

中國科學院高能物理研究所

Institute of High Energy Physics, CAS





中國科學院

CHINESE ACADEMY OF SCIENCES

Diquark correlations probed in heavy baryon hadronic weak decays

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ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

Theoretical motivations and empirical evidence for diquark correlations in hadrons

2026.10.19-23, ECT* Trento

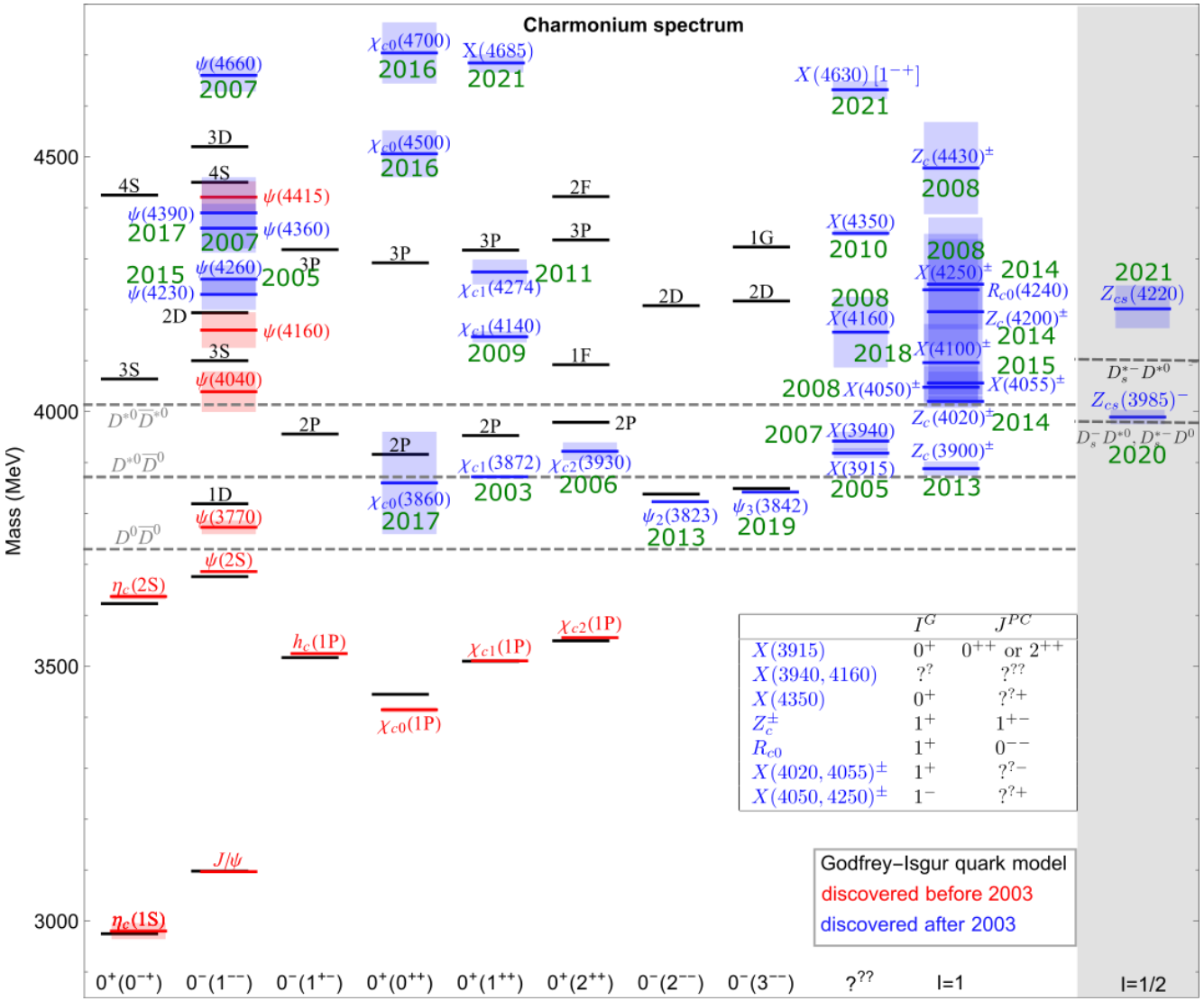
Outline

1. Background and motivation
2. Charmed baryon decays as a probe for the diquark correlation
   - **(I) Cabbibo favored decays:** $\Lambda_c \to \Lambda\pi^+, \Sigma^+\pi^0, \Sigma^0\pi^+$
   - **(II) Singly Cabbibo favored decays:** $\Lambda_c \to \Lambda K^+, \Sigma^0 K^+$
3. Summary

1. Background and motivation

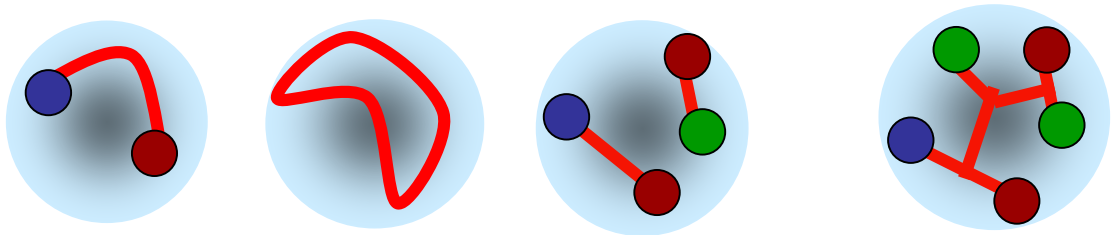
Conventional quark model & exotic hadrons

Charmonium(-like) mass spectra

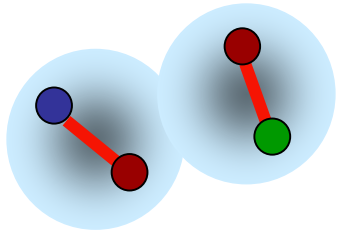


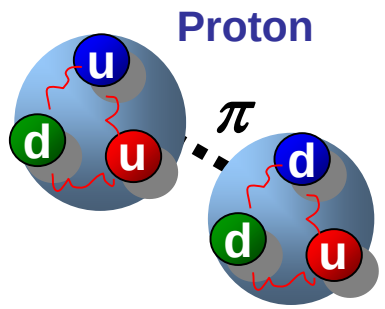
Exotic hadrons

Hybrids Glueballs Tetraquarks Pentaquarks



Hadronic molecules Deuteron

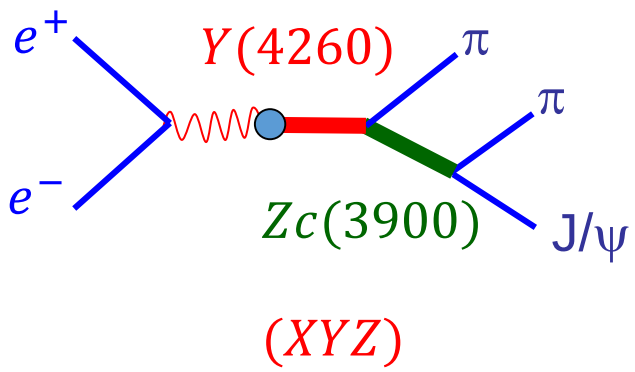




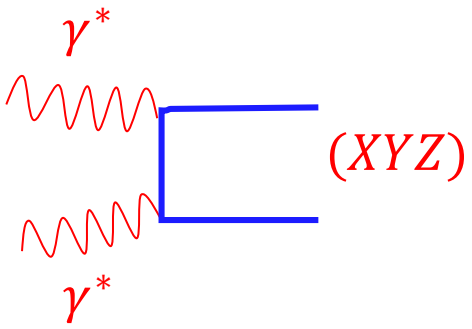
- **Exotic-hadron structure reveals the nature of nonperturbative QCD!**

Production mechanisms of exotic hadrons

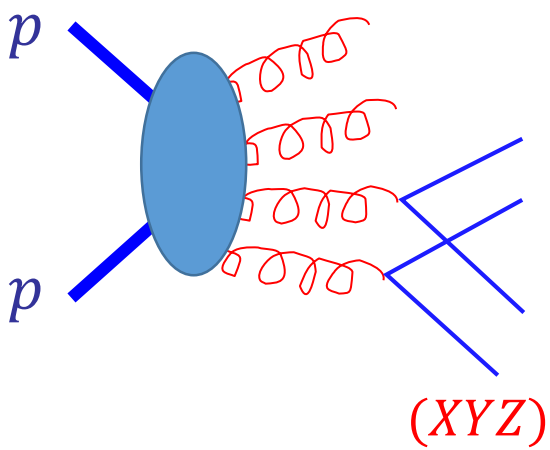
$e^+ e^-$ annihilation



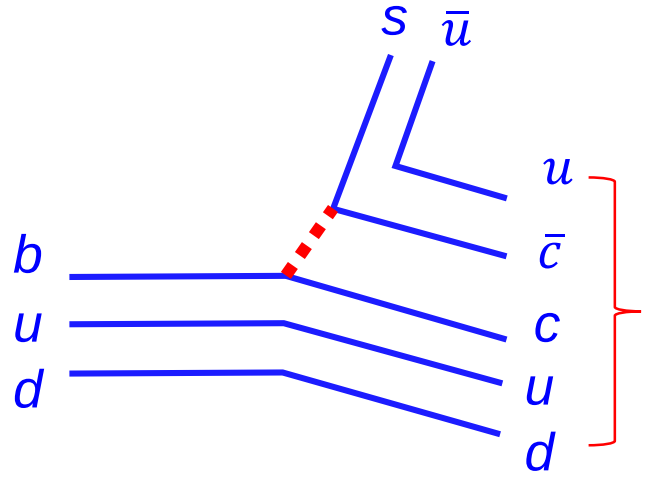
Two-photon collisions



Hadron collisions



Hadronic weak decays



Searching for exotic hadrons is a major goal of accelerator-based experiments.

