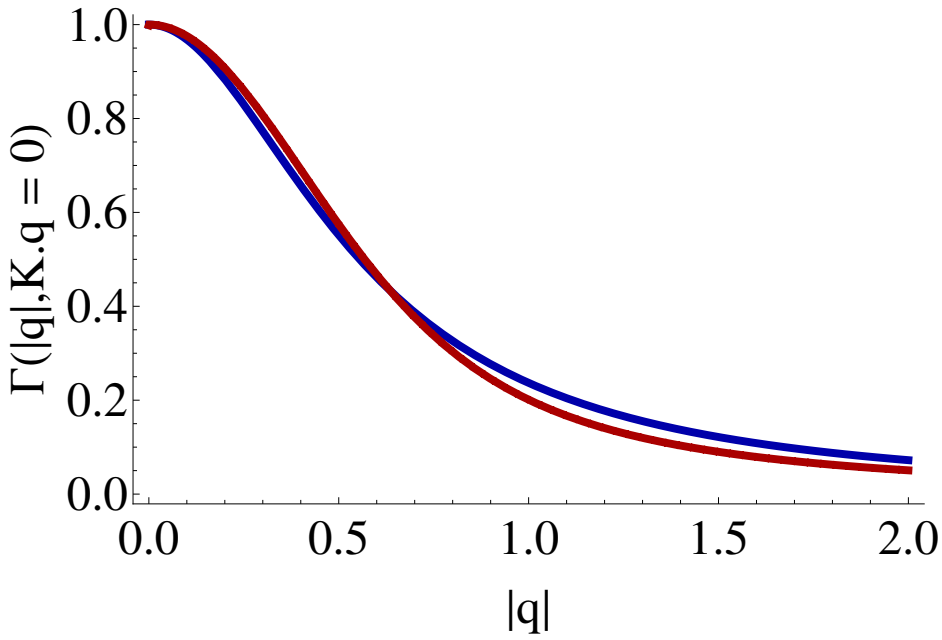
  - Avoid singularities in the complex plane
- Width of the diquark BSA $\Lambda_q = 3/5 M_N$ fitted on previous computations:



red curve from Segovia *et al.*,Few Body Syst. 55 (2014) 1185-1222

Scalar Diquark DA

- From that we can compute the scalar diquark DA as:

$$\phi(x, q_\perp) \propto \int d^{(2)}q \delta\left(q \cdot n - xK \cdot n\right) \mathrm{Tr}\left[S\Gamma^{0T} S^T L^{\downarrow} C^\dagger n \cdot \gamma L^{\uparrow}\right]$$

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- We compute Mellin moments $\rightarrow$ avoid difficulties with lightcone in euclidean space
- Nakanishi representation $\rightarrow$ analytic treatments of singularities and analytic reconstruction of the function from the moment

$$\phi(x, q_\perp) = \int_x^1 \mathrm{d}u \int_0^x \mathrm{d}v \frac{F(u, v, x)}{\left(M_{\mathrm{eff}}^2(u, v, x, M^2, \Lambda^2) + (q_\perp^{\mathrm{eff}}(u, v, x, q_\perp, K_\perp))^2 + K^2\right)^2}$$

F, M_{eff} and $q_\perp^{\mathrm{eff}}$ are computed analytically

First results for the diquark

- We present the first results at the level of the diquark DA
  - It depends on a single variable
  - It has been computed in the RL case

    Y. Lu *et al.*, Eur.Phys.J.A 57 (2021) 4, 115

    $\rightarrow$ we have a comparison point for our simple Nakanishi model.

