Electromagnetic Form Factors of the Nucleon at Large Momentum

Sergey Syritsyn (Stony Brook University)
with M.Engelhardt, J.Green, S.Krieg, S.Meinel,
A.Pochinsky, J.Negele, M.Rodekamp
(Lattice Hadron Physics collaboration)

Theoretical Motivations
and Empirical Evidence for Diquark Correlations in Hadrons"
ECT*, Trento, Oct 19-23, 2026





National Energy Research
Scientific Computing Center





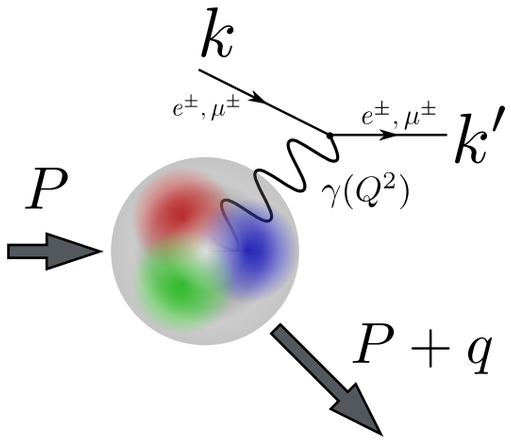




Outline

- Large-Q^2 Nucleon vector form factors
  - *Motivation*
  - *Methodology & challenges on a lattice*
- Preview of results
  - *Form factors and ratios*
  - *Light-Flavor contributions*
- Examining systematic effects
  - *G_E/G_M ratio*
  - *Disconnected diagrams*
- Summary & Outlook

Nucleon Elastic E&M Form Factors



$$Q^2 = -(k-k')^2 > 0$$

Elastic e–p amplitude

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[\overbrace{F_1(Q^2)}^{\text{(Dirac)}}\gamma^\mu + \overbrace{F_2(Q^2)}^{\text{(Pauli)}}\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

Sachs Electric $\quad G_E(Q^2) = F_1(Q^2) - \dfrac{Q^2}{4M^2}F_2(Q^2)$

Magnetic $\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

Elastic e–p cross-section

- $G_{E,M}$ from ϵ-dep. at fixed $\tau(Q^2)$ ("***Rosenbluth separation***")
- dominated by G_M at large Q^2
- 2γ corrections at $Q^2 \gtrsim 1$ GeV^2

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\rm Mott}}{1+\tau}\left[G_E^2 + \frac{\tau}{\epsilon}G_M^2\right]$$

$$\tau = \frac{Q^2}{4M_N^2} \qquad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

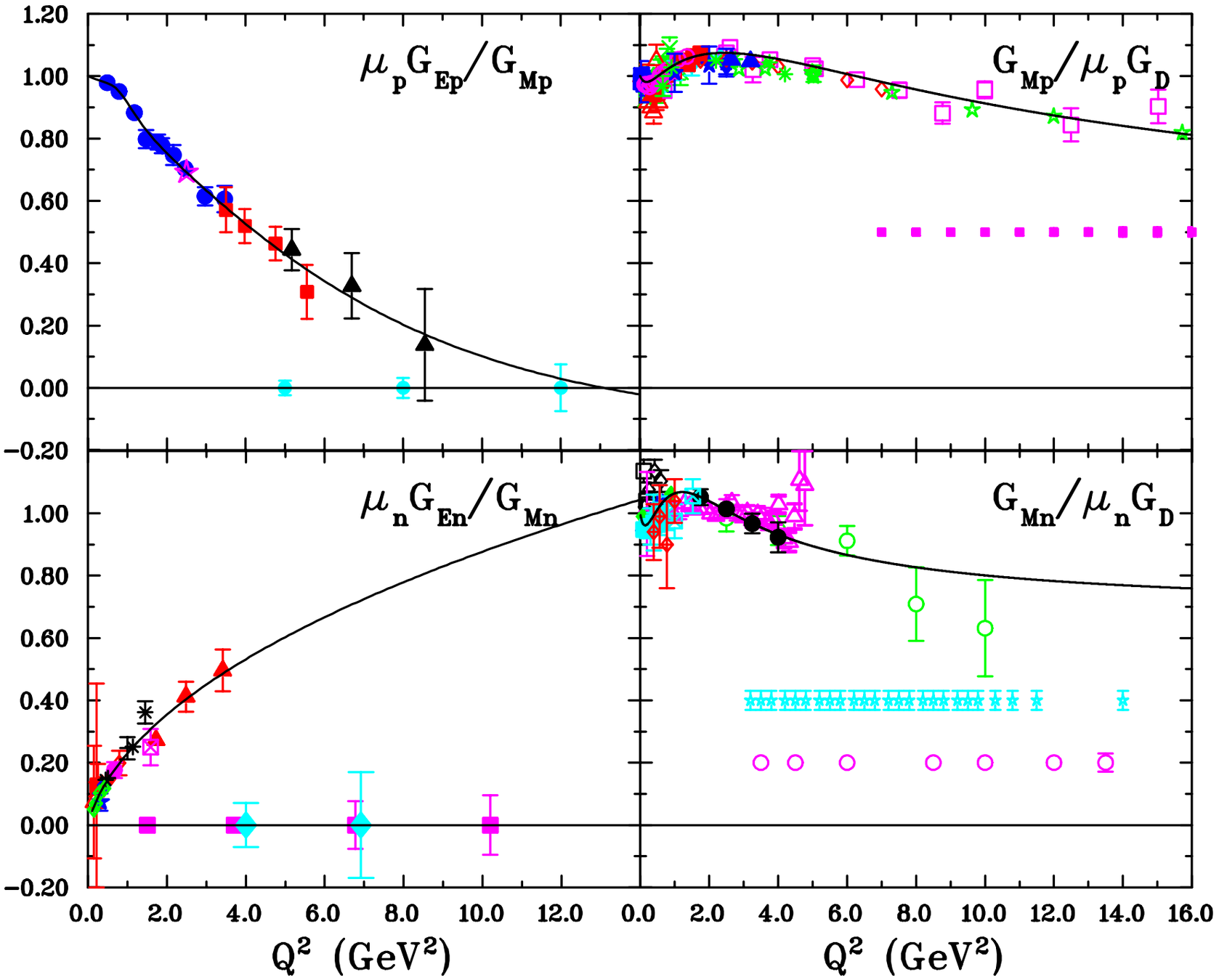
Polarization transfer: polarized e–beam + detect polarization of recoil nucleon

(alt.: transverse asymmetry on pol. target)

- G_E/G_M *ratio* (only small radiative corrections)

$$P_t/P_l \propto G_E/G_M$$

Recent Experiments @JLab : Projections



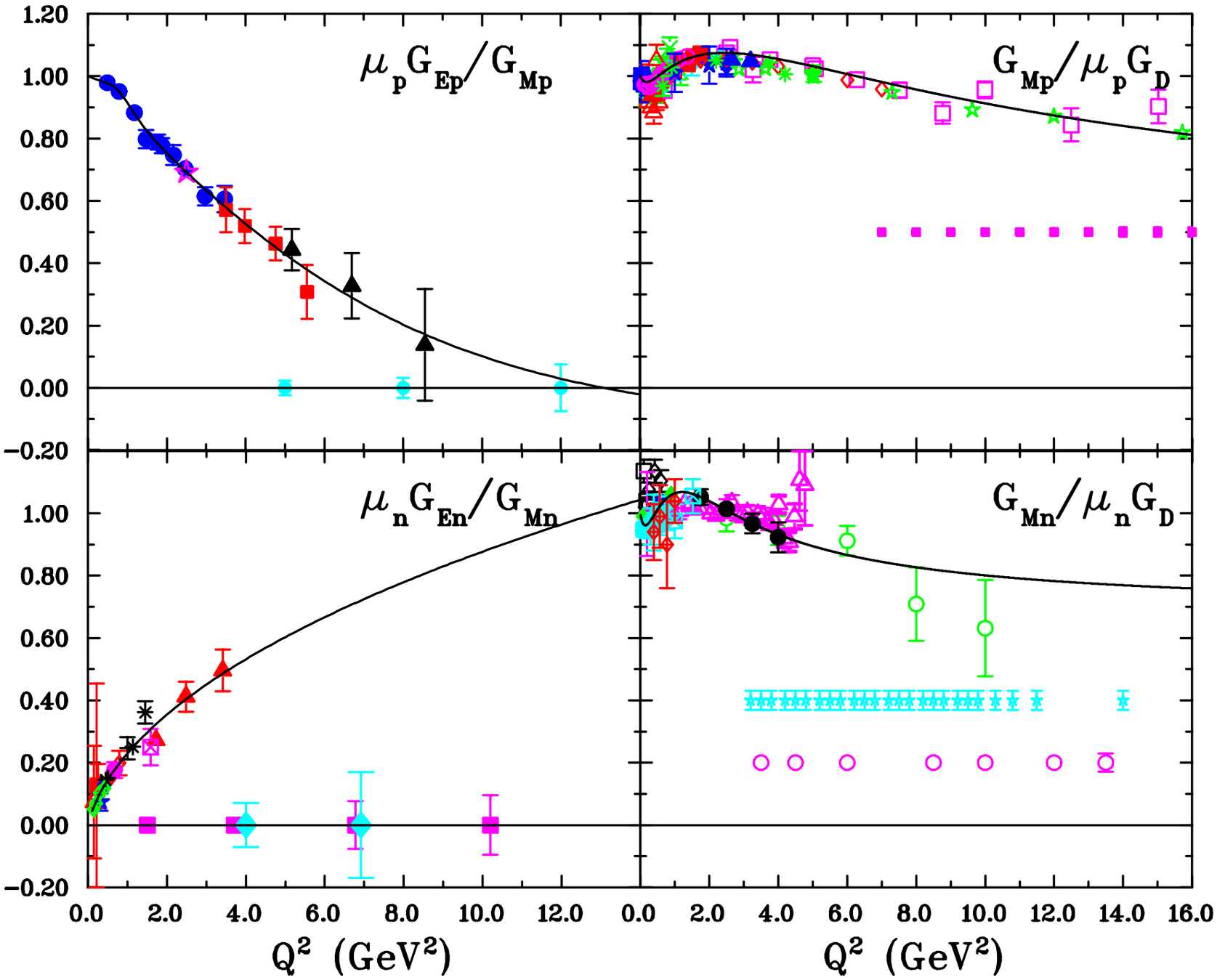
Projected new precision on proton & neutron form factors
[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]



Experiments at JLab@12GeV

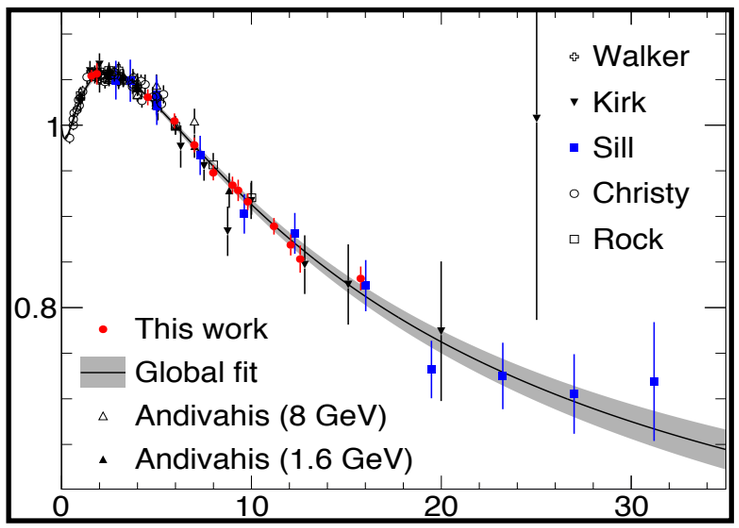
- Hall A (HRS, SBS):
  G_{Mp} @ $Q^2 \lesssim 17.5$ GeV^2
  G_{Ep}/G_{Mp} @ $Q^2 \lesssim 15$ GeV^2 ;
  G_{Mn} @ $Q^2 \lesssim 18$ GeV^2
  G_{En}/G_{Mn} @ $Q^2 \lesssim 10.2$ GeV^2 ;
- Hall B (CLAS12):
  G_{Mn} @ $Q^2 \lesssim 14$ GeV^2
- Hall C :
  G_{En}/G_{Mn} @ $Q^2 \lesssim 6.9$ GeV^2

Recent Experiments: Projections + G_{Mp} Result



Projected new precision on proton & neutron form factors
[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]

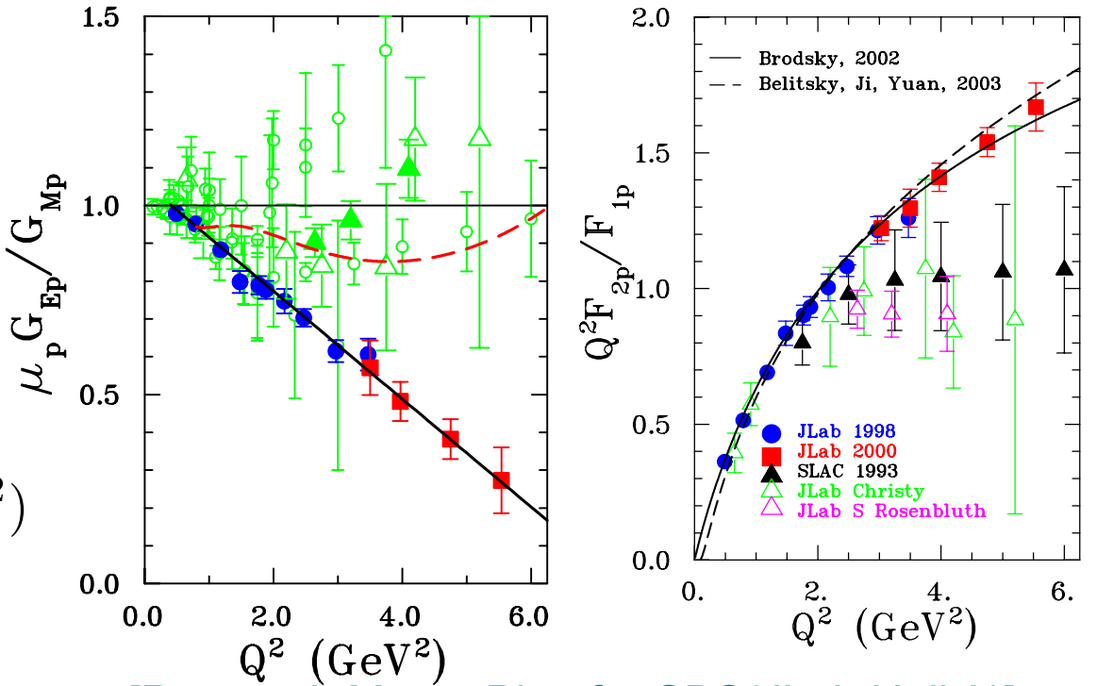




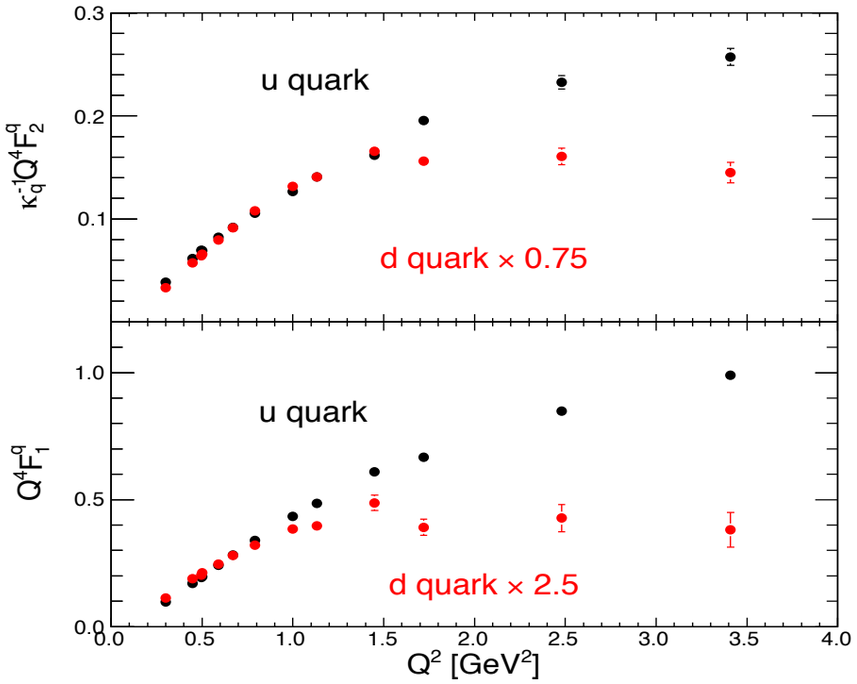
New G_{Mp} results (JLab Hall A)
[Christy et al, PRL'22]

Why Large-Q^2 Nucleon Form Factors?

