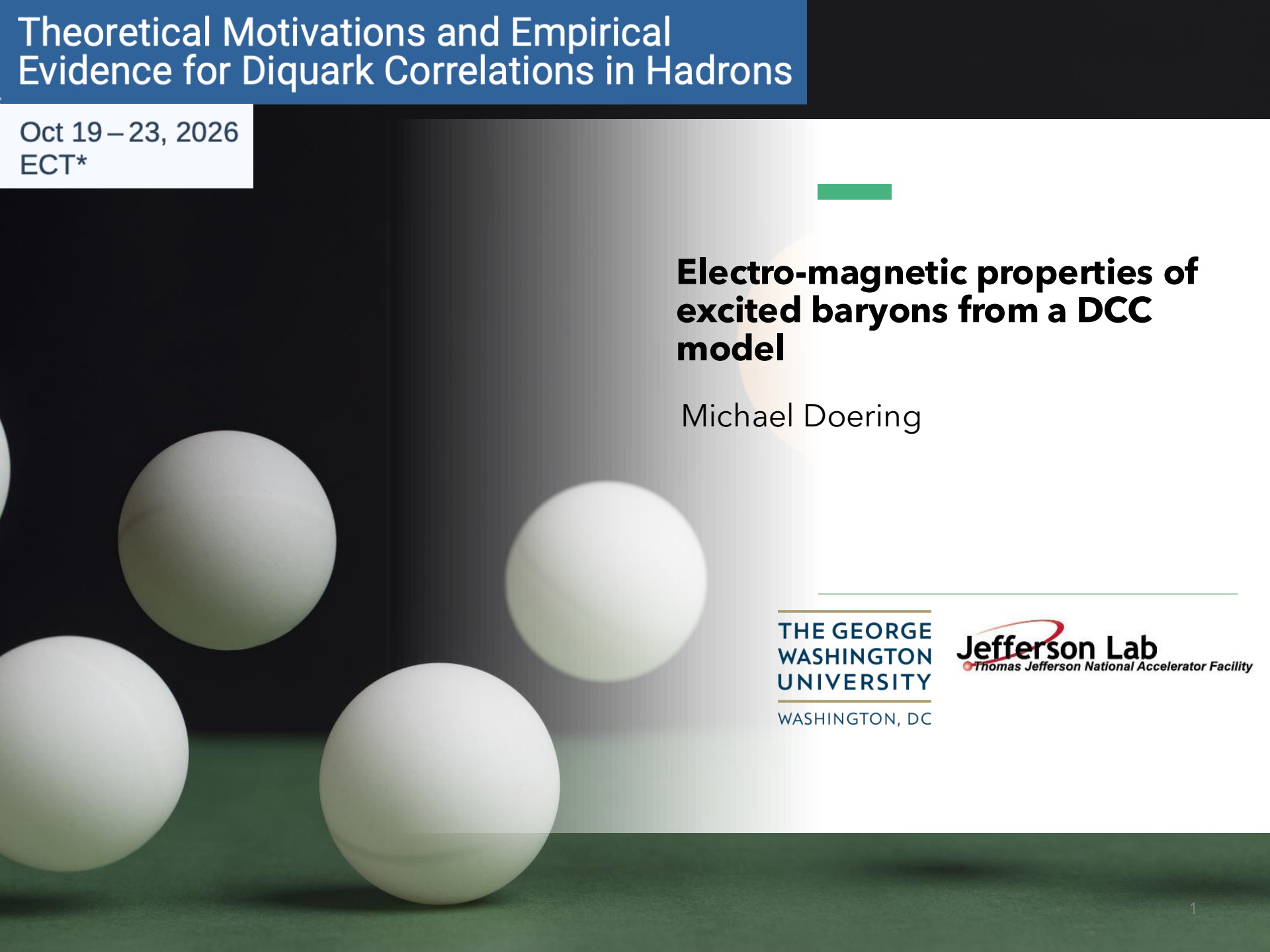
Theoretical Motivations and Empirical Evidence for Diquark Correlations in Hadrons

Oct 19 – 23, 2026
ECT*



Electro-magnetic properties of excited baryons from a DCC model

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WASHINGTON, DC

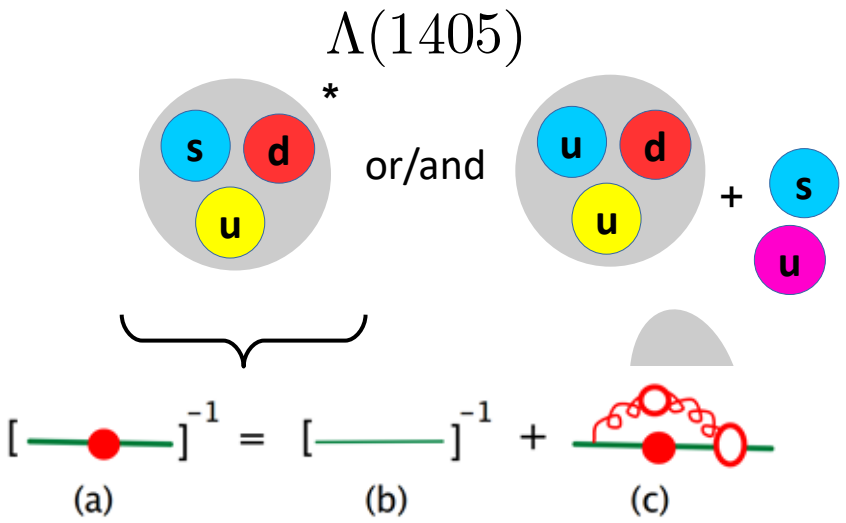
Jefferson Lab
Thomas Jefferson National Accelerator Facility

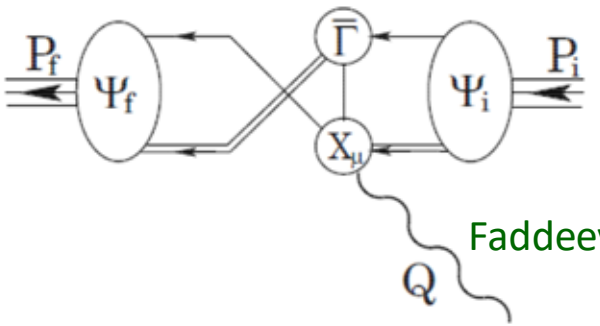
QCD at low energies

Non-perturbative dynamics → rich spectrum of excited states

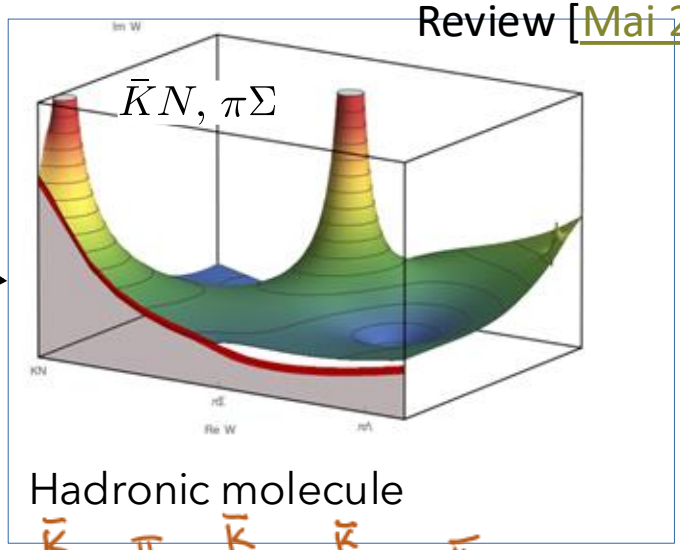
How many states are there? → missing resonance problem (does it exist?)

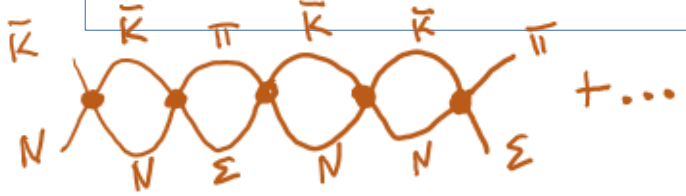
What are they? → 2-quark/3-quark, hadron molecules, …





Review [Mai 2021]

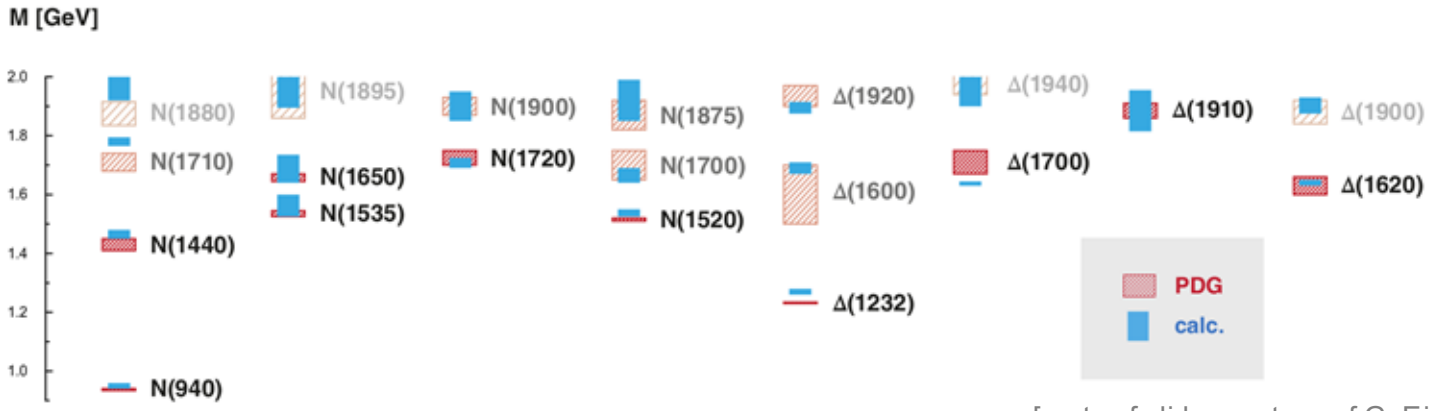




Faddeev Eq. / DSE (Binosi, Cloet, Chang, Eichman, Roberts,…);

Light baryons from diquark dynamics

Quark-diquark with reduced pseudoscalar + vector diquarks: [Eichmann (2016)]

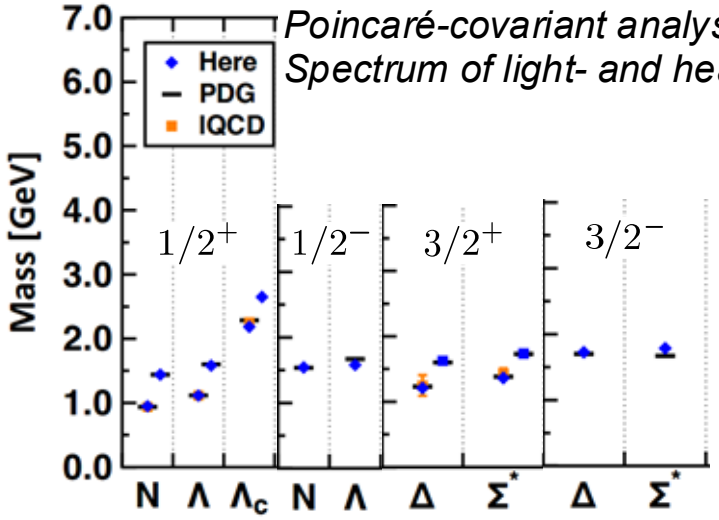


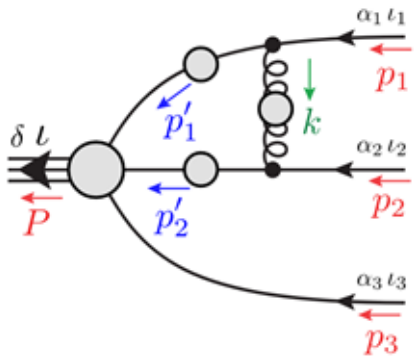
Not much of a missing resonance problem left…

[parts of slide courtesy of G. Eichmann, Few Body 2018]

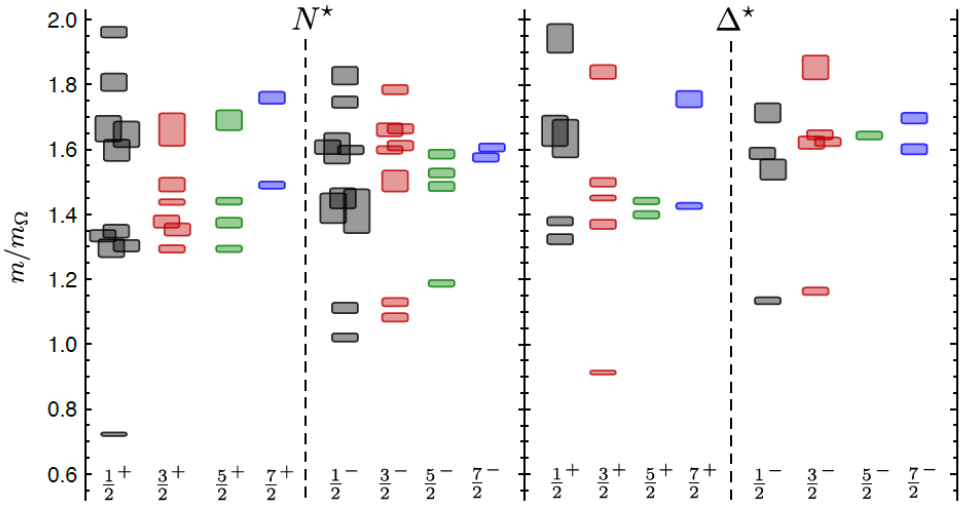
$J^P = \frac{1}{2}^+ \quad \frac{1}{2}^- \quad \frac{3}{2}^+ \quad \frac{3}{2}^- \quad \frac{3}{2}^+ \quad \frac{3}{2}^- \quad \frac{1}{2}^+ \quad \frac{1}{2}^-$

Poincaré-covariant analysis of heavy-quark baryons, [Qin 2018]
Spectrum of light- and heavy-baryons, [Qin 2019]



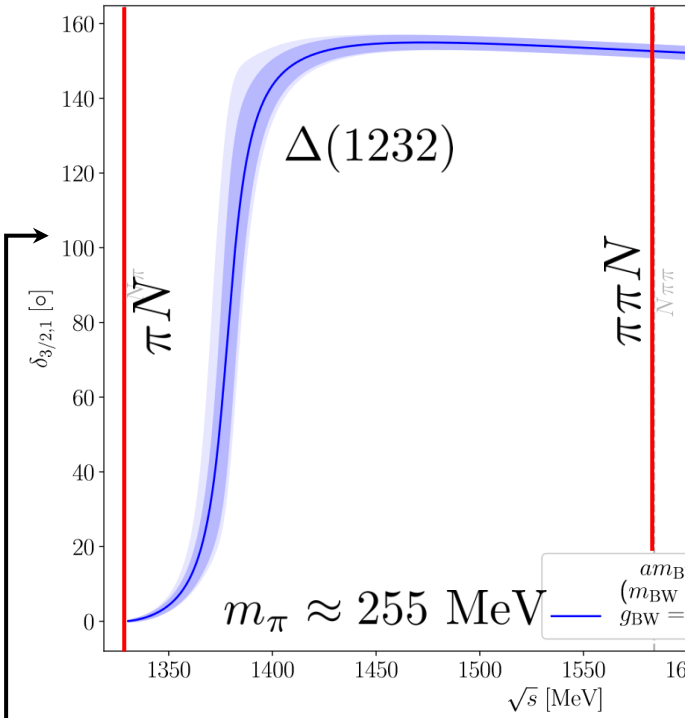


Lattice QCD for excited baryons



$m_\pi = 396$ MeV [Edwards *et al.*, Phys.Rev. D84 (2011)]

- Pioneering spectroscopic calculations
- Information on existence, width & properties of resonances requires
  - Meson-baryon interpolating operators
  - Detailed finite-volume analysis



[G. Silvi et. al., arXiv: 2101.00689]

See also: Bulava et al., [2208.03867] → Morningstar talk on Monday

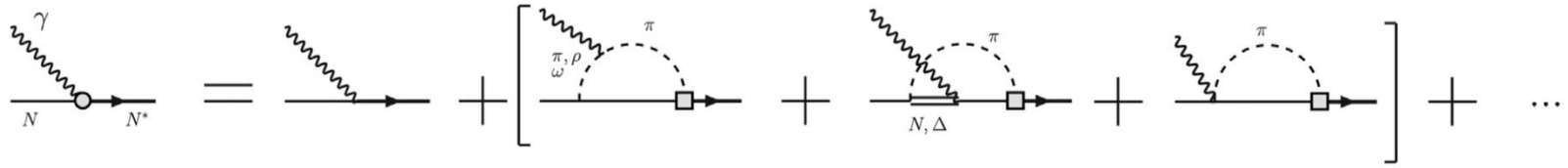
How about $\pi\pi N$? 3B Dynamics?

Phenomenology of the baryon spectrum

Different DCC approaches (baryons)

[https://collaborations.fz-juelich.de/ikp/meson-baryon/main
https://www.rcnp.osaka-u.ac.jp/~anl-osk/]

- EBAC and ANL/Osaka analysis.
- Juelich-Bonn (JuBo)-Washington/JBW analysis
  - Both share many aspects of channel space and formalism but differ in some details such as coupling of the photon. ANL/Osaka:



  - ANL/Osaka has been extended to analysis of $\pi\pi$N final, KY, and neutrino-induced reactions
  - JBW analysis focused on electroproduction reactions, also extensions to charm
- Related approaches in 3-body meson physics (Nakamura, IVU) and infinite-volume physics for lattice QCD (FVU)

Three-body unitarity with isobars *

[1706.06118]

$$\langle q_1,q_2,q_3|(\hat{T}-\hat{T}^\dagger)|p_1,p_2,p_3\rangle = i\int_P \langle q_1,q_2,q_3|\hat{T}^\dagger|k_1,k_2,k_3\rangle\langle k_1,k_2,k_3|\hat{T}|p_1,p_2,p_3\rangle$$

$$\times \prod_{\ell=1}^{3}\left[\frac{\mathrm{d}^4k_\ell}{(2\pi)^4}(2\pi)\delta^+(k_\ell^2-m^2)\right](2\pi)^4\delta^4\left(P-\sum_{\ell=1}^{3}k_\ell\right)$$

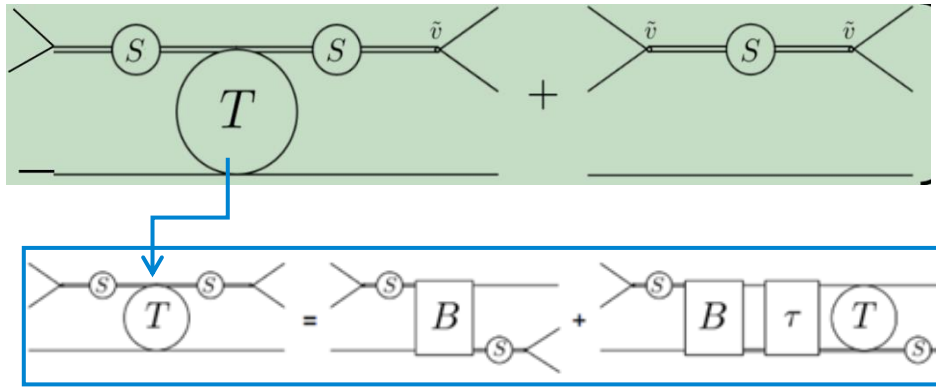
delta function sets all intermediate particles on-shell

Idea: To construct a 3B amplitude, start directly from unitarity (based on ideas of 60's); match a general amplitude to it

* "Isobar" stands for two-body sub-amplitude which can be resonant or not; can be matched to CHPT expansion to one loop if desired. Isobars are re-parametrizations of full 2-body amplitudes [Bedaque] [Hammer]

Three-body unitarity

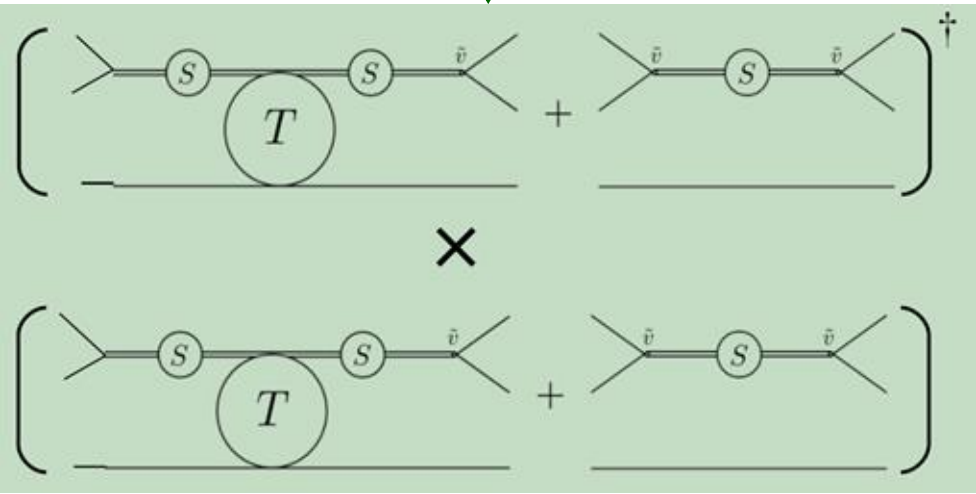
$$\langle q_1, q_2, q_3 | (\hat{T} - \hat{T}^\dagger) | p_1, p_2, p_3 \rangle = i \int_P \langle q_1, q_2, q_3 | \hat{T}^\dagger | k_1, k_2, k_3 \rangle \langle k_1, k_2, k_3 | \hat{T} | p_1, p_2, p_3 \rangle$$



General Ansatz for the isobar-spectator interaction
→ **B & τ** are **new** unknown functions