Weak processes probe hadron structure

Key challenges in hadron-structure studies:

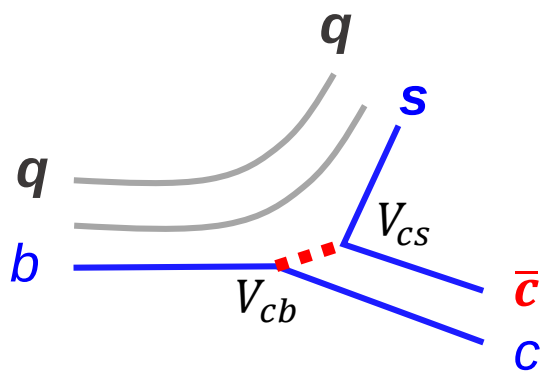
- **Hadron constituent number can vary—how do we describe the nonperturbative degrees of freedom?**
- **How can experiments reveal the dynamics of internal hadron structure?**

Why use weak processes to probe hadron structure:

• Flavor filtering

• A unique test of kinematic singularities, especially triangle singularities

• A probe of hadron spin structure and quark quantum correlations



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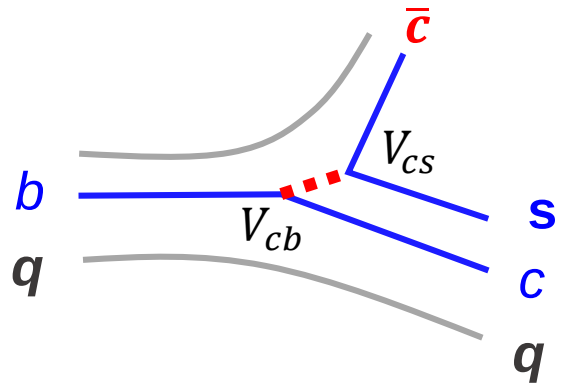

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- **A probe of hadron spin structure and quark quantum correlations**

Cabibbo–Kobayashi–Maskawa (CKM) matrix elements

$$V_{\mathrm{CKM}} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$

$$\begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

2. Charmed baryon decays as a probe for the diquark correlation

(I) Cabbibo favored decays: $\Lambda_c \to \Lambda\pi^+, \Sigma^+\pi^0, \Sigma^0\pi^+$

(II) Singly Cabbibo favored decays: $\Lambda_c \to \Lambda K^+, \Sigma^0 K^+$

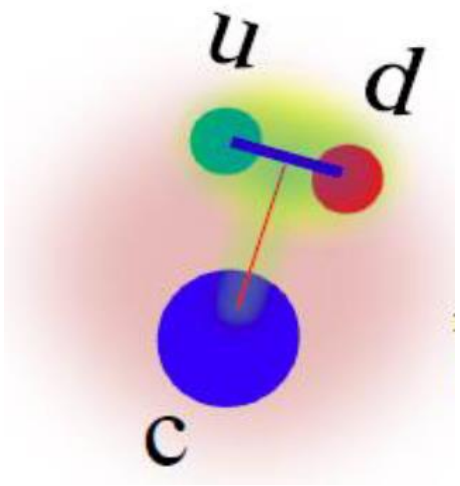
P.Y. Niu, J.-M. Richard, Q. Wang, Q. Zhao, PRD 102, 073005 (2020); Chin. Phys. C 45, no.1, 013101 (2021);
P.-Y. Niu, Q. Wang and Q. Zhao, PLB 826, 136916 (2022);
P.-Y. Niu, Q. Wang and Q. Zhao, PRD 113, 033001 (2026)

(I) Cabbibo favored decays

$\Lambda_c \to \Lambda\pi^+$

$\Lambda_c \to \Sigma^+\pi^0$

$\Lambda_c \to \Sigma^0\pi^+$



- **Theoretical studies based on different motivations**

Bag model and the current-algebra approach

Haiyang Cheng; Fanrong Xu ; F. Hussain …

Lightfront quark model

Wei Wang; Fusheng Yu …

SU(3) flavor symmetry

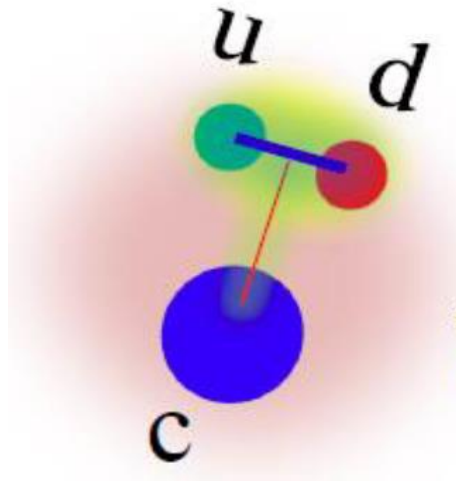
Chaoqiang Geng; Wei Wang; M. J. Savage; K. K. Sharma …

QCD sum rules

L. S. Kisslinger and B. Singha

……

Diquark correlation plays a crucial role in understanding the structure of heavy-light systems

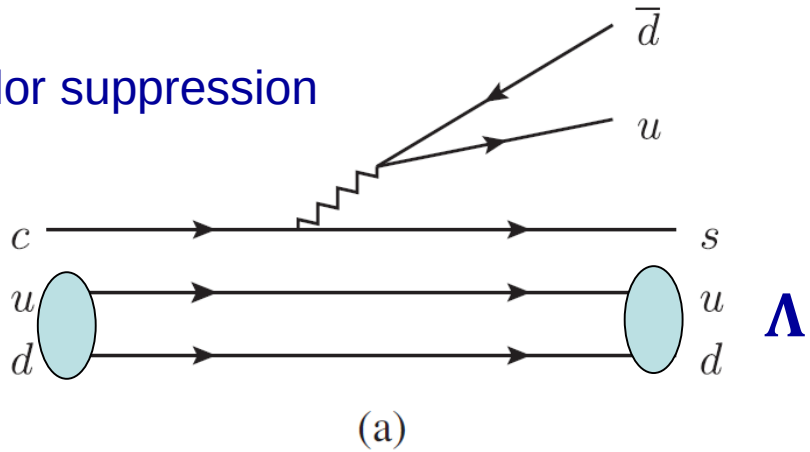


Color SU(3): $3 \otimes 3 \otimes 3 = (\bar{3} \oplus 6) \otimes 3 = (1 \oplus 8) \oplus (8 \oplus 10)$

Flavor SU(3): $\phi_c^{\bar{3}} = \frac{1}{\sqrt{2}}(ud - du)c$

ud diquark: $(I, S) = (0,0)$

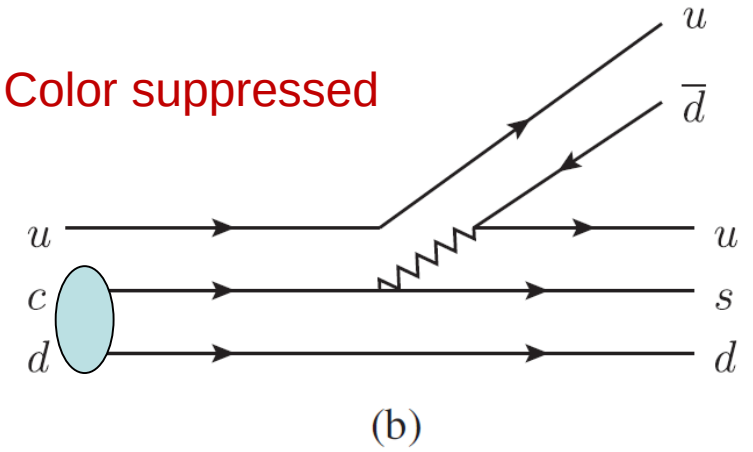
No color suppression



Λ_c Λ

(a)

Color suppressed

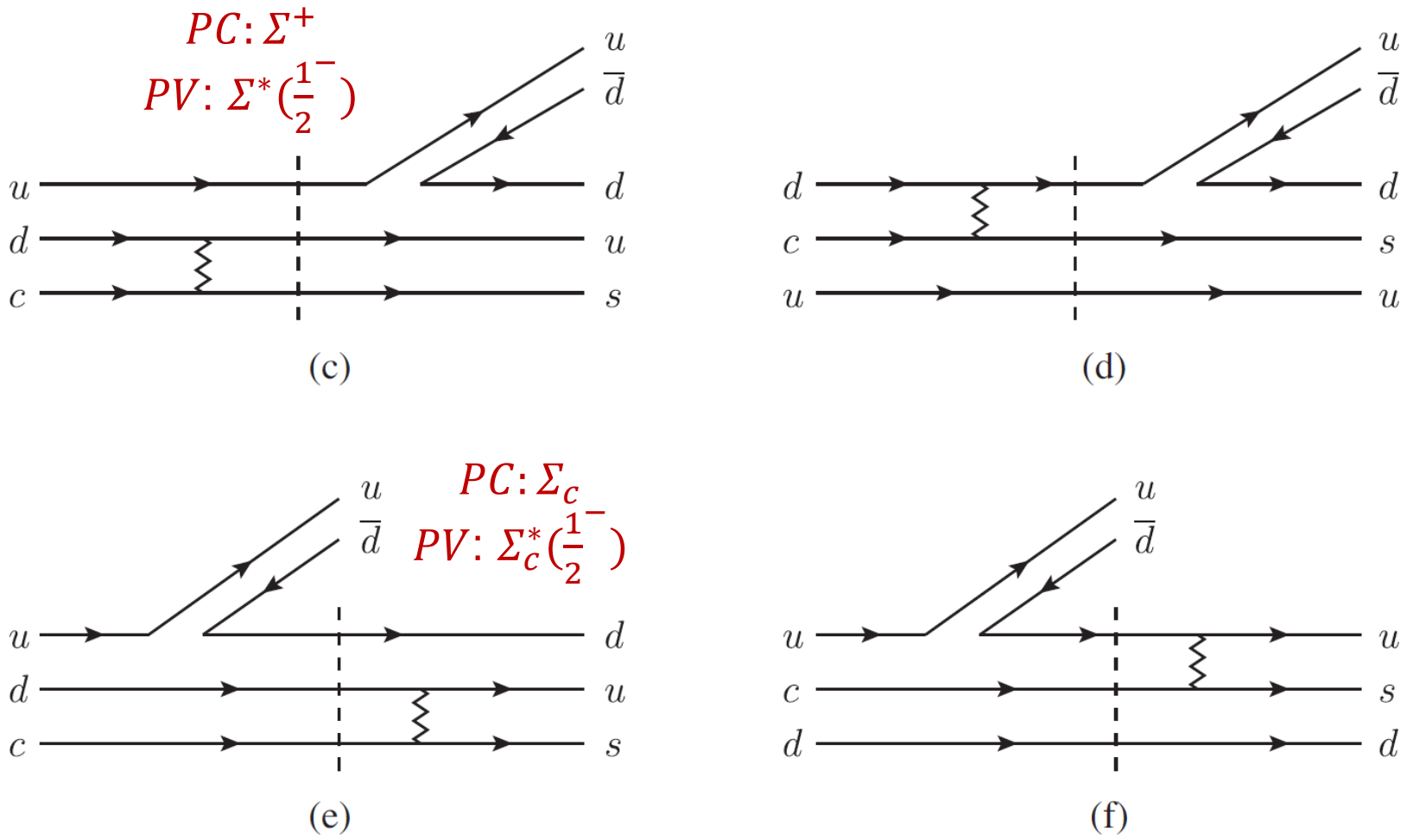


Λ, Σ

(b)