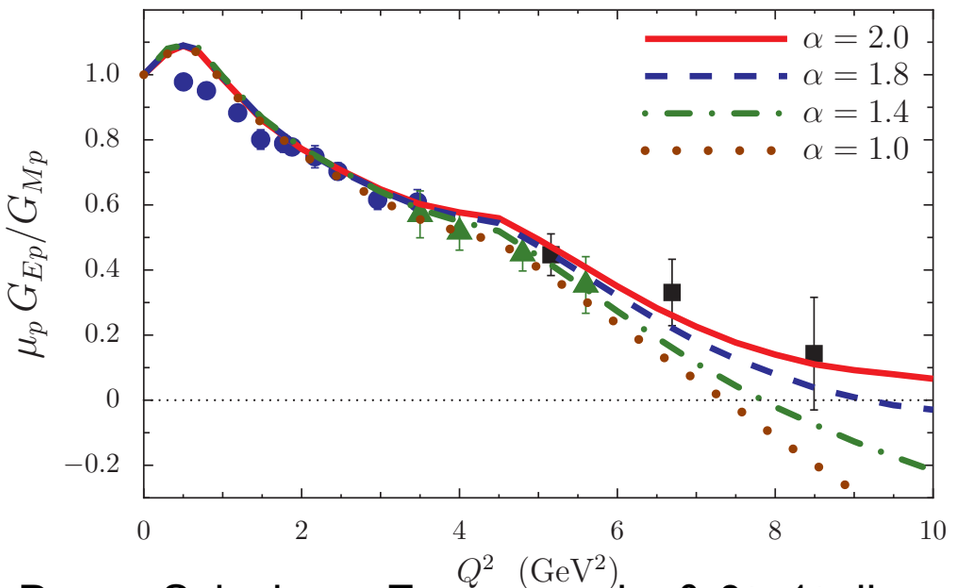
- *Rosenbluth / Pol.transfer discrepancy in G_E/G_M ?*
  Input from lattice QCD may help
- *Role of diquark correlations in elastic scattering ?*
  Neutron & proton G_E/G_M at/above Q^2 = 8 GeV^2
- *Scale of transition to perturbative QCD ?*
  (F_2/F_1) scaling at large Q^2 : $Q^2F_{2p}/F_{1p} \overset{?}{\propto} \log^2(Q^2/\Lambda^2)$
- *What are contributions from u and d flavors?*
  Proton and neutron data needed in wide Q^2 range



[Research Mgmt. Plan for SBS(JLab Hall A)]



[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]



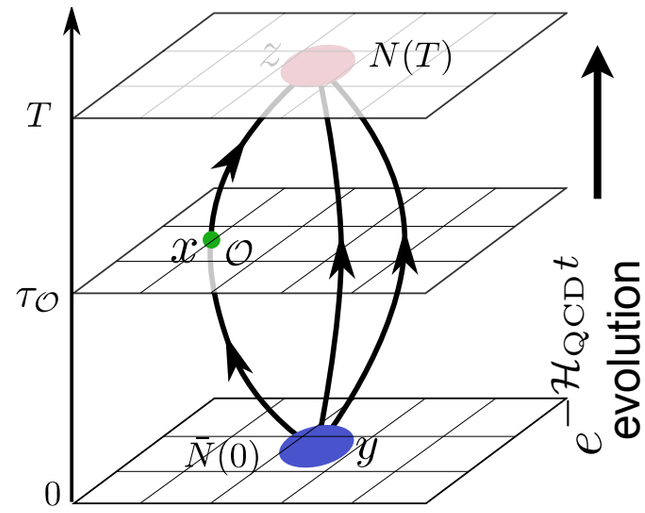
Dyson-Schwinger Eqns : quarks & 0$^+$, 1$^+$ diquarks
($\alpha \approx$ rate of transition const.quarks → pQCD with Q^2)
[Cloet, Roberts, Prog.Part.Nucl.Phys 77:1 (2014)]

Nucleon Structure from Lattice QCD (in a Nutshell)

- Compute nucleon correlation functions

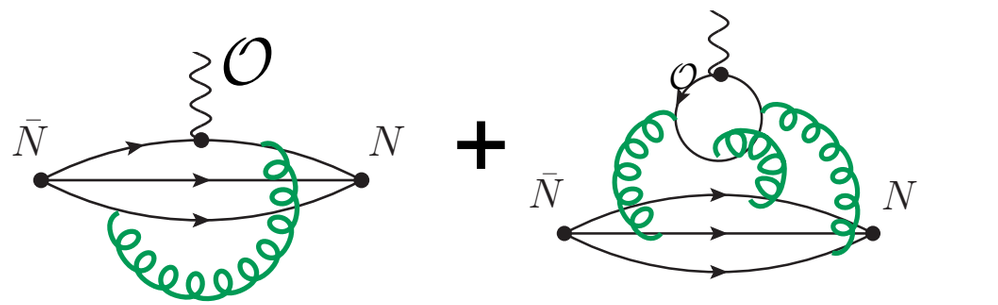
  Hadron Spectrum:

$$\langle N(\vec{p},T)\,\bar{N}(0)\rangle = \sum_{\vec{y}} \langle N(\vec{y},T)\,\bar{N}(0)\rangle$$
$$\sim |Z_0|^2 e^{-E_0 T}\left[1 + e^{-\Delta E_{10} T} + \ldots\right]$$



  Hadron Matrix Elements:

$$\langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle =$$



quark-connected **quark-disconnected**

Quark lines = $(\not{D} + m)^{-1} \cdot \psi$

- Extract ground state matrix elements

$$\langle N(T)\mathcal{O}(\tau)N(0)\rangle = \sum_{n,m} Z_m e^{-E_n(T-\tau)}\langle n|\mathcal{O}|m\rangle e^{-E_m\tau} Z_n^*$$
$$\xrightarrow{T\to\infty} Z_0 Z_{0'}^* e^{-E_0\tau - E_{0'}(T-\tau)}\left[\langle 0'|\mathcal{O}|0\rangle + \underbrace{O\left(e^{-\Delta E\{T,\tau,(T-\tau)\}}\right)}_{\text{excited states}}\right]$$

Ground state form factors

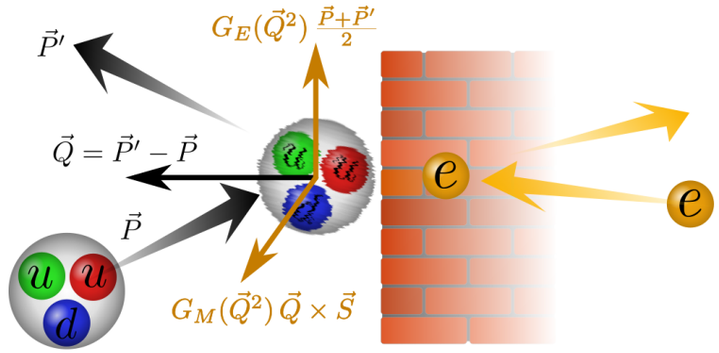
$$[\bar{u}'\gamma^\mu u]F_1 + [\bar{u}'\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}u]F_2$$

Fit and discard

Systematic uncertainties
- discretization effects
- unphysical-heavy pion mass
- finite volume
- excited state contributions
- quark-disconnected diagrams

Challenges at Large Q^2

- Excited-states gaps shrink
  - $E_1 - E_0 = \sqrt{M_1^2 + \vec{p}^2} - \sqrt{M_2^2 + \vec{p}^2} < M_1 - M_0$
  - N(~1500): $p_N \to 1.5$ GeV $\Rightarrow \Delta E = 500 \to 300$ MeV



- Stochastic noise grows faster with T [Lepage'89]:

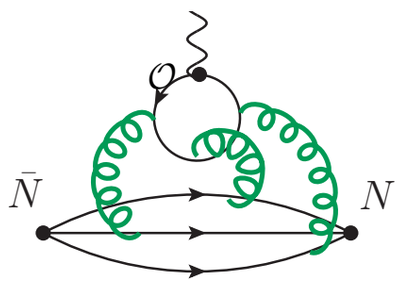
$$\begin{aligned} &\text{Signal} && \langle N(T)\bar{N}(0)\rangle && \sim e^{-E_N T} \\ &\text{Noise} && \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 && \sim e^{-3m_\pi T} \\ &\text{Signal/Noise} && && \sim e^{-(E_N - \frac{3}{2}m_\pi)T} \end{aligned}$$

SNR decay ~ O(10^{-4}) at T=1 fm/c
(phys.quarks, $Q^2 \approx 12$ GeV^2)

- Discretization effects:
  O(a) Correction to current operator

$$(V_\mu)_I = [\bar{q}\gamma_\mu q] + c_V\, a\, \underbrace{\partial_\nu [\bar{q} i\sigma_{\mu\nu} q]}_{\propto Q}$$

- Quark-disconnected contributions:
  negligible (≲1%) at $Q^2 \le 1$ GeV^2, unknown at large Q^2



- No reliable EFT/ChPT for mπ-, lattice-volume extrapolation at large-$p_{Nucleon}$

Many of the same challenges as for parton physics on a lattice
(LaMET , Quasi-/Pseudo- PDFs/DAs/TMDs)

Present QCD Calculation Parameters

QCD on lattice with $N_F = 2{+}1$ O(a²) dynamic quarks

Many thanks to [JLab , W&M , LANL , MIT] groups

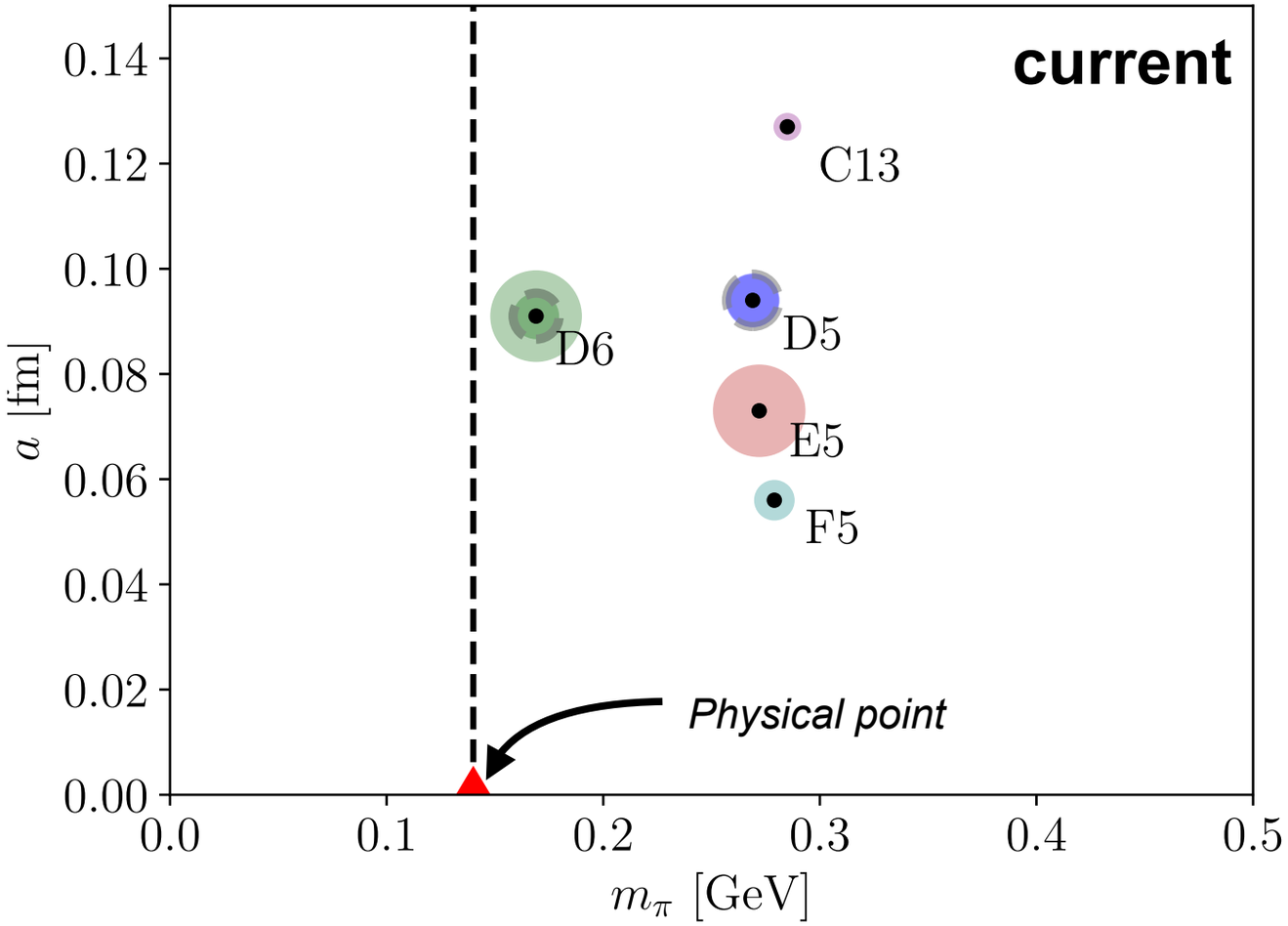
- Lattice spacing $a \approx 0.056 \div 0.091$ fm
- Nf=2+1 flavors ($SU(2)_f$ -symmetric + strange)
- Pion mass $m\pi = 170 \div 280$ MeV
- Physical volume $L \gtrsim 3.7\ (m\pi)^{-1}$
- Euclidean time $t_{sep} = 0.5 \div 1.1$ fm
  noise grows rapidly beyond 1fm