Analytic results

- In the specific case $M^2 = \Lambda_q^2$, the PDA can be analytically obtained:

$$\phi(x) \propto \frac{M^2}{K^2}\left[1 - \frac{M^2}{K^2}\frac{\ln\left[1 + \frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}\right]$$

C. Mezrag *et al.*, Springer Proc.Phys. 238 (2020) 773-781

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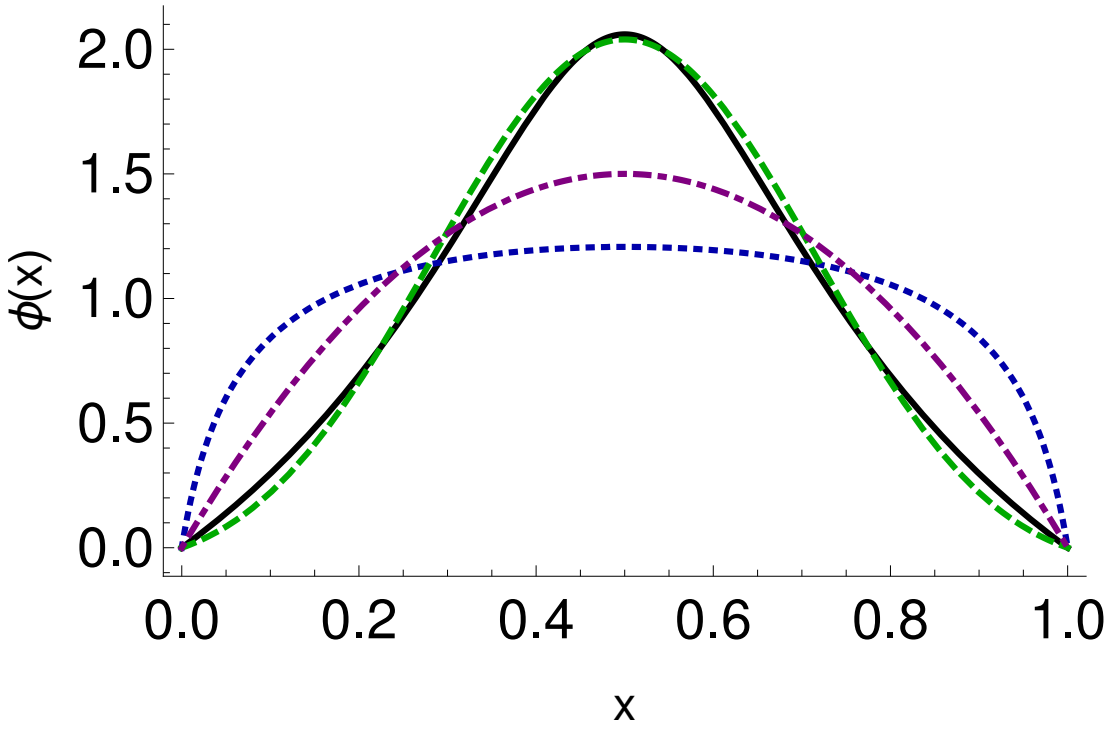
- Note that expanding the log, one get:

$$\phi(x) \propto \frac{1}{2}x(1-x) - \frac{1}{3}K^2/M^2x^2(1-x)^2 + \dots$$

so that:

  - at the end point the DA remains linearly decreasing (important impact on observable)
  - at vanishing diquark virtuality, one recovers the asymptotic DA

Comparison with DSE results



Model at K^2 = $-(3/4M_N)$^2

Model at K^2 = $4M_N$^2

RL Result

Asymptotic

RL results from Y. Lu *et al.*, Eur.Phys.J.A 57 (2021) 4, 115

Limitations

- Complex plane singularities for large timelike virtualities

$$\phi(x) \propto \frac{M^2}{K^2}\left[1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}\right]$$

  - Cut of the log reached for $K^2 \leq -4M^2$
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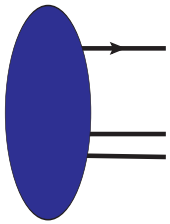
But overall, we expect to gain insights from this simple model

