Diquark correlations in the proton parton distribution amplitude

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Based on B. Almeida-Zamora, L. Albino, C. Mezrag and J. Segovia, `arXiv:2610.22994 [hep-ph]` (2026)

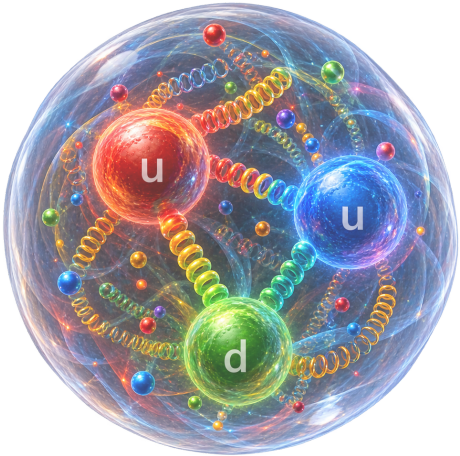






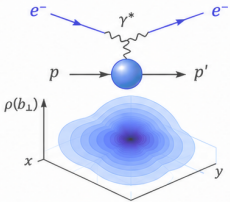
Understanding the internal structure of the proton

- QCD is the fundamental theory of the strong interaction.
- It successfully describes the interactions of **quarks** and **gluons**.
- How hadrons **emerges** from quarks and gluons?
- However, understanding how hadron structure emerges from QCD remains a major challenge.
- **How does the proton acquire its internal structure?**



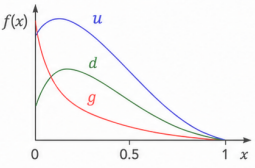
FORM FACTORS

Access the spatial distribution of charge and magnetization.



PARTON DISTRIBUTION FUNCTIONS (PDFS)

Give the probability of finding a parton carrying a given fraction x of the hadron's longitudinal momentum.

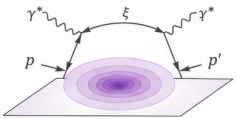




HADRON STRUCTURE

GENERALIZED PARTON DISTRIBUTIONS (GPDs)

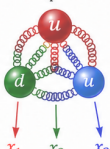
Encode the correlated information on longitudinal momentum and transverse spatial structure.



PARTON DISTRIBUTION AMPLITUDES (PDAs)

Describe how the longitudinal momentum of the hadron is shared among its valence quarks.

Proton P



For the proton (uud):

- x_1: bystander u quark
- x_2: d quark
- x_3: spectator u quark

Moments quantify the average momentum carried by each valence quark:

$$\langle x_1 \rangle = \int [dx]\, x_1\, \phi(x_i, \mu)$$
$$\langle x_2 \rangle = \int [dx]\, x_2\, \phi(x_i, \mu)$$
$$\langle x_3 \rangle = \int [dx]\, x_3\, \phi(x_i, \mu)$$

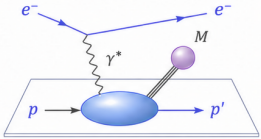
Momentum sharing

$$x_1 + x_2 + x_3 = 1$$

with $[dx] = dx_1\, dx_2\, dx_3\, \delta(1 - x_1 - x_2 - x_3)$

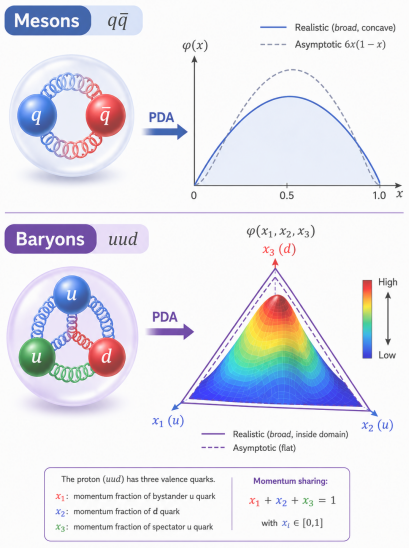
EXCLUSIVE AMPLITUDES (OBSERVABLES)

Accessed in exclusive reactions (*e.g.*, $\gamma^* p \to Mp'$, $ep \to epM$, ...).



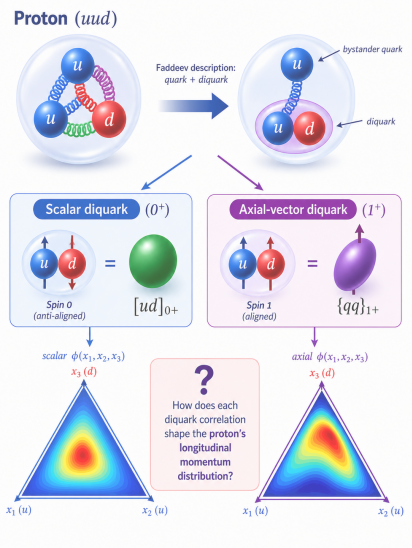
From Mesons to Baryons

- Meson PDAs have been extensively studied using continuum approaches and lattice QCD.
- At experimentally accessible scales, they are typically **broad and concave**, differing significantly from their asymptotic forms.
- In contrast, considerably less is known about **baryon PDAs**.
- In particular, the dynamical origin of the **momentum asymmetry** of the proton PDA remains incompletely understood.
- What is the **dynamical origin** of this momentum asymmetry?



