Planck scale signatures in exotic atoms

Nicola Bortolotti

Jagiellonian University in Kraków,
INFN-LNF

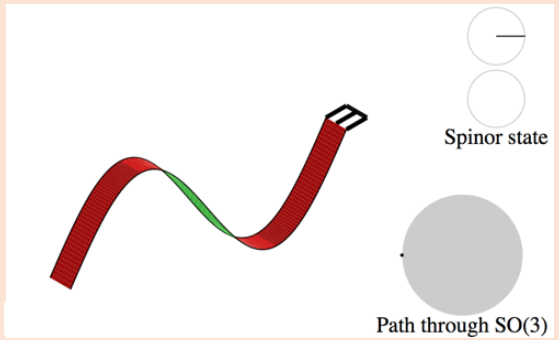
ECT* - 12-16/10/2026

New-EXOTICA: New Frontiers in Exotic Atoms: high-precision tests of QCD and QED and probes of beyond Standard Model physics

SPIN-STATISTICS CONNECTION AND QUANTUM GRAVITY

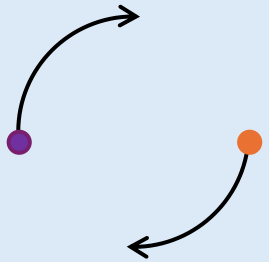
THE SPIN-STATISTICS CONNECTION (SSC)

Spin ↔ Geometry of Rotations



$d \geq 3$	1 rotation = 0 rotation →	**Integer spins**
	2 rotations = 0 rotation →	**Half-integer spins**
$d < 3$	All rotations differ →	**Arbitrary spins**

Statistics ↔ Indistinguishability



$$\psi(1,2) = e^{i\gamma(1,2)}\psi(2,1)$$

γ is a geometric phase

(Leinaas & Myrheim, Berry & Robbins)

$d \geq 3$	1 exchange = 0 exchange →	**Bosons**
	2 exchanges = 0 exchange →	**Fermions**
$d < 3$	All exchanges differ →	**Anyons**

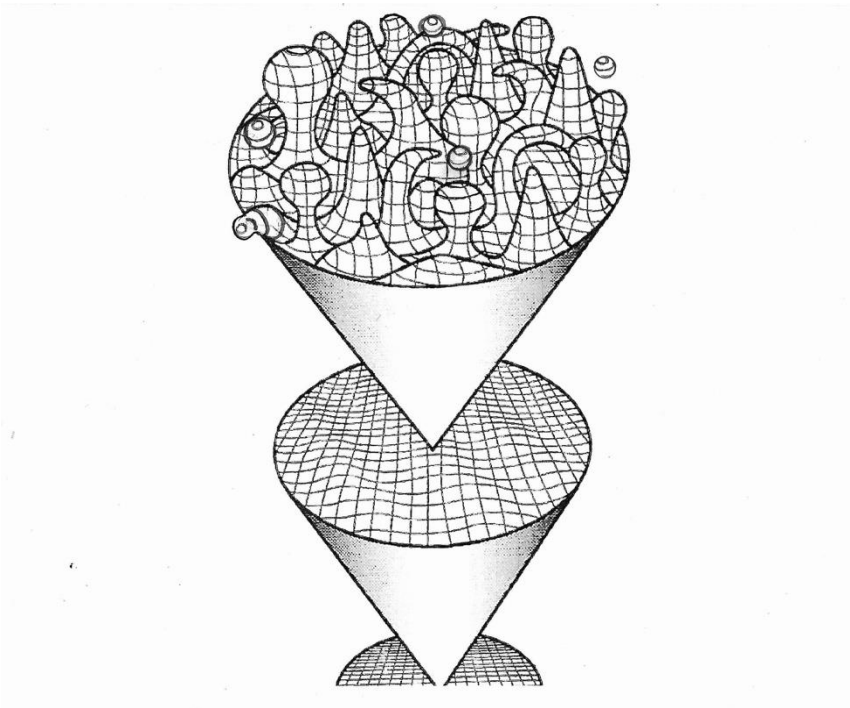
- Indistinguishability does not require SSC but **superselection rules** (SSRs):

$$\langle\psi_B|O|\psi_F\rangle = 0 \;\; \forall \text{ observables} \Rightarrow \rho = p_B|\psi_B\rangle\langle\psi_B| + p_F|\psi_F\rangle\langle\psi_F| = \textbf{ incoherent mixture of B\&F states}$$

- **SSC follows only by requiring locality,** i.e. $[O(x), O(y)] = 0$ if x, y spacelike:

integer spins: $p_B = 1,\ p_F = 0$ vs half-integer spins $p_B = 0,\ p_F = 1$

QUANTUM FOAM

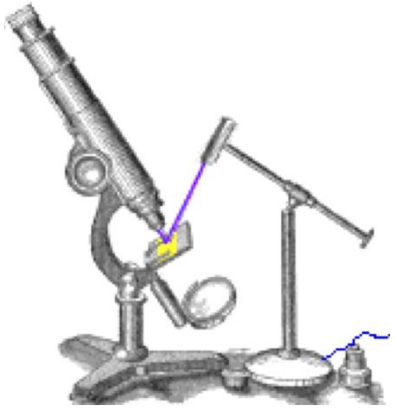


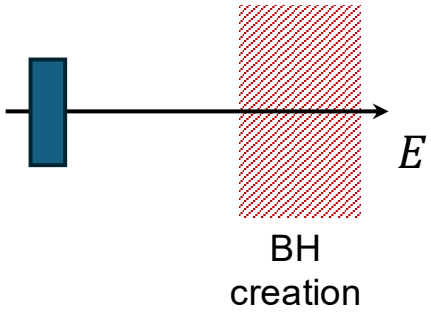
«To the transatlantic passenger flying above it, the ocean appears smooth. When the traveler's plane has dropped in altitude to a hundred meters above the surface he sees the waves. When he is in the lifeboat he sees the foam forming and breaking, breaking and forming. Space, smooth at the everyday level, and smooth even at distances comparable to atoms, atomic nuclei and elementary particles, is predicted at still smaller distances to show a foamlike structure.»

John Archibald Wheeler

QUANTUM GRAVITY AND LOCALITY

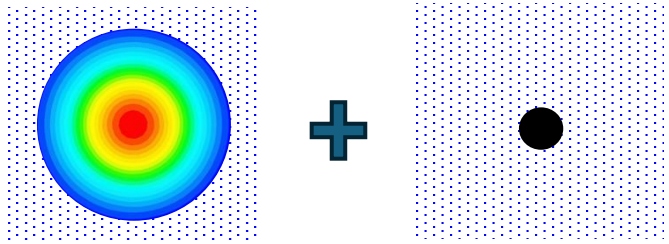
Quantum mechanics + general relativity **do not allow infinite localization!** **(Matvei Bronstein 1934)**





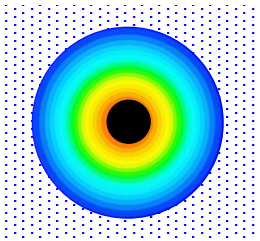
Heisenberg uncertainty

Schwarzschild radius



$\Delta x \sim 1/E$

$r_s \sim E$

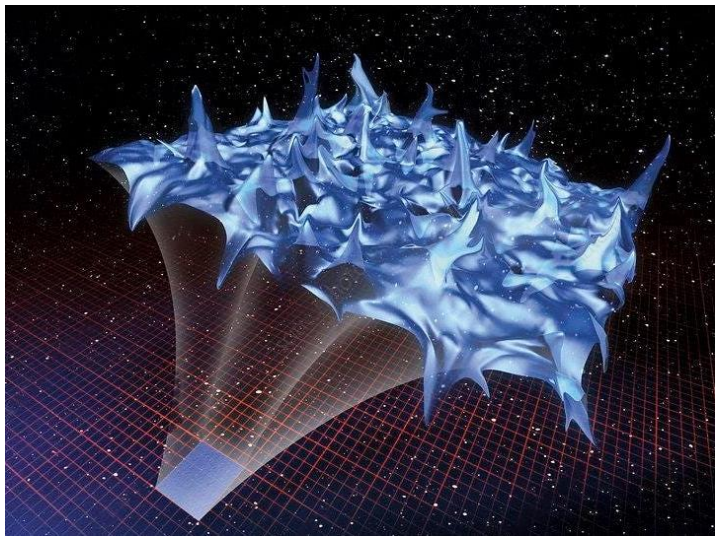


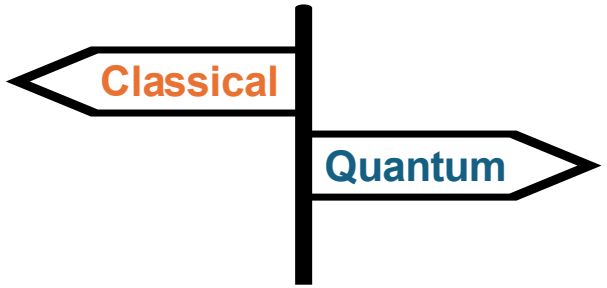
For $\Delta x < r_s$
BH creation:

$\boldsymbol{\Delta x_{\min} \sim \ell_P}$

$\boldsymbol{= 1.6\times10^{-34}\ \mathrm{m}}$

Intrinsic spacetime uncertainty





MINIMAL LENGTH AND LORENTZ INVARIANCE

- A general prediction of QG is the existence of a **minimal length** ℓ (LQG, string theory, causal set theory…)

It effectively manifests through

- Spacetime noncommutativity: $[x^\mu, x^\nu] = \ell^2 c^{\mu\nu}$ ⇒ $\Delta x^\mu \Delta x^\nu \geq \ell^2 c^{\mu\nu}$
- A generalized uncertainty principle (GUP): $[x, p] = i\hbar(1 + \ell^2 p^2)$ ⇒ $\Delta x \geq \ell$

These are **not Lorentz invariant**: different observer can measure her/his own minimal length!

