Machine learning in meson photoproduction

Dalibor Skoupil

skoupil@ujf.cas.cz

(in collaboration with P. Bydžovský, A. Cieplý, D. Petrellis, D. Gazda, D. Trnková)

Nuclear Physics Institute, Czech Academy of Sciences, Czech Republic



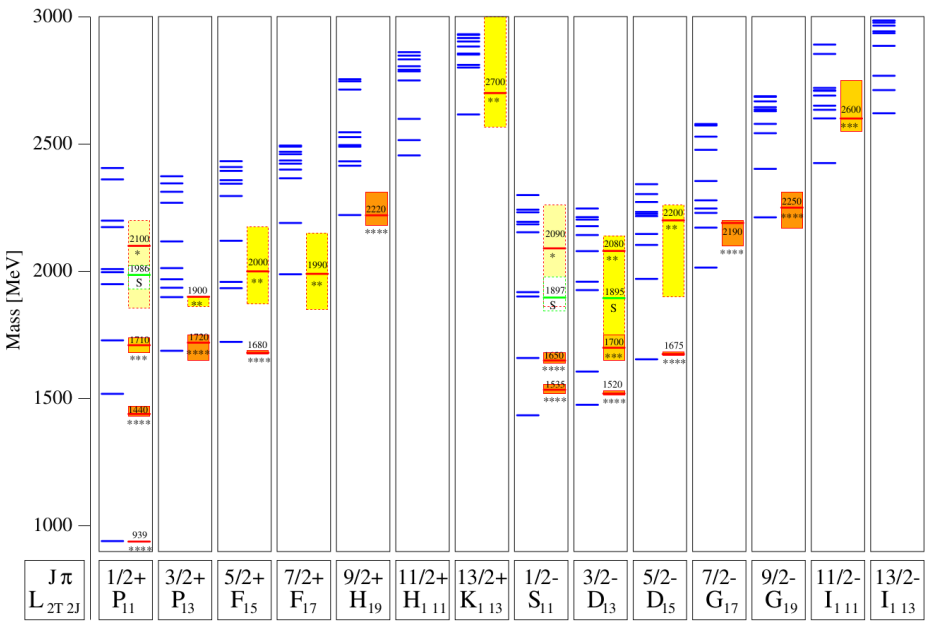
New-EXOTICA:
New Frontiers in Exotic Atoms: high-precision tests of QCD and QED
and probes of beyond Standard Model physics

ECT* – Villa Tambosi, Villazzano, Italy

12 - 16 October 2026

Motivation for focusing on meson (kaon) photoproduction

- Understanding the baryon spectrum
- "Missing" resonance problem:
  Constituent Quark Model predicts more N^* states than is observed in π production experiments.
- Hypernuclear physics calculations:
  PR C **106**, 044609 (2022),
  PR C **108**, 024615 (2023),
  PR C **112**, 024609 (2025)
  - studies of charge-symmetry breaking, baryon-baryon interaction, . . .



Bonn CQM [EPJ **A10** (2001) 395–446]

- New good-quality photoproduction data:
  CLAS, BGOOD, LEPS, GRAAL, MAMI, SAPHIR Collaborations
- Effective theories and models:
  a remedy for not being able to use perturbative QCD at low energies.

Introduction

How to produce strangeness (hyperons and hypernuclei)

a. Strangeness exchange process

$$K + N \rightarrow \pi + \Lambda$$

- $\sigma \sim$ mb; low luminosity

b. Strangeness creation process

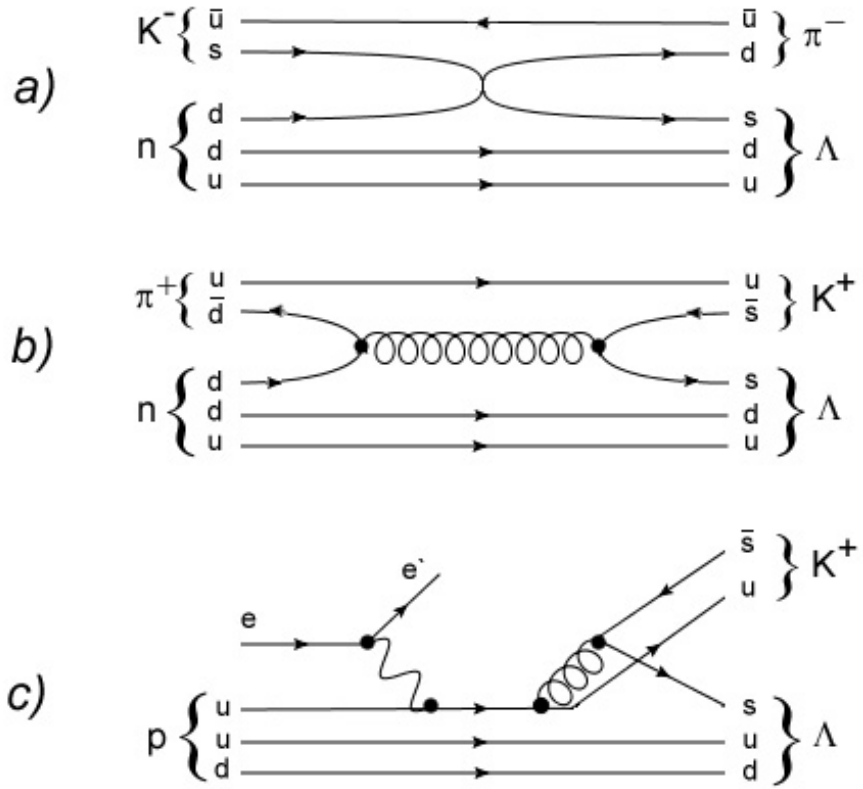
$$\pi + N \rightarrow K + \Lambda$$

- $\sigma \sim 10\,\mu$b

c. Electromagnetic production of strangeness

$$e + N \rightarrow e' + K + \Lambda$$

- $\sigma \sim 100$ nb
- low luminosity but high-intensity electron beam
- high energy resolution

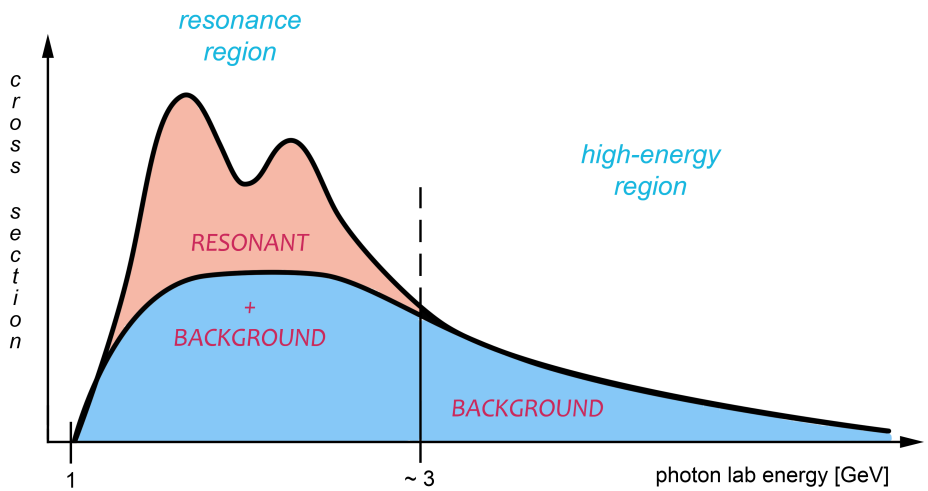


Introduction

Photoproduction process

$$p(p) + \gamma(k) \rightarrow K^+(p_K) + \Lambda(p_\Lambda)$$

- Threshold: $E_\gamma^{lab} = 0.911\,\text{GeV}$, $W = 1.609\,\text{GeV}$
- In the lowest order, the reaction is described by the exchange of hadrons.
  - *The 3rd nucleon-resonance region:*
    many resonant states and no dominant one in the kaon photoproduction
    $\rightarrow$ need to assume a large number of nucleon resonances with mass $< 2.5\,\text{GeV}$



- Resonance region:
  resonance contributions dominate (N^*)
- Background:
  - IM: a plenty of nonresonant contributions (p, K, Λ; K^* and Y^*)
  - RPR: exchange of kaon trajectories

Introduction

Methods of description of the meson photoproduction

Quark models [PR C **52**, 1648 (1995), PR C **56**, 1099 (1997), NP B **349**, 655 (1991), etc.]

- quark degrees of freedom; only a few free parameters
- contributions of resonances arise naturally as excited states of the quark system; "missing" resonance problem

Effective-Lagrangian calculations

Chiral models [NP A **612**, 297 (1997), EPJ A **34**, 161 (2007)]

- resonances generated dynamically by the chiral $\mathcal{L}_{eff}$; limitation to processes below 1 GeV above threshold

Coupled-channels models [PL B **57**, 101 (2001), PR C **59**, 460 (1999), EPJ A **54**, 110 (2018)]

- taking into account various intermediate processes between initial and final state; ambiguous determination of background, large number of unknown parameters

Single-channel models

- simplification: tree-level approximation; use of $\mathcal{L}_{eff}$, inner structure of hadrons described with help of hadron form factors
- **isobar model**
  [PR C **42**, 108 (1990), PR C **46**, 1617 (1992) , PR C **53**, 2613 (1996), PR C **61**, 012201(R) (1999), PR C **65**, 015201 (2001), PR C **80**, 065205 (2009), etc.]
- **Regge(-plus-resonance) model**
  [NP A **627**, 645 (1997), PR C **75**, 045204 (2007), PR C **86**, 015212 (2012), etc.]