(× %)	$\mathrm{BR}(\Lambda_c \to \Lambda\pi^+)$	$\mathrm{BR}(\Lambda_c \to \Sigma^0\pi^+)$	$\mathrm{BR}(\Lambda_c \to \Sigma^+\pi^0)$
PDG data [24]	1.30 ± 0.07	1.29 ± 0.07	1.24 ± 0.10
BESIII [20]	$1.24 \pm 0.07 \pm 0.03$	$1.27 \pm 0.08 \pm 0.03$	$1.18 \pm 0.10 \pm 0.03$

Pole terms due to internal conversions may play a non-negligible role



P.Y. Niu, J.-M. Richard, Q. Wang, Q. Zhao, Phys. Rev. D 102, 073005 (2020); Chin. Phys. C 45, no.1, 013101 (2021);
P.-Y. Niu, Q. Wang and Q. Zhao, Phys. Lett. B 826, 136916 (2022);

Quark and antiquark fields:

$$q(x) = \int \frac{d\boldsymbol{p}}{(2\pi)^{3/2}} \left(\frac{m}{p^0}\right)^{1/2} \sum_s [u_s(\boldsymbol{p}) b_s(\boldsymbol{p}) e^{ip\cdot x} + v_s(\boldsymbol{p}) d_s^\dagger(\boldsymbol{p}) e^{-ip\cdot x}],$$

$$\bar{q}(x) = \int \frac{d\boldsymbol{p}}{(2\pi)^{3/2}} \left(\frac{m}{p^0}\right)^{1/2} \sum_s [\bar{u}_s(\boldsymbol{p}) b_s^\dagger(\boldsymbol{p}) e^{-ip\cdot x} + \bar{v}_s(\boldsymbol{p}) d_s(\boldsymbol{p}) e^{ip\cdot x}].$$

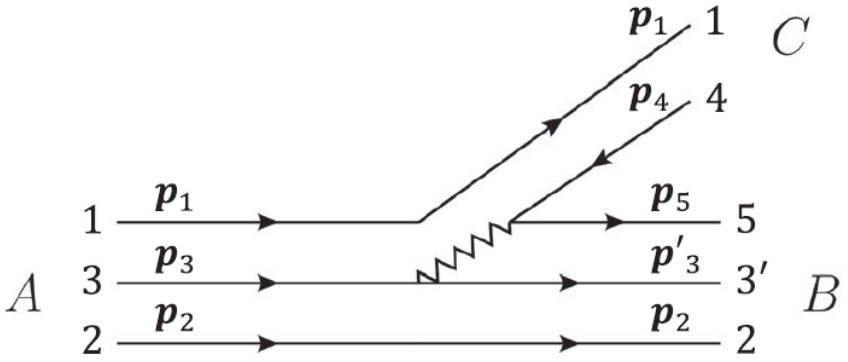
Meson and baryon in the quark model:

$$|M(\boldsymbol{P}_c)_{J,J_z}\rangle = \sum_{S_z,L_z;c_i} \langle L, L_z; S, S_z | J, J_z\rangle \int d\boldsymbol{p}_1 d\boldsymbol{p}_2 \delta^3(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{P}_c) \Psi_{N,L,L_z}(\boldsymbol{p}_1, \boldsymbol{p}_2) \chi^{S,S_z}_{s_1,s_2}$$
$$\times \frac{\delta_{c_1 c_2}}{\sqrt{3}} \phi_{i_1,i_2} b^\dagger_{c_1,i_1,s_1,\boldsymbol{p}_1} d^\dagger_{c_2,i_2,s_2,\boldsymbol{p}_2} |0\rangle,$$

$$|B(\boldsymbol{P}_c)_{J,J_z}\rangle = \sum_{S_z,L_z;c_i} \langle L, L_z; S, S_z | J, J_z\rangle \int d\boldsymbol{p}_1 d\boldsymbol{p}_2 d\boldsymbol{p}_3 \delta^3(\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}_3 - \boldsymbol{P}_c) \Psi_{N,L,L_z}(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3) \chi^{S,S_z}_{s_1,s_2,s_3}$$
$$\times \frac{\epsilon_{c_1 c_2 c_3}}{\sqrt{6}} \phi_{i_1,i_2,i_3} b^\dagger_{c_1,i_1,s_1,\boldsymbol{p}_1} b^\dagger_{c_2,i_2,s_2,\boldsymbol{p}_2} b^\dagger_{c_3,i_3,s_3,\boldsymbol{p}_3} |0\rangle,$$

Effective weak Hamiltonian:

$$H_W = \frac{G_F}{\sqrt{2}} \int dx \frac{1}{2} \{ J^{-,\mu}(x), J^{+}_{\mu}(x) \},$$



with

$$J^{+,\mu}(x) = (\,\bar{u} \quad \bar{c}\,)\gamma^{\mu}(1-\gamma_5)\begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix},$$

$$J^{-,\mu}(x) = (\,\bar{d} \quad \bar{s}\,)\begin{pmatrix} \cos\theta_C & -\sin\theta_C \\ \sin\theta_C & \cos\theta_C \end{pmatrix}\gamma^{\mu}(1-\gamma_5)\begin{pmatrix} u \\ c \end{pmatrix}.$$

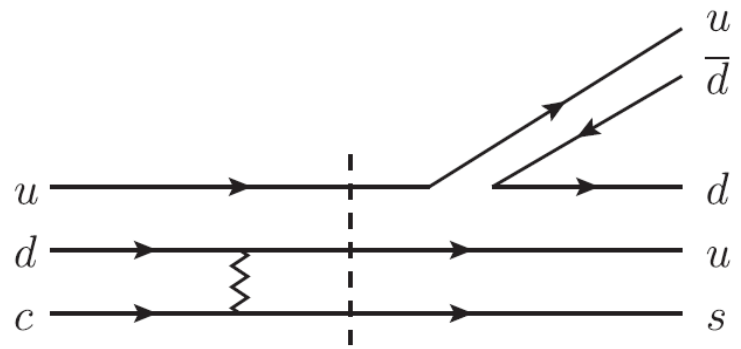
$$H_W = H_W^{PC} + H_W^{PV},$$

where

$$H_W^{PC} = \frac{G_F}{\sqrt{2}} \int dx [j^{-}_{\mu}(x) j^{+,\mu}(x) + j^{-}_{5,\mu}(x) j^{+,\mu}_{5}(x)],$$

$$H_W^{PV} = \frac{G_F}{\sqrt{2}} \int dx [j^{-}_{\mu}(x) j^{+,\mu}_{5}(x) + j^{-}_{5,\mu}(x) j^{+,\mu}(x)].$$

For the Cabbibo-favored 2 → 2 quark transition process, the relevant term is



$$H_{W,2\to2} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs} \frac{1}{(2\pi)^3} \delta^3(\boldsymbol{p}'_i + \boldsymbol{p}'_j - \boldsymbol{p}_i - \boldsymbol{p}_j) \bar{u}(\boldsymbol{p}'_i) \gamma_\mu (1-\gamma_5) u(\boldsymbol{p}_i) \bar{u}(\boldsymbol{p}'_j) \gamma^\mu (1-\gamma_5) u(\boldsymbol{p}_j).$$

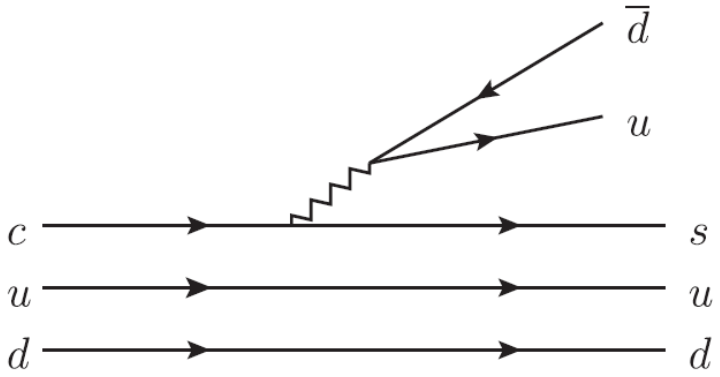
$$H^{PC}_{W,2\to2} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs} \frac{1}{(2\pi)^3} \sum_{i\neq j} \hat{\alpha}_i^{(-)} \hat{\beta}_j^{(+)} \delta^3(\boldsymbol{p}'_i + \boldsymbol{p}'_j - \boldsymbol{p}_i - \boldsymbol{p}_j)(1 - \langle s'_{z,i}|\boldsymbol{\sigma_i}|s_{z,i}\rangle \langle s'_{z,j}|\boldsymbol{\sigma_j}|s_{z,j}\rangle),$$

$$H^{PV}_{W,2\to2} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs} \frac{1}{(2\pi)^3} \sum_{i\neq j} \hat{\alpha}_i^{(-)} \hat{\beta}_j^{(+)} \delta^3(\boldsymbol{p}'_i + \boldsymbol{p}'_j - \boldsymbol{p}_i - \boldsymbol{p}_j)$$

$$\times \left\{ -(\langle s'_{z,i}|\boldsymbol{\sigma_i}|s_{z,i}\rangle - \langle s'_{z,i}|\boldsymbol{\sigma_j}|s_{z,j}\rangle) \left[\left(\frac{\boldsymbol{p}_i}{2m_i} - \frac{\boldsymbol{p}_j}{2m_j} \right) + \left(\frac{\boldsymbol{p}'_i}{2m'_i} - \frac{\boldsymbol{p}'_j}{2m'_j} \right) \right] \right.$$

$$\left. + i(\langle s'_{z,i}|\boldsymbol{\sigma_i}|s_{z,i}\rangle \times \langle s'_{z,i}|\boldsymbol{\sigma_j}|s_{z,j}\rangle) \left[\left(\frac{\boldsymbol{p}_i}{2m_i} - \frac{\boldsymbol{p}_j}{2m_j} \right) - \left(\frac{\boldsymbol{p}'_i}{2m'_i} - \frac{\boldsymbol{p}'_j}{2m'_j} \right) \right] \right\},$$