Similar to Galilean non-invariance of EM: **deform Lorentz transformations** s.t. ℓ becomes invariant

In particular the standard "coproduct" rule to transform two-particle states is modified: $\Delta_0(g) \neq g \otimes g$

- Both GUP and noncommutative spacetimes may have deep consequences on multiparticles states

FAILURE OF THE SPIN-STATISTICS CONNECTION

- **Locality** $\xrightarrow{QG}$ fails
- **Poincaré invariance** $\xrightarrow{QG}$ expected either deformations or violations

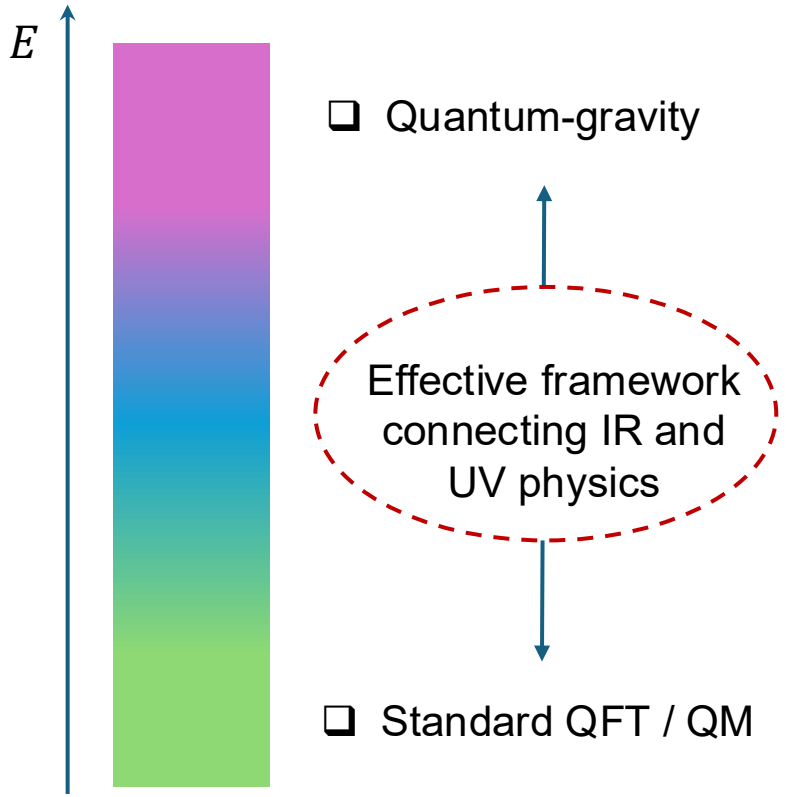
The SSC may fail in quantum gravity



The Pauli Exclusion Principle (PEP) may fail

PEP violations provide smoking gun for quantum gravity

QFT BEYOND THE SPIN-STATISTICS CONNECTION



Foundations of QG-effective theory:

- Quantum mechanics
- Nonlocality in the UV regime
- Relativistic invariance

$a_i a_j^\dagger - \sum_{kl} \eta_{kl} e^{i\gamma_{kl}} a_k^\dagger a_l = \delta_{ij}$, no relations for $a_i a_j$

- γ: geometric phase which defines statistics
- η: interpolation between different statistics

$-1 \leq \eta \leq 1$: no negative squared-norms

Most general oscillator algebra, it emerges in several types of noncommutative spacetime and GUP models

For more than 30 years η and γ have been studied independently, we shall see that in QG generalizaed statistics (γ) require nontrivial η

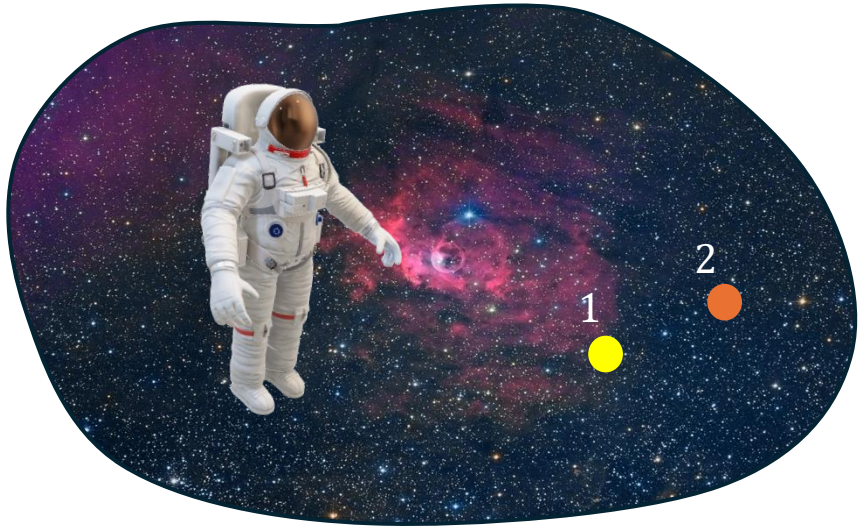
SPACETIME SYMMETRIES AND PARTICLE EXCHANGE

GOAL:

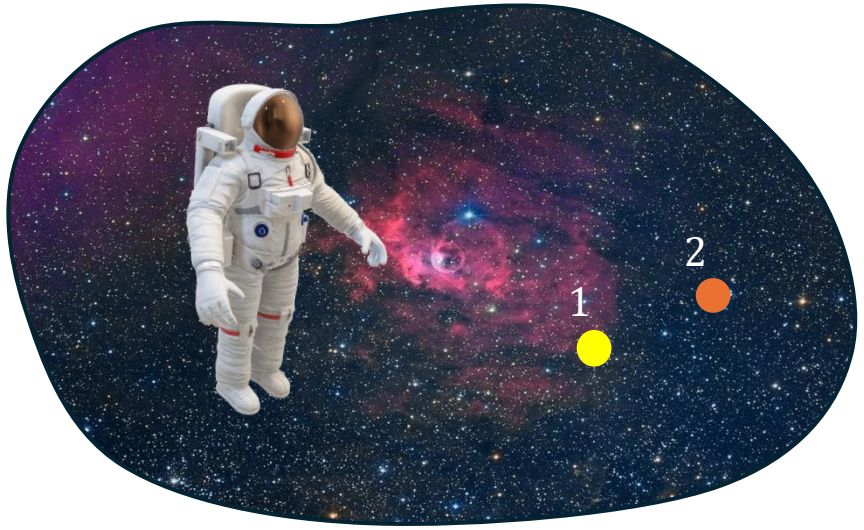
Find the exchange operators T compatible with spacetime symmetry group G

- $g \in \{\text{translations, Lorentz transformations}\}$
- Fix an action on multiparticles ($\Delta(g)$ rule)

$|1,2\rangle$



$|1,2\rangle$



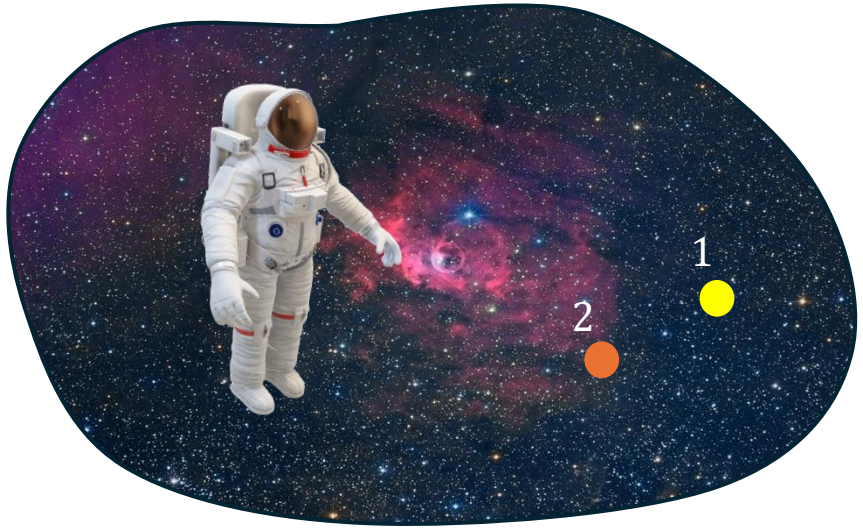
SPACETIME SYMMETRIES AND PARTICLE EXCHANGE

GOAL:

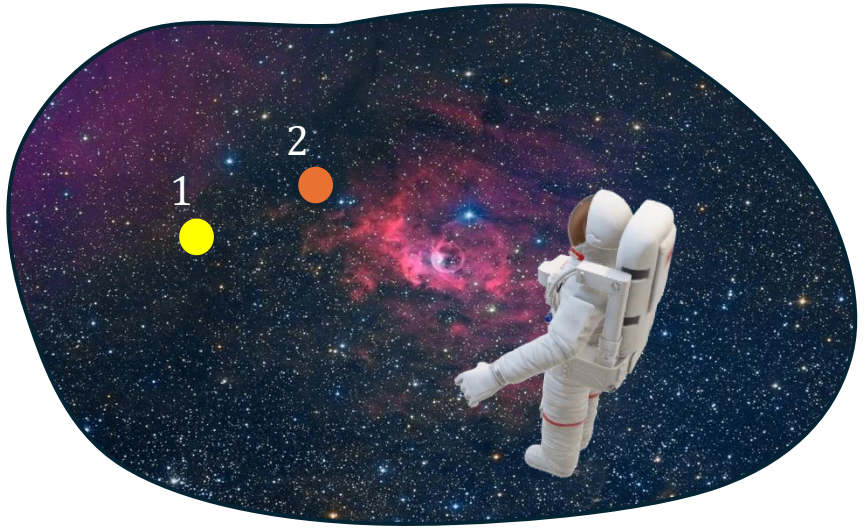
Find the exchange operators T compatible with spacetime symmetry group G

- $g \in \{\text{translations, Lorentz transformations}\}$
- Fix an action on multiparticles ($\Delta(g)$ rule)

$|1,2\rangle \rightarrow T|1,2\rangle$



$|1,2\rangle \rightarrow \Delta(g)|1,2\rangle$



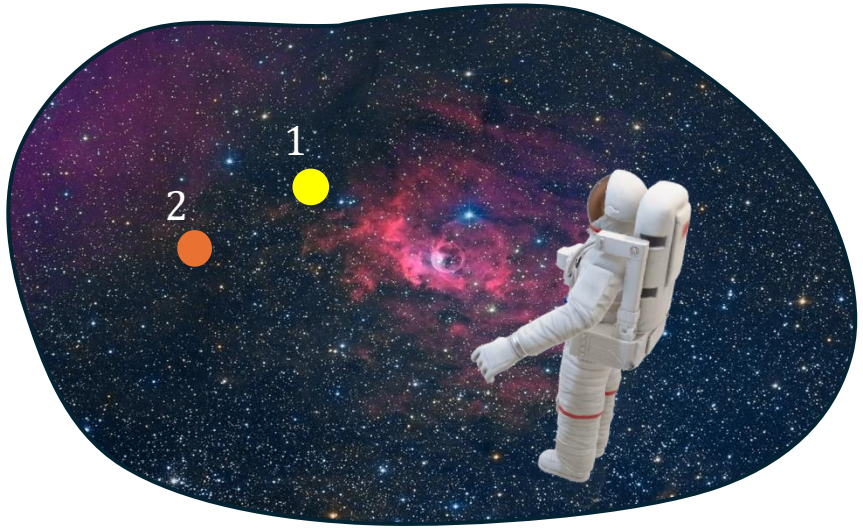
SPACETIME SYMMETRIES AND PARTICLE EXCHANGE

GOAL:

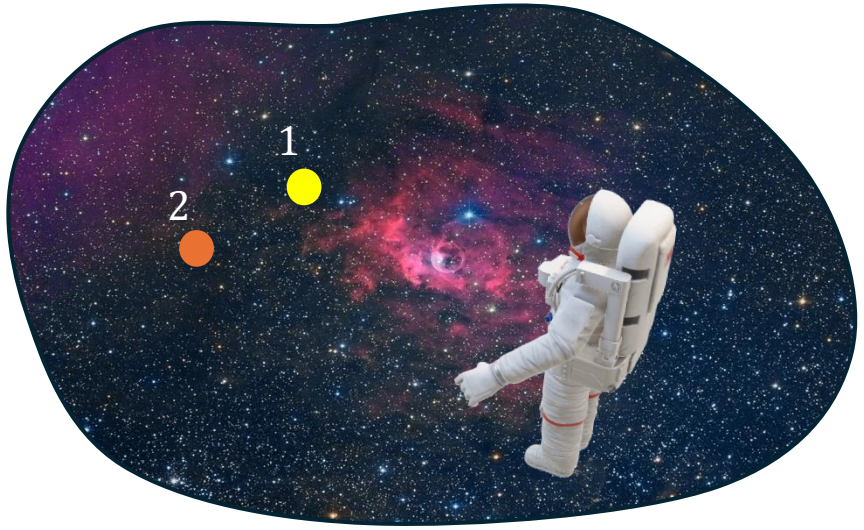
Find the exchange operators T compatible with spacetime symmetry group G

- $g \in$ {translations, Lorentz transformations}
- Fix an action on multiparticles ($\Delta(g)$ rule)

$$|1,2\rangle \rightarrow T|1,2\rangle \rightarrow \Delta(g)T|1,2\rangle$$



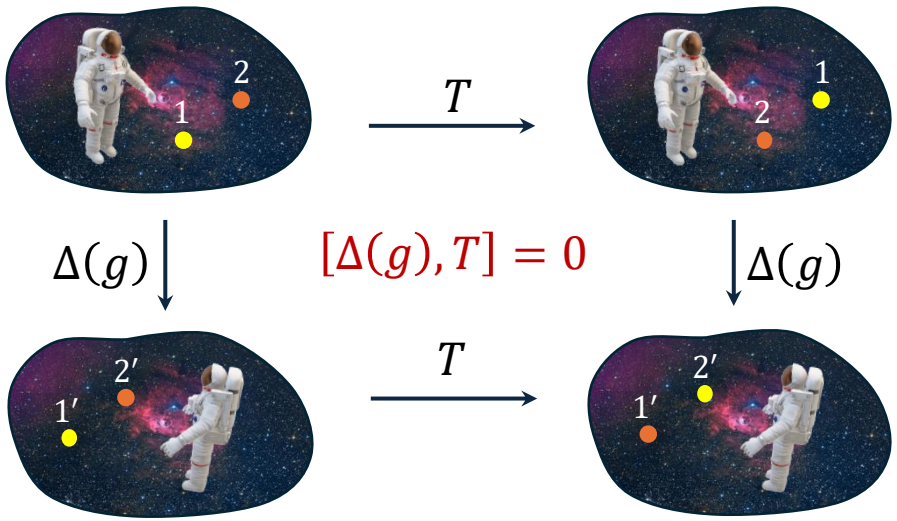
$$|1,2\rangle \rightarrow \Delta(g)|1,2\rangle \rightarrow T\Delta(g)|1,2\rangle$$



SPACETIME SYMMETRIES AND PARTICLE EXCHANGE

GOAL:

Find the exchange operators T compatible with spacetime symmetry group G $\left\{\begin{array}{l} g \in \{\text{translations, Lorentz transformations}\} \\ \text{Fix an action on multiparticles } (\Delta(g) \text{ rule}) \end{array}\right.$



Two types of operators admitted

$$\boldsymbol{\tau_R = \tau R \leftrightarrow \tau_R^2 = 1}$$

R fixed by symmetries

Important ex: $\boldsymbol{R_{1,2} = e^{i\gamma_{12}} = e^{-i\gamma_{21}}}$

vs

$$\boldsymbol{T = \tau_R \eta \leftrightarrow T^2 \neq 1}$$

$\eta(1,2)$ arbitrary Lorentz invariant function

PHYSICAL STATES

$a_1^\dagger a_2^\dagger|0\rangle \neq a_2^\dagger a_1^\dagger|0\rangle \longrightarrow$ $\psi(1,2)$ and $\psi(2,1)$ are independent states

PEPV states are normalizable: $\|\psi\|^2 = \frac{1}{2}\sum_{ij}\psi^*(i,j)\left(\psi(i,j)+\eta e^{i\gamma}\psi(i,j)\right) \to \frac{1}{2}\sum_{ij}|\varphi(i)|^2|\varphi(j)|^2\left(1+\eta_{ij}\cos\gamma_{ij}\right) > 0$

- **Superselection rules (SSRs)** $\langle\psi_{a_R}|O|\phi_{s_R}\rangle = 0$ may guarantee **indistinguishability**

Decompose $|\psi\rangle = \underbrace{\frac{1+\tau_R}{2}|\psi\rangle}_{|\psi\rangle_{s_R}} + \underbrace{\frac{1-\tau_R}{2}|\psi\rangle}_{|\psi\rangle_{a_R}}$ in generalized bosons and fermions $\psi_{s_R}(i,j) = \pm e^{i\gamma_{i,j}}\psi_{s_R}(j,i)$

$$\rho = p_{s_R}|\psi_{s_R}\rangle\langle\psi_{s_R}| + p_{a_R}|\psi_{a_R}\rangle\langle\psi_{a_R}|$$

$$p_{s_R,a_R} \propto \|\psi_{s_R,a_R}\| = \left[\sum_{ij}|\psi_{s_R,a_R}(i,j)|^2\left(\frac{1-\eta_{ij}}{2}\right)\right]^{1/2}$$

- A system *has* a given symmetry which is exactly preserved by all physical operations
- N copies of the system creates an incoherent mixture with different symmetries weighted by $p_\lambda(\eta,\gamma)$

EXAMPLES

Undeformed Poincaré

Greenberg quon model

$$a(\tilde{p})a^{\dagger}(\tilde{q}) - \eta(p,q)a^{\dagger}(\tilde{q})a(\tilde{p}) = \delta(\tilde{p},\tilde{q})$$

Physical states:

$$\psi_{s,a}(\tilde{p},\tilde{q}) = \pm\psi_{s,a}(\tilde{q},\tilde{p})$$

$$\psi_{a}^{\mathrm{PEPV}}(\tilde{p},\tilde{q}) = \varphi(\tilde{p})\varphi(\tilde{q}) - \varphi(\tilde{q})\varphi(\tilde{p}) = 0$$

