Doubly heavy tetraquark binding and diquarks in lattice QCD

Anthony Francis

From Craig's invitation: *If diquark correlations are real, then where should physics best look to find, understand, and exploit them?*

Theoretical Motivations and Empirical Evidence for Diquark Correlations in Hadrons

ECT*, Trento, 20.10.2026

New work with R. Lewis and M. Nagatsuka.

Review results based on: [2502.04701].

Own old results based on: [2407.08816],[2106.09080]

with P. de Forcrand, R. J. Hudspith, R. Lewis, K. Maltman

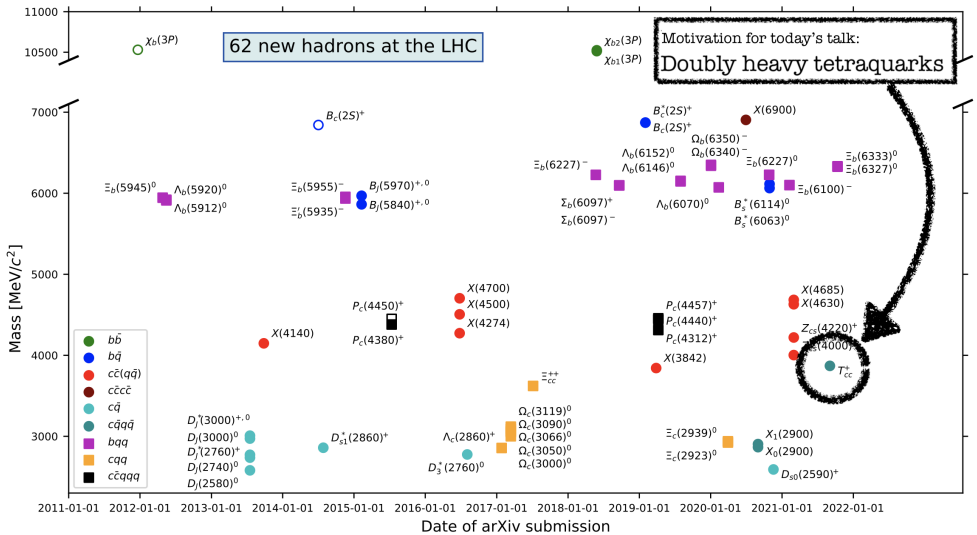
and a special shout-out to B. Colquhoun and W. Parrott.

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Heavy hadron spectrum today - A success story turned challenge to theory

Multi-decade-long theory-success-streak broken

- Before '03: Potential model and predictions worked.
- Now: Many new, unexpected, states observed.



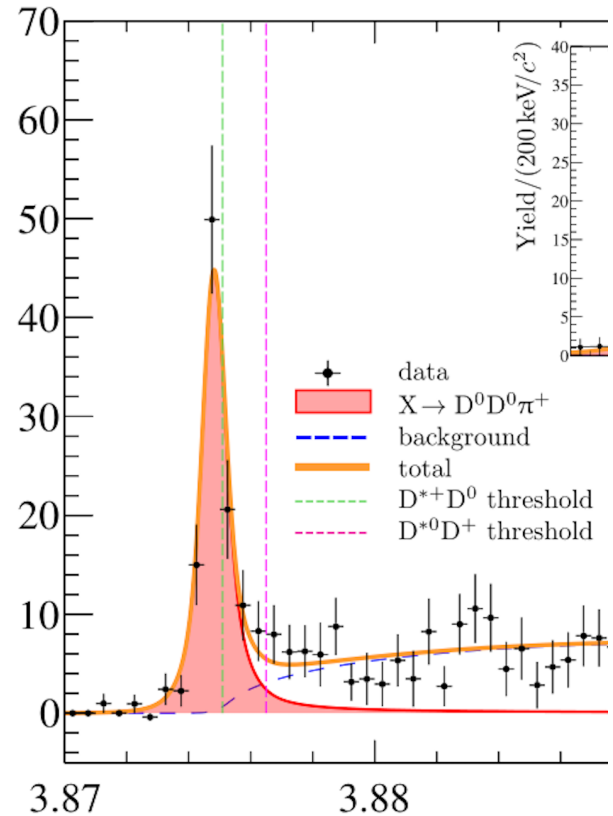
LHCb collaboration, 2021, 62 new hadrons, in 2023: 65, 2025: 79(!)

A new particle zoo with limited explanations in theory

1. QCD often approximated in models
2. Many extensions possible, many interpretations, some times contradictory statements
3. Insights through lattice QCD calculations? What role could effective QCD degrees of freedom play?

Especially 4-/5-quark states not expected before.

Like, for example:
T_{cc}, $\bar{u}\bar{d}cc$, $E_B = 0.3\text{MeV}$



⇝ LHCb-PAPER-2021-031

T_{QQ}: A new doubly-heavy tetraquark family(!)

Diquarks - An attractive concept for tetraquarks

- **Prelude:** A binding mechanism for 1^+ $\bar{u}dQQ$ tetraquarks based on diquarks.

"The concept of diquarks is almost as old as the quark model, and actually predates QCD [1]" $\rightsquigarrow$ arXiv:2203.16583; [1] PR 155, 1601 (1967)

- **Diquarks in QCD:** Formally the diquark interpolating operator may be written as

$$\boxed{D_\Gamma = q^c C \Gamma q'}$$

$\rightsquigarrow$ c, C=charge conjugation, Γ acts on Dirac space

Consequences for their properties:

- "good" $(\bar{3}_F, \bar{3}_c, J^P = 0^+)$ configuration
- quarks in "good" diquarks attract each other
- large mass splitting comparing good with bad and not-even-bad channels
- HQSS-limit: When $q \to Q = \infty$ a diquark acts as an antiquark $[QQ] \leftrightarrow \bar{Q}$.

$\rightsquigarrow$ "Effective mass-decomposition" of a tetraquark correlator

$$\boxed{C_\Gamma(t) \sim \exp\left[-t\left(m_{D_\Gamma} + 2m_Q + \mathcal{O}(m_Q^{-1})\right)\right]}$$

3 types of diquark:
good, bad and not-even bad

J^P	C	F	Op: Γ
0^+	$\bar{3}$	$\bar{3}$	γ_5, $\gamma_0\gamma_5$
1^+	$\bar{3}$	6	γ_i, σ_{i0}
0^-	$\bar{3}$	6	$\mathbb{1}$, γ_0
1^-	$\bar{3}$	$\bar{3}$	$\gamma_i\gamma_5$, σ_{ij}

This motivates 2 general operator structures for doubly-heavy tetraquarks.

1. Dimeson:

$$M = (\bar{b}_a \gamma_5 u_a)\,(\bar{b}_b \gamma_i d_b) - (\bar{b}_a \gamma_5 d_a)\,(\bar{b}_b \gamma_i u_b)$$

2. Diquark-Antidiquark:

$$D = \Big((q_a)^T (C\gamma_5) q'_b \Big) \times \Big[\bar{Q}_a (C\gamma_i) (\bar{Q}'_b)^T - a \leftrightarrow b \Big]$$

Model expectations:

- HQSS and effective $\mathcal{O}(1/m_Q)$ suppression $\Rightarrow$ prefer T_{bb} over T_{cc}.
  - Heavier $[\bar{Q}\bar{Q}']$ = more binding opportunity
- Diquark attractive effect stronger for lighter quarks involved $\Rightarrow$ prefer $\{ud\}$ type.
  - Lighter $\{qq'\}$ = more binding opportunity
- Total wave-fct should be anti-symmetric $\Rightarrow$ prefer $J^P(T_{QQ'}) = 1^+$

First-step lattice calculation:

1. Implement large operator basis* and calculate their correlation matrix.
2. Solve the GEVP and get the binding energies $\Delta E = E_{tetraquark} - E_{2\times meson}$, e.g.

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}, \; F(t)\nu = \lambda(t) F(t_0)\nu \; , \; \lambda_i(t) \propto e^{-E_i(t-t_0)}$$

Beware: Operator overlaps do not necessarily connect to the naively expected structures. Be careful when equating lattice correlators with trial-wave functions.