What Shapes the Proton PDA?

- The proton is a genuine **three-body bound state**, making its PDA considerably more complex than in mesons.
- Strong nonperturbative quark–quark correlations (diquarks) naturally emerge within a covariant Faddeev description.
- The dominant correlations are scalar diquarks **(0^+)**, and axial-vector diquarks **(1^+)**.
- **How does each diquark correlation shape the proton's longitudinal momentum distribution?**



The Proton as a Quark–Diquark System

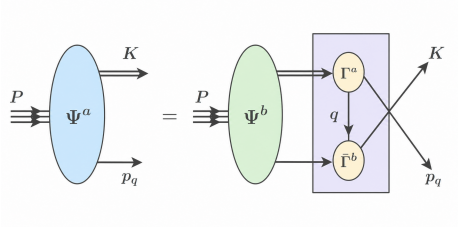
- The nucleon is described through a **Poincaré-covariant Faddeev equation**.
- Strong quark–quark correlations allow the three-body problem to be reorganized in terms of **quark–diquark configurations**.
- The Faddeev wave function contains three components, corresponding to each possible choice of the **bystander quark**:

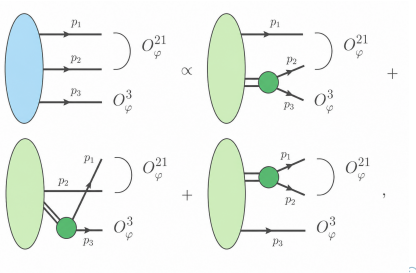
$$\Psi = \psi_1 + \psi_2 + \psi_3.$$

- Each component contains both

$$J^P = 0^+ \quad \text{and} \quad J^P = 1^+$$

diquark correlations.





From the diquark PDA to the proton PDA

In the quark–diquark picture, the proton PDA is built in two steps:

1 Diquark distribution amplitude

Internal momentum sharing (two quarks)

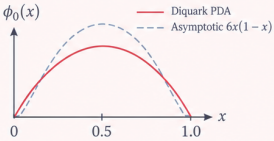


Scalar diquark (ud)

Compute its Mellin moments:

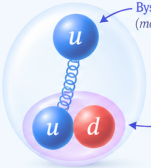
$$\langle x^n \rangle_{[ud]} = \int_0^1 dx\, x^n \phi_0(x)$$

Reconstruct the PDA:



2 Quark–diquark structure

Momentum sharing inside the proton (quark + diquark)



Compute the Mellin moments of the quark–diquark amplitude:

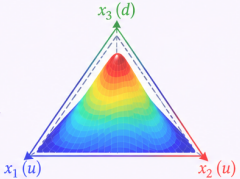
$$\langle x_3^m (1-x_3)^n \rangle = \int_0^1 dx_3\, x_3^m (1-x_3)^n \Phi(x_3)$$

Convolution

Combine both levels of momentum sharing

3 Proton PDA

Three-quark momentum sharing



Proton PDA from the convolution:

$$\phi_p^S(x_1,x_2,x_3) = \int_0^1 dz\, \phi_0(z)\, \Phi(x_1,x_2,x_3;z)$$

Equivalently, in terms of Mellin moments:

$$\langle x_1^l x_2^m x_3^n \rangle = \sum_{k=0}^{n} C_{lmn}^k \langle z^k \rangle_{[ud]} \langle x_3^{n-k} \rangle_{q-dq}$$

By first determining the diquark PDA and then convoluting it with the quark–diquark structure, we obtain the **scalar–diquark contribution** to the proton PDA.

Leading-Twist Proton PDA

- The leading-twist PDA follows from a vacuum-to-proton matrix element of a three-quark light-ray operator:

$$\langle 0|\epsilon^{ijk}\bar{u}^i_{\uparrow}(z_1)\not{n}u^j_{\downarrow}(z_2)\not{n}d^k_{\uparrow}(z_3)|P\rangle \propto f_N\int[dx]\,\phi(x_1,x_2,x_3;\mu)e^{-iP\cdot n\sum_i x_i z_i}.$$

- The momentum fractions satisfy

$$x_i \geq 0, \quad x_1+x_2+x_3=1, \quad \int[dx]\,\phi(x_i;\mu)=1.$$

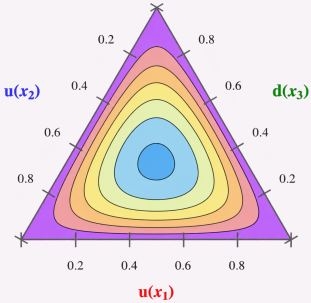
Physical content

$\phi(x_i;\mu)$ describes how the proton's longitudinal momentum is shared among the three valence quarks at the scale μ.

What we expect

A displacement of the $\phi_{\rm as}(x_i)$ reflects nonperturbative momentum sharing inside the proton.

