Multiquark states with functional methods

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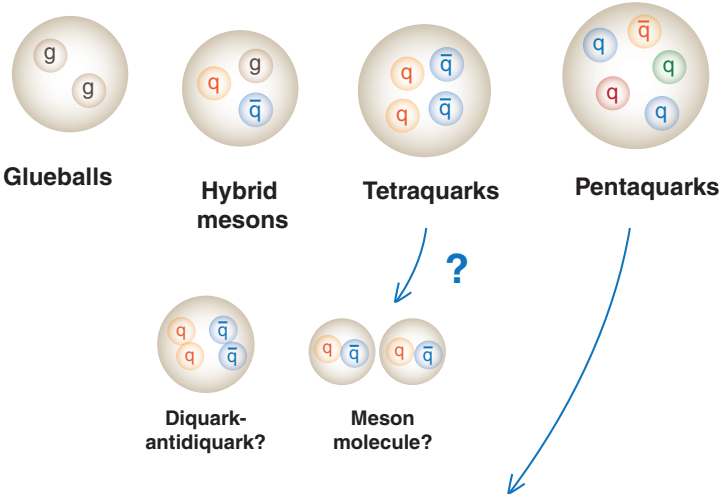
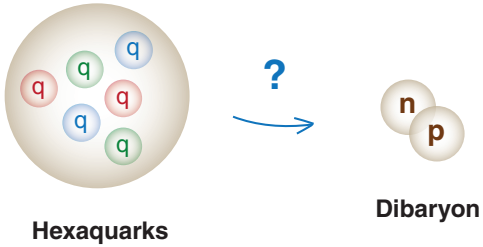
University of Graz

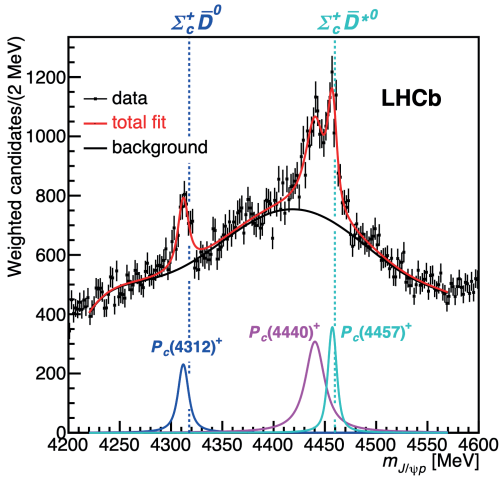
Theoretical Motivations and Empirical Evidence for Diquark Correlations in Hadrons
ECT*, Trento, Oct 21, 2026

Motivation

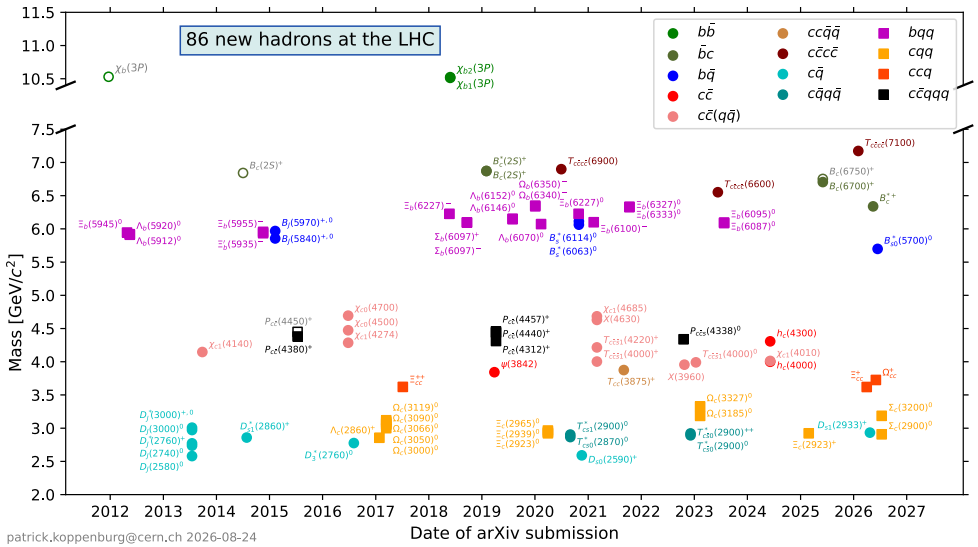
- **Understanding exotic hadrons**







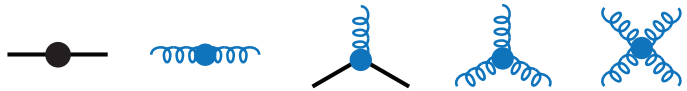
LHCb Collaboration,
PRL 122 (2019) 22, 22201
PRL 131 (2023) 3, 031901



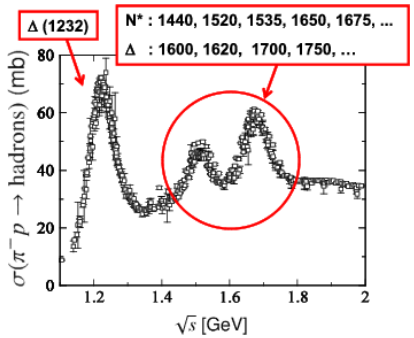
https://www.nikhef.nl/~pkoppenb/particles.html
https://qwg.ph.nat.tum.de/exoticshub

Theory tools

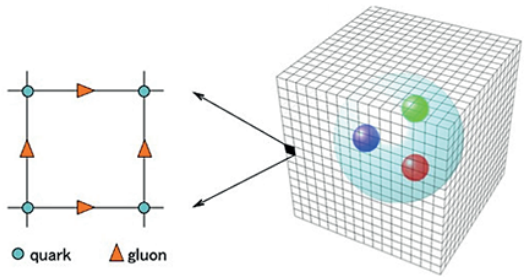
Functional methods
(DSEs & BSEs, FRG, ...)



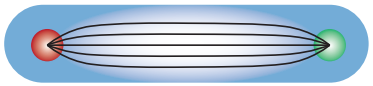
Amplitude analyses



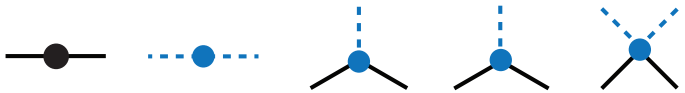
Lattice QCD



Phenomenological models

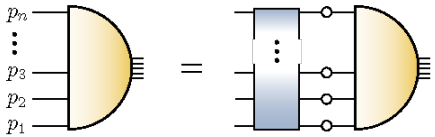


Effective theories
(ChPT, ...)



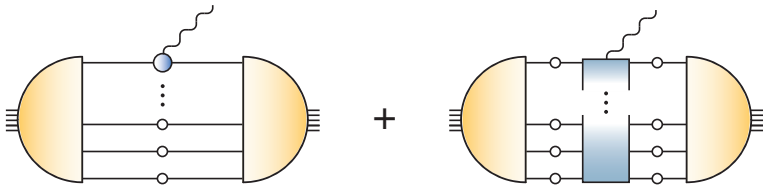
Functional methods

- Hadronic **bound-state equations** (Bethe-Salpeter & Faddeev eqs)

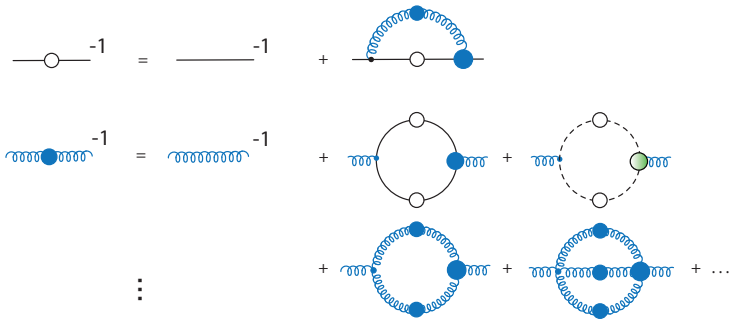


"QFT analogue of Schrödinger eq."
→ hadron masses & "wave functions"
→ **spectroscopy calculations**

- **Structure calculations:** form factors, PDFs, GPDs, TMDs, two-photon processes, ...



- Ingredients: **QCD's n-point functions,** computed from DSEs (quantum eqs. of motion) or FRG (functional renormalization group)

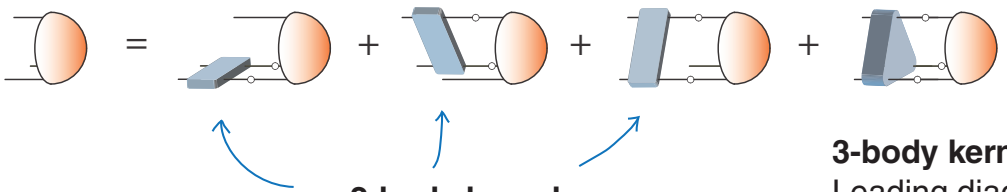


→ Dynamical mass generation, gluon mass gap, confinement, QCD phase diagram, ...

Baryons

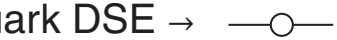
- **Three-quark BSE (Faddeev equation)** for baryons:

  Reviews: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016),
  Burkert, GE, Klempt, PPNP 146 (2026)

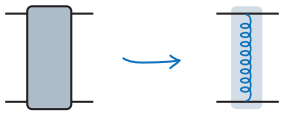


2-body kernel:
fixed in meson sector

3-body kernel:
Leading diagram
(3-gluon vertex)
vanishes by color trace,
2-quark correlations dominant?

- **Rainbow-ladder:**
  Solve quark DSE → 
  2-body kernel fixed in
  light meson sector (m_π, f_π):

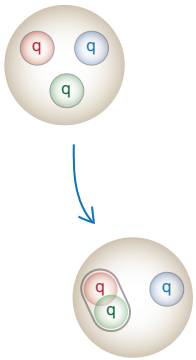
  Maris, Tandy, PRC 60 (1999),
  Qin, Chang, Liu, Roberts, Wilson, PRC 84 (2011)

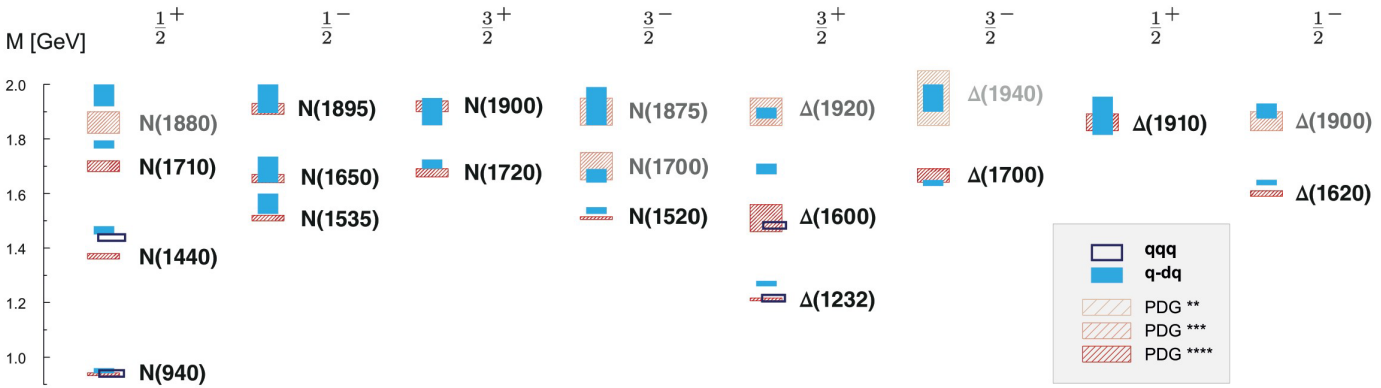


Respects chiral symmetry,
satisfies chiral properties

- **Diquark clustering** in baryons?

  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016), Barabanov et al., PPNP 116 (2021)

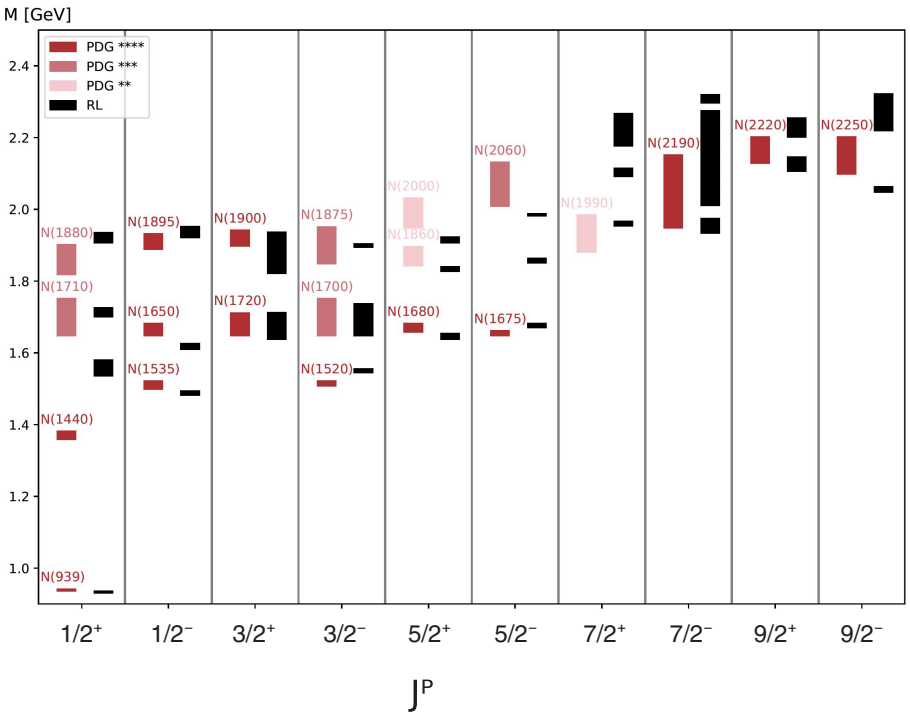




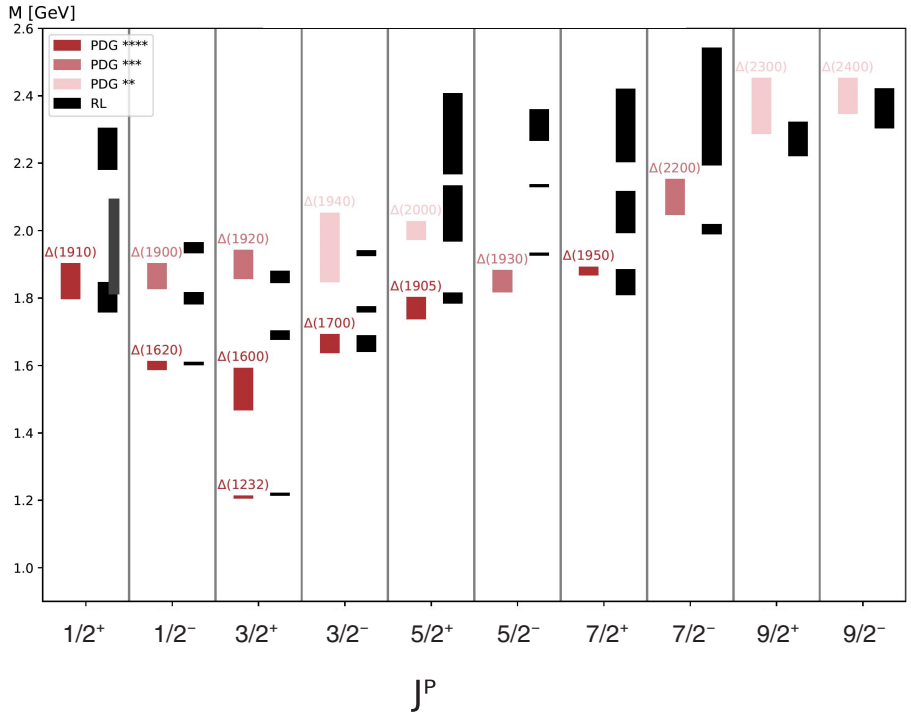
Baryons

Baryons with higher spin from quark-diquark calculations (+ spectral reconstruction):
Oberguggenberger, GE, Fischer, in preparation

Nucleon spectrum



Δ spectrum

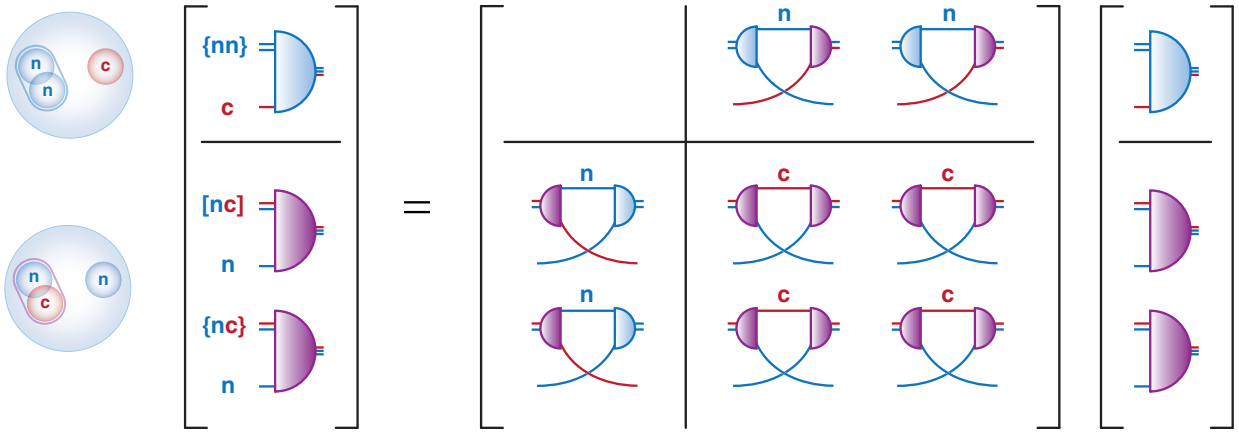


- Above 1.3 GeV extrapolated
- No signs for two N(1720)3/2+ states
- Parity doublets? Not obvious from the spectrum

Review on nucleon resonances:
Burkert, GE, Klempt, Prog. Part. Nucl. Phys. 146 (2026)

Heavy baryons

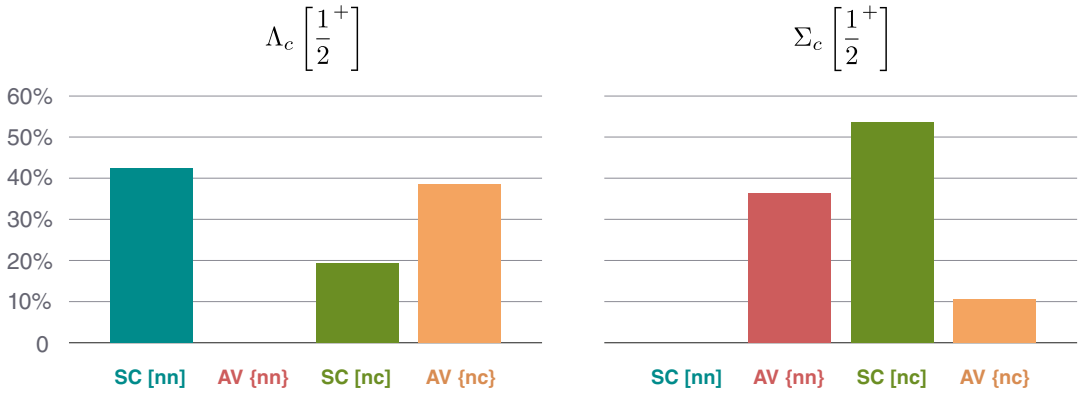
Quark-diquark BSE for Σ_c:

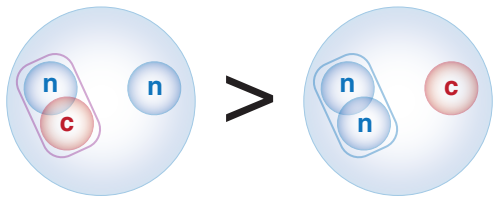


n[nc], n{nc} necessary, otherwise no equation. Presumably these are also dominant: cannot switch off n[nc], n{nc}, but c{nn}