*HadronSpectrum Coll. ('17)

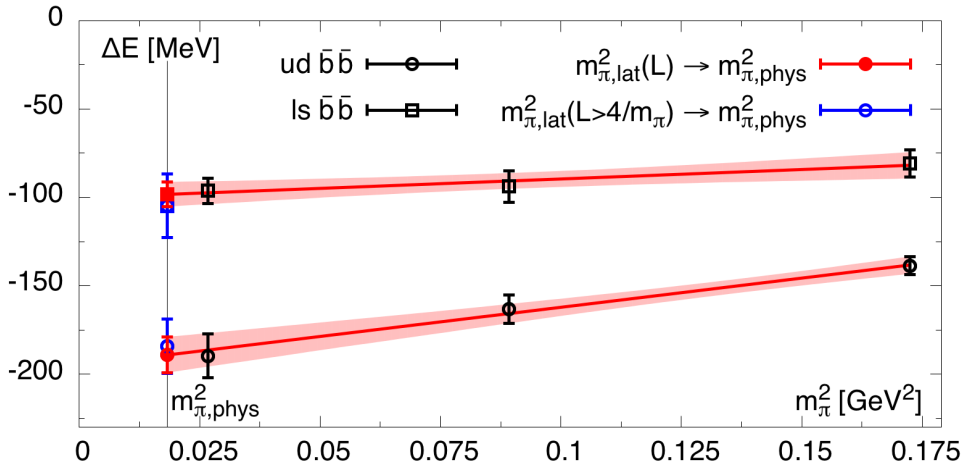
(Heavy) quark mass dependence of binding - Early studies

- $n_f = 2 + 1$ full QCD, $32^3 \times 64$, $a = 0.090$fm, $a^{-1} = 2.194$GeV
- $m_\pi = 164, 299, 415, 575$ and 707 MeV, $m_s \simeq m_s^{\text{phys}}$, NRQCD heavy quarks

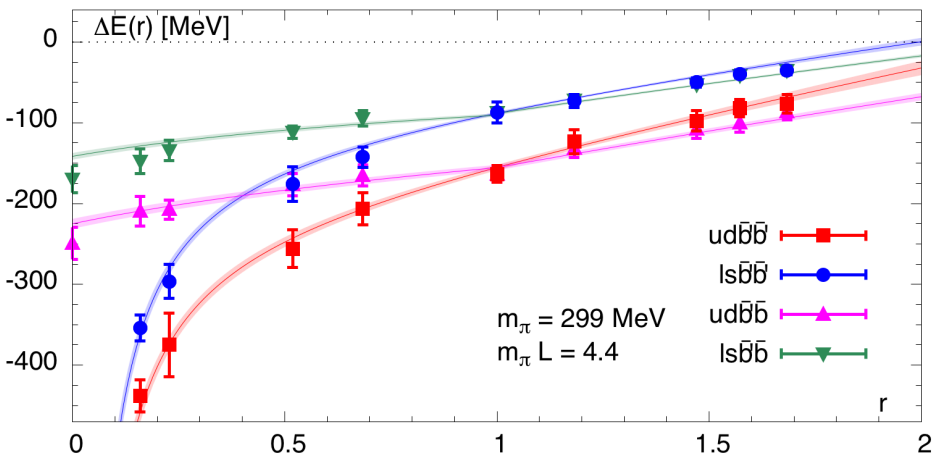
HQS-GDQ picture predictions:

- $J^P = 1^+$ *ground state* tetraquark
- Deeper binding with:
  $\rightarrow$ heavier Q in $[\bar{Q}'\bar{Q}]$
  $\rightarrow$ lighter q in $\{qq'\}$

All observed on the Lattice!

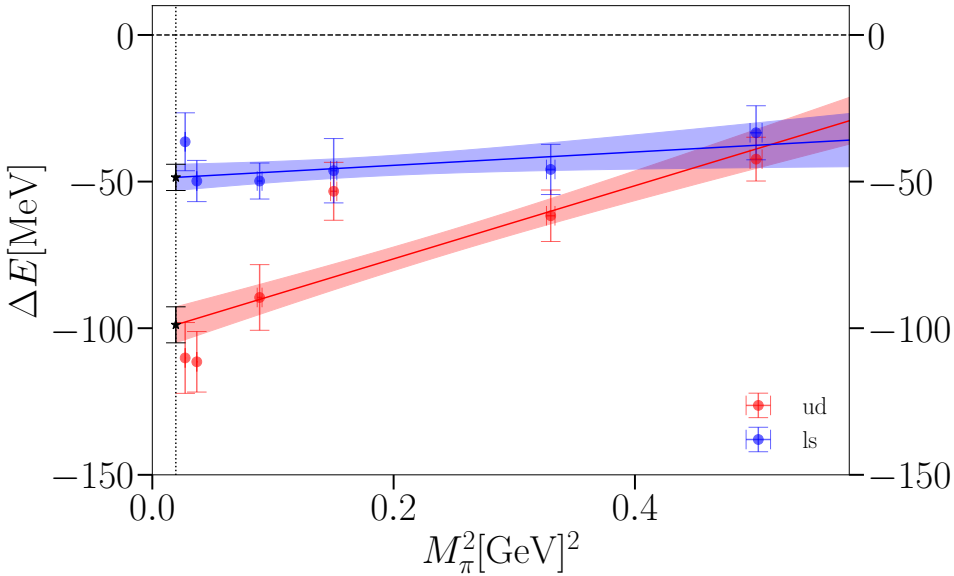


⇝AF et al. ('17)



⇝AF et al. ('18)

- Scans in $m_{b'}$ map out the heavy quark mass dependence.
- Away from physical masses the binding mechanism can be probed.
- 5 parameter pheno Ansatz describes dependence (see Appendix)
- **Problem: Systematics**
  These binding energies are much deeper than today's consensus



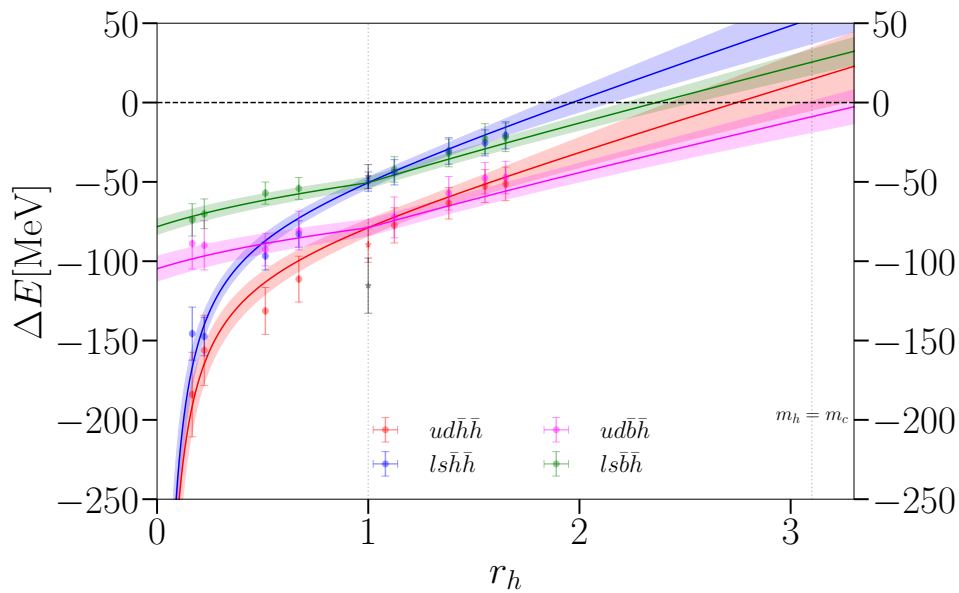
Updated analysis [2407.08816]:

$$\Delta E(T_{bb}^{ud}) = -189(10) \rightarrow -115(17) MeV$$
$$\Delta E(T_{bb}^{us}) = -98(10) \rightarrow -47(8) MeV$$

Changes:

- Re-analysis with box-sinks
- New m_π =165, 191MeV ensembles
- **Overall: Better control over systematics.**

- General and consistent upwards shift in all binding energies
- Significant systematic uncertainty addressed



Doubly-heavy tetraquarks - Experiment+Phenomenology+Lattice intersection

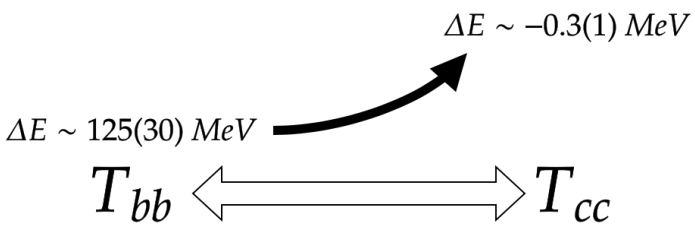
The T_{cc} discovery and the T_{QQ} family

- At the time, lattice QCD and phenomenology discussion* on doubly heavy tetraquarks like:

$$T_{QQ} \rightsquigarrow \bar{q}\bar{q}' QQ \ , \ J^P = 1^+ \text{ or } 0^-$$

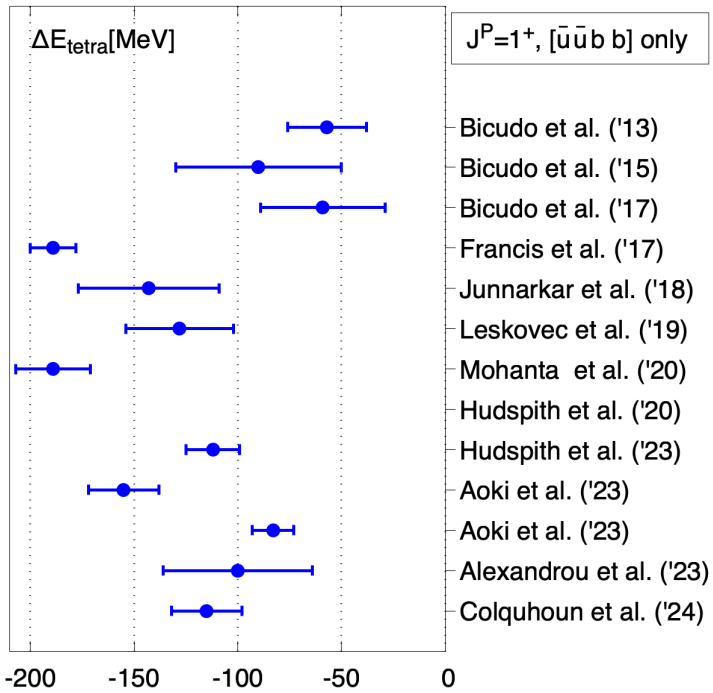
- Motivated binding mechanism in $J^P = 1^+$.
- **Phenomenology:** indications of bound T_{bb}^{ud}.
- **Lattice:** indications of bound T_{bb}^{ud} and T_{bb}^{us}.