Asymptotic PDA



$$\phi_{\rm as}(x_i)=120x_1x_2x_3$$

Its maximum lies at

$$(1/3,1/3,1/3).$$

Mellin Moments and Light-Front Projection

Moments of the PDA

$$\langle x_1^\ell x_2^m x_3^n \rangle_\mu = \int [dx]\, x_1^\ell x_2^m x_3^n \phi(x_1, x_2, x_3; \mu).$$

- First moments locate the average momentum sharing.
- Higher moments resolve widths, correlations, and finer structure.
- The sum rule gives

$$\langle x_1 \rangle + \langle x_2 \rangle + \langle x_3 \rangle = 1.$$

Projection of the Faddeev wave function

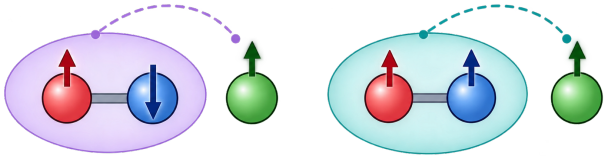
$$-\frac{f_N}{2} P\cdot n \not{n} N^+ \langle x_1^\ell x_2^m x_3^n \rangle$$
$$= \int \frac{d^4K}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \prod_{i=1}^{3} \delta(p_i \cdot n - x_i P\cdot n) \times x_1^\ell x_2^m x_3^n \chi_{uud} \mathcal{O}_{21} \mathcal{O}_3.$$

- The delta functions identify the light-front momentum fractions.
- The projectors select the leading-twist spinor structure.

Direct bridge between the covariant bound-state amplitude and the proton PDA.

The Faddeev wave function contains both $J^P = 0^+$ and $J^P = 1^+$ diquark components. We therefore separate these two sectors before evaluating their moments.

Scalar and Axial-Vector Faddeev Components



Scalar: $[ud]_{0^+}$

$$\mathcal{N}_3^{0^+} = \Gamma^{0^+}(k,K)\,\Delta^{0^+}(K)\big[\mathcal{S}(\ell;P)N(P)\big]$$

- Spin-isospin singlet $[ud]_{0^+}$.
- Produces the dominant asymmetry.

Axial-vector: $\{qq\}_{1^+}$

$$\mathcal{N}_3^{1^+} = \Gamma_\mu^{j;1^+}(k,K)\,\Delta_{\mu\nu}^{1^+}(K)\big[\mathcal{A}_\nu^j(\ell;P)N(P)\big]$$

- Isospin triplet with $\{ud\}$ and $\{uu\}$ channels.
- Generates spin-momentum structures.

We can calculate the scalar and axial-vector sectors independently and examine how each one changes the proton PDA.

Algebraic Model: Propagators and Faddeev Amplitudes

Dressed-quark propagator and spectral density function

$$S(p) = \frac{-i\gamma\cdot p + M}{p^2 + M^2}, \qquad \rho(z) = \frac{3}{4}(1-z^2), \qquad \hat{\sigma}_\Lambda(s) = \frac{\Lambda^2}{s+\Lambda^2}.$$

Scalar diquark (0^+)

Diquark Bethe–Salpeter amplitude

$$\Gamma^0(q,K)C^\dagger = i\gamma_5 \int_{-1}^{1} dz\, \rho(z)\hat{\sigma}_{\Lambda_\Gamma}(q_{+K}^2).$$

Diquark propagator

$$\Delta^{0^+}(K) = \frac{1}{K^2 + M_0^2}.$$

Quark–diquark Faddeev amplitude

$$n\mathcal{S}(\ell;P) = i\int_{-1}^{1} dz\, \rho(z)\hat{\sigma}_{\Lambda_{0p}}(K_{+P}^2).$$

Axial-vector diquark (1^+)

Diquark Bethe–Salpeter amplitude

$$n_1\Gamma_\mu^{1J}(k;K)C^\dagger = i\sigma^j\left[\gamma_\mu^T + r_1\mathfrak{f}(k;K)[\not{k},\gamma_\mu^T]\right]\int_{-1}^{1} dz\, \rho(z)\hat{\sigma}_{\Lambda_\Gamma}(k_{+K}^2).$$

Diquark propagator

$$\Delta_{\mu\nu}^{1^+}(K) = \frac{T_{\mu\nu}(K)}{K^2 + M_1^2}.$$

Quark–diquark Faddeev amplitude

$$\mathcal{A}_\mu^j = \frac{r_A\sigma^j}{6}\gamma_5(\gamma_\mu - ir_P P_\mu)\int_{-1}^{1} dz\, \rho(z)\hat{\sigma}_{\Lambda_{1p}}(K_{+P}^2).$$

Fixed model parameters ($m_p = 1$)

M	Λ_Γ	M_0	Λ_{0p}	M_1	Λ_{1p}	r_P	r_A
$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{3}$	1	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{13}{87}$	0.30(3)

Convolution Structure of the Proton PDA

- The calculation separates two longitudinal momentum partitions:
  1. sharing inside the correlated quark pair;
  2. sharing between the diquark and the bystander quark.
- The moments acquire the generic convolution form