PEPV atoms only in non-antisymmetric sectors

θ-Poincaré

$\theta^{\mu\nu}$ defines the *Moyal spacetime* $[x^{\mu}, x^{\nu}] = \theta^{\mu\nu}$

$$a(\tilde{p})a^{\dagger}(\tilde{q}) - \eta(p,q)e^{i\theta^{\mu\nu}p_{\mu}q_{\nu}}a^{\dagger}(\tilde{q})a(\tilde{p}) = \delta(\tilde{p},\tilde{q})$$

Physical states:

$$\psi_{s_\theta,a_\theta}(\tilde{p},\tilde{q}) = \pm e^{i\theta^{\mu\nu}p_{\mu}q_{\nu}}\psi_{s_\theta,a_\theta}(\tilde{q},\tilde{p})$$

$$\psi_{a_\theta}^{\mathrm{PEPV}}(\tilde{p},\tilde{q}) = \varphi(\tilde{p})\varphi(\tilde{q})(1 - e^{i\theta^{\mu\nu}p_{\mu}q_{\nu}}) \neq 0$$

PEPV atoms also in the antisymmetric sector

At low energy: $\eta(p,q) \sim \pm\left(1 - \left(\frac{\sigma(p,q)}{\Lambda_\eta}\right)^{\kappa}\right)$

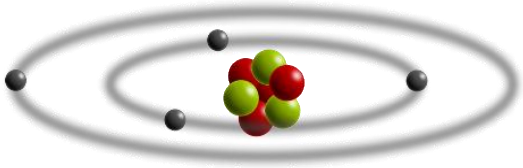
σ Lorentz invariant function with dimensions of energy,

κ a free parameter and Λ_η is the quon scale

VIOLATION OF THE PAULI EXCLUSION PRINCIPLE
Phenomenology and Experiments

PEP FORBIDDEN ATOMIC TRANSITIONS WITH SSRS

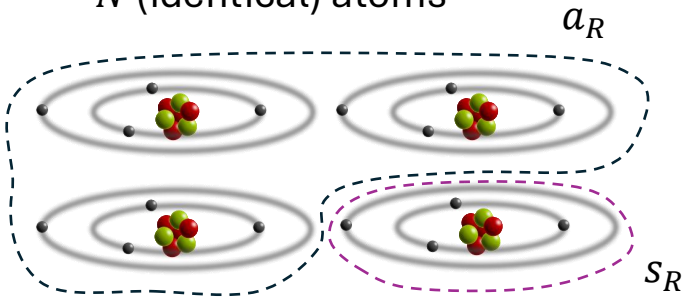
- One atom



Lorentz-invariant: $\psi_a^{\mathrm{PEPV}}(\tilde{p},\tilde{q}) = \varphi(\tilde{p})\varphi(\tilde{q}) - \varphi(\tilde{q})\varphi(\tilde{p}) = 0$

Lorentz-deformed: $\psi_{a_\theta}^{\mathrm{PEPV}}(\tilde{p},\tilde{q}) = \varphi(\tilde{p})\varphi(\tilde{q})(1 - e^{i\theta^{\mu\nu}p_\mu q_\nu}) \neq 0$

- N (identical) atoms



Fraction of atoms in antisymmetric states: $N_a/N = p_a = \dfrac{\|\psi_{a_R}\|^2}{\sum_{\gamma'}\|\psi_{\gamma'_R}\|^2} \sim 1$

Fraction of atoms with wrong symmetry: $N_w/N = p_w = \sum_{\lambda\neq a} p_\lambda \sim \left(\dfrac{\sigma}{\Lambda_\eta}\right)^\kappa \ll 1$

The **Lorentz-invariant** scenario allows PEP-violations only in the non-antisymmetric sectors: **suppressed probability**

The **Lorentz-deformed** scenario allows PEP-violations in the antisymmetric sector: **large probability**

Lorentz-deformed scenarios as noncommutative spacetimes **must beak SSRs**

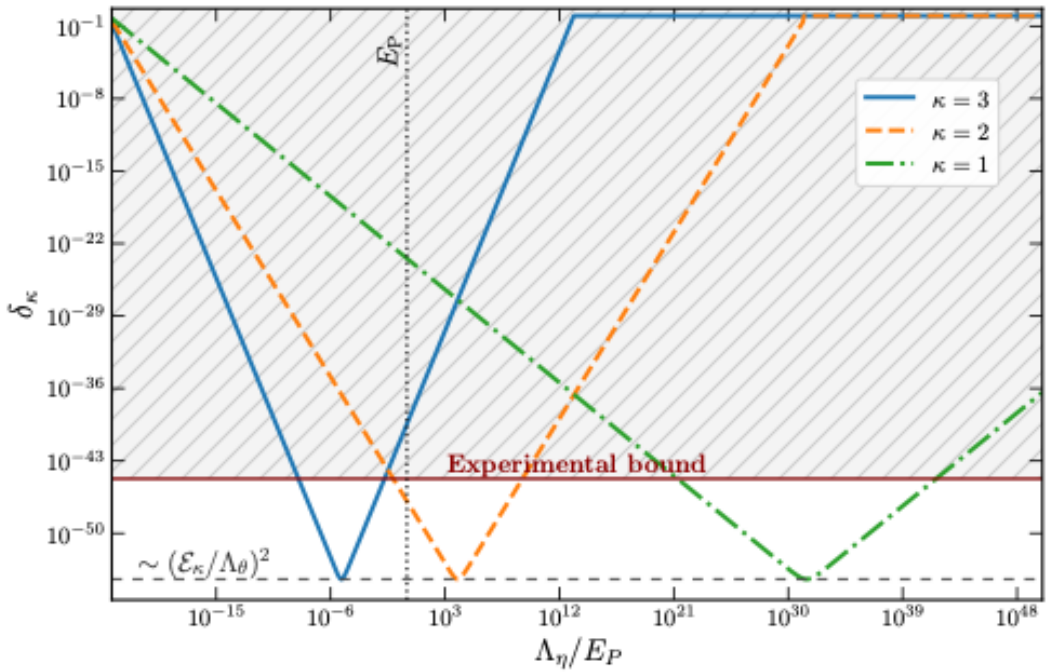
PEP FORBIDDEN ATOMIC TRANSITIONS WITHOUT SSRS

$$\boldsymbol{\Gamma_{PEPV} = \delta(E_{at}, \Lambda_{NC}, \Lambda_\eta) \cdot \Gamma_0,}$$

- E_{at} is an atomic energy scale
- Λ_{NC} is the UV noncommutative scale
- Λ_η is the UV quon scale

δ depends on the specific form of $\eta(p, q)$ and spacetime symmetries

In the Moyal NC case: $[x^\mu, x^\nu] = i\frac{c^{\mu\nu}}{\Lambda_\theta^2}$

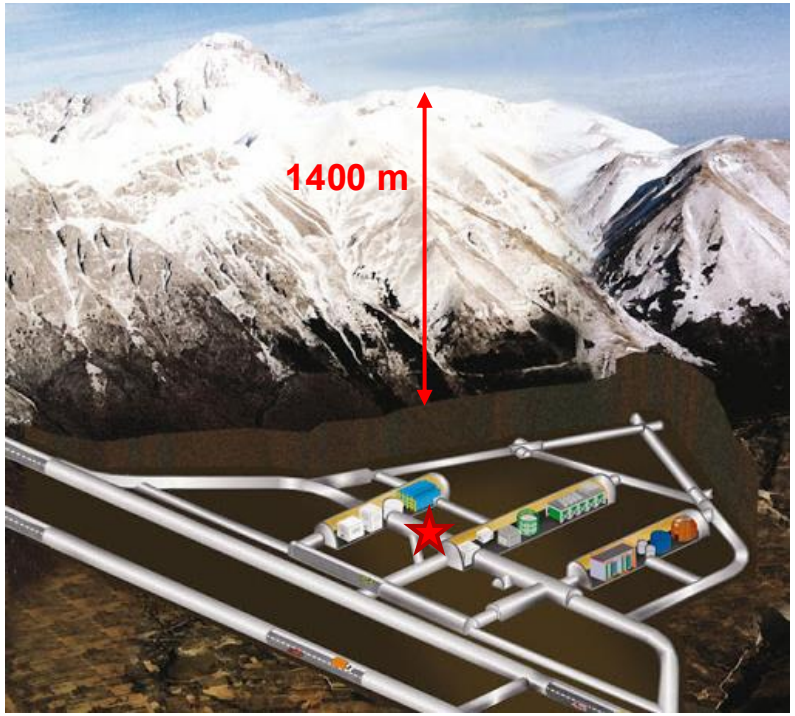


- Maximal suppression of the rate scales as $\delta_{\max} \sim (E_{at}/\Lambda_\theta)^2$
- Suppression requires enough quon deformation $1 - |\eta| \sim (\sigma/\Lambda_\eta)^\kappa$ but not too much

VIP EXPERIMENTS

Search for anomalous X-ray emission in the low-background environment of the underground Gran Sasso National Laboratory (INFN)

- Overburden corresponding to a minimum thickness of 3100 m w.e.
- **Cosmic ray flux reduced by 10^{-6}**