Analogous for Λ_c and hyperons

Results: **wave function contributions** Torcato, Arriaga, GE, Peña, FBS 64 (2023)

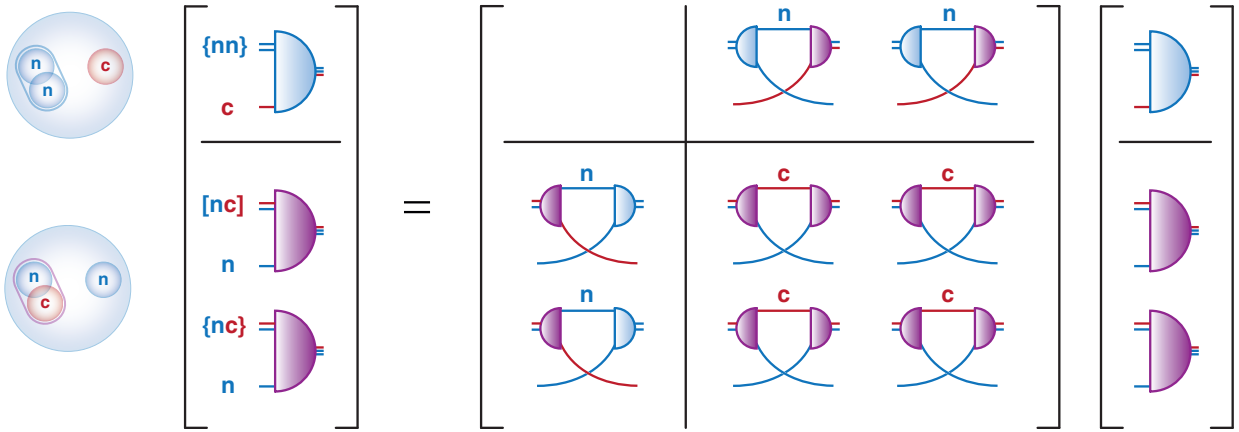




~60% ~40%

Heavy baryons

Quark-diquark BSE for Σ_c:



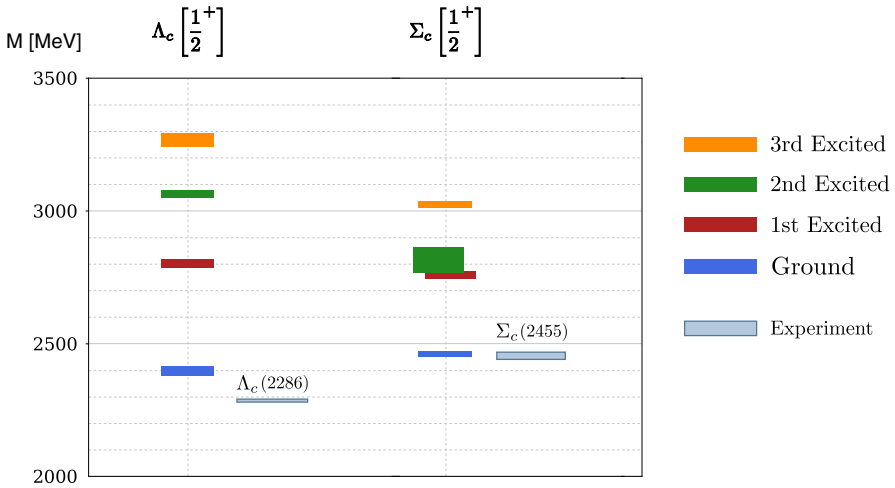
n[nc], n{nc} necessary, otherwise no equation. Presumably these are also dominant: cannot switch off n[nc], n{nc}, but c{nn}

Analogous for Λ_c and hyperons

Results: **spectrum**

Torcato, Arriaga, GE, Peña, FBS 64 (2023)

see also:
Yin, Chen, Krein, Roberts, Segovia, PRD 100 (2019)

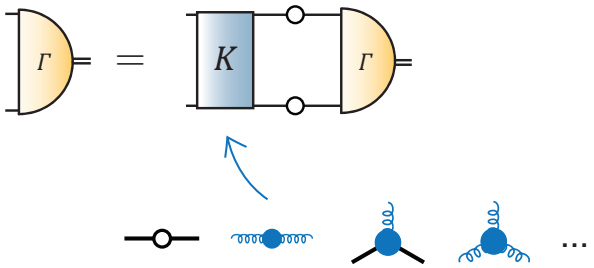


Complete calculation underway:
nnn, nns, nss, sss,
nnc, ncc, ccc,
ssc, scc, nsc

André Torcato, Arriaga, GE, Peña, in preparation

Towards ab-initio

- **Goal:** go towards ab-initio calculations by calculating **higher n-point functions**

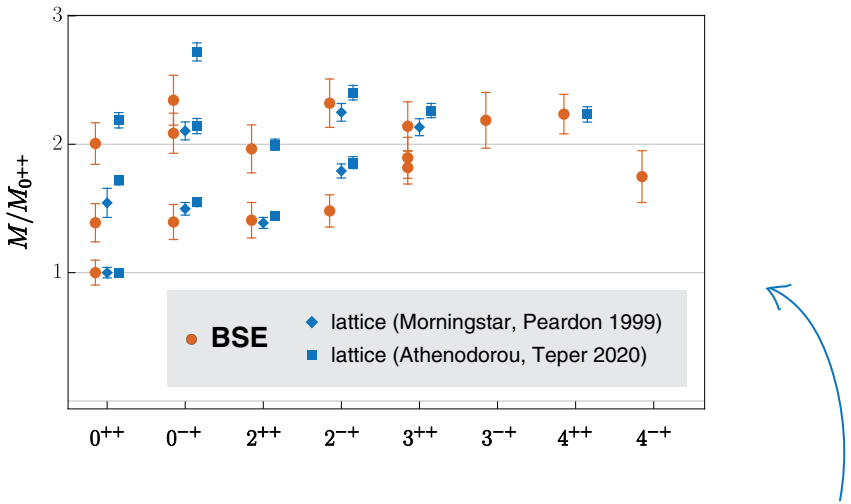


. . . ,
Williams, Fischer, Heupel, PRD 93 (2016),
Cyrol et al., PRD 97 (2018),
Oliveira, Silva, Skullerud, Sternbeck, PRD 99 (2019),
Aguilar et al., EPJ C 80 (2020),
Huber, PRD 101 (2020),
Qin, Roberts, Chin. Phys. Lett. 38 (2021),
GE, Pawlowski, Silva, PRD 104 (2021),
Miramontes, Morgado, Papavassiliou, Pawlowski, EPJ C 85 (2025)
Huber, Fischer, Sanchis-Alepuz, EPJ C 85 (2025)
Alkofer, Ferreira, Miramontes, Morgado, Papavassiliou, PLB 880 (2026),
. . .

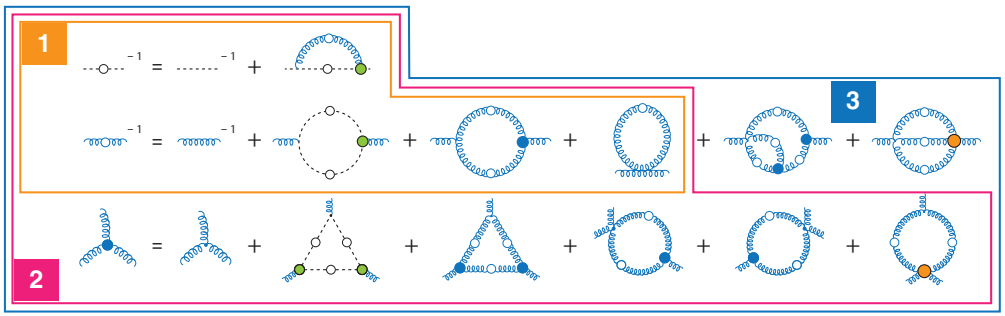
truncation error:

1 60% 2 10% 3 4%

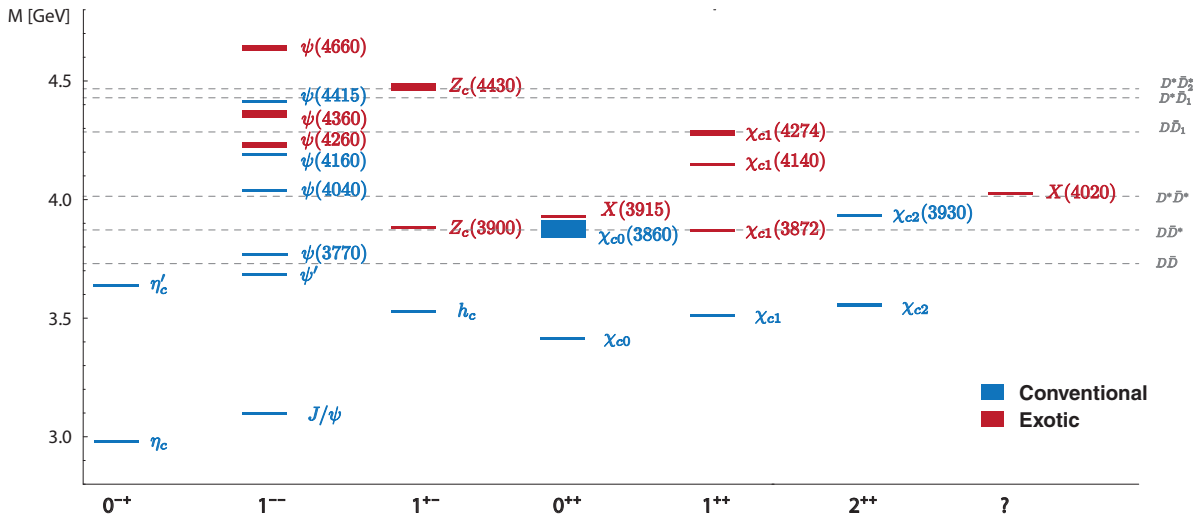
- **Glueball spectrum** agrees with lattice QCD
  Huber, Fischer, Sanchis-Alepuz, EPJ C 80 (2020), EPJ C 81 (2021)



- Coupled **Yang-Mills DSEs** GE, Pawlowski, Silva, PRD 104 (2021)



Exotic mesons



- Several tetraquark candidates in **charmonium spectrum**: X(3872), X(3915), Zc(3900), ...
- Z states cannot be $c\bar{c}$ since they carry charge
- Recent additions: all-charm X(6900), open-charm T_{cc}^{+}, ...
- Oldest tetraquark candidates: **light scalar mesons**

Reviews:

Chen, Chen, Liu, Zhu,
Phys. Rept. 639 (2016), 1601.02092

Lebed, Mitchell, Swanson
PPNP 93 (2017), 1610.04528

Esposito, Pilloni, Polosa,
Phys. Rept 668 (2017), 1611.07920

Guo, Hanhart, Meißner et al.,
Rev. Mod. Phys. 90 (2018), 1705.00141

Ali, Lange, Stone,
PPNP 97 (2017), 1706.00610

Olsen, Skwarnicki, Zieminska,
Rev. Mod. Phys. 90 (2019), 1708.04012

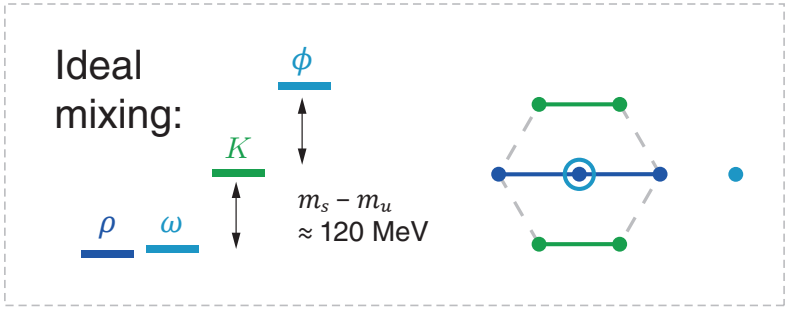
Liu, Chen, Chen, Liu, Zhu,
PPNP 107 (2019), 1903.11976

Brambilla, Eidelman, Hanhart et al.,
Phys. Rept. 873 (2020)

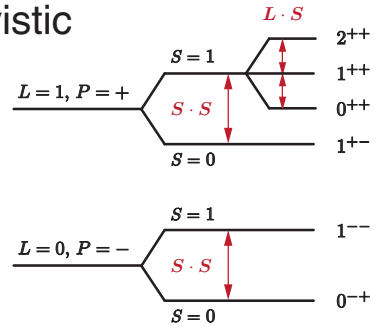
...

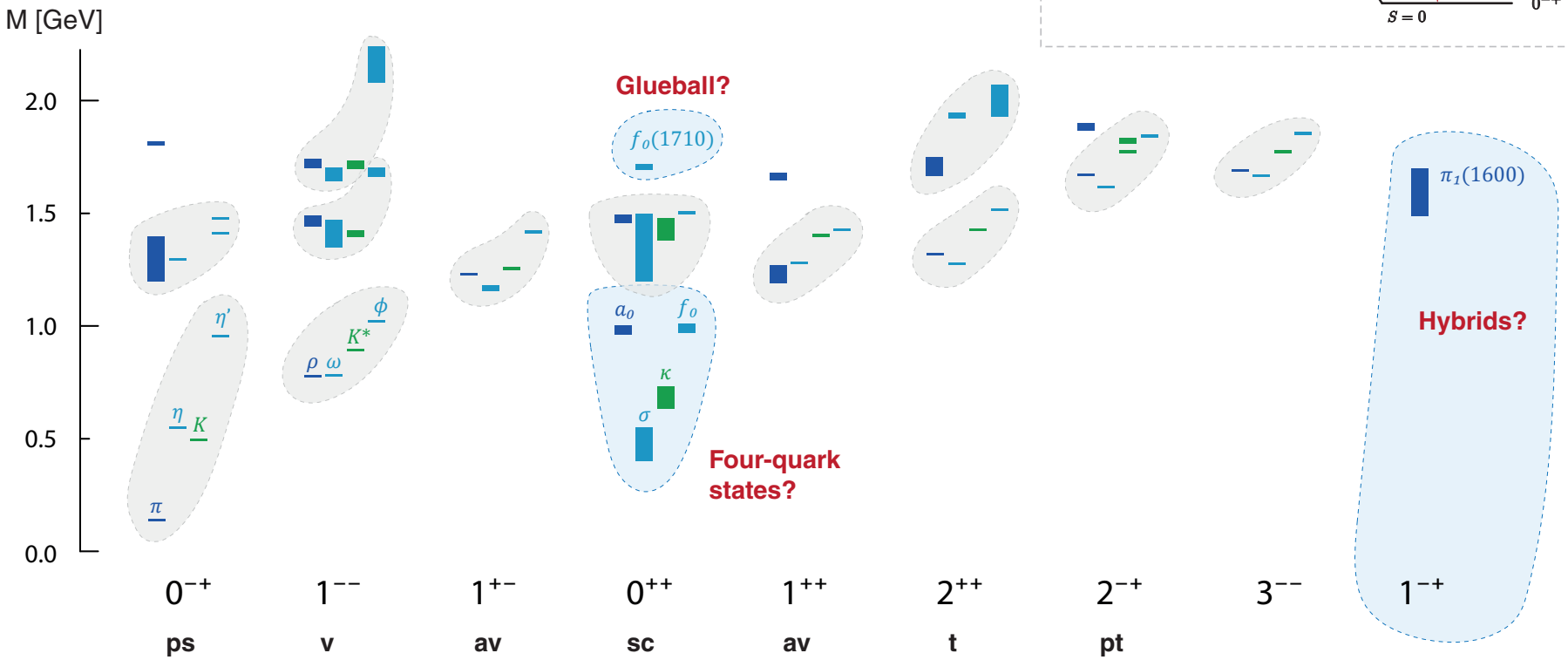
Light exotic mesons

Light meson spectrum
(PDG 2020)



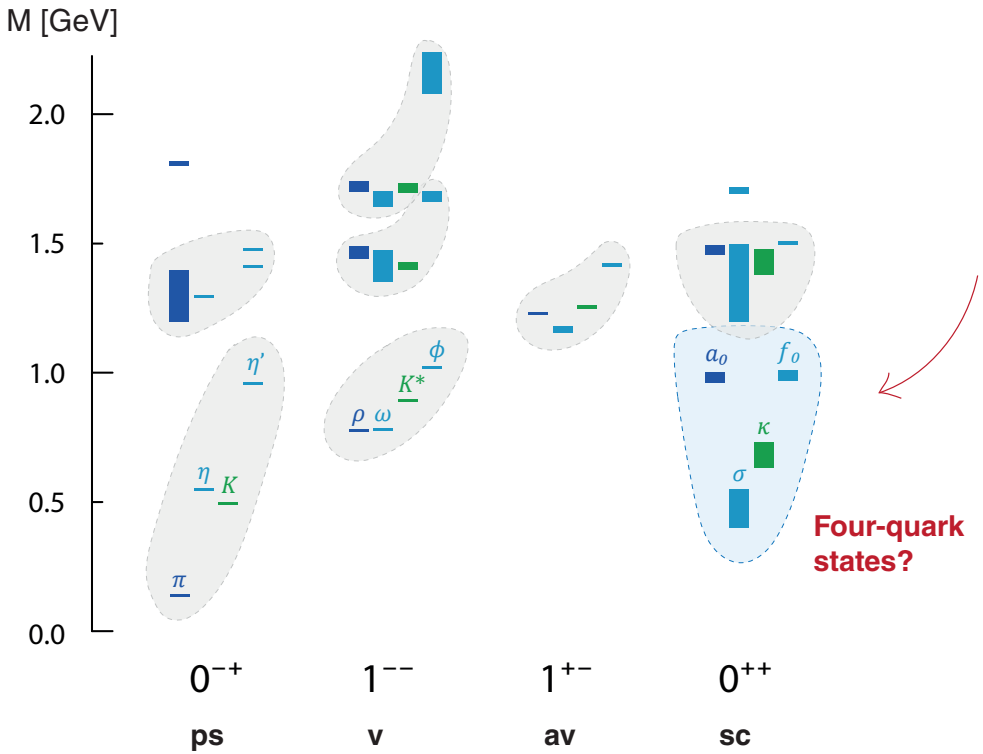
Non-relativistic level ordering



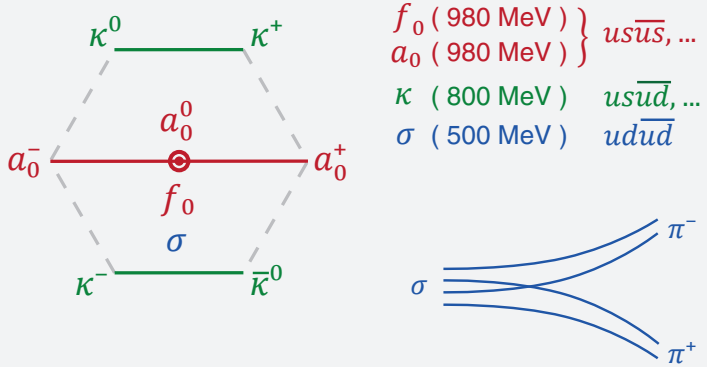


Light exotic mesons

Light meson spectrum
(PDG 2020)



- **Diquark-antidiquark**?
  Explains mass ordering & decay widths
  Jaffe 1977, Close, Tornqvist 2002,
  Maiani, Polosa, Riquer 2004

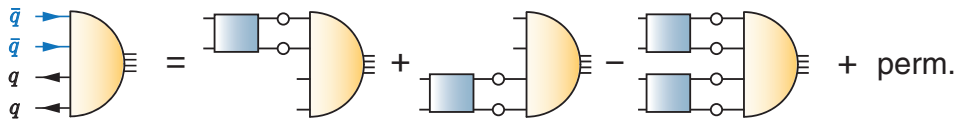


- **Meson molecules?**
  Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993