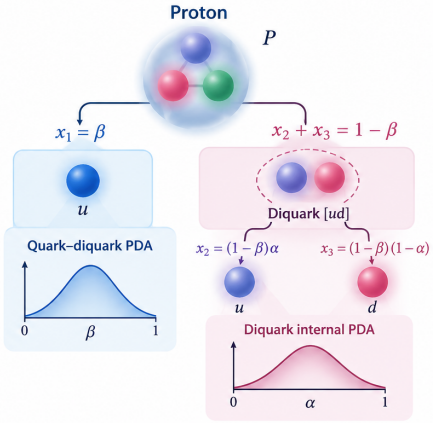
$$\langle x_1^{\ell} x_2^{m} x_3^{n} \rangle = \int_0^1 d\beta \, \mathscr{W}_{\ell mn}(\beta) \, D_{m+n}(K^2),$$

where D_{m+n} is a moment of the internal diquark PDA and $\mathscr{W}_{\ell mn}(\beta)$ contains the quark-diquark part of the Faddeev amplitude.

- PTIR representations reduce the covariant loop integrals to finite integrals over spectral and Feynman parameters.

Interpretation

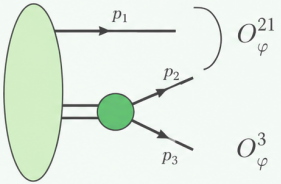
The proton PDA retains information about both the internal structure of the diquark and its motion relative to the bystander quark.



Scalar Diquark Contribution (0^+)

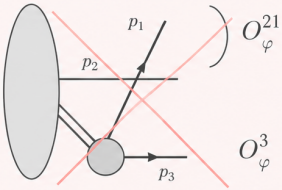
Leading-twist projection (Mellin moments)

(i) Bystander u with $[ud]_{0+}$ diquark
Survives at leading twist



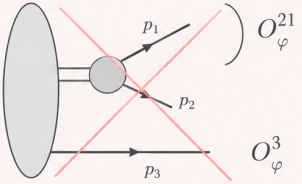
- The scalar $[ud]$ diquark correlation contributes to the proton PDA at **leading twist**.

(ii) Permutation 1
Does not contribute at leading twist
(requires nonzero orbital angular momentum)



- This contribution is suppressed at **leading twist** and **requires nonzero orbital** angular momentum.

(iii) Permutation 2
Vanishes
($[uu]_{0+} = 0$, *isoscalar diquark required*)



- This contribution vanishes because a **scalar** $[uu]$ diquark correlation is zero.

Contributes at leading twist | Does not contribute at leading twist | Forbidden (vanishes or higher twist)

Only the configuration with a bystander u quark and a scalar $[ud]$ diquark contributes to the proton PDA at leading twist.

Scalar-Diquark Distribution Amplitude

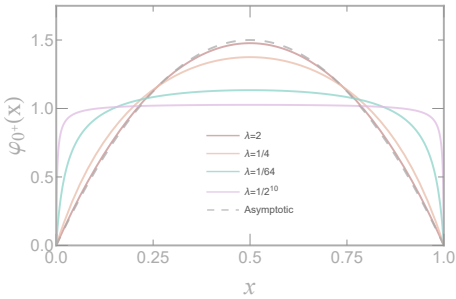
- The scalar-diquark PDA is obtained from the Mellin moments

$$\langle x^m \rangle_{0^+} = \int_0^1 dx\, x^m\, \phi_0(x)$$

- The PDA depends explicitly on the diquark virtuality through $\lambda = \frac{M^2}{K^2}$.

- As the diquark virtuality increases, the PDA becomes **broader** than the asymptotic form.

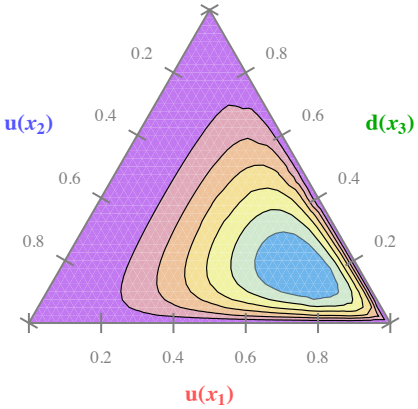
$$\phi_0(x,\lambda) = \lambda \left[1 - \frac{\lambda}{x(1-x)} \ln\left(1 + \frac{x(1-x)}{\lambda} \right) \right].$$



The internal structure of the scalar correlation has a significant effect on **how** longitudinal momentum is shared between its two quarks.

Proton PDA: Scalar-Diquark Approximation

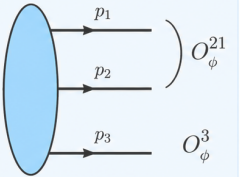
- The proton PDA combines two momentum scales:
  - momentum sharing inside the $[ud]_{0^+}$ diquark,
  - momentum sharing between the diquark and the bystander u quark.
- The resulting PDA is **broader and more asymmetric** than the asymptotic distribution.
- It is skewed toward large x_1: the **bystander u-quark carries, on average, more longitudinal momentum**.



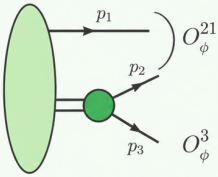
The preferred configurations are those in which the bystander quark carries a larger fraction of the proton's longitudinal momentum.