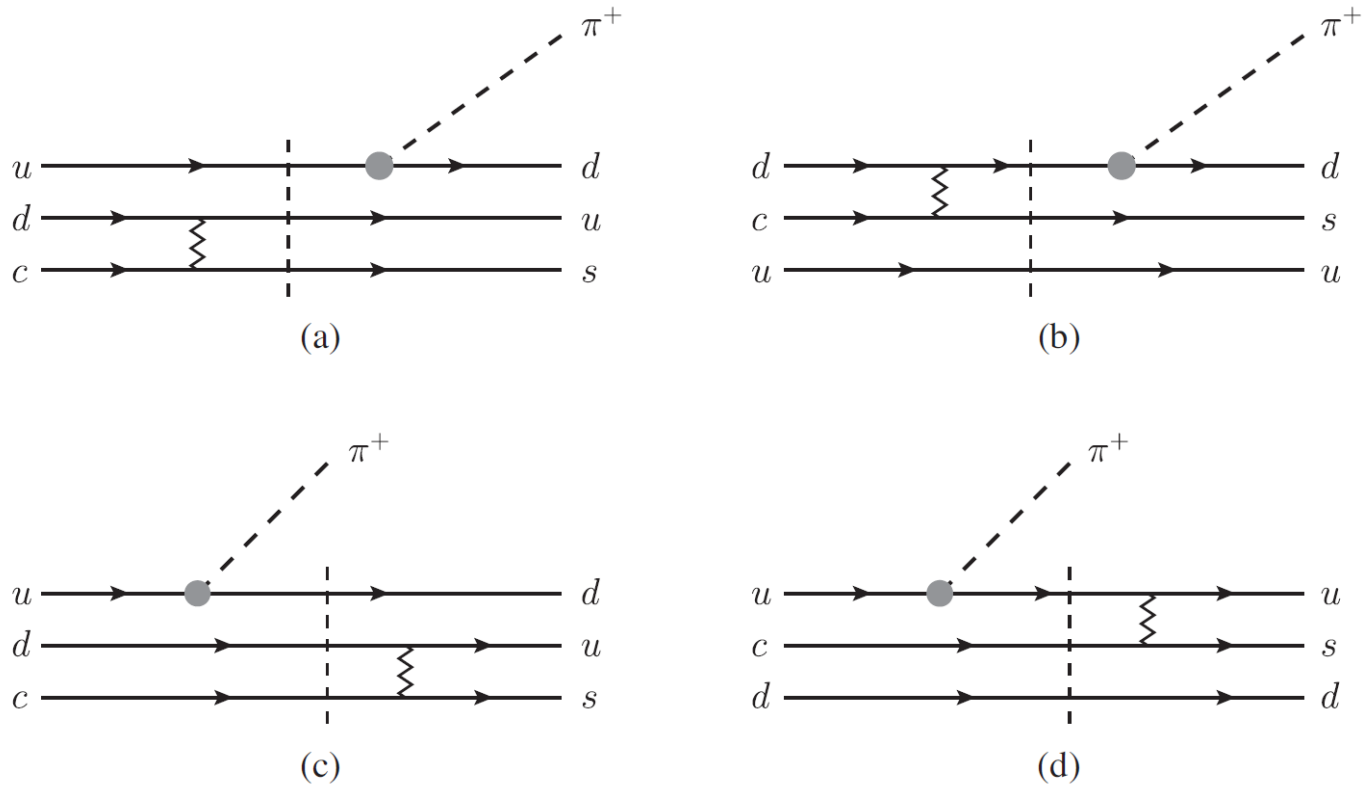
Direct emission:



$$H^{PC}_{W,1\to 3} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs} \frac{\beta}{(2\pi)^3} \delta^3(\boldsymbol{p}_3 - \boldsymbol{p}_3{}' - \boldsymbol{p}_4 - \boldsymbol{p}_5) \Bigg\{ \langle s_3'|I|s_3\rangle \langle s_5\bar{s}_4|\boldsymbol{\sigma}|0\rangle \left(\frac{\boldsymbol{p}_5}{2m_5} + \frac{\boldsymbol{p}_4}{2m_4}\right)$$
$$- \left[\left(\frac{\boldsymbol{p}_3'}{2m_3{}'} + \frac{\boldsymbol{p}_3}{2m_3}\right)\langle s_3'|I|s_3\rangle - i\langle s_3'|\boldsymbol{\sigma}|s_3\rangle \times \left(\frac{\boldsymbol{p}_3}{2m_3} - \frac{\boldsymbol{p}_3'}{2m_3'}\right)\right]\langle s_5\bar{s}_4|\boldsymbol{\sigma}|0\rangle$$
$$- \langle s_3'|\boldsymbol{\sigma}|s_3\rangle \left[\left(\frac{\boldsymbol{p}_5}{2m_5} + \frac{\boldsymbol{p}_4}{2m_4}\right)\langle s_5\bar{s}_4|I|0\rangle - i\langle s_5\bar{s}_4|\boldsymbol{\sigma}|0\rangle \times \left(\frac{\boldsymbol{p}_4}{2m_4} - \frac{\boldsymbol{p}_5}{2m_5}\right)\right]$$
$$+ \langle s_3'|\boldsymbol{\sigma}|s_3\rangle \left(\frac{\boldsymbol{p}_3'}{2m_3'} + \frac{\boldsymbol{p}_3}{2m_3}\right)\langle s_5\bar{s}_4|I|0\rangle \Bigg\} \hat{\alpha}_3^{(-)} \hat{I}'_\pi,$$

$$H^{PV}_{W,1\to 3} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs} \frac{\beta}{(2\pi)^3} \delta^3(\boldsymbol{p}_3 - \boldsymbol{p}_3' - \boldsymbol{p}_4 - \boldsymbol{p}_5)(-\langle s_3'|I|s_3\rangle \langle s_5\bar{s}_4|I|0\rangle + \langle s_3'|\boldsymbol{\sigma}|s_3\rangle \langle s_5\bar{s}_4|\boldsymbol{\sigma}|0\rangle)\hat{\alpha}_3^{(-)} \hat{I}'_\pi,$$

Pole term contributions:



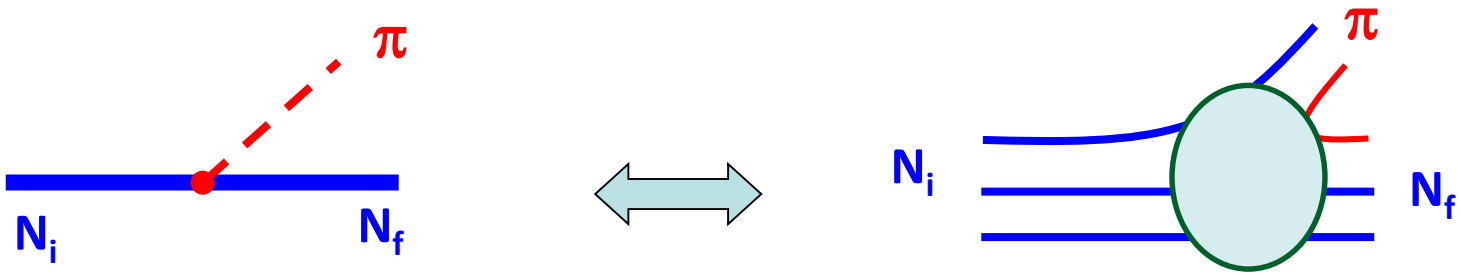
Intermediate charmed baryon states

Intermediate hyperon states

$$M^{J_f,J_f^z;J_i,J_i^z}_{\text{Pole,A};PC} = \langle B_f(\boldsymbol{P}_f;J_f,J_f^z)|H_\pi|B_m(\boldsymbol{P}_i;J_i,J_i^z)\rangle \frac{i}{\not{p}_{B_m} - m_{B_m} + i\frac{\Gamma_{B_m}}{2}} \langle B_m(\boldsymbol{P}_i;J_i,J_i^z)|H^{PC}_{W,2\to 2}|B_c(\boldsymbol{P}_i;J_i,J_i^z)\rangle,$$

$$M^{J_f,J_f^z;J_i,J_i^z}_{\text{Pole,A};PV} = \langle B_f(\boldsymbol{P}_f;J_f,J_f^z)|H_\pi|B'_m(\boldsymbol{P}_i;J_i,J_i^z)\rangle \frac{i}{\not{p}_{B'_m} - m_{B'_m} + i\frac{\Gamma_{B'_m}}{2}} \langle B'_m(\boldsymbol{P}_i;J_i,J_i^z)|H^{PV}_{W,2\to 2}|B_c(\boldsymbol{P}_i;J_i,J_i^z)\rangle,$$

Chiral quark model for light meson coupling to light flavors:



$$H_m = \sum_j \int dx \frac{1}{f_m} \bar{q}_j(x) \gamma_\mu^j \gamma_5^j q_j(x) \partial^\mu \phi_m(x),$$

The non-relativistic expansion gives:

$$H_\pi = \frac{1}{\sqrt{(2\pi)^3 2\omega_k}} \frac{1}{f_\pi} \left[k_0 \left(\frac{\boldsymbol{\sigma}_j \cdot \boldsymbol{p}'_j}{2m_j} + \frac{\boldsymbol{\sigma}_j \cdot \boldsymbol{p}_j}{2m_j} \right) - \boldsymbol{\sigma}_j \cdot \boldsymbol{k} \right] I_j \delta^3(\boldsymbol{p}'_j + \boldsymbol{k} - \boldsymbol{p}_j).$$

A. Manohar and H. Georgi, Nucl. Phys. B234, 189 (1984). Z. P. Li, H. X. Ye, and M. H. Lu, Phys. Rev. C 56, 1099 (1997); Q. Zhao, J. S. Al-Khalili, Z. P. Li, and R. L.Workman, Phys. Rev. C 65, 065204 (2002); X. H. Zhong and Q. Zhao, Phys. Rev. D 77, 074008 (2008)

Partial widths Γ and parity asymmetry α

$$\Gamma(A \to B + C) = 8\pi^2 \frac{|\boldsymbol{k}| E_B E_C}{M_A} \frac{1}{2J_A + 1} \times \sum_{\text{spin}} \left(|M_{\text{PC}}|^2 + |M_{\text{PV}}|^2 \right)$$

The decay width receives coherent contributions from DME, CS, WS, and SW amplitudes.

$$\alpha = \frac{2\text{Re}[M_{\text{PV}}^* M_{\text{PC}}]}{|M_{\text{PC}}|^2 + |M_{\text{PV}}|^2}$$

The asymmetry parameter is sensitive to the relative phase and interference of parity-conserving (PC) and parity-violating (PV) amplitudes.