But: +++ How do we get from T_{bb} to T_{cc}? +++



- How and does the binding mechanism change?
- What are the relevant dynamics?
- What are the other family members?
- What are their properties?

Current T_{bb} status



Recently gathered in: PPNP 140 (2025) 104143

*Discussion in pheno and lattice: Pheno: *Karliner, Rosner ('17); Eichten, Quigg ('17); Czarnecki, Leng, Voloshin ('18); Mehen ('17); Maiani ('19)*, Lattice: *AF ('16-'24); Bicudo, Wagner ('11-'22); Mathur ('18); Prelovsek, Manugopalan ('22), and more...*

⇒ Are diquarks being formed and do they explain the dynamics?

Lattice roadmap

Research milestones:

1. **Spectrum:** *Probe mass dependence of binding energy.*
2. **Mechanism:** *Consider static case and isolate diquark contribution.*
3. **Structure:** *Go towards tetraquark structure calculations.*

Diquarks on the lattice - Approaching the binding mechanism

Problem: A diquark is a colored object and not observable.
Path forward: Embed diquark in baryons / mesons with a static spectator quark.
$\rightsquigarrow$ spectator cancels off in appropriate mass differences.

$\rightsquigarrow$ hep-lat/0510082, hep-lat/0509113, hep-lat/0609004, arxiv:1012.2353

Mass difference 1

Differences of two $qq'Q$ baryons:

$$M_{\Gamma}^{qq'Q}(t) - M_{\gamma_5}^{qq'Q}(t)$$

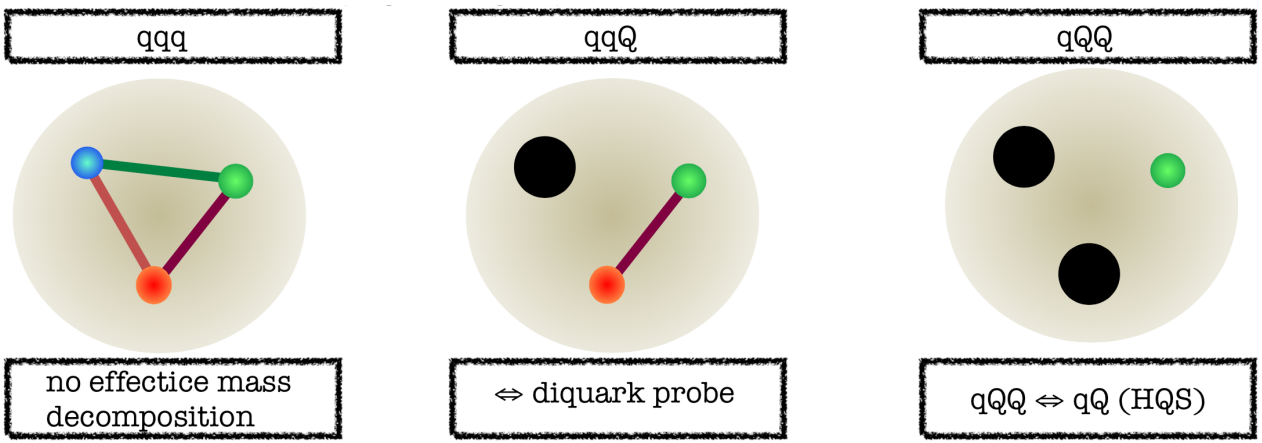
$\rightsquigarrow$ diquark-diquark mass difference

Mass difference 2

$qq'Q$ baryon and qQ meson difference:

$$M_{\Gamma=\gamma_5}^{qq'Q}(t) - M_{\gamma_5}^{q'\bar{Q}}(t)$$

$\rightsquigarrow$ diquark-quark mass difference



$\rightsquigarrow$baryon pictures from Hosaka, 2013

"These mass differences are fundamental characteristics of QCD, which should be measured carefully on the lattice." $\rightsquigarrow$ Jaffe, arXiv:hep-ph/0409065; Phys. Rept. 409 (2005)

Lattice spectroscopy - Diquark-(di)quark differences

We consider $qq'Q$ baryon differences:

$$M_{\Gamma}^{qq'Q}(t) - M_{\gamma_5}^{qq'Q}(t)$$

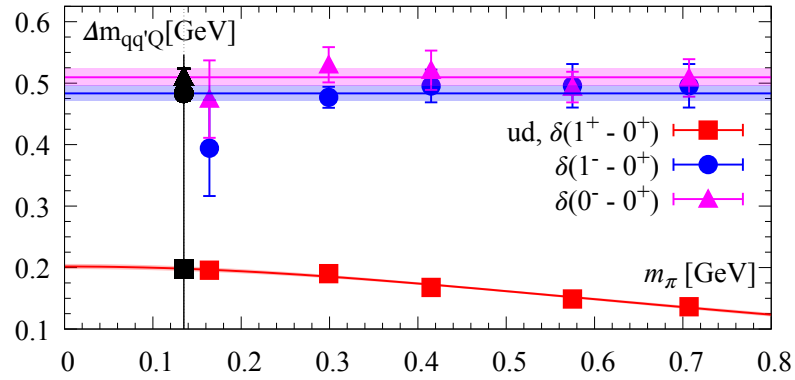
Special status of good diquark observed

- Good ud diquark lowest in spectrum
- Pattern repeated in ℓs and ss'

... and $qq'Q$ baryon qQ meson differences

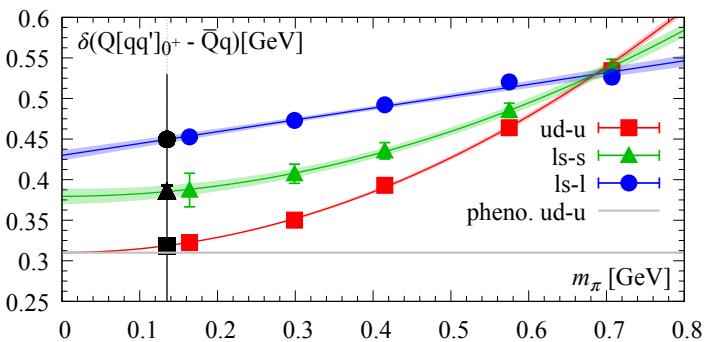
$$M_{\Gamma=\gamma_5}^{qq'Q}(t) - M_{\gamma_5}^{q'\bar{Q}}(t)$$

ud 0^+ versus 1^+, 0^- and 1^-



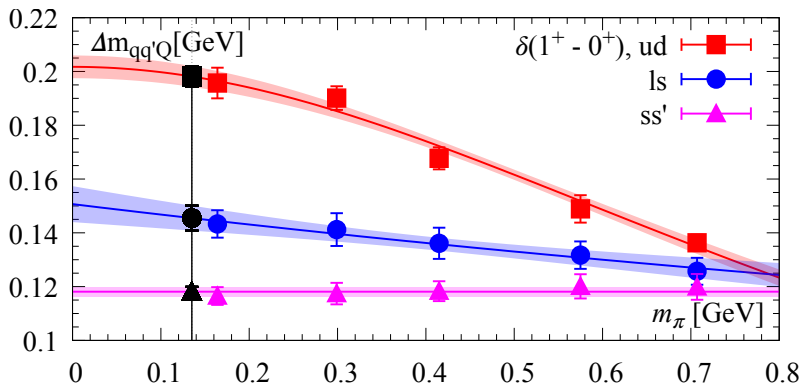
$Qqq' - \bar{Q}q'$ splittings

$\rightsquigarrow$ useful for 4-/5-quark states.



$$\delta(Q[q_1q_2]_{0^+} - \bar{Q}q_2) = C\left[1 + (m_\pi/D)^{n\in 0,1,2}\right]$$

$(1^+ - 0^+)_{qq'}$ splitting



$$\delta(1^+ - 0^+)_{q_1q_2} = A/\left[1 + \left(m_\pi/B\right)^{n\in 0,1,2}\right]$$

Diquark spectroscopy - Comparing results

- We want to compare our results with phenomenology
  - Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
  - For pheno estimates combine charm and bottom hadron masses such that leading $\mathcal{O}(1/m_Q)$ ($Q = c, b$) cancel

- The main spectroscopy results are summarised as:

All in [MeV]	$\delta E_{\text{lat}}(m_\pi^{\text{phys}})$	δE_{pheno}	$\delta E_{\text{pheno}}^{\text{bottom}}$	$\delta E_{\text{pheno}}^{\text{charm}}$
$\delta(1^+ - 0^+)_{ud}$	198(4)	206(4)	206	210
$\delta(1^+ - 0^+)_{\ell s}$	145(5)	145(3)	145	148
$\delta(1^+ - 0^+)_{ss'}$	118(2)			
$\delta(Q[ud]_{0^+} - \bar{Q}u)$	319(1)	306(7)	306	313
$\delta(Q[\ell s]_{0^+} - \bar{Q}s)$	385(9)	397(1)	397	398
$\delta(Q[\ell s]_{0^+} - \bar{Q}\ell)$	450(6)			

$\rightsquigarrow$ use the bottom estimate for static

$\rightsquigarrow$ use charm-bottom difference as estimate for deviation from static

$\Rightarrow \lesssim \mathcal{O}(7)$MeV deviation

- Overall, very good agreement observed.