Axial–Vector Diquark Contribution (1^+)

Full projection (Mellin moments)

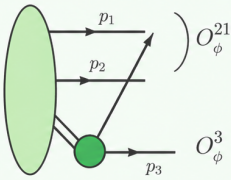


(i) Bystander u with $\{ud\}_{1+}$ (axial–vector diquark)



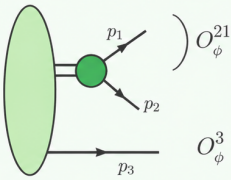
- This configuration contributes at leading twist.

(ii) Bystander d with $\{uu\}_{1+}$ (axial–vector diquark)



- This configuration contributes at leading twist.

(iii) Bystander u with $\{ud\}_{1+}$ (axial–vector diquark)



- This configuration contributes at leading twist.

Contributes at leading twist | Does not contribute at leading twist | Forbidden (vanishes or higher twist)

All three permutations contribute to the proton PDA at leading twist for the axial–vector diquark.

Axial–vector diquark: isotriplet $(\{ud\}_{1+}, \{uu\}_{1+})$

Axial-Vector Diquark PDAs

- The spin–1 structure generates two independent leading-twist projections:

$$\phi_{1^+}^{\mathrm{even}}(x), \qquad \phi_{1^+}^{\mathrm{odd}}(x).$$

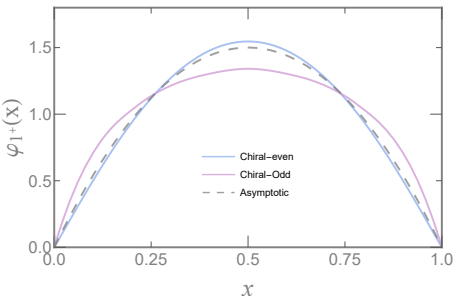
- Both distributions are symmetric under

$$x \leftrightarrow 1-x,$$

but exhibit different shapes.

- The chiral-even PDA is relatively close to the asymptotic form.

- The chiral-odd PDA shows a stronger **nonperturbative broadening**.

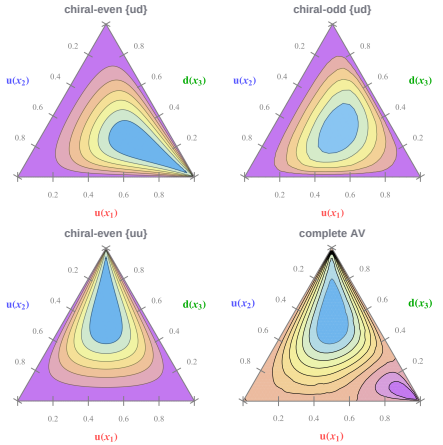


Proton PDA: Axial-Vector Diquark Approximation

- In both chiral-even PDA, the bystander quark carries the largest momentum fraction, while the chiral-odd PDA is broader.
- The three axial-vector spectator configurations are combined according to their spin–isospin weights:

$$\phi_{\mathrm{AV}} = \frac{1}{2}\left(-\phi^1_{\mathrm{AV}} + \phi^2_{\mathrm{AV}} + 2\phi^3_{\mathrm{AV}}\right).$$

- The resulting PDA is **broader and less asymmetric** than in the scalar-diquark approximation.
- Localized **negative regions** arise from destructive interference among the different spectator contributions.



Complete Proton PDA

- The complete PDA is constructed with the scalar/axial-vector balance

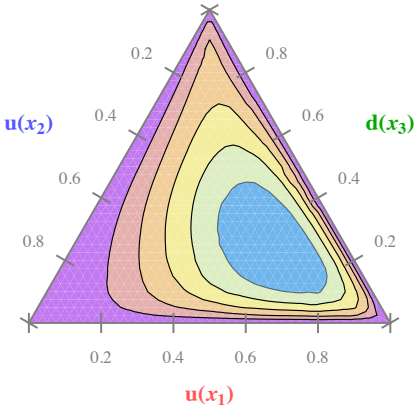
$$\phi = 0.65\,\phi_{\mathrm{S}}^{\mathrm{norm}} + 0.35\,\phi_{\mathrm{AV}}^{\mathrm{norm}}.$$

- AV correlations **redistribute longitudinal momentum** and reduce the strong asymmetry generated by the scalar sector.
- The first Mellin moments are

$$\boxed{\langle x_1\rangle = 0.404, \quad \langle x_2\rangle = 0.265, \quad \langle x_3\rangle = 0.331}$$

- The maximum remains far from the conformal limit:

$$(x_1, x_2, x_3)_{\max} \simeq (0.55,\ 0.20,\ 0.25).$$



Comparison with Other Approaches

- All nonperturbative approaches predict **asymmetric momentum sharing** in the proton.
- The scalar-diquark contribution produces a **strong enhancement of** $\langle x_1 \rangle$.
- Including axial-vector correlations **redistributes longitudinal momentum** and moves the moments away from the scalar-only limit.