2022/25:

- Large Statistics (≳250k) on D6 ($m\pi$ =170 MeV),
  E5 ($m\pi$ =280 MeV, a=0.073 fm)
- Disconnected contractions on D5, D6

2026:

- initial Statistics ≳47k on F5,
  ($m\pi$ =280 MeV, a=0.056 fm)



Present QCD Calculation Parameters

QCD on lattice with $N_F = 2+1$ O(a²) dynamic quarks

Many thanks to [JLab / W&M / LANL / MIT]

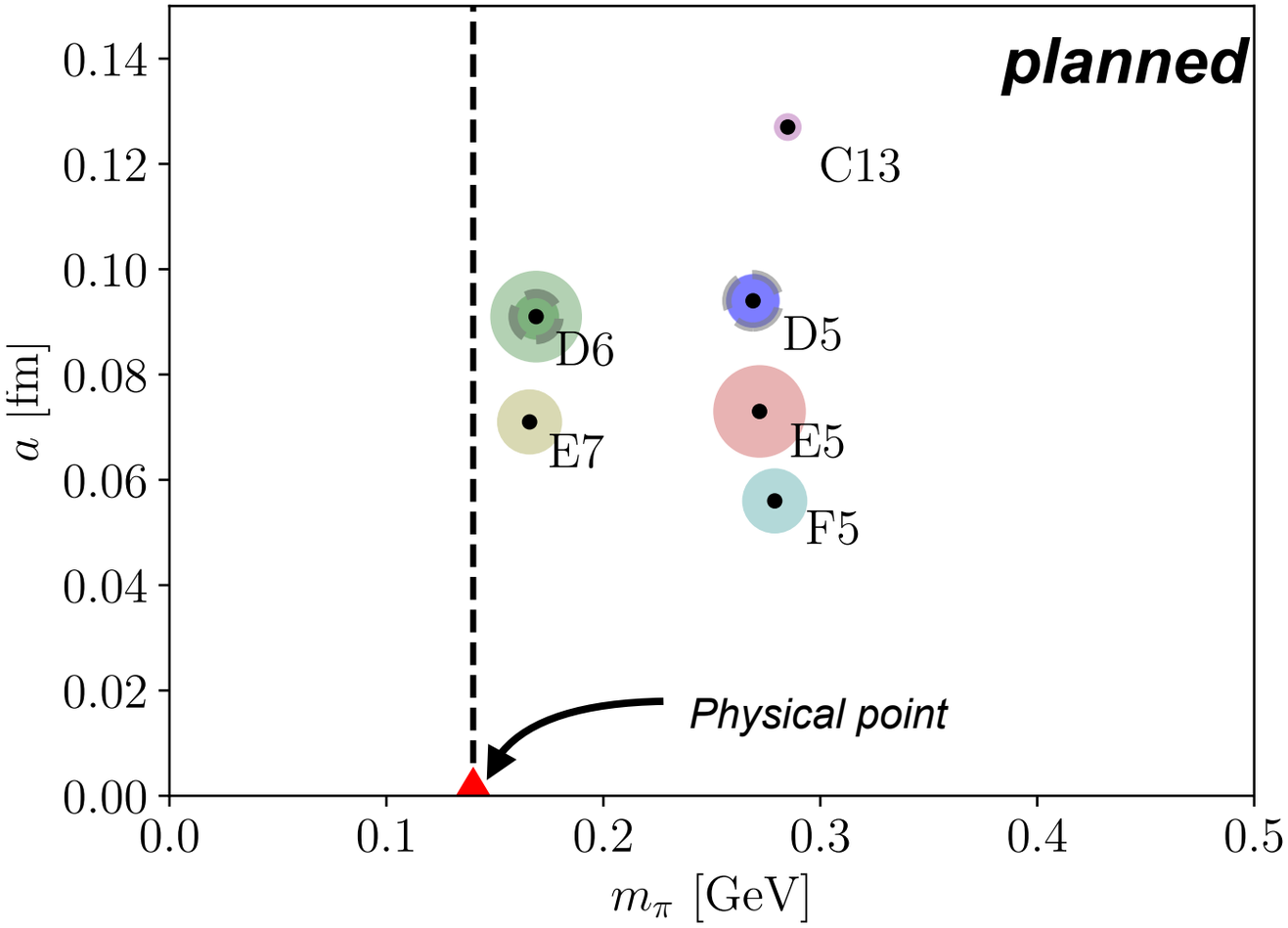
- Lattice spacing $a \approx 0.056 \div 0.091$ fm
- SU(2)$_f$ -symmetric + strange quarks
- Pion mass $m\pi = 170 \div 280$ MeV
- Physical volume $L \gtrsim 3.7\ (m\pi)^{-1}$
- Euclidean time $t_{sep} = 0.5 \div 1.1$ fm

2022/25:

- Large Statistics (≈250k) on D6 ($m\pi$ =170 MeV), E5 ($m\pi$ =280 MeV, a=0.073 fm)
- Disconnected contractions on D5, D6

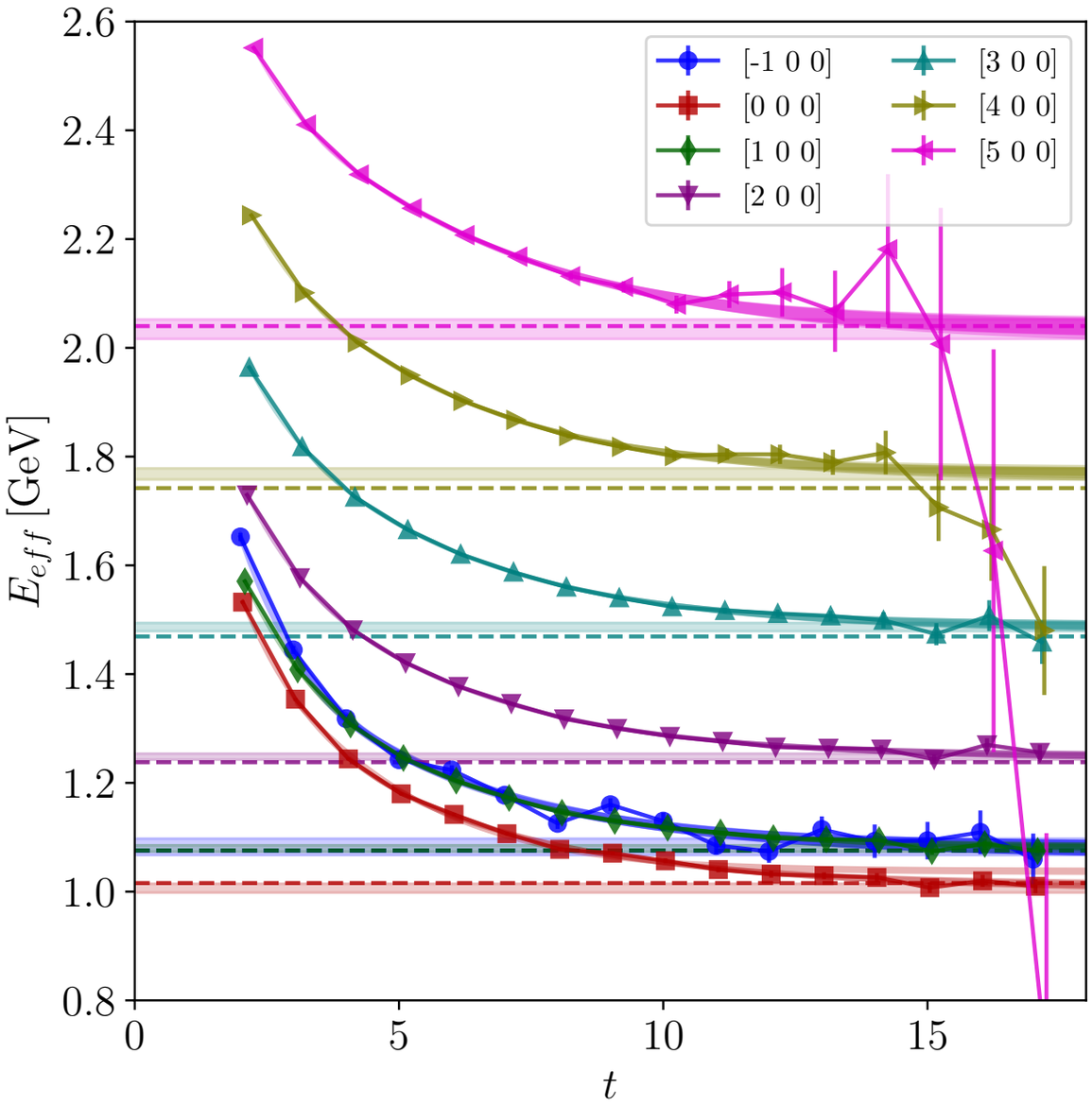
2026:

- initial Statistics ≈47k on F5 ($m\pi$ =280 MeV, a=0.056 fm)



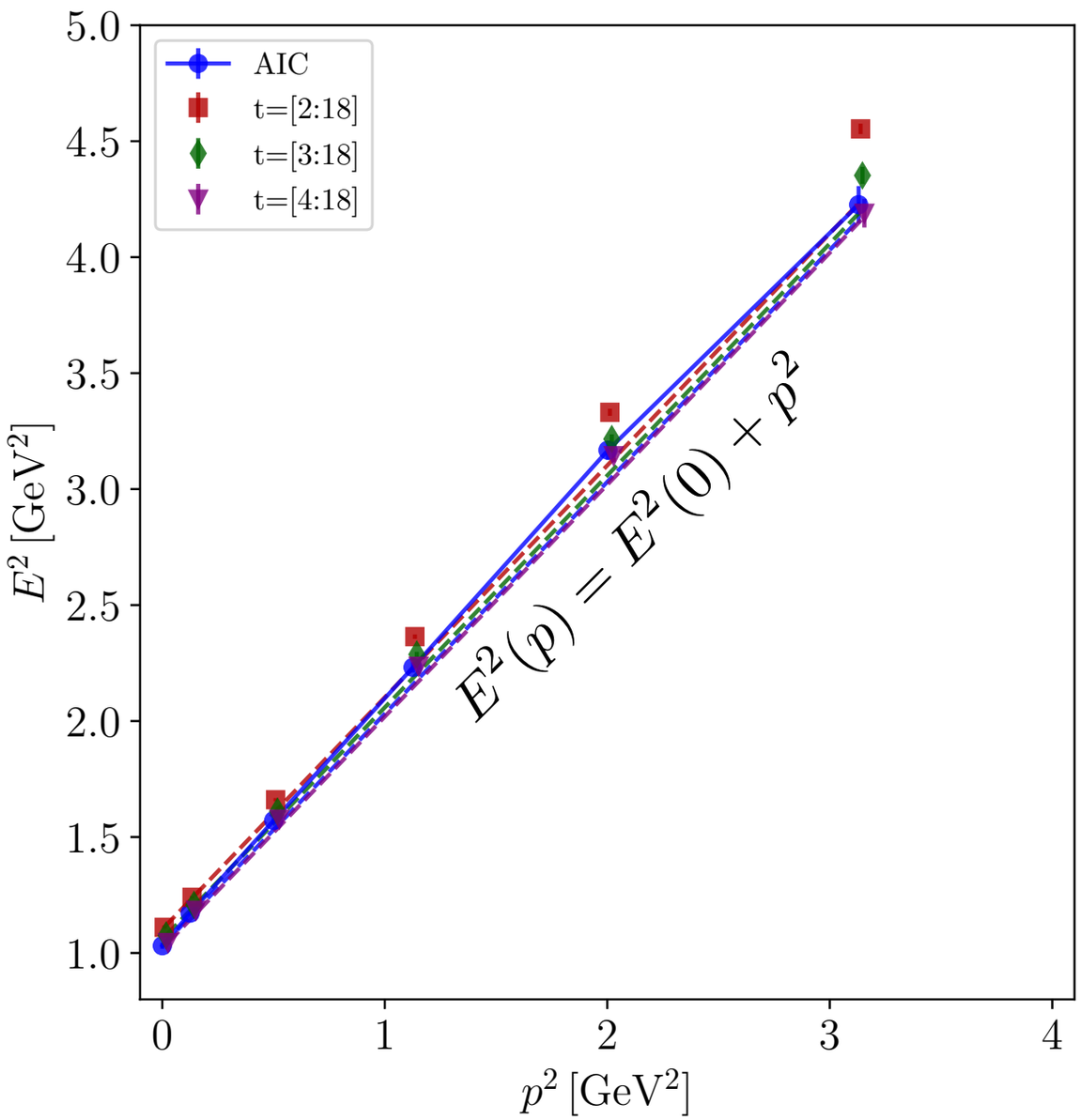
Lattice Nucleon Energy & Dispersion Relation (E5)

- E5 : $m\pi = 272$ MeV , spacing $a = 0.073$ fm , 266k MC samples



- 2-state fits compared to Effective energy

$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$



- Lattice energies vs. continuum dispersion relation (dashed)

Analysis Minutiae

- Filter outliers: large deviation from median
  (~2% at a=0.09 fm, ~0.15% at a=0.07 fm)
  - *suppress noise in covariance estimates*

  [A.Agadjanov et al, Phys.Rev.Lett. 131 (2023) 261902]

$$|y - y_{median}| \geq 25 \cdot \underbrace{\Delta y}_{\text{(median abs. deviation)}}$$

energies and ground-state amplitudes from ⟨*N* $\bar{N}$⟩:

$$\langle N(t)\,\bar{N}(0)\rangle \sim c_0 e^{-E_0 t} + c_1 e^{-(E_0+\Delta E_1)t} + \ldots$$