Three-body unitarity

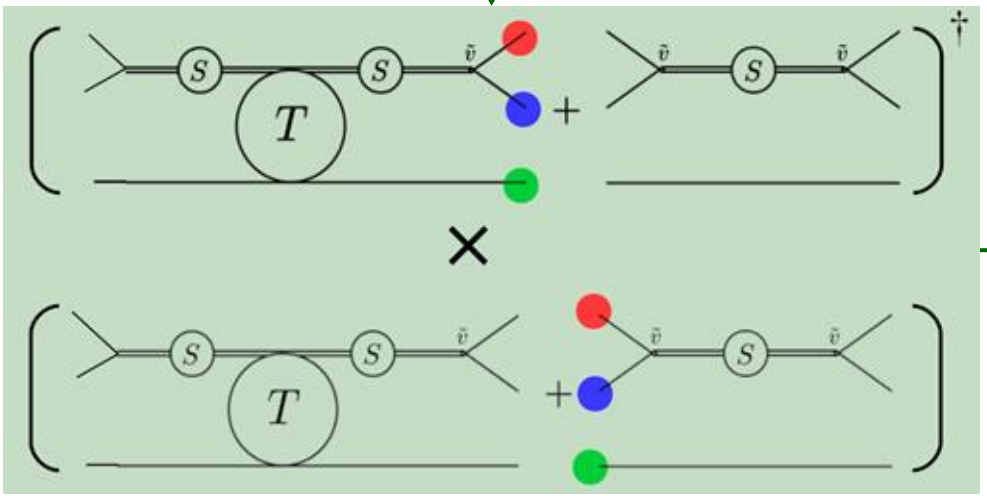
$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1, p_2, p_3\rangle = i\int_P \langle q_1, q_2, q_3|\hat{T}^\dagger|k_1, k_2, k_3\rangle\langle k_1, k_2, k_3|\hat{T}|p_1, p_2, p_3\rangle$$

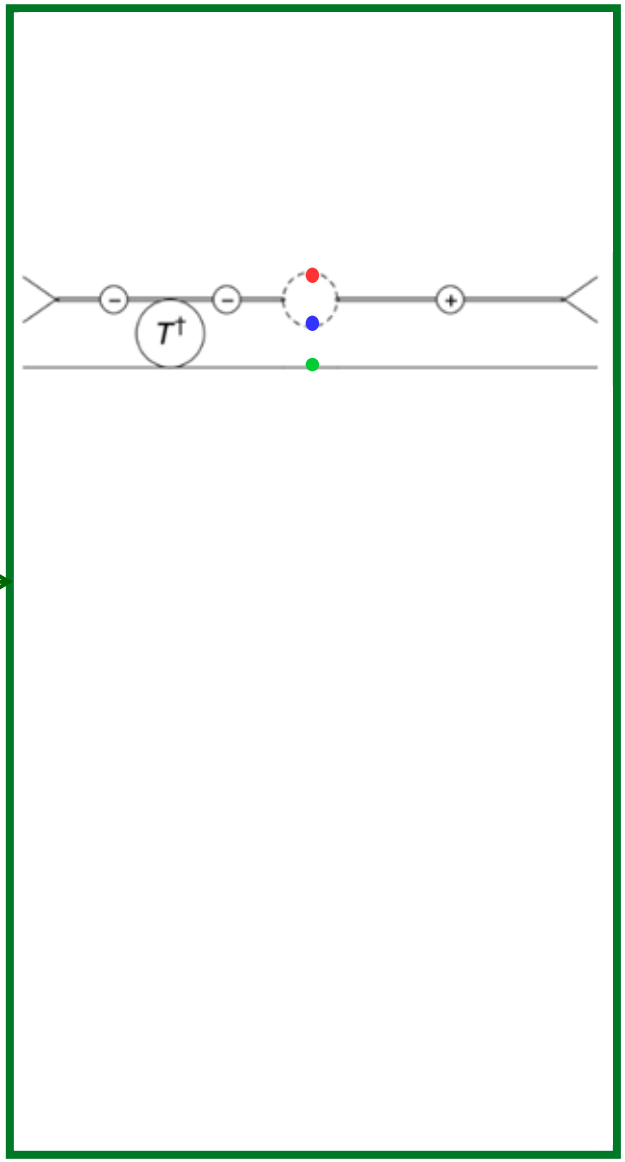


General connected-disconnected structure

Three-body unitarity

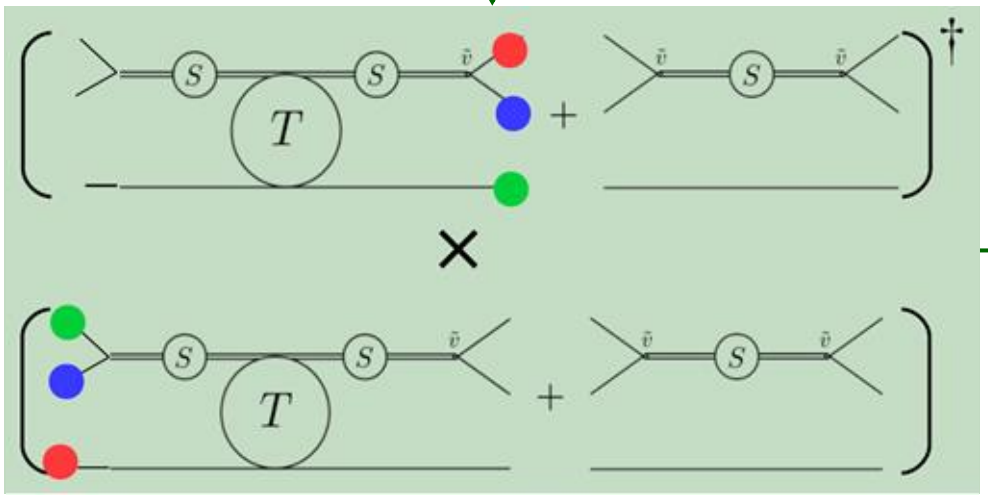
$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1, p_2, p_3\rangle = i\int_P \langle q_1, q_2, q_3|\hat{T}^\dagger|k_1, k_2, k_3\rangle\langle k_1, k_2, k_3|\hat{T}|p_1, p_2, p_3\rangle$$

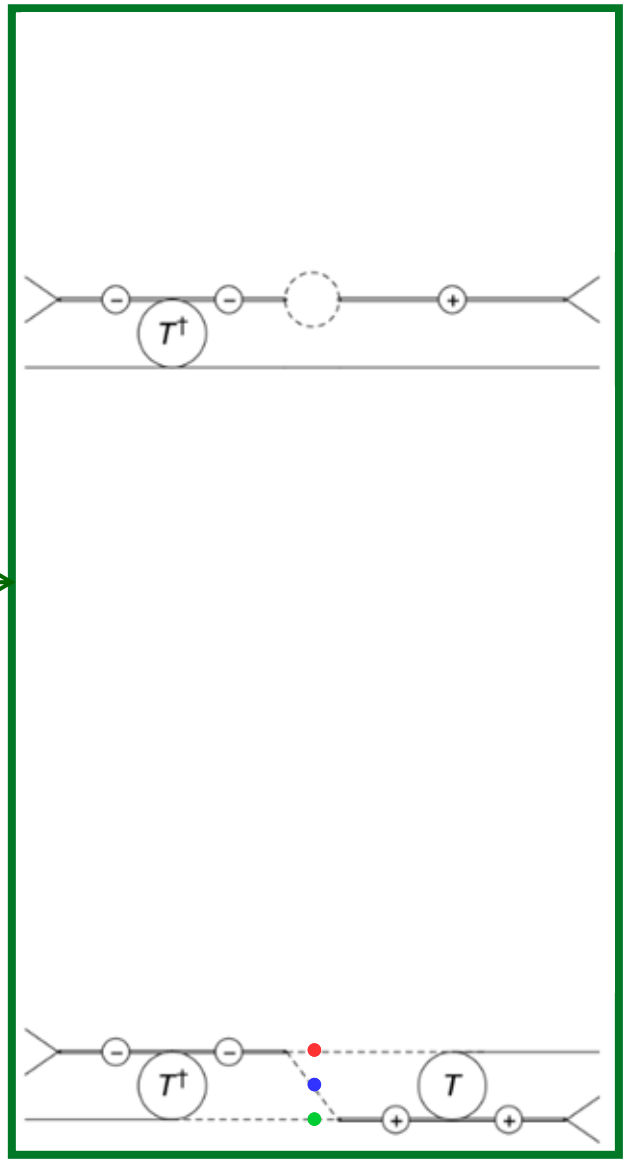




Three-body unitarity

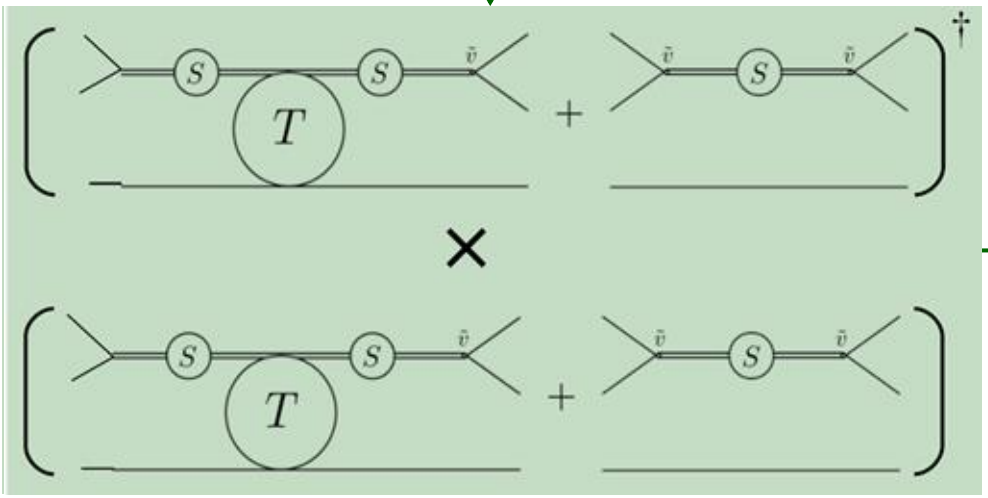
$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1, p_2, p_3\rangle = i\int_P \langle q_1, q_2, q_3|\hat{T}^\dagger|k_1, k_2, k_3\rangle\langle k_1, k_2, k_3|\hat{T}|p_1, p_2, p_3\rangle$$



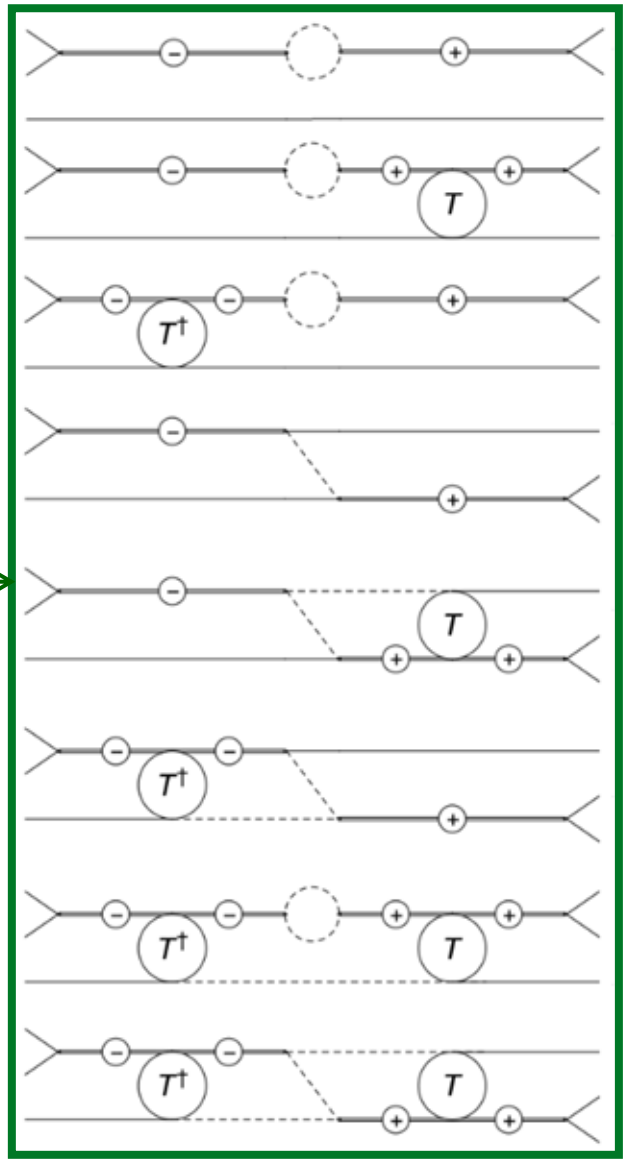


Three-body unitarity

$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1, p_2, p_3\rangle = i\int_P \langle q_1, q_2, q_3|\hat{T}^\dagger|k_1, k_2, k_3\rangle\langle k_1, k_2, k_3|\hat{T}|p_1, p_2, p_3\rangle$$

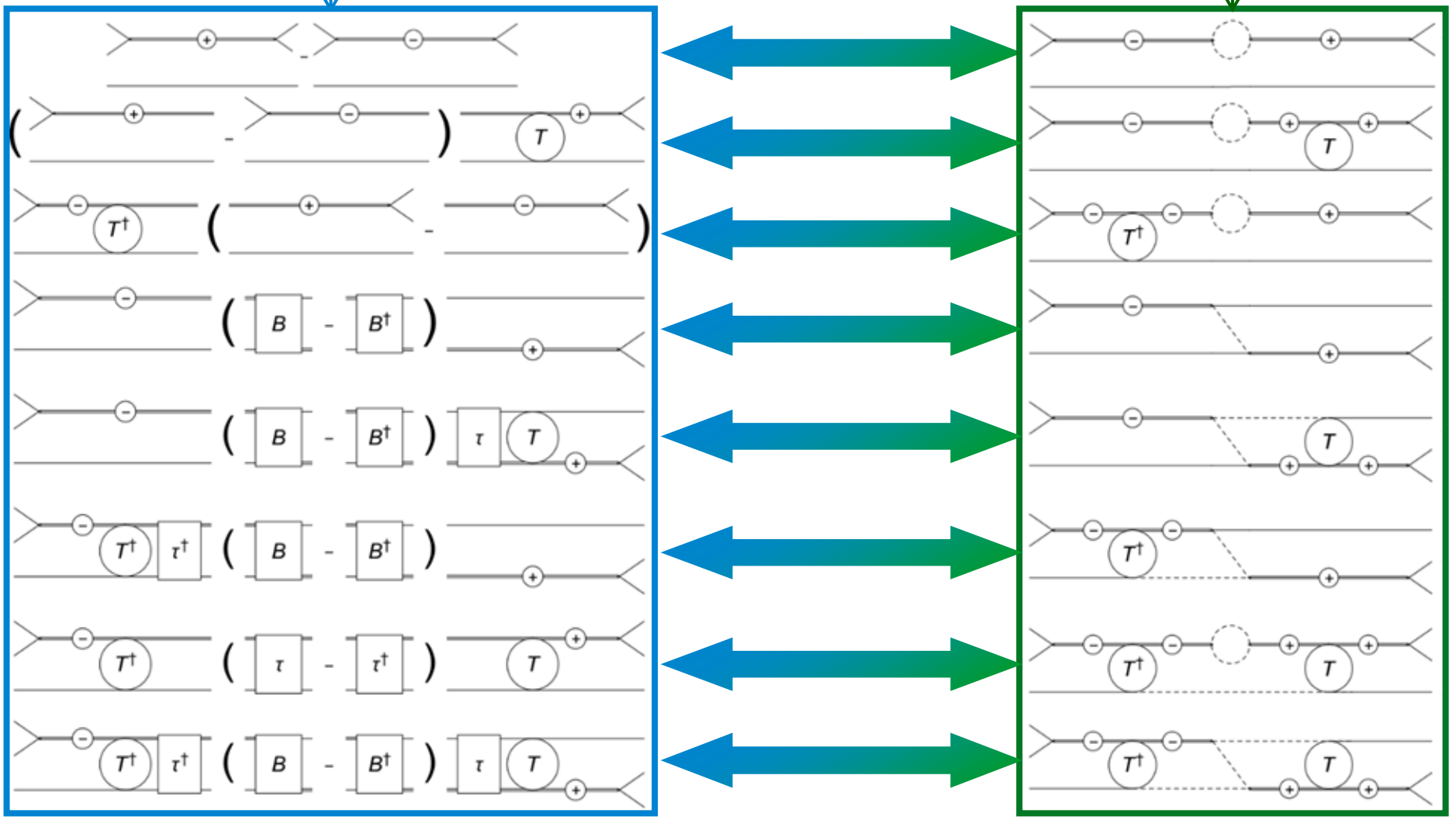


8 topologies



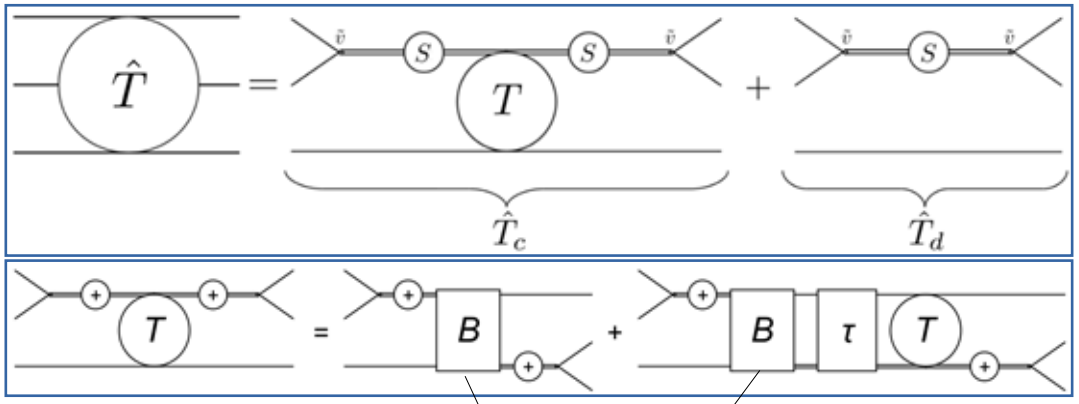
Three-body unitarity

$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1, p_2, p_3\rangle = i\int_P \langle q_1, q_2, q_3|\hat{T}^\dagger|k_1, k_2, k_3\rangle\langle k_1, k_2, k_3|\hat{T}|p_1, p_2, p_3\rangle$$



Scattering amplitude

3 → 3 scattering amplitude is a 3-dimensional integral equation



LS-type

- Imaginary parts of ***B, S*** are fixed by **unitarity/matching**
- B, S are determined **consistently** through 8 different relations

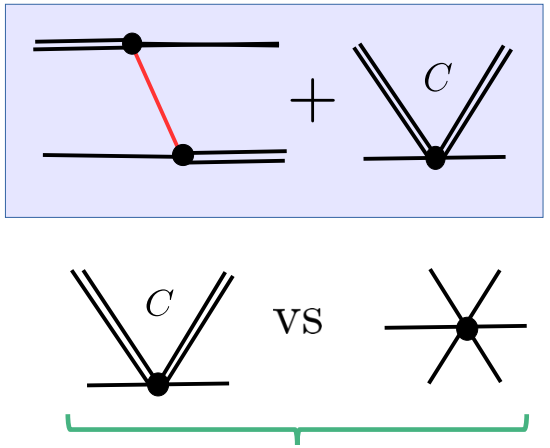
Matching → $\text{Disc}\, B(u) = 2\pi i \lambda^2 \dfrac{\delta\left(E_Q - \sqrt{m^2+\mathbf{Q}^2}\right)}{2\sqrt{m^2+\mathbf{Q}^2}}$

- un-subtracted dispersion relation

$$\langle q|B(s)|p\rangle = -\frac{\lambda^2}{2\sqrt{m^2+\mathbf{Q}^2}\left(E_Q-\sqrt{m^2+\mathbf{Q}^2}+i\epsilon\right)} + C$$

- one-**π** exchange in TOPT → *RESULT, NOT INPUT !*

- One <u>can</u> map to field theory but does not have to. Result is a-priori dispersive.