	$\langle x_1 \rangle_u$	$\langle x_2 \rangle_u$	$\langle x_3 \rangle_d$
Conformal	0.333	0.333	0.333
LQCD	0.396	0.311	0.293
LQCD	0.400	0.309	0.290
scalar only	**0.450**	**0.250**	**0.250**
scalar + AV	**0.404**	**0.265**	**0.331**

Low-scale input: $\mu_0 = 0.51\,\mathrm{GeV}$.

Our result is defined at μ_0, while the lattice results are usually quoted at 2 GeV. Therefore, we require a renormalization scale; we need to **evolve** the proton PDA.

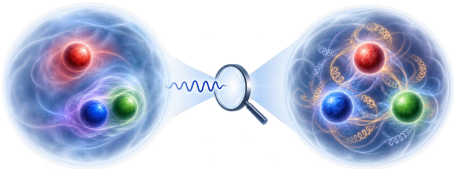
Why the PDA Must Be Evolved

- The light-ray operator defining the PDA requires renormalization:

$$\phi(x_i) \longrightarrow \phi(x_i;\mu).$$

$$\mu\frac{d}{d\mu}\phi(x_i;\mu) = -\frac{\alpha_s(\mu)}{2\pi}\int [dy]\, V(x_i,y_i)\phi(y_i;\mu).$$

- The algebraic quark–diquark calculation defines the input at $\mu_0 = 0.51\,\mathrm{GeV}$.
- Lattice-QCD results and phenomenological analyses commonly quote moments at $\mu = 2\,\mathrm{GeV}$.
- We therefore perform LO evolution of the **normalized shape**, keeping $\int [dx]\,\phi = 1$.



Increasing μ resolves the proton at progressively shorter distance scales.

1. Project the numerical PDA

At the hadronic scale, we write

$$\phi(x_i;\mu_0) = 120 x_1 x_2 x_3 \left[1 + \sum_{N=1}^{N_{\max}} \sum_{n=0}^{N} \frac{B_{Nn}(\mu_0)}{\mathcal{N}_{Nn}} P_{Nn}(x_i)\right].$$

The coefficients follow from

$$B_{Nn}(\mu_0) = \int [dx]\, P_{Nn}(x_i)\, \phi(x_i;\mu_0).$$

Conformal basis

$$P_{Nn}(x_i) = (N+n+4)(x_2+x_3)^n P_{N-n}^{(2n+3,1)}(2x_1-1) C_n^{3/2}\left(\frac{x_3-x_2}{x_2+x_3}\right).$$

Jacobi: x_1 vs. x_2+x_3, Gegenbauer: x_2 vs. x_3.

2. Diagonalize and evolve

At fixed degree N, the $N+1$ coefficients mix:

$$B_N = (B_{N0}, \ldots, B_{NN})^{\mathrm{T}}.$$

We rotate to the eigenbasis of the LO kernel:

$$E_N = V_N B_N.$$

Each eigenmode then evolves multiplicatively:

$$E_{Nk}(\mu) = E_{Nk}(\mu_0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{(\gamma_{Nk}-\gamma_{00})/\beta_0}.$$

Evolution parameters

$$\mu_0 = 0.51\ \mathrm{GeV} \longrightarrow \mu = 2\ \mathrm{GeV},$$

$$n_f = 4, \qquad \beta_0 = \frac{25}{3}, \qquad \Lambda_{\mathrm{QCD}} = 0.234\ \mathrm{GeV}$$

3. Rotate back and reconstruct

$$B_N(\mu) = V_N^{-1} E_N(\mu),$$

The subtraction of γ_{00} removes the common evolution of the three-quark normalization, so that

$$\int [dx]\, \phi(x_i;\mu) = 1, \qquad \phi(x_i;\mu) \underset{\mu\to\infty}{\longrightarrow} 120 x_1 x_2 x_3.$$

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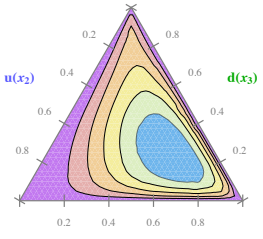
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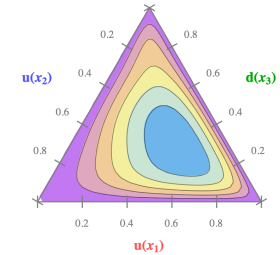
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Proton PDA at 0.51 and 2 GeV



$\mu_0 = 0.51\,\mathrm{GeV}$



$\mu = 2\,\mathrm{GeV}$

Moment	$\mu_H = 0.51\,\mathrm{GeV}$	$\mu = 2\,\mathrm{GeV}$
$\langle x_1 \rangle$	0.404	0.385
$\langle x_2 \rangle$	0.265	0.285
$\langle x_3 \rangle$	0.331	0.330

	$\mu_H = 0.51\,\mathrm{GeV}$	$\mu = 2\,\mathrm{GeV}$
$x_1^{\max}$	0.550	0.425
$x_2^{\max}$	0.200	0.225
$x_3^{\max}$	0.250	0.330

- Evolution produces a smoother PDA, shifting its dominant region toward a more balanced momentum distribution.
- The PDA **remains asymmetric** and significantly differs from the asymptotic form.