- **Non-$q\bar{q}$ nature** supported by various approaches
  Pelaez, Phys. Rept. 658 (2016)

Four-quark states

- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states:**
  GE, Fischer, Heupel, PLB 753 (2016)



$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2,q^2,k^2,\{\omega_j\},\{\eta_j\}\right)\,\tau_i(p,q,k,P) \;\otimes\; \text{Color} \;\otimes\; \text{Flavor}$$

9 Lorentz invariants:

$$p^2,\quad q^2,\quad k^2,\quad P^2=-M^2$$

$$\omega_1 = q\cdot k \qquad \eta_1 = p\cdot P$$
$$\omega_2 = p\cdot k \qquad \eta_2 = q\cdot P$$
$$\omega_3 = p\cdot q \qquad \eta_3 = k\cdot P$$

256 Dirac-Lorentz tensors

2 Color tensors:

$3\otimes\bar{3},\quad 6\otimes\bar{6}$ or
$1\otimes 1,\quad 8\otimes 8$
(Fierz-equivalent)

$$K\,\psi_i = \lambda_i\,\psi_i$$

	dim K	memory
Mesons	10^3	20 MB
Baryons	10^8	10^7 GB
Tetraquarks	10^{13}	10^{18} GB

Four-quark states

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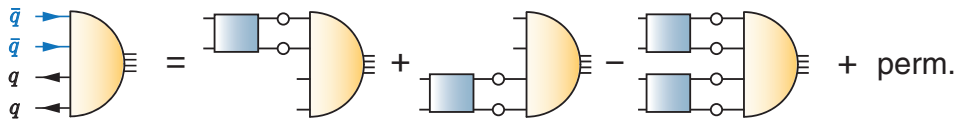
  GE, Fischer, Heupel, PLB 753 (2016)



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9 Lorentz invariants:

$$p^2,\quad q^2,\quad k^2,\quad P^2=-M^2$$

$$\omega_1 = q\cdot k \qquad \eta_1 = p\cdot P$$
$$\omega_2 = p\cdot k \qquad \eta_2 = q\cdot P$$
$$\omega_3 = p\cdot q \qquad \eta_3 = k\cdot P$$

256 Dirac-Lorentz tensors

2 Color tensors:

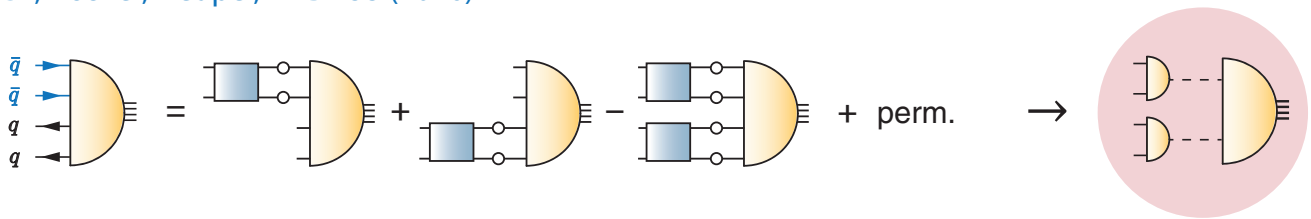
$3\otimes\bar{3},\quad 6\otimes\bar{6}$ or
$1\otimes 1,\quad 8\otimes 8$
(Fierz-equivalent)

- Group momentum variables into multiplets of **permutation group S4:** can switch off groups of variables without destroying symmetries
  GE, Fischer, Heupel, PRD 92 (2015)

$$f_i\left(\mathcal{S}_0, \triangledown, \text{⧋}, \bigcirc\right)$$

Four-quark states

- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states:**
  GE, Fischer, Heupel, PLB 753 (2016)



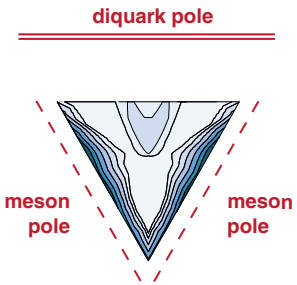
- BSE dynamically generates **meson poles** in BS amplitude:

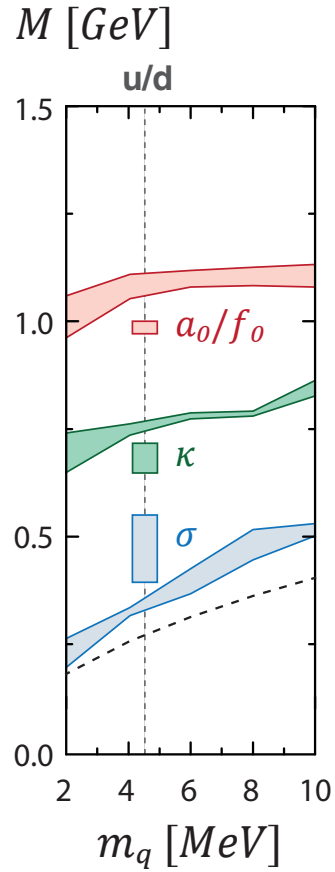
$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ 1500 MeV

$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ 1500 MeV

$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ 1200 MeV

$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ **350 MeV !**

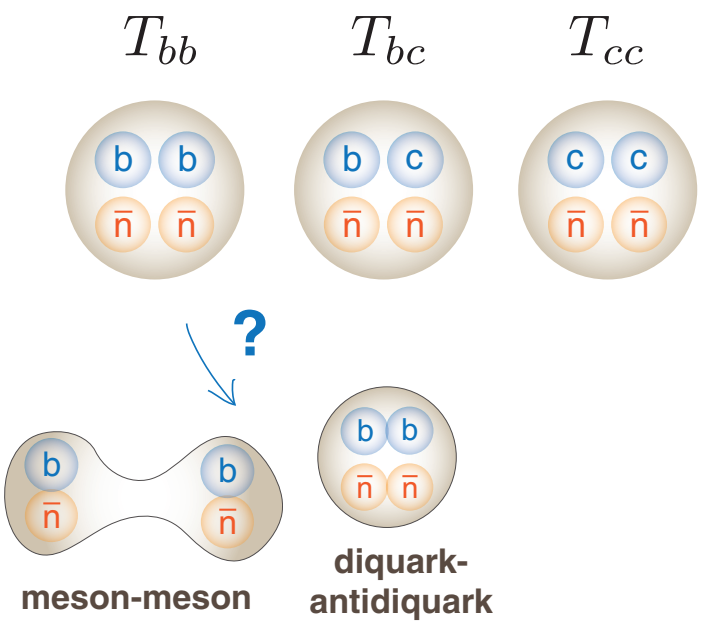




- "Light scalar mesons" look like **meson molecules,**
  diquark-antidiquark components almost negligible.
  Lightness is inherited from pseudoscalar Goldstone bosons!

Four-quark states

Open-flavor four-quark states:
cannot mix with quark-antiquark!

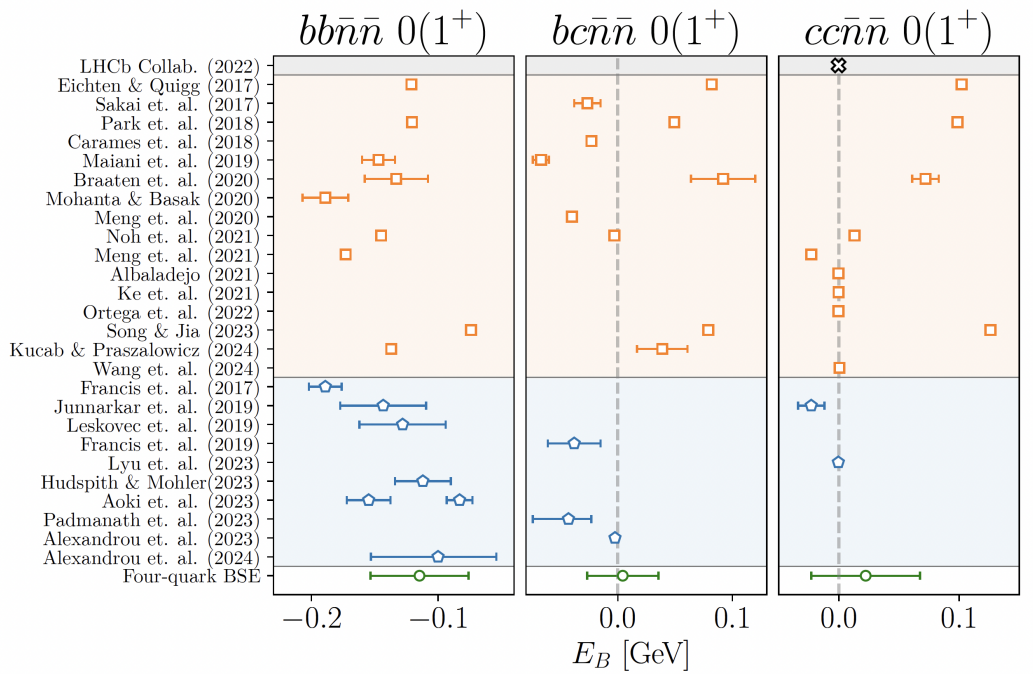


$$f_i\,(\,\mathcal{S}_0\,,\,\triangledown\,,\,\diamondsuit\,,\,\bigcirc\,)$$

important not important

Efficient approximation: $f_i(\mathcal{S}_0)\times$ **poles,**
keep "physical" tensors: meson-meson, dq-$\overline{\text{dq}}$, ...
4-quark BSE determines admixture dynamically!

Binding energies: Hoffer, GE, Fischer, PRD 111 (2025)

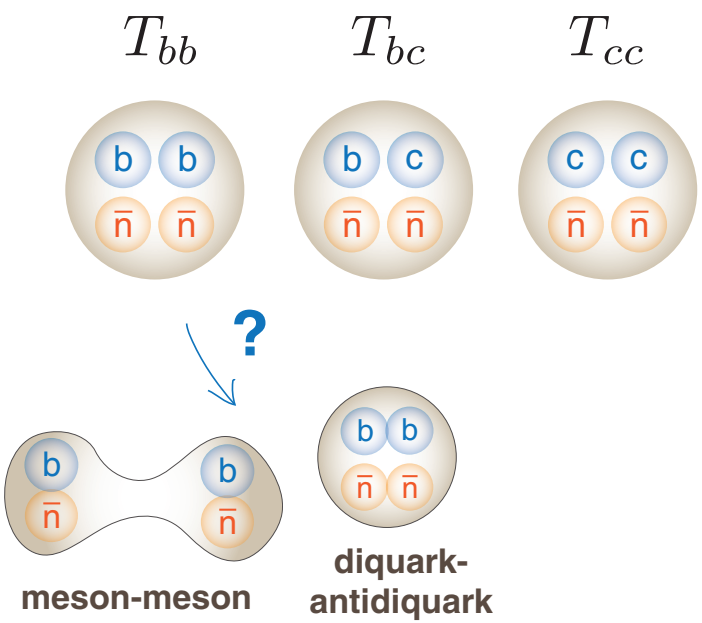


T_{bb} is bound

T_{cc} sits on top of threshold

Four-quark states

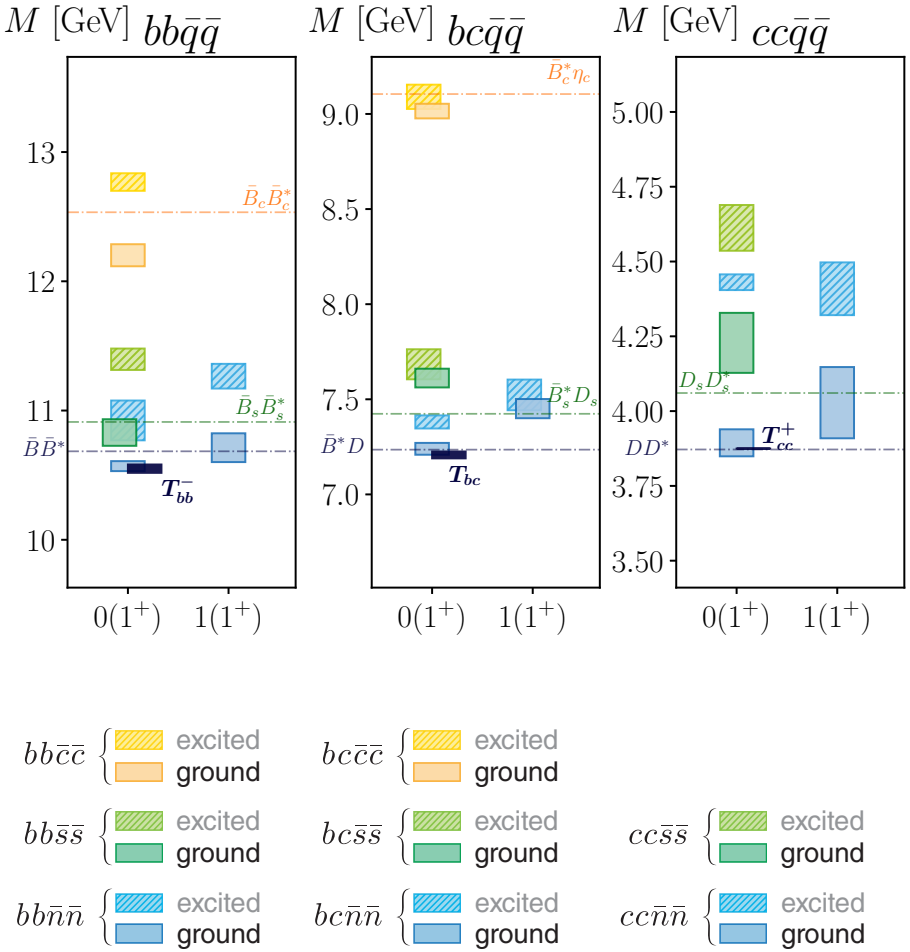
Open-flavor four-quark states:
cannot mix with quark-antiquark!



$$f_i\,(\,\mathcal{S}_0\,,\underbrace{\triangledown\,,\,\diamondsuit}_{\text{important}}\,,\underbrace{\bigcirc}_{\text{not important}}\,)$$

Efficient approximation: $f_i(\mathcal{S}_0)\times$ **poles,**
keep "physical" tensors: meson-meson, dq-$\overline{\text{dq}}$, ...
4-quark BSE determines admixture dynamically!

Spectrum: Hoffer, GE, Fischer, PRD 111 (2025)

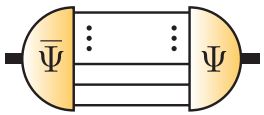


Four-quark states

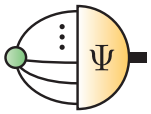
Open-flavor four-quark states:
cannot mix with quark-antiquark!

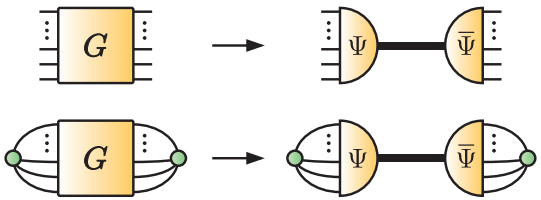
Internal composition can be
quantified in two ways,
both look very similar
Hoffer, GE, Fischer, Wagner, in preparation

- **Norm contributions**



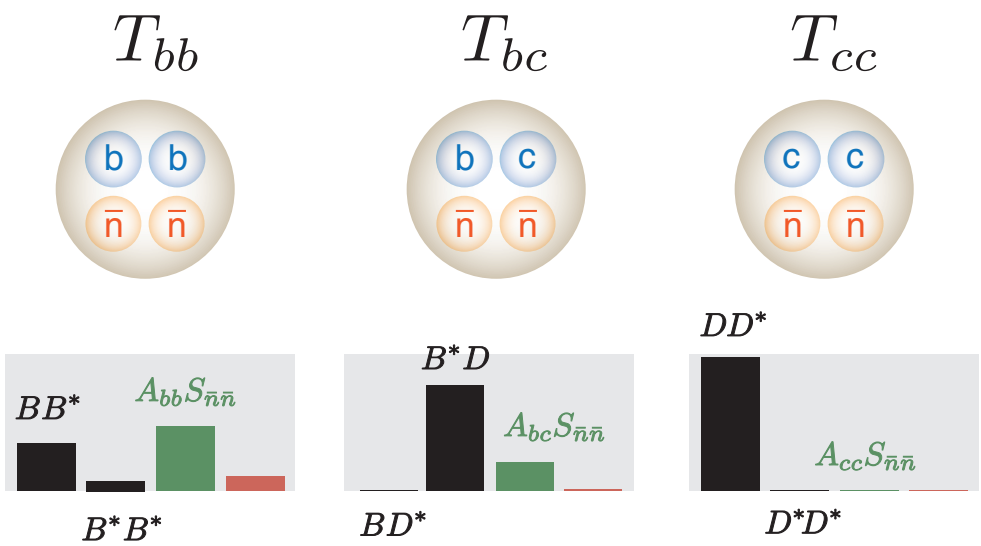
- **Overlap factors**
  (also accessible on lattice)





Composition: Hoffer, GE, Fischer, PRD 111 (2025)

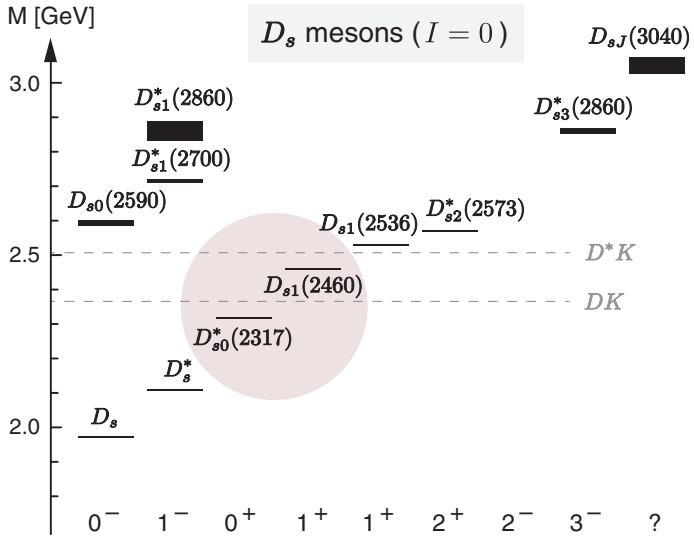
Meson-meson
Diquark-antidiquark
$\mathbf{8} \otimes \mathbf{8}\,,\, \mathbf{6} \otimes \bar{\mathbf{6}}$

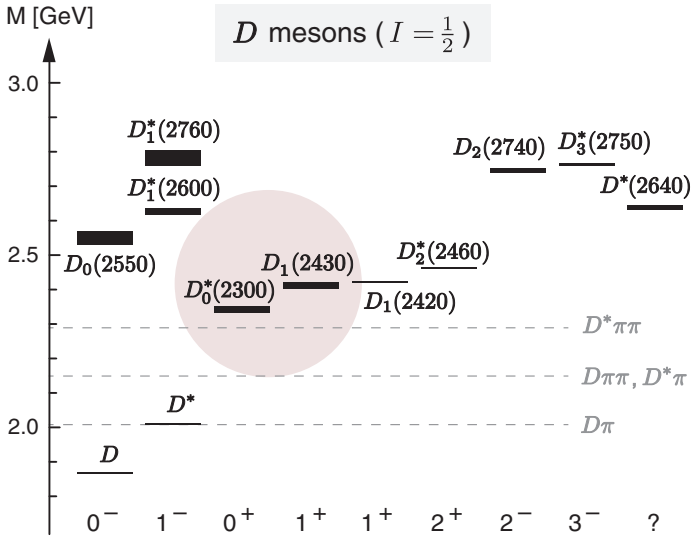


T_{bb} is complicated
mixture of molecule
& diquark-antidiquark

T_{cc} sits on top of
threshold, almost
pure molecule

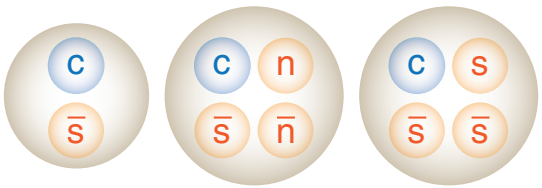
Single-charm states





- **D_s mesons**: masses much below quark model. Hadronic molecules?