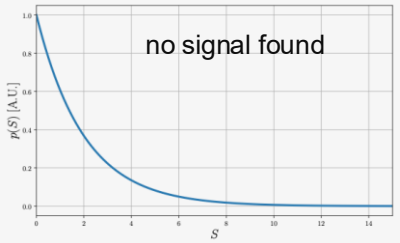
Bayesian analysis to extract pdf $p(S)$ for the # of expected counts S

$$p(S) = \int d\boldsymbol{\theta}\, p(S, \boldsymbol{\theta}|data) \longleftarrow p(S, \boldsymbol{\theta}|data) = \frac{\mathcal{L}(data|S, \boldsymbol{\theta}) P_0(S) P_0(\boldsymbol{\theta})}{\int dS d\boldsymbol{\theta} \mathcal{L}(data|S, \boldsymbol{\theta}) P_0(S)\, P_0(\boldsymbol{\theta})}$$

- $\mathcal{L}(data|S, \boldsymbol{\theta}) = \prod_i \mathrm{Pois}\,(\lambda_i)$

  $\lambda_i = \int_{\Delta E_i} dE\, f(E; S, \boldsymbol{\theta})$: exp counts at bin-i

  fit function

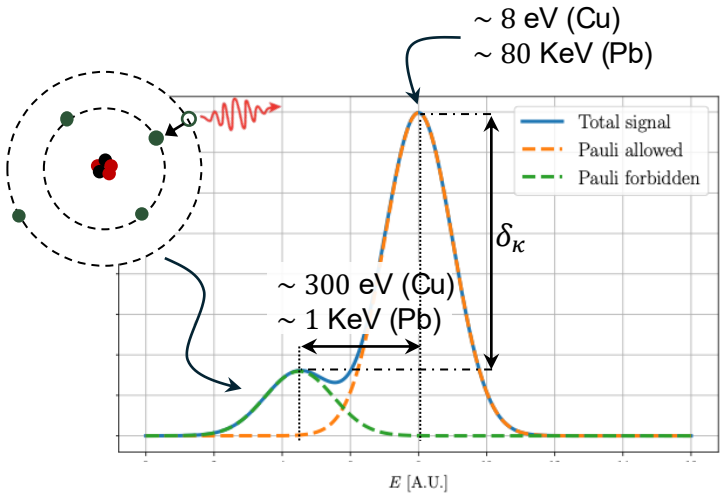


- P_0: prior
- Markov-Chain MC numerical integration: $\int_0^{\bar{S}} dS\, p(S) = 0.9$

Connect S to model's parameters ⟹ **constraint theory**

VIP EXPERIMENTS

- Search for the X-rays signature of PEP violating K transitions in Cu and Pb
- **Transition energies are shifted downward** with respect to the standard ones due to additional shielding of the nuclear field

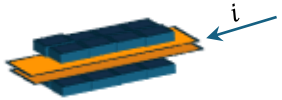


$$N_{\text{events}} = N_{at} \cdot P_{PEPV}^{atom}$$

VIP-Cu "open system": superselection rules (GUP)

Fast PEPV transitions but only for a small fraction p_w of atoms: **use a current** to enhance the numbers of PEPV transitions

32 Silicon Drift Detectors (SDDs) facing two target copper strips (76x20x0.25 mm)



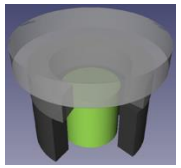
Background provided by data acquisition without current

$S = p_w N_{\text{new}} N_{\text{int}} \epsilon \sim \left(E_{\text{at}}/\Lambda_{QG}\right)^{\kappa}$ ⟹ **lower limit on** Λ_{UV}

VIP-Pb "closed system": no superselection rules (NCST)

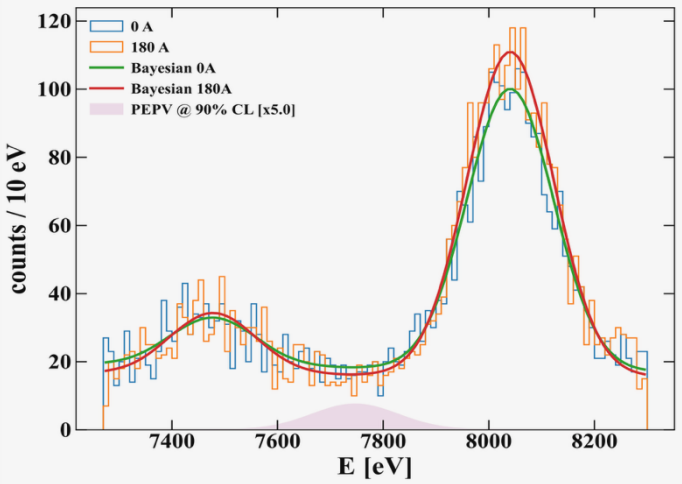
PEPV probability suppressed by $\sim \left(E_{\text{at}}/\Lambda_{\text{QG}}\right)^{a}$ but for all atoms

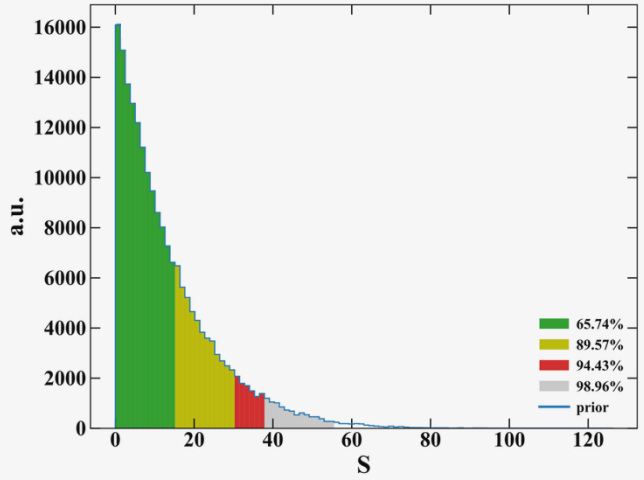
High-purity germaniumn (HPGe) detector (8x8 cm)
Target: 3 cylindrical sections of radio-pure Roman lead completely surrounding the detector



$S \sim \left(E_{\text{at}}/\Lambda_{QG}\right)^{a}$ ⟹ **lower limit on** Λ_{UV}

TEST OF GUP WITH VIP-Cu





$$S = p_w N_{\text{new}}\, N_{\text{int}}\, \epsilon$$

$$\Rightarrow \quad p_w < 6.74\times10^{-43}$$

Strongest limit on PEPV in open systems

A. Porcelli, **N.B.** et al. *EPJC* 84.3 2024

A. Porcelli, **N.B.** et al. *Scientific Reports* 15.1 2025

GUP PEPV probability: $p_w = \beta \underbrace{\frac{m^5}{\Delta p^3} \sum_{nl} BR_{nl} I_{nl}}$

known parameters

$$\beta \lesssim \ell_p^2$$

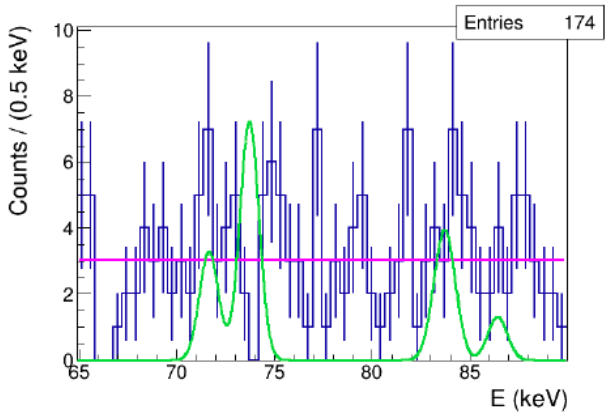
Previous strongest limit:
$\beta/\ell_p^2 < 10^{16}$**!**

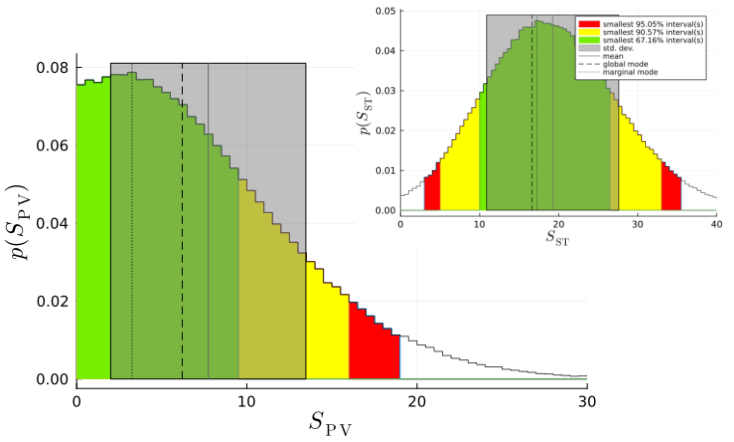
Paper under finalization N.B., P. Bosso, C. Curceau, F. Illuminati, A. Marcianò, L. Petruziello, K. Piscicchia, A. Porcelli

TEST OF DIFFERENT CLASSES OF NCSTS WITH VIP-Lead

$$\Gamma^{\max}_{PEPV} = \left(\frac{E_{\mathrm{at}}}{\Lambda_{NC}}\right)^{a} \Gamma_0$$

$Z_{Pb} = 82 \rightarrow$ test Λ at the highest transition energy





MODEL	Relevant for κ-Minkowski (LQG)	Moyal spacetime (strings with background B-field)	Relevant for Triply Special Relativity
Scaling	Linear	Quadratic	Cubic **(first bound ever)**
$\Lambda_{QG}/\mathrm{E_P} >$	$\mathbf{4.2\times10^{21}}$	$\mathbf{1.6\times10^{-1}}$	$\mathbf{5.6\times10^{-9}}$