- Varying fit windows

  Averaged wiith AIC

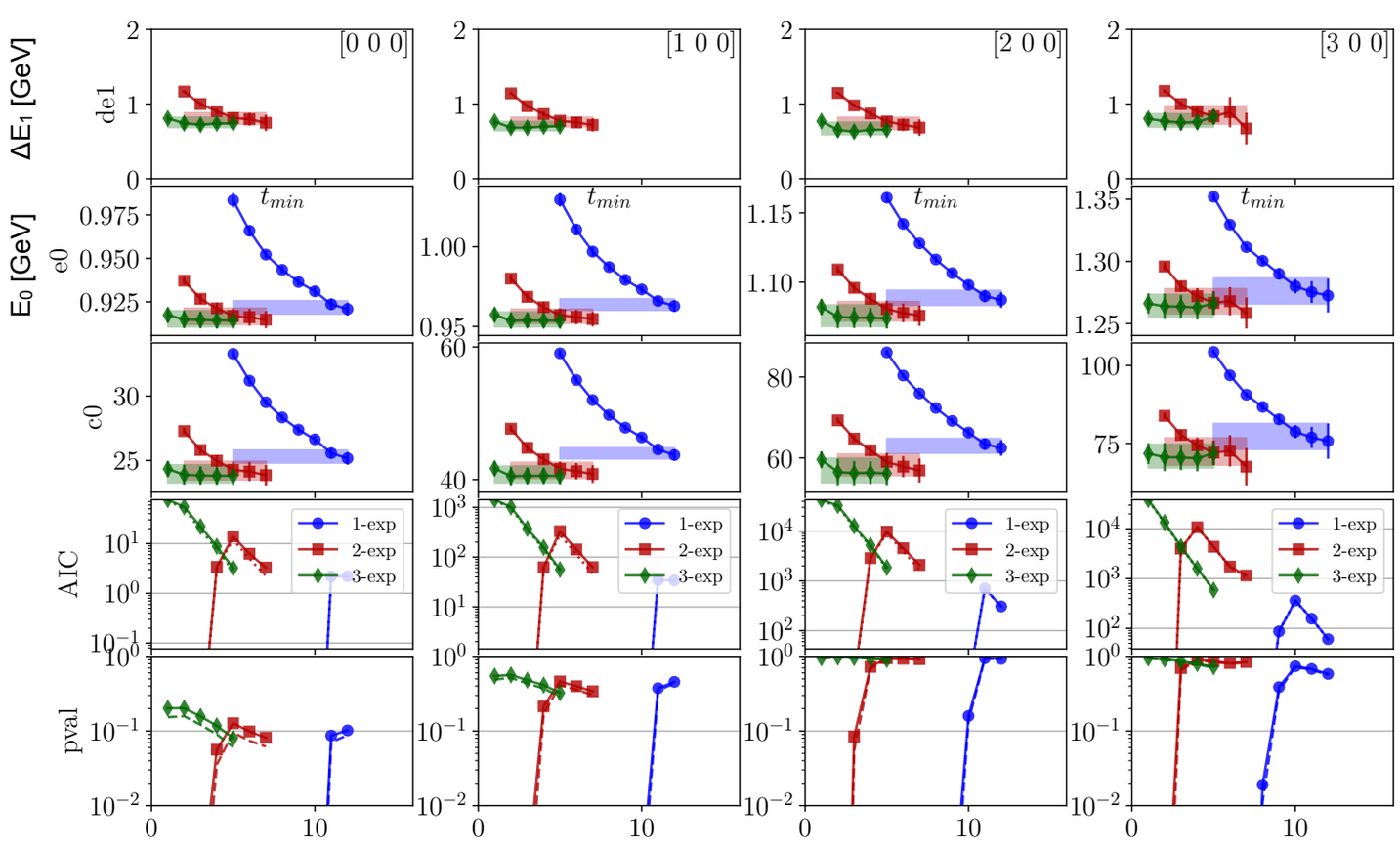
  [W.Jay, E.Neil PRD 103:114502(2021); E.Neil, J.W.Sitison, 109:014510(2024)].

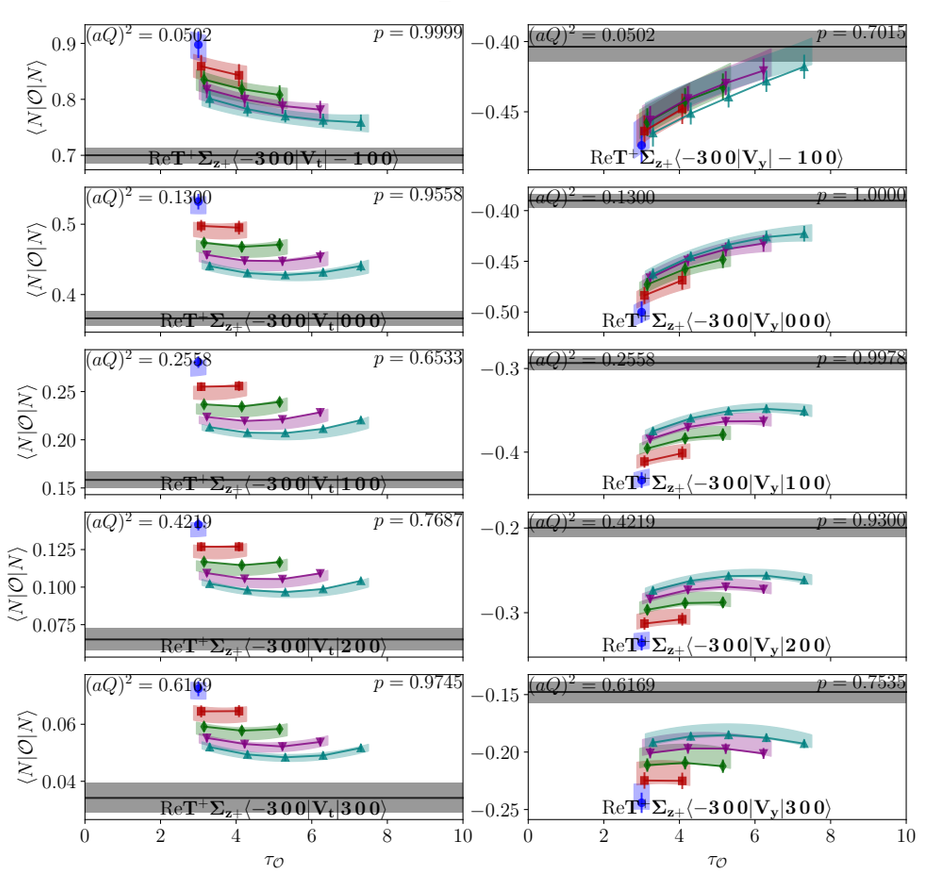
$$w_i \propto \exp\left(-\chi_i^2 + N_i^{\text{dof}}\right)$$

current matrix elements from ⟨*N* J $\bar{N}$⟩ :

$$\begin{cases} \langle J_t \rangle & \propto G_E \\ \langle J_{\vec{q}\times\vec{S}} \rangle & \propto G_M \end{cases}$$

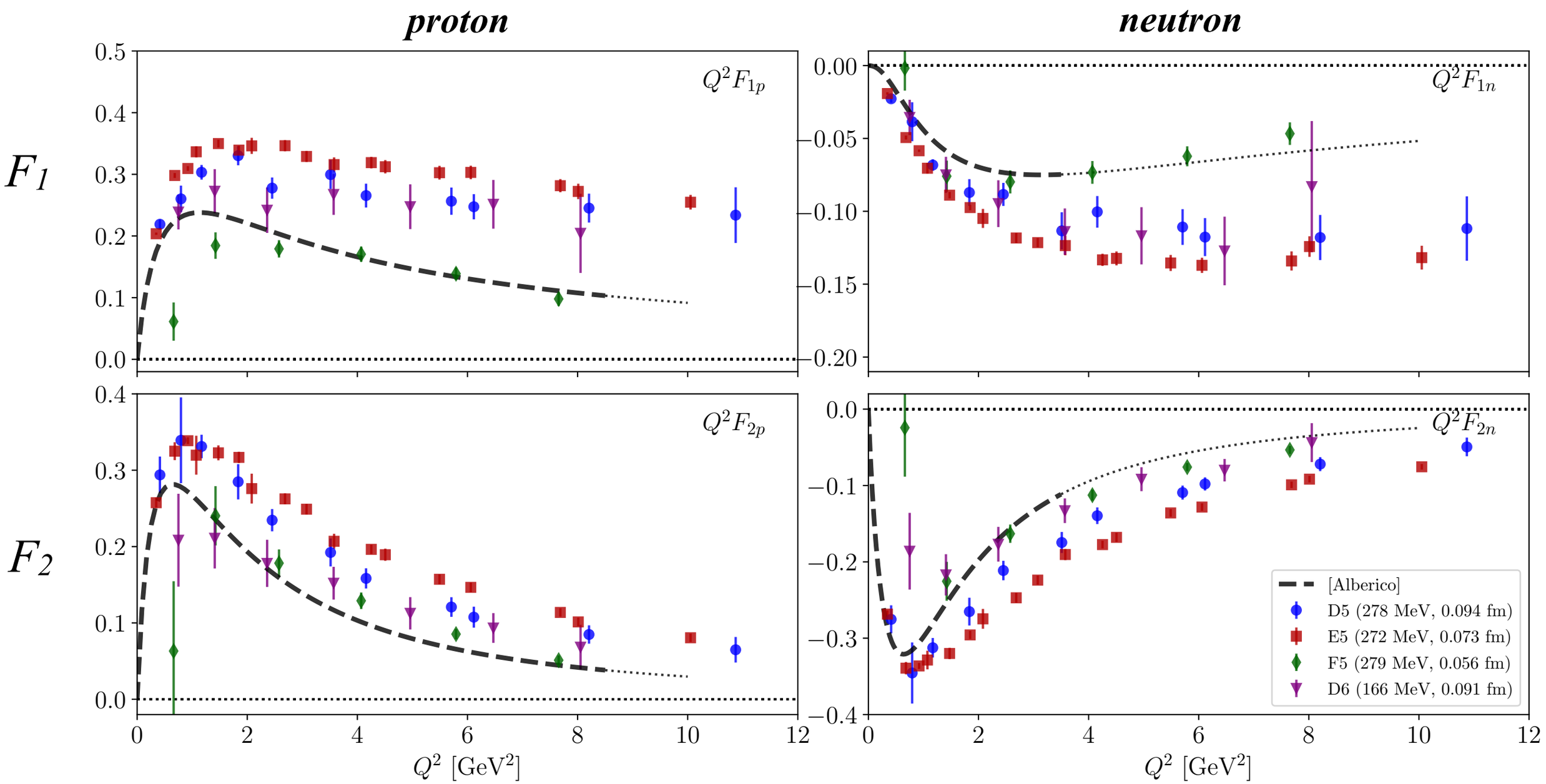
$$\langle N'|\,J(\tau)\,|N\rangle \sim \mathcal{A}_{00}\left[1 + O(e^{-\Delta E_1^{(\prime)} t} + \ldots)_{\text{exc}}\right]$$





Nucleon Form Factors: Dirac and Pauli

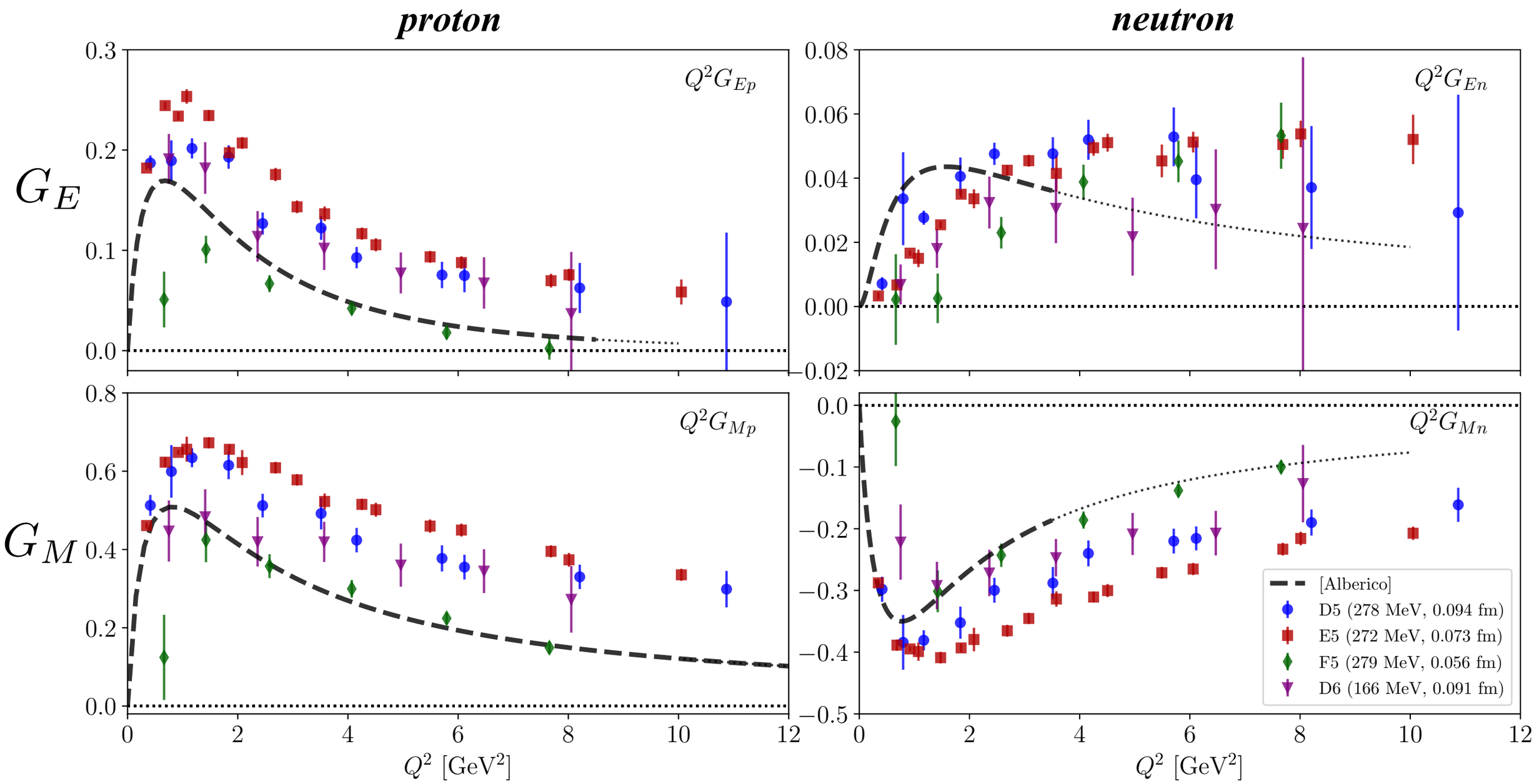
- Comparison of 4 ensembles (**D5** : 86k, **E5** : 266k, **F5** : 47k, **D6** : 261k)
- Lattice error bars: stat. + sys[high exc. states]
- *(dashed line)* Phenomenological fit [Alberico et al, PRC79:065204 (2008)]



- *No disconnected diagrams*

Nucleon Form Factors: Sachs Electric and Magn.