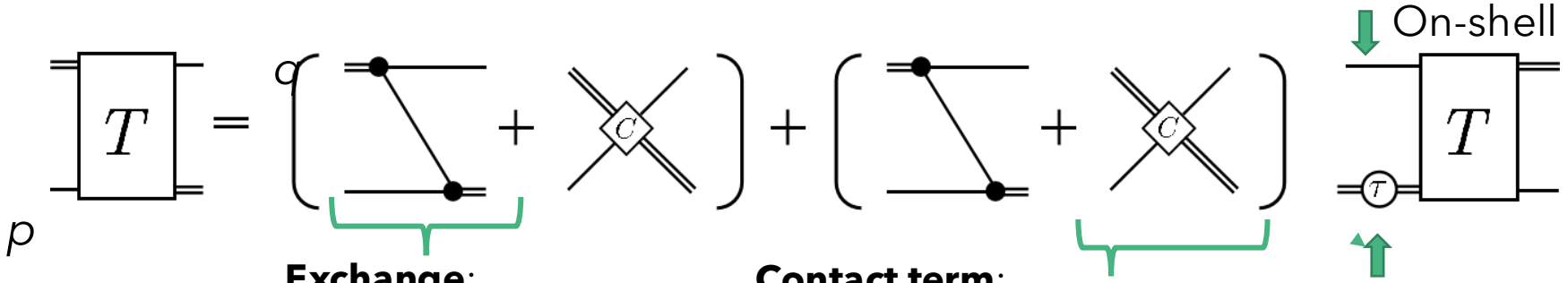


Add. Steps to map to theory might be needed [Brett (2021)]

Scattering equation

- 4-dim BSE

$$\langle q|\,T(s)\,|p\rangle = \langle q|\,B(s)\,|p\rangle + \langle q|\,C(s)\,|p\rangle + \int \frac{d^4k}{(2\pi)^4}\,\langle q|\,(B(s)+C(s))\,|k\rangle\,\tau(\sigma(k))\,\langle k|\,T(s)\,|p\rangle\ ,$$



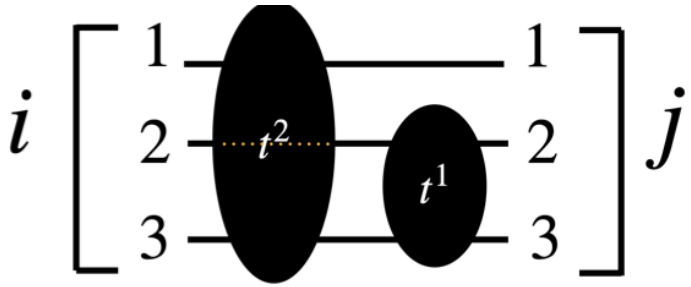
- → 3 dim by setting spectator on-shell

$$T_{ji}(s,\boldsymbol{p}',\boldsymbol{p}) = \tilde{B}_{ji}(s,\boldsymbol{p}',\boldsymbol{p}) + \int \frac{\mathrm{d}^3 l}{(2\pi)^3\,2E_l}\,\tilde{B}_{jk}(s,\boldsymbol{p}',\boldsymbol{l})\,\tilde{\tau}_{kk'}(\sigma_l)\,T_{k'i}(s,\boldsymbol{l},\boldsymbol{p})$$

- Avoids overlapping on-shell singularities in 4D but remains a true integral equation as required by unitarity (never algebraic)

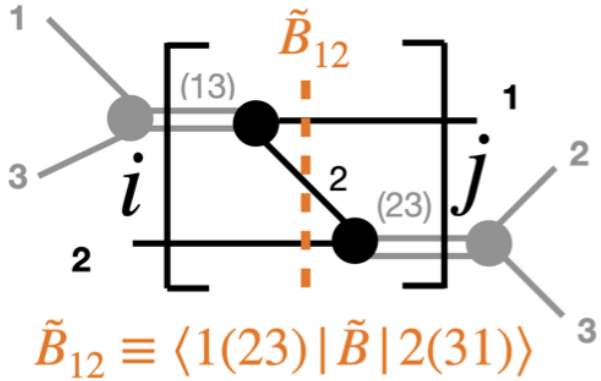
Comparison with Faddeev amplitude

[MD, Khemchandani, Martinez, 2025]

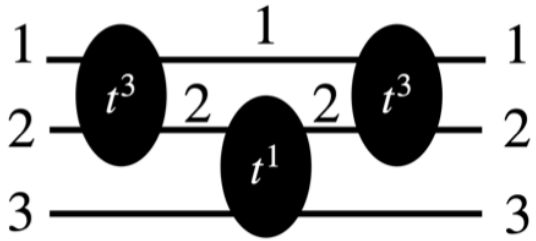


$\langle j | t^1 g^{12} t^2 | i \rangle$

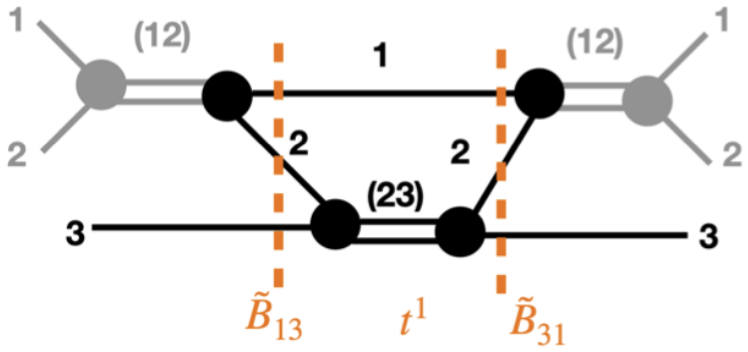
≡



$\tilde{B}_{12} \equiv \langle 1(23) | \tilde{B} | 2(31) \rangle$



≡



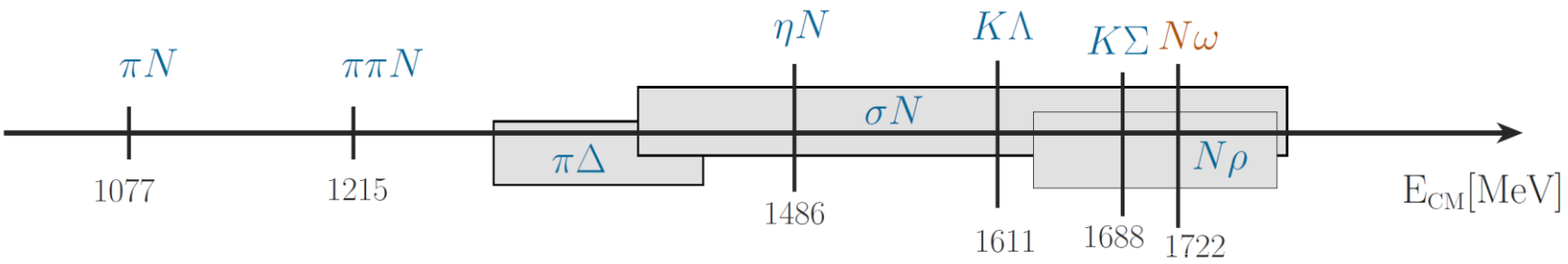
JBW DCC approach (Jülich-Bonn-Washington)

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq\; q^2\; \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$

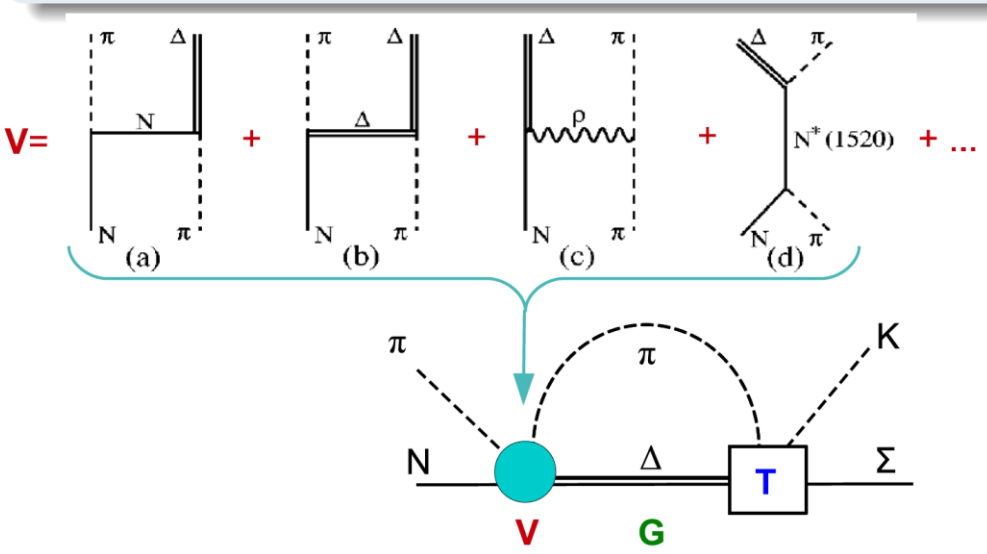
- channels ν, μ, γ:



JBW DCC approach (Jülich-Bonn-Washington)

The scattering equation in partial-wave basis

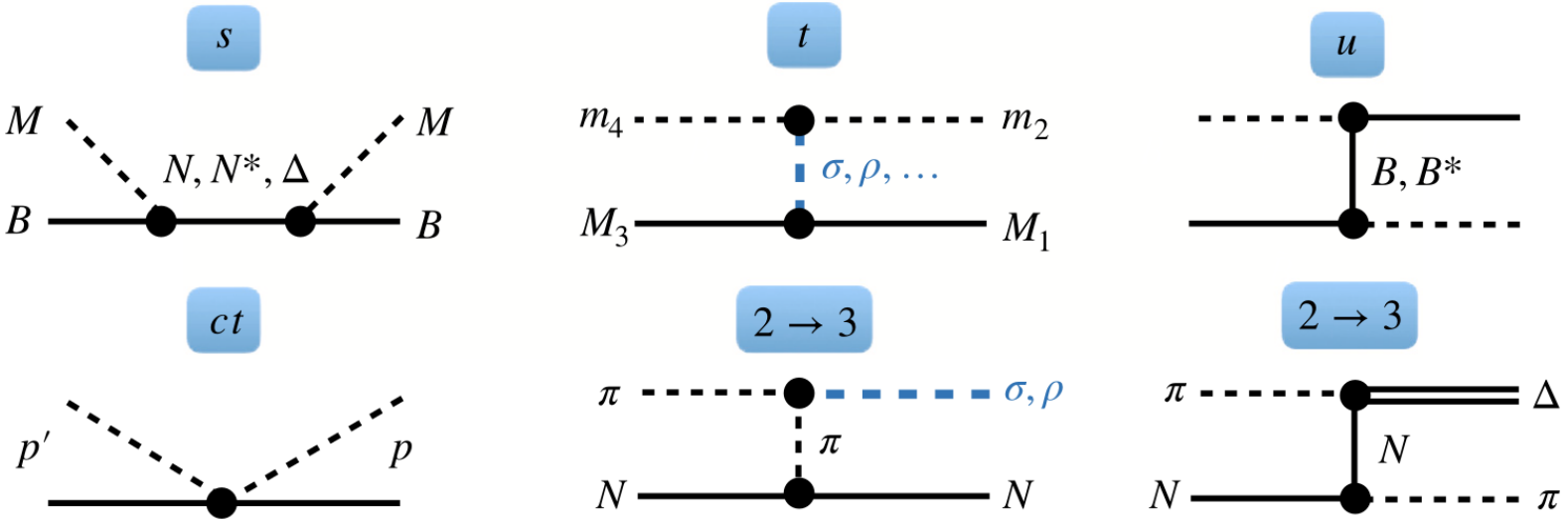
$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq\; q^2\; \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E-E_\gamma(q)+i\epsilon}\langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$



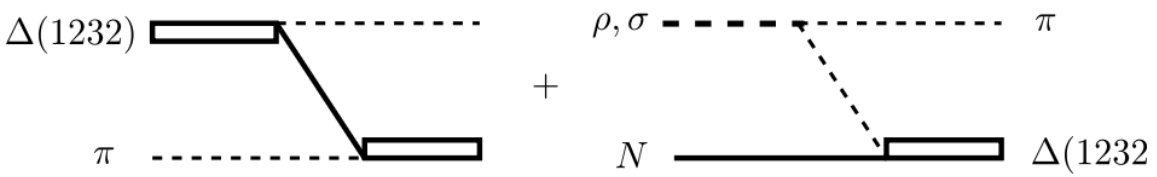
- potentials V constructed from effective $\mathcal{L}$
- s-channel diagrams: T^P
  genuine resonance states
- t- and u-channel: T^{NP}
  dynamical generation of poles
  partial waves strongly correlated
- contact terms

Details channel transitions

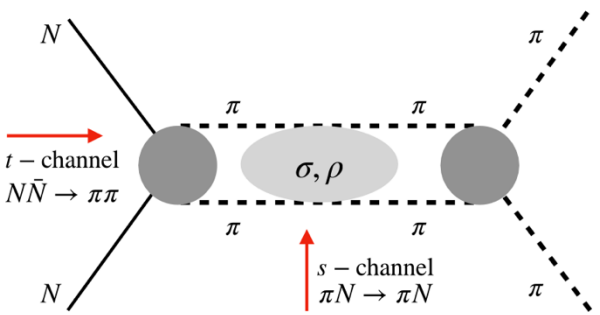
- 2→2 and 2→3 transitions



- 3→3 transitions



- crossed channels



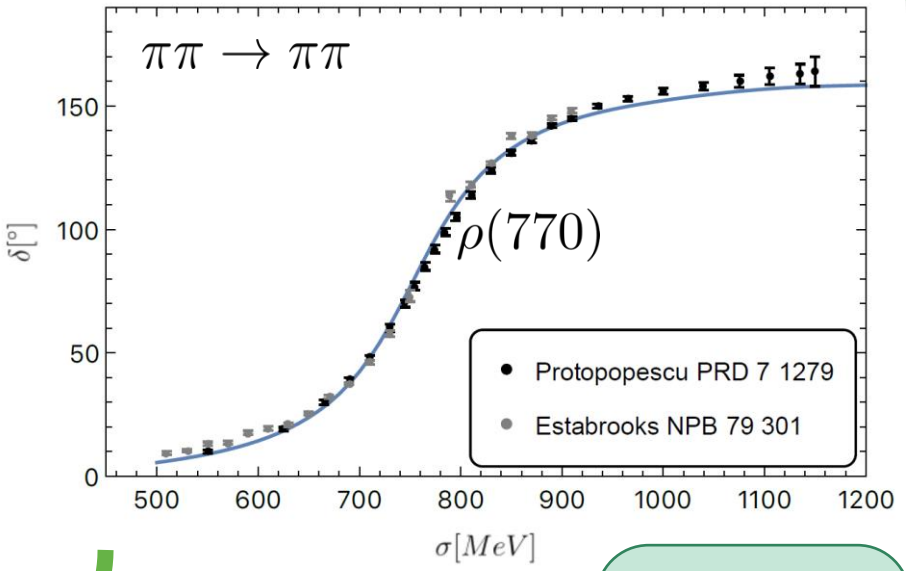
Transitions in s, t, and u-channels

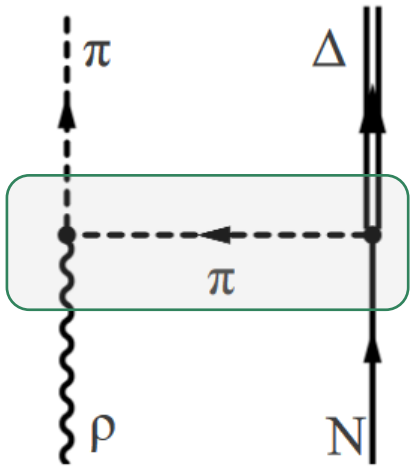
- 21 s-channel exchanges (resonance)
- Contact terms
- t and u-channel exchanges (ANL/Osaka & JuBo/JBW):

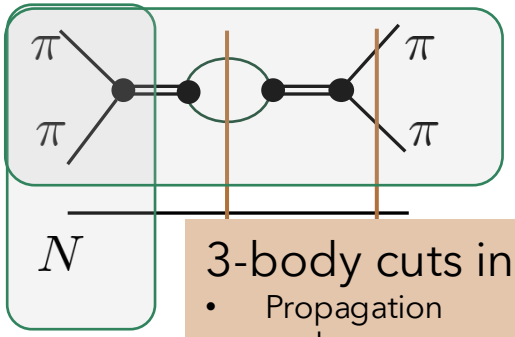
μ	πN	ηN	$K\Lambda$	$K\Sigma$	ωN	$\pi\Delta$	σN	ρN
πN	N, Δ, $(\pi\pi)_\sigma$, σ, f_0 $(\pi\pi)_\rho$, ρ	a_0, N	K^*, Σ, Σ^*, κ	K^*, Λ, Σ, Σ^*, κ	ρ, N	ρ, N, Δ	π, N	π, ω, a_1, N, Δ, C
ηN		N, f_0	K^*, Λ, κ	K^*, Σ, Σ^*, κ	ω, N		N	N
$K\Lambda$			ω, f_0, ϕ, Ξ, Ξ^*	ρ, a_0, Ξ, Ξ^*	K, K^*, Λ			
$K\Sigma$				ρ, ω, ϕ, f_0, a_0, Ξ, Ξ^*	K, K^*, Σ, Σ^*			
ωN					σ, N			
$\pi\Delta$						ρ, N, Δ	π	π, N
σN							σ, N	N
ρN								ρ, N, Δ, C

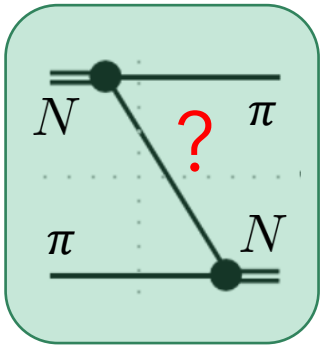
Three-body channels $\sigma N, \pi\Delta, \rho N$

- Resonant sub-channels
- Fit 2→2 amplitude to 2→ 2 scattering data
- Include as sub-channel in 3-body amplitude:
- 3-body unitarity: Requires, e.g.







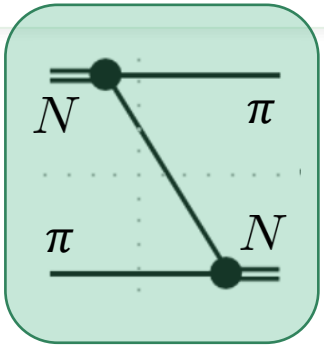


3-body cuts in
- Propagation
- exchange

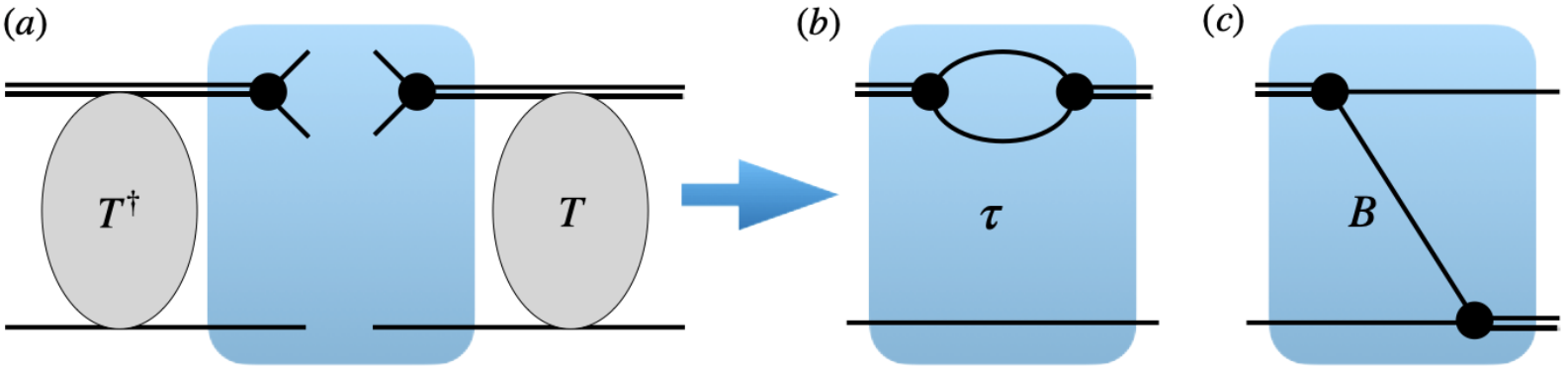
Three-body channels: π-Roper

[Jinzi Wu, MD, M. Mai, in preparation]

- Nucleon exchange requires….
- ($N\pi$) + spectator pion channel



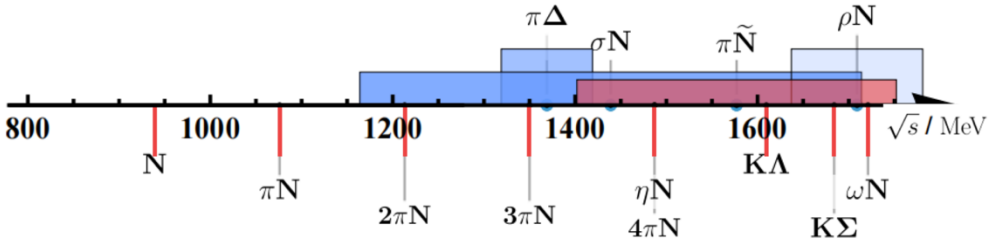
3-body cuts in
- Propagation
- exchange

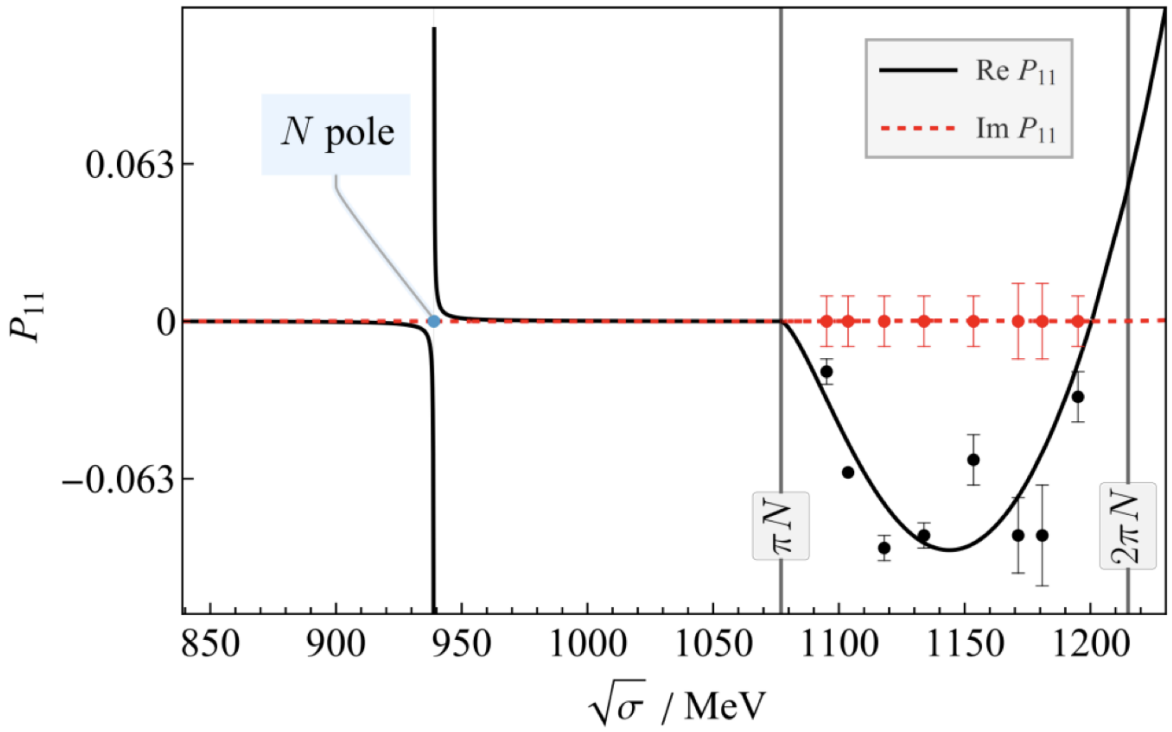


Three-body channels: π-Roper

[Jinzi Wu, MD, M. Mai, in preparation]