K. Piscicchia, **N.B.**, et al *Universe,* 9.7, 2023

Stronger bounds arise in the hadronic sector from DAMA/LIBRA, Borexino and Kamiokande
(A. Addazi, et al, Chinese Physics C 42 094001 2018)

Test of both sectors are needed as in principle different particles may have different couplings to noncommutativity

CURRENT EXPERIMENTAL IMPROVEMENTS

Improve on efficiency: **use Ge detector as active material** → gain of about four orders of magnitude

2 O.o.m. improvement on Λ_{QG} from PEPV

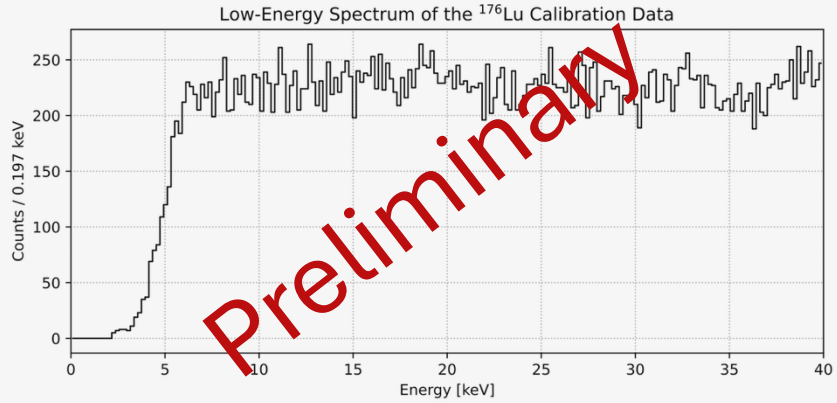
For Ge standard $K_\alpha = 9.9$ keV

Use a **Broad Energy Germanium (BEGe) detector to push down the energy range of the spectrum**

- Ranges from ~ 1 keV up to several MeV
- Excellent resolution and efficiency at low energies







Ongoing work:

- MC characterization of the background
- ML-based event selection. Already **improved SNR** by 14% and **reduced the energy threshold**
  (S. Manti, **N.B.**, et al, JINST 21.01 01036 2026)
- Pulse-shape analysis for **directionality effects**

P. Kristian**, N.B.**, et al Condensed Matter 9.2 2024

CONCLUSIONS

- **Quon field theories** emerge naturally in many quantum gravity scenarios, **with both standard and deformed relativistic spacetime symmetries**
- **Superselection rules** play a crucial role leading to two alternative phenomenological/experimental scenarios
- **Underground X-ray experiments** provide the opportunity to explore the quantum-gravity connection at unprecedented scales
- **Scenarios with deformed symmetries** (e.g. noncommutative Moyal space) predict an unphysical rate of PEPV unless a quon deformation is introduced and **superselection rules are broken**
- **GUP**-inspired QFT constrained **up to the Planck scale**
- **Ongoing improvements** on the **experimental set up** will either put stronger limits or hopefully see QG effects

Thank you for your attention!

Backup slides

❖ What happens if we use **wrong spin-statistics**?

E.g. scalar fields: $\langle 0|[\varphi_a(x), \varphi_b(y)]_+|0\rangle = \Delta_+(x-y;m^2) + \int_{M^2}^{\infty} d\mu^2 \rho(\mu^2)\Delta_+(x-y;\mu^2) \neq 0 \quad (x-y)^2 < 0$

$$\Delta_+(\boldsymbol{x}, t; m^2) \sim \frac{\exp\left(-m\sqrt{|\boldsymbol{x}|^2 - t^2}\right)}{|\boldsymbol{x}|^2 - t^2} \to 0, \quad -(x-y)^2 \gg m^{-2}$$

Violation of locality appears only at distances comparable to the **Compton wavelength**

COVARIANT FOCK SPACE

The state space of a system of identical particles is the **full Fock space** $\mathfrak{F}(\mathfrak{H}) = \bigoplus_{n=0}^{\infty} \mathfrak{H}^{\otimes n}$

The **inner product** of each $\mathfrak{H}^{\otimes n}$ is **fixed by the T:**

$$a(\tilde{p})a^\dagger(\tilde{q}) - \eta(p,q)e^{i\gamma(p,q)}a^\dagger(\tilde{q})a(\tilde{p}) = \delta(\tilde{p},\tilde{q})$$

No relations between $a^\dagger(\tilde{p})a^\dagger(\tilde{q})$

No negative squared-norms: $-1 \leq \eta \leq 1$

QUON ALGEBRA (Greenberg '91)

The **flip operator τ_R** defines **sectors** $\mathfrak{H}^{\otimes n} = \oplus_\lambda \mathfrak{H}_\lambda^{\otimes n}$:

$$|\psi\rangle = \frac{1+\tau_R}{2}|\psi\rangle + \frac{1-\tau_R}{2}|\psi\rangle \equiv |\psi\rangle_{s_R} + |\psi\rangle_{a_R}$$

$$\psi_{s_R}(p,q) = e^{i\gamma(p,q)}\psi_{s_R}(q,p)$$

$$\psi_{a_R}(p,q) = -e^{i\gamma(p,q)}\psi_{a_R}(q,p)$$

Indistinguishability requires **superselection rules (SSRs)**:

$$_{a_R}\langle\psi|O|\phi\rangle_{s_R} = 0 \ \ \forall O \quad \Rightarrow \quad \rho = \sum_\lambda p_\lambda(T)\,|\psi_{\lambda_R}\rangle\langle\psi_{\lambda_R}|$$

(Messiah & Greenberg)

Covariant Quon Field Theory

$$\Psi^{(l)}(x) = \sum_{\sigma} \int \widetilde{dp} \left[a_{\sigma}^{(l)}(\boldsymbol{p}) u_{\sigma}^{(l)}(\boldsymbol{p}) e^{-ip\cdot x} + b_{\sigma}^{(l)\dagger}(\boldsymbol{p}) v_{\sigma}^{(l)}(\boldsymbol{p}) e^{ip\cdot x} \right]$$

$$i\partial_{\mu}\Psi^{(l)}(x) = \left[\Psi^{(l)}(x), P_{\mu} \right]$$

Q-deformed algebra, with $|Q_{lm}| < 1$:

$$a^{(l)}(\tilde{p}) a^{(m)\dagger}(\tilde{q}) - Q_{lm}^{*}(p,q) a^{(m)\dagger}(\tilde{q}) a^{(l)}(\tilde{p}) = \delta_{lm}\delta(\tilde{p},\tilde{q})$$
$$b^{(l)}(\tilde{p}) b^{(m)\dagger}(\tilde{q}) - Q_{lm}^{*}(p,q) b^{(m)\dagger}(\tilde{q}) b^{(l)}(\tilde{p}) = \delta_{lm}\delta(\tilde{p},\tilde{q})$$
$$a^{(l)}(\tilde{p}) b^{(m)\dagger}(\tilde{q}) - Q_{lm}(p,q) b^{(m)\dagger}(\tilde{q}) a^{(l)}(\tilde{p}) = 0$$

- Additivity of energy, momentum, charge etc. requires that **operators have infinite expansions in $\boldsymbol{a}^{\#}$**

$$P_{\mu} = \int_{\tilde{p}} p_{\mu} N(\tilde{p}), \quad N(\tilde{p}) = \sum_{l} \sum_{x=a,b} x^{(l)\dagger}(\tilde{p}) x^{(l)}(\tilde{p}) + \sum_{lm} \sum_{xy} \int_{\tilde{q}} \frac{1}{1 - |Q|_{lm}^{2}(p,q)} \left[y^{(m)\dagger}(\tilde{q}), x^{(l)\dagger}(\tilde{p}) \right]_{Q_{mn}} \underbrace{\left[x^{(l)}(\tilde{p}), y^{(m)}(\tilde{q}) \right]_{Q_{nm}}}_{x^{(l)}(\tilde{p}) y^{(m)}(\tilde{q}) - Q_{lm}(p,q) y^{(m)}(\tilde{q}), x^{(l)}(\tilde{p})} + \cdots$$

- Q-deformed commutation relations yield a **Wick's theorem,** CPT theorem and cluster decomposition hold (Greenberg)
- **Causality breaks down:** $[\mathcal{H}_I(x), \mathcal{H}_I(y)] \neq 0$ at space-like separations
- **But the *S*-matrix is invariant:** $\int dxdy\, x[\mathcal{H}_I(\boldsymbol{x},0), \mathcal{H}_I(\boldsymbol{y},0)] \neq 0$, which ensures $U(\Lambda)^{\dagger} S U(\Lambda) = S$

$|Q| < 1$ ***might*** be crucial to obtain UV completion

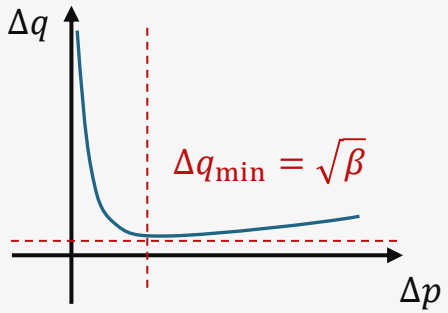
Quon realization from generalized uncertainty principle

By modifying the HUP a **minimal length** appears

$$[\hat{q},\hat{p}] = i[1+\beta\hat{p}^2]$$

$$\Delta q\Delta p = \frac{1}{2}[1+\beta\Delta p^2]$$

from **string theory**



Deformed ladder operators for the h.o.