Isobar model

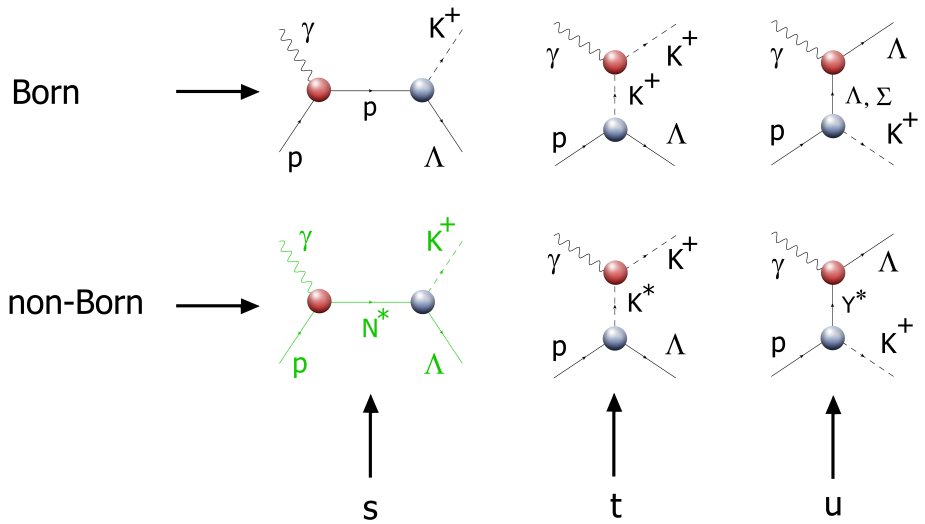
Photoproduction off protons with K^+ and Λ in the final state

Single-channel approximation

- higher-order contributions (rescattering, FSI) included, to some extent, by means of effective values of the coupling constants

Use of effective hadron Lagrangian

- hadrons either in their ground or excited states
- amplitude constructed as a sum of tree-level Feynman diagrams
  - background and resonant part



- hadronic form factors account for the inner structure of hadrons

Free parameters (couplings, hff's cutoffs) adjusted to experimental data.

Satisfactory agreement with the data in the energy range $W = 1.6 - 2.5\,\mathrm{GeV}$.

Isobar model

Calculation procedure

- Reaction amplitude: sum of s-, t-, and u-channel (non) Born amplitudes

$$\mathbb{M} = \sum_x \mathbb{M}_x, \text{ where } x \equiv s,\ t,\ u,\ N^*,\ K^*,\ Y^*$$

- Each contribution can be rewritten in a compact form

$$\mathbb{M}(p, p_\Lambda, k) = \bar{u}(p_\Lambda)\gamma_5 \left(\sum_{j=1}^{6} \mathcal{A}_j(s,t)\mathcal{M}_j \right) u(p),$$

where $\mathcal{A}_j$ are (complex) scalar amplitudes and $\mathcal{M}_j$ are gauge-invariant operators,

$$\begin{aligned}
\mathcal{M}_1 &= \tfrac{1}{2}\left[\not{k}\not{\varepsilon} - \not{\varepsilon}\not{k}\right], & \mathcal{M}_2 &= (p\cdot\varepsilon) - (k\cdot p)\tfrac{(k\cdot\varepsilon)}{k^2}, \\
\mathcal{M}_3 &= (p_\Lambda\cdot\varepsilon) - (k\cdot p_\Lambda)\tfrac{(k\cdot\varepsilon)}{k^2}, & \mathcal{M}_4 &= \not{\varepsilon}(k\cdot p) - \not{k}(p\cdot\varepsilon), \\
\mathcal{M}_5 &= \not{\varepsilon}(k\cdot p_\Lambda) - \not{k}(p_\Lambda\cdot\varepsilon), & \mathcal{M}_6 &= \not{k}(k\cdot\varepsilon) - \not{\varepsilon}k^2.
\end{aligned}$$

[terms with $(k\cdot\varepsilon)$ and k^2 appear only in electroproduction with virtual photons]

- When N diagrams contribute, then each of them contributes to $\mathbb{M}$ and

$$\mathcal{A}_j = \sum_{k=1}^{N} \mathcal{A}_j^k.$$

- Works for hadronic part of any EM pseudoscalar meson production amplitude!

Isobar model

Calculation procedure

- CGLN amplitudes $f_i(k^2, s, t)$ (in cms) [PR **106**, 1345-1355 (1957)]

$$\mathbb{M} = \chi_\Lambda^\dagger \mathcal{F} \chi_p; \quad \mathcal{F} = f_1(\vec{\sigma}\cdot\vec{\varepsilon}) - \mathrm{i} f_2(\vec{\sigma}\cdot\hat{p}_K)[\vec{\sigma}\cdot(\hat{k}\times\vec{\varepsilon})] + f_3(\vec{\sigma}\cdot\hat{k})(\hat{p}_K\cdot\vec{\varepsilon}) + f_4(\vec{\sigma}\cdot\hat{p}_K)(\hat{p}_K\cdot\vec{\varepsilon}) + f_5(\vec{\sigma}\cdot\hat{k})(\hat{k}\cdot\vec{\varepsilon}) + f_6(\vec{\sigma}\cdot\hat{p}_K)(\hat{k}\cdot\vec{\varepsilon}),$$

  where *e.g.*

$$f_1 = N^*[-(W - m_p)\mathcal{A}_1 + (k\cdot p)\mathcal{A}_4 + (k\cdot p_\Lambda)\mathcal{A}_5 - k^2\mathcal{A}_6]$$

- Response functions, *e.g.* **transverse cross section**

$$\frac{d\sigma}{d\Omega} = \sigma_T = C\left\{|f_1|^2 + |f_2|^2 - 2\,\mathrm{Re}\, f_1 f_2^* \cos\theta_K + \sin^2\theta_K\left[\frac{1}{2}(|f_3|^2 + |f_4|^2) + \mathrm{Re}\,(f_1 f_4^* + f_2 f_3^* + f_3 f_4^* \cos\theta_K)\right]\right\},$$

  (for other response functions see Z. Phys. A **352** (1995) 327)

- **Single polarization asymmetries** A **(**P**,** Σ**,** T**)** have the form

$$A = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-},$$

  where $d\sigma$ denotes the differential cross section and the superscript ($+$ or $-$) the polarization state (parallel or antiparallel) with respect to the corresponding quantization axis

Isobar model

Important features of the isobar model

Exchanges of high-spin resonant states

- Rarita-Schwinger (RS) propagator for the spin-3/2 field

$$S_{\mu\nu}(q) = \frac{\not{q} + m}{q^2 - m^2 + im\Gamma}\mathcal{P}^{(3/2)}_{\mu\nu} - \frac{2}{3m^2}(\not{q} + m)\mathcal{P}^{(1/2)}_{22,\mu\nu} + \frac{1}{m\sqrt{3}}\left(\mathcal{P}^{(1/2)}_{12,\mu\nu} + \mathcal{P}^{(1/2)}_{21,\mu\nu}\right)$$

  - spin-3/2 projection operator: $\mathcal{P}^{(3/2)}_{\mu\nu} = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3q^2}(\not{q}\gamma_\mu q_\nu + q_\mu\gamma_\nu\not{q})$.
  - spin-1/2 projection operators: $\mathcal{P}^{(1/2)}_{22,\mu\nu} = \frac{q_\mu q_\nu}{q^2}$, $\mathcal{P}^{(1/2)}_{12,\mu\nu} = \frac{q^\rho q_\nu \gamma_{\mu\rho}}{q^2\sqrt{3}}$, $\mathcal{P}^{(1/2)}_{21,\mu\nu} = \frac{q_\mu q^\rho \gamma_{\rho\nu}}{q^2\sqrt{3}}$

- $\mathcal{L}_{int}$ invariant under local gauge transformation of the RS field [PR D **58**, 096002 (1998)]
  - interaction vertices are transverse: $q_\mu\Gamma^\mu(q,\ldots) = 0$

    $\Rightarrow$ spin-1/2 projection operators decouple in the amplitude:

$$\Gamma^\mu(q,\ldots)S_{\mu\nu}(q)\Gamma^\nu(q,\ldots) = \Gamma^\mu(q,\ldots)\frac{\not{q}+m}{q^2-m^2+im\Gamma}\mathcal{P}^{(3/2)}_{\mu\nu}\Gamma^\nu(q,\ldots).$$

. . . analogously in the case of higher-spin fields [PR C **84**, 045201 (2011)]

Isobar model

Important features of the isobar model

Energy-dependent decay widths of nucleon resonances

$\rightarrow$ (partial) restoration of unitarity

$$\Gamma(W) = \Gamma_{N^*} \frac{W}{m_{N^*}} \sum_i x_i \left(\frac{|\vec{q}_i|}{|\vec{q}_i^{N^*}|} \right)^{2l+1} \frac{D(|\vec{q}_i|)}{D(|\vec{q}_i^{N^*}|)},$$

Extension from photoproduction to electroproduction

- Phenomenological form factors in the electromagnetic vertex [PR C **66**, 045501 (2002)]
- Longitudinal couplings of N^*'s to γ^* (crucial at small Q^2)

$$V^{EM}(N^*_{1/2} p\gamma) = -i \frac{g_3^{EM}}{(m_R + m_p)^2} \Gamma_\mp \gamma_\beta \mathcal{F}^\beta,$$

$$V^{EM}_\mu(N^*_{3/2} p\gamma) = -i \frac{g_3^{EM}}{m_R(m_R + m_p)^2} \gamma_5 \Gamma_\mp (\not{q} g_{\mu\beta} - q_\beta \gamma_\mu) \mathcal{F}^\beta,$$

$$V^{EM}_{\mu\nu}(N^*_{5/2} p\gamma) = -i \frac{g_3^{EM}}{(2m_p)^5} \Gamma_\mp (q_\alpha q_\beta g_{\mu\nu} + q^2 g_{\alpha\mu} g_{\beta\nu} - q_\alpha q_\nu g_{\beta\mu} - q_\beta q_\nu g_{\alpha\mu}) p^\alpha \mathcal{F}^\beta.$$

Fitting the data in the $K^+\Lambda$ channel

Minimization of χ^2/n.d.f. with help of MINUIT library

Resonance selection

- s channel: spin-1/2, 3/2, and 5/2 N^* with mass $< 2.5\,\text{GeV}$;
- t channel: $K^*(892)$, $K_1(1272)$
- u channel: $Y^*(1/2)$ and $Y^*(3/2)$

Free parameters ($\approx 30 + 10$):

- SU(3)_f : $-4.4 \leq g_{K\Lambda N}/\sqrt{4\pi} \leq -3.0$, $0.8 \leq g_{K\Sigma N}/\sqrt{4\pi} \leq 1.3$
- K^*'s have vector and tensor couplings
- spin-1/2 resonance $\rightarrow$ 1 parameter; spin-3/2 and 5/2 resonance $\rightarrow$ 2 parameters
- 2 cut-off parameters for the hff
- 1 longitudinal coupling for each N^*
- 2 cut-off parameters for the emff of K^* and K_1