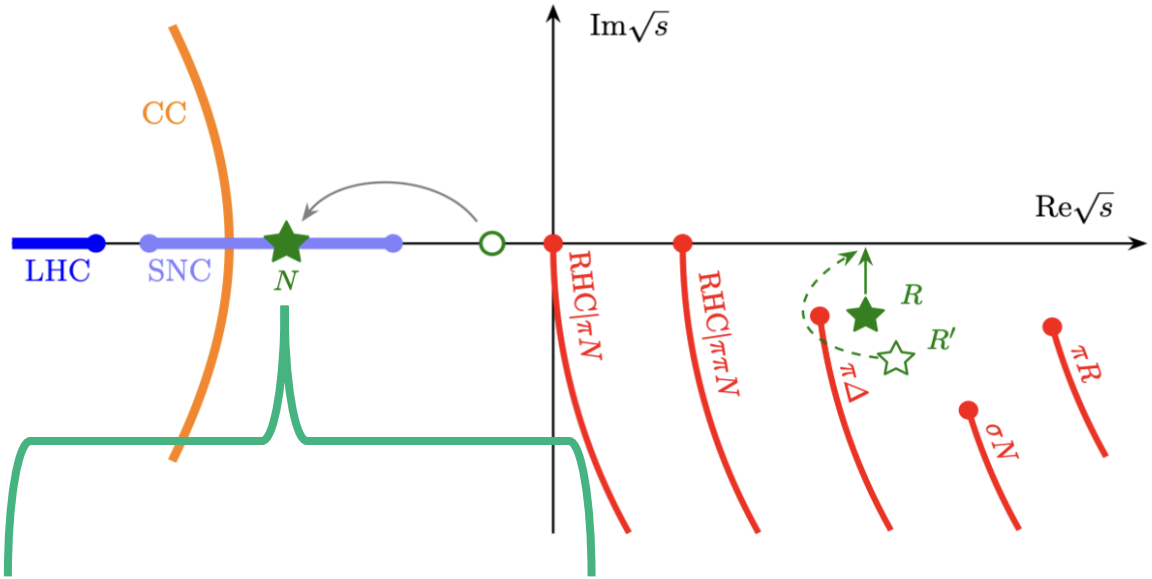
- No explicit πN channel any more
- πN arises as bound state-spectator scattering

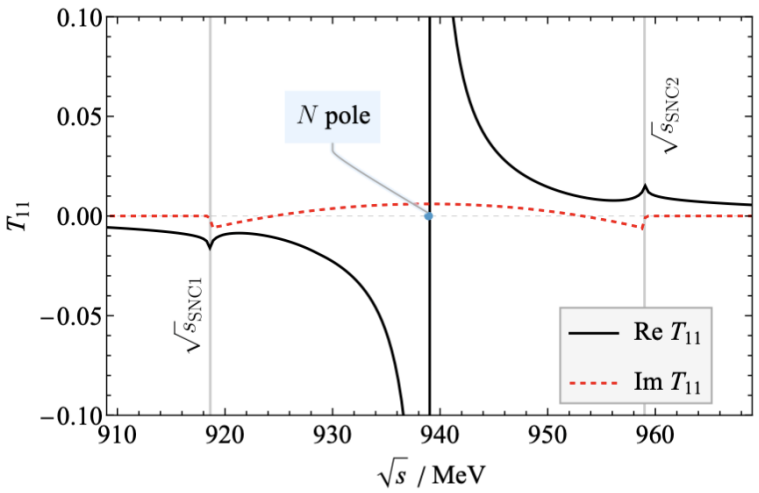




Analytic structure of the amplitude

[Jinzi Wu, MD, M. Mai, in preparation]

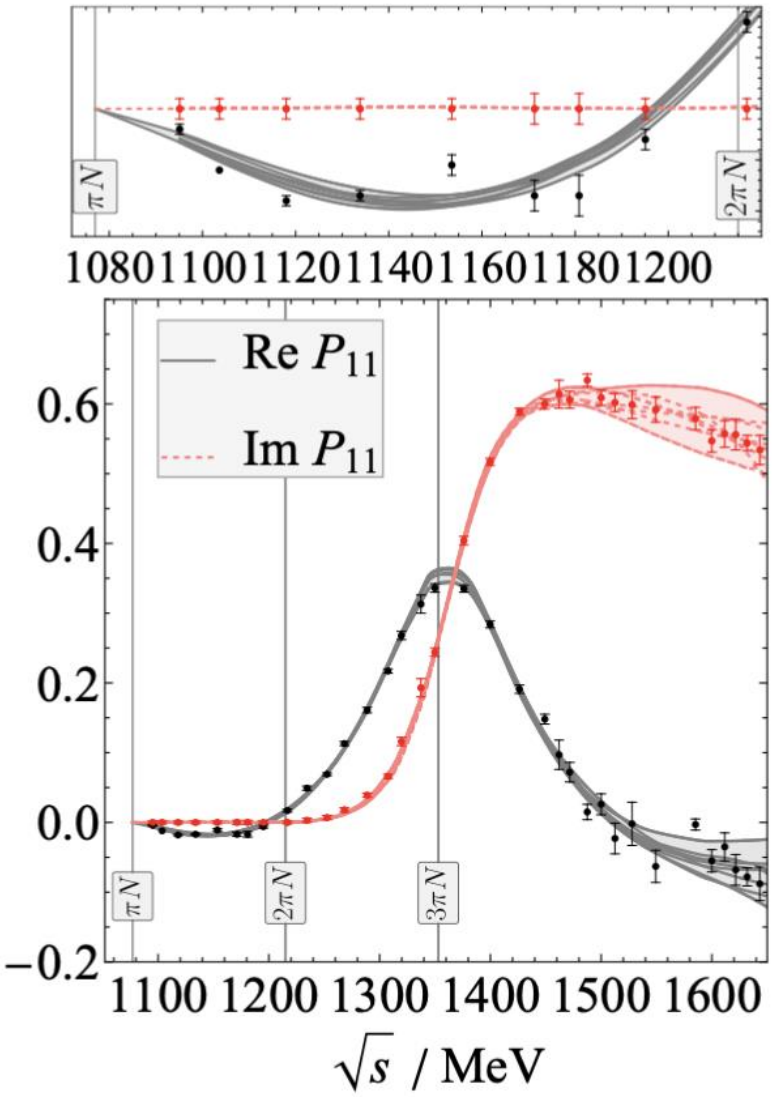


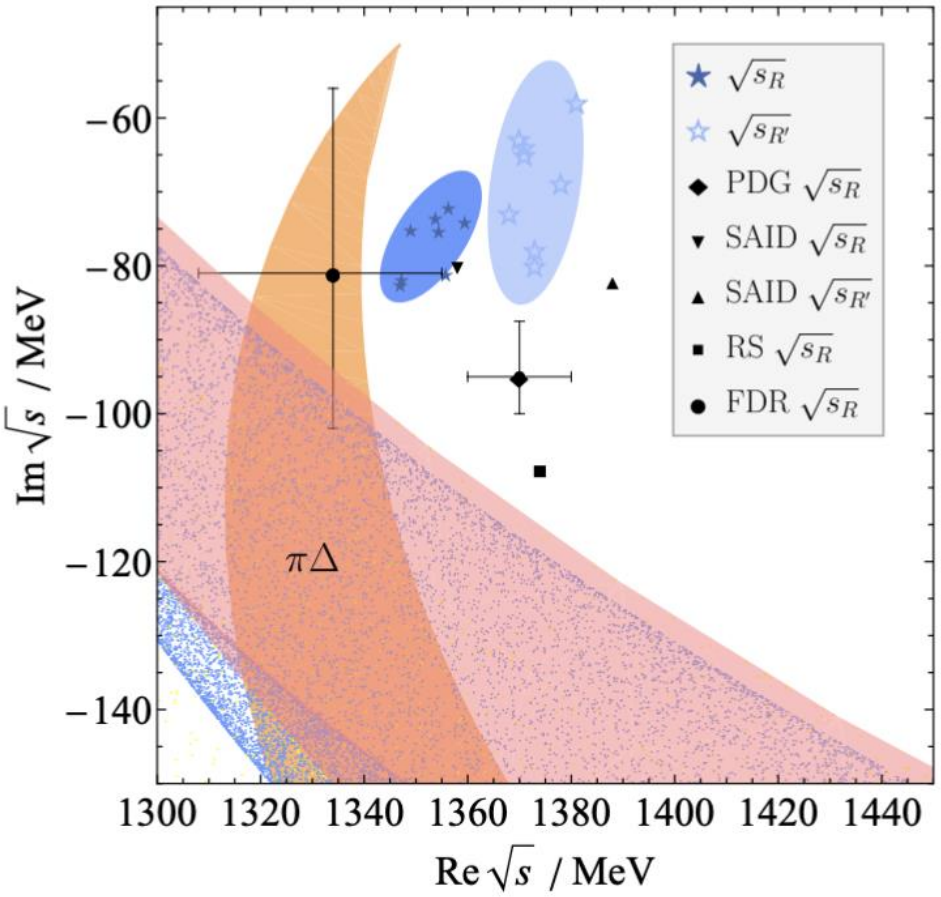


- Real and complex thresholds
- Sub-threshold cuts
- Two-pole structure of the Roper [SAID]

Pole position Roper resonance

[Jinzi Wu, MD, M. Mai, in preparation]





JuBo: Data base

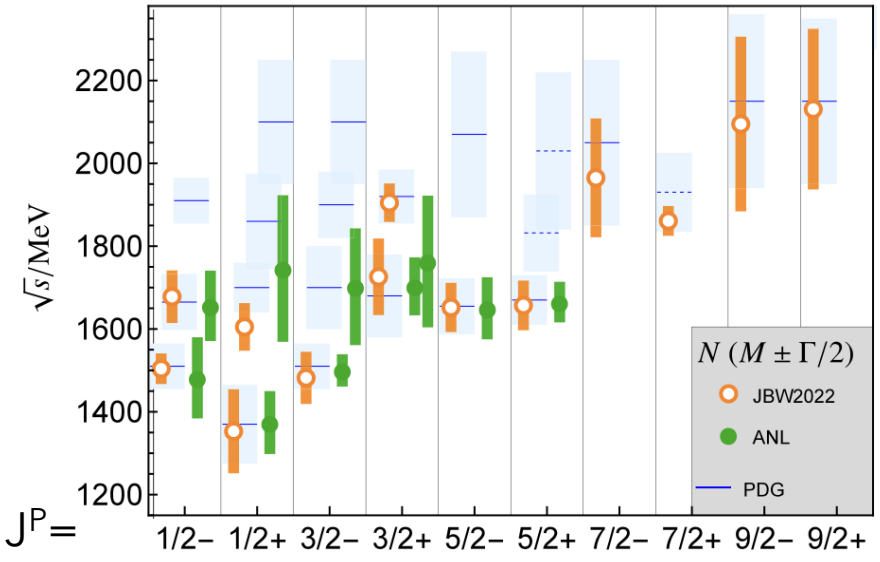
- $\pi N \to X$: $>$ **7,000** data points ($\pi N \to \pi N$: GW-SAID WI08 (ED solution))
- $\gamma N \to X$:

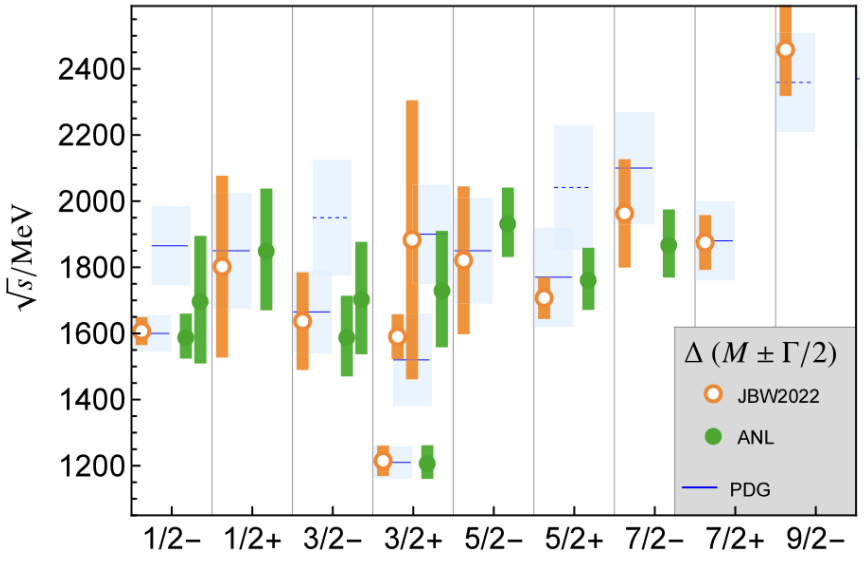
update: $\pi N \to \omega N$ [2208.03061/Wang]
Upcoming data from JParc

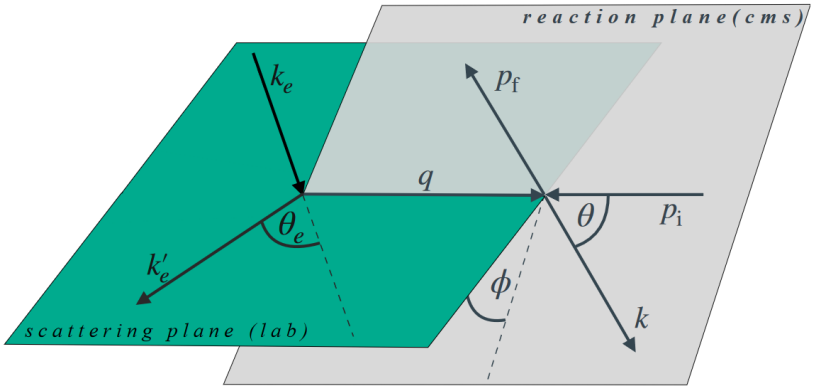
Reaction	Observables (# data points)	p./channel
$\gamma p \to \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (2927), P (768), T (1404), $\Delta\sigma_{31}$ (140),	
	G (393), H (225), E (467), F (397), $C_{x'_L}$ (74), $C_{z'_L}$ (26)	25,542
$\gamma p \to \pi^+ n$	$d\sigma/d\Omega$ (5961), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231),	
	G (86), H (128), E (903)	9,748
$\gamma p \to \eta p$	$d\sigma/d\Omega$ (9112), Σ (403), P (7), T (144), F (144), E (129)	9,939
$\gamma p \to K^+\Lambda$	$d\sigma/d\Omega$ (2478), P (1612), Σ (459), T (383),	
	$C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	5,936
$\gamma p \to K^+\Sigma^0$	$d\sigma/d\Omega$ (4271), P (422), Σ (280), T (127), $C_{x',z'}$ (188), $O_{x,z}$ (254)	5,542
$\gamma p \to K^0\Sigma^+$	$d\sigma/d\Omega$ (242), P (78)	320
	in total	**57,027**

New interface [https://jbw.phys.gwu.edu/]

Results: Resonance spectrum

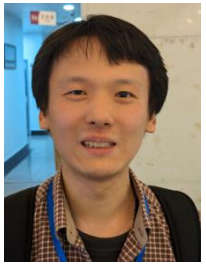







J. Hergenrather


M. Mai


Yu-Fei Wang

Coupled-Channel Electroproduction

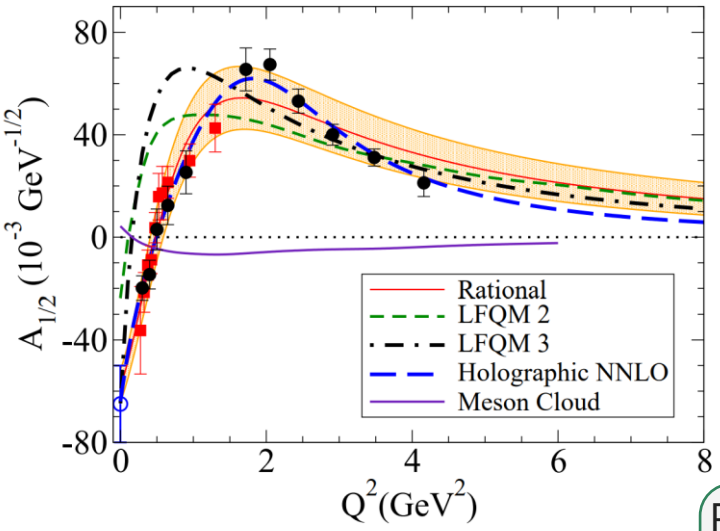
First coupled-channel electroproduction analysis with different final states

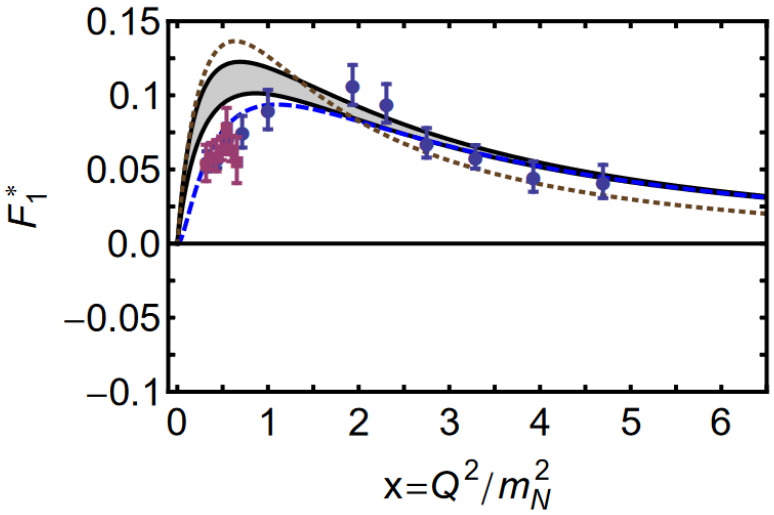
[M. Mai, Yu-Fei Wang et al. (2020 -)]

Theory:

- Siegert's theorem manifestly fulfilled (consequence of gauge invariance)
- Watson theorem fulfilled
- Coupled-channel unitarity fulfilled
- General expansion of electroproduction kernel in Laurent series
- Resonances, background, channels have independent Q^2 dependence for flexible fit

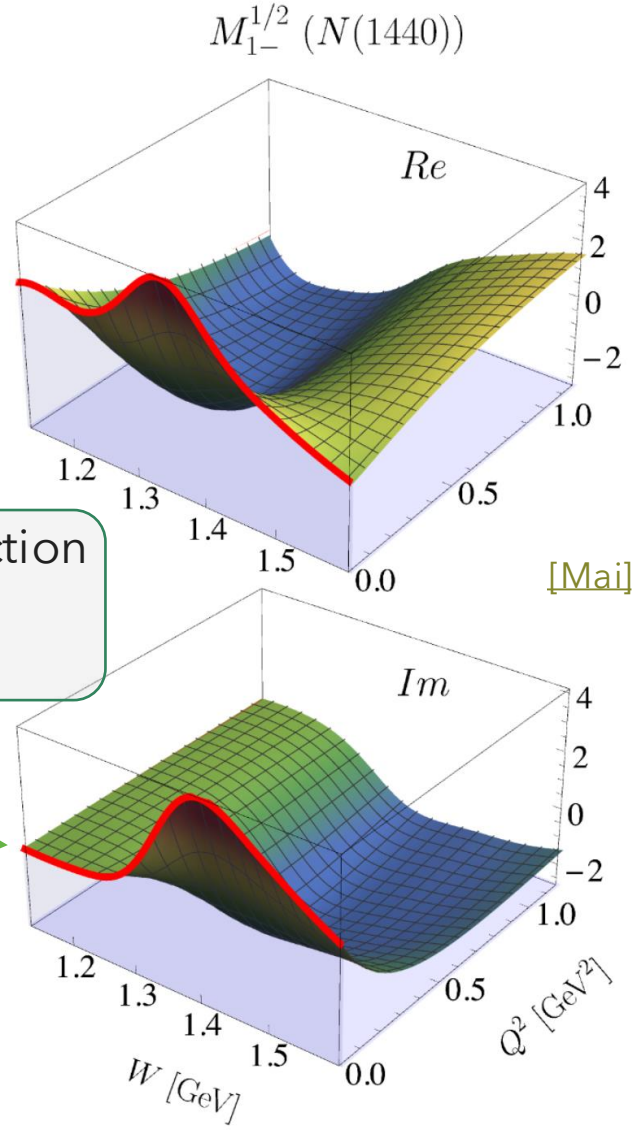
Electroproduction reveals resonance structure





Proton-Roper Transition [Burkert] [Segovia]

Photoproduction included as boundary



[Mai]