$$\hat{c}^\dagger|n\rangle = \sqrt{\frac{E_{n+1}-E_0}{\hbar\omega}}|n+1\rangle, c|n\rangle = \sqrt{\frac{E_n-E_0}{\hbar\omega}}|n-1\rangle$$

$$H = \hbar\omega c^\dagger c + E_0\mathbb{I}, \qquad \left[c,c^\dagger\right] = \sqrt{1+2\beta mH} + \frac{\beta\hbar\omega m}{2}$$

(P. Bosso, G.G. Luciano *EPJC* **81**, 982 2021)

QFT can be constructed using such operators

Spinor fields: $a(\tilde{p})a^\dagger(\tilde{q}) + \eta_\beta(p,p')a^\dagger(\tilde{q})a(\tilde{p}) = \delta^{(3)}(\tilde{p},\tilde{q})$

$$\{a(\tilde{p}),a(\tilde{q})\} = 0, \quad \{a^\dagger(\tilde{p}),a^\dagger(\tilde{q})\} = 0, \quad \text{for } \tilde{p}\neq\tilde{q}$$

Quon deformation of fermions: $\eta_\beta(\boldsymbol{p},\boldsymbol{p}') = 1-\beta mE_p^4\delta^{(3)}(\boldsymbol{p}-\boldsymbol{p}')$

Infinite statistics and UV physics

❖ Same corr. functions as a **matrix model** in the 't Hooft limit (Voiculescu, Gopakumar & Gross, Douglas, Aref'eva & Volovich,…)

$$Q = 0: \quad \underbrace{\langle 0|\Psi(x_1)\cdots\Psi(x_n)|0\rangle}_{\text{Master field}} = \underbrace{\lim_{N\to\infty} \frac{1}{\mathrm{N}^{1+k/2}} \langle\Omega|\mathrm{Tr}\big(M(x_1)\cdots M(x_n)\big)|\Omega\rangle}_{\textbf{Free string theory}}, \text{ where } M(x) \in Mat(N,N)$$

$Q \neq 0$ introduces interactions

❖ **Quantum Boltzmann statistical properties** provides interesting connections with **holography**

In string theory **black hole entropy** is derived if the microstates have infinite statistics (Strominger, BFKS, Volovich, Minic,…)

Holographic construction of **dark matter/energy** requires particles with infinite statistics (Jack Ng, Minic et al)

Quons interpolate between IR point-like and UV **extended quanta**

Energy dependent quon statistics approaching $\mathbf{Q \sim 0}$ **at** $\boldsymbol{\Lambda_{UV} \sim E_P}$
as a manifestation of gravitational physics

Quantum statistics:
A (partial) hystorical perspective



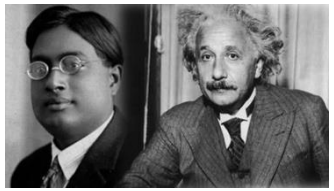
1911 — W. Natanson discovers **indistinguishability** of Planck's quanta



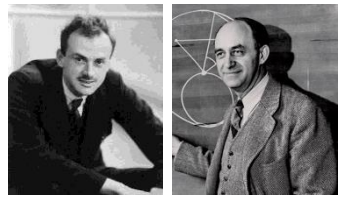
1925 — **Pauli's exclusion principle (PEP)**

1934 — Fermi investigates the case of **"slightly non identical"** particles

1924 — Bose-Einstein statistics for photons and matter



1926 — Fermi-Dirac statistics for particles that obey the PEP



Large violations of statistics



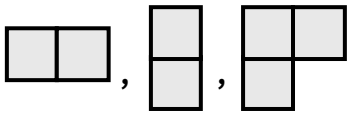
1940

Spin-statistics theorem

G. Gentile Jr looked at intermediate statistics

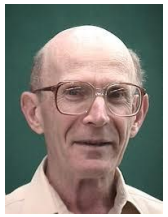


1953

$[a_i, [a_j, a_k]_{\pm}] = 0$: 

H.S. Green shows that **local QFT** is compatible with **parastatistics**





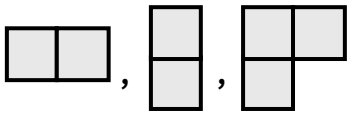
1964

Messiah & Greenberg: **indistinguishability = parastatistics + selection rules**

Color degree of freedom

Large violations of statistics



$[a_i,[a_j,a_k]_{\pm}]=0$: 

H.S. Green shows that **local QFT** is compatible with **parastatistics**

1940 — 1953 — 1964



Spin-statistics theorem

G. Gentile Jr looked at intermediate statistics



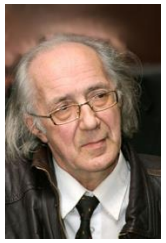


Messiah & Greenberg: **indistinguishability = parastatistics + selection rules**

Color degree of freedom

All known particles are either bosons or fermions → Is this the end of the story?

Small violations of statistics



Vadim Kuzmin estimates the **QG-induced PEPV**

Greenberg formulates the **quon model**

Connection with QG frameworks

1974 — 1985 — 1987 — 1991 — 2001 — Ongoing…

Reines & Sobel: first test of the PEP in “closed systems”

Ignatiev & Kuzmin toy model

Ramberg & Snow experiment in “open system”

VIP-Cu “open system” experiment: best limit on quon

Hilbert space structure

❖ The state space of a system of **distinguishable** identical particles is the **full Fock space** $\mathfrak{F}(\mathfrak{H}) = \bigoplus_{n=0}^{\infty} \mathfrak{H}^{\otimes n}$

For all n, finite linear combinations of product states $|\xi_1, \dots, \xi_n\rangle \equiv |\xi_1\rangle \otimes \cdots \otimes |\xi_n\rangle$ span the n-particle Hilbert space $\mathfrak{H}^{\otimes n}$

❖ The inner product of $\mathfrak{H}^{\otimes n}$ can always be written as

$\langle \xi_n, \dots, \xi_1 | \chi_1, \dots, \chi_n \rangle = (\xi_n, \dots, \xi_1 | W_{(n)} | \chi_1, \dots, \chi_n)$

$(\xi_n, \dots, \xi_1 | \chi_1, \dots, \chi_n) = \langle \xi_1 | \chi_1 \rangle \cdots \langle \xi_n | \chi_n \rangle$

$W_{(n)}$ is positive: squared norms are non-negative

In second quantization, **$W_{(n)}$ is determined by the exchange relation**

$$a(\tilde{p})a^\dagger(\tilde{q}) - \int_{\tilde{p}\tilde{q}} T(\tilde{k}, \tilde{l} | \tilde{p}, \tilde{q}) a^\dagger(\tilde{k}) a(\tilde{l}) = \delta(\tilde{p}, \tilde{q})$$

e.g. $W_{(2)} = \frac{1}{2}(1 + T)$

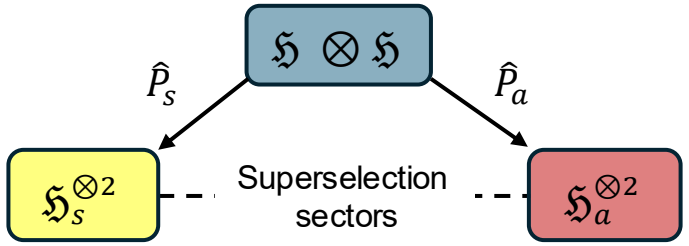
Indistinguishability of identical particles

❖ At least at energies ≪ Planck scale, identical particles are indistinguishable

Indistinguishability DOES NOT require the symmetrization postulate: $|\xi,\chi\rangle \nrightarrow |\chi,\xi\rangle_{\lambda_R}$, $a(\tilde{p})a(\tilde{q}) = \int_{\tilde{p}\tilde{q}} R(\tilde{k},\tilde{l}|\tilde{p},\tilde{q})a(\tilde{k})a(\tilde{l})$

It is sufficient the existence of **superselection rules (SSRs):** ${}_{a_R}\langle\psi|O|\phi\rangle_{s_R} = 0 \;\; \forall$ osservables O

- $\hat{\rho}$ can be written in **block-diagonal** form! $\hat{\rho} = \sum_{\gamma} p_\gamma |\psi_{\gamma_R}\rangle\langle\psi_{\gamma_R}|$
- **States are incoherent mixtures of different permutation symmetries, with weights** $\boldsymbol{p_\gamma = \frac{\|\psi_{\gamma_R}\|^2}{\sum_{\gamma'}\|\psi_{\gamma'_R}\|^2}}$