Experimental data

3383 $p(\gamma, K^+)\Lambda$ data

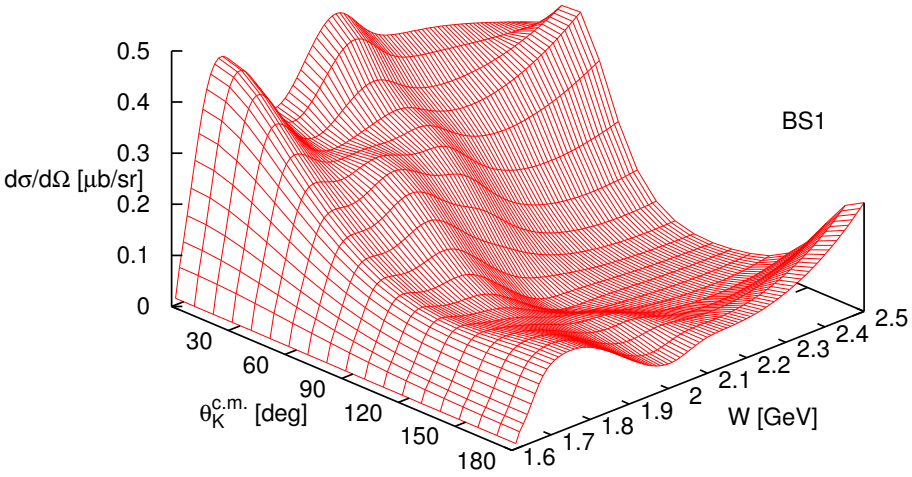
- cross section for $W < 2.355\,\text{GeV}$ (CLAS 2005 & 2010; LEPS, Adelseck-Saghai)
- hyperon polarisation for $W < 2.225\,\text{GeV}$ (CLAS 2010)
- beam asymmetry (LEPS)

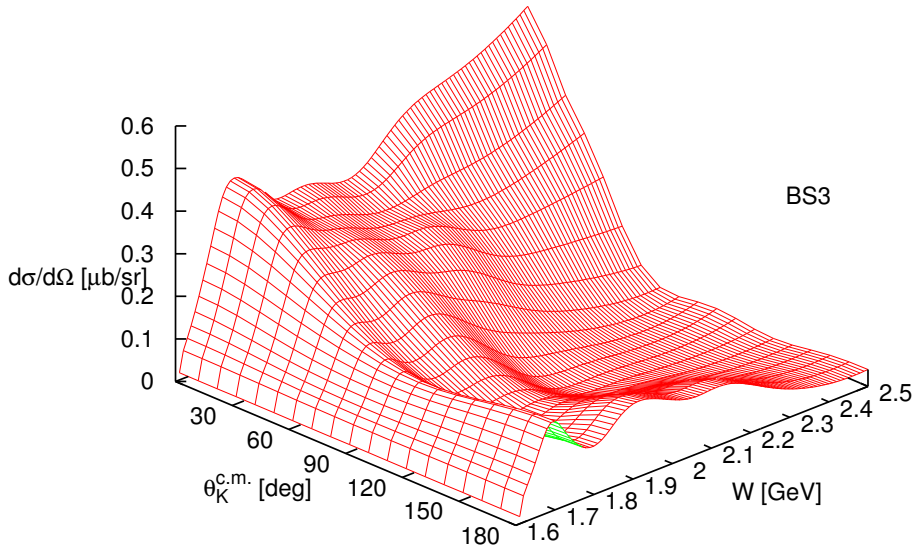
171 $p(e, e' K^+)\Lambda$ data

- $\sigma_U, \sigma_T, \sigma_L, \sigma_{LT'}, \sigma_K$

Resulting models for the $K^+\Lambda$ photo- and electroproduction

BS models (BS = Bydžovský, Skoupil) [PR C **93**, 025204 (2016), PR C **97**, 025202 (2018)]





BS1 model ($\chi^2/n.d.f. = 1.64$)

- $S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{13}(1720)$, $F_{15}(1860)$, $D_{13}(1875)$, $F_{15}(2000)$;
- $K^*(892)$, $K_1(1272)$;
- $\Lambda(1520)$, $\Lambda(1800)$, $\Lambda(1890)$, $\Sigma(1660)$, $\Sigma(1750)$, $\Sigma(1940)$;
- multidipole form factor:

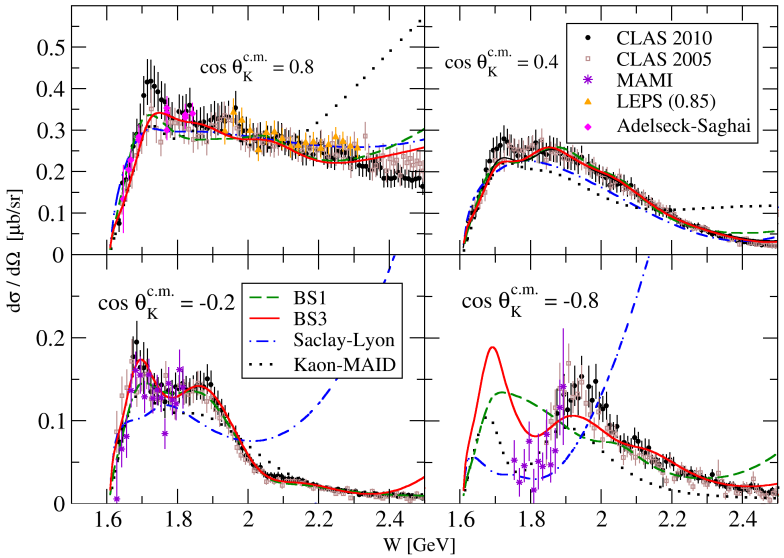
  $\Lambda_{bgr} = 1.88\,\text{GeV}$, $\Lambda_{res} = 2.74\,\text{GeV}$

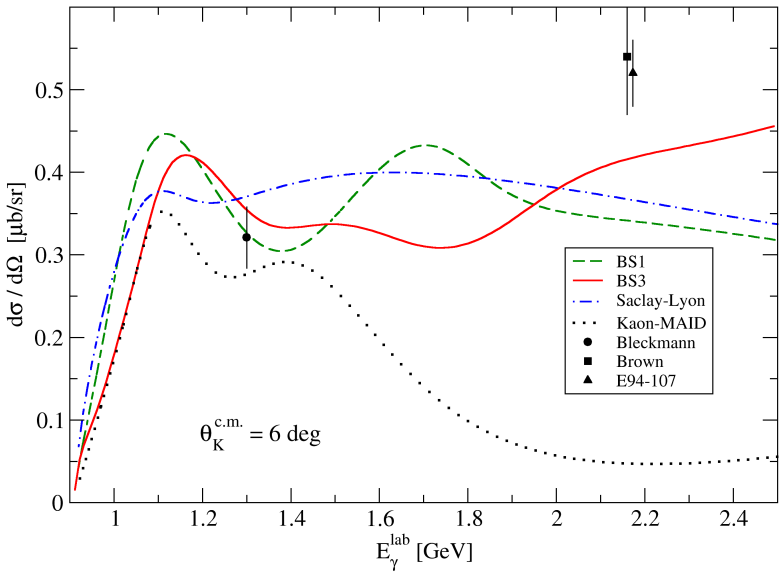
BS3 model ($\chi^2/n.d.f. = 1.74$)

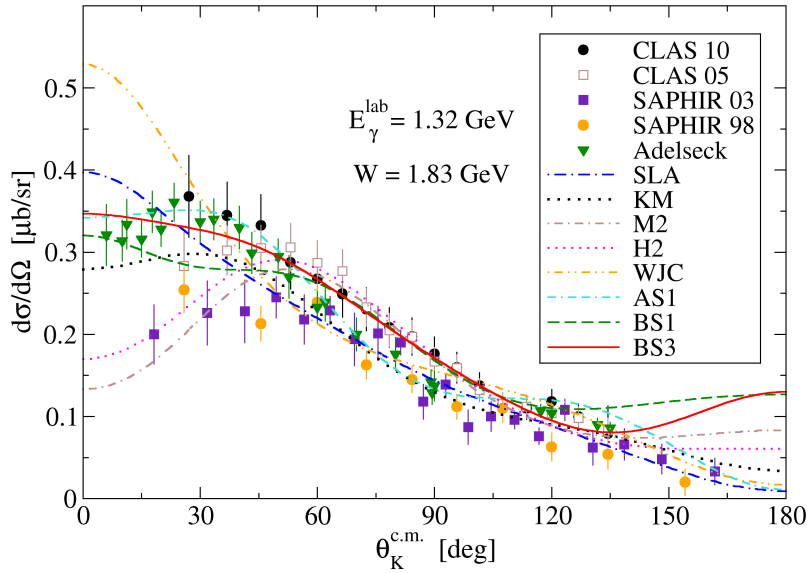
- $S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{11}(1710)$, $P_{13}(1720)$, $F_{15}(1860)$, $D_{13}(1875)$, $P_{13}(1900)$, $F_{15}(2000)$, $D_{13}(2120)$;
- $K^*(892)$, $K_1(1272)$;
- $\Lambda(1405)$, $\Lambda(1600)$, $\Lambda(1890)$, $\Sigma(1670)$;
- dipole form factor:

  $\Lambda_{bgr} = 1.24\,\text{GeV}$, $\Lambda_{res} = 0.89\,\text{GeV}$

Cross sections of $p(\gamma, K^+)\Lambda$







← Brown [$Q^2 = 0.18\,(\mathrm{GeV}/c)^2$]

E94-107 [$Q^2 = 0.07\,(\mathrm{GeV}/c)^2$]:
data for $p(e, e' K^+)\Lambda$ with:

$\sigma_L \sim Q^2$,

$\sigma_{TT} \sim \sin^2 \theta_K^{c.m.}$,

$\sigma_{LT} \sim \sqrt{Q^2} \sin \theta_K^{c.m.} \Rightarrow \sigma \approx \sigma_T$

Remedy for over-fitting: regularization

Least Absolute Shrinkage and Selection Operator (LASSO)