  Guo, Hanhart, Meißner, Wang, Zhao, Zou, RMP 90 (2018)

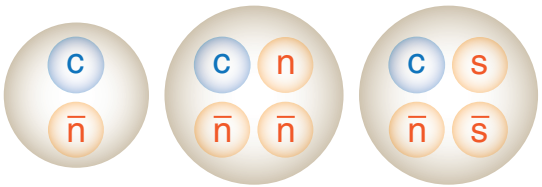


Compact diquark-antidiquark states?

Maiani, Polosa, Riquer, PRD 100 (2019)

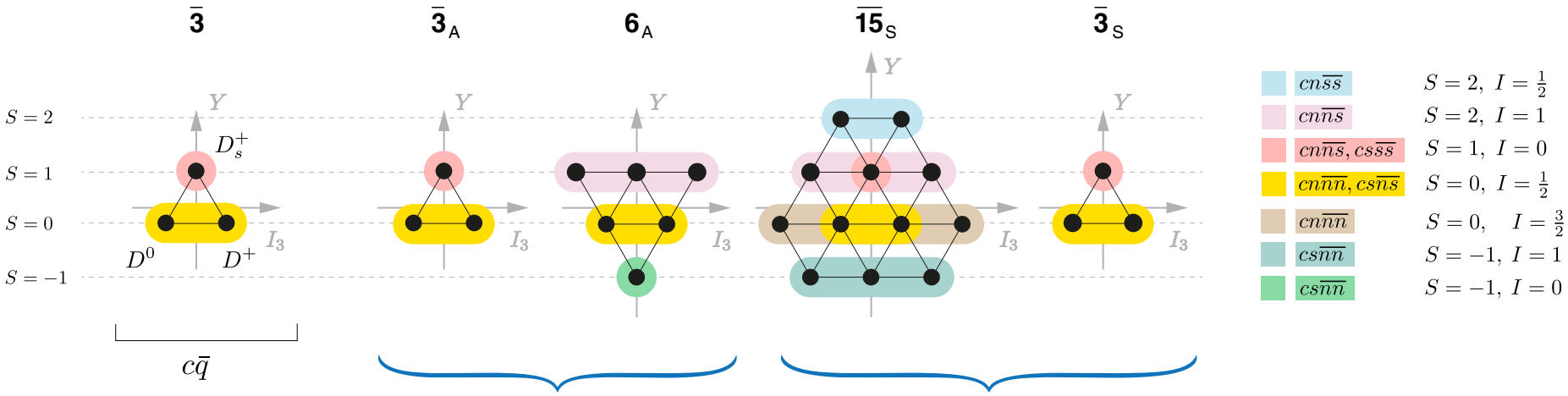
- **D mesons:** resonances. Two poles for each state, hadronic molecules? Supported by lattice

  Du, Albaladejo, Fernandez-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PRD 98 (2018), . . .
  Gregory, Guo, Hanhart, Krieg, Luu, EPJA 71 (2025)



Single-charm states

SU(3) multiplets for light & strange quarks in $c\bar{q}\bar{q}q$: $\bar{3} \otimes \bar{3} \otimes 3 = \bar{3}_A \oplus 6_A \oplus \overline{15}_S \oplus \bar{3}_S$



Antisymmetric in $\bar{q}\bar{q}$: scalar diquarks

Symmetric in $\bar{q}\bar{q}$: axialvector diquarks

Diquark-antidiquark picture

- **Expectation:** Guo, Hanhart, Commun. Theor. Phys. 77 (2025)

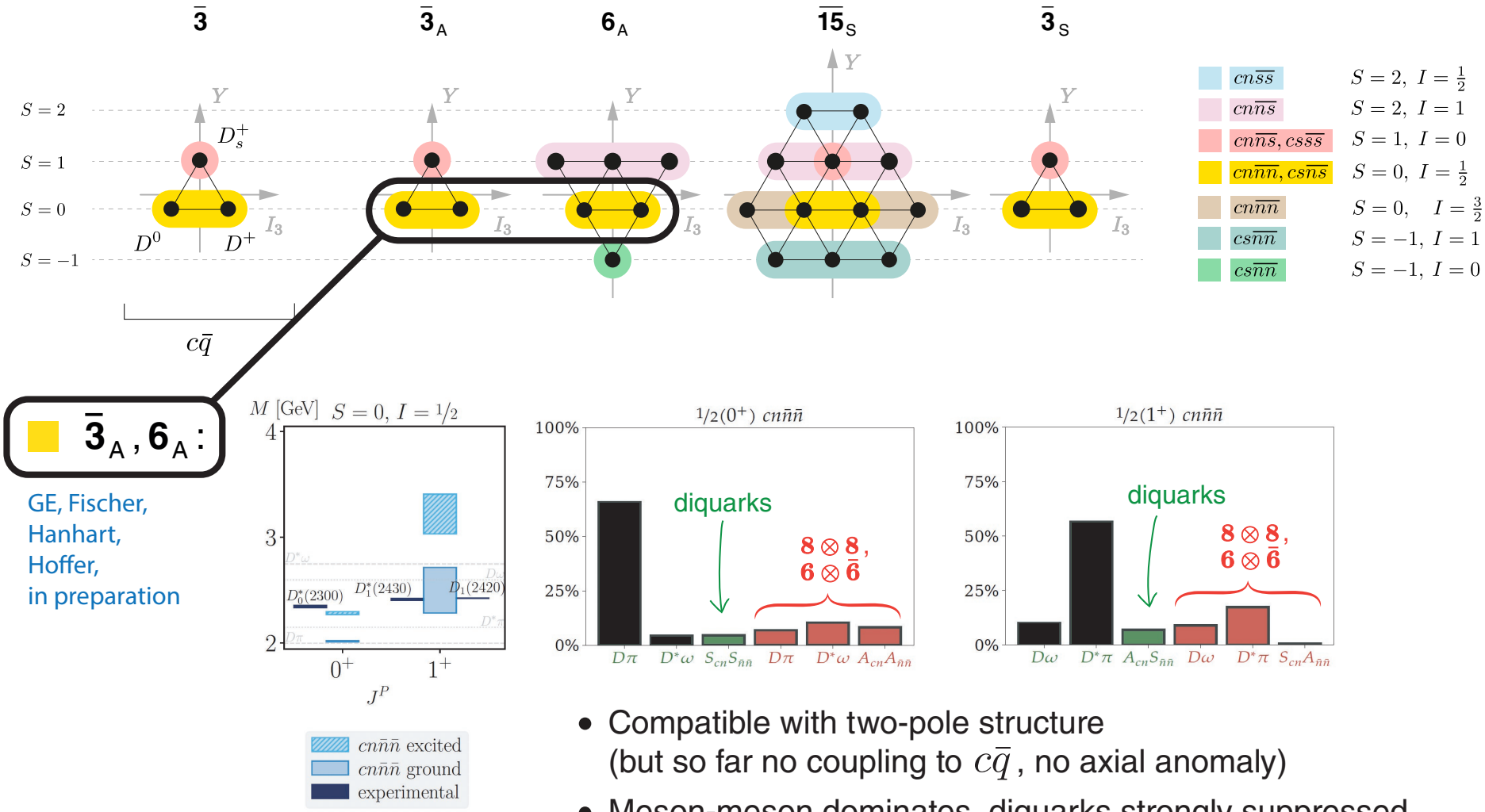
  (strongly) attractive | (strongly) attractive | (strongly) repulsive | strongly repulsive

Meson-molecule picture

- Similar trend on lattice Gregory, Guo, Hanhart, Krieg, Luu, EPJA 71 (2025)
- Functional methods? No preference on diquark/antidiquark or meson-meson, determined dynamically → investigate

Single-charm states

SU(3) multiplets for light & strange quarks in $c\bar{q}\bar{q}q$: $\quad \bar{3} \otimes \bar{3} \otimes 3 = \bar{3}_A \oplus 6_A \oplus \overline{15}_S \oplus \bar{3}_S$



$\bar{3}_A, 6_A$:

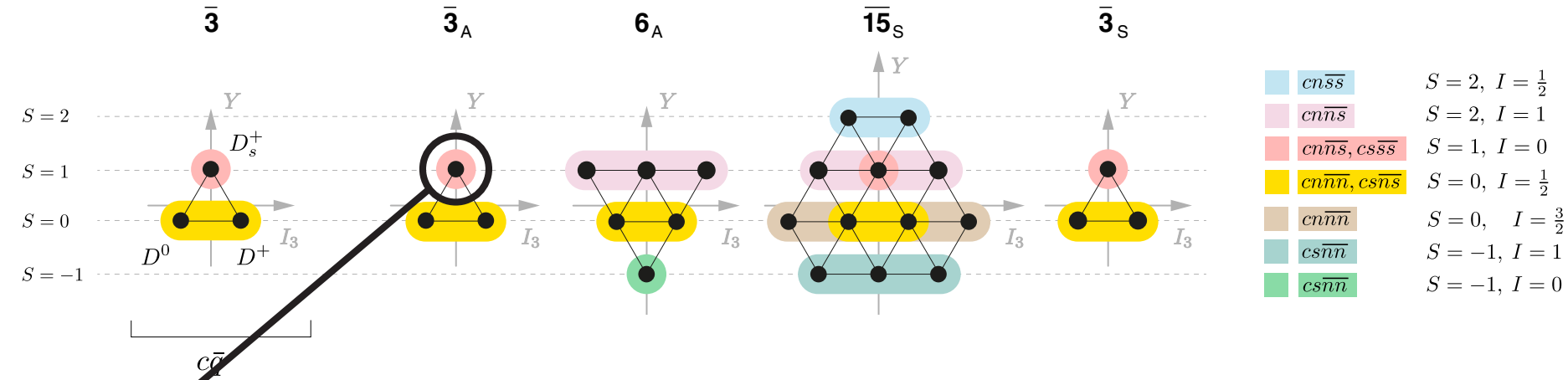
GE, Fischer, Hanhart, Hoffer, in preparation

$cn\bar{n}\bar{n}$ excited
$cn\bar{n}\bar{n}$ ground
experimental

- Compatible with two-pole structure
  (but so far no coupling to $c\bar{q}$, no axial anomaly)
- Meson-meson dominates, diquarks strongly suppressed

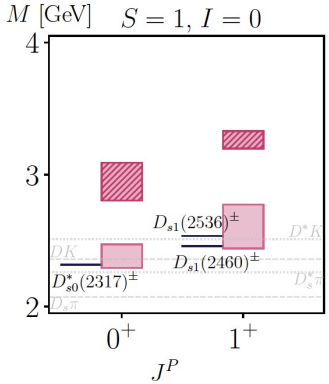
Single-charm states

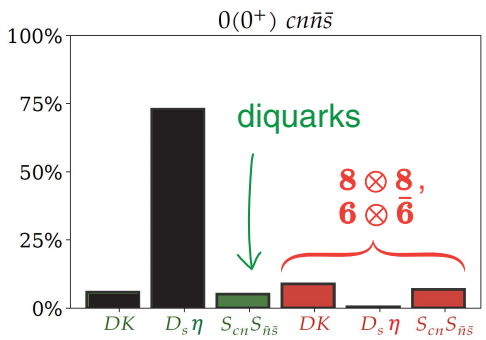
SU(3) multiplets for light & strange quarks in $c\bar{q}\bar{q}q$: $\quad \bar{3} \otimes \bar{3} \otimes 3 = \bar{3}_A \oplus 6_A \oplus \overline{15}_S \oplus \bar{3}_S$

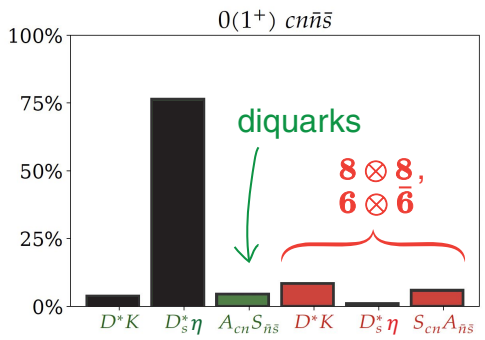


$\bar{3}_A$:

GE, Fischer, Hanhart, Hoffer, in preparation

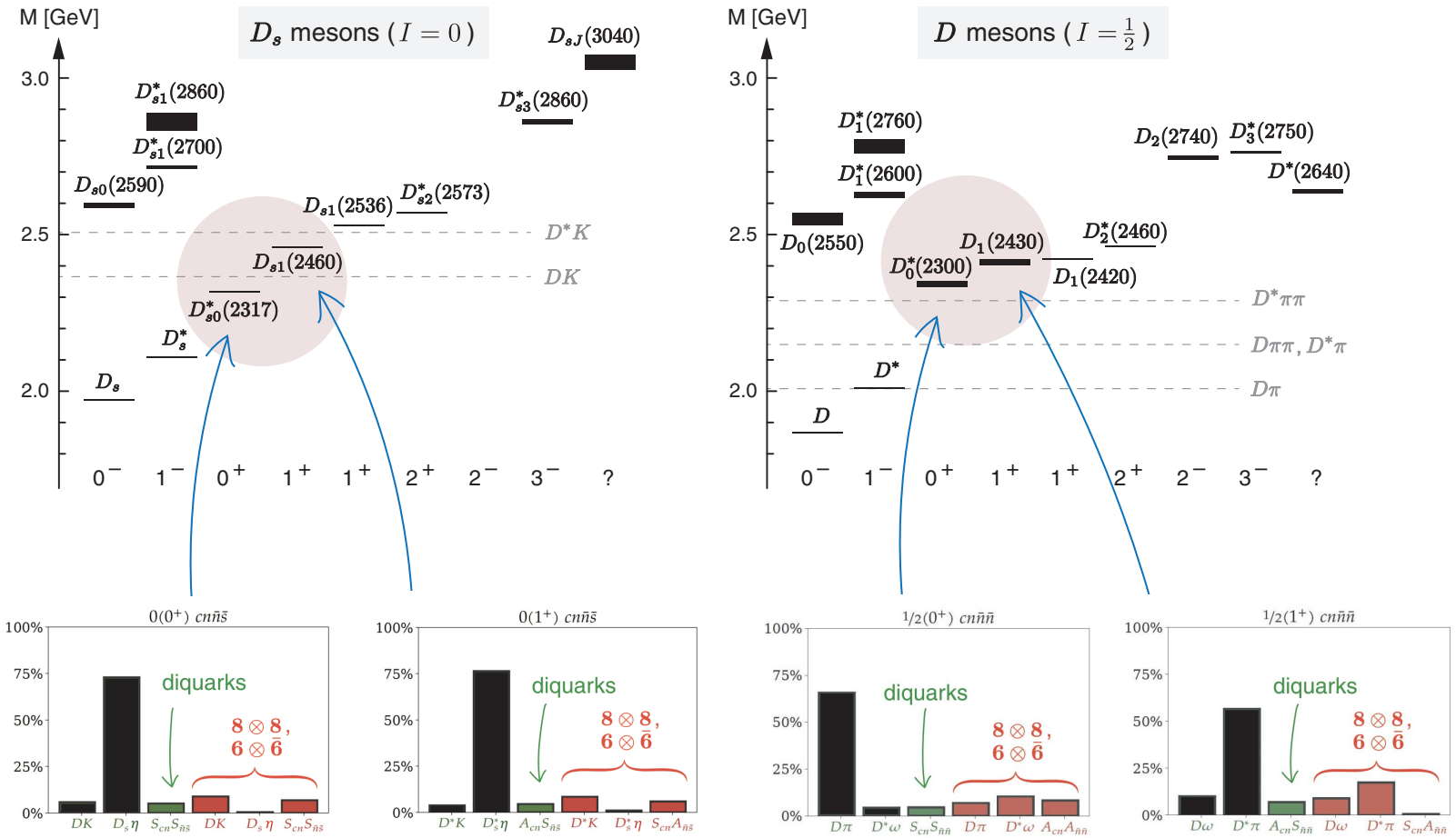






- Compatible with experiment
  (but so far no coupling to $c\bar{q}$, no axial anomaly)
- Meson-meson dominates, diquarks strongly suppressed

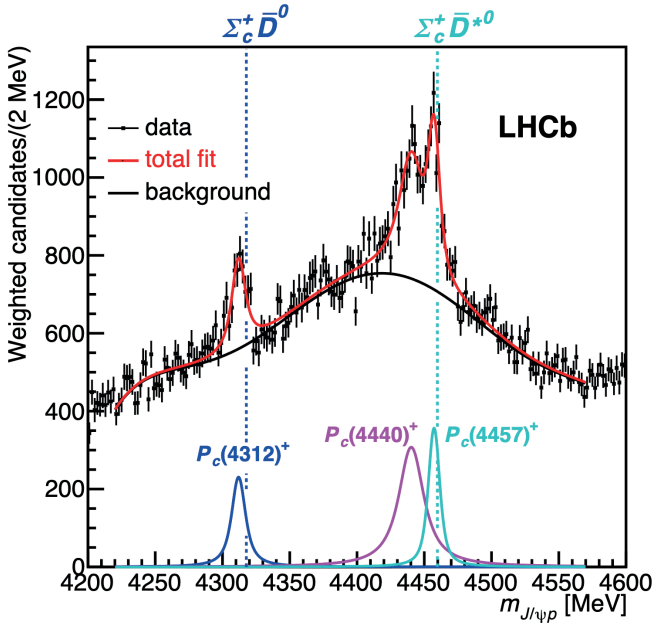
Single-charm states



Four-quark calculations (so far no coupling to $c\bar{q}$, no axial anomaly):
meson-meson components dominate, diquarks strongly suppressed

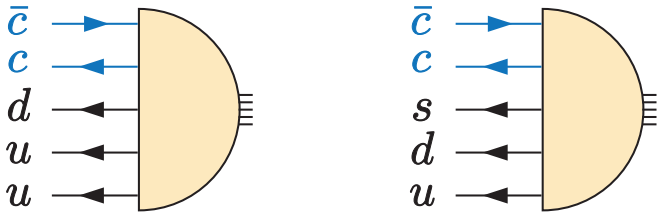
GE, Fischer, Hanhart, Hoffer, in preparation

Pentaquarks?



LHCb Collaboration,
PRL 122 (2019) 22, 22201
PRL 131 (2023) 3, 031901

$P_c^+(uudc\bar{c})$ $\quad$ $P_{cs}^0(udsc\bar{c})$



- Well-established states:

$P_c(4312)^+$	$P_{cs}(4338)^0$
$P_c(4440)^+$	$P_{cs}(4459)^0$
$P_c(4457)^+$	

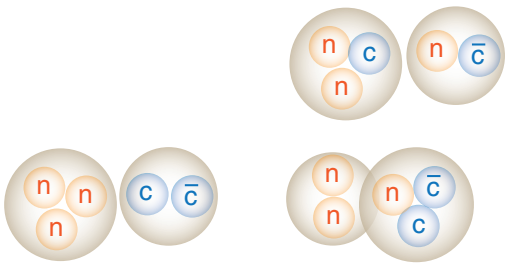
narrow (10... 20 MeV)

$P_c(4337)^+$ ... only ~3σ, threshold effect?
$P_c(4380)^+$ ... broad (~200 MeV)

- Proximity to thresholds suggests molecular nature:

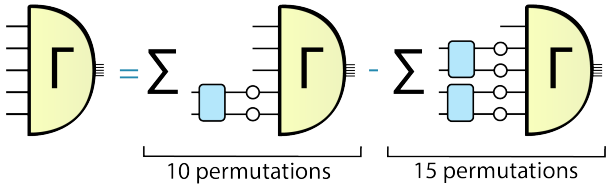
$\Sigma_c\, D$, $\quad \Xi_c\, D$,
$\Lambda_c\, D$, $\quad \Lambda_c\, D_s$,
$N\, \eta_c$ (?), $\quad \Lambda\, \eta_c$ (?),
diquark-triquark (?), . . .