Summary

- We constructed the leading-twist proton PDA within a **Poincaré-covariant quark–diquark framework**.
- **Scalar-diquark correlations** generate the dominant asymmetry, favoring configurations in which the bystander quark carries a large fraction of the longitudinal momentum.
- **Axial-vector correlations** introduce additional spin–momentum structures, redistribute longitudinal momentum, and **reduce the scalar-induced asymmetry**.
- At $\mu_0 = 0.51\,\mathrm{GeV}$, the complete PDA remains broad and asymmetric, with $(\langle x_1\rangle, \langle x_2\rangle, \langle x_3\rangle) = (0.404, 0.265, 0.331)$.
- LO evolution to $2\,\mathrm{GeV}$ suppresses nonasymptotic modes and reduces the momentum imbalance, while preserving a visible imprint of the low-scale quark–diquark dynamics.
- A quantitative comparison still depends on the restricted Dirac and momentum structure retained in the algebraic Faddeev amplitude.

Outlook: Ground and Excited Baryons

Roper resonance: $N(1440)\,1/2^+$

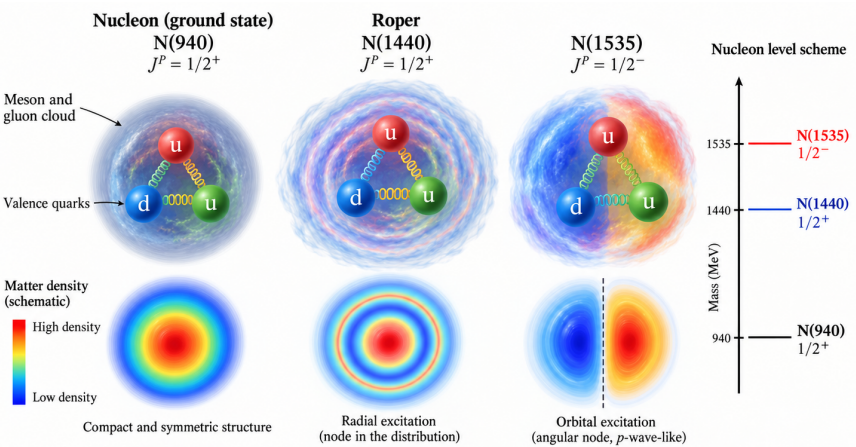
- Construct its leading-twist PDA from the corresponding excited-state Faddeev amplitude.
- Determine how the radial excitation and its nodal structure are reflected in longitudinal momentum space.
- Compare its scalar and axial-vector diquark content with that of the proton ground state.

Negative-parity state: $N(1535)\,1/2^-$

- Extend the light-front projections to a negative-parity Faddeev amplitude.
- Identify which diquark correlations control its momentum-sharing pattern.
- Separate effects associated with parity from those generated by radial excitation.

Connection with observables

After evolution to a common factorization scale, the ground- and excited-state PDAs can provide input for large-Q^2 elastic and transition form factors and other hard exclusive processes.



Ground state: N(940)

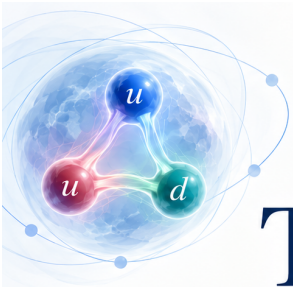
- $J^P = 1/2^+$
- three valence quarks (uud or udd)
- meson and gluon cloud
- compact and symmetric structure
- no nodes

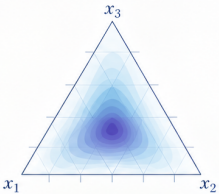
Roper: N(1440)

- $J^P = 1/2^+$
- same parity as the nucleon
- mainly interpreted as a radial excitation
- node in the amplitude distribution
- more extended structure

N(1535)

- $J^P = 1/2^-$
- opposite parity to the nucleon
- mainly interpreted as an orbital excitation (p-wave-like component)
- angular node in the distribution
- asymmetric structure (lobes)





Thank You

I could take questions, but I suspect we're all more interested in lunch right now.

$$\Phi(x_1, x_2, x_3, \mu) = \langle 0 | \varepsilon^{abc} (u^a C\gamma_5 d^b) u^c | p \rangle$$

Backup Slides

Additional definitions, numerical checks, and model assumptions

Scope and Systematic Limitations

- The calculation retains the dominant scalar and axial-vector diquark correlations in an algebraic Poincaré-covariant Faddeev framework.
- The algebraic amplitudes contain restricted momentum dependence and do not include the complete set of Dirac covariants of a numerical Faddeev solution.
- The **65/35** scalar–axial-vector balance is fixed from the adopted model input. A systematic variation would quantify this source of model dependence.
- Evolution from $\mu_0 = 0.51\,\mathrm{GeV}$ uses the leading-order three-quark kernel. The low starting scale introduces a perturbative matching uncertainty.
- The polynomial reconstruction uses $N_{\max} = 5$. Comparisons with degrees four, six, and seven provide a numerical stability test.