- penalty term: $\lambda \sum_{j=1}^{K} |w_j|$
- uses L1 norm (absolute values)
- shrinks coefficients and **can set some of them to zero**
- performs automatic feature selection and produces **sparser models**
- constraint region is a diamond with sharp corners $\rightarrow$ solution often ends up on axes $\rightarrow$ some coefficients become exactly zero

Ridge regression

- penalty term: $\lambda \sum_{j=1}^{K} w_j^2$
- uses L2 norm (squared coefficients)
- shrinks coefficients but **do not set them exactly to zero**
- keeps all predictors in the model
- constraint region is a circle (or hypersphere) which is smooth $\rightarrow$ solution only rarely ends up exactly on axes $\rightarrow$ coefficients rarely become exactly zero

Elastic net

- combines both penalties: $\lambda \sum_{j=1}^{K} |w_j| + \lambda \sum_{j=1}^{K} w_j^2$
- performs feature selection (like LASSO)
- handles correlated predictors better (like Ridge)

$K^+\Sigma^-$ channel within the isobar model

Least Absolute Shrinkage and Selection Operator (LASSO)

Fitting procedure with MINUIT library: **minimizing the** χ^2

$$\chi^2 = \sum_{i=1}^{N} \frac{[d_i - p_i(c_1, \ldots, c_n)]^2}{\sigma_{d_i}^2},$$

$(c_1, \ldots, c_n)$ – set of free parameters, $(d_1, \ldots, d_N)$ – set of data points, p_i – theory, σ_{d_i} – error

Problem: χ^2 minimization cannot prevent overfitting

Remedy to the overfitting issue: regularization

- introduce a penalty term to the $\chi^2 \rightarrow$ penalization of large parameter values

$$\chi_P^2 = \chi^2 + P(\lambda)$$

- **penalty term:** $P(\lambda) = \lambda^4 \sum_{i=1}^{N_{res}} |g_i|$

  λ – regularization parameter, g_i – resonances' couplings

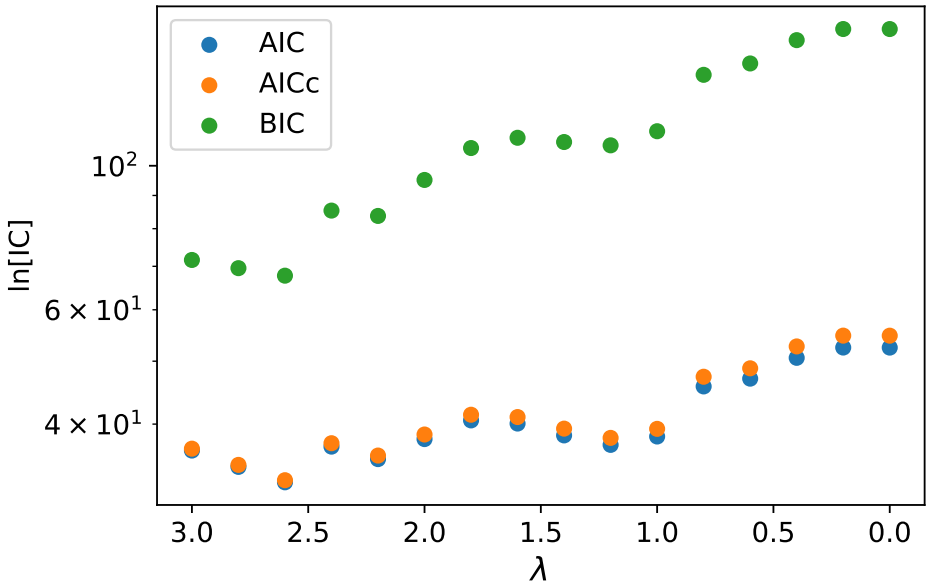
- Least Absolute Shrinkage Selection Operator (LASSO)
  forces some of the parameters to zero $\rightarrow$ selection of a subset of the fit parameters

$K^+\Sigma^-$ channel within the isobar model

Information criteria:

- Akaike information criterion
  $\mathrm{AIC} = 2n_i + \chi^2_P$
- Corrected Akaike information criterion
  $\mathrm{AICc} = \mathrm{AIC} + \frac{2n_i(n_i+1)}{N-n_i-1}$
- Bayesian information criterion
  $\mathrm{BIC} = n_i \ln(N) + \chi^2_P$

n_i - no. of parameters corresponding to λ_i
N - number of data points



Applying the information criteria – forward selection

1. start with the full model: parameters initialized within $\langle -1; +1 \rangle$; use λ_{max}
2. perform LASSO χ^2_P minimization and compute IC
3. in each run reduce λ and run LASSO with the values of the previous run as starting values
4. repeat until λ_{min} is reached

Optimal λ occurs at the minimum of the IC.

$K^+\Sigma^-$ channel within the isobar model

PR C **104**, 065202 (2021)

Fitting procedure

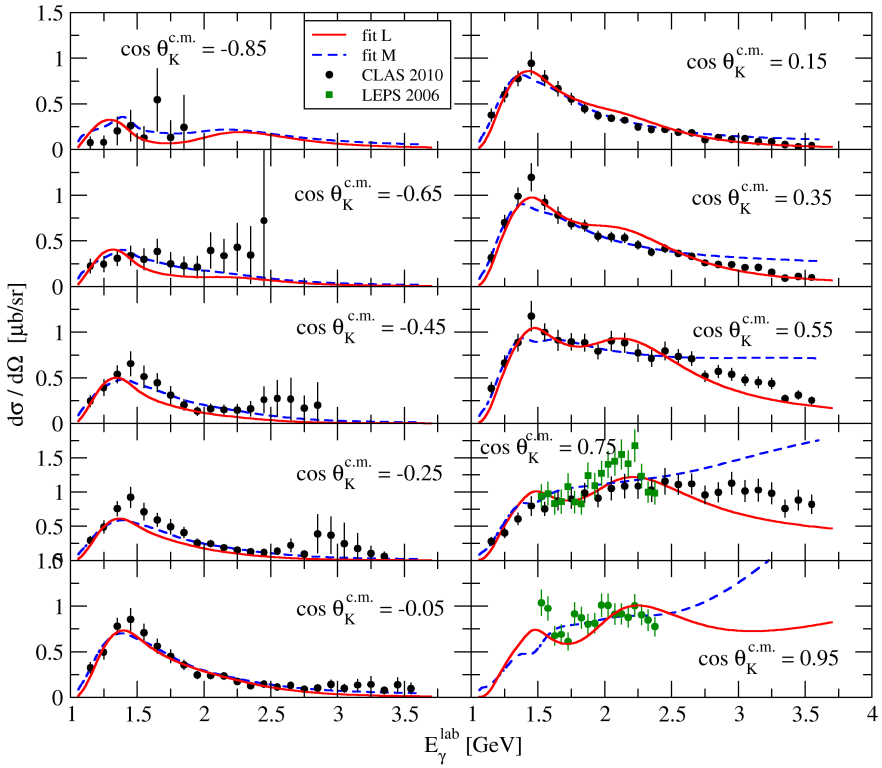
- resonance selection: motivation from previous analysis of $K^+\Lambda$ channel
- non resonant part: Born terms and exchanges of K^* and K_1 and Σ^*'s
- resonant part: exchanges of N^*'s and Δ^*'s in the s channel
- around 600 data utilized to fit $\leq$ 25 parameters
  (new CLAS data on $\Sigma \rightarrow$ main motivation for this study)
- result with the smallest $\chi^2/\text{ndf} = 2.3 \rightarrow$ **fit M** (25 parameters, 14 resonances)
- **LASSO** applied at fit M: $\chi^2_P/\text{ndf} = 3.4 \rightarrow$ **fit L** (17 parameters, 9 resonances)

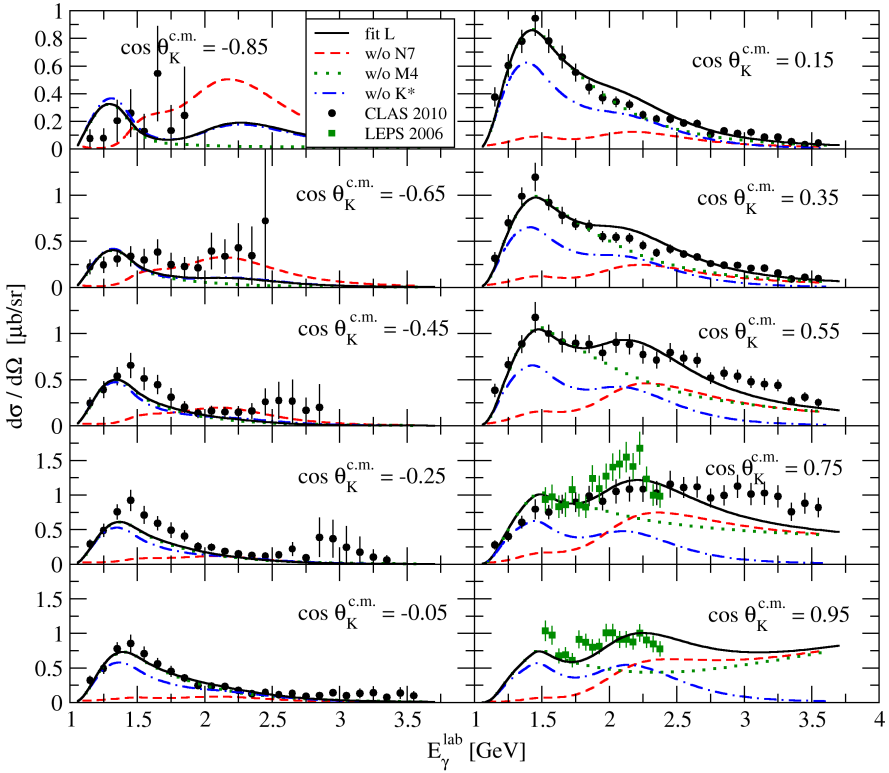
Characteristics of models

- only one Δ resonance introduced
- no hyperon resonances needed for reliable data description
- results in very good agreement with the cross-section and beam-asymmetry data
- fit L is very economical

$K^+\Sigma^-$ channel within the isobar model

Differential cross section in dependence on the photon lab energy





Notation: N7: N(1720)$3/2^+$, M4: N(2060)$5/2^-$

Ridge regression in the $K^+\Lambda$ channel

PR C **107**, 045206 (2023)

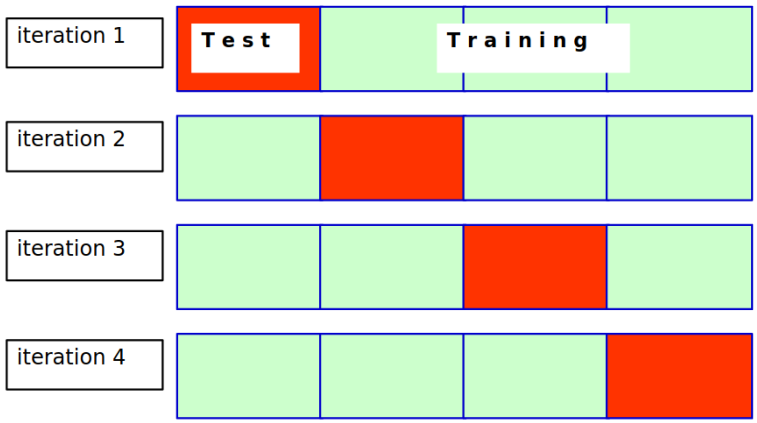
Why?