Wu, Molina, Oset, Zou, PRL 105 (2010), PRC 84 (2011), . . .

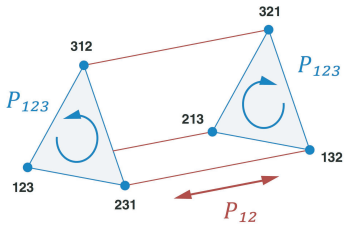


Five-body systems

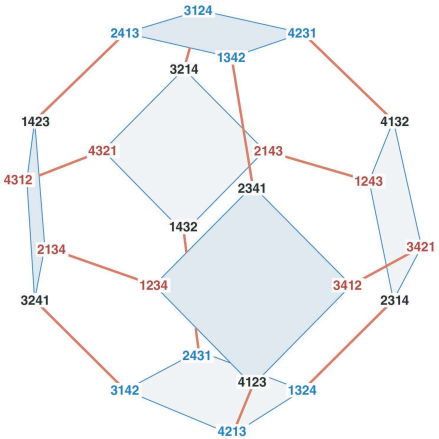
- **Five-body equation**
  GE, Torres, PRD 111 (2025),
  GE, Peña, Torres, PLB 866 (2025)



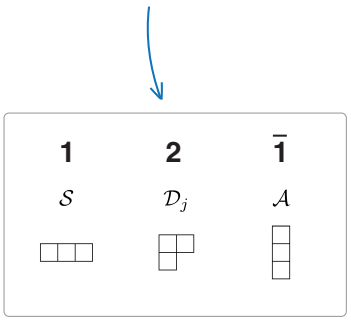
- Multiplets of permutation group S_5?

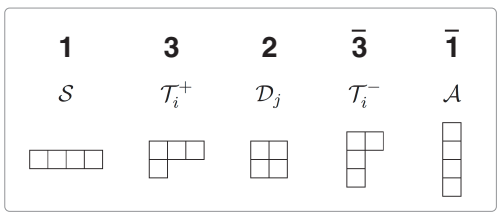


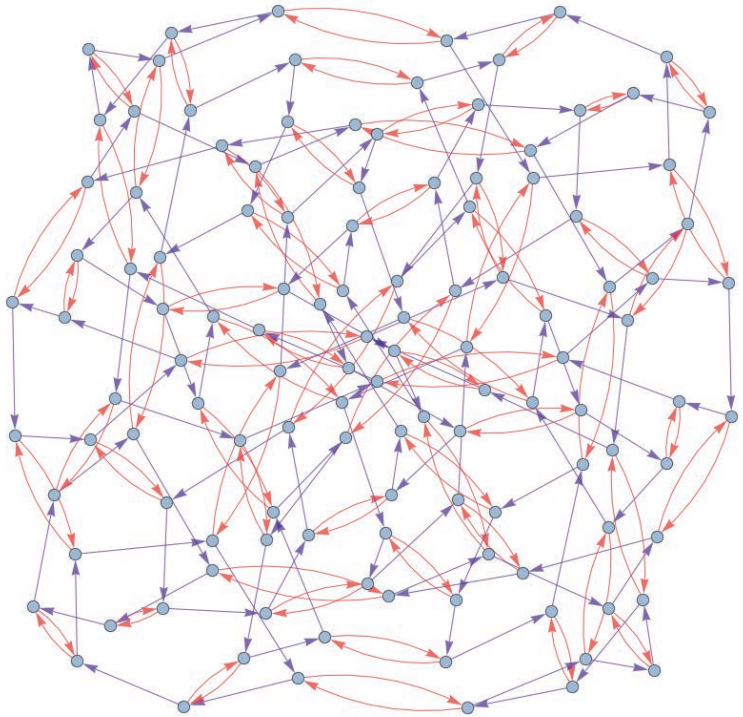
Cayley graph of S_3



Cayley graph of S_4



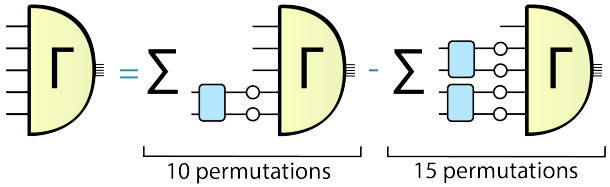




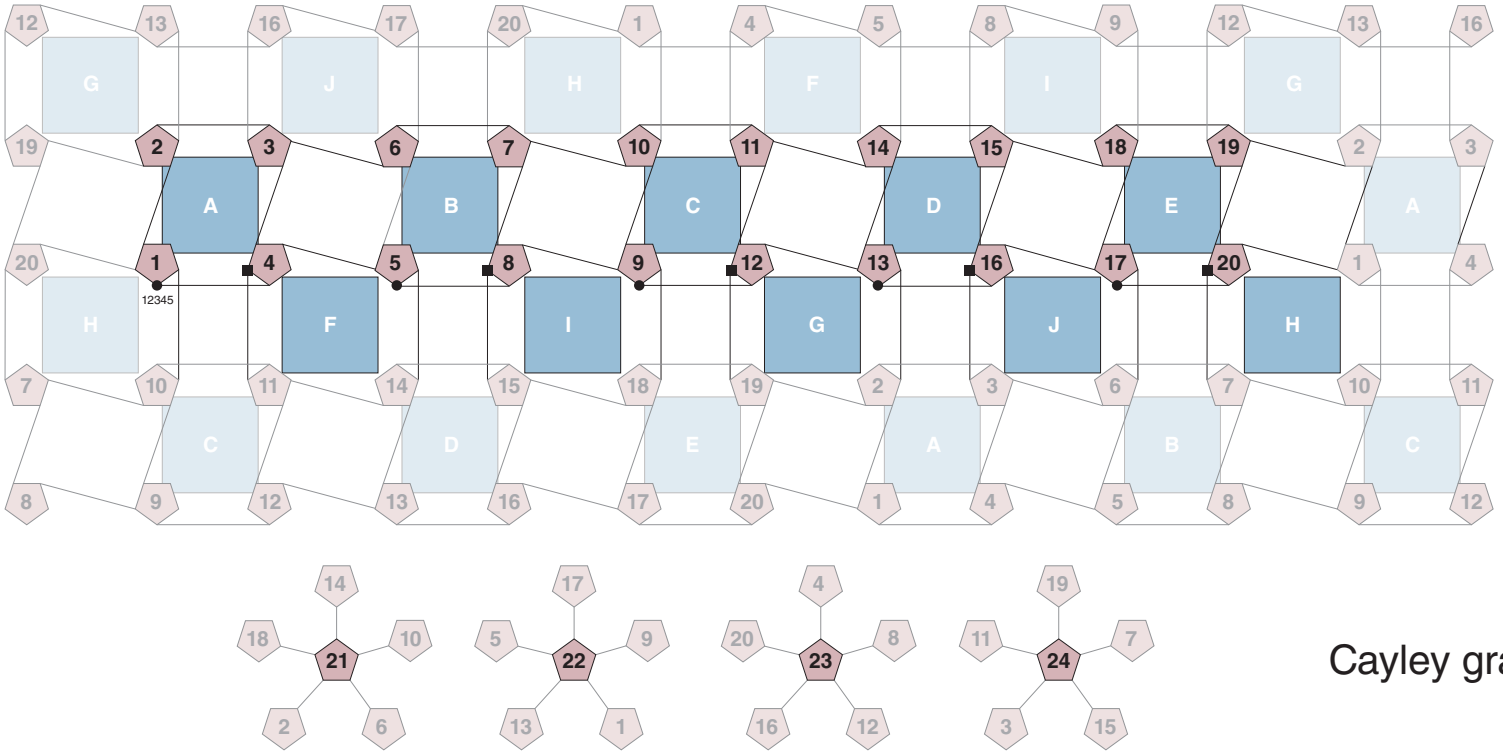
Cayley graph of S_5

Five-body systems

- **Five-body equation**
  GE, Torres, PRD 111 (2025),
  GE, Peña, Torres, PLB 866 (2025)



- Multiplets of permutation group S_5?



Cayley graph of S_5

Five-body systems

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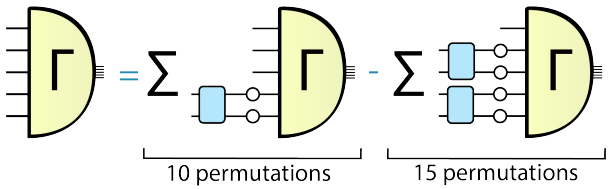
  GE, Torres, PRD 111 (2025),
  GE, Peña, Torres, PLB 866 (2025)

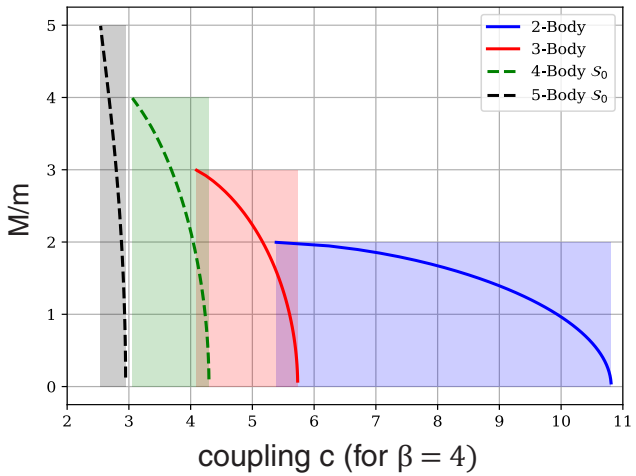


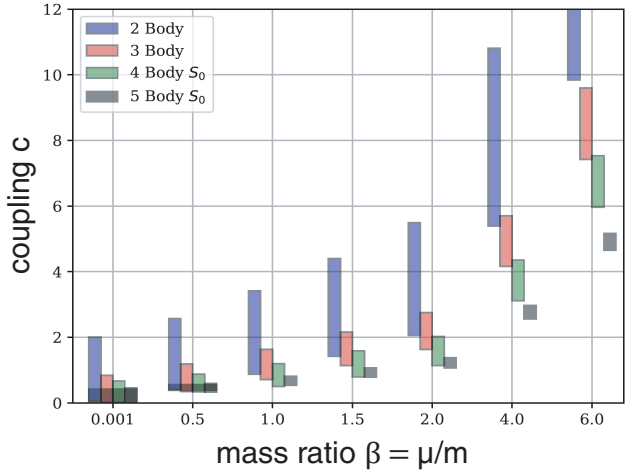
$$\Gamma(p,q,k,l,P) = \sum_i f_i(p^2,q^2,k^2,l^2,P^2,\{\omega_i\},\{\eta_i\})\,\tau_i(p,q,k,l,P)$$

15 Lorentz invariants,
group into multiplets of S_5:
1 + 1 + 4 + 4 + 5

+ dimensional constraint:
only 14 are independent

- Scalar (massive Wick-Cutkosky) model:
  Parameters are coupling strength c, exchange particle mass ratio $\beta = \mu/m$





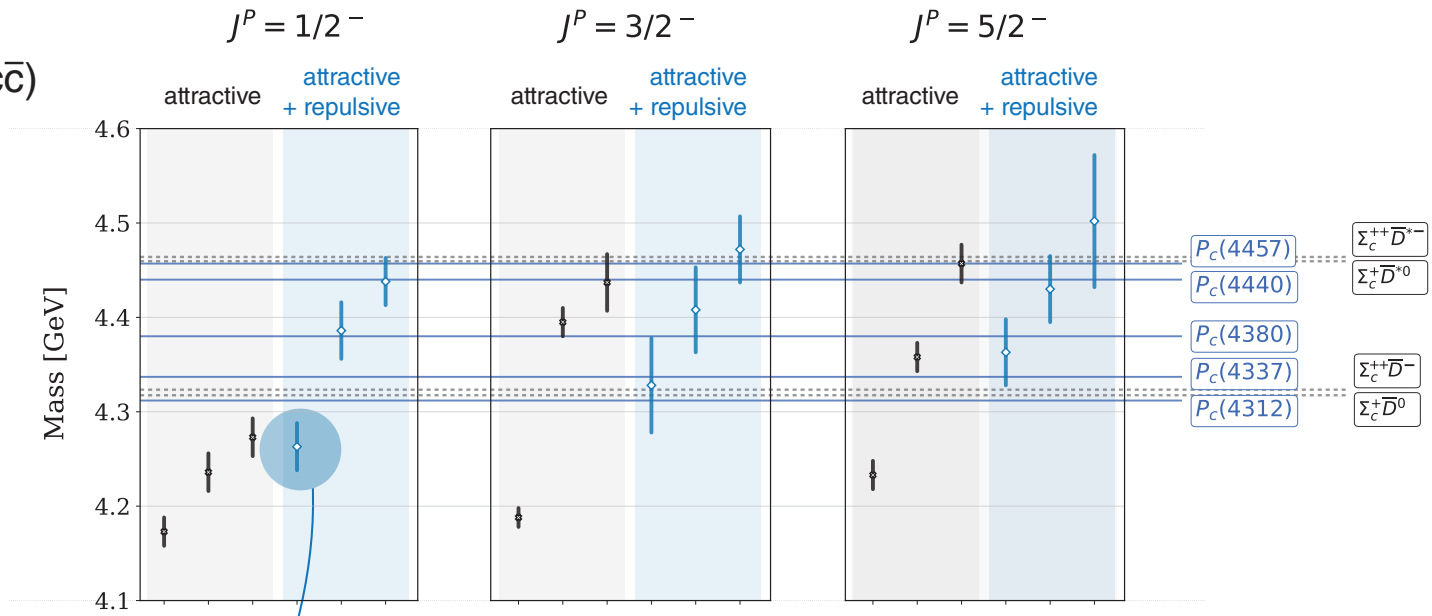
Regions in c, β where system is **Borromean:** 5-body eq. has solution but 2- and 3-body eqs. do not

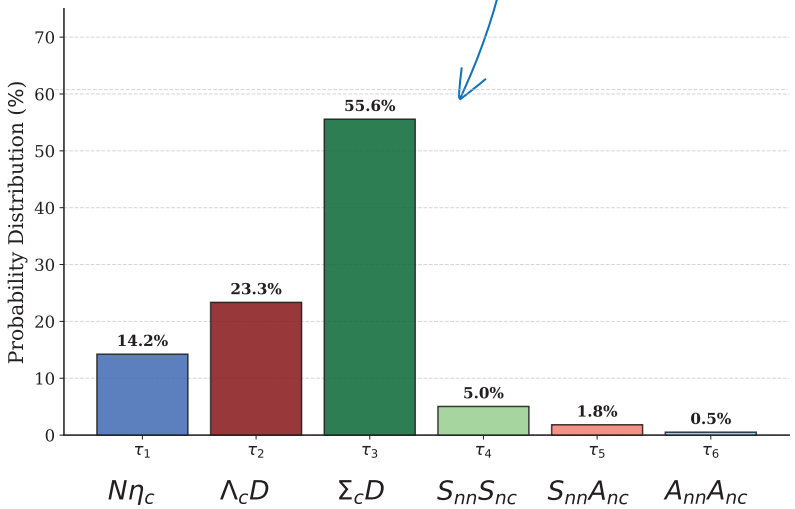
LHCb pentaquarks

QCD pentaquarks:
First results for $P_c^+(uudc\bar{c})$ (preliminary)

Torres, GE, Peña, in preparation

Raúl Torres





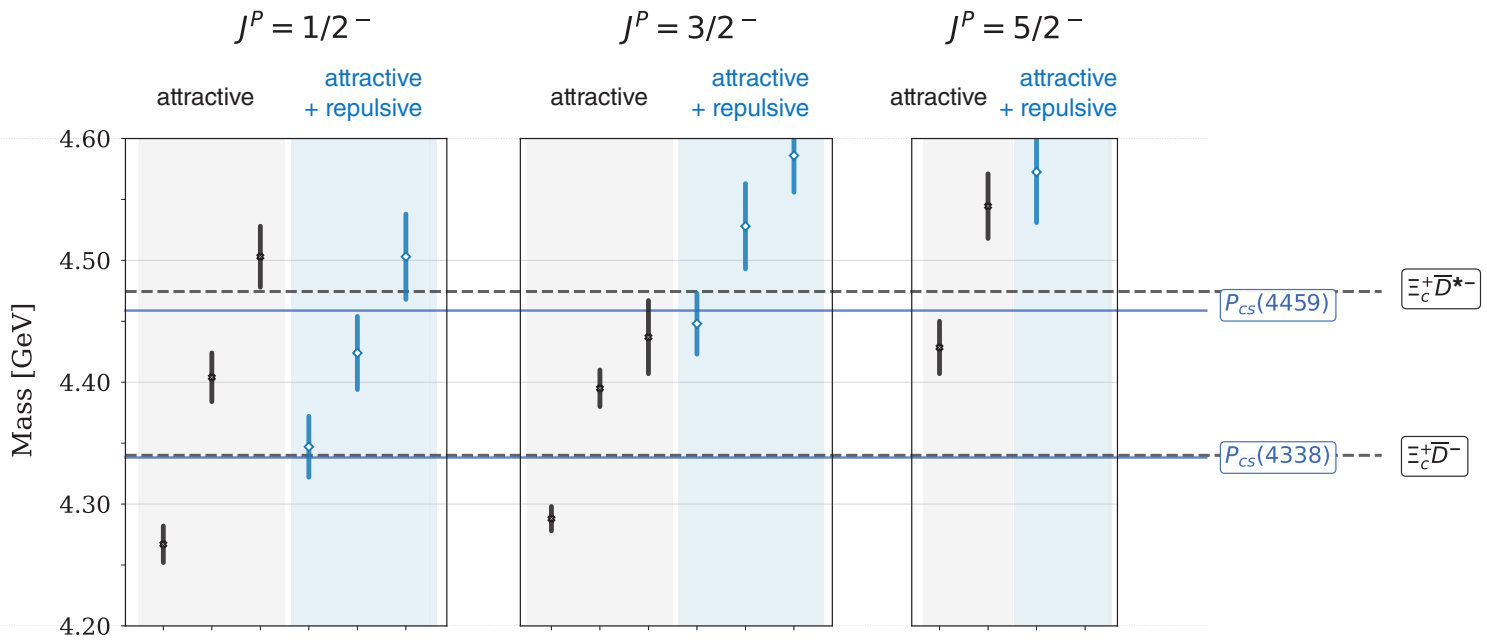
- Repulsive color channels significant
- Can test different internal structures:
  - **Baryon-meson**: Σ_cD, Λ_cD, Nη_c
  - **Diquark-triquark**: (nn)(nc)$\bar{c}$
- Lowest $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ pentaquark is dominated by Σ_cD, exotic components small

LHCb pentaquarks

QCD pentaquarks: P_{cs}^{0} ($udsc\bar{c}$), preliminary

Torres, GE, Peña, in preparation

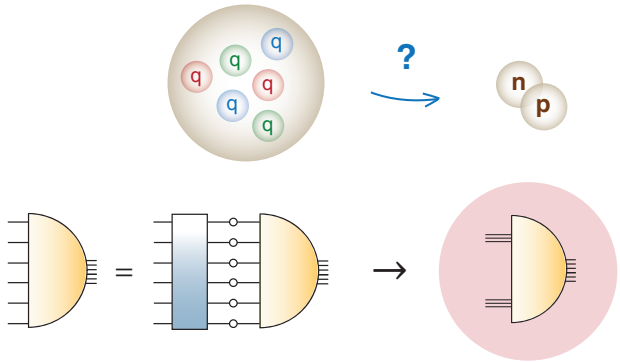
Raúl Torres



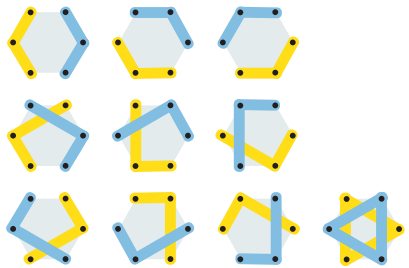
- $1/2^-$ state agrees with $P_{cs}(4338)$
- $1/2^-$ and $3/2^-$ states in vicinity of $P_{cs}(4459)$

Six-body systems

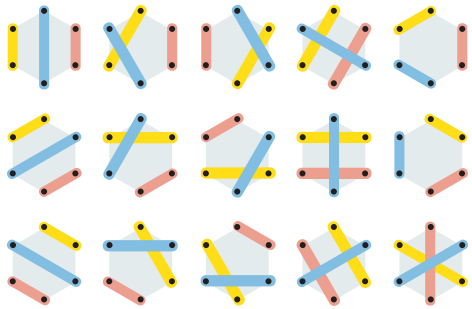
- 6-body equations: Transition from quarks & gluons to **light nuclei**
  GE, **Hoffer**, in preparation



- 10 possible 3-body configurations: (123)(456), ...
  → **baryon clusters**



- 15 possible 2-body configurations: (12)(34)(56), ...
  → **diquark clusters**



- $\Gamma^{(\mu)}(p,q,k,l,r,P) = \sum_i f_i(\underbrace{\text{18 variables}}) \; \tau_i^{(\mu)}(p,q,k,l,r,P) \otimes \underbrace{\text{Flavor}}_{\square\square\square} \otimes \underbrace{\text{Color}}_{\square\square\square}$

  1 + 1 + 5 + 5 + 9
  (+ 3 constraints)

  $\approx f_i(S_0) \times$ **poles**

  J = 0: 4096 tensors (320 s waves)
  J = 1: 12288 tensors (576 s waves)

  5 (deuteron)

  $\overline{\mathbf{5}}$

  $\overline{\mathbf{1}}, \overline{\mathbf{5}}, \overline{\mathbf{9}}, \overline{\mathbf{10}}$

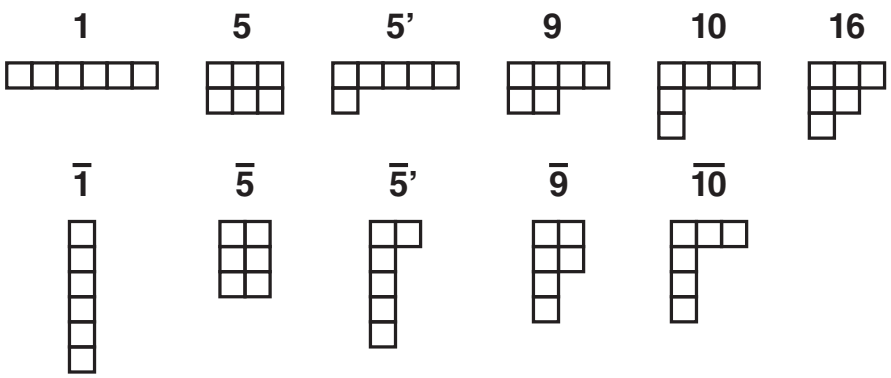
Joshua Hoffer

Permutation group S_6

- 720 elements: 123456, 234561, ...