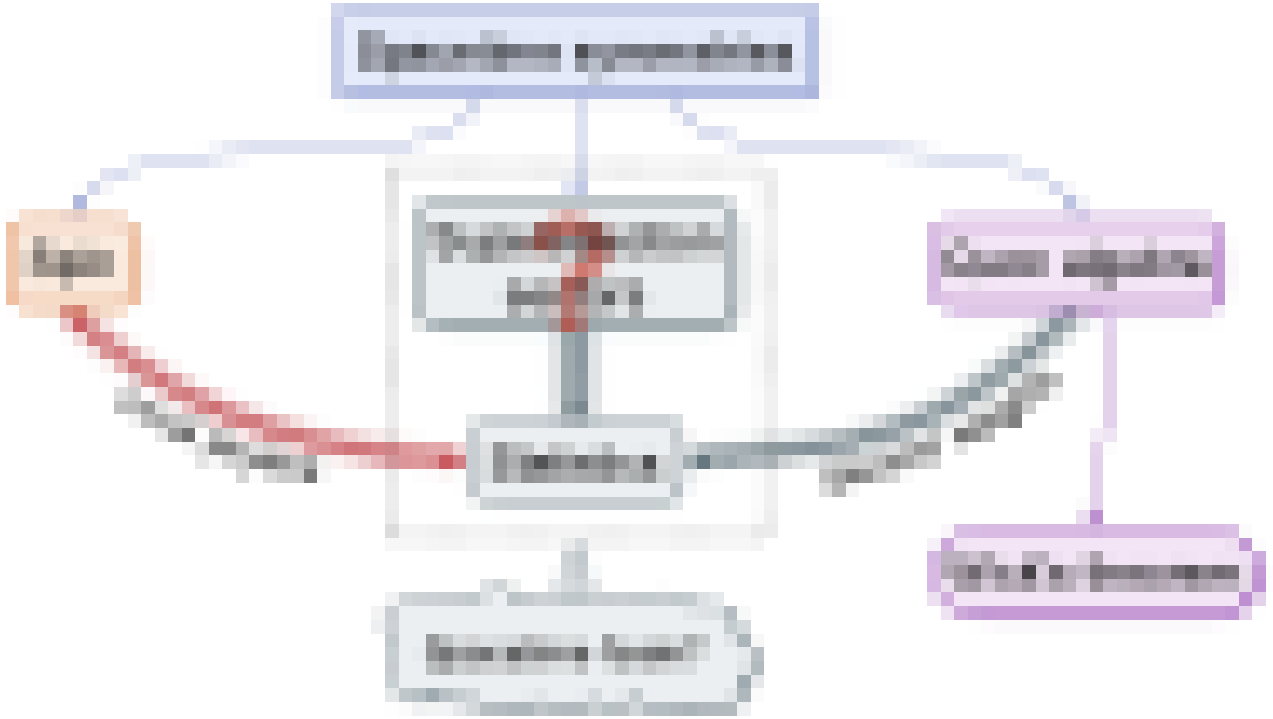


- $\|\psi_{\gamma_R}\|^2 = \langle\psi_{\gamma_R}|\psi_{\gamma_R}\rangle \Longrightarrow$ **p_γ depend on the T operator**
- Superselection sectors $\mathfrak{H}_\lambda^{\otimes n}$ + weights $p_\gamma =$ **statistics**

Spacetime symmetries

Let $U(g)$ be a unitary representation of the Poincaré group on $\mathfrak{H}^{\otimes n}$, where $g = \{a, \Lambda\}$. Then:

- The inner product must be invariant $(U(g)\psi|W|U(g)\phi) = (\psi|U(g)^{\dagger}WU(g)|\phi)$ ➡ $U(g)T = TU(g)$
- Permutation of particles must be compatible with spacetime transformations ➡ $U(g)\tau_R = \tau_R U(g)$

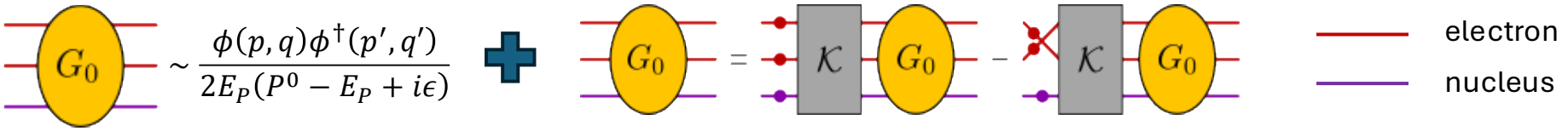


Atomic states

Rigorous relativistic treatment of atomic states is based on Green functions

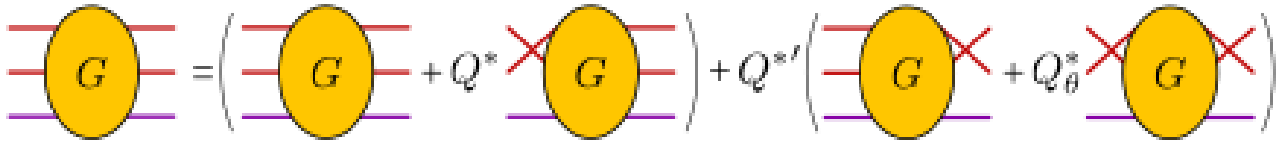
- pole structure contains bound states
- treat interactions nonperturbatively
- provide dynamical evolution

Bethe-Salpeter (BS) amplitudes: $\phi(x,y) = \langle \Omega | T\{\Psi(x)\Psi(y)\} | B \rangle$

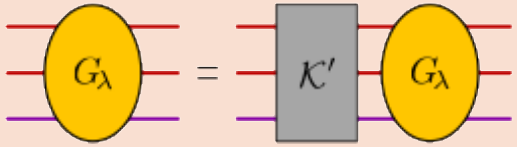


BS equation for the amplitude $\boldsymbol{\phi(p,q)}$

❖ The generalization to twisted-quons leads to **two distinct scenarios:**

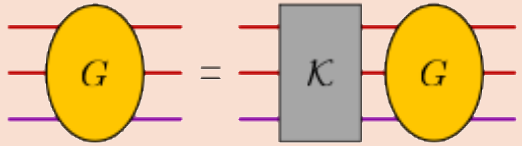


The kernel preserves superselection rules



The physical states are those with **twisted statistics**

The kernel breaks superselection rules



Identical particles are slightly different

Poincaré Hopf-algebra

Poincaré **algebra**

$$[P_\mu, P_\nu] = 0$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = i\left(\eta_{\nu\rho}M_{\mu\sigma} - \eta_{\mu\rho}M_{\nu\sigma} - \eta_{\nu\sigma}M_{\mu\rho} + \eta_{\mu\sigma}M_{\nu\rho}\right)$$
$$[M_{\mu\nu}, P_\rho] = -i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu)$$

Action of the generators on **single particle states**

Poincaré **co-algebra**

$$\Delta(Y) = Y \otimes \mathbb{1} + \mathbb{1} \otimes Y\ , \quad Y = \{P_\mu, M_{\mu\nu}\}$$

Action of the generators on **multi particle states**

Pauli forbidden atomic transitions without SSRs

$\psi(p_1, p_2)$ and $\psi(p_2, p_1)$ describe different physical states:

Initial state
$\psi^{\pm}(\boldsymbol{p}, \boldsymbol{q}) = \psi_s(\boldsymbol{p}, \boldsymbol{q}) \pm \psi_a(\boldsymbol{p}, \boldsymbol{q})$

PEPV state
$\psi'(\boldsymbol{p}, \boldsymbol{q}) = \psi'_s(\boldsymbol{p}, \boldsymbol{q})$

$$d\Gamma^{\pm}_{\mathrm{PEPV}} = 2\pi\delta(E' - E + \omega)\left|H^s_{if} \pm H^a_{if}\right|^2 \frac{d^3k}{(2\pi)^3}$$

$$H^{\gamma}_{if} = \frac{e}{\sqrt{2\omega}} \int d^3p d^3q \; D^{\gamma}\psi'_s(\boldsymbol{p} - \boldsymbol{k}, \boldsymbol{q}) \; \boldsymbol{\alpha}_{(1)} \cdot \boldsymbol{\varepsilon}^* \psi_{\lambda}(\boldsymbol{p}, \boldsymbol{q})$$

Deformation factor due to modified statistics

QED matrix elements are deformed by

$$D^{s,a} = \frac{[1 \pm Q(p,q)][1 + Q(p-k,q)] \pm [1 - \eta^2(p-k,q)]}{\|\psi'_s\|\|\psi\|}$$

Pauli forbidden atomic transitions without SSRs

$\psi(p_1, p_2)$ and $\psi(p_2, p_1)$ describe different physical states:

Initial state
$\psi^{\pm}(\boldsymbol{p}, \boldsymbol{q}) = \psi_s(\boldsymbol{p}, \boldsymbol{q}) \pm \psi_a(\boldsymbol{p}, \boldsymbol{q})$

PEPV state
$\psi'(\boldsymbol{p}, \boldsymbol{q}) = \psi'_s(\boldsymbol{p}, \boldsymbol{q})$

$$d\Gamma^{\pm}_{\mathrm{PEPV}} = 2\pi\delta(E' - E + \omega)\left|H^s_{if} \pm H^a_{if}\right|^2 \frac{d^3k}{(2\pi)^3}$$

$$H^{\gamma}_{if} = \frac{e}{\sqrt{2\omega}} \int d^3p d^3q \; D^{\gamma} \psi'_s(\boldsymbol{p} - \boldsymbol{k}, \boldsymbol{q}) \; \boldsymbol{\alpha}_{(1)} \cdot \boldsymbol{\varepsilon}^* \psi_{\lambda}(\boldsymbol{p}, \boldsymbol{q})$$

Deformation factor due to modified statistics

QED matrix elements are deformed by

$$D^{s,a} = \frac{[1 \pm Q(p,q)][1 + Q(p-k,q)] \pm [1 - \eta^2(p-k,q)]}{\|\psi'_s\|\|\psi\|}$$

$$\boldsymbol{\Gamma_{\mathrm{PEPV}} = \delta\left(E_{at}, \Lambda_{\mathrm{NC}}, \Lambda_{\eta}\right) \cdot \Gamma_0},$$

- E_{at} is an atomic energy scale
- Λ_{NC} is the UV noncommutative scale
- Λ_{η} is the UV quon scale

δ depends on the specific form of $\eta(p,q)$ and spacetime symmetries