A step further - Diquark structure

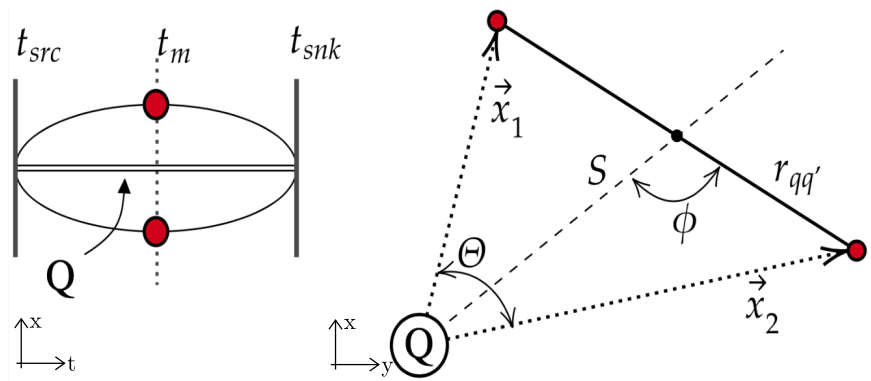
We can access (good) diquark structure information through density-density correlations:

$$C_\Gamma^{dd}(\vec{x}_1, \vec{x}_2, t) = \left\langle \mathcal{O}_\Gamma(\vec{0}, 2t)\ \rho(\vec{x}_1, t)\rho(\vec{x}_2, t)\ \mathcal{O}_\Gamma^\dagger(\vec{0}, 0) \right\rangle$$

$\rightsquigarrow \mathcal{O}_\Gamma = q^c C\Gamma q$ and $\rho(\vec{x}, t) = \bar{q}(\vec{x}, t)\gamma_0 q(\vec{x}, t)$

$\rightsquigarrow t_m = (t_{snk} + t_{src})/2$ to minimize excited states

Main tool: Correlations between two light quarks' relative positions to the static quark



$\rightsquigarrow \vec{r}_{ud} = \vec{x}_2 - \vec{x}_1$ and $\vec{S} = (\vec{x}_1 - \vec{x}_2)/2$

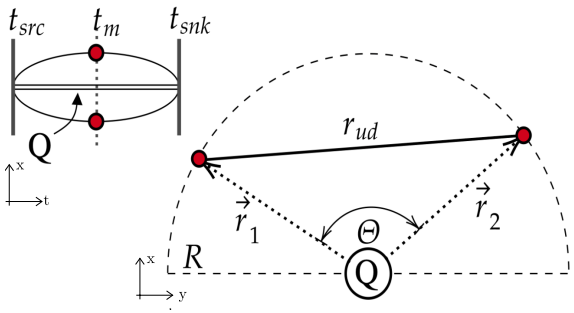
$$\rho_2(r_{ud}, S, \phi; \Gamma) = C_\Gamma^{dd}(\vec{x}_1, \vec{x}_2, t_m)$$

Note, when S and r_{ud} fixed, distance between static quark Q and light quarks q, q' is

- Minimized for $\phi = \pi$, possible disruption due to Q is largest
- Maximized for $\phi = \pi/2$, possible disruption due to Q is smallest

Good diquarks - Attraction, radius and spatial properties

Mode 1: Attraction and radius



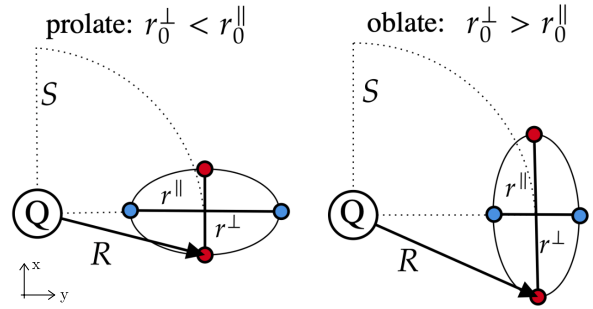
Setting $\phi = \pi/2$:

- $|\vec{x}_1| = |\vec{x}_2| = R$, use R, Θ:
$$\rho_2^{\perp}(R, \Theta) = \rho_2(r_{ud}, S, \pi/2)$$
- Distance between quarks:
$$r_{ud} = R\sqrt{2\,(1 - \cos(\Theta))}$$

Goals:

- Probe diquark attraction effect.
$\Rightarrow$ Visible increase in $\rho_2^{\perp}$ for small Θ at any fixed R.
- $\rho_2^{\perp}(R, r_{ud}) \sim \exp(-r_{ud}/r_0)$
$\Rightarrow$ "characteristic diquark size" r_0.

Mode 2: Radial/Tangential shape



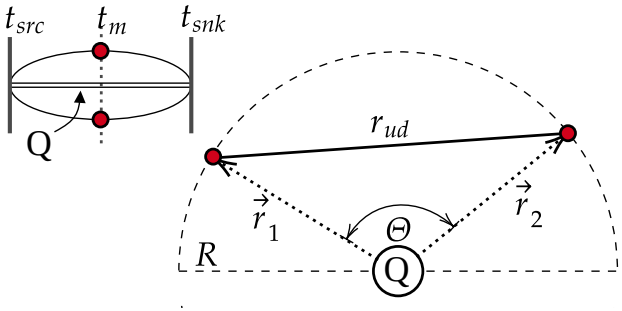
As opposed to before $R \neq$ *fixed*:

- $\phi = \pi$: radial correlation, size $\rightsquigarrow r_0^{\parallel}$
- $\phi = \pi/2$: tangential, size $\rightsquigarrow r_0^{\perp}$
- $r_0^{\perp}/r_0^{\parallel}$ gives information on shape:
  - $= 1$, spherical
  - $\neq 1$, prolate/oblate

Goals:

- Probe $J = 0$ nature of good diquark (spherical, S-wave expectation)
- Diquark polarisation through heavy quark?

Mode 1 - Diquark attraction effect



Two limiting cases for the two quarks:

- $\cos(\Theta) = 1$ on top of each other
- $\cos(\Theta) = -1$ opposite each other

"Lift" as qualitative criterion:

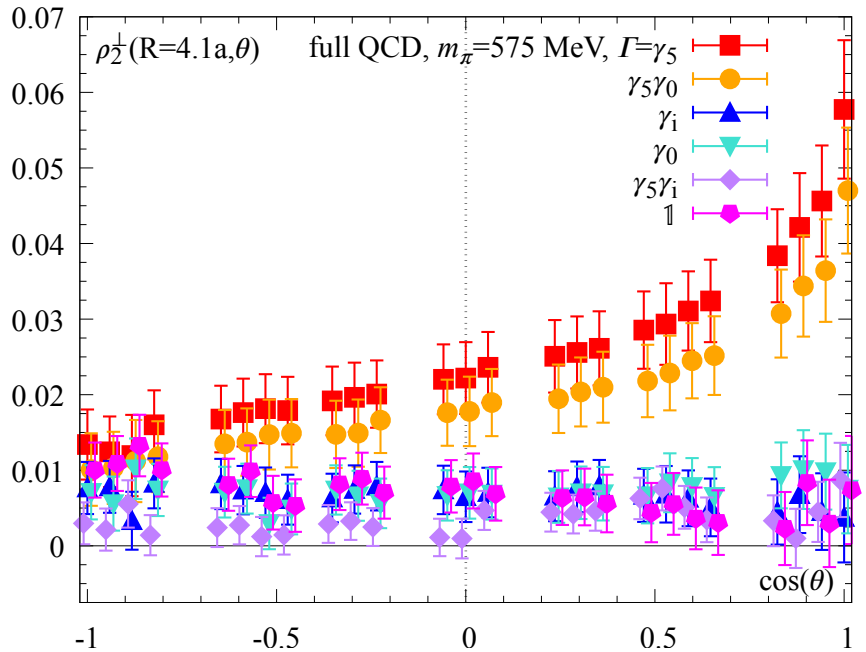
$$\frac{\rho_2^{\perp}(R, \Theta = 0, \Gamma)}{\rho_2^{\perp}(R, \Theta = \pi/2, \gamma_5)}$$