Non-factorizable pole terms will contribute to the imaginary part of the amplitude and are numerically important.

Eqs. (20), (22)

TABLE I. The baryon masses and widths taken from PDG [24] in the calculation. Only the central values of the masses and widths are listed. Note that the $J^P = 1/2^-$ states for both charmed and strange baryons have not been well determined. We assign $\Sigma_c^{*0}(2806)$ and $\Sigma_c^{*+}(2792)$ for the charmed states and $\Sigma^{*+}(1620)$ and $\Sigma^{*+}(1750)$ for the strange baryons with $J^P = 1/2^-$.

Particles	Λ	Λ_c	Σ^0	Σ^+	$\Sigma^{*+}(1620)$	$\Sigma^{*+}(1750)$	Σ_c^0	Σ_c^{*0}	Σ_c^+	Σ_c^{*+}
$I(J^P)$	$0(1/2^+)$	$0(1/2^+)$	$1(1/2^+)$	$1(1/2^+)$	$1(1/2^-)$	$1(1/2^-)$	$1(1/2^+)$	$1(1/2^-)$	$1(1/2^+)$	$1(1/2^-)$
Mass (GeV)	1.116	2.286	1.193	1.189	1.62	1.75	2.453	2.806	2.452	2.792
Width (GeV)	...	...	...	...	0.050	0.050	0.00183	0.072	0.0046	0.062

TABLE V. The amplitudes with $J_f^z = J_i^z = -1/2$ for different processes and the unit is 10^{-9} GeV$^{-1/2}$. Amplitudes $A1(PV)$ and $A2(PV)$ are given by the parity-violating intermediate states $\Sigma^{*+}(1620)$ ($[\mathbf{70}, {}^2\mathbf{8}]$) and $\Sigma^{*+}(1750)$ ($[\mathbf{70}, {}^4\mathbf{8}]$), respectively.

Processes	$A(PC)$	$A1(PV)$	$A2(PV)$	$B(PC)$	$B(PV)$	$CS(PC)$	$CS(PV)$	$DPE(PC)$	$DPE(PV)$
$\Lambda_c \to \Lambda\pi^+$	-16.50	$0.74 - 0.023i$	$-2.57 + 0.10i$	$22.33 + 0.021i$	$-10.72 - 0.33i$	3.50	-4.17	-42.47	24.07
$\Lambda_c \to \Sigma^0\pi^+$	19.67	$-3.21 + 0.10i$	$-2.23 + 0.090i$	$-40.73 - 0.040i$	$19.16 + 0.60i$	-6.04	7.53	0	0
$\Lambda_c \to \Sigma^+\pi^0$	19.64	$-3.15 + 0.098i$	$-2.19 + 0.088$	$-40.65 - 0.10i$	$19.28 + 0.52i$	-6.04	7.51	0	0

TABLE VI. The calculated branching ratios (in %) of the Λ_c decays in this work are compared with experimental data [20,24] and other model calculations [4,39].

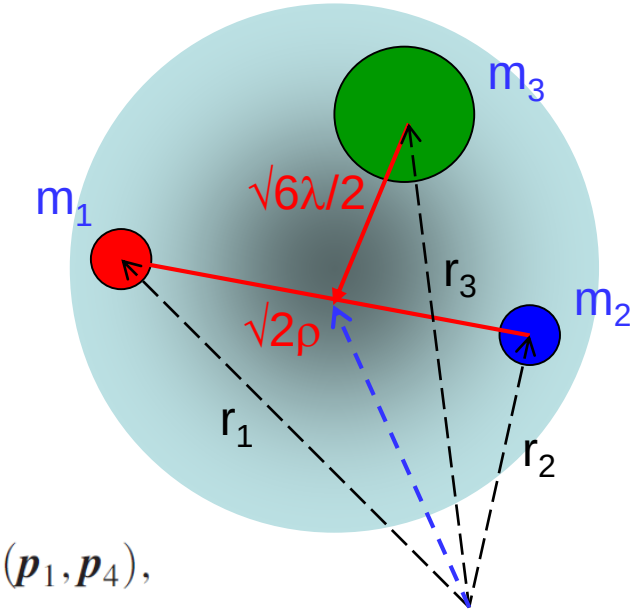
	$\mathrm{BR}(\Lambda_c \to \Lambda\pi^+)$	$\mathrm{BR}(\Lambda_c \to \Sigma^0\pi^+)$	$\mathrm{BR}(\Lambda_c \to \Sigma^+\pi^0)$
PDG data [24]	1.30 ± 0.07	1.29 ± 0.07	1.24 ± 0.10
BESIII [20]	$1.24 \pm 0.07 \pm 0.03$	$1.27 \pm 0.08 \pm 0.03$	$1.18 \pm 0.10 \pm 0.03$
SU(3) [39]	1.3 ± 0.2	1.3 ± 0.2	1.3 ± 0.2
Pole model [4]	1.30 ± 0.07	1.29 ± 0.07	1.24 ± 0.10
Current algebra [4]	1.30 ± 0.07	1.29 ± 0.07	1.24 ± 0.10
This work	1.30	1.24	1.26

The parity asymmetry parameter α and its uncertainties caused by the quark model parameters with 20% errors

	$\Lambda_c \to \Lambda\pi^+$	$\Lambda_c \to \Sigma^0\pi^+$	$\Lambda_c \to \Sigma^+\pi^0$
PDG data [24]	-0.91 ± 0.14	$\cdots$	-0.45 ± 0.32
Pole model [4]	-0.95	0.78	0.78
Current algebra [4]	-0.99	-0.49	-0.49
This work	-0.16 ± 0.27	-0.46 ± 0.20	-0.47 ± 0.19

Quark correlation in terms of wavefunction convolutions:

$$L_{CS}^{0,0;0,0}(\boldsymbol{p}_\rho) = \int d\boldsymbol{p}_1 d\boldsymbol{p}_4 \int d\boldsymbol{p}_\lambda d\boldsymbol{p}'_\rho d\boldsymbol{p}'_\lambda \delta^3(\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_4)$$
$$\times \delta^3\left(\frac{m_q}{M}\boldsymbol{P}_i + \frac{1}{2}\boldsymbol{p}_\lambda + \boldsymbol{p}_\rho - \boldsymbol{p}_1\right)\delta^3\left(\frac{m_q}{M}\boldsymbol{P}_i + \frac{1}{2}\boldsymbol{p}_\lambda - \boldsymbol{p}_\rho - \frac{m_q}{M'}\boldsymbol{P}_f - \frac{1}{2}\boldsymbol{p}'_\lambda + \boldsymbol{p}'_\rho\right)$$
$$\times \delta^3\left(\frac{m_c}{M}\boldsymbol{P}_i - \boldsymbol{p}_\lambda - \frac{2m_q}{M'}\boldsymbol{P}_f + \frac{1}{2}\boldsymbol{p}'_\lambda - \boldsymbol{p}'_\rho - \boldsymbol{p}_4\right)\Psi_{0,0,0}(\boldsymbol{p}_\rho,\boldsymbol{p}_\lambda)\Psi^*_{0,0,0}(\boldsymbol{p}'_\rho,\boldsymbol{p}'_\lambda)\Psi^*_{0,0,0}(\boldsymbol{p}_1,\boldsymbol{p}_4),$$



Jacobi coordinate

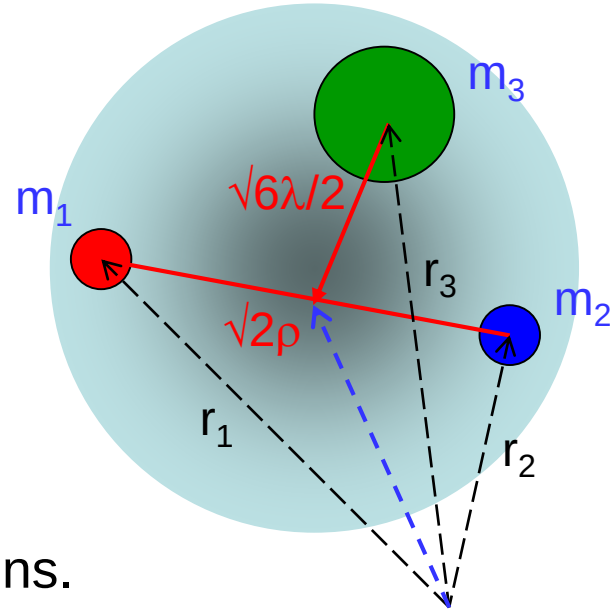
$$L_{DPE}^{0,0;0,0}(\boldsymbol{p}_\rho) = \int d\boldsymbol{p}_4 d\boldsymbol{p}_5 \int d\boldsymbol{p}_\lambda d\boldsymbol{p}'_\rho d\boldsymbol{p}'_\lambda \delta^3(\boldsymbol{k}-\boldsymbol{p}_4-\boldsymbol{p}_5)$$
$$\times \delta^3\left(\frac{m_q}{M}\boldsymbol{P}_i + \frac{1}{2}\boldsymbol{p}_\lambda + \boldsymbol{p}_\rho - \frac{m_q}{M'}\boldsymbol{P}_f - \frac{1}{2}\boldsymbol{p}'_\lambda - \boldsymbol{p}'_\rho\right)\delta^3\left(\frac{m_q}{M}\boldsymbol{P}_i + \frac{1}{2}\boldsymbol{p}_\lambda - \boldsymbol{p}_\rho - \frac{m_q}{M'}\boldsymbol{P}_f - \frac{1}{2}\boldsymbol{p}'_\lambda + \boldsymbol{p}'_\rho\right)$$
$$\times \delta^3\left(\frac{m_c}{M}\boldsymbol{P}_i - \boldsymbol{p}_\lambda - \frac{m_q}{M'}\boldsymbol{P}_f + \boldsymbol{p}'_\lambda - \boldsymbol{k}\right)\Psi_{0,0,0}(\boldsymbol{p}_\rho,\boldsymbol{p}_\lambda)\Psi^*_{0,0,0}(\boldsymbol{p}'_\rho,\boldsymbol{p}'_\lambda)\Psi^*_{0,0,0}(\boldsymbol{p}_4,\boldsymbol{p}_5),$$

Harmonic oscillator wavefunctions from Godfrey-Isgur quark model are adopted.