Quark-diquark amplitude

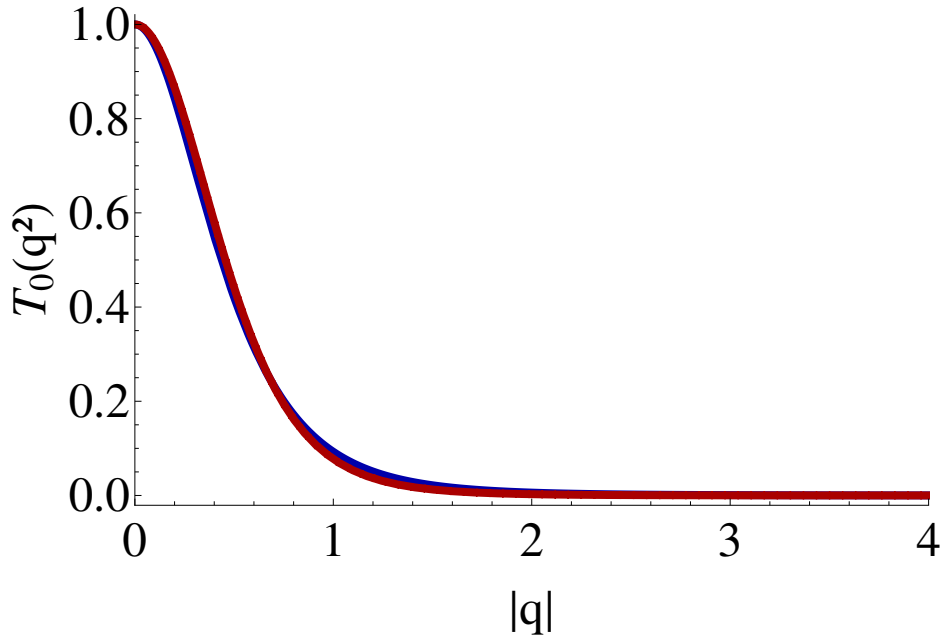
Nucleon Quark-Diquark Amplitude

Scalar diquark case



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \qquad \tilde{\rho}(z) = \prod_j (z-a_j)(z-\bar{a}_j)$$

Fits of the parameters through comparison to Chebychev moments:



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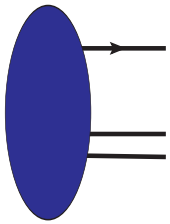

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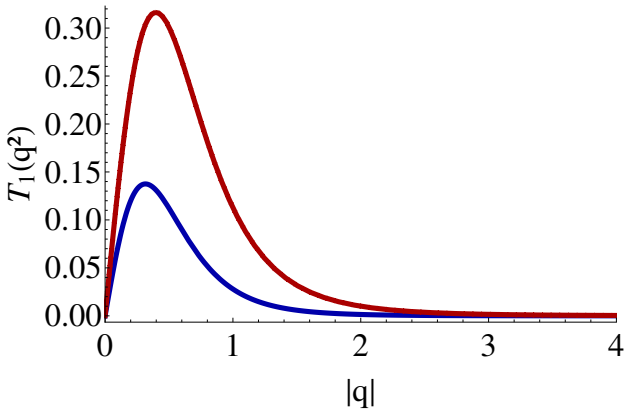