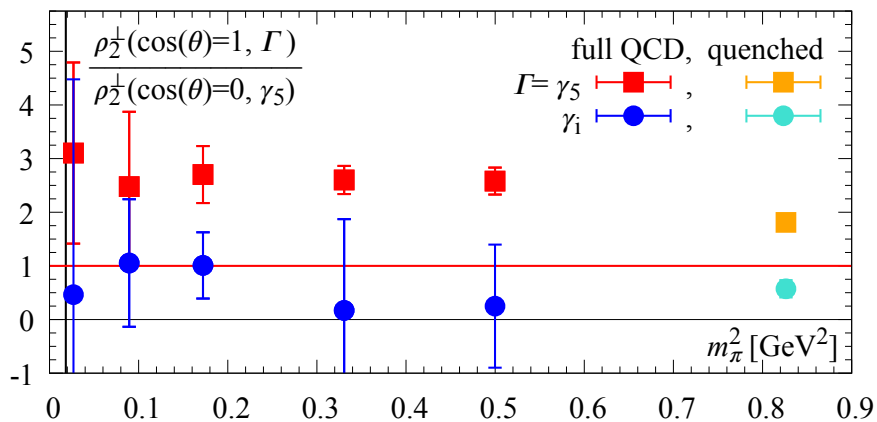
Increase observed in good diquark only

In the good diquark channel

$\Rightarrow$ Quarks spatially correlated.
$\Rightarrow$ Indication of attraction.

Spatial correlation over Θ

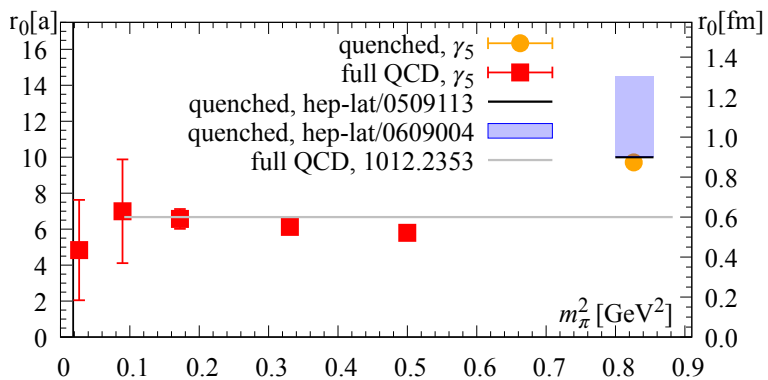




Mode 1 - Good diquark size

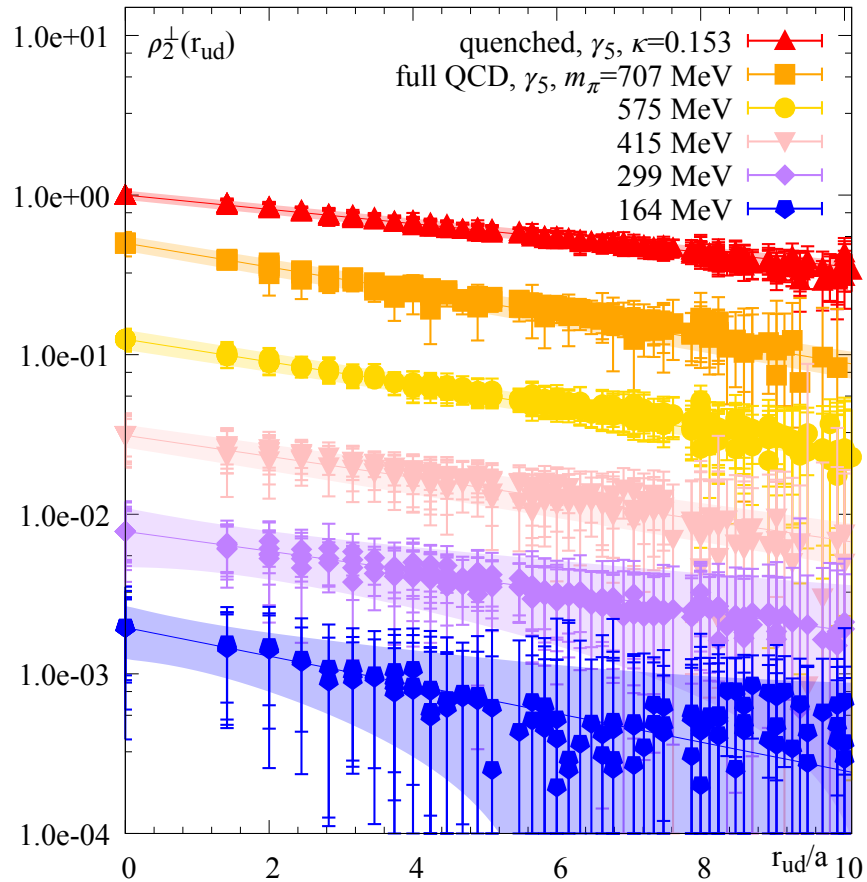
- Expectation:
  $\rho_2^{\perp}(R, r_{ud}) \sim \exp(-r_{ud}/r_0)$
- Need to control:
  - Q-interference $\rightsquigarrow$ keep $r_{ud} < R$
  - periodicity effects $\rightsquigarrow$ $L = 5r_0$ ok
- Fit form:
  $A(R, r_{ud} = 0) \sim \exp(-R/r_0)$
  $\Rightarrow$ Data well described by (single) exponential Ansatz

Size dependence $r_0(m_\pi)$



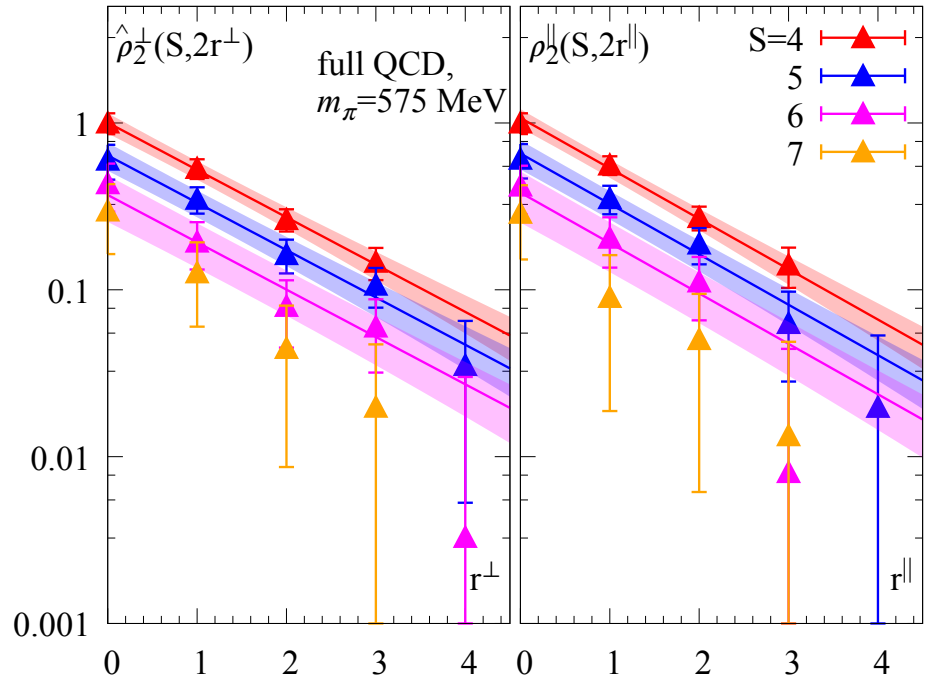
$r_0 \simeq \mathcal{O}(0.6)\text{fm} \sim r_{\text{meson, baryon}}$ (1604.02891)

Spatial correlation over r_{ud}



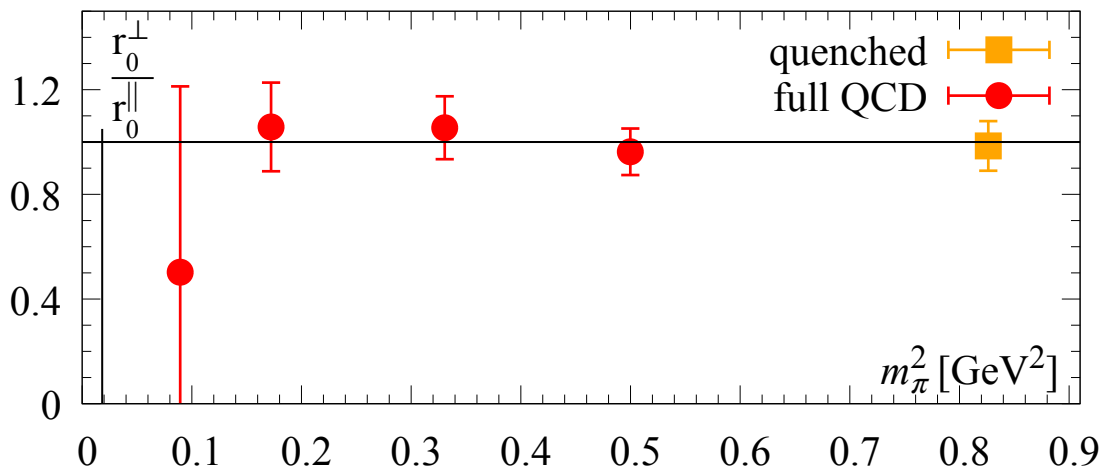
- $r_{ud} = 0$ normalised, offset for each m_π
- all R shown simultaneously
- combined fits over $\forall R$ with shared r_0

Mode 2 - Diquark shape



Radial/Tangential size results at $m_\pi = 575$MeV.