Evaluation of NRCQM spatial structure

- Jacobi coordinates $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ for the light-light and heavy-light modes.

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\boldsymbol{r}_1 - \boldsymbol{r}_2) \qquad \boldsymbol{\lambda} = \sqrt{\frac{2}{3}}\left(\frac{m_1}{m_1 + m_2}\boldsymbol{r}_1 + \frac{m_2}{m_1 + m_2}\boldsymbol{r}_2 - \boldsymbol{r}_3\right)$$



Jacobi coordinate

- The harmonic-oscillator strengths α_ρ and α_λ control the spatial distributions.

$$\langle \rho^2 \rangle = \frac{3}{2}\frac{1}{\alpha_\rho^2} \qquad \langle \lambda^2 \rangle = \frac{3}{2}\frac{1}{\alpha_\lambda^2}$$

$$\langle r_{cq}^2 \rangle = \frac{9}{4}\frac{1}{\alpha_\lambda^2} + \frac{3}{4}\frac{1}{\alpha_\rho^2}$$

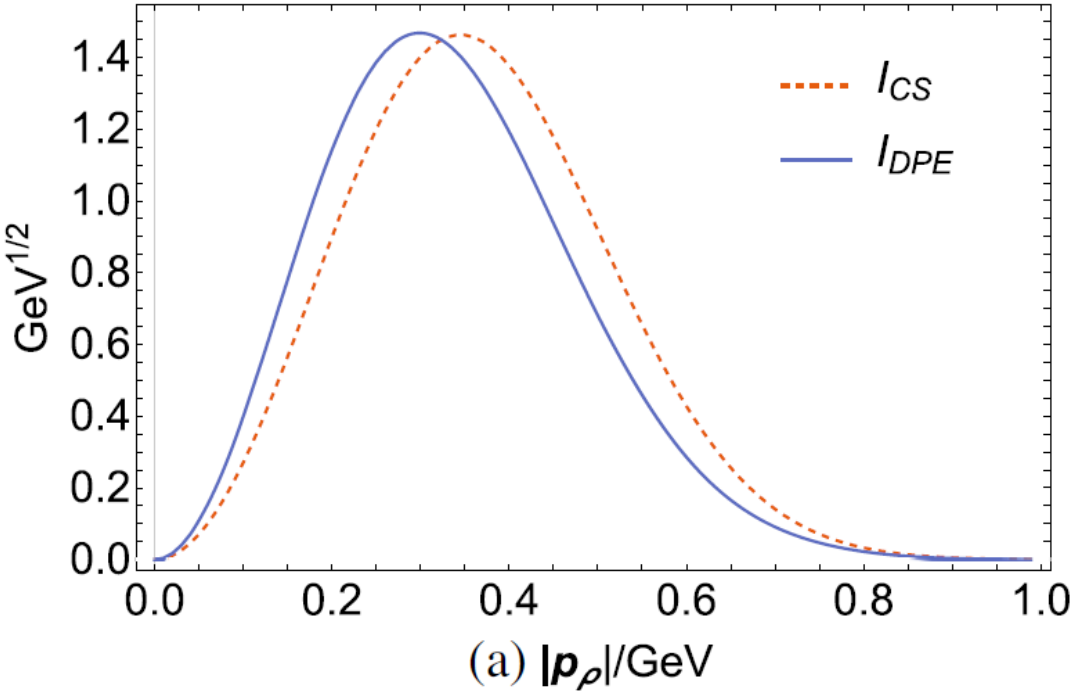
$$\frac{\alpha_\lambda}{\alpha_\rho} = \frac{1}{\sqrt{2}}\left[\frac{3(m_1 + m_2)m_3}{(m_1 + m_2 + m_3)m_1 m_2}\right]^{1/4}$$

Larger α_ρ or α_λ means a more compact spatial distribution.

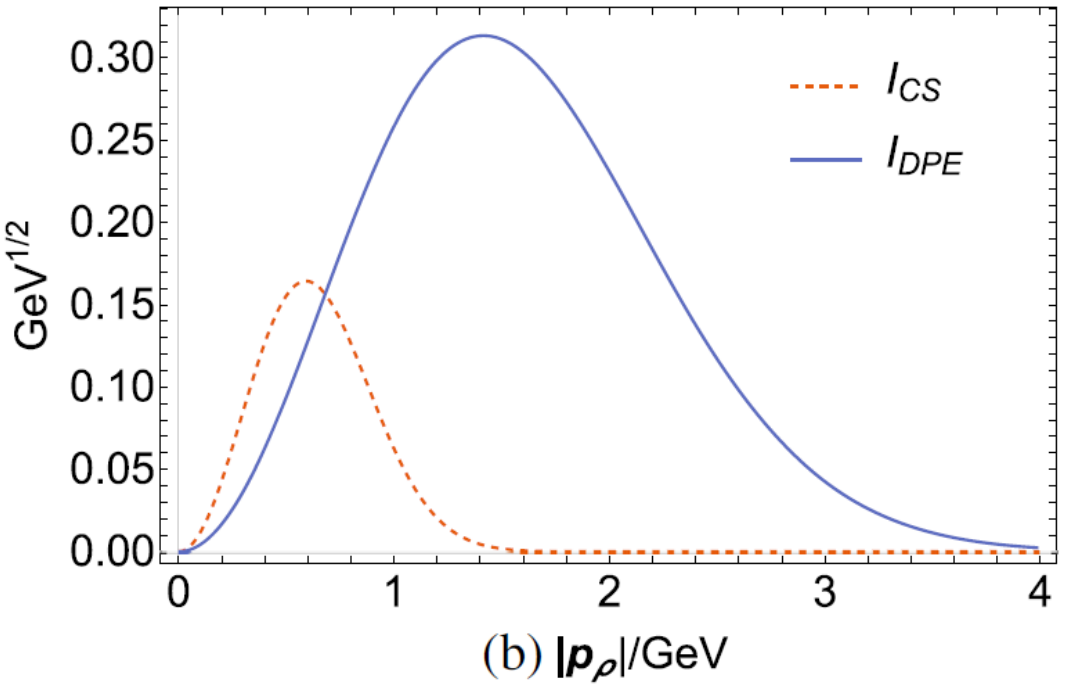
P.Y. Niu, J.-M. Richard, Q. Wang, Q. Zhao, Phys. Rev. D 102, 073005 (2020); Chin. Phys. C 45, no.1, 013101 (2021);
P.-Y. Niu, Q. Wang and Q. Zhao, Phys. Lett. B 826, 136916 (2022);

- The spatial wave function convolutions of the **DPE** process (blue line) and **CS** process (brown line). The left panel shows the results with $\alpha_\rho = \alpha'_\rho = 0.45$ GeV and the right one with $\alpha_\rho = \alpha'_\rho = 2$ GeV.
- For small α_ρ, one gets a loose Gaussian, and for large α_ρ, the ud diquark becomes more compact, and the CS transition will be highly suppressed.

Comparable $\Lambda_c \to \Lambda\pi^+, \Sigma^0\pi^+, \Sigma^+\pi^0$

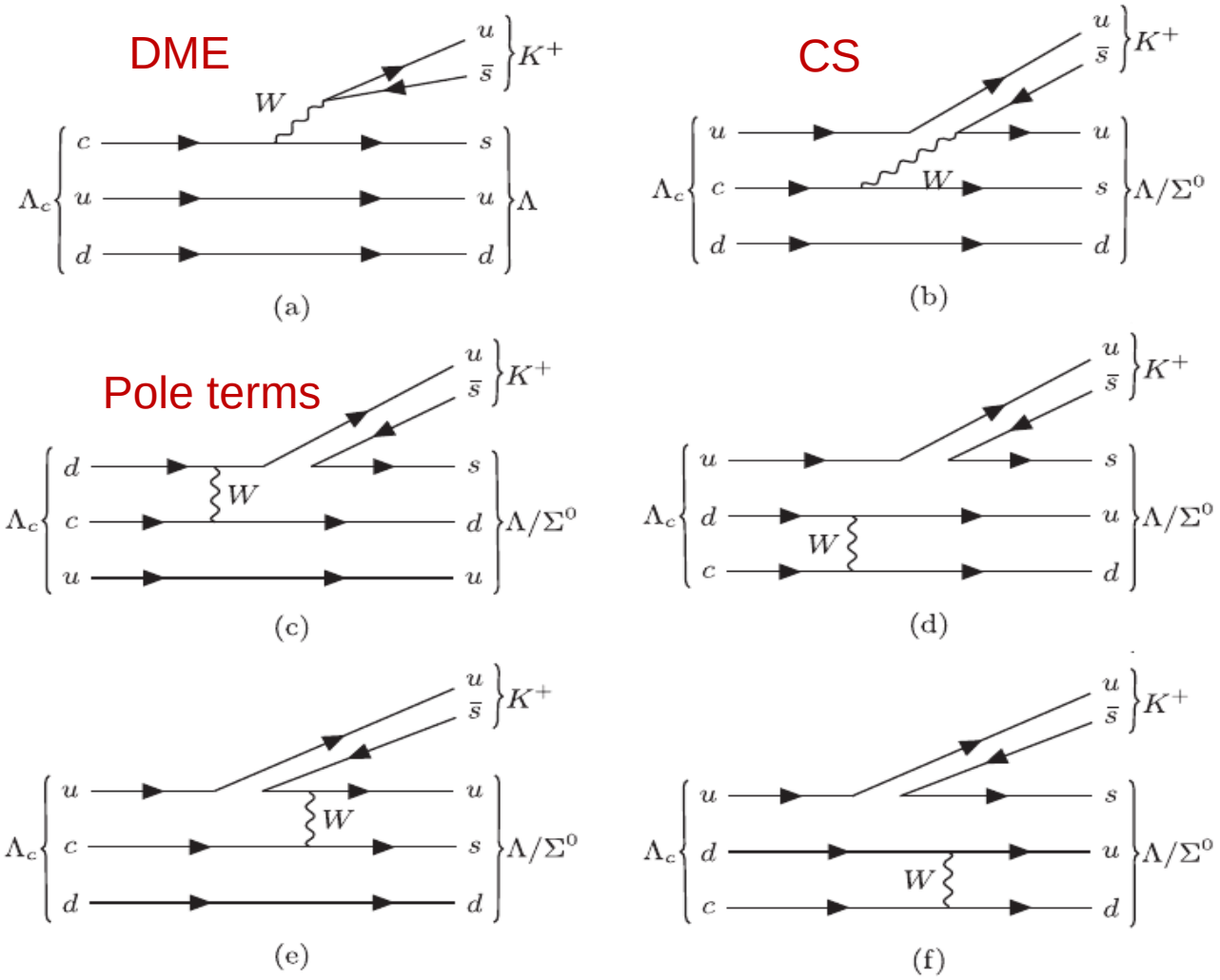


$\Gamma(\Lambda_c \to \Lambda\pi^+) \gg \Gamma(\Lambda_c \to \Sigma^0\pi^+, \Sigma^+\pi^0)$



The data for $\Lambda_c \to \Lambda\pi^+, \Sigma^0\pi^+, \Sigma^+\pi^0$ strongly favor a quantum correlation between ud which should not be a spatially compact object.