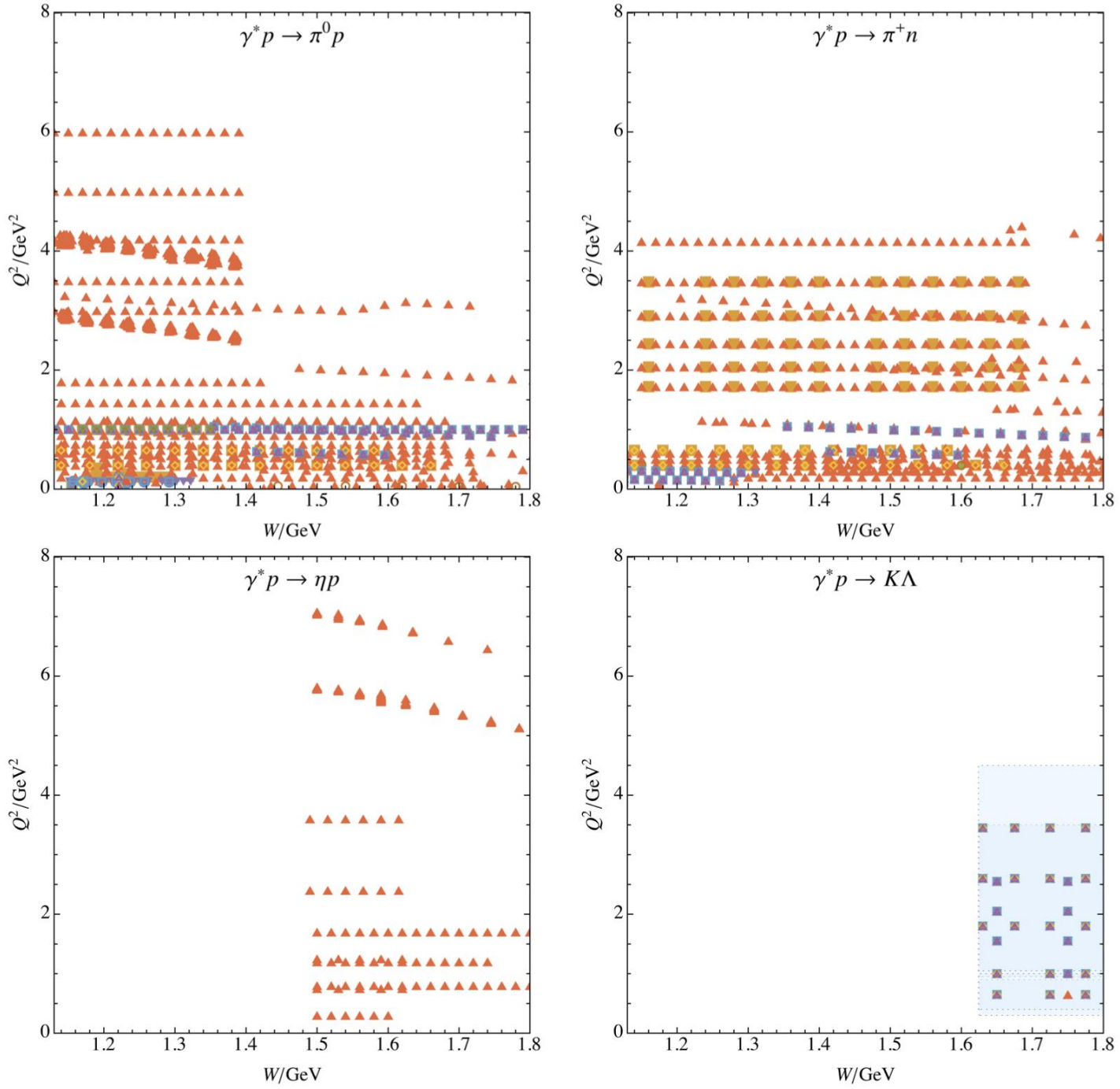
Electroproduction Analysis efforts

- **MAID**: electroproduction of pions, eta mesons, and kaons in separate approaches [Tiator 2007]
- **JM (JLab)** approach: single-pion analysis, double pion analysis [Mokeev, PRC 2023]
- **ANL-Osaka:** Single-pion electroproduction, using multi-channel model. [Kamano, Lee, Nakamura, Sato, 2016]
- **JBW**: simultaneous analysis of multiple electroproduction final states, using multi-channel model
- **Bonn-Gatchina**: Upcoming calculations

JBW Electroproduction data base

Type	$N^{\pi^0 p}_{\text{data}}$	$N^{\pi^+ n}_{\text{data}}$	$N^{\eta p}_{\text{data}}$	$N^{K\Lambda}_{\text{data}}$
ρ_{LT}	45	–	–	–
$\rho_{LT'}$	2768	5068	–	–
σ_L	–	2	–	–
$d\sigma/d\Omega$	48135	44266	3665	2055
$\sigma_T + \epsilon\sigma_L$	384	182	–	204
σ_T	30	2	–	–
σ_{LT}	373	138	–	204
$\sigma_{LT'}$	214	208	–	156
σ_{TT}	327	123	–	204
K_{D1}	1527	–	–	–
P_Y	–	2	–	–
Total	53804	49989	3665	2823

- Data base grown over decades with recent input mostly by CLAS, MAMI.
- Far from complete: Kinematic gaps & consistency issues. Need to combine information from different (W, Q^2) regions
- Need to combine information from simultaneous analysis of different final states ($\pi N/\eta N/K Y/\pi\pi N, \ldots$) to extract resonance helicity couplings


$\gamma^* p \to \pi^0 p$
$\gamma^* p \to \pi^+ n$
$\gamma^* p \to \eta p$
$\gamma^* p \to K\Lambda$
Q^2/GeV^2
W/GeV
0
2
4
6
8
1.2
1.3
1.4
1.5
1.6
1.7
1.8

Fit details: Weighted vs. unweighted χ^2

- Meson production data bases are heterogeneous:
  - A few polarization measurements with large error bars (small weight in χ^2)
  - Many cross section data with smaller error bars (large weight in χ^2)
  - … but those **few** polarization possess **great** power to discriminate solutions
- Introduce **weighted** vs. **unweighted** χ^2:

$$\chi^2_{\rm wt} = \sum_{j\in\{\pi^0 p,\pi^+ n,\eta p\}} \frac{N_{\rm all}}{3N_j} \sum_{i=1}^{N_j} \left(\frac{\mathcal{O}_{ji}^{\rm exp} - \mathcal{O}_{ji}}{\Delta_{ji}^{\rm stat} + \Delta_{ji}^{\rm syst}} \right)^2 .$$

$$\chi^2_{\rm reg} = \sum_{i=1}^{N_{\rm all}} \left(\frac{\mathcal{O}_{i}^{\rm exp} - \mathcal{O}_{i}}{\Delta_{i}^{\rm stat} + \Delta_{i}^{\rm syst}} \right)^2$$

- Quote results for both cases

Fit Strategies (πN)

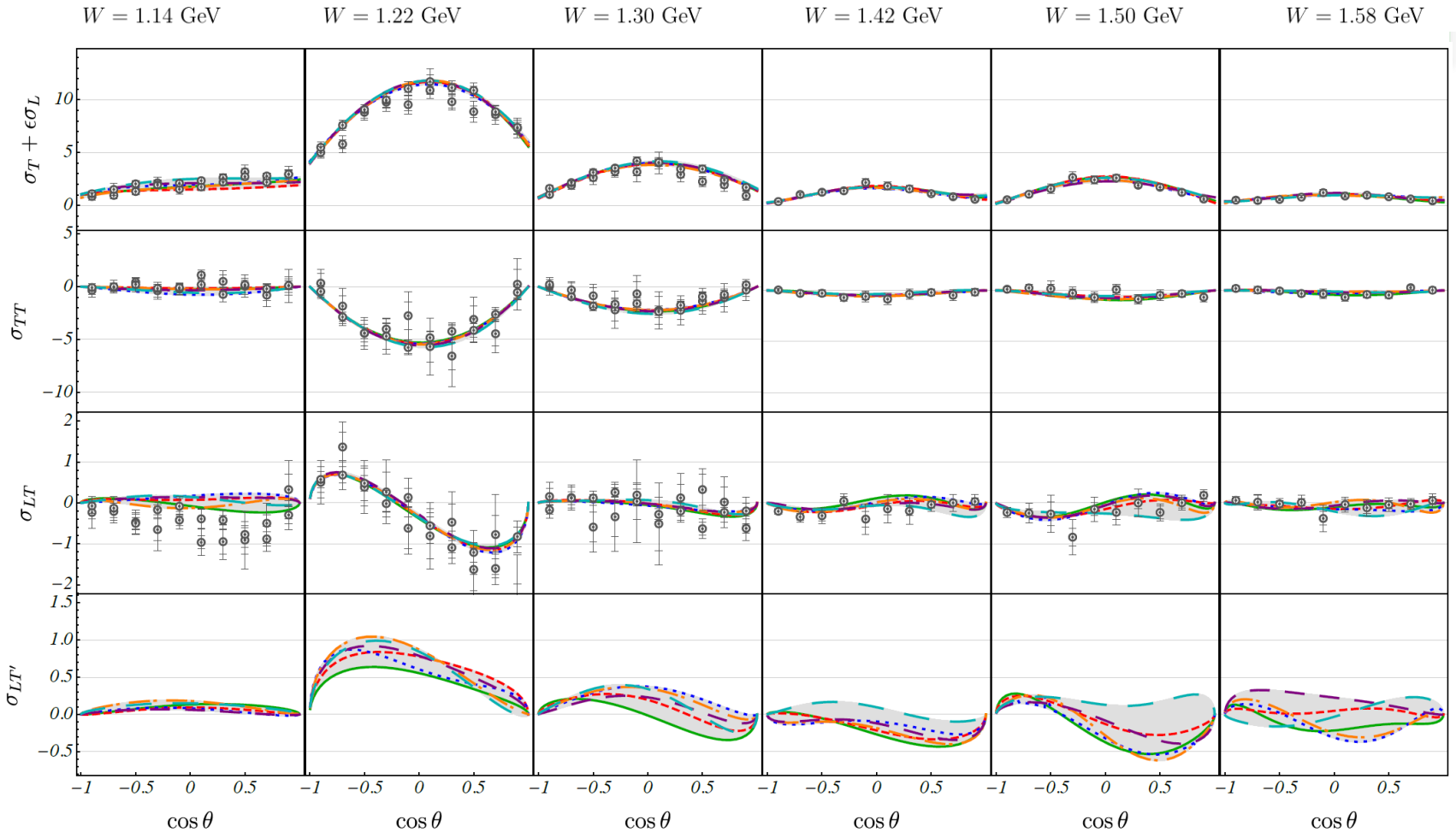
[M. Mai et al, 2021]

- Different fit strategies for $N \approx 85{,}000$ data in $\gamma^* N \to \pi N, \eta N$:
  - Sequential S → S+P → S+P+D waves;
  - Subsets of data until full data set reached
  - Simultaneous fit all parameters (209) set to zero without any (!) guidance
  - Extend data range from $0 < Q^2 < 4$ Gev^2 to $0 < Q^2 < 6$ Gev^2 to check for stability

Fit	σ_L		$d\sigma/d\Omega$		$\sigma_T + \epsilon\sigma_L$		σ_T		σ_{LT}		$\sigma_{LT'}$		σ_{TT}		K_{D1}		P_Y		ρ_{LT}		$\rho_{LT'}$		$\chi^2_{\rm dof}$
	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	
$\mathfrak{F}_1$	–	9	65355	53229	870	418	87	88	1212	133	862	762	4400	251	4493	–	234	–	525	–	3300	10294	1.77
$\mathfrak{F}_2$	–	4	69472	55889	1081	619	65	78	1780	150	1225	822	4274	237	4518	–	325	–	590	–	3545	10629	1.69
$\mathfrak{F}_3$	–	8	66981	54979	568	388	84	95	1863	181	1201	437	3934	339	4296	–	686	–	687	–	3556	9377	1.81
$\mathfrak{F}_4$	–	22	63113	52616	562	378	153	107	1270	146	1198	1015	4385	218	5929	–	699	–	604	–	3548	11028	1.78
$\mathfrak{F}_5$	–	20	65724	53340	536	528	125	81	1507	219	1075	756	4134	230	5236	–	692	–	554	–	3580	11254	1.81
$\mathfrak{F}_6$	–	18	71982	58434	1075	501	29	68	1353	135	1600	1810	3935	291	5364	–	421	–	587	–	3932	11475	1.78

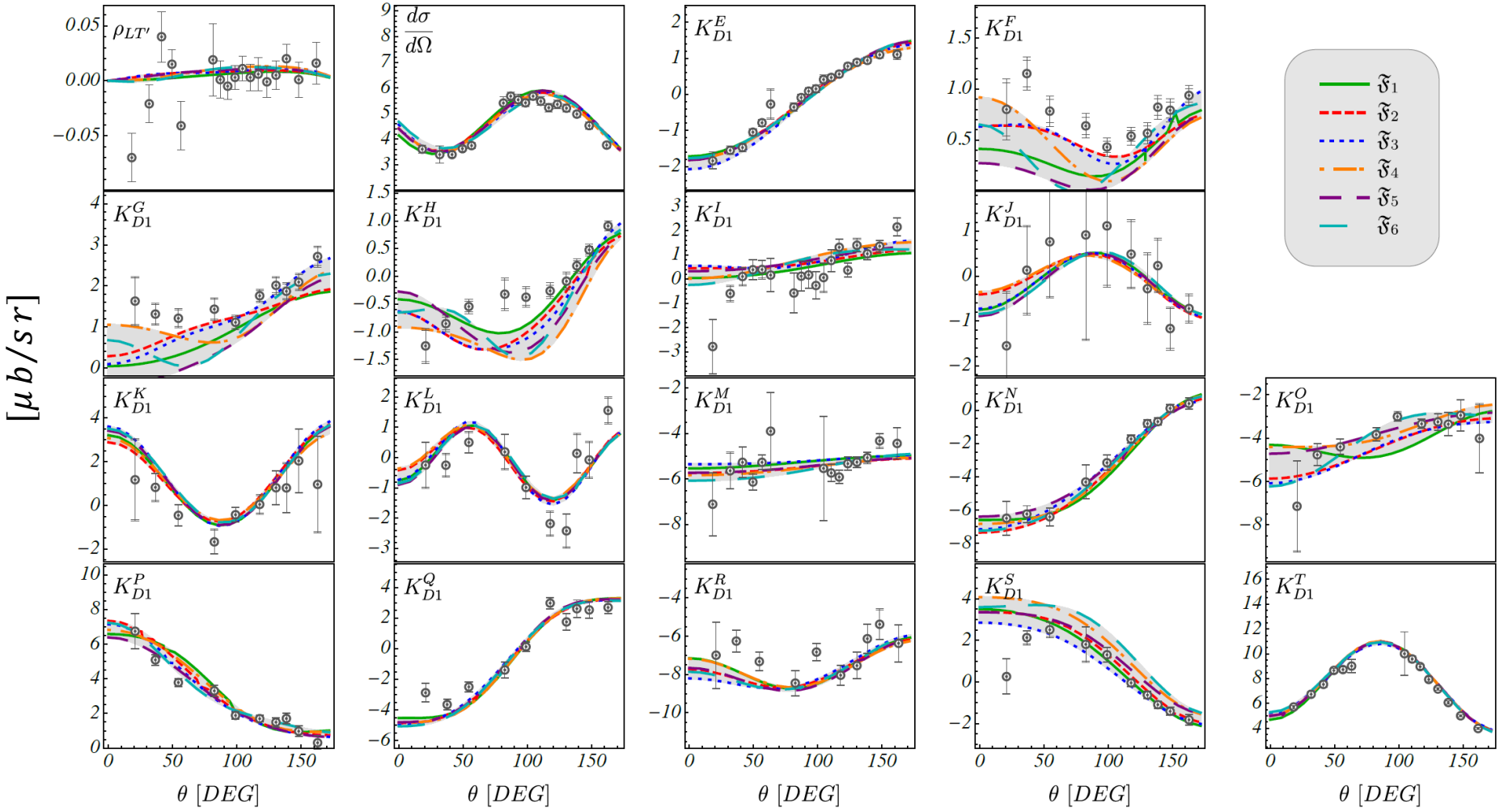
χ^2

Structure functions $\pi^0 p$ (not fitted)



Data: CLAS, Phys. Rev. C (2003) 0301012 [nucl-ex], Phys. Rev. Lett. (2002) 0110007 [hep-ex]

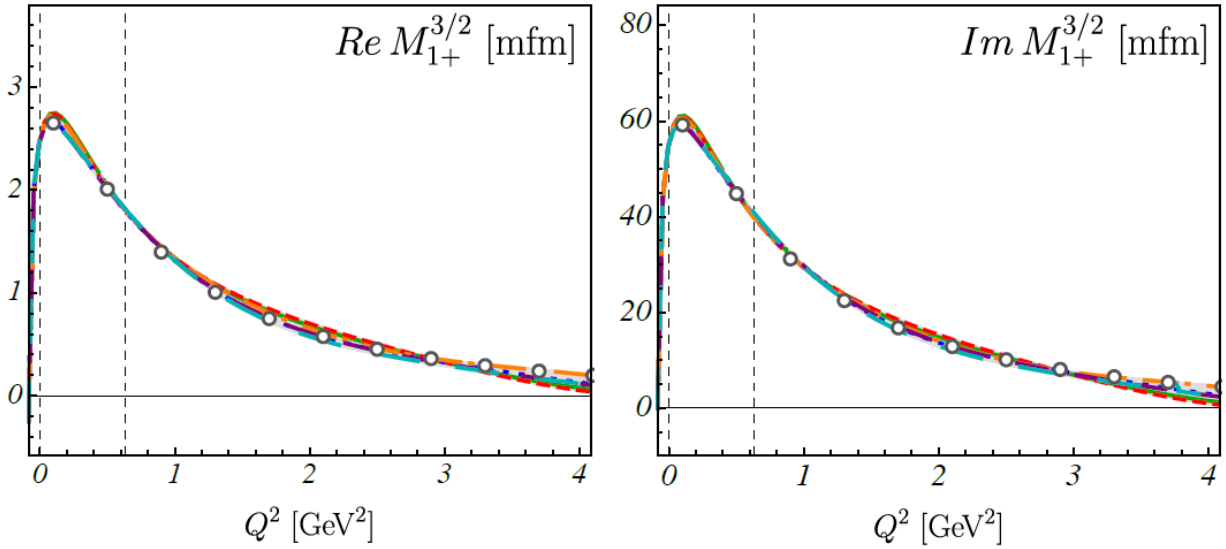
Description of Polarization Observables (πN)

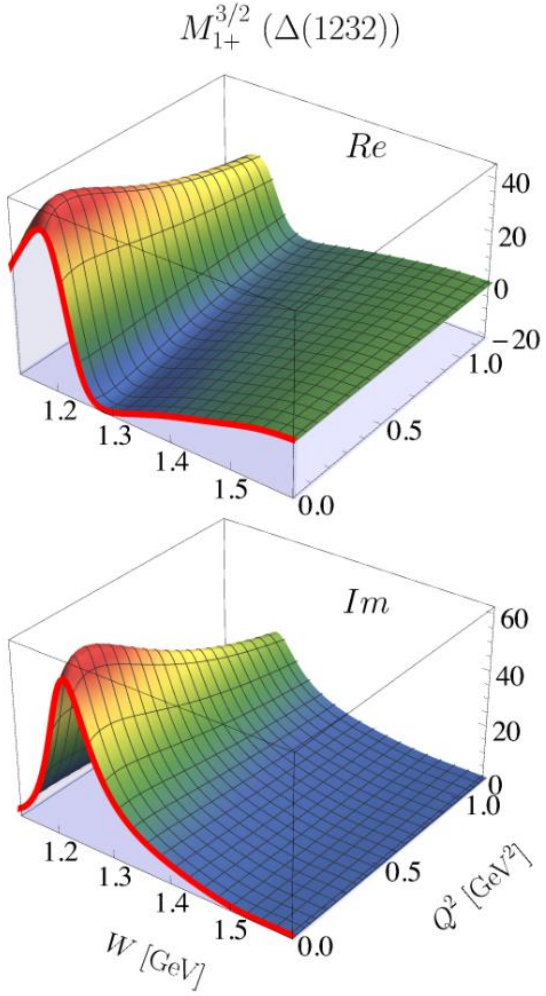


$\pi^0 p$, Q^2=1 GeV^2, W=1.23 GeV, ϕ=15^0

J. J. Kelly, Phys. Rev. Lett. 95 (2005).

Large Multipoles



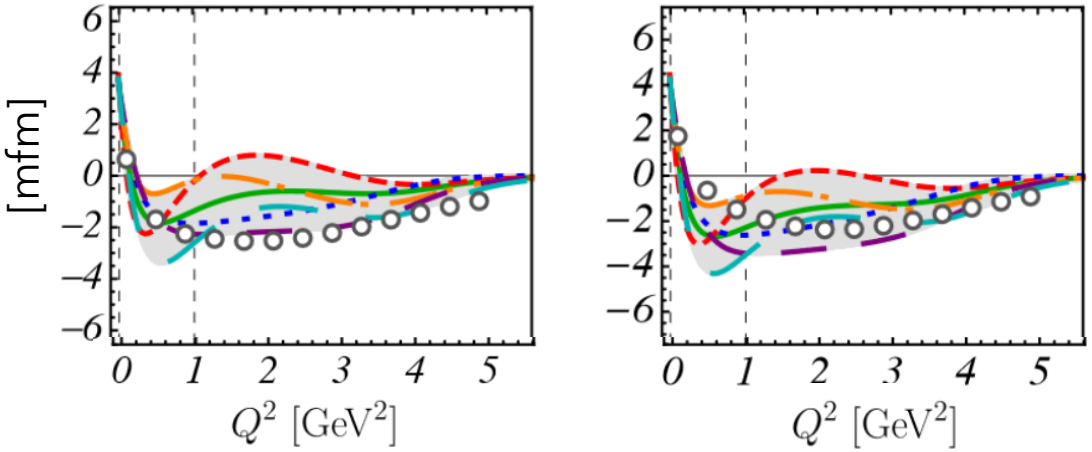


Fit strategies 1-6 together with MAID (open dots) for the magnetic multipole of the $\Delta(1232)$ Drechsel et al., EPJA (2007) 0710.0306 [nucl-th]

Prominent multipoles are well determined

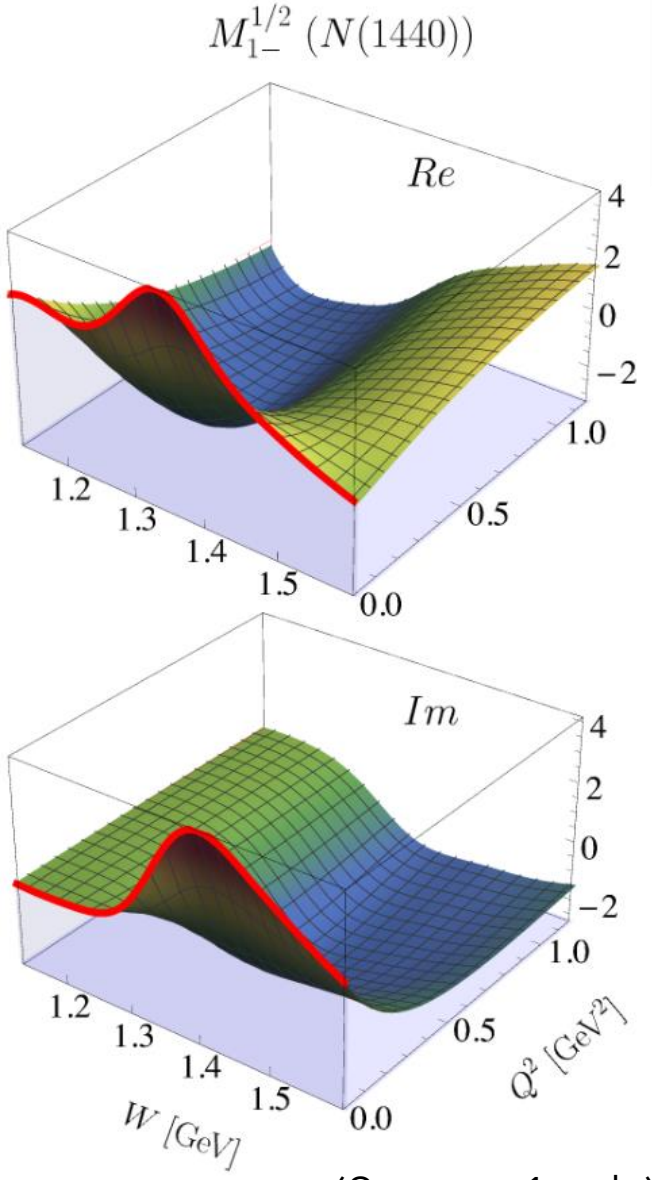
(Strategy 1 only)