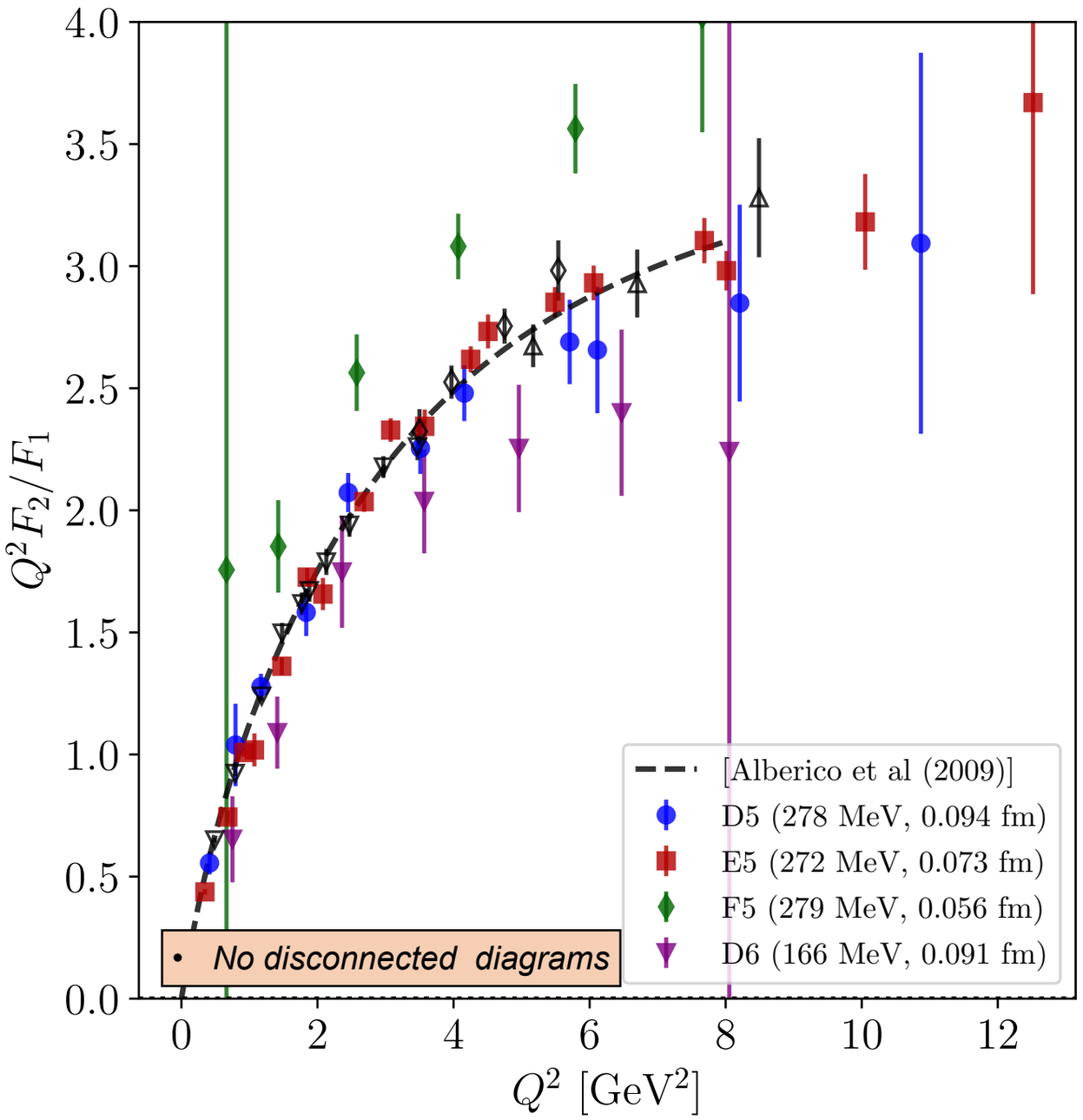
- Comparison of 4 ensembles (**D5** : 86k, **E5** : 266k, **F5** : 47k, **D6** : 261k)
- Lattice error bars: stat. + sys[high exc. states]
- *(dashed line)* Phenomenological fit [Alberico et al, PRC79:065204 (2008)]



- *No disconnected diagrams*

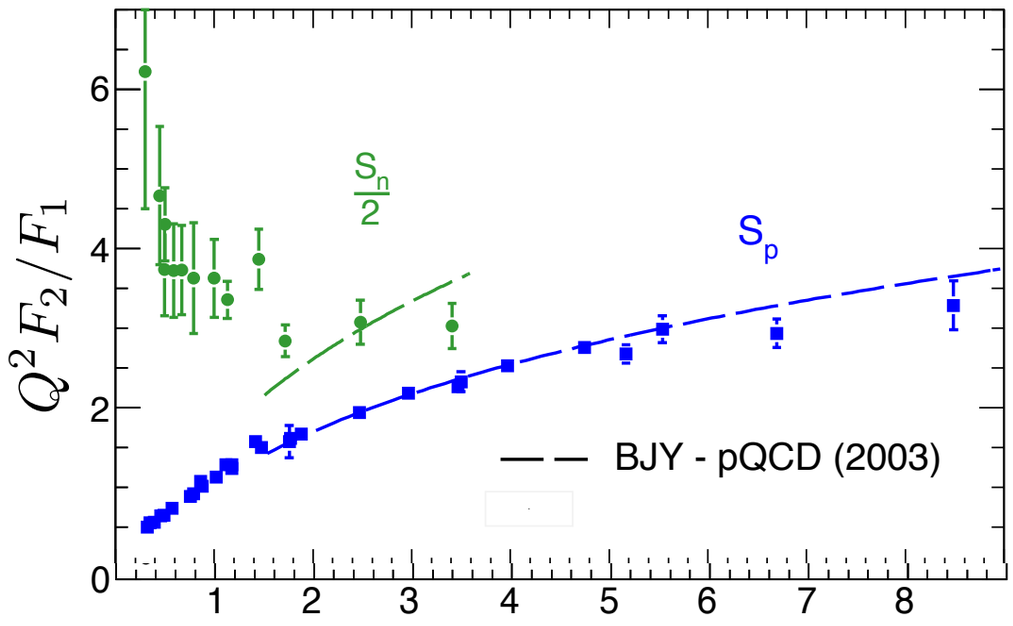
Proton F_2/F_1 Ratio

- Comparison of 4 ensembles (**D5** : 86k, **E5** : 266k, **F5** : 47k, **D6** : 261k)
- *(dashed line)* Phenomenological fit [Alberico et al, PRC79:065204 (2008)]
- (black points) Experimental data for proton ($Q^2 \lesssim 8.5$ GeV^2)



- Prediction from pQCD + quark OAM [Balitsky, Ji, Yuan (2003)]

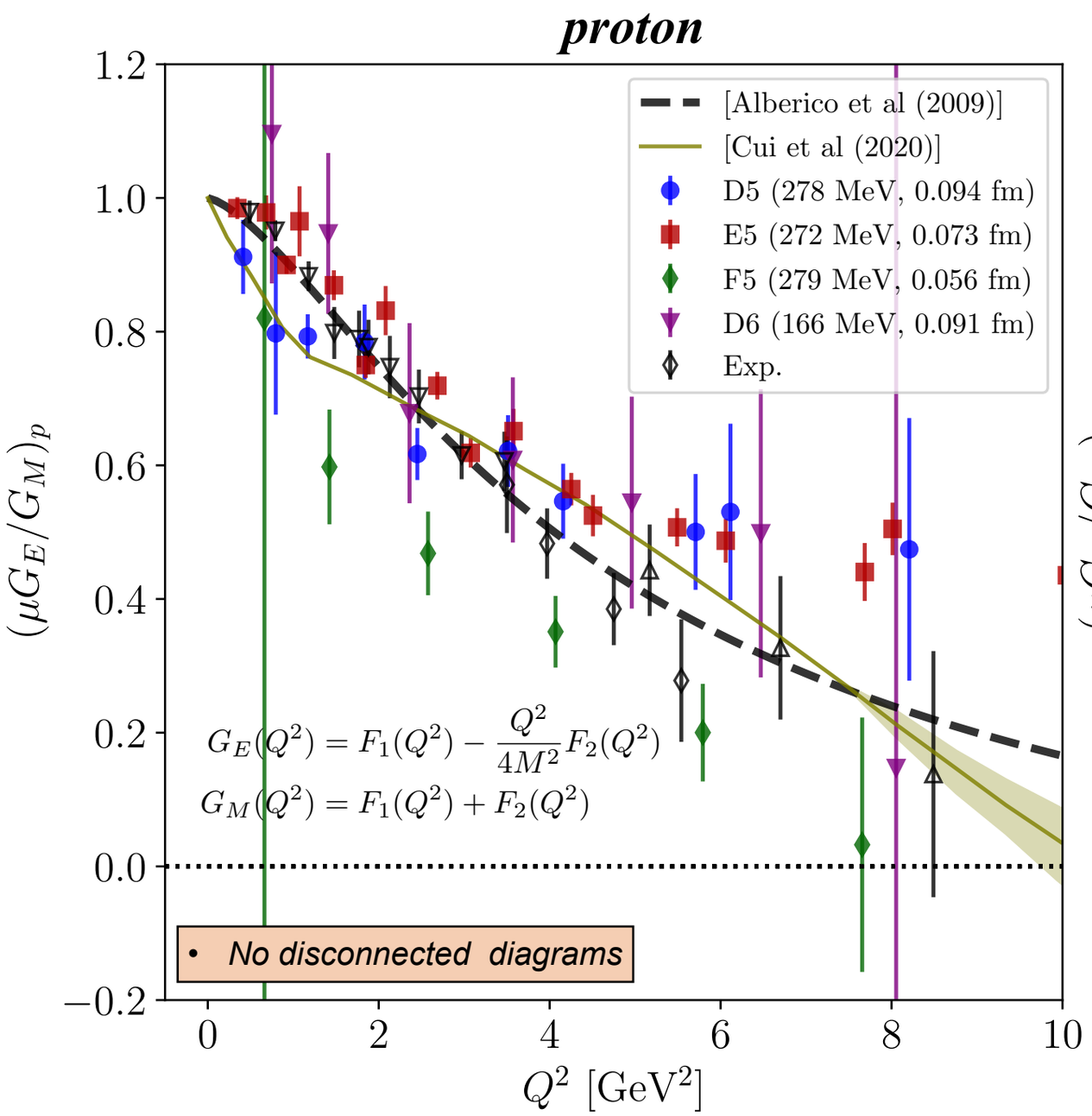
$$Q^2 F_{2p}/F_{1p} \overset{?}{\propto} \log^2(Q^2/\Lambda^2)$$

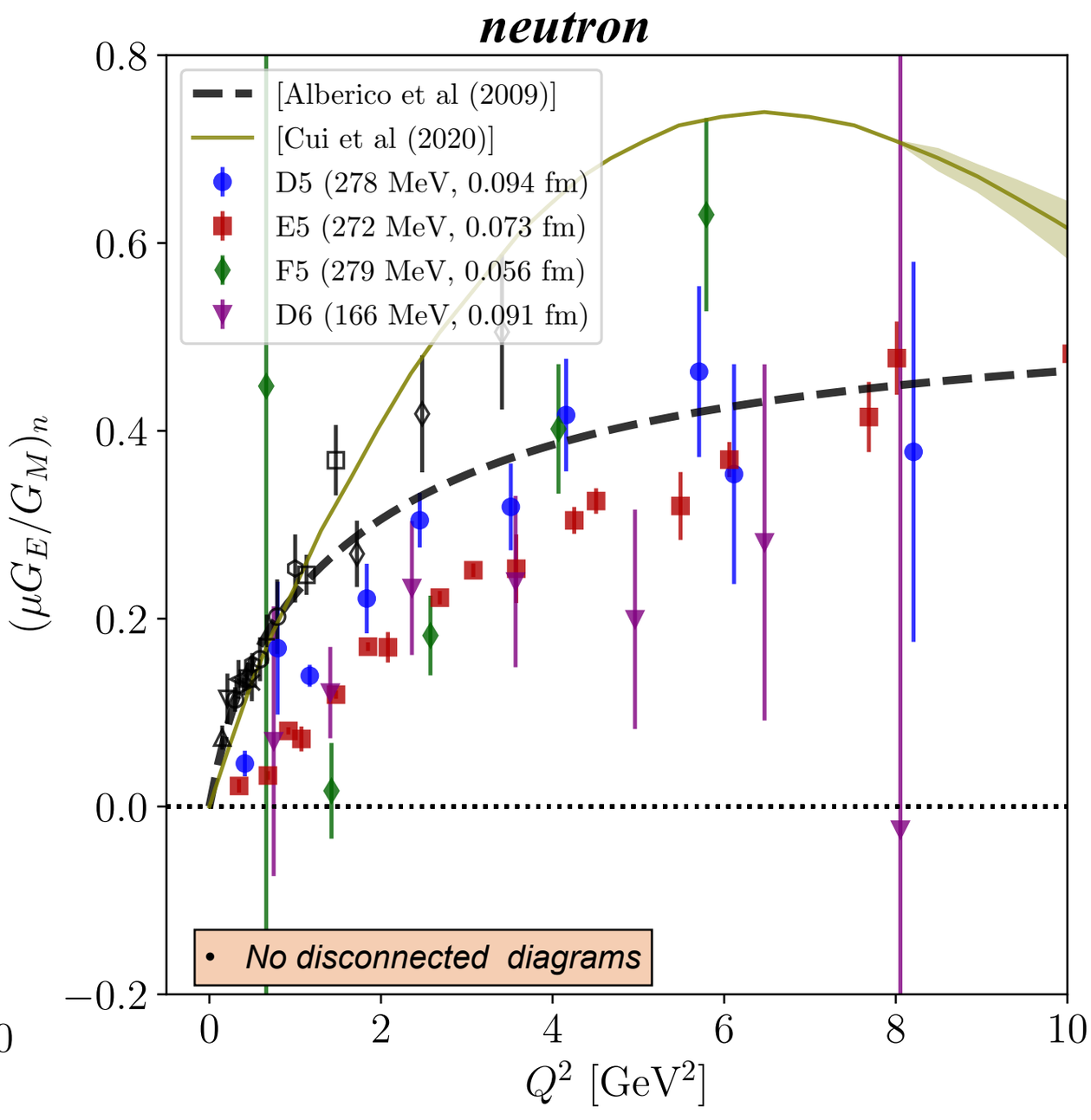


[G.D.Cates, et al, PRL106:252003 (2011)]

Proton & Neutron G_E/G_M Ratio

- Comparison of 4 ensembles (**D5** : 86k, **E5** : 266k, **F5** : 47k, **D6** : 261k)
- *(dashed line)* Phenomenological fit [Alberico et al, PRC79:065204 (2008)]
- *(black points)* Experimental data for proton ($Q^2 \lesssim 8.5$ GeV^2) and neutron ($Q^2 \lesssim 3.4$ GeV^2)