(II) Singly Cabbibo favored decays



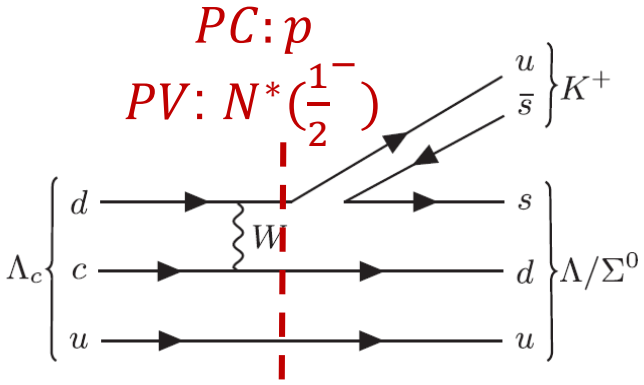
Similar mechanism in:

$$\begin{cases} \Lambda_c \to \Lambda K^+ \\ \Lambda_c \to \Sigma^0 K^+ \end{cases}$$

- DME: the quark pair from the W is hadronized into one final-state hadron.
- CS: the W-created pair is hadronized into different final hadrons.
- Pole terms : WS and SW sequences contain intermediate baryon states.

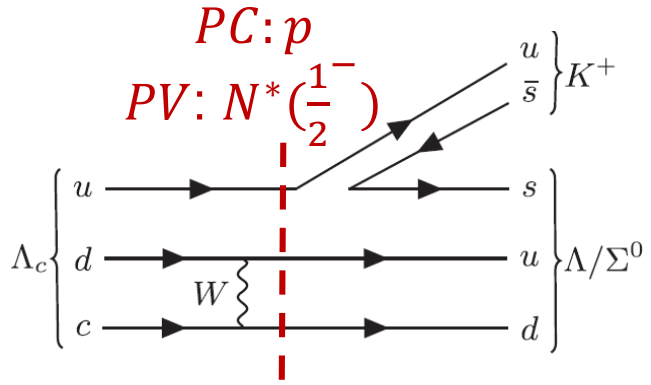
P.-Y. Niu, Q. Wang and Q. Zhao, PRD 113, 033001 (2026)

But some different features with the pole terms



Intermediate nonstrange baryons

$$\begin{pmatrix} N(1535) \\ N(1650) \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |70, {}^2\!8, 1, 1, \frac{1}{2}\rangle \\ |70, {}^4\!8, 1, 1, \frac{1}{2}\rangle \end{pmatrix}.$$



Λ selection rule ($N^* \to \Lambda K^{(*)}$)

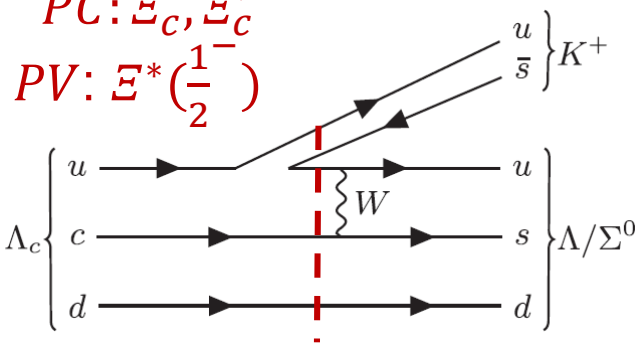
- Λ: $S_{ud} = 0$. In the spectator approximation, S_{ud} is conserved.
- Hence $[70, {}^4\!8]$ ($S_{ud} = 1$) $\to$ $[56, {}^2\!8]$ ΛK is forbidden; $[70, {}^2\!8]$ is allowed.
- $N(1535)$, $N(1650)$ are θ-mixed states (matrix above). At $\theta = 0$, $N(1650)$ gives no WS pole term in $\Lambda_c \to \Lambda K$; mixing affects the PV amplitude and parity asymmetry α.

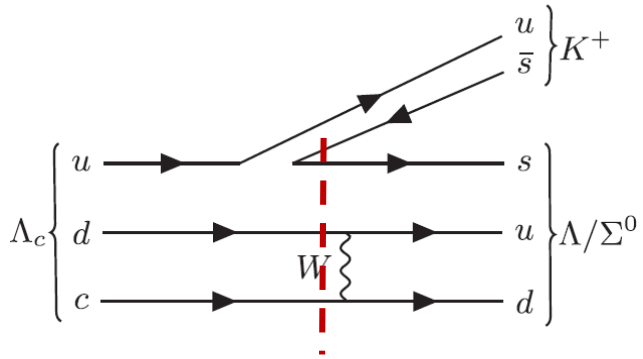
[1] P.-Y. Niu, Q. Wang, Q. Zhao, PRD 113, 033001 (2026).
[2] Q. Zhao and F. E. Close, PRD 71, 094022 (2005) .

Intermediate Ξ_c baryons

$PC: \Xi_c, \Xi_c'$

$PV: \Xi^*(\frac{1}{2}^-)$





$$\Psi_{\bar{3}} = \begin{cases} \left|{}^2S, \frac{1}{2}^+\right\rangle = \Psi^s_{000}\chi^\rho_{S_z}\phi^{\bar{3}}_c \\ \left|{}^2P_\lambda, \frac{1}{2}^-\right\rangle = \Psi^\lambda_{11L_z}\chi^\rho_{S_z}\phi^{\bar{3}}_c \\ \left|{}^2P_\rho, \frac{1}{2}^-\right\rangle = \Psi^\rho_{11L_z}\chi^\lambda_{S_z}\phi^{\bar{3}}_c \\ \left|{}^4P_\rho, \frac{1}{2}^-\right\rangle = \Psi^\rho_{11L_z}\chi^s_{S_z}\phi^{\bar{3}}_c \end{cases} \qquad \Psi_6 = \begin{cases} \left|{}^2S, \frac{1}{2}^+\right\rangle = \Psi^s_{000}\chi^\lambda_{S_z}\phi^6_c \\ \left|{}^2P_\lambda, \frac{1}{2}^-\right\rangle = \Psi^\lambda_{11L_z}\chi^\lambda_{S_z}\phi^6_c \\ \left|{}^2P_\rho, \frac{1}{2}^-\right\rangle = \Psi^\rho_{11L_z}\chi^\rho_{S_z}\phi^6_c \\ \left|{}^4P_\lambda, \frac{1}{2}^-\right\rangle = \Psi^\lambda_{11L_z}\chi^s_{S_z}\phi^6_c \end{cases}.$$

$$\phi^{\bar{3}}_c = \begin{cases} \frac{1}{\sqrt{2}}(ud-du)c, & \text{for } \Lambda_c^+, \\ \frac{1}{\sqrt{2}}(us-su)c, & \text{for } \Xi_c^+, \\ \frac{1}{\sqrt{2}}(ds-sd)c, & \text{for } \Xi_c^0, \end{cases} \qquad \phi^6_c = \begin{cases} uuc, & \text{for } \Sigma_c^{++}, \\ \frac{1}{\sqrt{2}}(ud+du)c, & \text{for } \Sigma_c^+, \\ ddc, & \text{for } \Sigma_c^0, \\ \frac{1}{\sqrt{2}}(us+su)c, & \text{for } \Xi_c'^+ \\ \frac{1}{\sqrt{2}}(ds+sd)c, & \text{for } \Xi_c'^0, \\ ssc, & \text{for } \Omega_c^0. \end{cases}$$

TABLE IV. The values of the $\Lambda_c \to \Lambda/\Sigma^0 K^+$ amplitudes (in unit of 10^{-9} GeV$^{-1/2}$). For the SW processes, there are two kinds of weak transitions processes, i.e., $cs \to su$ and $cd \to ud$, and the relevant results are labeled with (a, b). Here, a is the amplitude for $cs \to su$, and b is the amplitudes for $cd \to ud$. The imaginary part of the amplitudes less than 10^{-11} GeV is set as zero.

		States	$\Lambda_c \to \Lambda K^+$		$\Lambda_c \to \Sigma^0 K^+$	
Parity	Processes	$\cdots$	$\theta = 0°$	$\theta = 30°$	$\theta = 0°$	$\theta = 30°$
PC	DPE	$\cdots$	7.78	5.96	0	0
	CS	$\cdots$	0.82	0.50	−1.42	−0.85
	WS	p	−4.46	−4.88	0.85	0.94
	SW	Ξ_c^0	(0,0)	(0, 0)	(0, 0)	(0,0)
		$\Xi_c'^0$	(1.69, 3.22)	(2.28, 4.32)	(−3.19, 0)	(−4.09, 0)
	Total	$\cdots$	9.05	8.18	−3.76	−4.00
PV	DPE	$\cdots$	−4.71	−4.11	0	0
	CS	$\cdots$	−1.19	−0.80	2.19	1.43
	WS	$N(1535)$	$4.11 - 0.23i$	$2.22 - 0.13i$	$0.92 - 0.052i$	$-0.85 + 0.098i$
		$N(1650)$	0	$0.59 - 0.033i$	$8.91 - 0.82i$	$5.46 - 0.49i$
	SW	$\Xi_c^0\|^2P_\rho\rangle$	(−0.57, 1.10)	(−1.11, 2.11)	(1.10, 0)	(2.04, 0)
		$\Xi_c^0\|^2P_\lambda\rangle$	(0, 0)	(0,0)	(0, 0)	(0, 0)
		$\Xi_c^0\|^4P_\rho\rangle$	(2.27, −4.31)	(4.35, −8.26)	(−4.30, 0)	(−8.01, 0)
		$\Xi_c'^0\|^2P_\rho\rangle$	(0, 0)	(0, 0)	(0, 0)	(0, 0)
		$\Xi_c'^0\|^2P_\lambda\rangle$	(−0.56, −1.06)	(−1.08, −2.06)	(1.04, 0)	(1.98, 0)
		$\Xi_c'^0\|^4P_\lambda\rangle$	(2.17, 4.14)	(4.26, 8.08)	(−4.09, 0)	(−7.77, 0)
	Total	$\cdots$	$1.38 - 0.23i$	$4.17 - 0.16i$	$5.78 - 0.87i$	$-5.72 - 0.40i$

Λ selection rule

Light quarks:
$0^+ \not\to 1^- + 0^-$

Branching ratios: comparison with data

Branching ratios in units of ($\times 10^{-4}$).