Joshua Hoffer

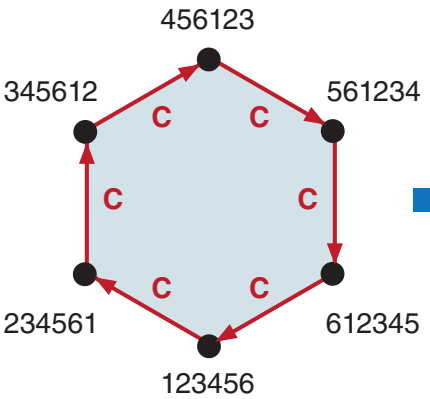
- Irreps of S_6:



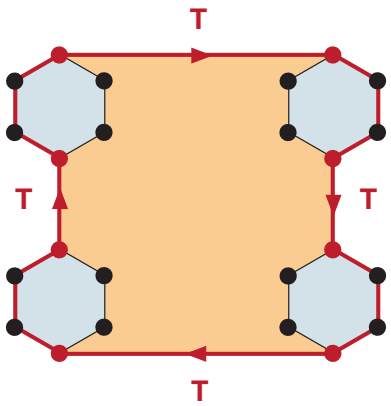
Cumbersome expressions...
e.g., each quintet has 5 entries which are linear combinations of 720 permutations

- **Transpositions T** (1 ↔ 2) and **cyclic permutations C** (1 → 2 → ... → 6 → 1) span whole group:

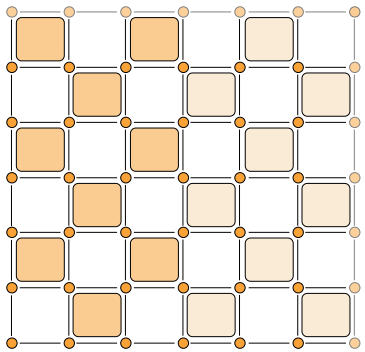
120 **hexagons**



90 "**plaquettes**" with 8 unique elements



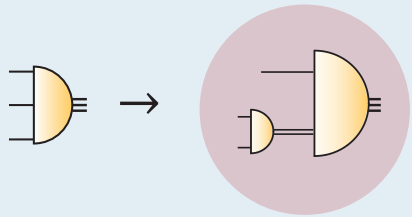
10 **chessboard planes** with 18 plaquettes



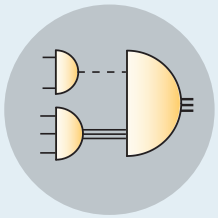
Pick 9 plaquettes per chessboard ⇒ 90 plaquettes with 8 permutations = **720**

Summary: Diquark correlations in ...

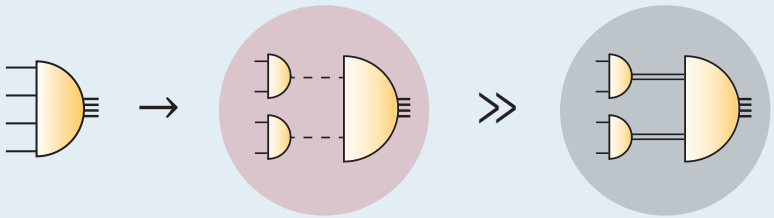
- **Baryons?** Yes, unavoidable (only clustering options for 3q):



5q components could also be significant:

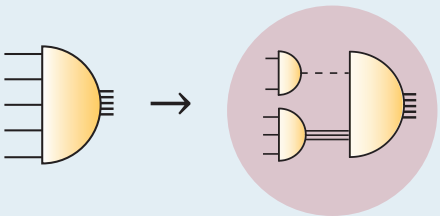


- **Tetraquarks? Meson-meson** correlations dominate

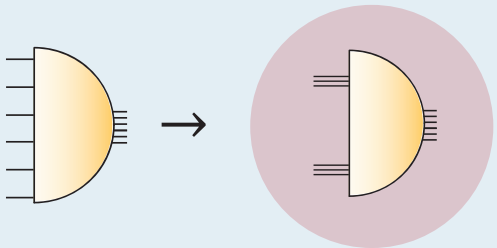


$nn\bar{n}\bar{n}$. . . light scalar mesons
$cn\bar{n}\bar{n}$. . . single-charm states
$c\bar{c}n\bar{n}$. . . hidden-flavor states
$cc\bar{n}\bar{n}$. . . open-flavor states, **exception**: T_{bb}

- **Pentaquarks? Meson-baryon** correlations dominate, so far no evidence for diquark clusters



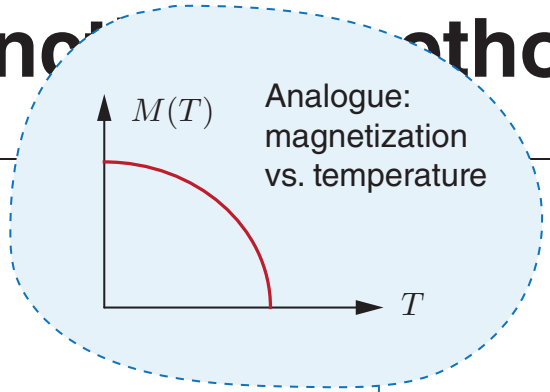
- **Hexaquarks? Baryon-baryon** correlations must dominate, maybe also diquark clusters?



Thank you!

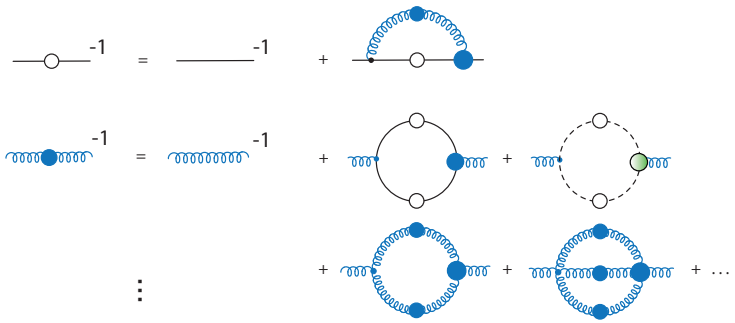
Backup slides

Functional methods



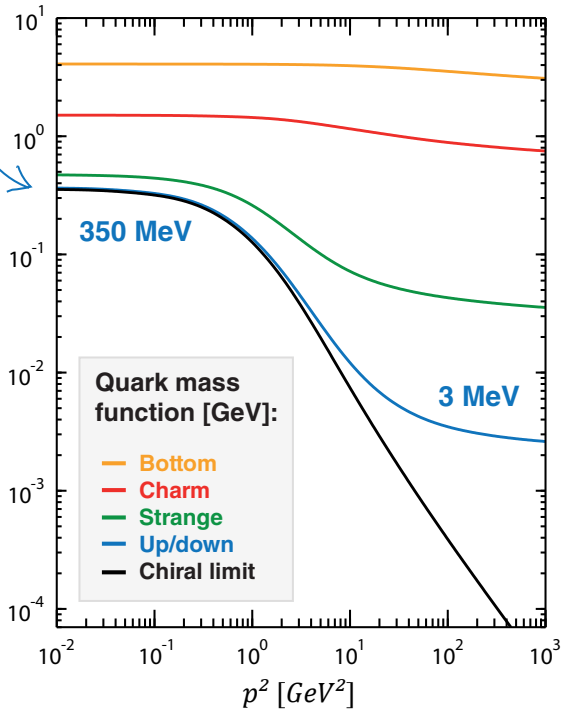
- Ingredients: **QCD's n-point functions,** Satisfy quantum eqs. of motion (DSEs)

Quark mass



→ Dynamical mass generation, gluon mass gap, confinement, ...

"constituent-quark mass"

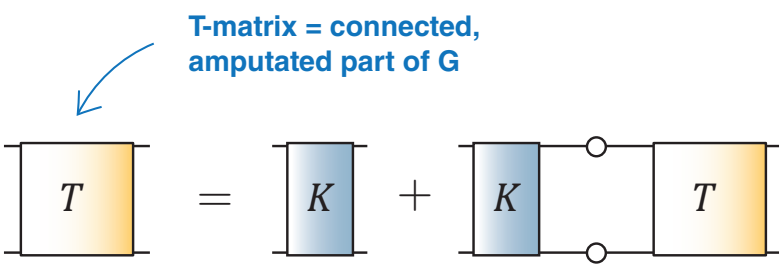


current-quark mass

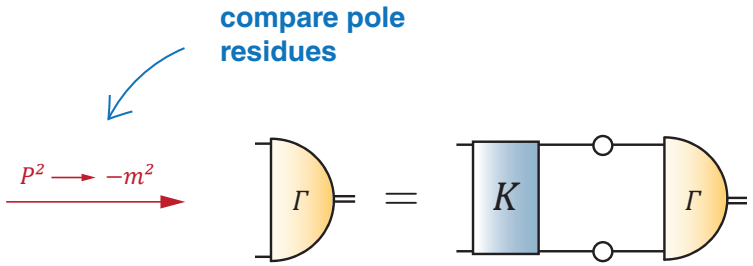


Bethe-Salpeter equations

Write down **inhomogeneous BSE**:

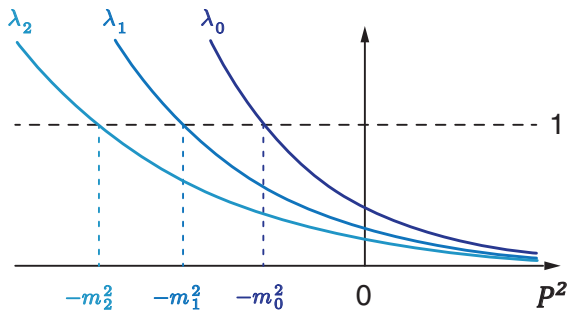


Homogeneous BSE at pole:



BSE = eigenvalue equation,
pole in T $\Leftrightarrow$ **eigenvalue = 1**

$$K G_0 \Gamma_i = \lambda_i \Gamma_i$$



Explicitly:

$$\Gamma(p,P) = \int \frac{d^4q}{(2\pi)^4} \mathbf{K}_{\alpha\gamma,\delta\beta}(p,q,P)\,[S(q_+)\,\Gamma(q,P)\,S(q_-)]_{\gamma\delta}$$

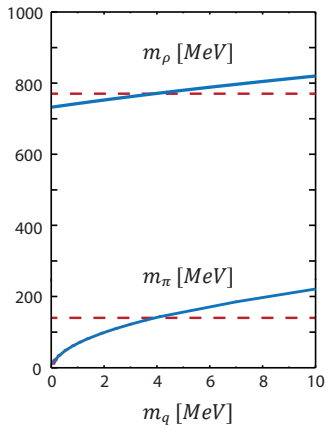
Basis decomposition:

$$\Gamma(p,P) = \sum_{i=1}^{n} f_i(p^2, \hat{p}\cdot\hat{P}, P^2)\,\tau_i(p,P)$$

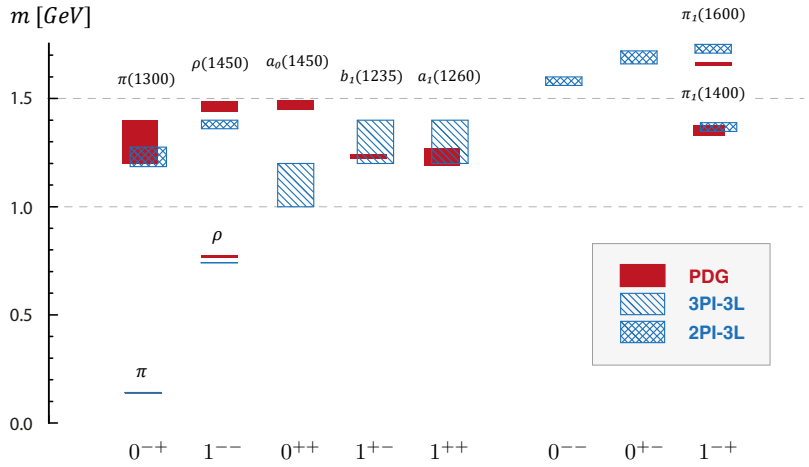
$\Rightarrow$ Coupled Lorentz-invariant equations
for the dressing functions f_i

Mesons

- Pion is **Goldstone boson:** $m_\pi{}^2 \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder

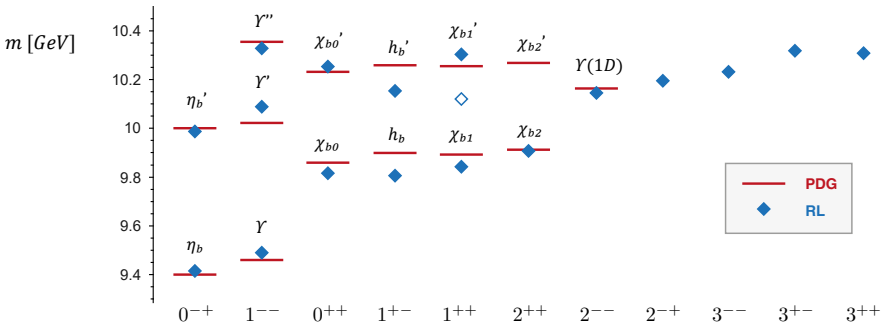


Williams, Fischer, Heupel, PRD 93 (2016)

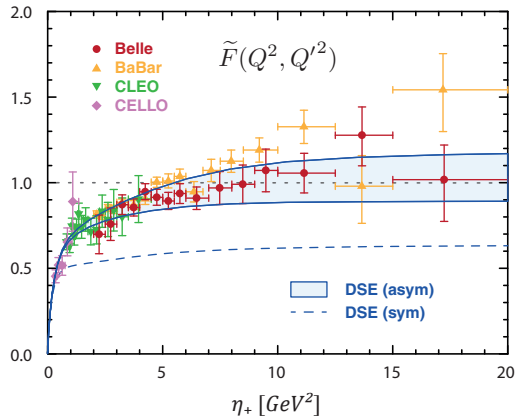
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **Bottomonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)



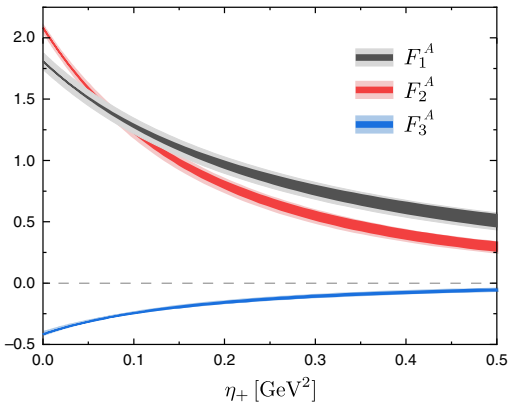
- **Pion transition form factor**



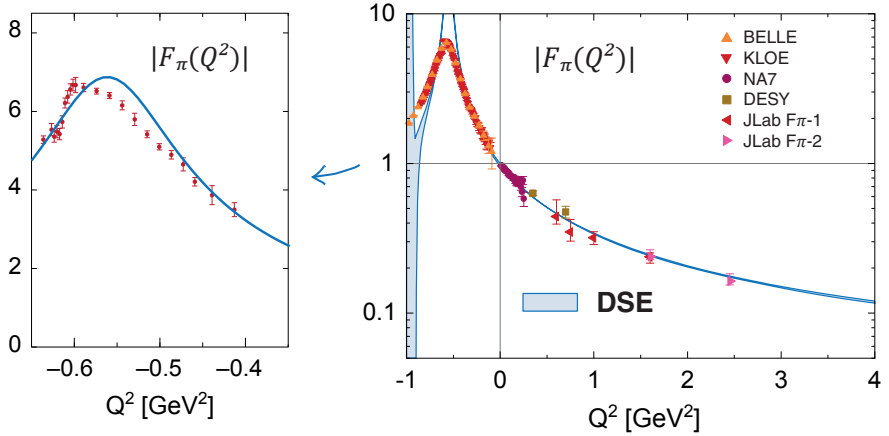
GE, Fischer, Weil, Williams, PLB 774 (2017)

Form factors

- **Axial & scalar → 2γ FFs** for muon g-2
  GE, Fischer, Häuser, Regenfelder, EPJC 85 (2025)

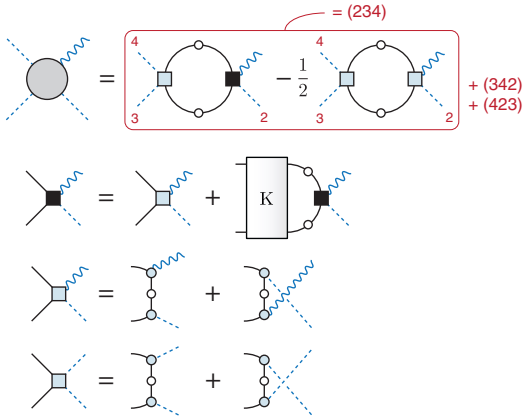


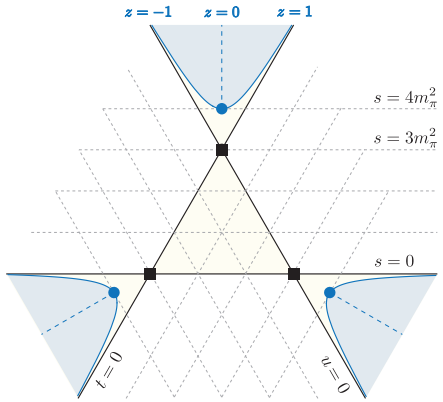
- **Timelike FFs** dynamically develop ρ pole with $\pi\pi$ decay channel
  Miramontes, Sanchis-Alepuz, Alkofer, PRD 103 (2021)