Roper Multipole



(W=1.38 GeV fixed)

- Zero-transition (agrees with MAID)
- Extensive exploration of parameter space reveals ambiguities in PWA and reflects systematic uncertainties



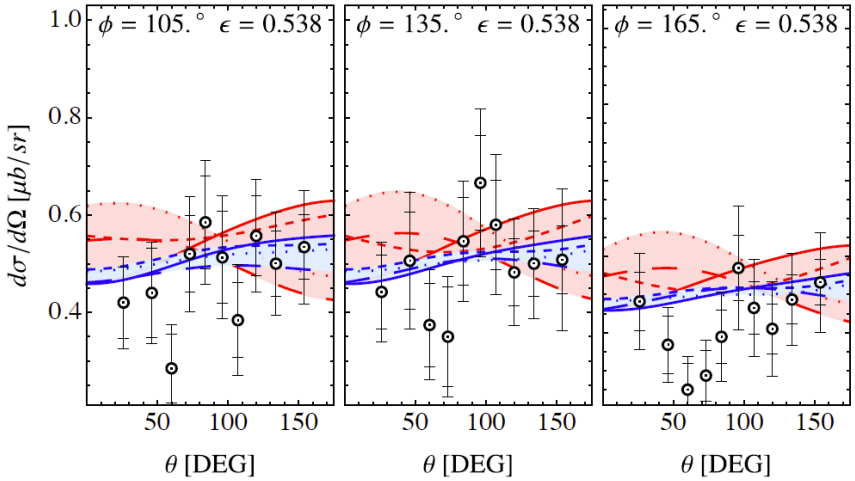
(Strategy 1 only)

η Electroproduction

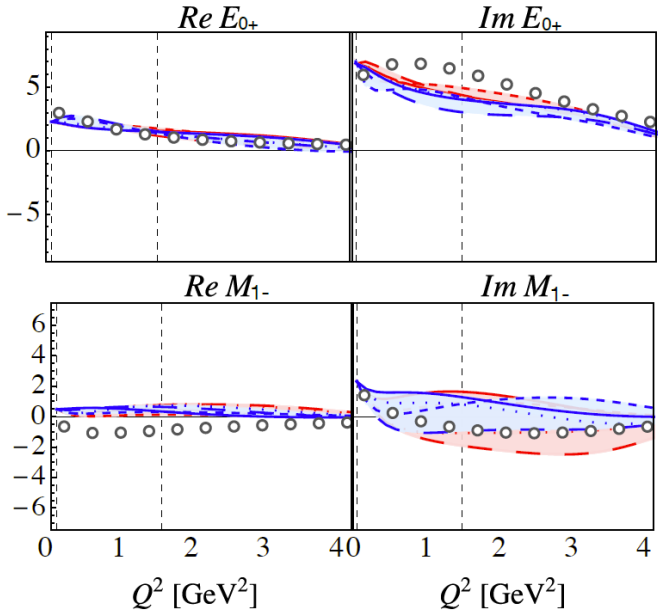
[M. Mai et al., PRC (2022)]

- $N_{data}^{\eta p} = 1,874$ (only $d\sigma/d\Omega$) (84,842 in total)
- kinematic range: $0 < Q^2 < 4$ GeV^2, $1.13 < W < 1.6$ GeV
- 8 different fit strategies: 4 with standard χ^2, 4 with weighted χ^2 to account for the smaller $N_{data}^{\eta p}$
  $\rightarrow$ better data description with weighted fit strategies:

Selected fit results: $\gamma^* p \rightarrow \eta p$ at $W = 1.5$ GeV, $Q^2 = 1.2$ GeV^2. Data: Denizli et al. (CLAS) PRC 76 (2007)



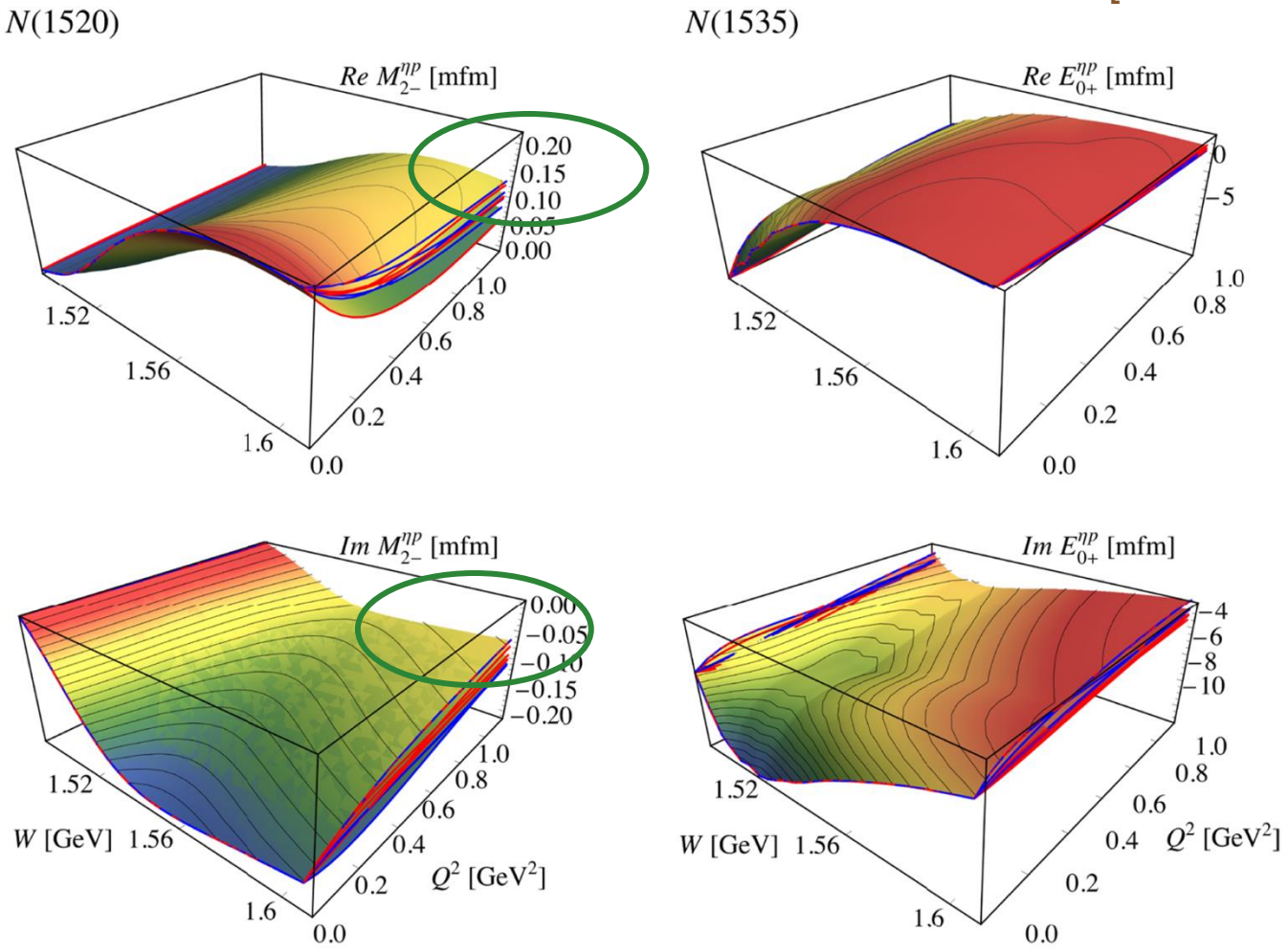
Selected multipoles at $W = 1535$ MeV



Dots: eta-MAID, arXiv:0110034

η Multipoles: Resonances disappear at high Q^2

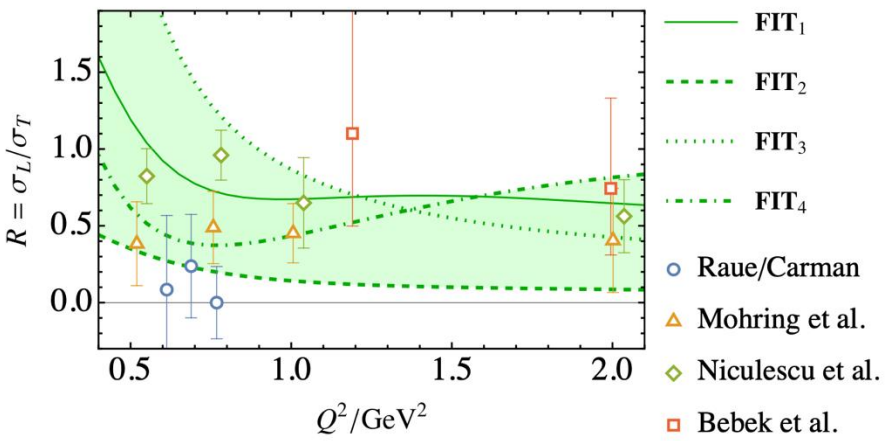
[M. Mai et al., PRC (2022)]

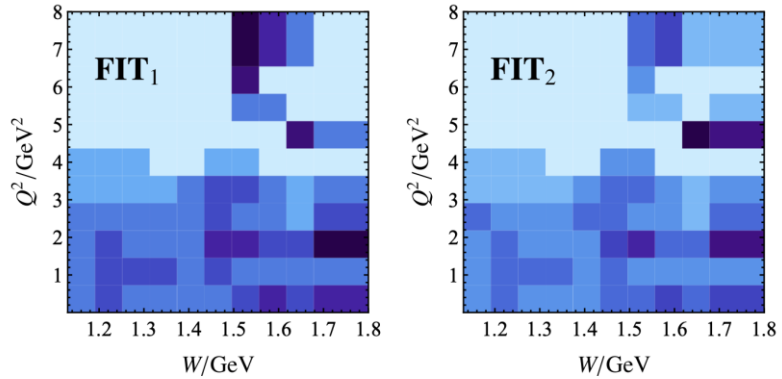


Kaon electroproduction

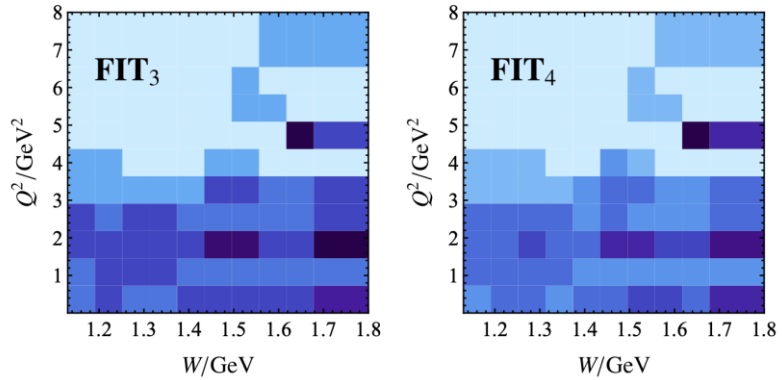
[M. Mai et al., EPJA 2023]

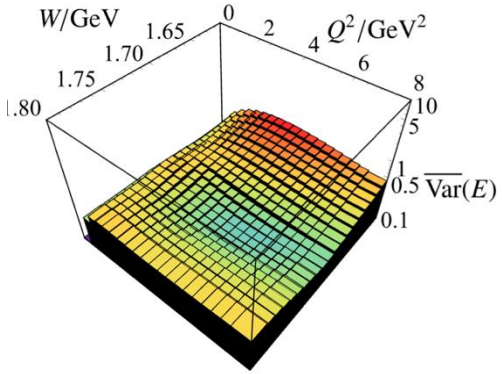
	$\chi^2_{\rm dof}$	$\chi^2_{\rm pp}(\pi^0 p)$	$\chi^2_{\rm pp}(\pi^+ n)$	$\chi^2_{\rm pp}(\eta p)$	$\chi^2_{\rm pp}(K^+\Lambda)$
FIT$_1$	1.42	1.40	1.47	1.49	0.70
FIT$_2$	1.35	1.38	1.35	1.40	0.58
	$\chi^2_{\rm wt,dof}$	$\chi^2_{\rm pp}(\pi^0 p)$	$\chi^2_{\rm pp}(\pi^+ n)$	$\chi^2_{\rm pp}(\eta p)$	$\chi^2_{\rm pp}(K^+\Lambda)$
FIT$_3$	1.12	1.44	1.61	1.08	0.33
FIT$_4$	1.06	1.42	1.44	1.09	0.32

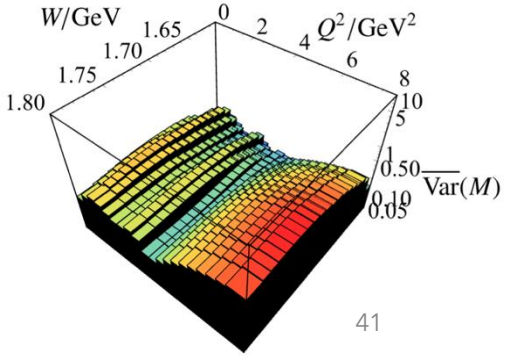




$$\overline{\rm Var}(W,Q^2)(X) := \sum_{\ell\pm} \frac{\rm Var\{|X_{\ell\pm,i}|\}}{\rm Mean\{|X_{\ell\pm,i}|\} + \varepsilon}$$



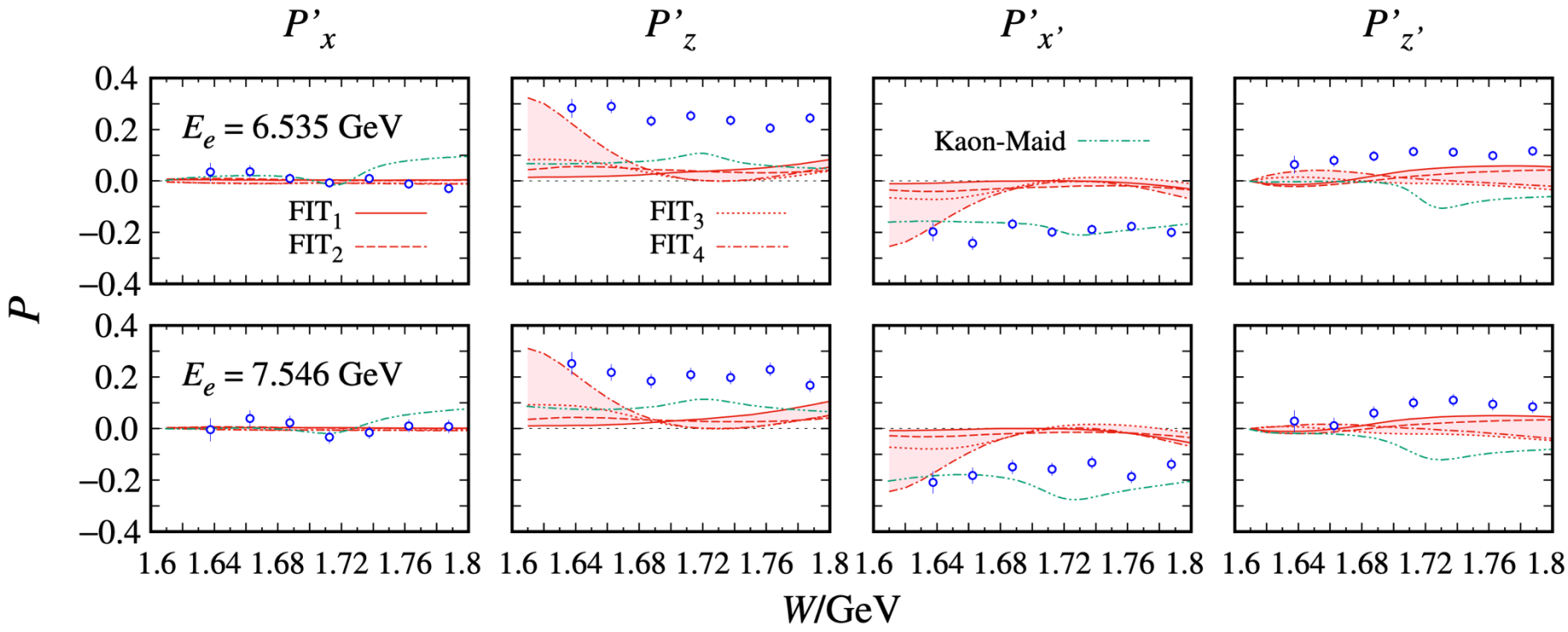




K Λ Challenges: Polarization measurements

- During analysis, first polarization data became available by CLAS:
- Predicted, not fitted

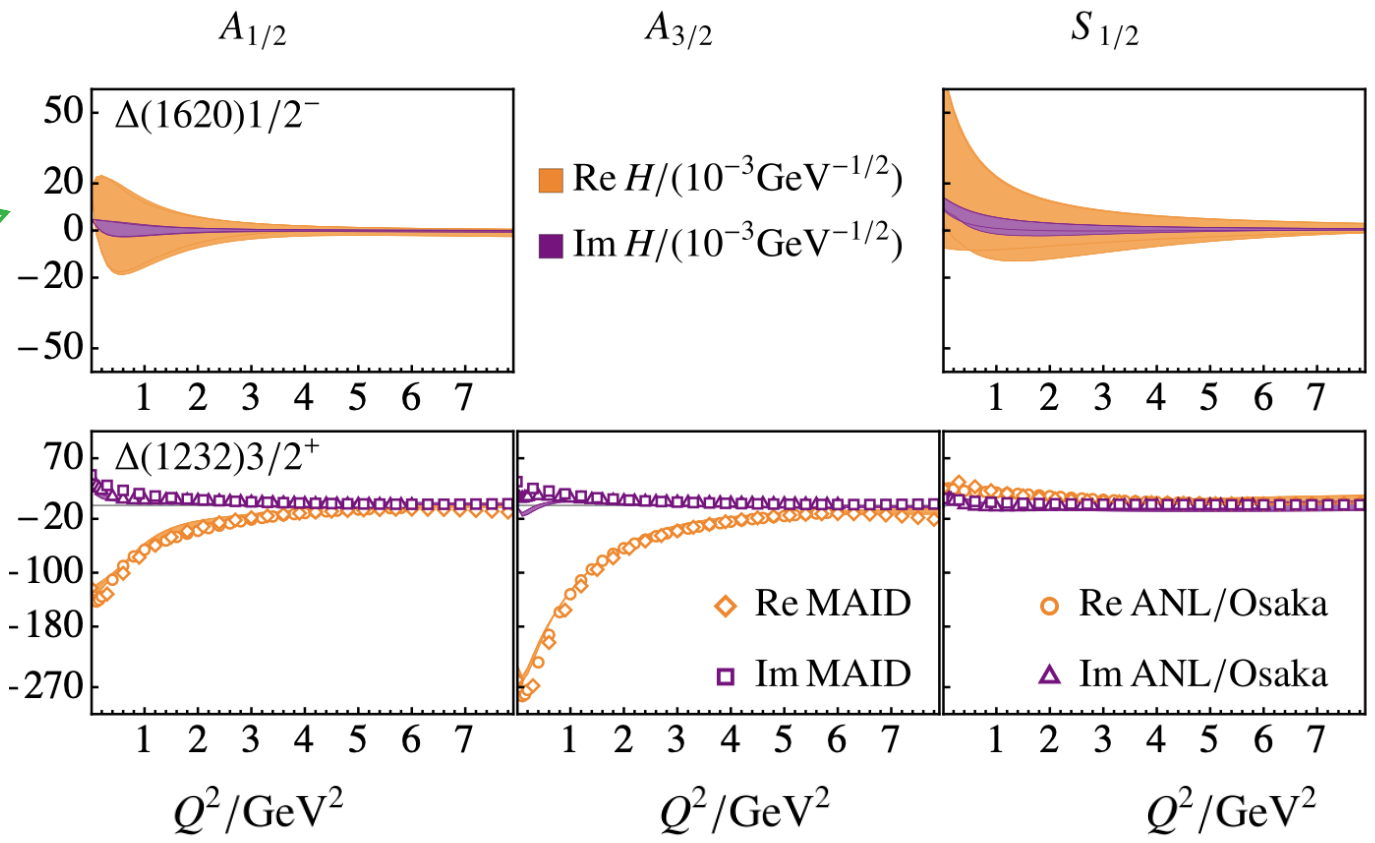
Beam-recoil polarization transfer
[Carman et al. 2022]



Helicity Couplings

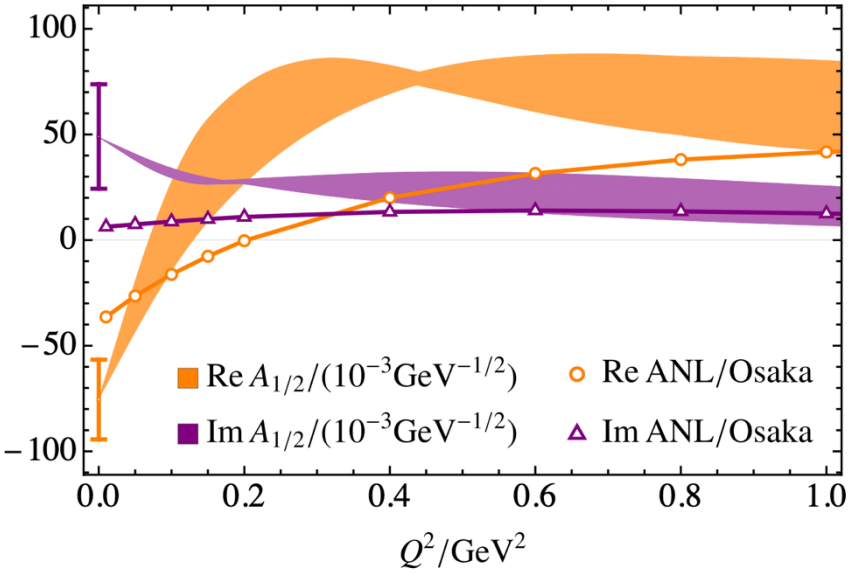
[Yu-Fei Wang et al., 2024]

- (Selected results)



Compares qualitatively with [Mokeev et al., 2022]

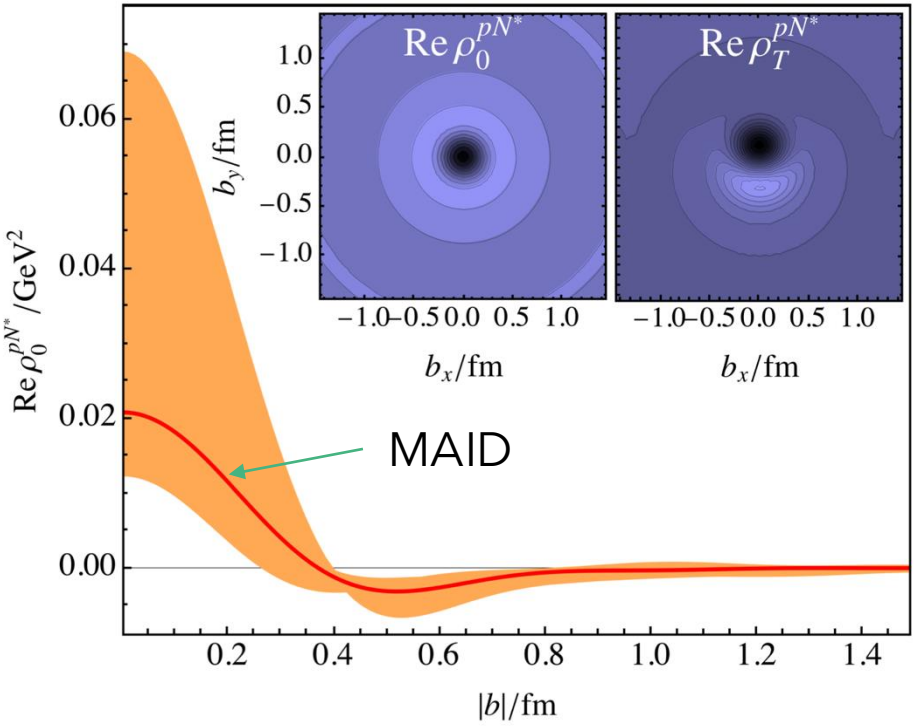
Results for the Roper resonance



- Zero-transition is there
- At quite low Q^2
- Breit-Wigner version?