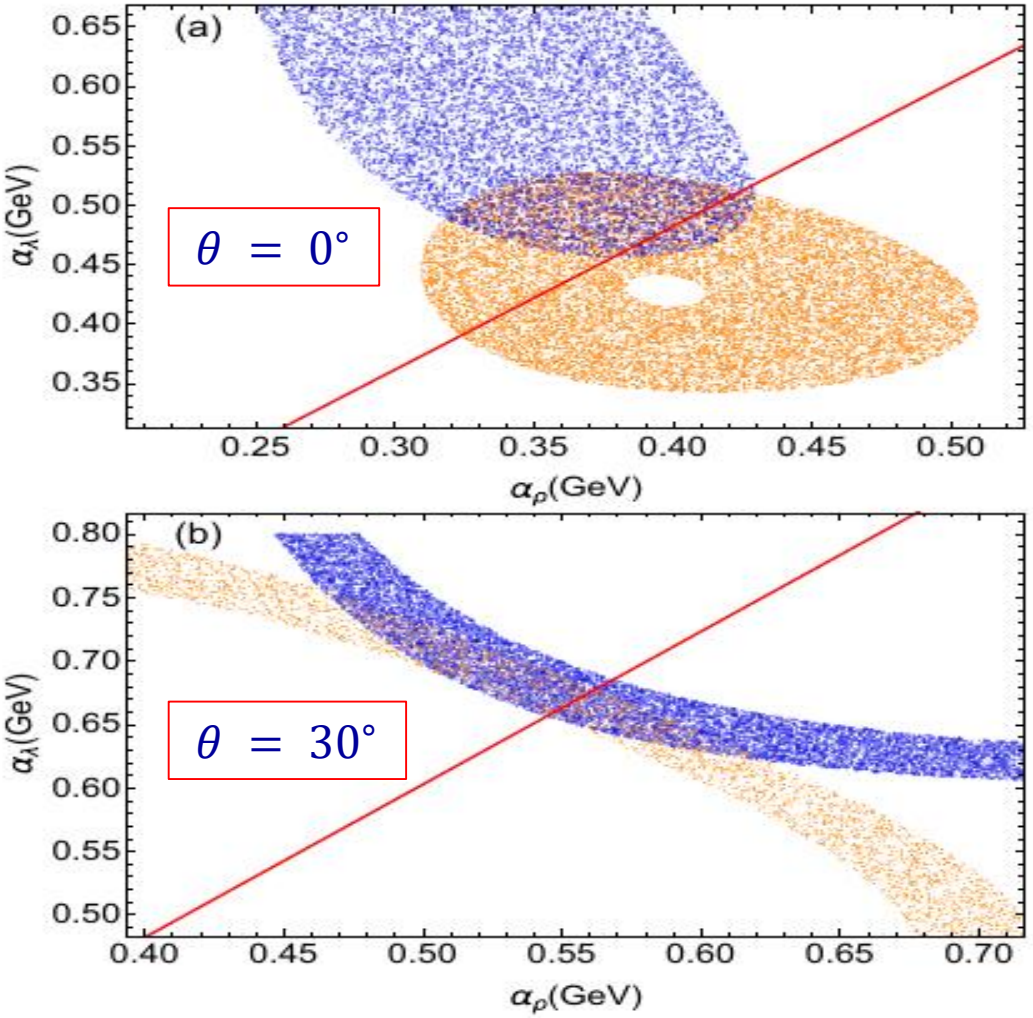
Channel	Experiment	$\theta = 0°$	$\theta = 30°$
$\Lambda_c \to \Lambda K^+$	6.42 ± 0.31	6.51	6.55
$\Lambda_c \to \Sigma^0 K^+$	3.70 ± 0.31	3.47	3.51

$$R_\Gamma = \frac{\mathrm{Br}(\Lambda_c \to \Lambda K^+)}{\mathrm{Br}(\Lambda_c \to \Sigma^0 K^+)}$$

- A naive isospin-only expectation $R_\Gamma \approx 3$ is modified by non-identical diagrams and interference.
- The ratio may serve as a discriminator for the light-diquark configuration.

Definition: Eq. (31)

Allowed $\alpha_\rho - \alpha_\lambda$ parameter regions



Orange: $\Lambda_c \to \Lambda K^+$;blue: $\Lambda_c \to \Sigma^0 K^+$

- The harmonic-oscillator strengths α_ρ and α_λ control the spatial distributions.

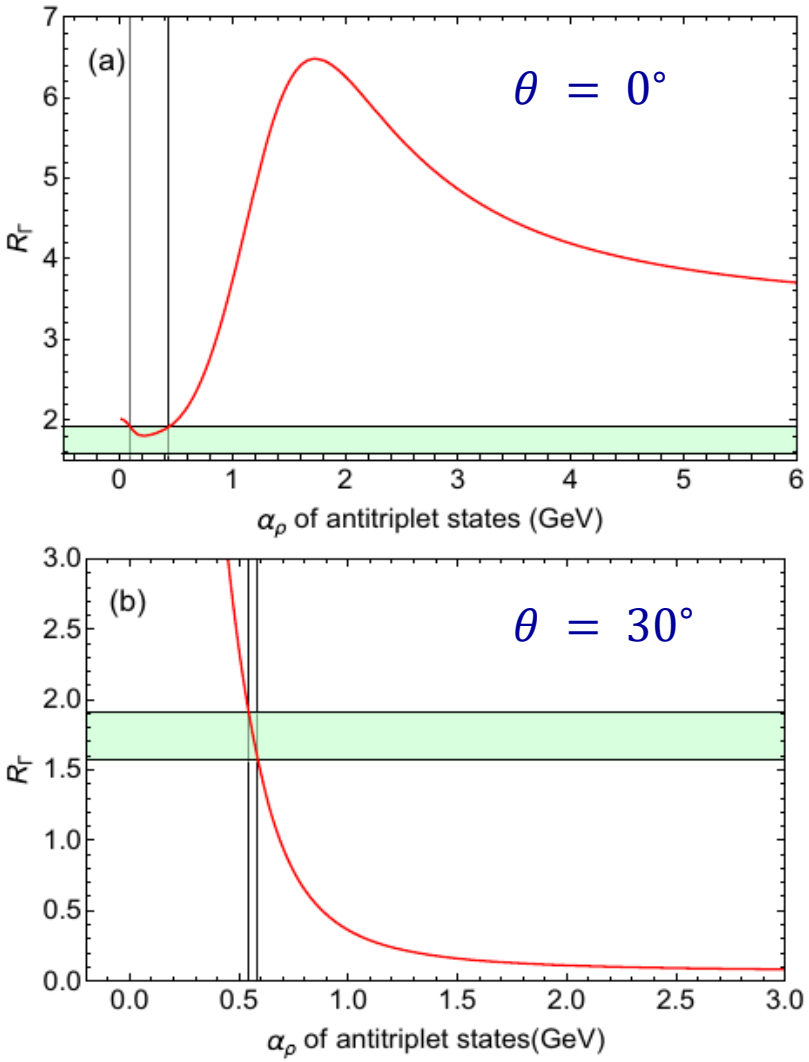
$$\langle \rho^2 \rangle = \frac{3}{2}\frac{1}{\alpha_\rho^2} \qquad \langle \lambda^2 \rangle = \frac{3}{2}\frac{1}{\alpha_\lambda^2} \qquad \langle r_{cq}^2 \rangle = \frac{9}{4}\frac{1}{\alpha_\lambda^2} + \frac{3}{4}\frac{1}{\alpha_\rho^2}$$

$$\frac{\alpha_\lambda}{\alpha_\rho} = \frac{1}{\sqrt{2}}\left[\frac{3(m_1+m_2)m_3}{(m_1+m_2+m_3)m_1m_2}\right]^{1/4}$$

- Both decay widths restrict the parameter space to a small region close to standard NRCQM values.
- The constraints remain compatible whether $N(1535)$ and $N(1650)$ are mixed or treated as unmixed states.

The preferred α_ρ and α_λ values are not large: the data do not favor a pointlike $[ud]$ or $[cq]$ diquark.

The ratio R_Γ and interference



$$R_\Gamma = \frac{\mathrm{Br}(\Lambda_c \to \Lambda K^+)}{\mathrm{Br}(\Lambda_c \to \Sigma^0 K^+)}$$

- For identical single-diagram contributions, weak isospin suggests $R_\Gamma \approx 3$.
- The full calculation deviates strongly because the diagrams, intermediate states, and form-factor parameters are not identical.
- The mixing of $[70, {}^2 8]$ and $[70, {}^4 8]$ components changes the parity-violating interference pattern.

The observed ratio is a dynamical result, not a pure isospin factor.

3. Summary

• $\Lambda_c \to \Lambda\pi^+, \Sigma^+\pi^0, \Sigma^0\pi^+$ and $\Lambda_c \to \Lambda K^+, \Sigma^0 K^+$ provide a combined probe of the spatial distributions of $[ud]$ and $[cq]$.

• The data can only be accounted for with reasonable values for α_ρ and α_λ, which are within the conventional NRCQM inputs.

• Neither the light diquark nor the heavy-light diquark is favored to be pointlike and compact.

• Selection rules and non-factorizable pole terms are essential to the branching-ratio pattern.

• Similar combined analyses can be carried out to further examine the diquark correlation in other processes.

Thanks for your attention!