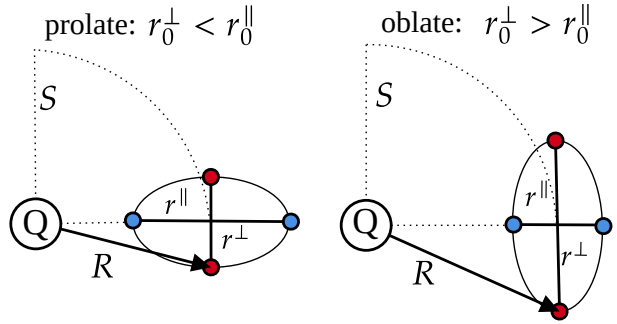
- Get radial and tangential radii $r_0^{\parallel}$, $r_0^{\perp}$
- Ratio $r_0^{\perp}/r_0^{\parallel}$ sensitive to distortions
  - $= 1$, spherical
  - $\neq 1$, prolate/oblate
- Ratio $\simeq 1$ for all $m_\pi \Rightarrow$ spherical
- Consistent w/ scalar, $J = 0$, shape
- No diquark polarization through Q observed

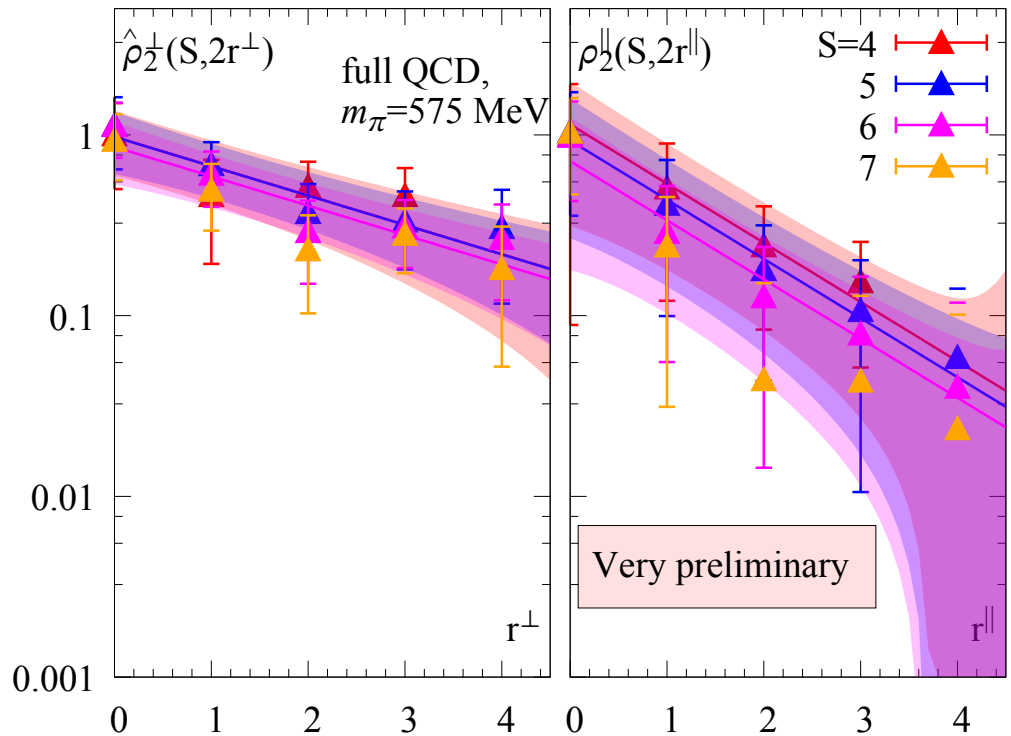


Mode 2 - Testing polarization with lighter spectator quarks

[New and preliminary]

- Do we see the diquark attraction also when $m_Q \neq \infty$?
- Expecting significant polarization effects with lighter spectator quarks.
  - Recall: Ratio $r_0^\perp / r_0^\parallel$ sensitive to distortions
  - Q =static quark: $r_0^\perp \simeq r_0^\parallel$
  - Test replacing static quark with strange quark: $m_Q \simeq m_s$





Radial/Tangential size results at m_π = 575MeV where Q=strange quark.

- Low statistics calculation at $m_\pi = 575$ MeV
- Preliminary finding: $r_0^\perp \neq r_0^\parallel$
- "Deformation" of diquark attraction(?)

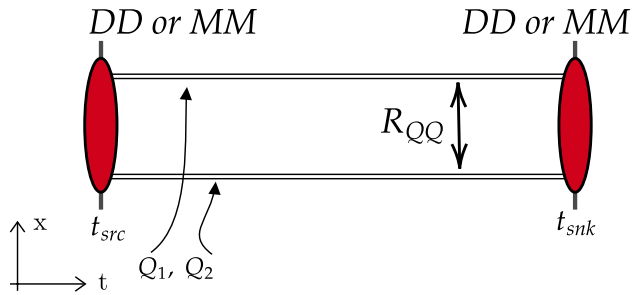
$\Rightarrow$ More work required! Big question: Systematics and localization of the spectator quark.

see also [hep-lat/9208025], [0902.4046]

- Our studies show that lattice QCD can access diquark probes effectively.
- They reveal:
  - The splittings, which match closely with experimental findings.
  - Spatial quark-quark correlation consistent with an attraction in the "good" diquark configuration.
  - No distortion of this attraction with heavy spectator quarks.
  - Preliminary studies with light spectator quarks indicate distortion.
- What about diquarks in tetraquarks?

[New and preliminary]

Mode 3: Effective binding energy



Embed in the T_{QQ} tetraquark:

- The two static quarks Q_1, Q_2 are separated by a distance R_{QQ}.
- Compute the 2-point function:

$$C_{\mathcal{O}}(t, R_{QQ}) \ , \quad \mathcal{O} \in DD, MM$$

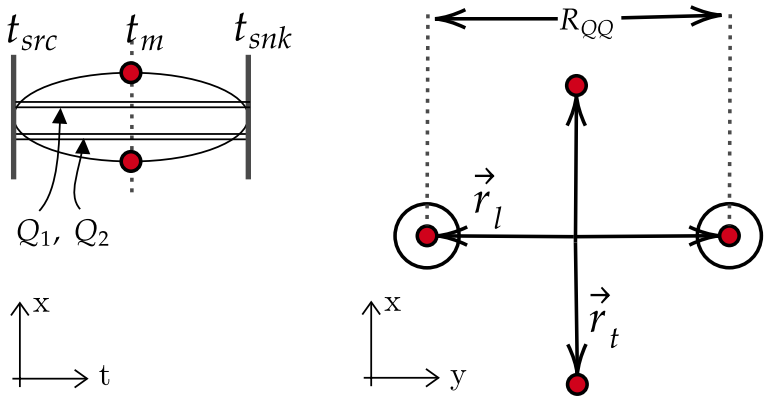
- Effective binding energy (=splitting):

$$\Delta E(R_{QQ}) = E_{\mathcal{O}}(R_{QQ}) - 2 \cdot E_{qQ}$$

Idea:

- Track binding over R_{QQ}.
- Connection to BO EFT.
- Later step: GEVP analysis.

Mode 4: Diquarks in T_{QQ}



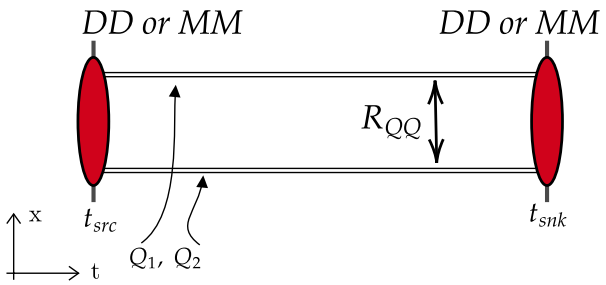
QQ-axis defines special direction:

- Correlation transverse to axis ($\vec{r}_t$).
- Correlation longitudinal to axis ($\vec{r}_l$). Here: At sample location as the static quarks.