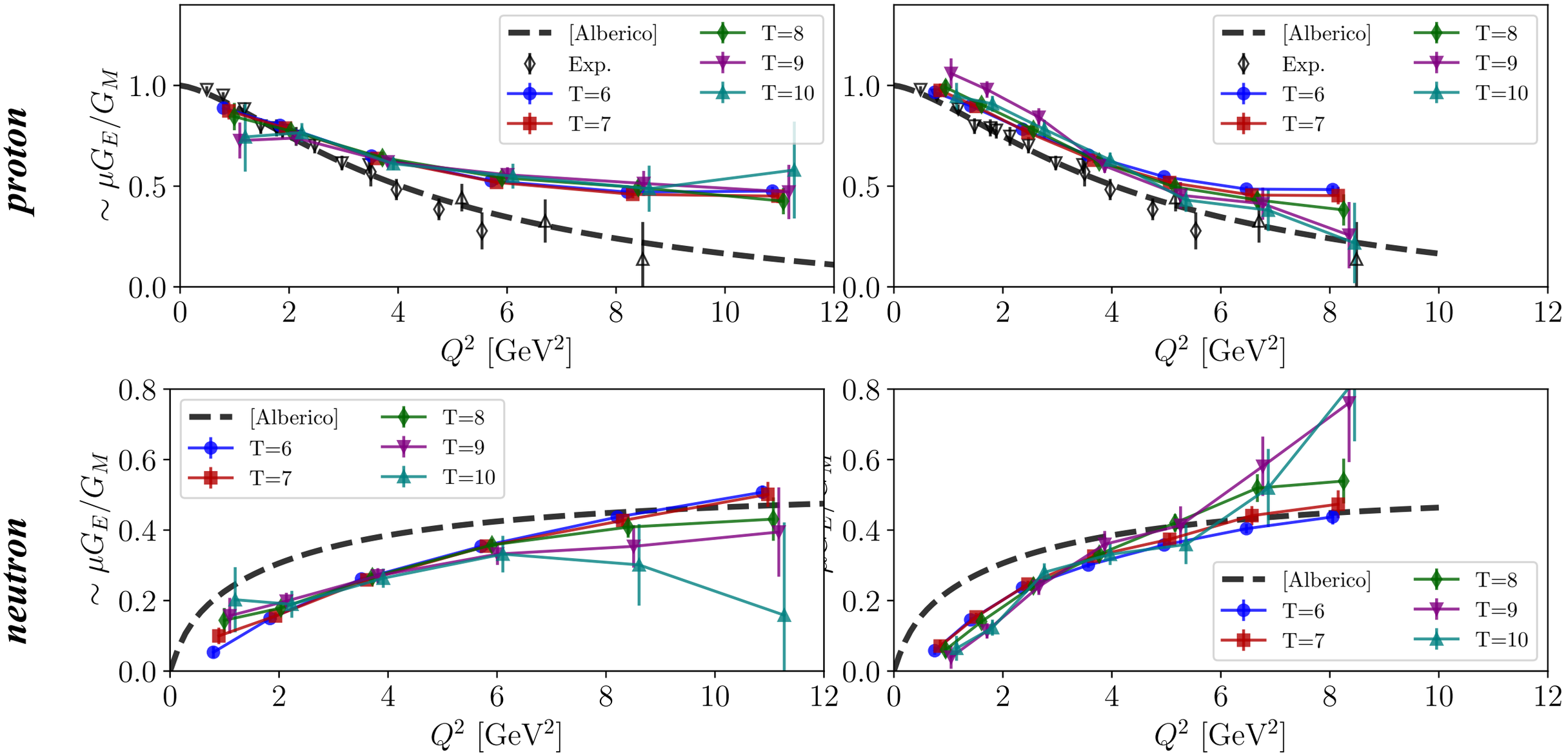




Proton&Neutron G_E/G_M from $\langle NJ\bar{N}\rangle$ Correlators

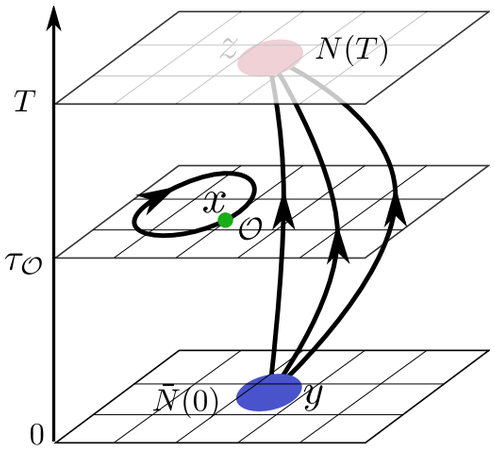
$$\begin{pmatrix} \sinh\frac{\lambda'-\lambda}{2} \\ \cosh\frac{\lambda'+\lambda}{2} \end{pmatrix} \frac{\mathrm{Re}\langle N_\uparrow(p'_x,T)\, J_t(T/2)\, \bar{N}_\uparrow(p_x,0)\rangle}{\mathrm{Re}\langle N_\uparrow(p'_x,T)\, J_y(T/2)\, \bar{N}_\uparrow(p_x,0)\rangle} \overset{T\to\infty}{=} G_E/G_M \qquad \text{with} \begin{pmatrix} p^{(\prime)} & = m_N \sinh\lambda^{(\prime)} \\ E^{(\prime)} & = m_N \cosh\lambda^{(\prime)} \end{pmatrix}$$

D5(mπ =278 MeV, a = 0.094 fm) **D6(mπ =170 MeV, a = 0.094 fm)**

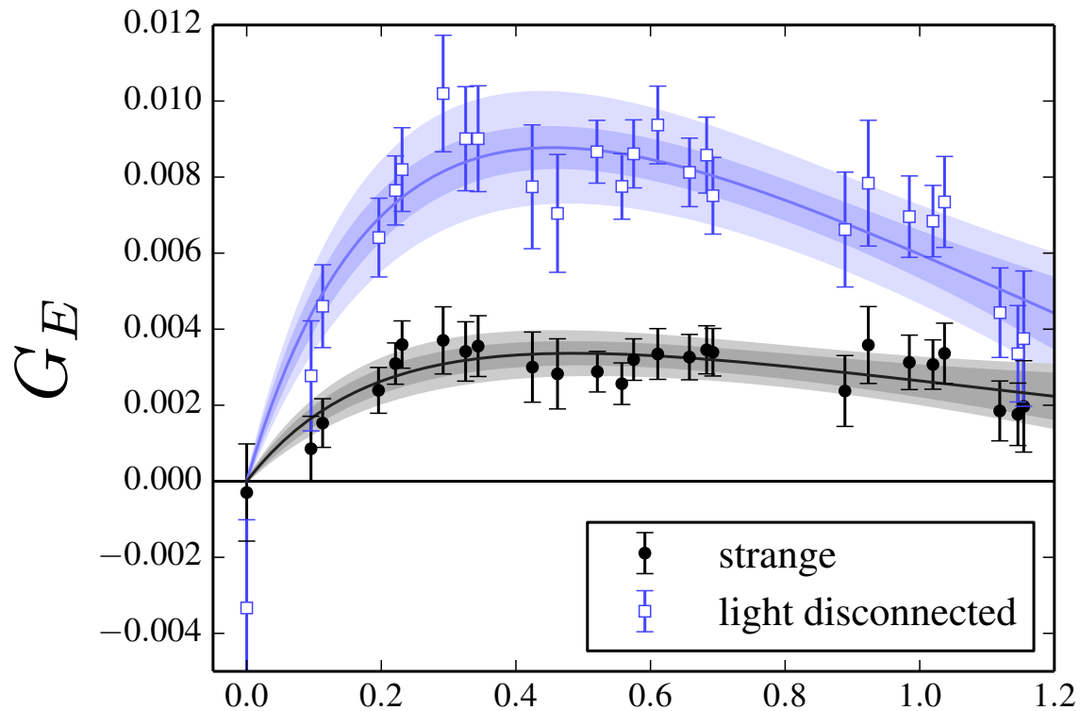


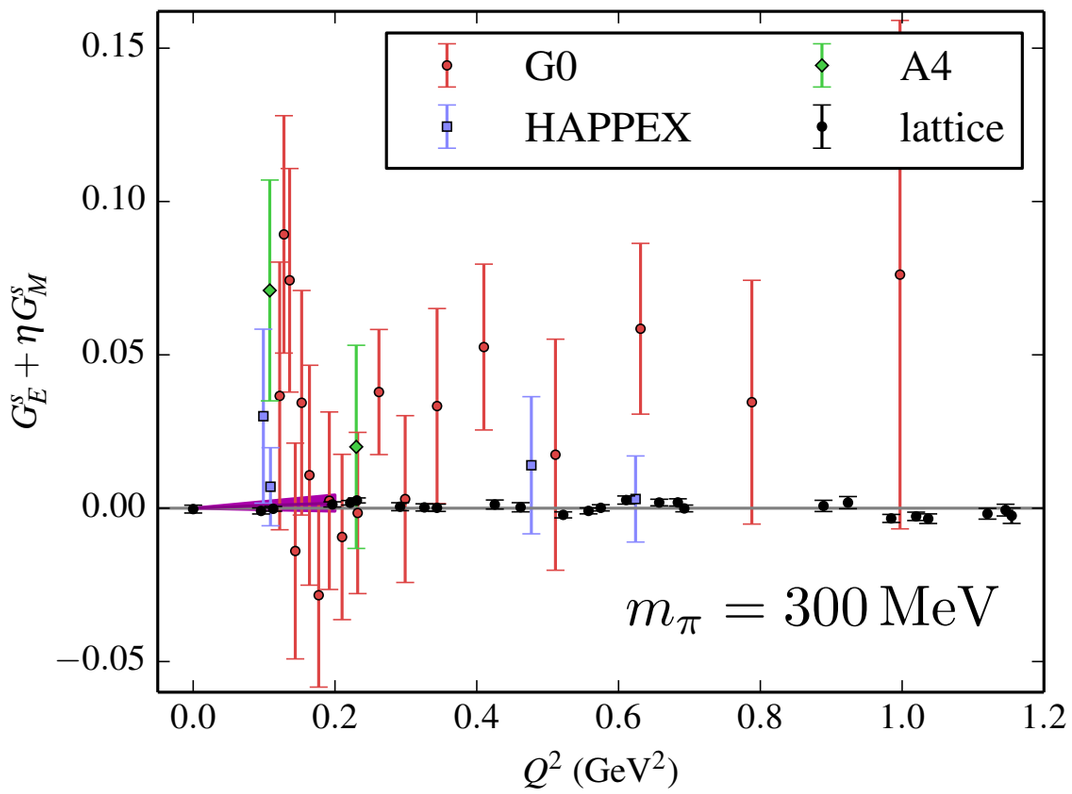
- Comparison of 4 ensembles (**D5** : 86k, **D6** : 261k); varying tsep = amount of exc.states
- *Nucleon excited state(s) have very similar GE/GM at large Q2?*

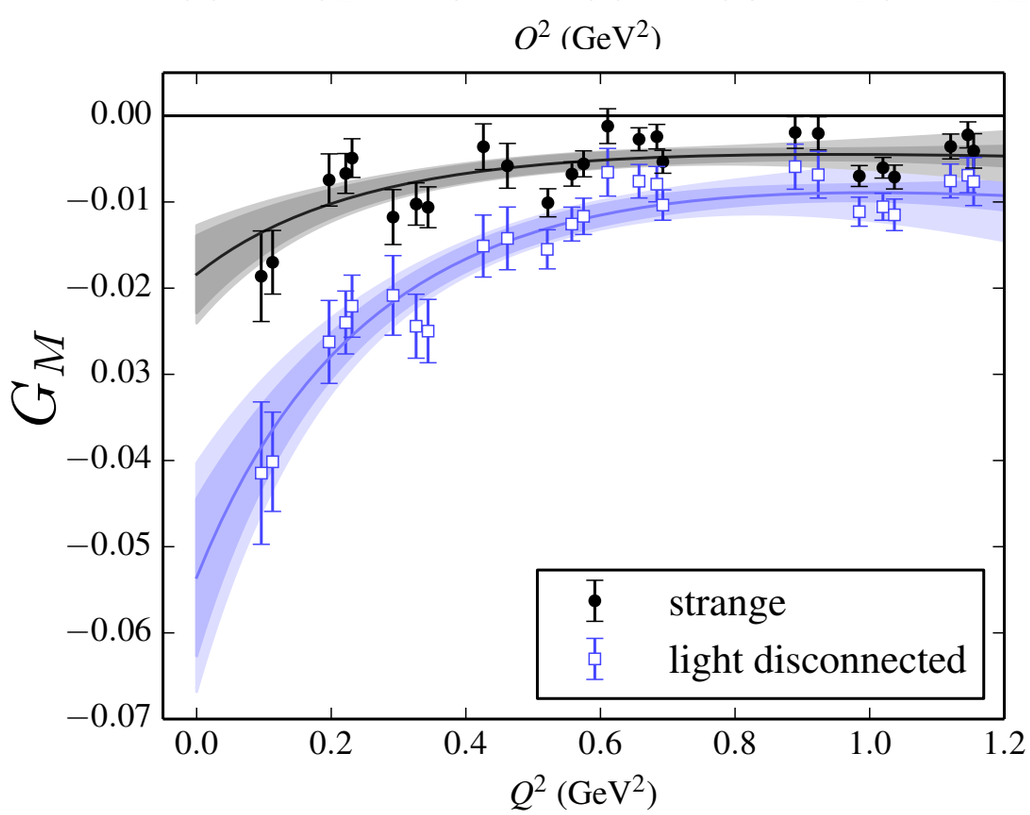
Disconnected Contributions to Vector FFs?