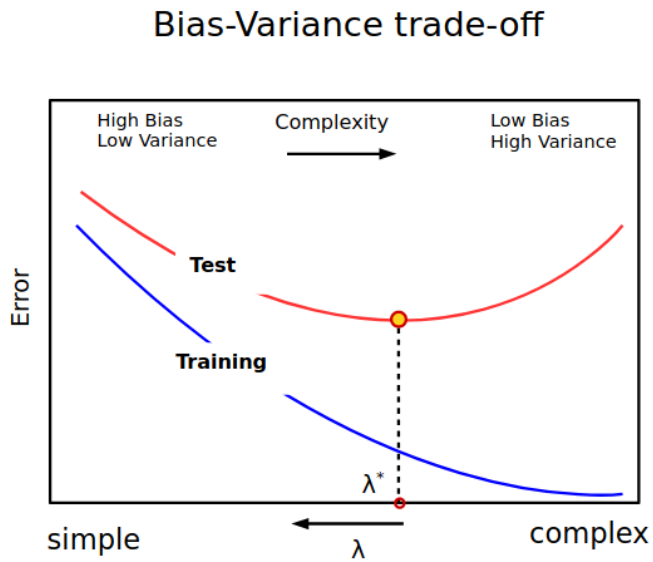
- include recent measurements of polarization observables (PR C **93**, (2016) 065201)
- need to investigate the role of hyperon resonances in KY photoproduction
- large values of hyperon couplings: ridge regression to suppress them during the fitting procedure

Ridge regression

- $\chi^2_P = \chi^2 + \lambda^4 \sum_{i=1}^{n_\Lambda} g_i^2$, ($n_\Lambda$ = no. of Y couplings)
- parameter values reduced but they are *not* reduced to zero

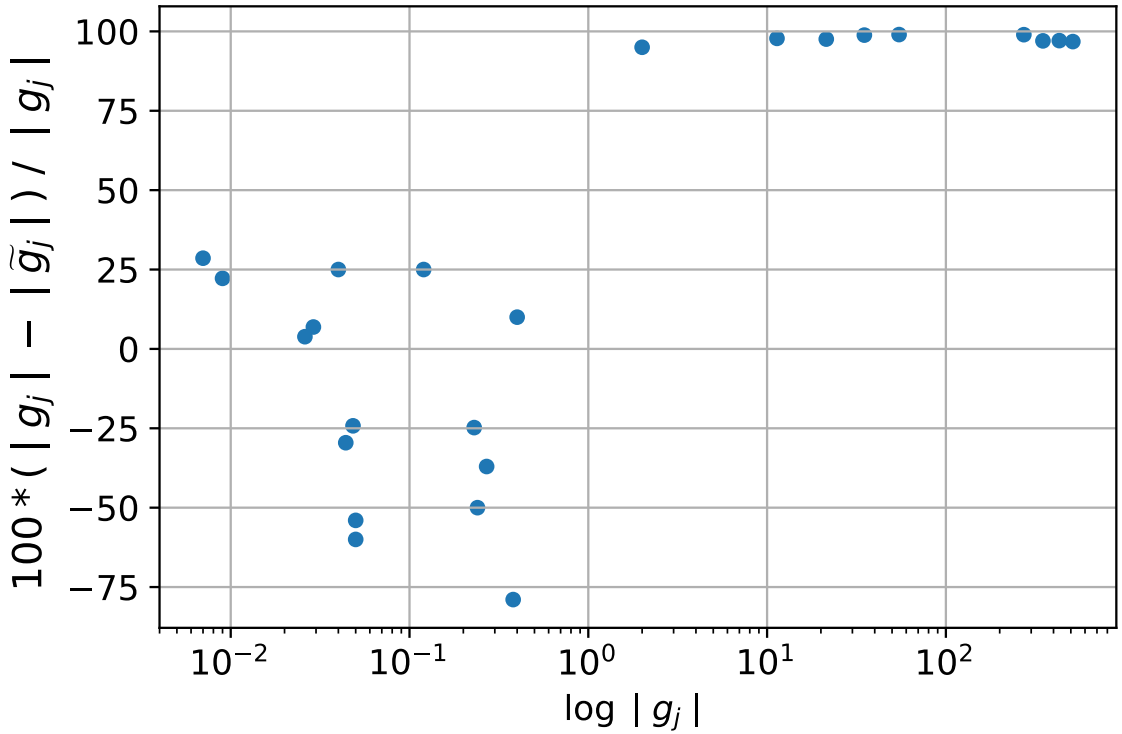
Cross validation (here 4-fold)





Ridge regression in the $K^+\Lambda$ channel

Relative percentage reduction of the resonance couplings



BS2 BS2r

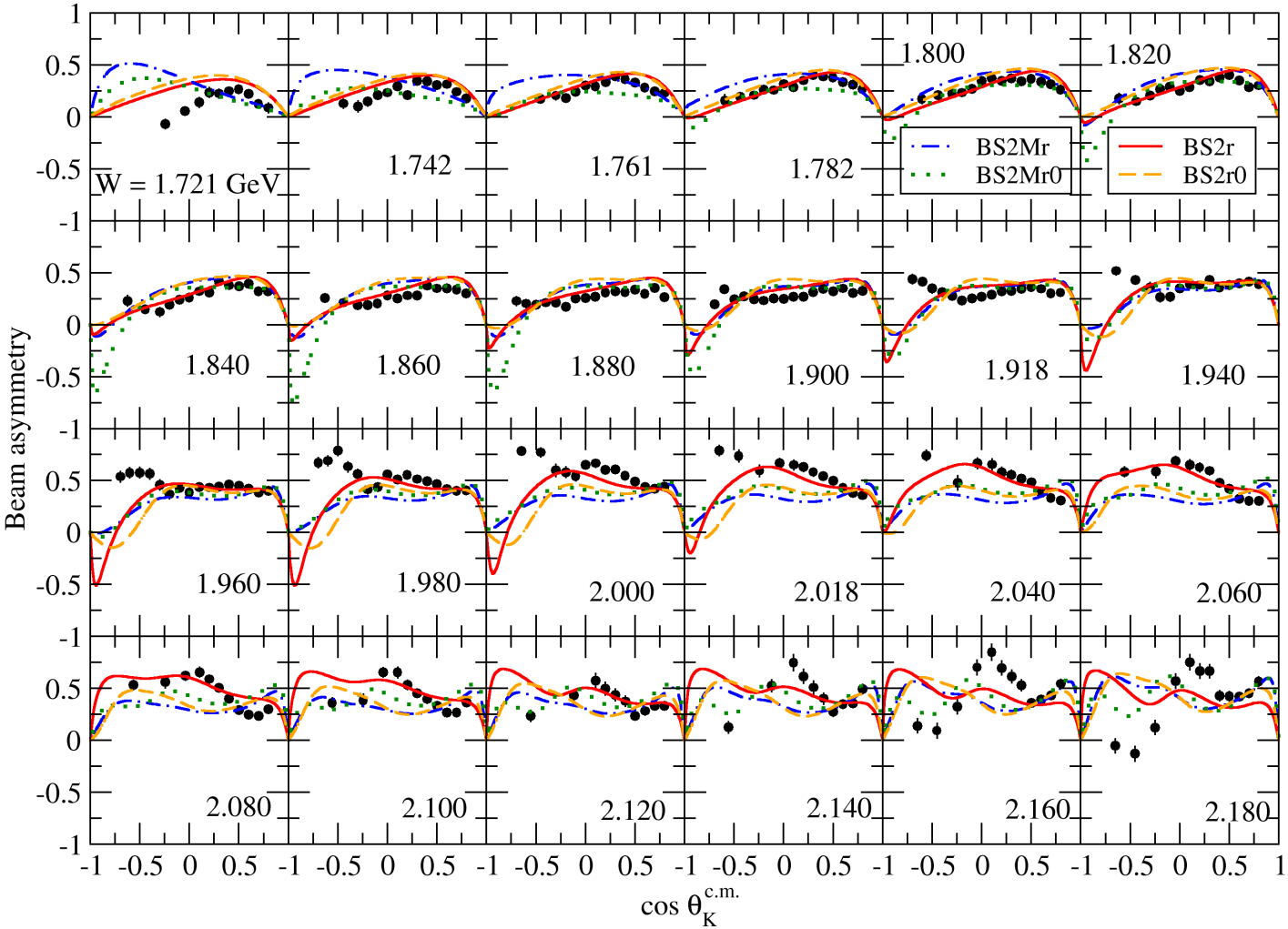
Tag	Resonance	g_1	g_2		
L1	$\Lambda(1405)\ 1/2$	9.67	–		2.624
S1	$\Sigma(1660)\ 1/2^+$	-8.09	–		−5.925
L4	$\Lambda(1800)\ 1/2^-$	-11.55	–		−1.409
S4	$\Sigma(1940)\ 3/2^-$	-0.86	0.18		−0.685 0.079