Idea:

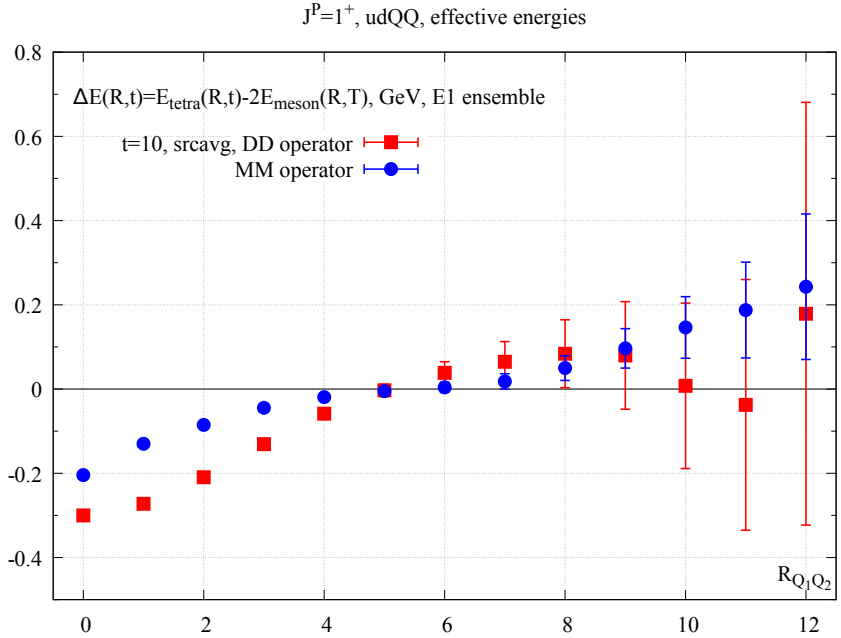
- Probe the quark-quark correlation shape (diquark effect?).
- Does the quark-quark attraction persist? Are there special directions?

Mode 3 - Effective binding energy



[New and preliminary]

Effective binding energy over R_{QQ}



- E1 ensemble: m_π =707 MeV.
- Only operator diagonal *DD* and *MM* correlators.
- Proper statistical analysis pending:

$$E(R) = E_{eff}(t = 10, R)$$

Preliminary:

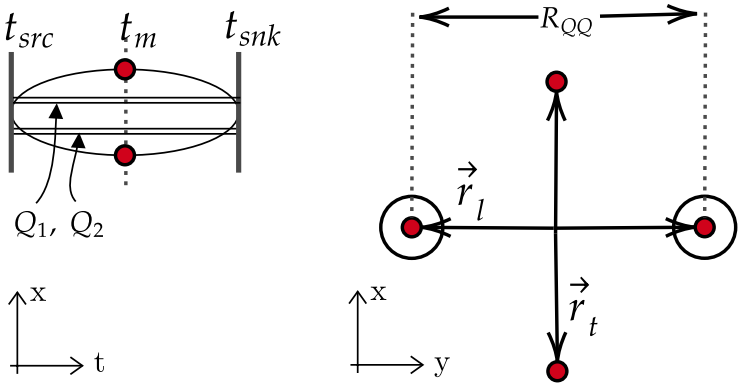
- Only one heavy ensemble analyzed.
- Only operator diagonal *DD* and *MM* correlators, no GEVP.
- Instead of fitting the energy a **representative value in the plateau** of the binding correlator effective energy was picked.

Observations to study and confirm:

- $R_{QQ} \lesssim 4-6$:
  DD lowest in $\Delta E(R_{QQ}) < 0$,
- $R_{QQ} > 6$:
  MM tracks $\Delta E(R_{QQ}) = 0$.
- $R_{QQ} > 8$:
  Loss of statistical significance.

Mode 4 - Diquarks in tetraquarks

[New and preliminary]



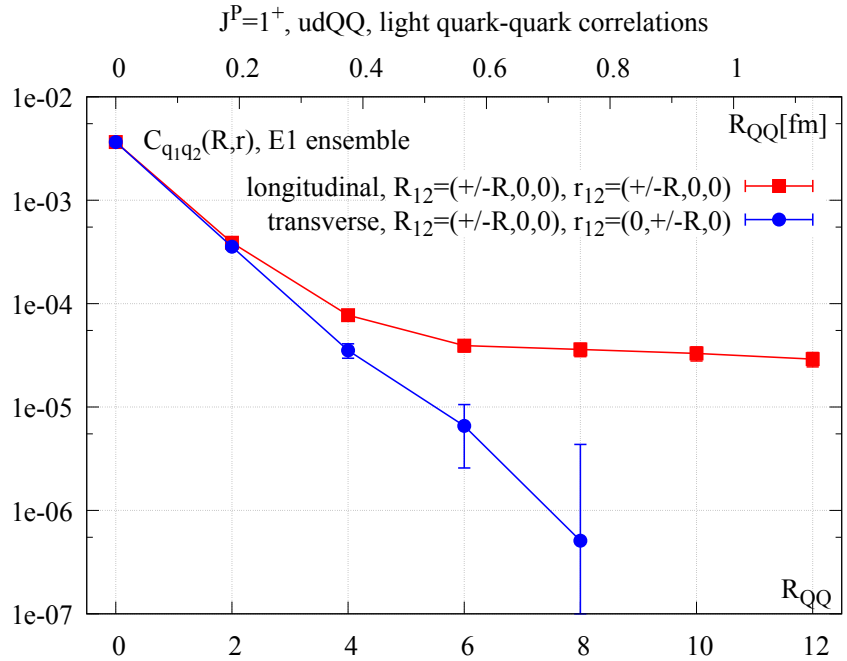
Preliminary:

- Only one heavy ensemble analyzed.
- Only operator diagonal DD correlators.
- Only small subset of possible tomography points selected $C(R_{QQ}, r_{l,t} = R_{QQ})$.

Observations to study and confirm:
$R_{QQ} \lesssim 4$:

- Longitudinal and transverse match.
- Diquark attraction like in baryon case?

Quark-quark correlations over R_{QQ}



- E1 ensemble: m_π =707 MeV.
- Only operator diagonal DD correlators.

$R_{QQ} \gtrsim 4$:

- Longitudinal correlation flattens.
- Transverse continues exponential drop.
- Re-arrangement of light quark d.o.f?

Summary - Diquarks and tetraquarks in lattice QCD

T_{QQ} tetraquarks and diquark picture predictions

- Community results are converging.
- In general, all diquark-HQSS binding mechanism predictions observed.
- But how do we go from T_{bb} to T_{cc}? Is it really the diquarks?

Diquarks in lattice QCD

Studying diquarks in isolation using static spectators reveals:

- Good diquark attraction is observed and behaves as predicted for static specators.
- Non-static spectators may induce deformation of diquark correlations (preliminary).
- Still an effective mechanism for T_{cc}?

Diquarks in tetraquarks

Studying diquarks in tetraquarks with static spectators

- Gives insight into the internal structure of T_{QQ} tetraquarks.
- Combining **quark-quark correlation measurements** with heavy quark-heavy quark **separation dependent energies** gives a picture for diquark formation at short distances in T_{QQ}.
- Current studies do not confirm or permit an interpretation yet. But work is ongoing.

If diquark correlations are real, then where should physics best look to find, understand, and exploit them?

From lattice QCD we learn:

- Static-light-light baryons provide a controlled environment for isolating and measuring diquark correlations.
- Spectroscopy and density-density measurements provide complementary evidence.
- Varying the spectator mass probes the environmental dependence of those correlations.
- Doubly-heavy tetraquarks, with controlled heavy-quark separation, offer a route toward connecting diquark correlations to multi-quark binding.

Thank you for your attention.



Further material

One more thing: Study HQSS through spectrum with static spectator

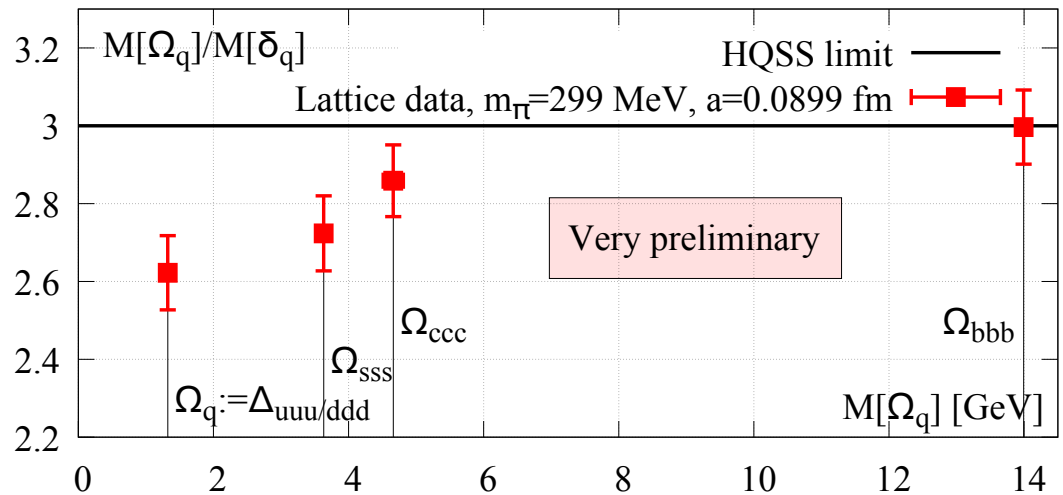
New and very preliminary

- Consider the $qq'Q$ and $q\bar{Q}$ difference (where $q \in l, s, c$):

$$\delta_q(t) = C^{qqQ}_{\gamma_i}(t) - C^{q\bar{Q}}_{\gamma_i}(t) \quad \rightsquigarrow \text{ determine on the lattice: } M[\delta_q]$$