Disconnected-U,D and S:
≲ 1% of nucleon FFs at $Q^2 \lesssim 1$ GeV^2
[J. Green, S. Meinel, S.S. et al; PRD92:031501 (2015)]





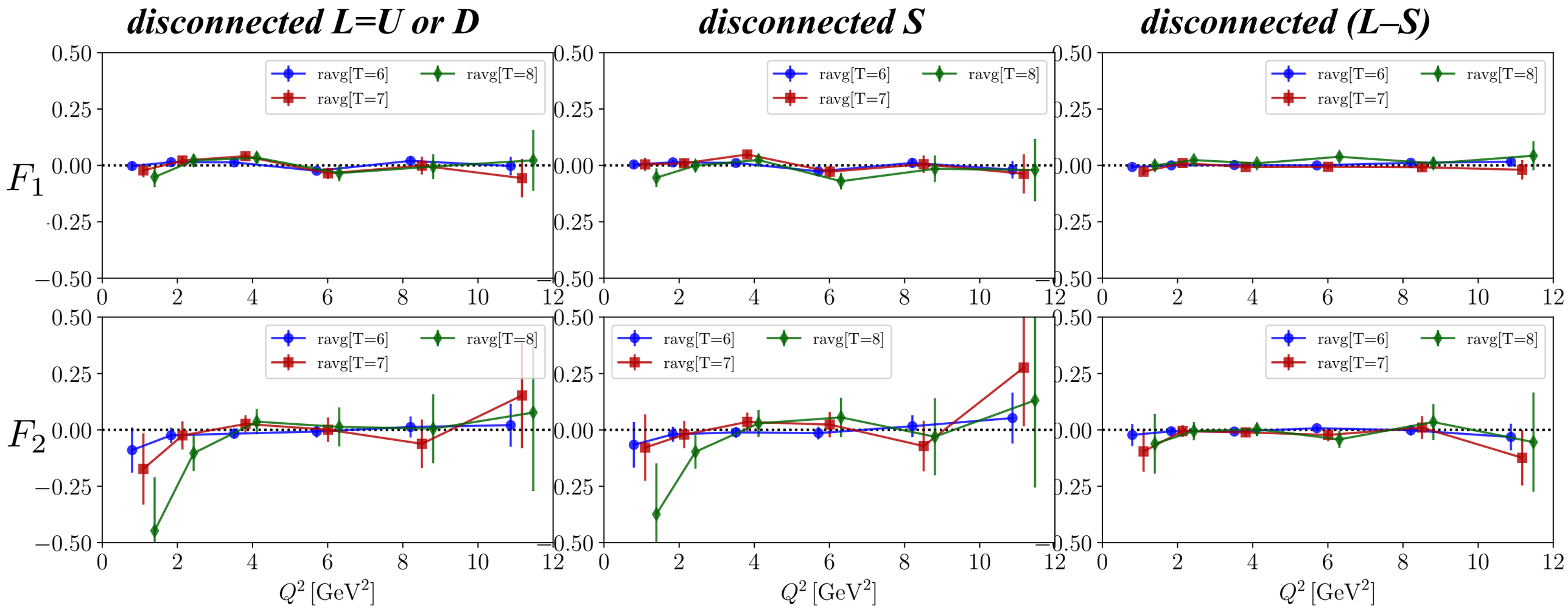


Disconnected-U,D & Strange vs. Connected (D5)

- *relative correction* $|\delta F_{1,2}^{disc} / F_{1,2}^{conn}| \lesssim 10\%$ *but quickly grows with Euclidean time*
- partial noise cancellation in (L–S) contributing to proton & neutron

$$P = \frac{1}{3}\left[2U - D\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

$$N = \frac{1}{3}\left[2D - U\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$



D5 ensemble (mπ=280 MeV, a=0.094 fm), 86k samples of $\langle N\bar{N}\rangle$

G_E/G_M : Systematics from Disconnected U,D & S

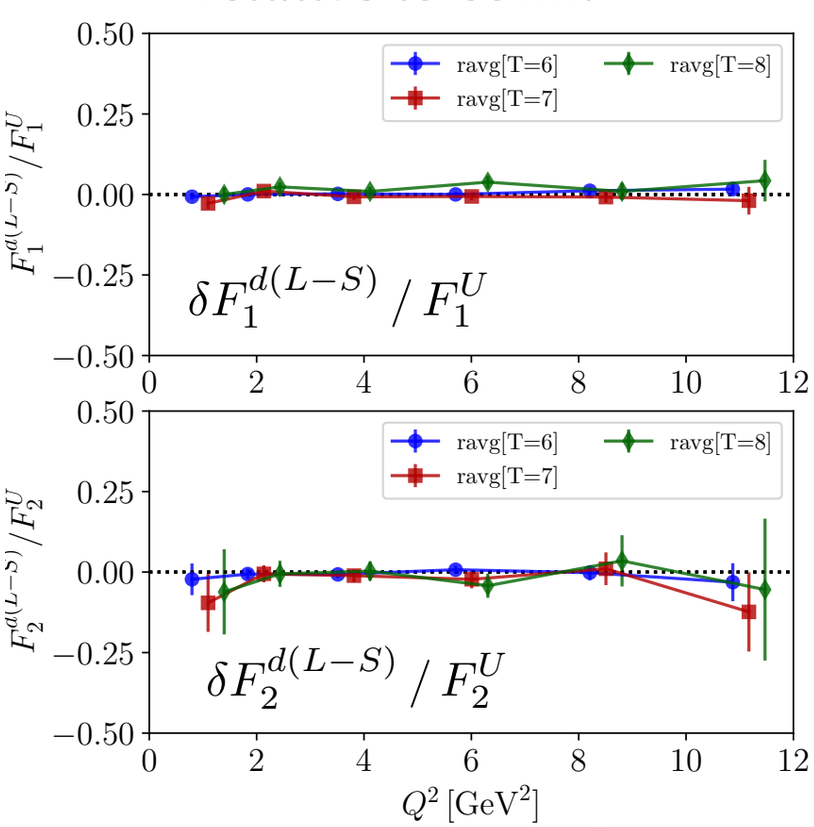
- partial noise cancellation in (L–S) contributing to proton & neutron

$$P = \frac{1}{3}\left[2U - D\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

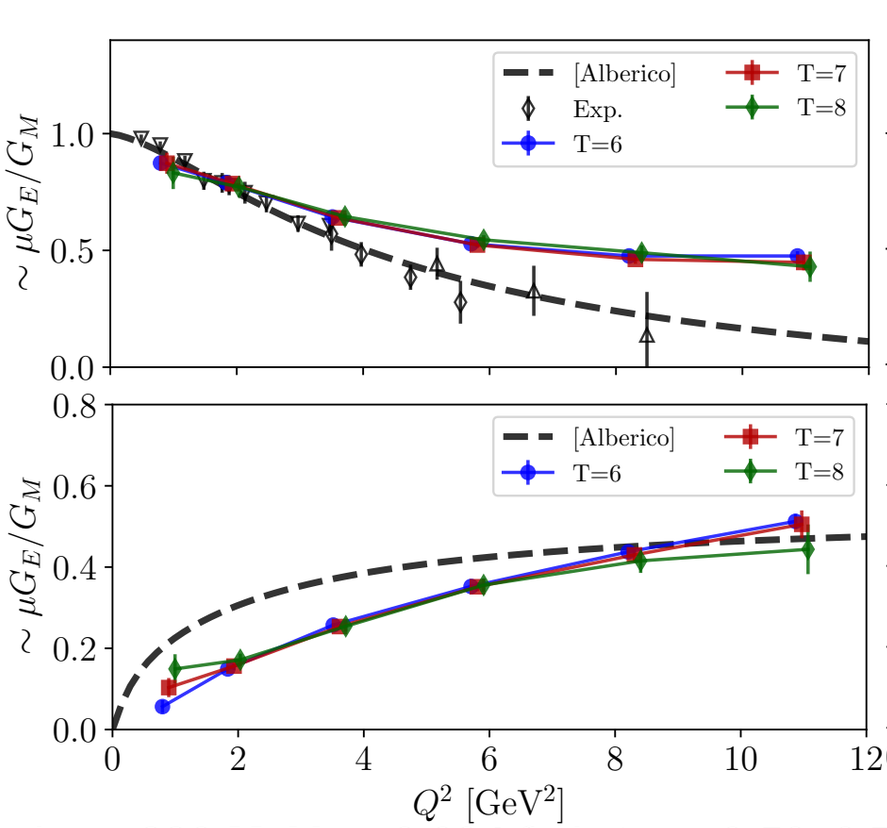
$$N = \frac{1}{3}\left[2D - U\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

- limited effect on G_E/G_M – *within stat. noise*
- larger effect on flavor-decomposition of $F_{1,2}$

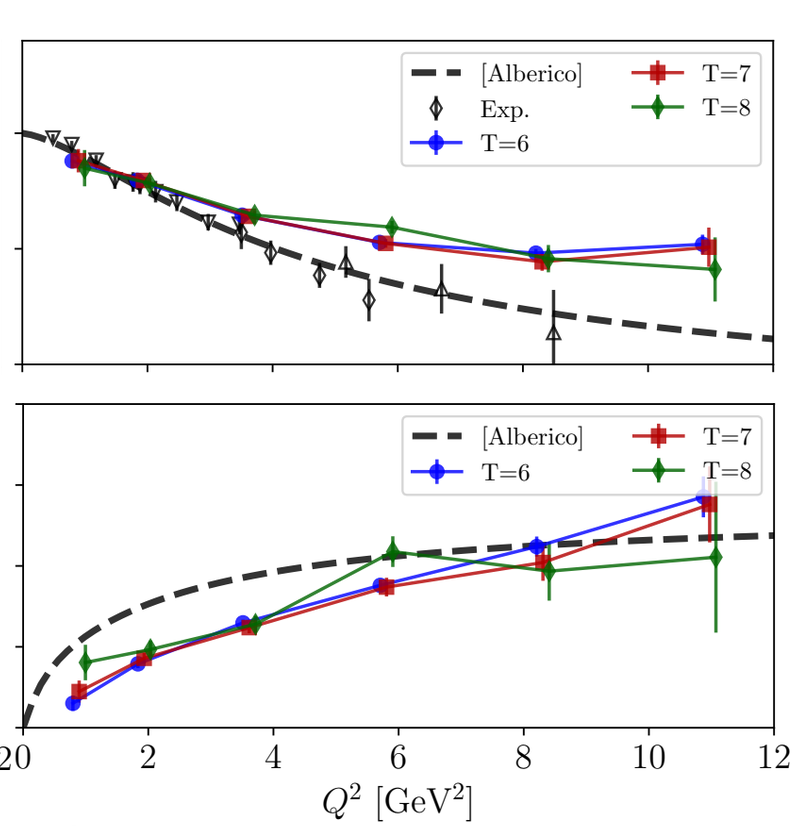
disconnected (L–S), relative to conn.



G_E/G_M, CONN. ONLY

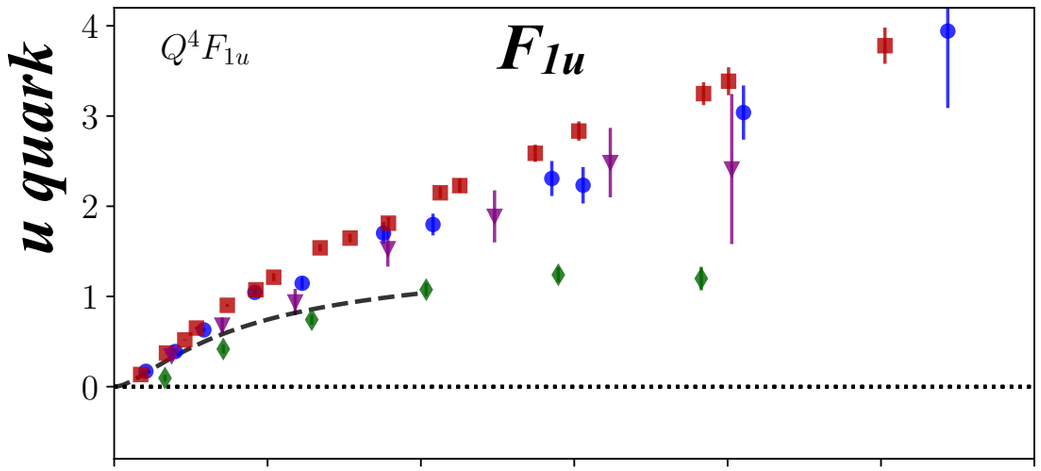


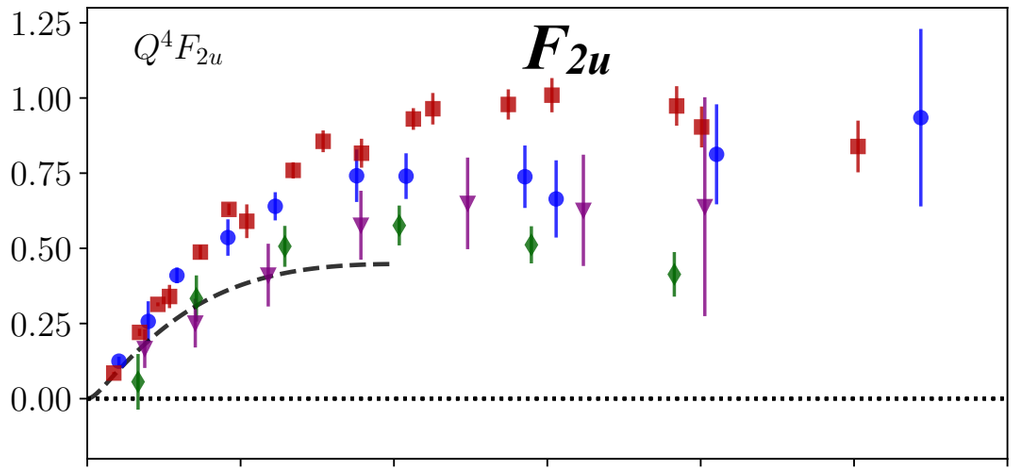
G_E/G_M, FULL

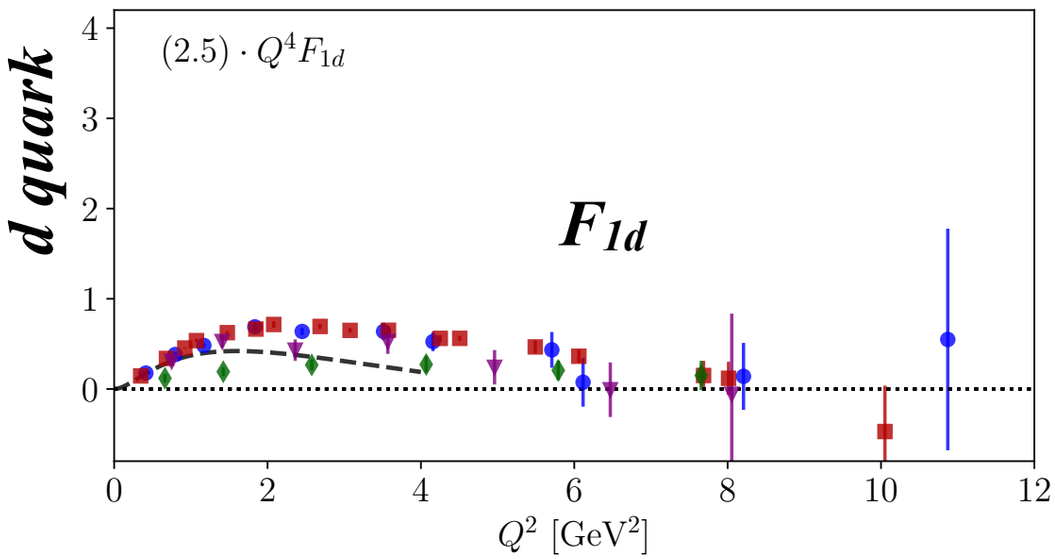


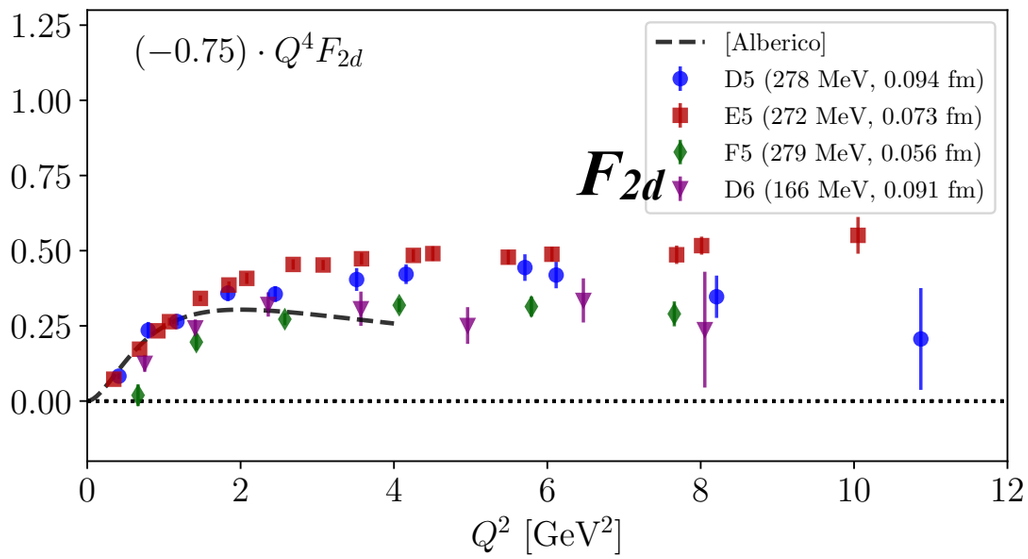
D5 ensemble (mπ=280 MeV, a=0.094 fm), t_{sep}= 0.50÷0.75 fm