g_j - values from the unregularized fitting

$\tilde{g}_j$ - values after performing Ridge regularization

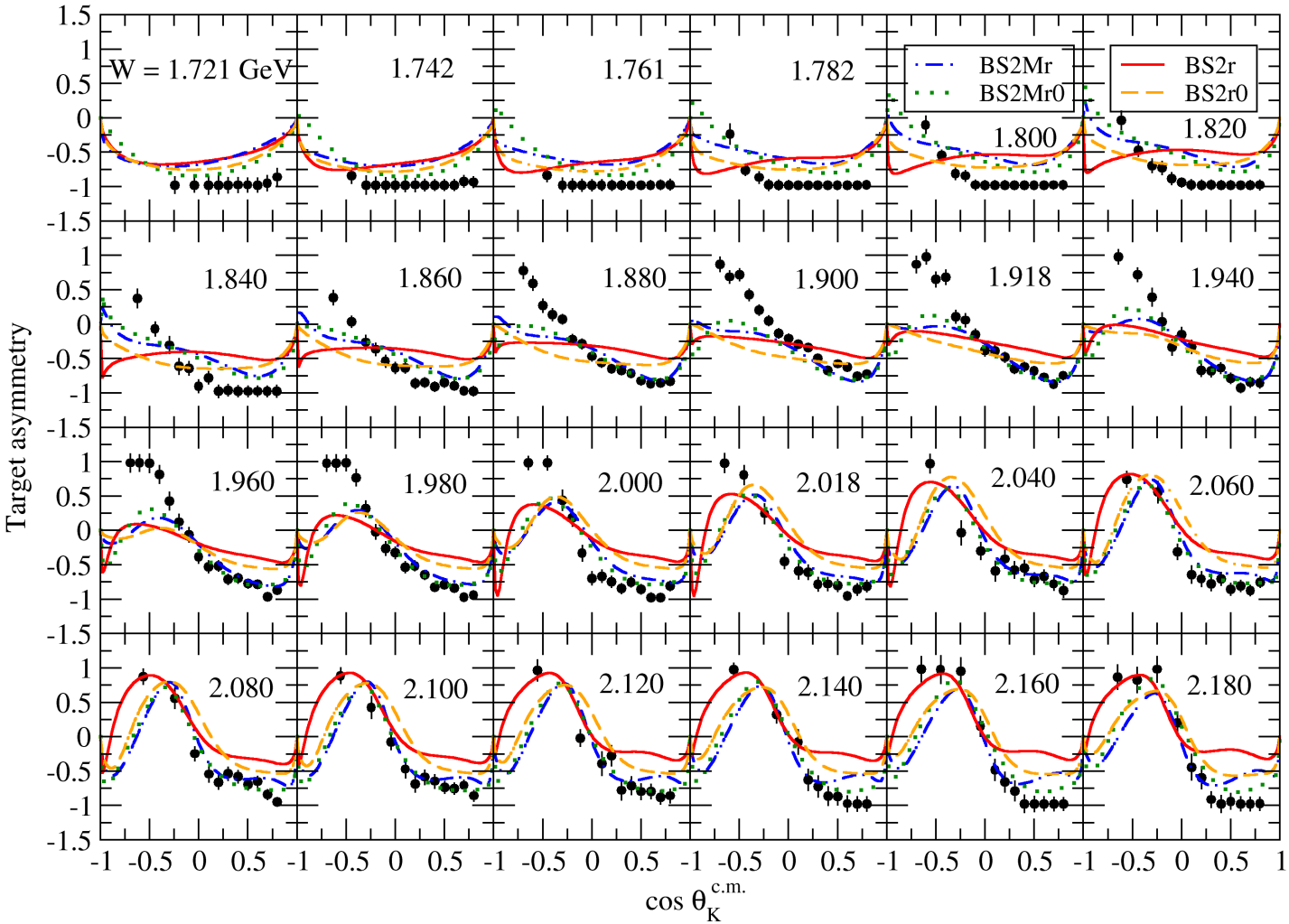
Ridge regression in the $K^+\Lambda$ channel

Beam asymmetry Σ



Ridge regression in the $K^+\Lambda$ channel

Target asymmetry T



$K^+\Sigma^0$ channel within the isobar model

Models A (w/o LASSO) and B (w/ LASSO) [PR C **110**, 065204 (2024)]

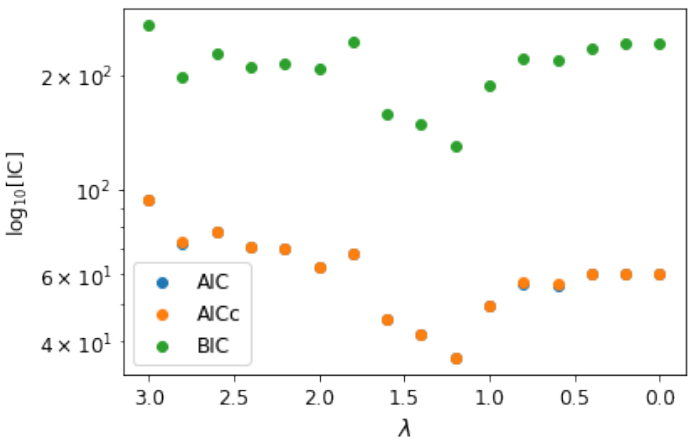
Resonances in models A and B:

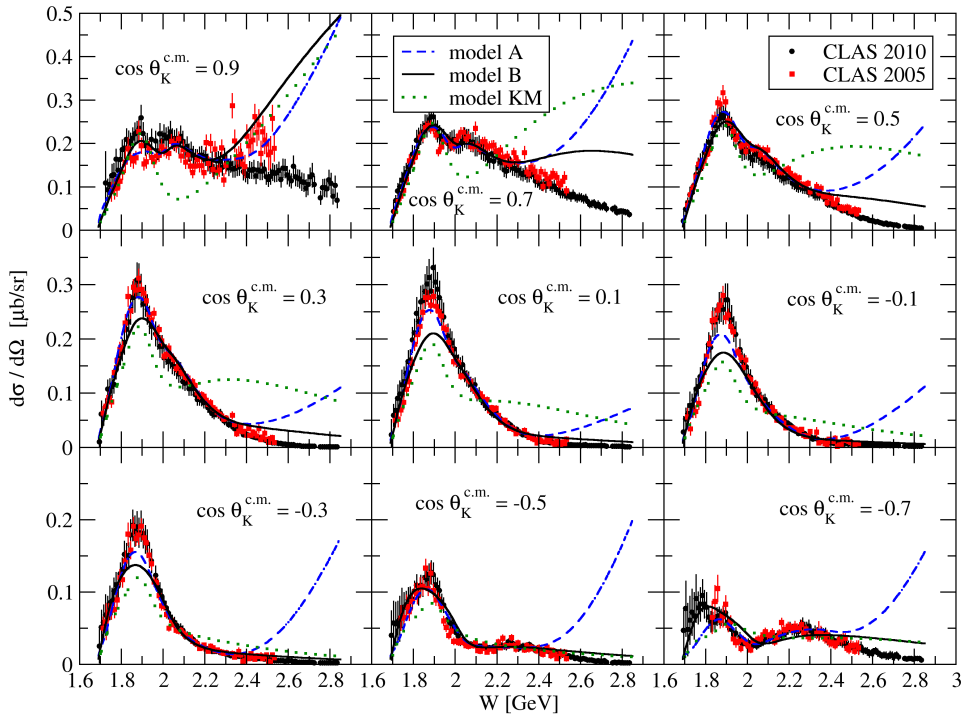
- $N3(1535)1/2^-$, $N4(1650)1/2^-$, $N9(1680)5/2^+$, $N7(1720)3/2^+$, $P5(1820)5/2^+$, $M4(1860)1/2^-$, $P4(1875)3/2^-$, $P2(1900)3/2^+$, $\Delta(1900)3/2^+$, $P3(2000)5/2^+$, $M2(2300)1/2^+$;
- $K^*(892)$, $K_1(1272)$;
- $\Lambda(1890)$, $\Sigma(1660)$.

Model	no. of resonances	no. of parameters	$\chi^2_{(P)}$/n.d.f.
A	15	29	2.02
B	7	14	4.67

(initial motivation for selecting resonances:

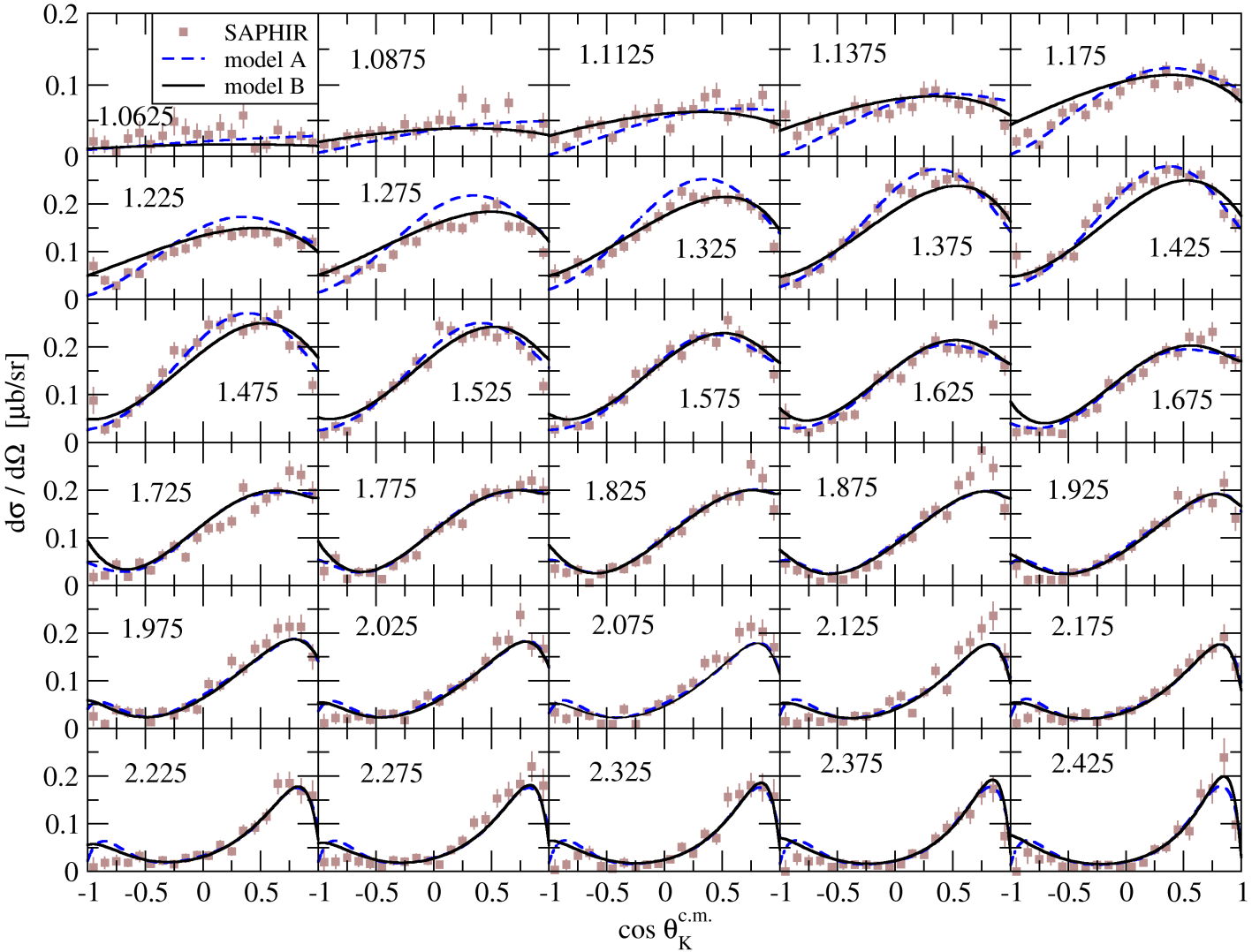
PR C **66**, 035202 (2002))