- Diquark attraction not expected $\rightsquigarrow$ Opens up new avenue to study HQSS in hadrons.
- For example, relate the $qqQ - qQ$ to the corresponding Ω-type baryon:

$$\frac{M[qqq]}{(M[qqQ] - M[q\bar{Q}])} \sim M[\Omega_q]/M[\delta_q] \quad \overset{\text{HQSS limit}}{\longrightarrow} \quad M[\Omega_q]/M[\delta_q] = 3$$



- At $m_\pi = 299$ MeV we find (low statistics)
  - HQSS expectation well fulfilled for b-type quark masses
  - Deviation for c-type quark masses and lower

Technical note: b-type calculation performed using NRQCD, masses determined via dispersion in $\vec{p}^2$.

5-parameter pheno model for heavy quark mass dependence

$\bar{b}'\bar{b}'$:

$$\Delta E_{ud\bar{b}'\bar{b}'} = \frac{C_0}{2r} + C_1^{ud} + C_2^{ud}\,(2r) + (23\ \text{MeV})\,r,$$
$$\Delta E_{\ell s\bar{b}'\bar{b}'} = \frac{C_0}{2r} + C_1^{\ell s} + C_2^{\ell s}\,(2r) + (24\ \text{MeV})\,r$$

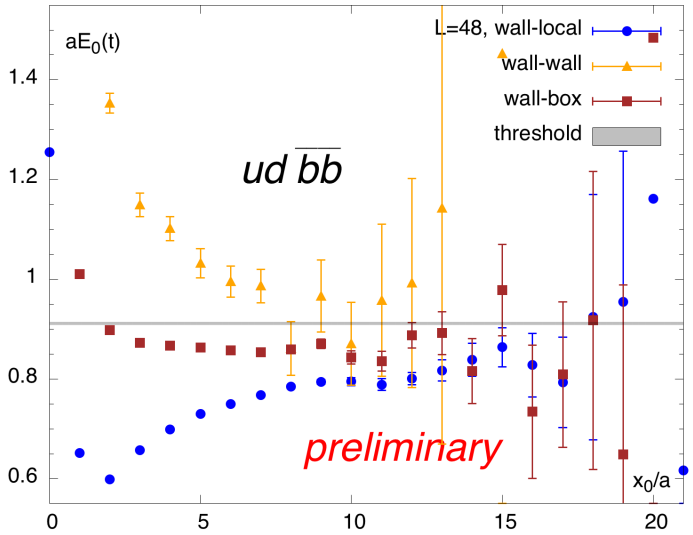
$\bar{b}'\bar{b}$, $r < 1$:

$$\Delta E_{ud\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}\,(1+r) + (34\ \text{MeV} - 11\ \text{MeV}\,r)\,,$$
$$\Delta E_{\ell s\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}\,(1+r) + (34\ \text{MeV} - 12\ \text{MeV}r)$$

$\bar{b}'\bar{b}$, $r > 1$:

$$\Delta E_{ud\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}\,(1+r) + (34\ \text{MeV}\,r - 11\ \text{MeV})\,,$$
$$\Delta E_{\ell s\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}\,(1+r) + (36\ \text{MeV}\,r - 11\ \text{MeV})$$

(Heavy) quark mass dependence of binding - Revisit systematics



⇝Hudspith, AF et al. ('20)

- Systematic **overestimate** of binding energies?
- Our studies used Coulomb gauge-fixed wall-local correlators
- Issue: WL-correlators approach the ground state from below.
- **Update:** Study that includes correlators using a wall-box approach to increase ground state overlap.

Box-sink construction

Correlators made of Coulomb gauge fixed wall sources and local sinks are known to have exceptionally good signal-to-noise ratios. Properties, benefits and drawbacks:

a. little resource requirements
b. benefit from mom. proj.
c. alternating sign in spec. decomp.
d. ground state reached from below!
e. GEVP is non-Hermitian
f. wall-wall correlators very noisy, but do not have problems *c*./*d*.

- Addressing *c*./*d*.

  → Sum the propagator over a sphere in R at the sink:

$$S^B(x,t) = \frac{1}{N} \sum_{r^2 \leq R^2} S(x+r,t)$$

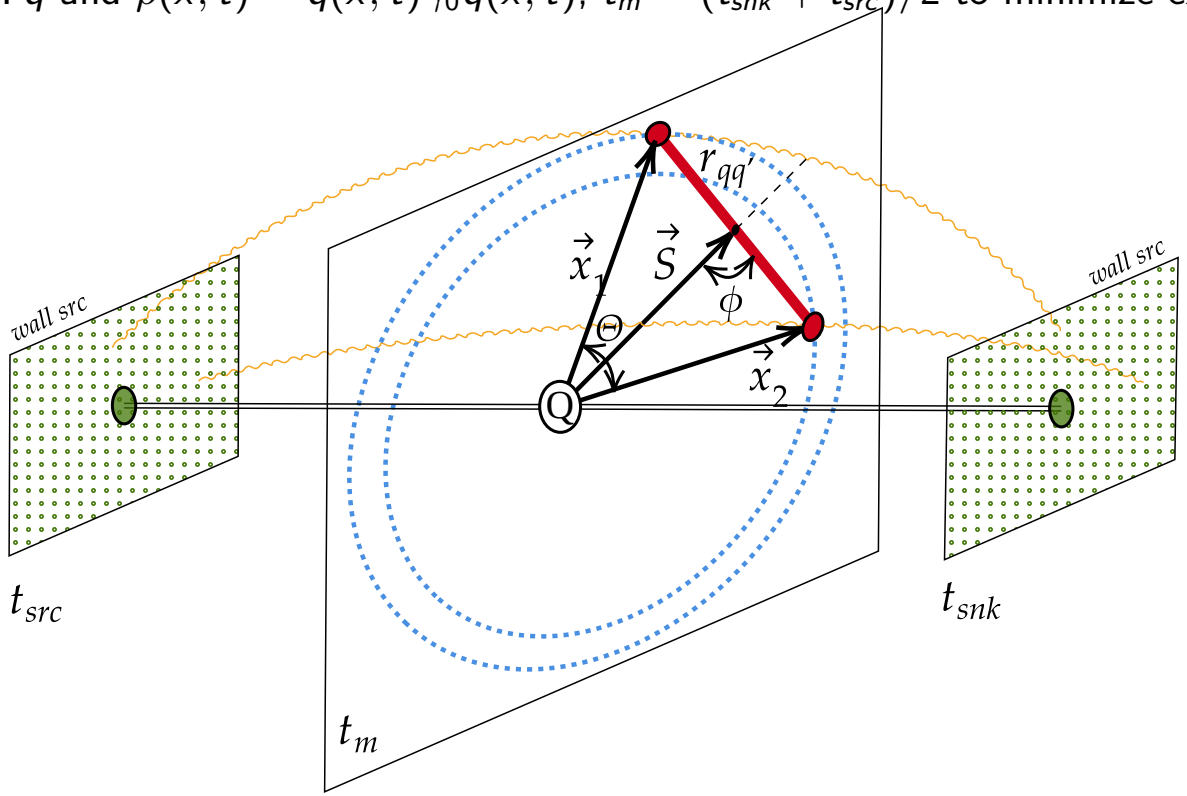
  → Tune R to reduce excited states

Diquarks - spatial correlations

We can access (good) diquark structure information through density-density correlations:

$$C_\Gamma^{dd}(\vec{x}_1, \vec{x}_2, t) = \left\langle \mathcal{O}_\Gamma(\vec{0}, 2t)\ \rho(\vec{x}_1, t)\rho(\vec{x}_2, t)\ \mathcal{O}_\Gamma^\dagger(\vec{0}, 0) \right\rangle := \rho_2(r_{ud}, S, \phi; \Gamma)$$

$\rightsquigarrow$ $\mathcal{O}_\Gamma = q^c C\Gamma q$ and $\rho(\vec{x}, t) = \bar{q}(\vec{x}, t)\gamma_0 q(\vec{x}, t)$, $t_m = (t_{snk} + t_{src})/2$ to minimize excited states



Main tool: Correlations between two light quarks' relative positions to the static quark.
Note, when S and r_{ud} fixed, distance between static quark Q and light quarks q, q' is

- Minimized for $\phi = \pi$, possible disruption due to Q is largest
- Maximized for $\phi = \pi/2$, possible disruption due to Q is smallest