Interpretation of the results

The calculation identifies the qualitative impact of scalar and axial-vector correlations on the proton PDA. Quantitative precision remains controlled by the retained Faddeev structures and the evolution accuracy.

Momentum-Fraction Convention

$$x_1 + x_2 + x_3 = 1,$$
$$x_1 = x_{u_{\text{bystander}}},$$
$$(x_2, x_3) = (x_u, x_d)_{\text{correlated pair}}.$$

Physical interpretation

Large x_1 corresponds to configurations in which the bystander u quark carries the largest longitudinal momentum fraction.

The conformal basis uses

$$2x_1 - 1$$

for momentum sharing between the bystander and the pair, and

$$\frac{x_3 - x_2}{x_2 + x_3}$$

for momentum sharing inside the correlated pair.

Convention dependence

Permuting $x_2 \leftrightarrow x_3$ reflects the PDA in the internal diquark coordinate. The basis coefficients and mixing matrices must be transformed with the same convention.

Scalar and Axial-Vector Balance

The scalar and axial-vector sectors are normalized independently before they are combined:

$$\phi(x_i;\mu_0) = 0.65\,\phi_S^{\mathrm{norm}}(x_i) + 0.35\,\phi_{AV}^{\mathrm{norm}}(x_i).$$

The relative strength follows the scalar-diquark content used in continuum Faddeev-equation studies. In the algebraic amplitude this corresponds to

$$r_A = 0.30 \pm 0.03.$$

Contribution	$\langle x_1 \rangle$	$\langle x_2 \rangle$	$\langle x_3 \rangle$
Scalar only	0.450	0.250	0.250
Complete PDA	0.404	0.265	0.331

Role of the axial-vector sector

It broadens the distribution and redistributes momentum away from the strongly asymmetric scalar-diquark limit.

Complete Conformal Basis

$$P_{Nn}(x_i) = (N+n+4)(x_2+x_3)^n P_{N-n}^{(2n+3,1)}(2x_1-1) \times C_n^{3/2}\left(\frac{x_3-x_2}{x_2+x_3}\right), \qquad 0 \le n \le N,$$

with the weighted norms

$$\mathcal{N}_{Nn} = \int [dx], 120 x_1 x_2 x_3 \, P_{Nn}^2(x_i).$$

The projected moments are

$$B_{Nn}(\mu_0) = \int [dx], P_{Nn}(x_i)\phi(x_i;\mu_0),$$

and the reconstructed PDA is

$$\phi^{(N_{\max})}(x_i;\mu) = 120 x_1 x_2 x_3 \left[1 + \sum_{N=1}^{N_{\max}} \sum_{n=0}^{N} \frac{B_{Nn}(\mu)}{\mathcal{N}_{Nn}} P_{Nn}(x_i)\right].$$

Example of the Evolution Eigensystem

For the first two polynomial degrees, the change-of-basis matrices and anomalous dimensions are

$$V_1 = \begin{pmatrix} 0.6 & 1 \\ -1.8 & 1 \end{pmatrix}, \qquad \gamma_1 = \begin{pmatrix} 10/3 \\ 26/9 \end{pmatrix},$$

$$V_2 = \begin{pmatrix} -2/7 & 10/7 & 1 \\ -1 & -5/7 & 1 \\ 3/2 & -5/14 & 1 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 16/3 \\ 46/9 \\ 38/9 \end{pmatrix}.$$

$$\begin{aligned} E_N &= V_N B_N, \\ E_{Nk}(\mu) &= E_{Nk}(\mu_0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{(\gamma_{Nk} - \gamma_{00})/\beta_0}, \\ B_N(\mu) &= V_N^{-1} E_N(\mu). \end{aligned}$$

Convergence of the Polynomial Reconstruction

$N_{\max}$	Integrated absolute error	Relative L^2 error
3	0.0820	0.0902
4	0.0426	0.0496
5	0.0396	0.0444
6	0.0314	0.0367
7	0.0321	0.0368

Central truncation

We use $N_{\max} = 5$. It captures the first moments and the global shape while avoiding sensitivity to small local oscillations at higher degree.

- Including degree five changes the evolved distribution by approximately **2.4%** relative to the degree-four result.
- The first Mellin moments remain stable when degrees six and seven are included.

Retained Faddeev Structures

Scalar correlation

$$\mathcal{N}_3^{0^+} = \Gamma^{0^+}(k,K)\,\Delta^{0^+}(K) \times \left[\mathcal{S}(\ell;P)N(P)\right].$$

The scalar $[ud]_{0^+}$ channel generates the dominant displacement toward large bystander momentum.

Axial-vector correlation

$$\mathcal{N}_3^{1^+} = \Gamma_\mu^{j;1^+}(k,K)\,\Delta_{\mu\nu}^{1^+}(K) \times \left[\mathcal{A}_\nu^j(\ell;P)N(P)\right].$$

The $\{ud\}$ and $\{uu\}$ channels generate chiral-even and chiral-odd leading-twist projections.

$$\Psi_N = \sum_{a=1}^{3}\left(\mathcal{N}_a^{0^+} + \mathcal{N}_a^{1^+}\right)$$

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