$K^+\Sigma^0$ channel within the isobar model

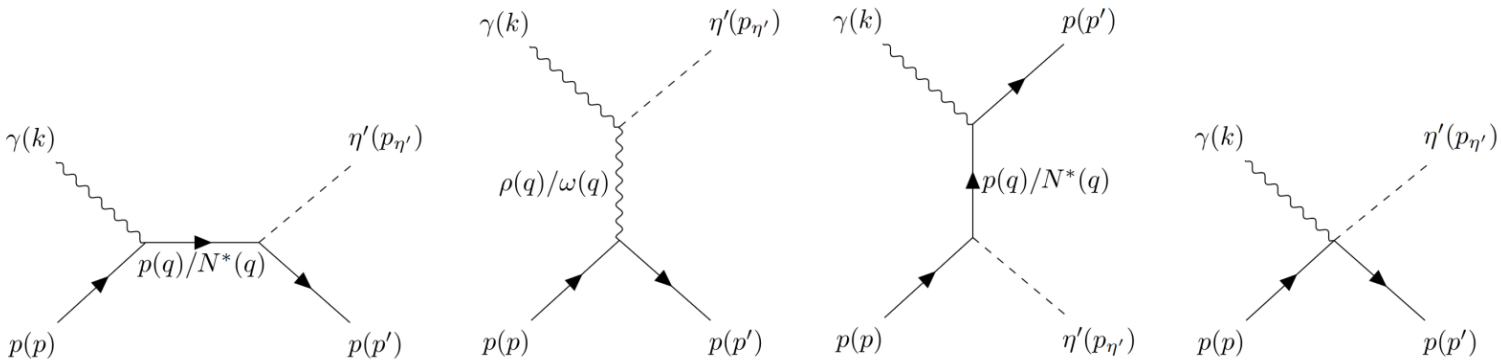
Models A (w/o LASSO) and B (w/ LASSO) – comparison with SAPHIR data (not fitted)



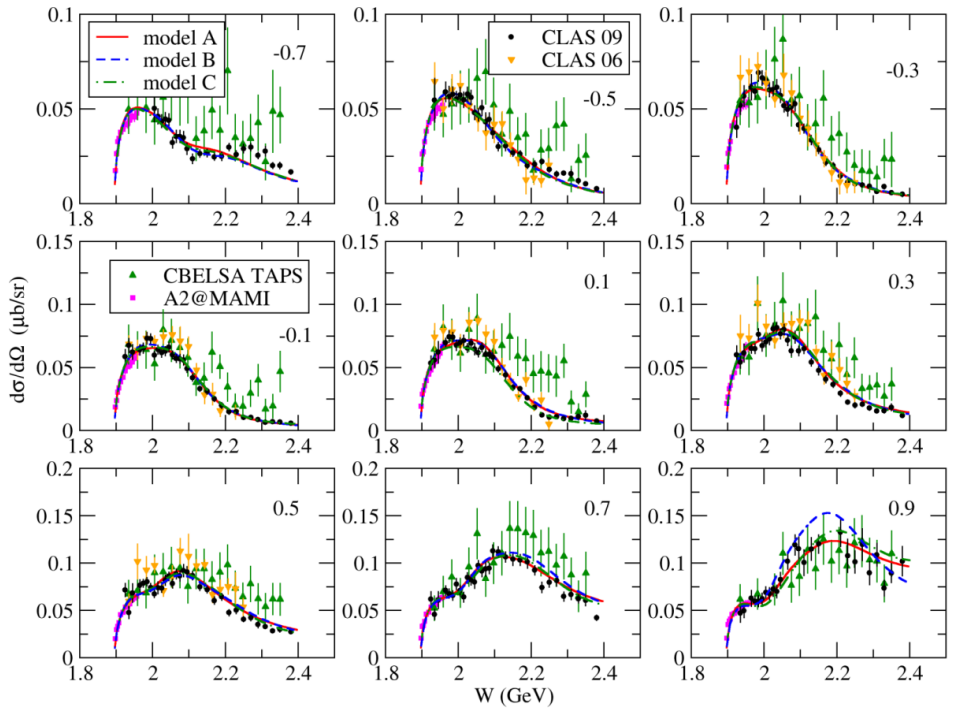
Photoproduction of η' mesons off protons

PR C **112**, 025202 (2025), PTEP 2026 043D04

Exchanges of p/N^* (s and u channels), and ρ/ω mesons in t channel + contact term



Resonance	Mass (MeV)	Width (MeV)	Status	BR
$N(1875)3/2^-$	1850–1920 (1875)	120–250 (250)	***	
$N(1895)1/2^-$	1870–1920 (1870)	80–200 (200)	****	10–40
$N(1900)3/2^+$	1890–1950 (1890)	100–320 (200)	****	4–8
$N(2100)1/2^+$	2050–2150 (2050)	200–320 (200)	***	5–11
$N(2120)3/2^-$	2060–2160 (2120)	260–360 (300)	***	2–6



- model A: $\chi^2/\text{n.d.f.} = 1.90$, PS, fit on $d\sigma$
- model B: $\chi^2/\text{n.d.f.} = 2.15$, PS, fit on $d\sigma + \Sigma$
- model C: $\chi^2/\text{n.d.f.} = 2.21$, PV, fit on $d\sigma + \Sigma$

From $K^+\Sigma^0$ channel to $K^0\Sigma^+$ channel

Strong vertices: isospin symmetry

- $K^+\Sigma^0 p \;\rightarrow\; K^0\Sigma^+ p$
- $g_{K^{*0}\Sigma^+ p} = \sqrt{2}\, g_{K^{+*}\Sigma^0 p}$

Electromagnetic vertices:

- s channel: γpp in both channels
- t channel: $K^* \rightarrow K\gamma$ decay width

$$\Gamma = \frac{1}{24}\frac{g^2_{K^*K\gamma}}{4\pi M^2}\left[m_{K^*}\left(1-\frac{m_K^2}{m_{K^*}^2}\right)\right]^3$$

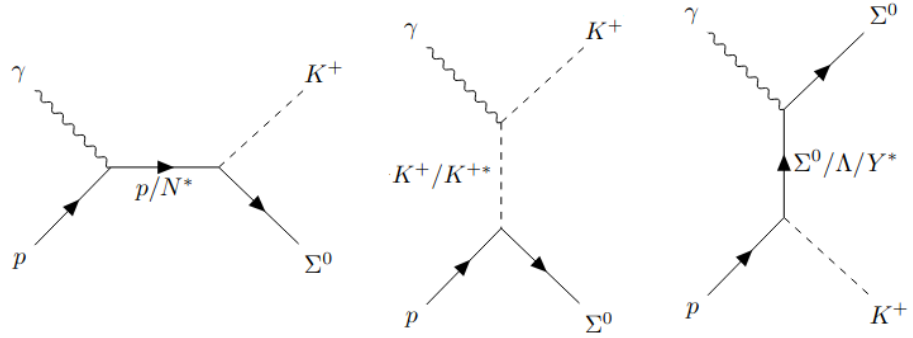
$$\Gamma_{K^{+*}\rightarrow K^+\gamma} = 50 \pm 5\,\text{keV},$$
$$\Gamma_{K^{0*}\rightarrow K^0\gamma} = 116 \pm 10\,\text{keV},$$
$$\Rightarrow g_{K^{0*}K^0\gamma} = -1.52\, g_{K^{+*}K^+\gamma}$$

- u channel: not enough info on

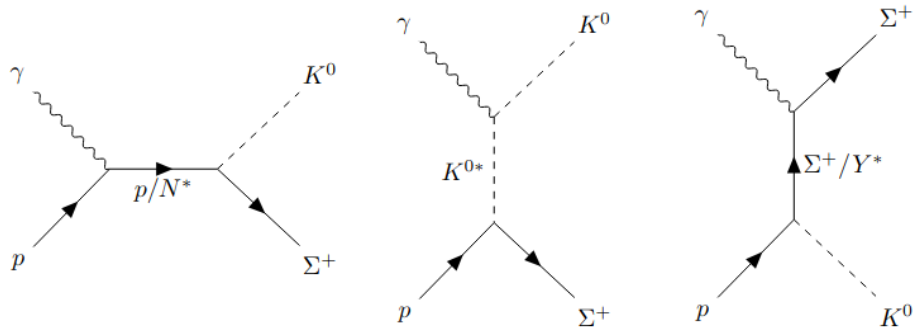
$\gamma\Sigma^0\Sigma^0 \;\rightarrow\; \gamma\Sigma^+\Sigma^+$

$\gamma\Sigma^0 Y^* \;\rightarrow\; \gamma\Sigma^+ Y^*$

Feynman diagrams for $\gamma p \rightarrow K^+\Sigma^0$



Feynman diagrams for $\gamma p \rightarrow K^0\Sigma^+$



$K^0\Sigma^+$ channel (accepted in PR C)