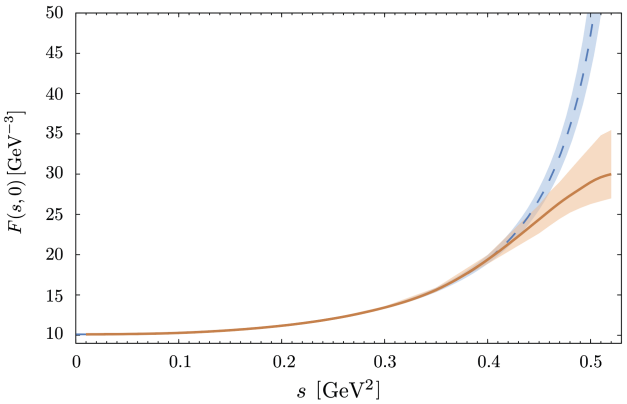


GE, Fischer, Williams, PRD 101 (2020)

- **Timelike form factor** γπ → ππ
  Miramontes, GE, Alkofer, 2504.20899

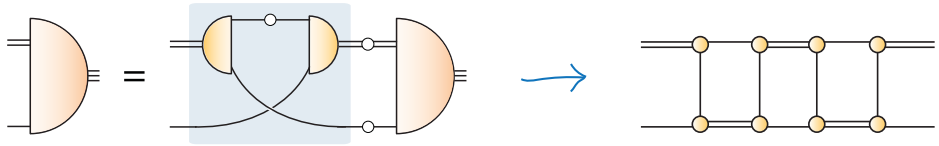






Diquark correlations

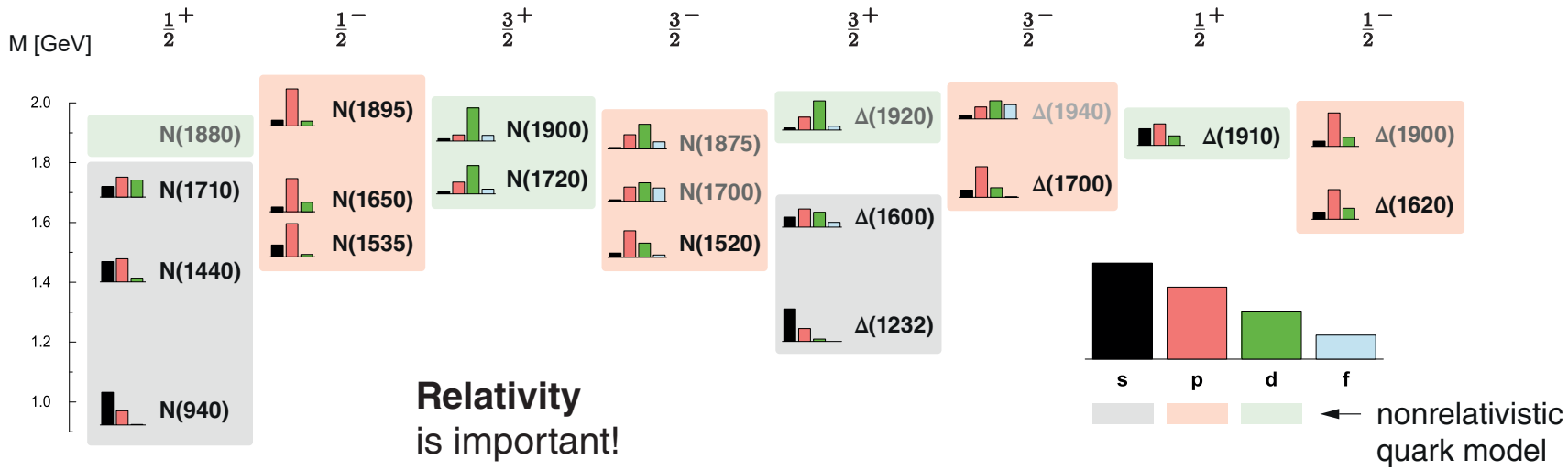
- **Quark-diquark** (two-body) equation
  Oettel et al., PRC 58 (1998), GE et al., Ann. Phys. 323 (2008), Cloet et al., FBS 46 (2009), Segovia et al., PRL 115 (2015), Chen et al., PRD 97 (2018)



Diquark clustering in baryons?
Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021)



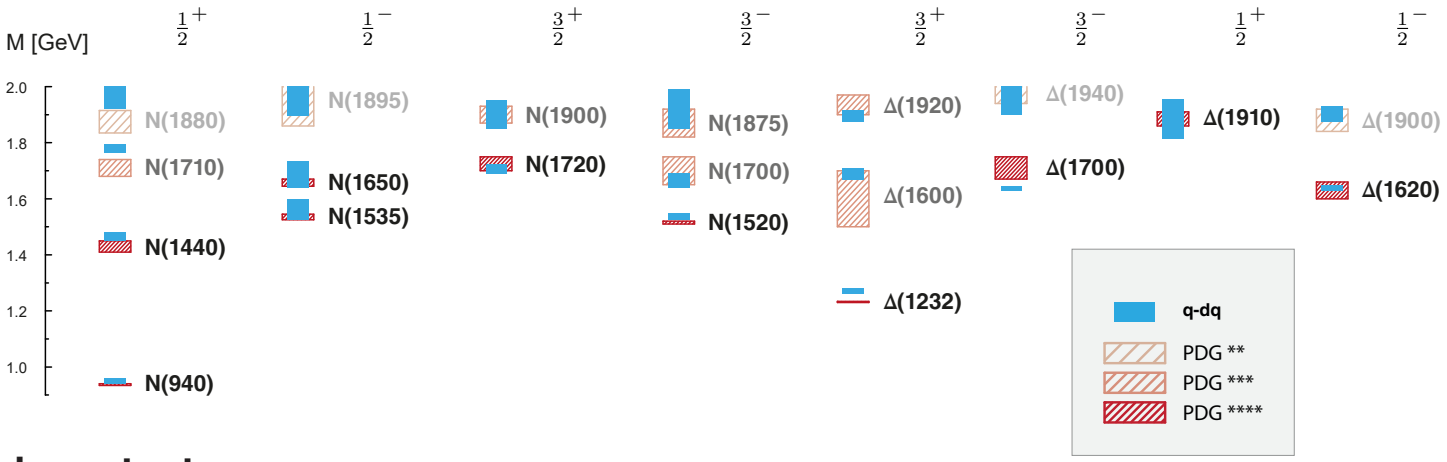
- **Three-quark** and **quark-diquark** results very similar
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016), GE, FBS 63 (2022)



Diquark correlations

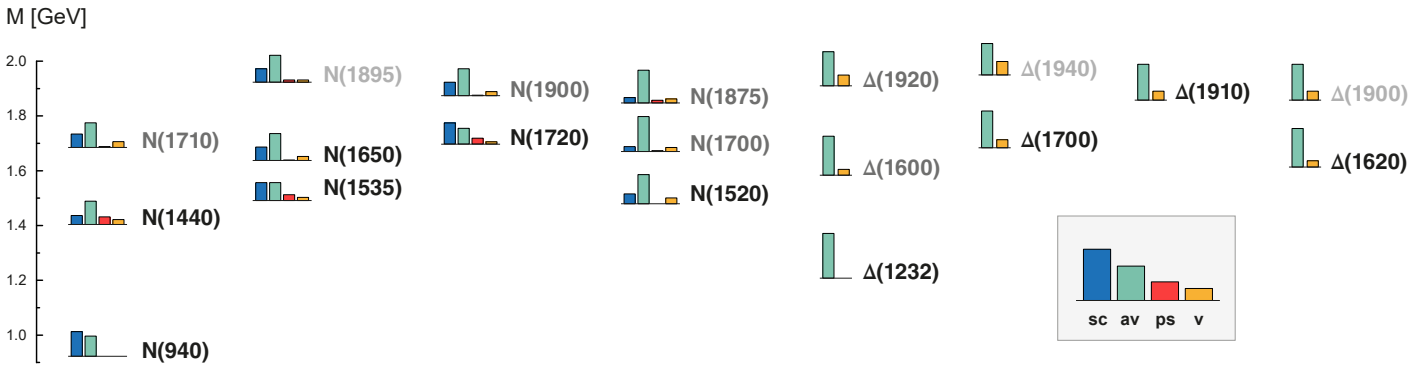
Light baryon spectrum

GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

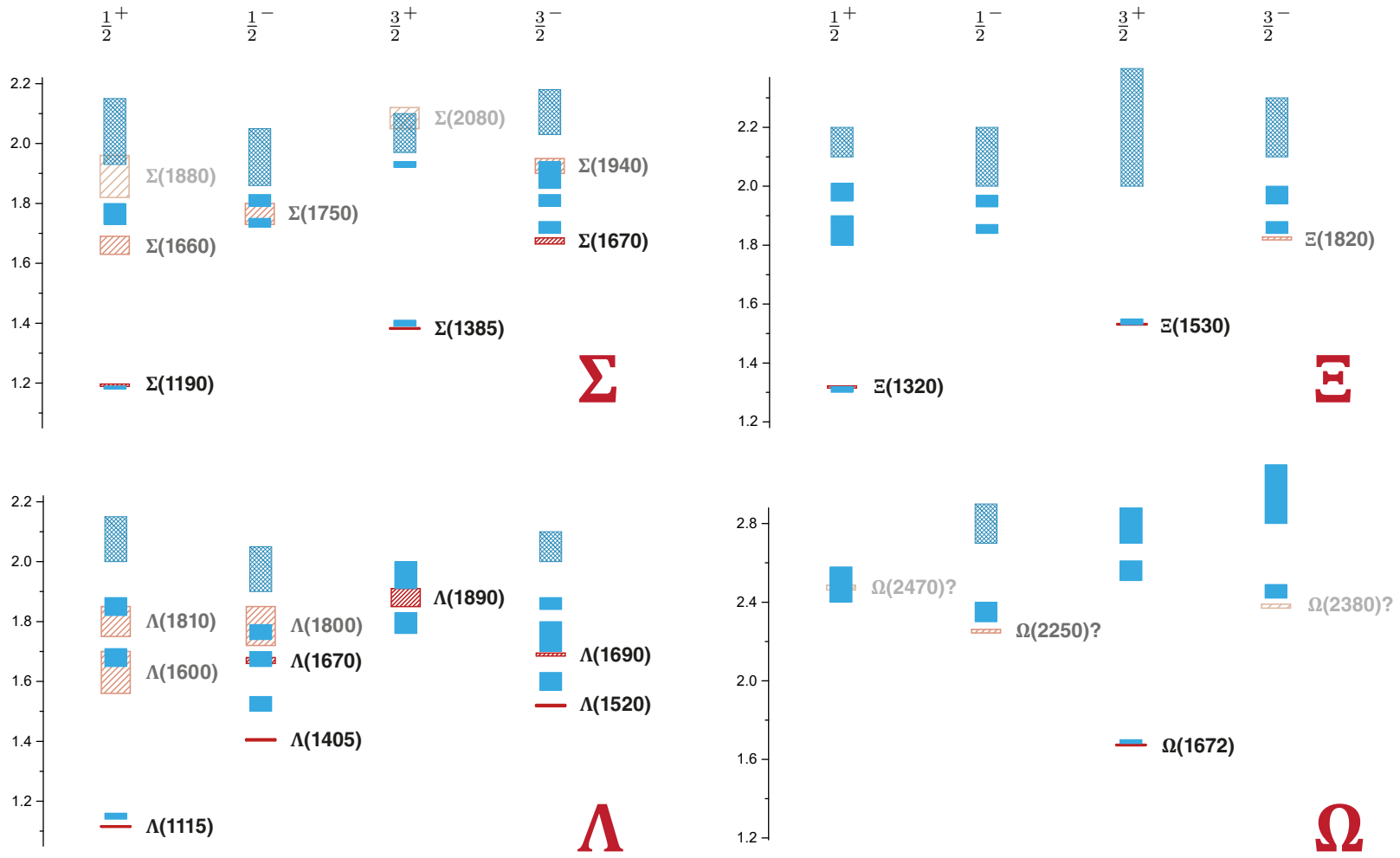


Diquark content:

Barabanov et al., PPNP 116 (2021)

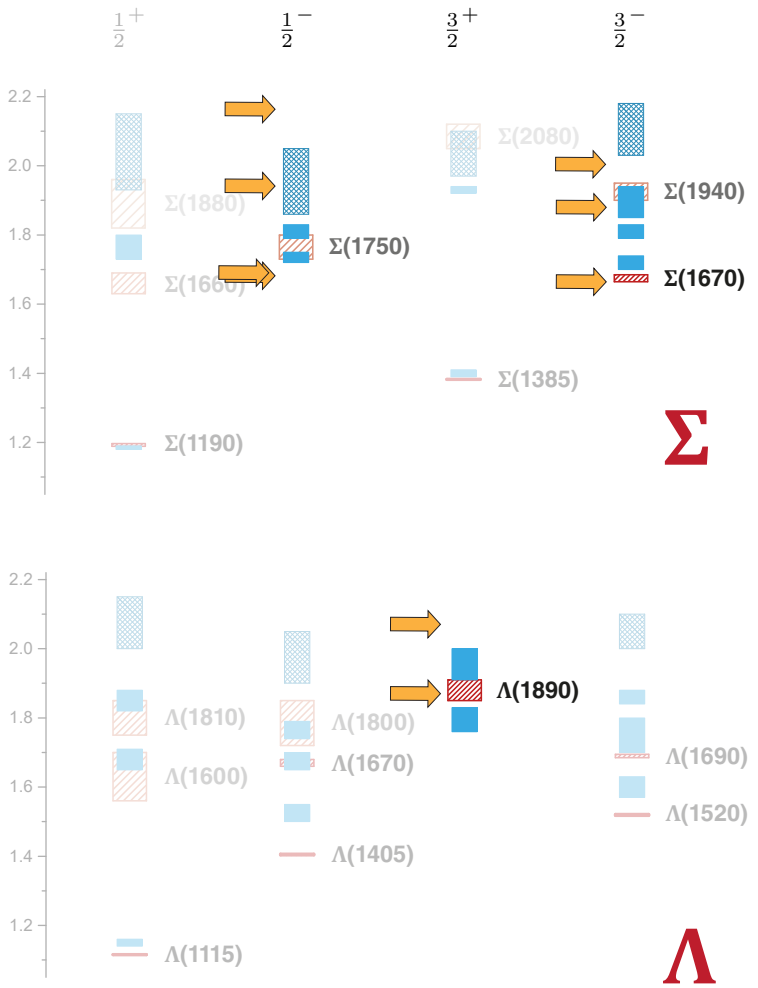


Strange baryons



GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Strange baryons



New states from Bonn-Gatchina

Sarantsev et al., 1907.13387 [nucl-ex]

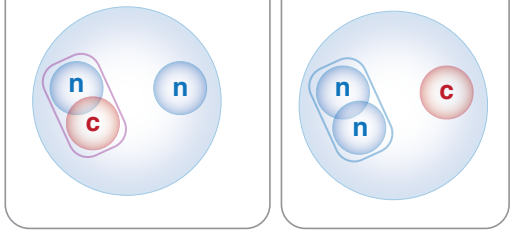
GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Heavy baryons

Quark content	q-dq	Isospin	contributes to
nnc	n [nc]	0, 1	Λ_c , Σ_c
↑ n = u, d	n {nc}	0, 1	Λ_c , Σ_c
	c [nn]	0	Λ_c
	c {nn}	1	Σ_c

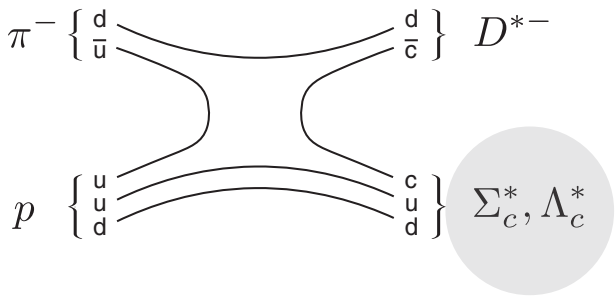
⇒ Λ_c: n [nc], n {nc}, c [nn]

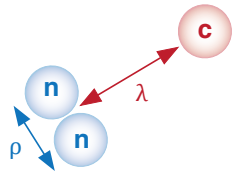
Σ_c: n [nc], n {nc}, c {nn}



These are often assumed as dominant components

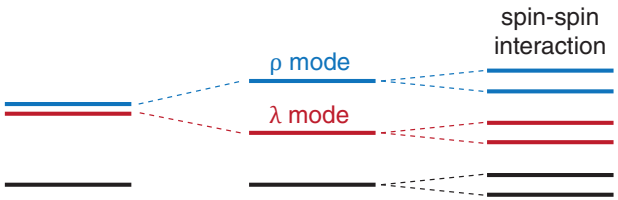
Kim, Hosaka, Kim, Noumi, Shirotori, PTEP 2014 (2014), 10, 103D01,
Shim, Hosaka, Kim, PTEP 2020 (2020) 5, 053D01





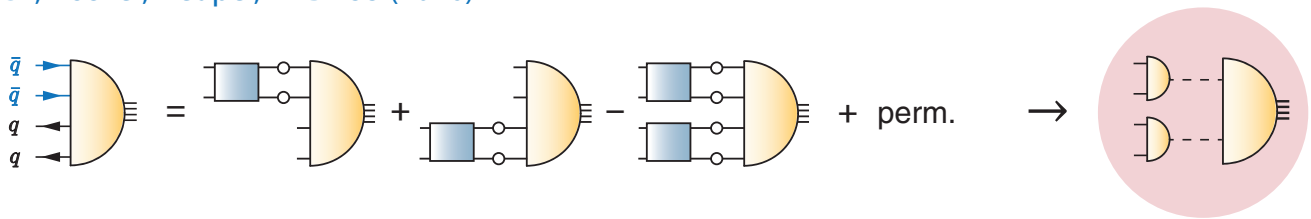
Decay of ρ mode → $\Sigma_c\pi$
Decay of λ mode → pD

Assumptions on spectrum & production rates



Four-quark states

- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states:**
  GE, Fischer, Heupel, PLB 753 (2016)



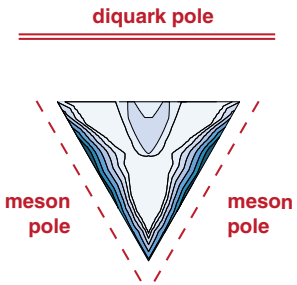
- BSE dynamically generates **meson poles** in BS amplitude:

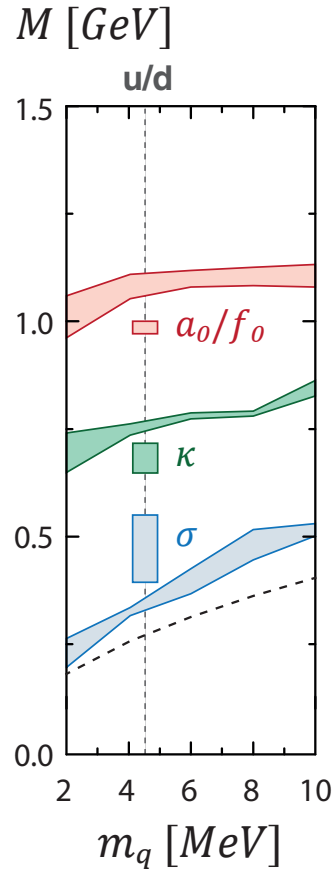
$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ 1500 MeV

$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ 1500 MeV

$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ 1200 MeV

$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ **350 MeV !**



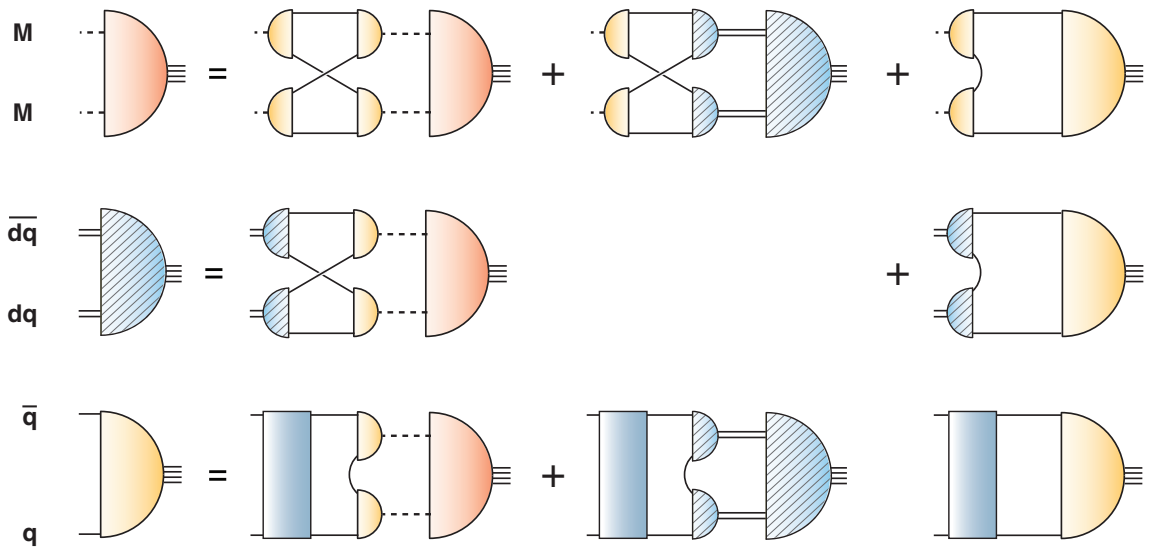


- "Light scalar mesons" look like **meson molecules,**
  diquark-antidiquark components almost negligible.
  Lightness is inherited from pseudoscalar Goldstone bosons!