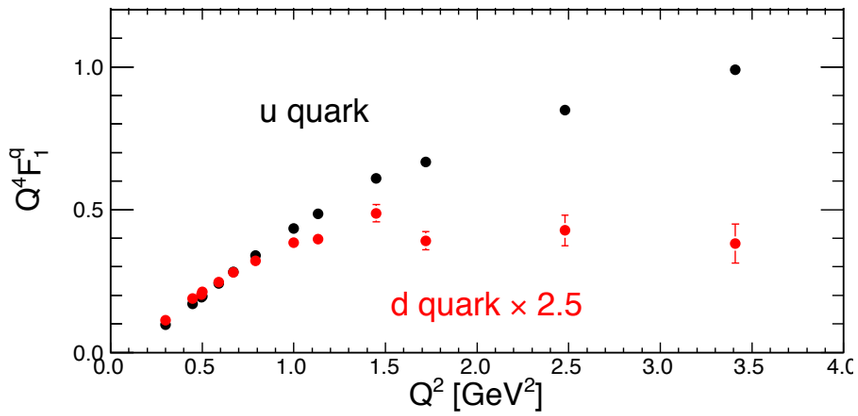
Light-flavor Contributions to Proton Form Factors

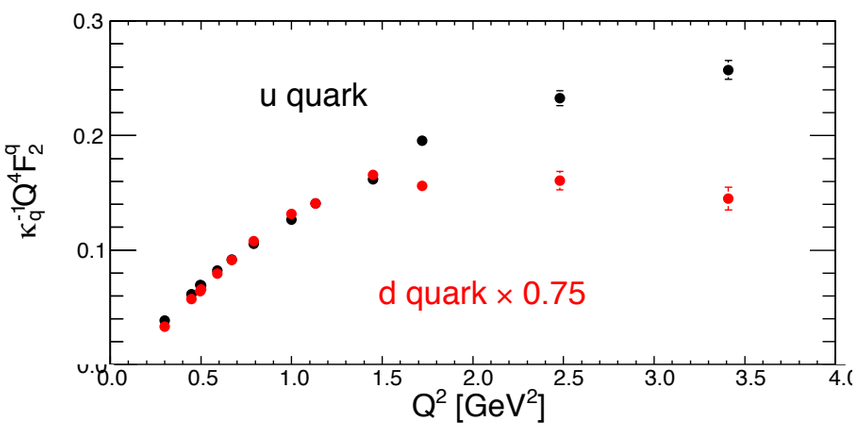






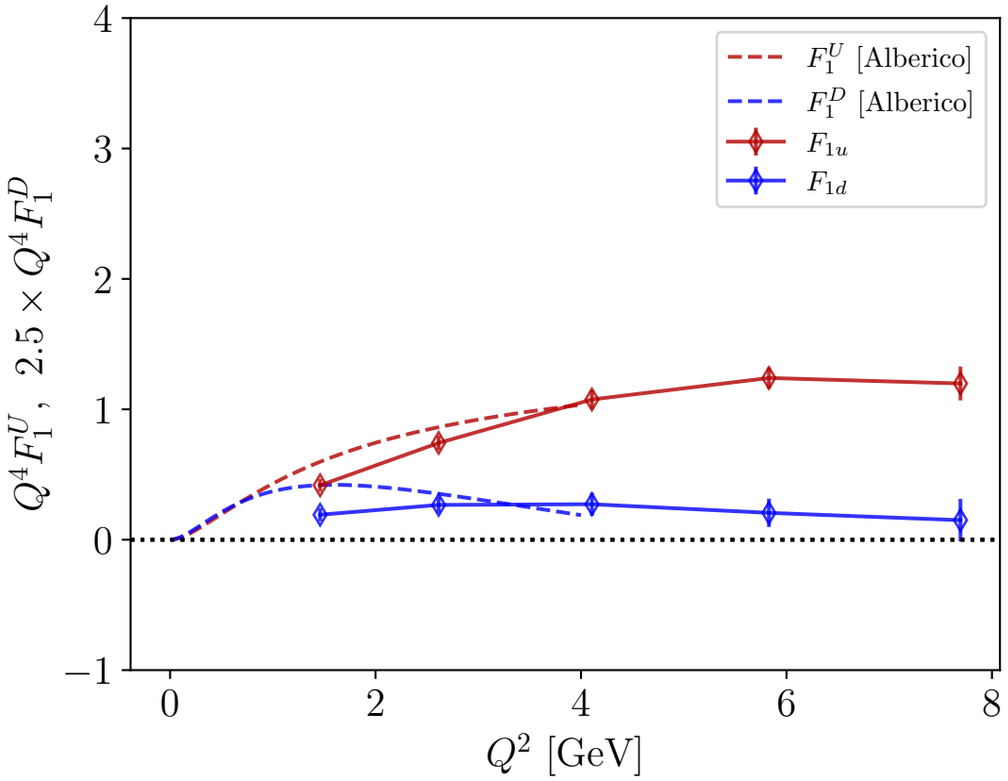


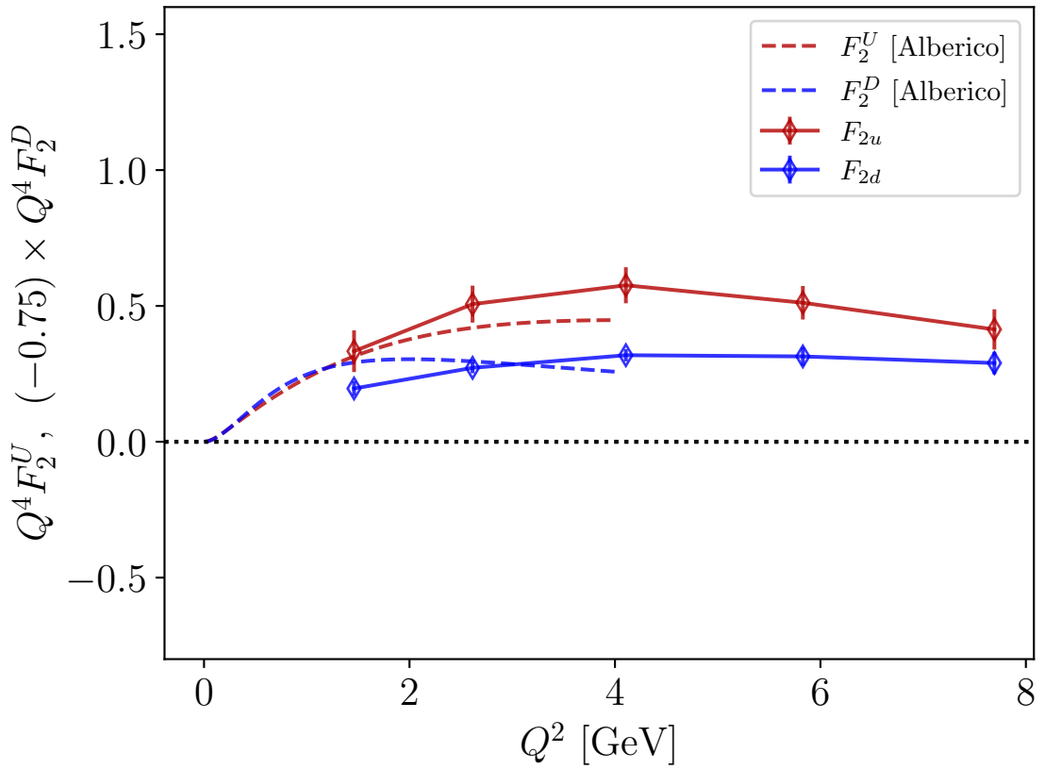


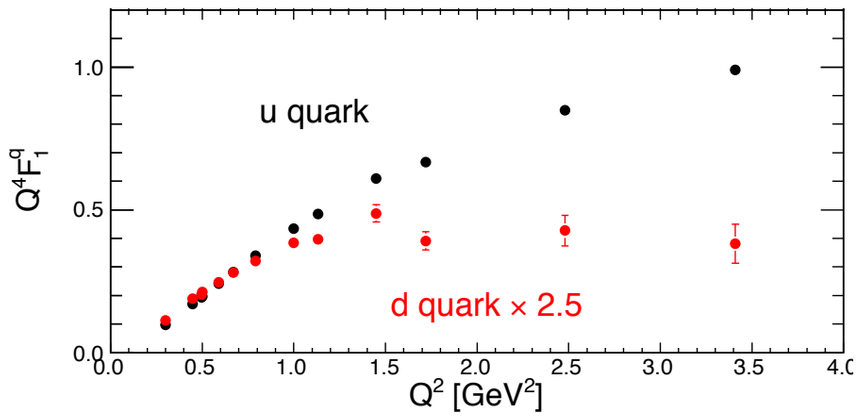


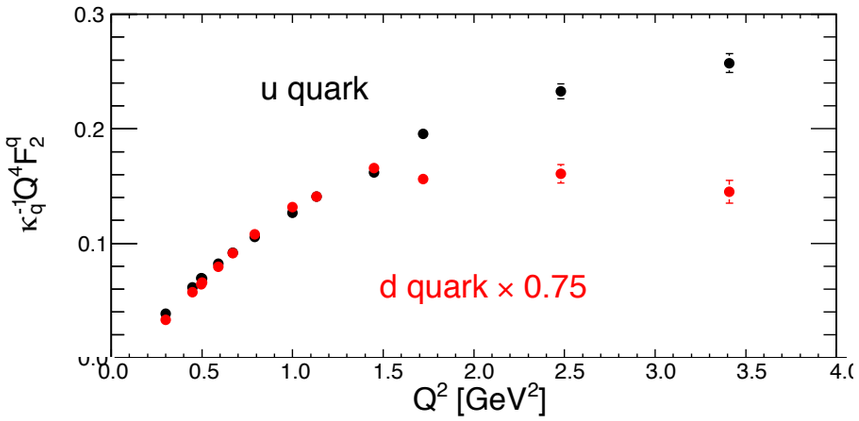
- Comparison of 4 ensembles (**D5** : 86k, **E5** : 266k, **F5** : 47k, **D6** : 261k)
- *(dashed lines)* flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]

Light-flavor Contrib's to Proton FFs (a=0.056fm)









- Initial statistics on **F5** : (47k)
- *(dashed lines)* flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]

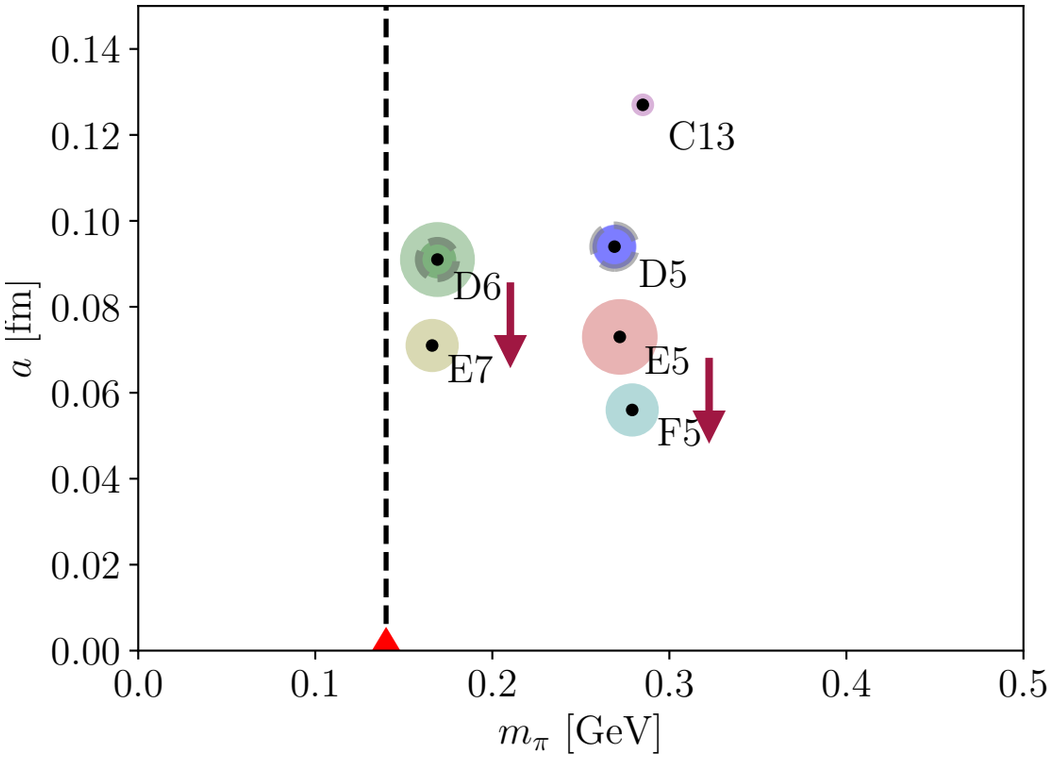
Summary

- High-statistics lattice calculations of nucleon form factors at high-momentum
  - up to $Q^2 \lesssim 10$ GeV^2
  - three lattice spacings $a \gtrsim 0.056$ fm
  - two pion masses $m\pi \gtrsim 170$ MeV

- Form factor results overshoot experimental data x(2 ... 2.5) ;
  G_E/G_M ratios in qualitative agreement
  - "quark-disconnected" contractions → little effect for $Q^2 \lesssim 6$ GeV^2 , uncertain for higher-Q^2
  - Non-physical quarks masses?
  - Discretization?
  - Excited states (most likely) — *need improved/variational nucleon operators*

Outlook

- Upcoming next-gen computing facilities (e.g. Jupiter at Julich SC)
  - More statistics (x10) to control excited state systematics
  - Smaller lattice spacings (0.09 → 0.07 → 0.05 fm); physical and 2x physical pion mass
  - Improved lattice vector-current operator



- *Effective theory for large-momentum form factors of nucleon-like states?*
- *Explanation for potential G_E/G_M universality – similar structure of excited N-like states?*