Charge density structure

[approx./ following Tiator et al., (2009)]

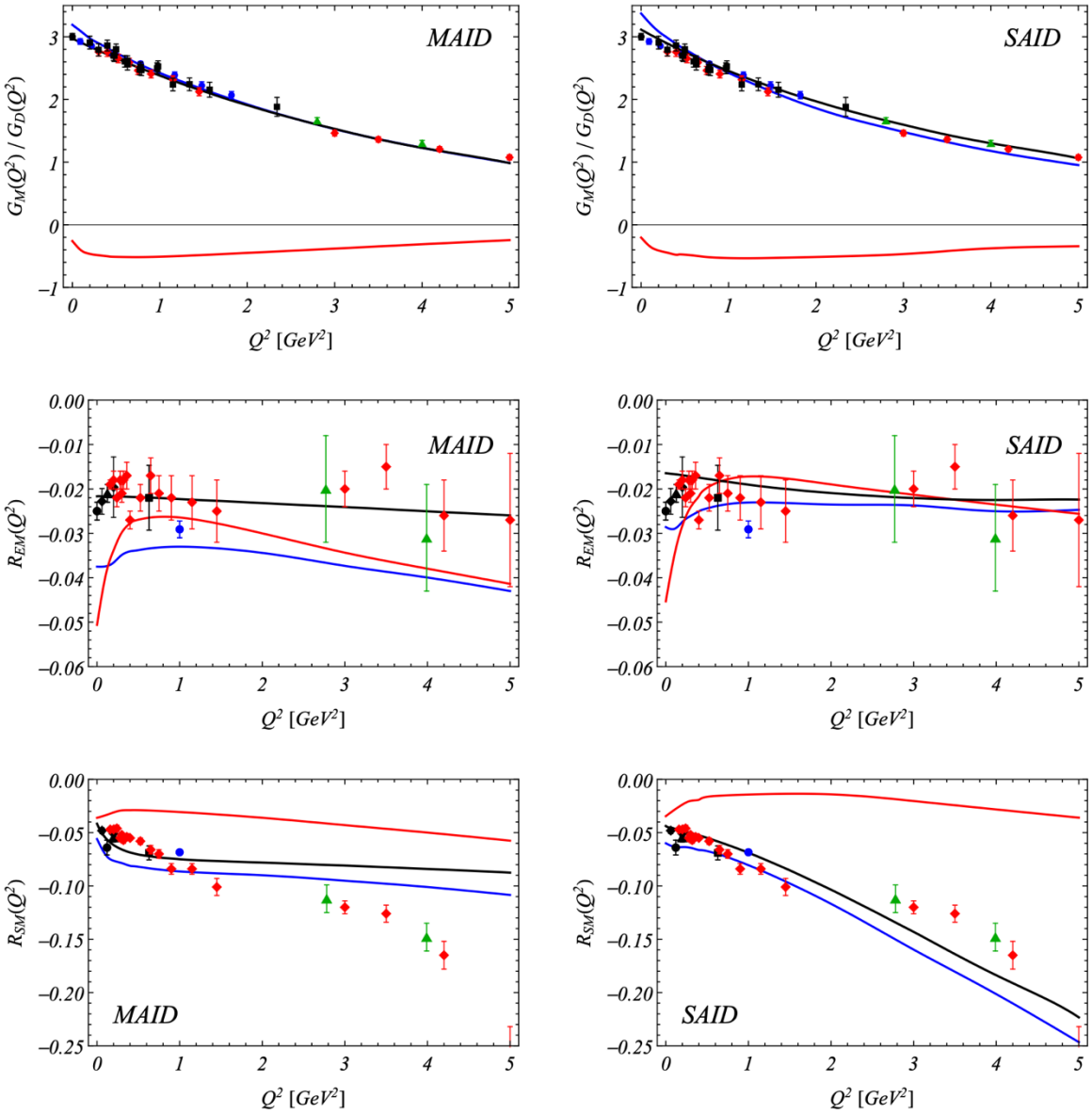


- Transverse charge density ~ Fourier transform of the form factor

Breit-Wigner helicity couplings

- Helicity couplings couplings/Transition form factors at the pole are reaction independent & independent of background modeling
- Consequence of factorization property of the T-matrix at pole
- How much do they differ from Breit-Wigner helicity couplings?

BW & pole definitions compared - Δ(1232)

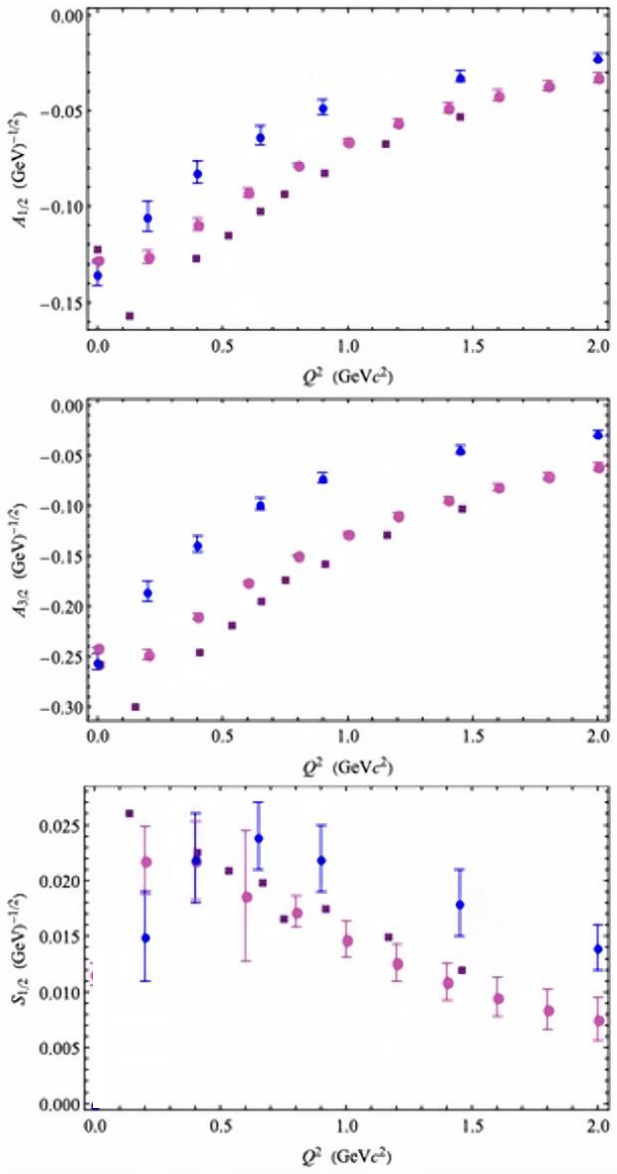


[Tiator, MD et al., 2016]

- Magnetic FF, E/M and S/M ratios
- Real part very similar for ideal resonance case
- Imaginary parts provide some additional information
- Other resonances?

BW helicity couplings from JBW (Prelim.)

[Sadie Burks, MD, Worman, in prep]



- $A_{1/2}$, $A_{3/2}$, $S_{1/2}$
- Fit (s, Q^2)-dependent JBW multipoles
- Uncertainties from different JBW solutions
- Additional uncertainties from fitted energy windows
- Uncertainties from different background
- Compared to MAID
  - Similar parametrization
  - Substantially different data base compared to 2007

Summary

- Juelich-Bonn-Washington/JBW model: Phenomenology of excited baryons through coupled-channels, two- and three-body dynamics.
  - Three-body unitarity restored by consequently including the $\boldsymbol{\pi}$-Roper isobar [Jinzi Wu et al.]
- Renewed effort to explore additional reaction channels in the last years:
  - $\gamma p \to K\Sigma,\ \pi N \to \omega N$
  - $\gamma^* p \to \pi N, \eta N, K\Lambda$ (Electroproduction)
- First global electroproduction analysis of different final states
- Extensive exploration of parameter space with good χ^2 (better than MAID) leads to *significant* variance of some multipoles.
- Many Transition Form factors at the pole extracted for the first time.
- Many hyperon polarization data changed (α_- decay parameter of Λ changed)

[D.G. Ireland et al., PRL, 1904.07616]

- How to find a minimal resonance spectrum? Model selection.

[J. Landay et al., PRD, 1810.00075]

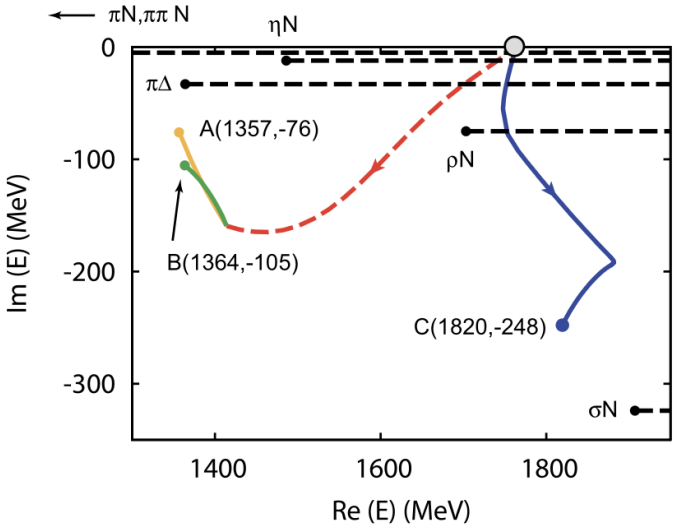
- Data aspects: How to get solid statistical statements out of a heterogeneous data base dominated by systematic errors?

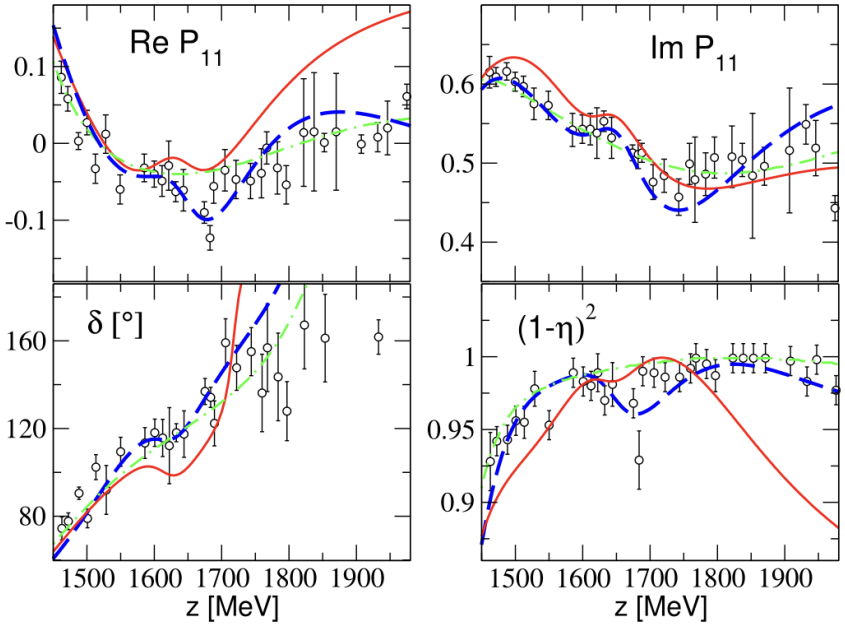
Spare slides

Highlight: P11

- **ANL/Osaka** [0909.1356/nucl-th]
- Roper and N(1710) (?) in ANL/Osaka from one genuine s-channel state & channel dynamics

- **JuBo** [1211.6998/nucl-th]
- Roper generated from ρ t-channel but also strong coupling to σN channel [9911080/nucl-th]
- But genuine pole in IVU/Wu's talk
- N(1710) comes from fitting KΛ, and appears right in elastic πN (!) → not entirely stable result across all analyses

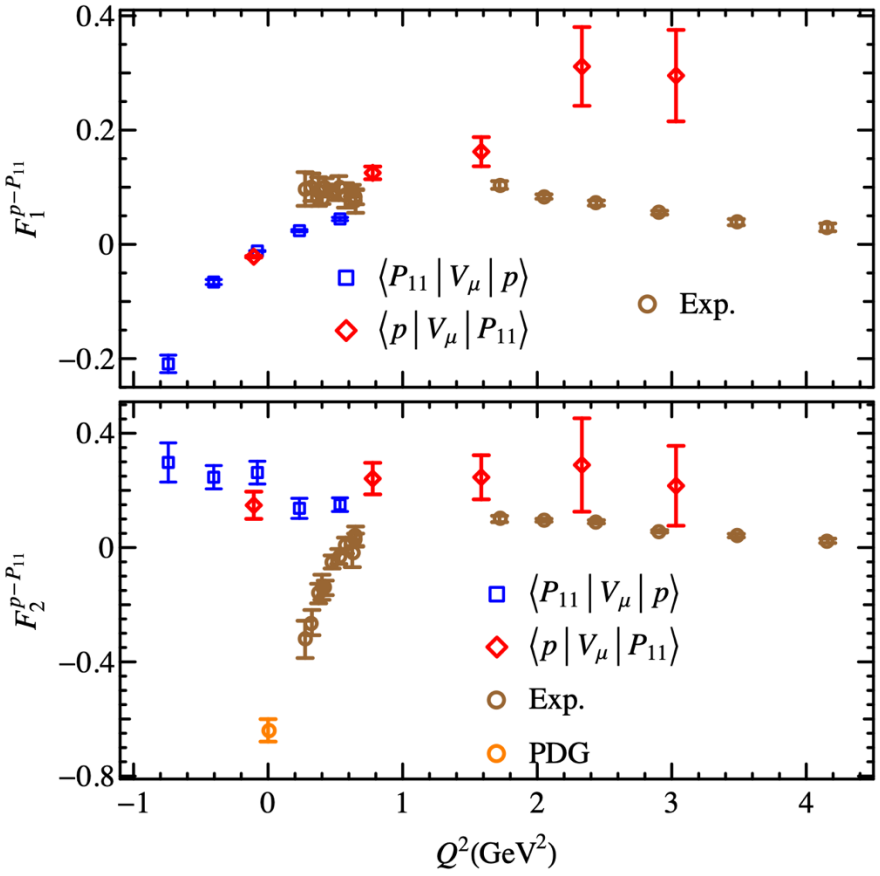




Lattice explorations of TFFs

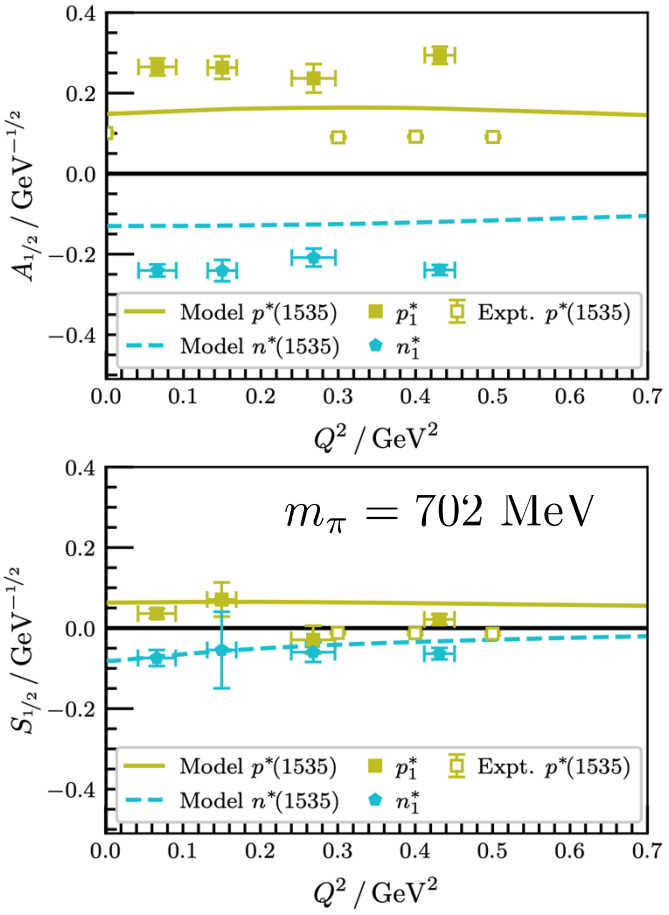
- Proton-Roper

  [H.-W. Lin et al. (2008)]



- N(1535)1/2$^-$

  [F.-M. Stokes et al. (2024)]



Electroproduction Parametrization

- Dependence on virtuality: Channel and resonance-wise:

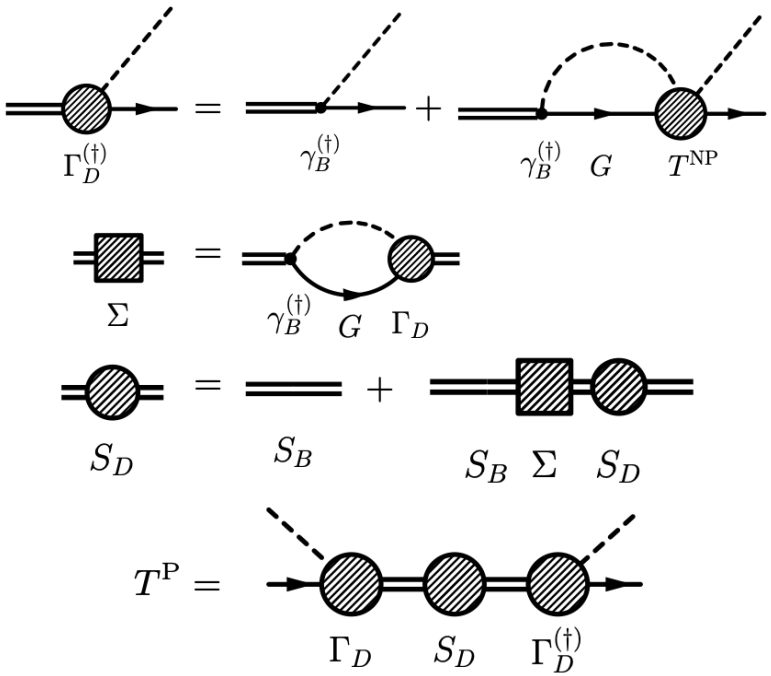
$$\tilde{F}_\mu(Q^2) = \tilde{F}_D(Q^2)\, e^{-\beta_\mu^0 Q^2/m^2}\; P^N(Q^2/m^2, \vec{\beta}_\mu)$$
$$\tilde{F}_i(Q^2) = \tilde{F}_D(Q^2)\, e^{-\delta_i^0 Q^2/m^2}\; P^N(Q^2/m^2, \vec{\delta}_i)$$

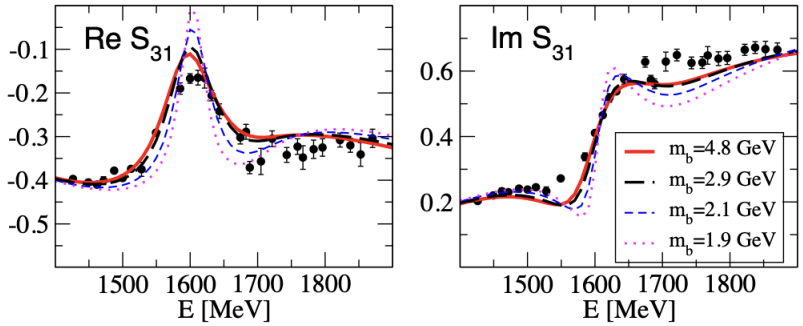
- Factorization:

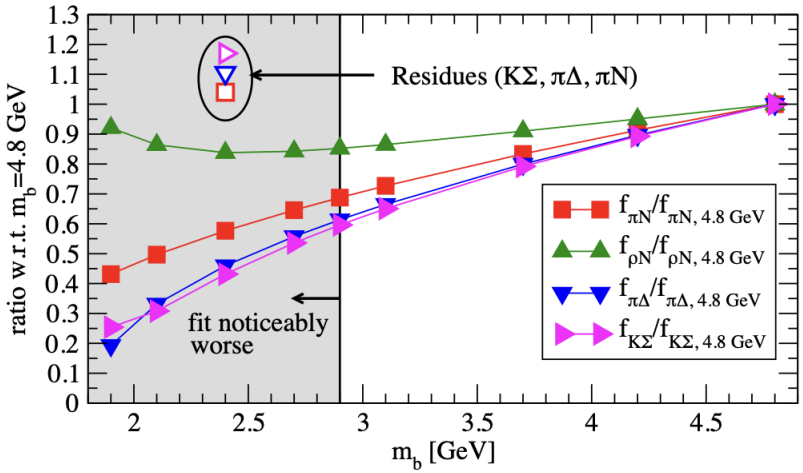
$$\alpha^{NP}_{\mu\gamma^*}(p, W, Q^2) = \tilde{F}_\mu(Q^2)\alpha^{NP}_{\mu\gamma}(p, W)$$
$$\gamma^c_{\gamma^*;i}(W, Q^2) = \tilde{F}_i(Q^2)\gamma^c_{\gamma;i}(W)$$

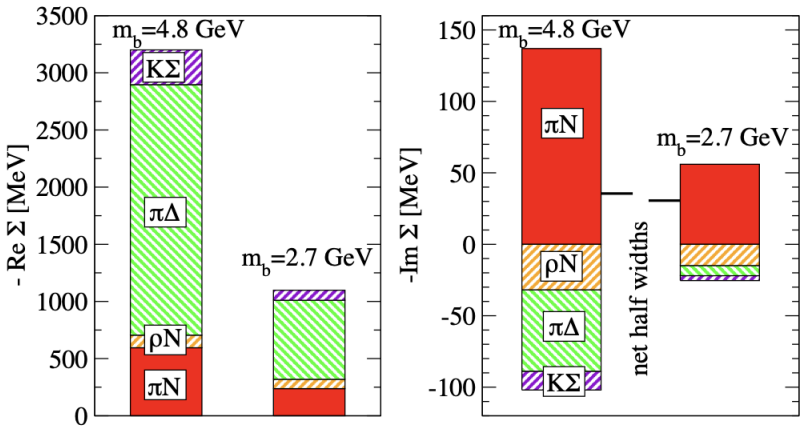
Bare states & resonances

- The bare state masses heavily depend on
  - Regularization
  - Included channels
  - Indirectly, also on the non-resonant interactions



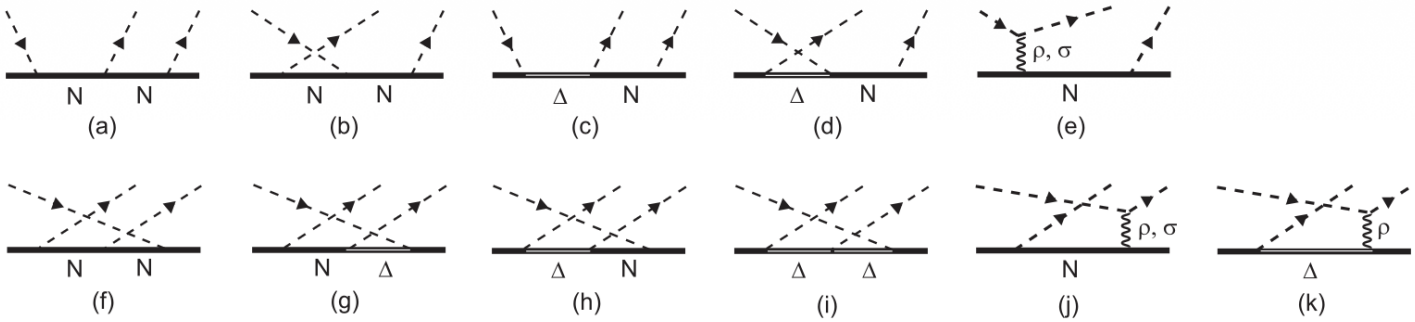




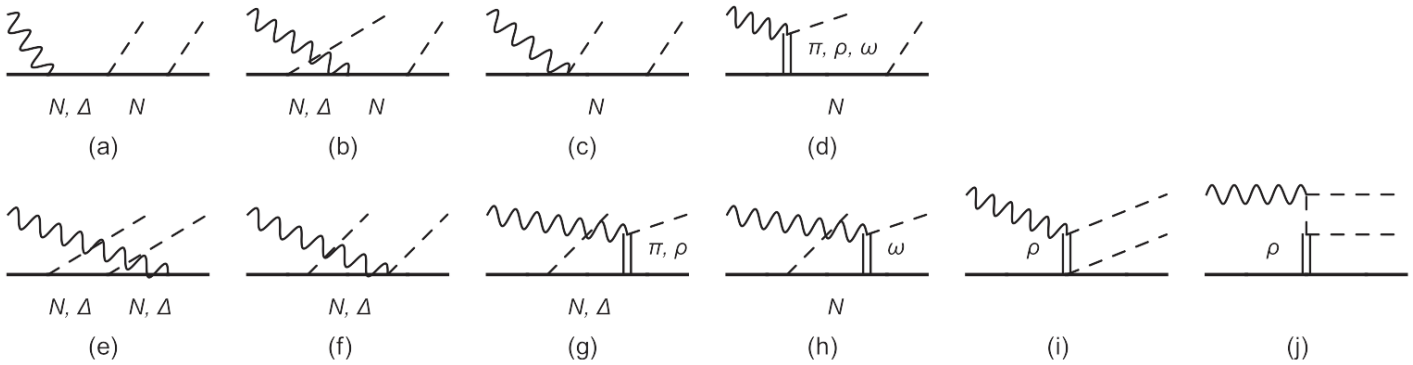


EBAC: $\pi\pi N$ final states

$\pi N \to \pi\pi N$



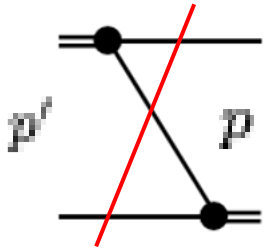
$\gamma N \to \pi\pi N$



How to solve the scattering equation

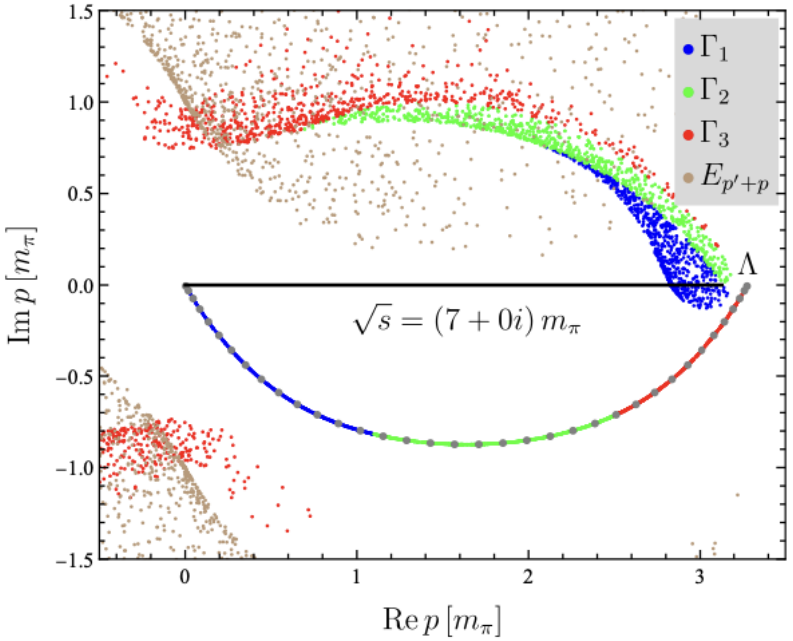
$$\tilde{T}_{ji}(s,p',p) = \tilde{B}_{ji}(s,p',p) + \tilde{C}_{ji}(s,p',p) + \int_0^\Lambda \frac{\mathrm{d}l\, l^2}{(2\pi)^3\, 2E_l} \left(\tilde{B}_{jk}(s,p',l) + \tilde{C}_{jk}(s,p',l)\right) \tilde{\tau}_k(\sigma_l)\, \tilde{T}_{kj}(s,l,p)$$

- **Three-body cuts**



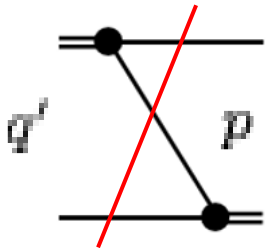
$$\tilde{B}_{ji}(s,\boldsymbol{p}',\boldsymbol{p}) = \frac{(\tilde{\boldsymbol{I}}_F)_{ji}\, v_j^*(p,P-p-p')v_i(p',P-p-p')}{2E_{p'+p}(\sqrt{s}-E_p-E_{p'}-E_{p'+p})+i\epsilon}$$

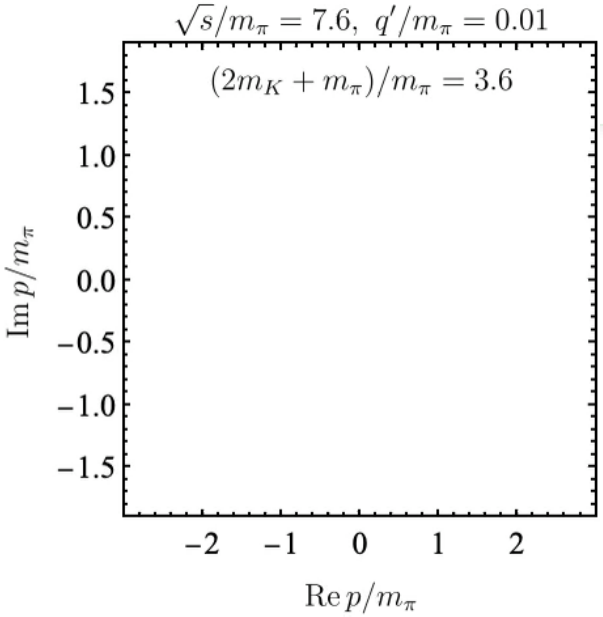
- Angle, energy dependent
- Depend also on p' and p
- Solve LSE for complex momenta on a deformed contour

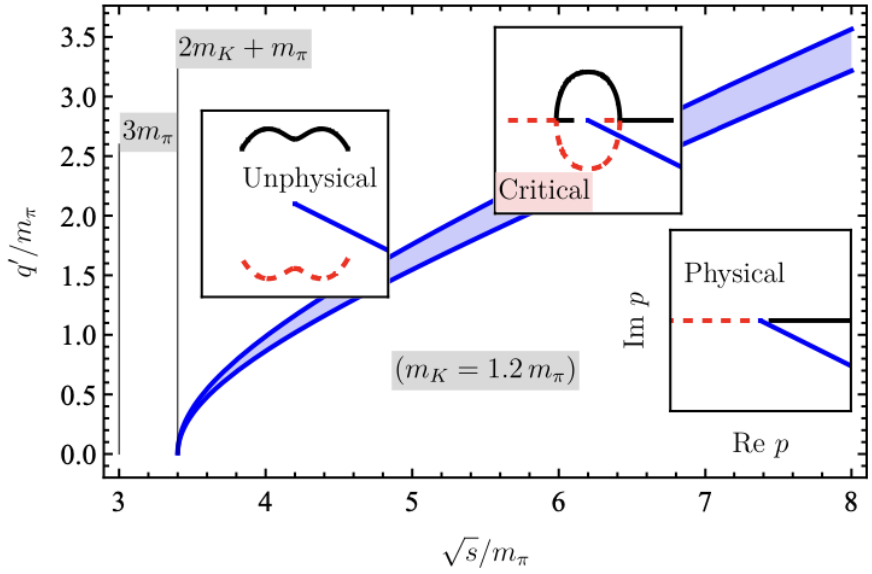


How to get the solution for real, physical momenta

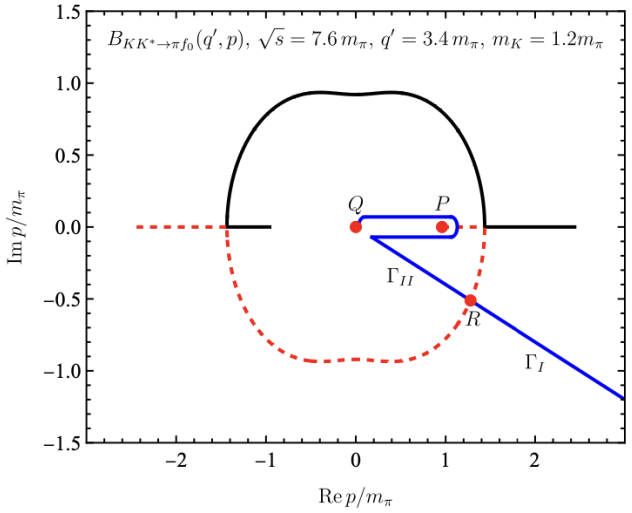
- No solution for real momenta in"critical region"







Solution 1: Contour deformation [Cahill & Sloane]



How to get the solution for real, physical momenta

Solution 2: Direct inversion [Ziegelmann et al.]

Production amplitude $$\tilde{\Gamma}^T_j(s,q') = D_j(s,q') + \int_0^\Lambda \frac{dq\, q^2}{(2\pi)^3 2E_q} \tilde{B}ji(s,q',q)\, \tilde{\tau}_i(\sigma(q))\, \tilde{\Gamma}^T_i(s,q)$$

Ansatz $$\tilde{\Gamma}^T(q) \approx \sum_{i=1}^{N} \tilde{\Gamma}^T(q_i)\, H_i(q)$$ with Lagrange polynomials $$H_i(q) = \frac{\prod_{j\neq i}^{N}(q-q_j)}{\prod_{j\neq i}^{N}(q_i-q_j)}$$

Makes integral equation a matrix equation $$\tilde{\Gamma}^T(q_j) = D(q_j) + A_{ji}\tilde{\Gamma}^T(q_i)$$

With singular integrals $$A_{ji} = \int_0^\Lambda \frac{dq\, q^2}{(2\pi)^3 2E_q} \tilde{B}(q_j,q)\, \tilde{\tau}(\sigma(q))\, H_i(q)$$

… for which many established algorithms exist

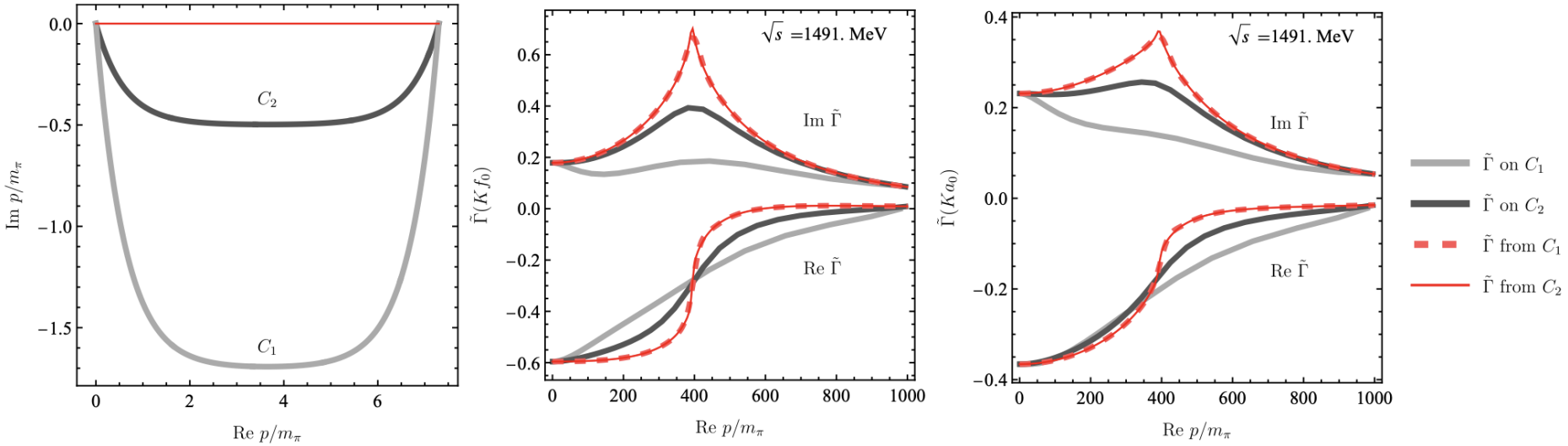
How to get the solution for real, physical momenta

Solution 3: Continued fractions

- Not rigorous as methods 1 and 2, but very good extrapolation properties
- Sample the solution on complex contour and evaluate the continued fraction at real momenta

$$f(x) = f(x_1) + \cfrac{x - x_1}{\rho(x_1, x_2) + \cfrac{x - x_2}{\rho_2(x_1, x_2, x_3) - f(x_1) + \cfrac{x - x_3}{\rho_3(x_1, x_2, x_3, x_4) - \rho(x_1, x_2) + \cdots}}}$$

[example: 2511.02543 ,
MD, K. Khemchandani, A. Martinez



Partial-wave decomposition

- Plane-wave basis

$$T_{\lambda'\lambda}(\boldsymbol{p},\boldsymbol{q}_1) = (B_{\lambda'\lambda}(\boldsymbol{p},\boldsymbol{q}_1) + C) + \sum_{\lambda''} \int \frac{d^3\boldsymbol{l}}{(2\pi)^3 2E_l} (B_{\lambda'\lambda''}(\boldsymbol{p},\boldsymbol{l}) + C)\,\tau(\sigma(l))T_{\lambda''\lambda}(\boldsymbol{l},\boldsymbol{q}_1)$$

$$B^J_{\lambda\lambda'}(q_1,p) = 2\pi \int_{-1}^{+1} \mathrm{d}x\, d^J_{\lambda\lambda'}(x) B_{\lambda\lambda'}(\boldsymbol{q}_1,\boldsymbol{p}) \qquad B^J_{LL'}(q_1,p) = U_{L\lambda} B^J_{\lambda\lambda'}(q_1,p) U_{\lambda' L'}$$

- JLS basis:

$$T^J_{LL'}(q_1,p) = \left(B^J_{LL'}(q_1,p) + C_{LL'}(q_1,p)\right) + \int_0^\Lambda \frac{\mathrm{d}l\, l^2}{(2\pi)^3 2E_l} \left(B^J_{LL''}(q_1,l) + C_{LL''}(q_1,l)\right) \tau(\sigma(l)) T^J_{L''L'}(l,p)$$

Analytic cont. 3-body

[Sadasivan (2021)]
[Doering (2009)]

SMC

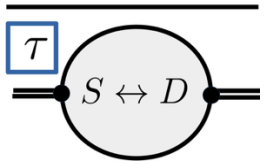
$$T^J_{LL'}(q_1,p) = \left(B^J_{LL'}(q_1,p) + C_{LL'}(q_1,p)\right) + \int_0^\Lambda \frac{\mathrm{d}l\, l^2}{(2\pi)^3 2E_l} \left(B^J_{LL''}(q_1,l) + C_{LL''}(q_1,l)\right) \tau(\sigma(l)) T^J_{L''L'}(l,p)$$

$$\tau^{-1}(\sigma) = K^{-1} - \Sigma\,,$$

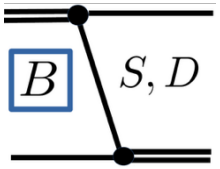
$$\Sigma = \int_0^\infty \frac{\mathrm{d}k\, k^2}{(2\pi)^3} \frac{1}{2E_k} \frac{\sigma^2}{\sigma'^2} \frac{\tilde{v}(k)^* \tilde{v}(k)}{\sigma - 4E_k^2 + i\epsilon}$$

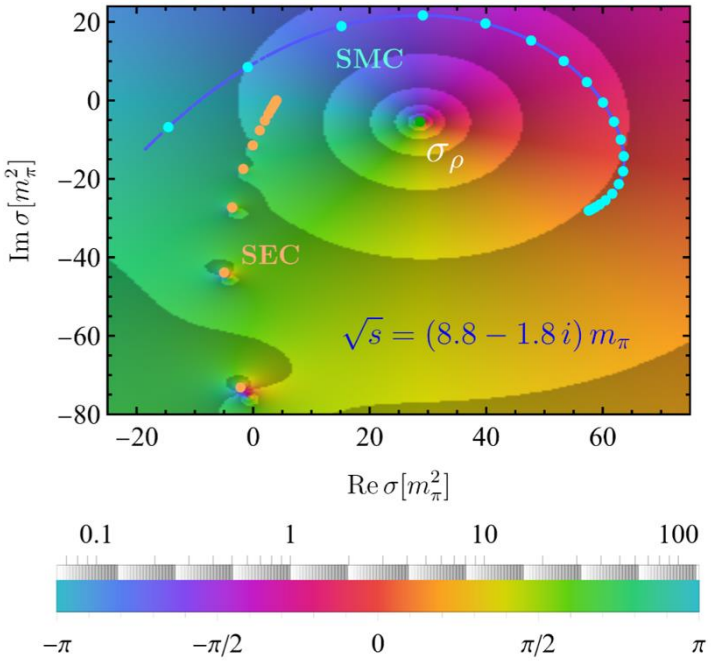
$$B_{\lambda\lambda'}(\boldsymbol{p},\boldsymbol{p}') = \frac{v^*_\lambda(P-p-p',p)\, v_{\lambda'}(P-p-p',p')}{2E_{p'+p}\left(\sqrt{s} - E_p - E_{p'} - E_{p'+p} + i\epsilon\right)}$$

SEC



Singularities





- Two contours (SMC and SEC)
- Deform both "adiabatically" to go to complex s
- Set of rules:
  - Contours cannot intersect with each others
  - Contours cannot intersect with (3-body) cuts
- Passing singularities left or right determines sheet