Akaike differences and weights

Akaike information criterion:

$$\mathrm{AIC} = 2n + \chi^2$$

Akaike difference:

$$\Delta_i = \mathrm{AIC}_i - \mathrm{AIC}_{\mathrm{min}}$$

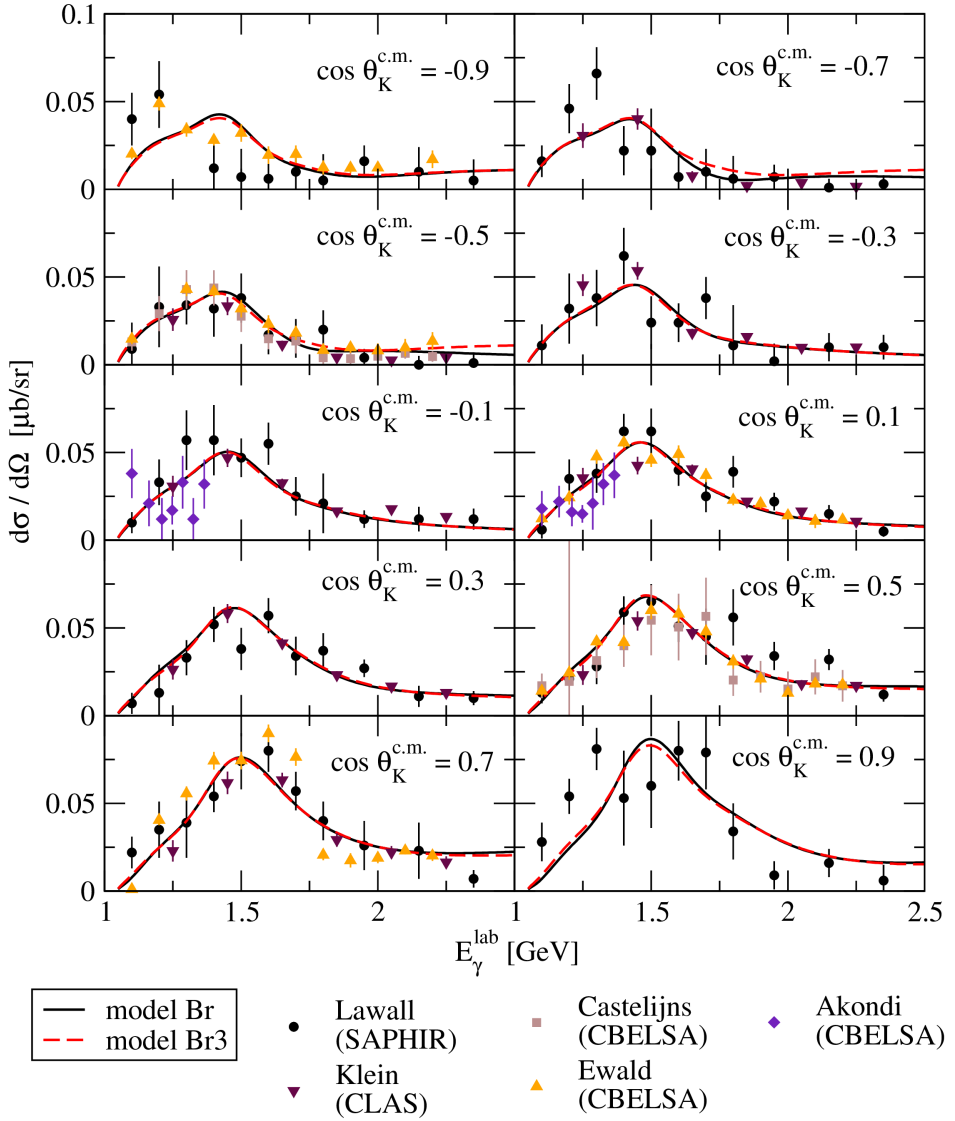
Akaike weight:

$$w_i = \frac{\exp(-\Delta_i/2)}{\sum_{r=1}^{R}(-\Delta_r/2)}$$

Model	χ^2	AIC_i	Δ_i	w_i
Br	1586.38	1614.38	32.95	10^{-7}
Br3	1543.43	1581.43	0	0.999

Model Br:
resonances as in model B in $K^+\Sigma^0$

Model Br3:
$+\Delta(1900)3/2^+$, $N(1685)5/2^+$



$K^0\Sigma^+$ channel (accepted in PR C)

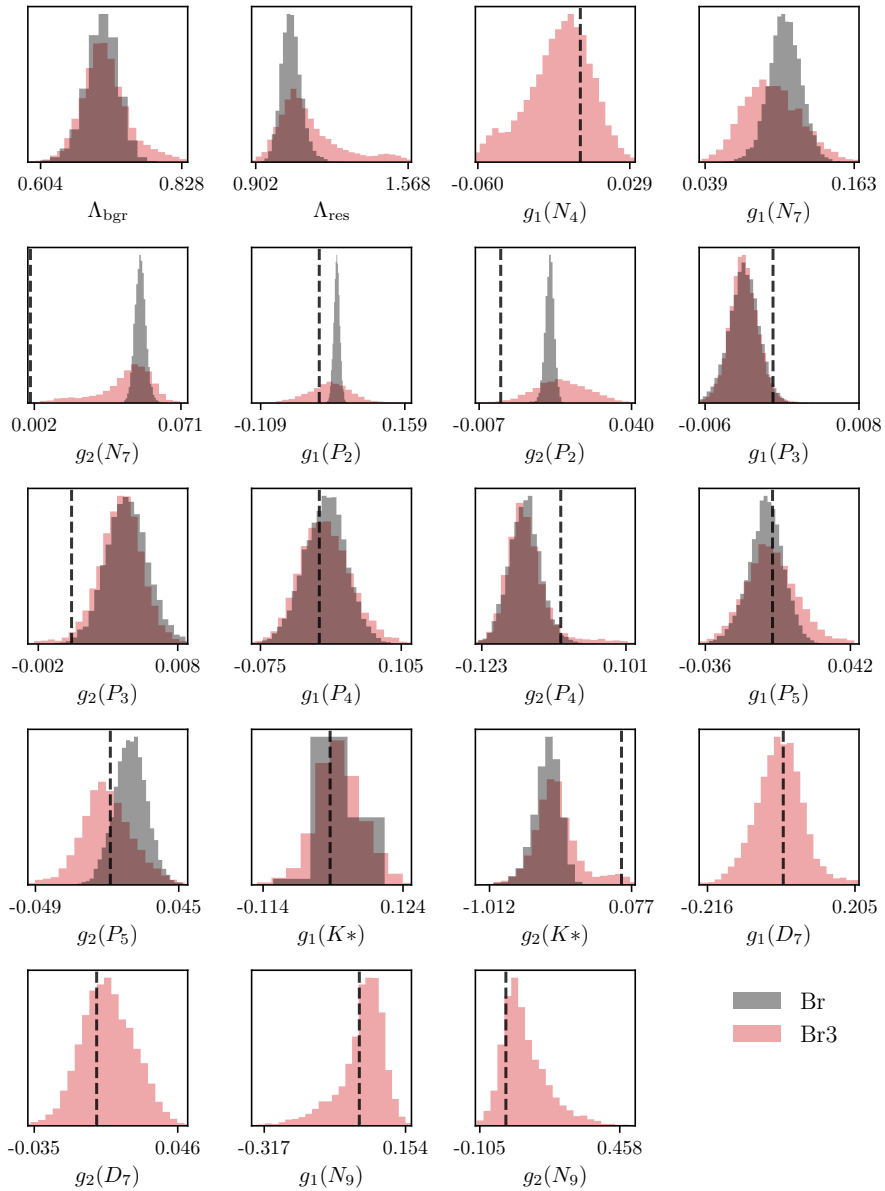
Bootstrap: correlation coefficients & parameter distributions

- Bootstrap belongs to the most powerful tools for exposing **parameter overfitting**
- here: 5000 bootstrap iterations
- Pearson correlation coefficients:

$$r_{xy} = \frac{\mathrm{cov}(x,y)}{\sigma_x \sigma_y}$$

$r_{xy} \gtrsim 0.7 \Rightarrow$ high interdependence
$r_{xy} \gtrsim 0.9 \Rightarrow$ redundancy

Parameter pair		r_{xy}	
x	y	Br	Br3
Λ_{bgr}	$g_2(K*)$	0.95	0.91
Λ_{res}	$g_2(P_2)$	−0.78	0.71
Λ_{res}	$g_2(K*)$	–	0.77
$g_1(N_4)$	$g_2(N_7)$	–	0.91
$g_1(P_2)$	$g_2(P_2)$	0.74	–
$g_1(P_2)$	$g_1(D_7)$	–	0.98
$g_1(P_2)$	$g_2(D_7)$	–	0.71
$g_2(P_2)$	$g_2(D_7)$	–	0.94
$g_1(P_3)$	$g_2(P_3)$	−0.90	−0.88
$g_1(P_4)$	$g_2(P_4)$	0.83	0.79
$g_1(P_5)$	$g_2(P_5)$	−0.85	−0.89
$g_1(D_7)$	$g_2(D_7)$	–	0.78
$g_1(N_9)$	$g_2(N_9)$	–	−0.91



Summary

Isobar model and Regge-plus-resonance model (not shown here) for

- $K^+\Lambda$ photoproduction and electroproduction

  $\rightarrow$ further use in hypernuclei electroproduction

  [PR C **106**, 044609 (2022), PR C **108**, 024615 (2023), PR C **112**, 024609 (2025)]

- $K\Sigma$ photoproduction

  $\rightarrow$ $K^+\Sigma^0$, $K^+\Sigma^-$, and $K^0\Sigma^+$ channels

- charm photoproduction (not mentioned in this talk) [PR D **102**, 074009 (2020)]

Regularization methods introduced as a remedy for overfitting and as model selection tools.

Outlook

- simulation based inference [PR D **114**, 016002 (2026)]
- extending the analysis of electroproduction of $K^+\Lambda$ beyond $Q^2 = 1\,\mathrm{GeV}^2$
- describing electroproduction of η' mesons
- photoproduction of $K^0\Lambda$, $K\Lambda(1405)$, $K\Lambda(1520)$...

Thank you for your attention!