Four-quark states

Two-body formulation: **meson-meson / diquark-antidiquark,**
follows from four-quark eq. (analogue of quark-diquark for baryons)

Heupel, GE, Fischer, PLB 718 (2012)



Include **mixing with $q\bar{q}$:**
$\pi\pi$ still dominant

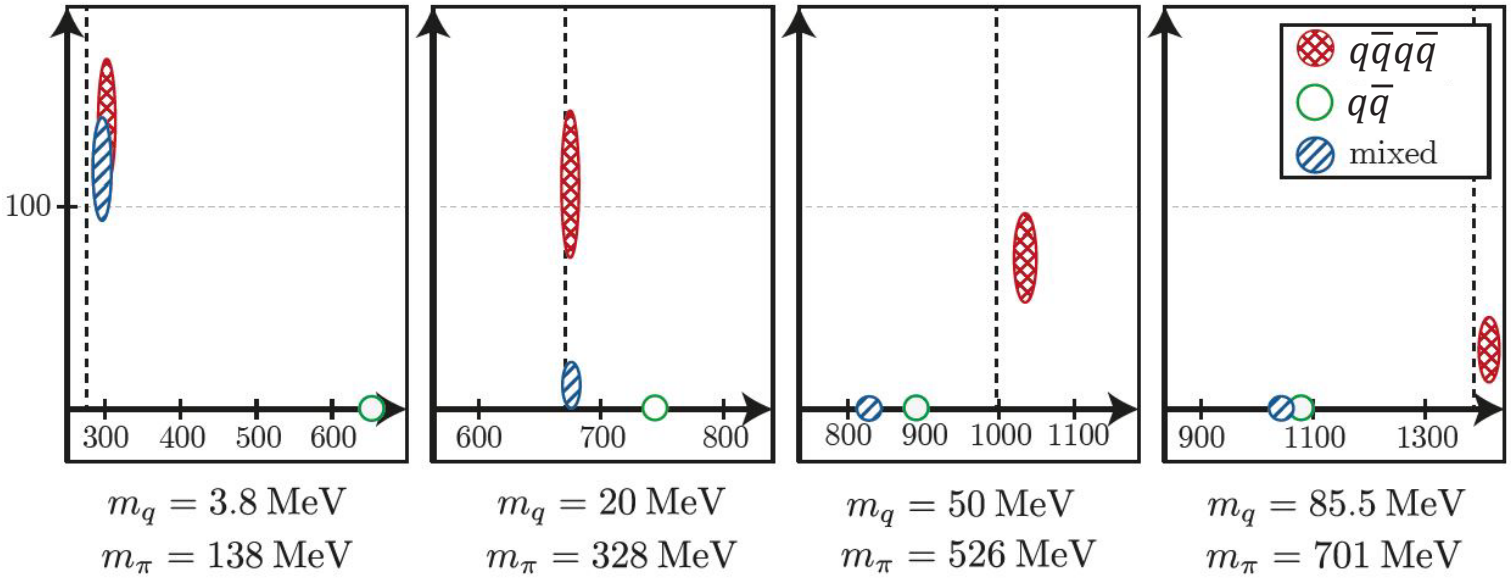
Santowsky, GE, Fischer, Wallbott, Williams, PRD 102 (2020)

[MeV]	ground state mass	first excitation
$\pi\pi$	416 ± 26	970 ± 130
$\pi\pi + 0^+0^+$	416 ± 26	970 ± 130
$q\bar{q}$	667 ± 2	1036 ± 8
$\pi\pi + q\bar{q}$	472 ± 22	1080 ± 280
$\pi\pi + 0^+0^+ + q\bar{q}$	456 ± 24	1110 ± 110

Four-quark states

Four-quark vs. $q\bar{q}$ dominance

Santowsky, Fischer, PRD 105 (2022)



Light quarks:
$\pi\pi$ dominant

Strange quarks:
$q\bar{q}$ dominant

Towards ab-initio

Gluon propagator:

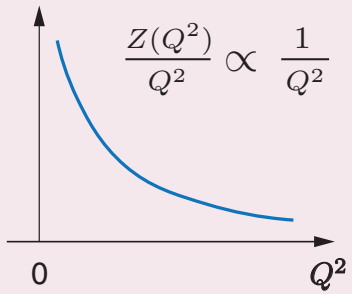
transverse dressing

longitudinal dressing = 1

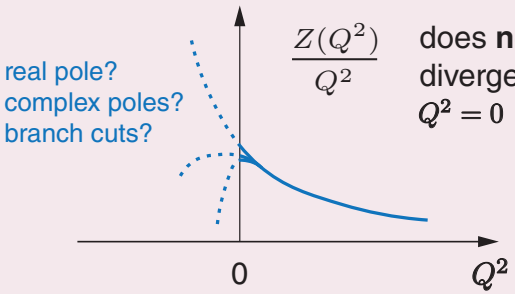


$$D^{\mu\nu}(Q) = \frac{Z(Q^2)}{Q^2}\left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) + \xi\,\frac{L(Q^2)}{Q^2}\,\frac{Q^\mu Q^\nu}{Q^2}$$

- **Perturbation theory:** Massless gluon pole



- **Nonperturbative calculations:** Massless pole disappears!



Family of **"decoupling"** solutions, also seen in lattice QCD

Cucchieri, Maas, Mendes, PRD 77 (2008)
Boucaud et al., JHEP 06 (2008)
Bogolubsky et al., PLB 676 (2009)
Fischer, Maas, Pawlowski, Ann. Phys. 324 (2009)
Duarte, Oliveira, Silva, PRD 94 (2016)
Aguilar et al., EPJ C 80 (2020)

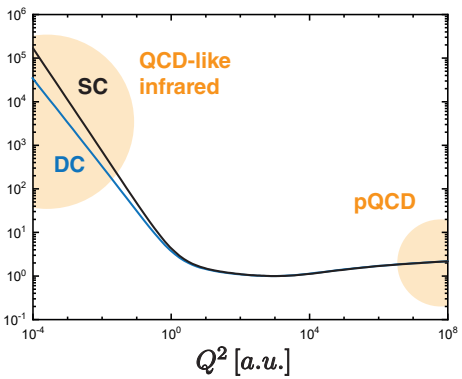
Endpoint is **"scaling"** solution, confinement manifest

Lerche, Smekal, PRD 65 (2002)
Fischer, Alkofer, PLB 536 (2002)
Alkofer, Fischer, Llanes-Estrada, MPLA 23 (2008)

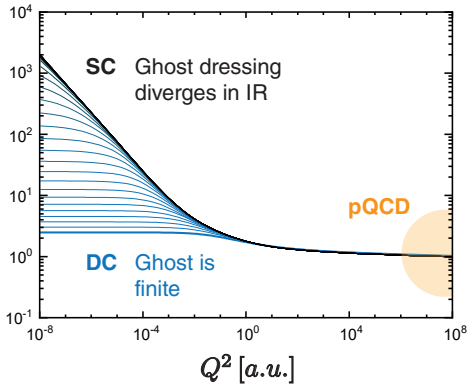
All solutions show **gluon mass gap**

$$\lim_{r\to\infty} \int \frac{d^3Q}{(2\pi)^3}\,\frac{Z(Q^2)}{Q^2}\,e^{i\,\boldsymbol{x}\cdot\boldsymbol{Q}} \propto e^{-m_{\rm gap}\,r}$$

Gluon $1/Z(Q^2)$



Ghost $G(Q^2)$



Gluon mass gap

Gluon propagator:



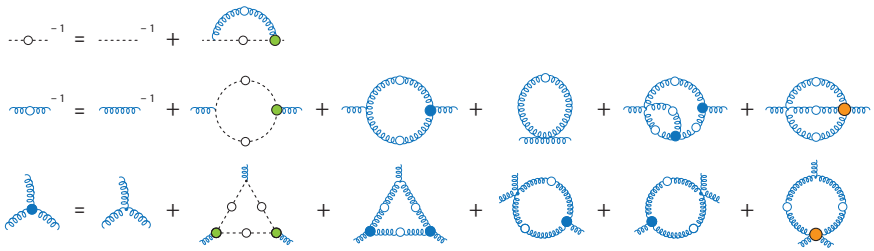
transverse dressing

longitudinal dressing = 1

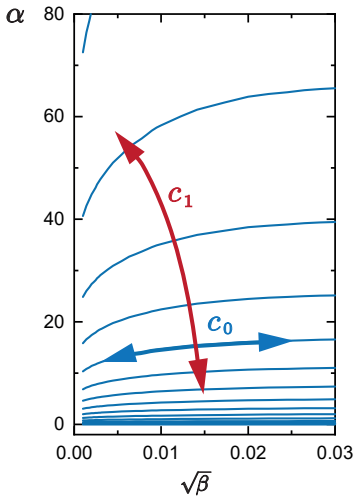
$$D^{\mu\nu}(Q) = \frac{Z(Q^2)}{Q^2}\left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) + \xi \frac{L(Q^2)}{Q^2}\frac{Q^\mu Q^\nu}{Q^2}$$

Coupled **Yang-Mills DSEs**

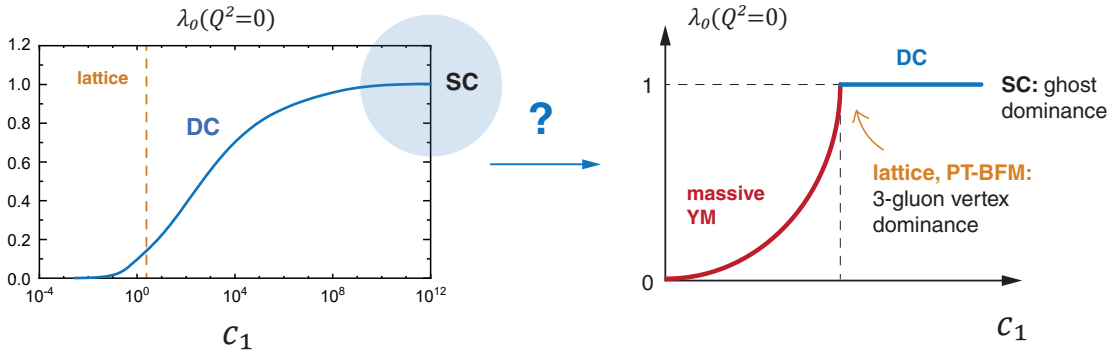
Huber, PRD 101 (2020), GE, Pawlowski, Silva, PRD 104 (2021)



- SC & DC solutions distinguished by c_1 (combination of coupling α and mass parameter β)
- Scaling solution: $c_1 \to \infty$



Gauge consistency (L = 1) satisfied only by **SC solution**



Possible scenario for consistency of DC solutions:

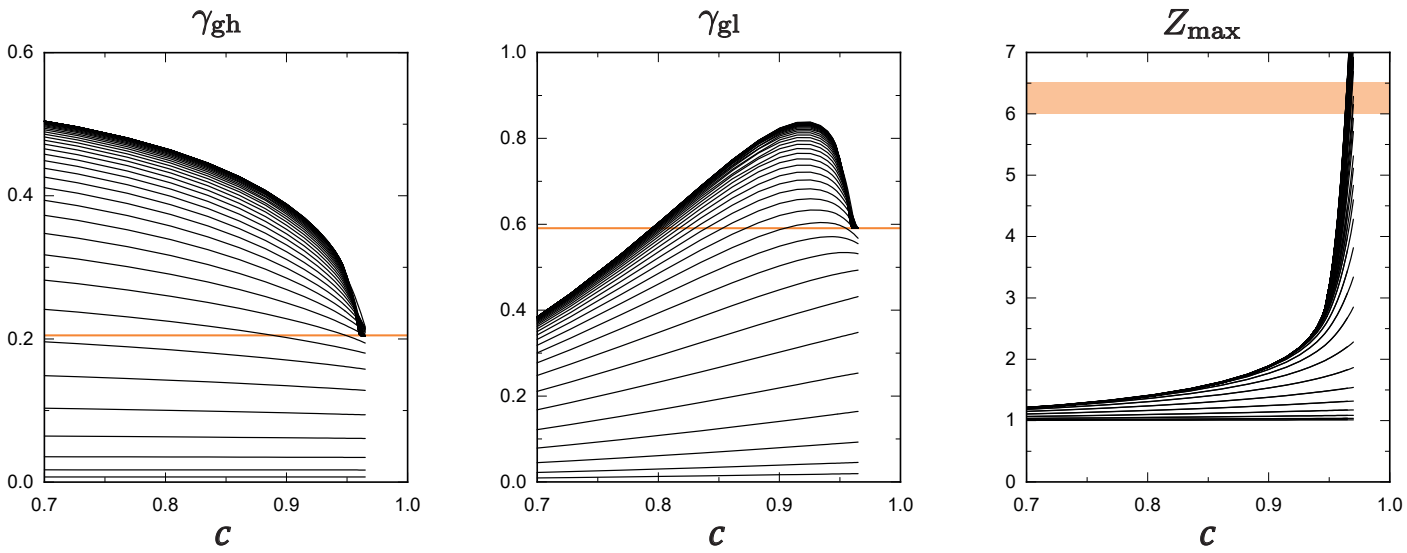
Then all massless (SC, DC) solutions would be physically equivalent; distinguished from massive solutions

Truncation error

- Set $Z_{3g} \to c\, Z_{3g}$ ... quantifies deviation from STI
  (without truncation: $c = 1$)

- Anomalous dimensions reproduced for
  1 c ~ 0.4
  2 c ~ 0.9
  3 c ~ 0.96
  $\Rightarrow$ identifies "physical point" for each truncation

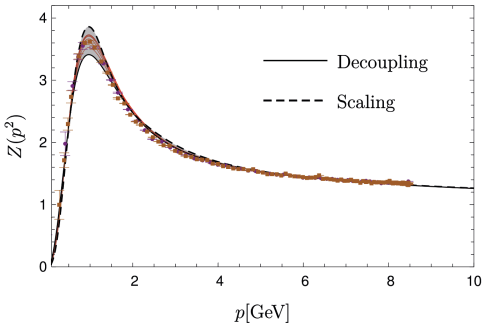
  GE, Pawlowski, Silva, PRD 104 (2021)



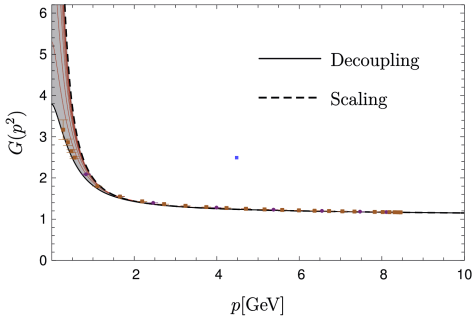
Towards precision

Markus Huber, ACHT 2025; Phys. Rept. 879 (2020) & PRD 101 (2020)

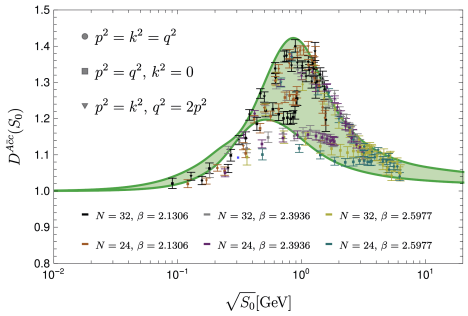
- Gluon propagator



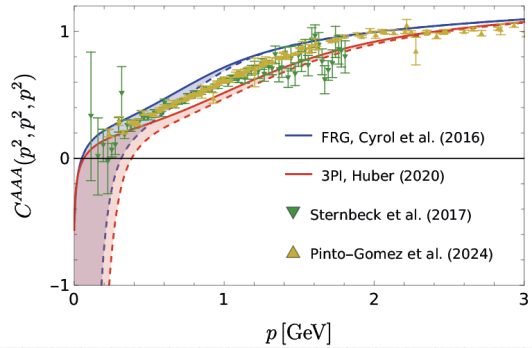
- Ghost propagator

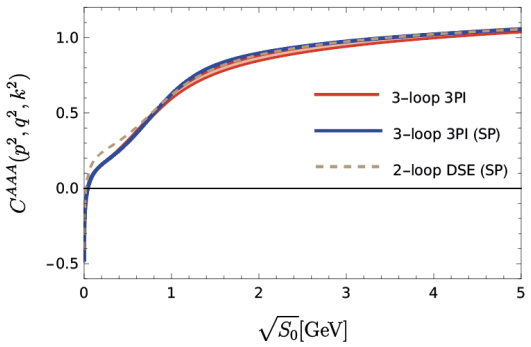


- Ghost-gluon vertex



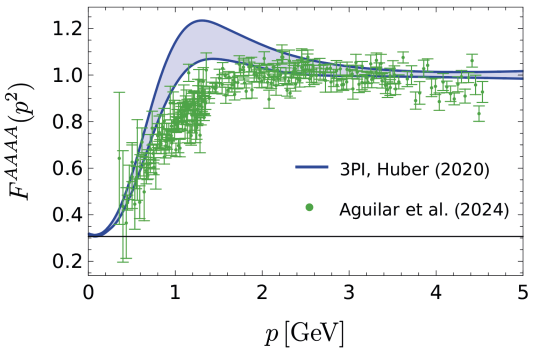
- Three-gluon vertex: lattice vs. FRG vs. DSE





Lattice: Pinto-Gómez, de Soto, Rodriguez-Quintero, PLB 838 (2023), Sternbeck et al., Pos LATTICE 2016; **FRG:** Cyrol et al., PRD 94 (2016); **DSE:** Huber, PRD 101 (2020), Huber, Fischer, Sanchis-Alepuz, 2503.03821

- Four-gluon vertex



Lattice: Aguilar et al., PLB 858 (2024)
DSE: Huber, PRD 101 (2020)