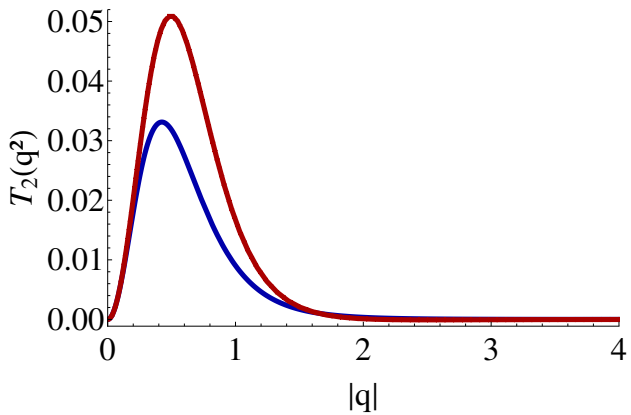

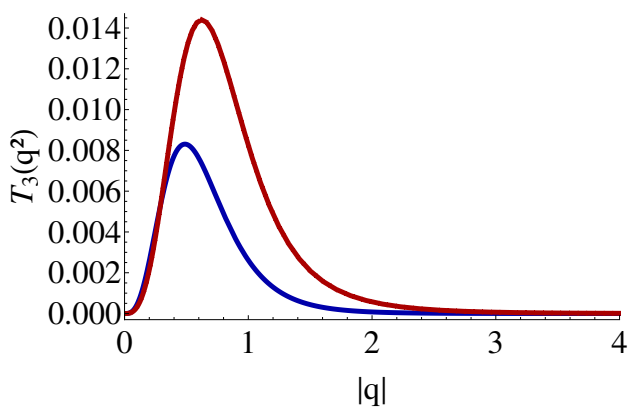

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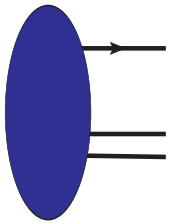

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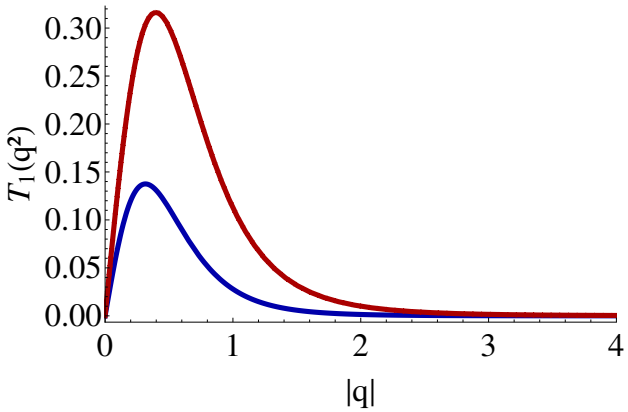

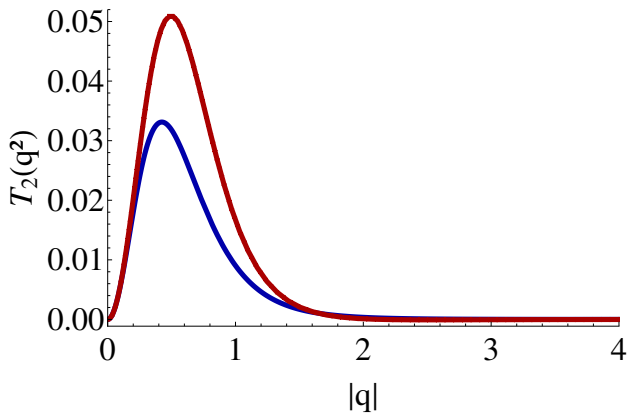

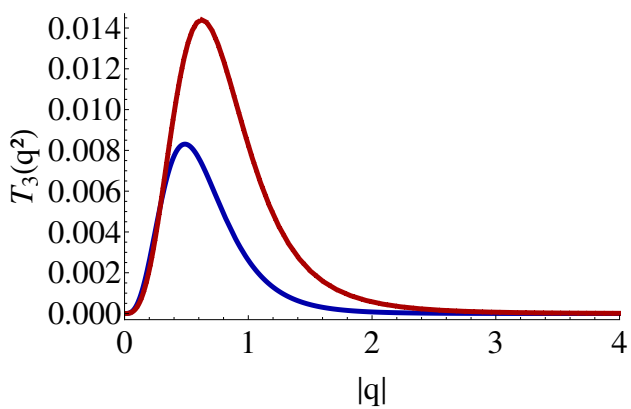

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Modification of the $\tilde{\rho}$ Ansatz ? $\tilde{\rho}(z) \to \tilde{\rho}(\gamma, z)$?