Perturbative Insights in Crossing Violation and the Fixed-point Structure of 3d Systems

Relativistic fermions in Flatland
From Moiré materials to gauge field theories

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Relativistic 3d systems and perturbative approaches

$$\mathcal{L} \sim \underbrace{\frac{1}{\boldsymbol{g}^2} FF}_{\text{gauge}} + \underbrace{\boldsymbol{k}(A\wedge dA + A\wedge A\wedge A)}_{\text{Chern-Simons}} + \underbrace{(D\phi)^2 + m_\phi^2\phi^2 - V(\phi)}_{\text{scalar matter}}$$

$$+ \underbrace{i\bar{\psi}\not{D}\psi - m_\psi\bar{\psi}\psi}_{\text{fermionic matter}} + \underbrace{\boldsymbol{y}\phi\bar{\psi}\psi}_{\text{Yukawa}} + \underbrace{\boldsymbol{\rho}\phi^2\bar{\psi}\psi}_{\text{marginal-}\phi^2\psi^2} + \text{non-renormalisable}$$

$$[g] = [E]^{1/2} \qquad [y] = [E]^{1/2} \qquad [k] = [E]^0 \qquad [\rho] = [E]^0$$

Theories flow to strong coupling in the IR, difficult to control perturbatively

Perturbative frameworks: ϵ-expansion, large-k, large-N,…

Chern–Simons matter theory

- topological theories
- part of a web of dualities [Aharony 15]
- ϵ-expansion not possible, but large-k/N possible

Perturbative Fixed Points

- often non-trivial fixed points
- existence depends on N_F, N_c
- ϵ-expansion can provide estimates

Outline

I. Crossing and unitarity in Chern–Simons matter theory

- What changes in an anyonic S matrix?
- Naive crossing violates unitarity $\rightarrow$ modify naive S matrix
- Test conjecture with full S-matrix computation at $1/k^2$ and finite N

II. Fixed-point structure of 3d systems

- High-loop RGEs?
- Landscape of perturbative fixed points?

III. Conclusions

Crossing and unitarity in Chern–Simons matter theories

Large-k at finite-N tests of crossing violation

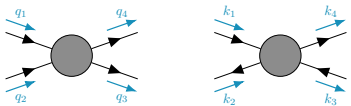
$$\mathcal{L} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho + \frac{2i}{3}A_\mu A_\nu A_\rho\right) + \begin{cases} i\bar{\psi}\not{D}\psi - m_\psi\bar{\psi}\psi \\ (D_\mu\phi)^2 - m_\phi^2\phi^\dagger\phi \end{cases}$$

[With L. Di Pietro, E. Di Salvo and M. Uetrecht (soon on arXiv)]

The S Matrix: unitarity, analyticity, and crossing

$$S = {}_{\text{out}}\langle f | i \rangle_{\text{in}} = \mathbf{1} + iT$$

- $\mathbf{1}$: forward scattering T: transfer matrix (includes scattering)
- probability conservation ➜**Unitarity**: $S^\dagger S = \mathbf{1}$
- analyticity in kinematic variables ➜**Crossing Symmetry/Relations**

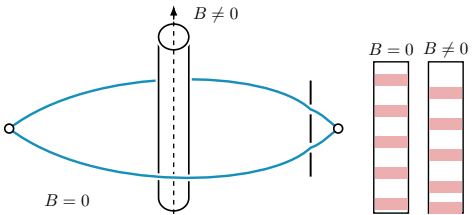


$\mathcal{M}_{PP\to PP}(q_1, q_2; q_3, q_4)$ $\leftrightarrow$ crossing $\mathcal{M}_{P\bar{P}\to \bar{P}P}(k_1, k_2; k_3, k_4)$

$$\mathcal{M}_{P\bar{P}\to \bar{P}P}(k_1, k_2; k_3, k_4) \overset{!}{=} \underbrace{\mathcal{M}_{PP\to PP}(k_1, -k_3; -k_2, k_4)}_{\text{analytic continuation}}$$

Aharonov–Bohm effect

Must the 1 in S be a 1? ➜ Input from non-relativistic QM



- Phase from winding around magnetic flux
  (even though $\vec{B} = 0$ along path)

[Aharonov, Bohm 59]

- Different paths, different phases
  $\Psi = \Psi_0 \left(e^{i\nu_1} + e^{i\nu_2}\right)$ with $\nu_i \propto q \int_\gamma \vec{A} \cdot d\vec{r}$

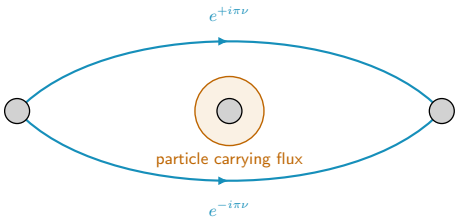
- Average over paths left and right of solenoid ($e^{\pm i\nu} \to \cos(\nu)$)

Seems incompatible with $S = 1 + iT$ decomposition

Aharonov–Bohm and Chern–Simons matter theory

$$\mathcal{L} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\big(A_\mu\partial_\nu A_\rho + \frac{2i}{3}A_\mu A_\nu A_\rho\big) + \begin{cases} i\bar{\psi}\not{D}\psi - m_\psi\bar{\psi}\psi \\ (D_\mu\phi)^2 - m_\phi^2\phi^\dagger\phi \end{cases}$$

- Abelian CS matter theory provides a relativistic version of AB [Boz, Fainberg, Pak 96]
- Charged matter in CS carries magnetic flux [e.o.m $A_\mu J^\mu_{\text{matter}} \propto A \wedge F$]
- Each particle acts as an AB flux tube for the other



$$e^{+i\pi\nu} + e^{-i\pi\nu} \quad \rightsquigarrow \quad \cos(\pi\nu)\times 1$$ **Modification of forward identity term**

ν depends on two-particle scattering channel, M [Jain et al 15]

$2 \to 2$ scattering channels

	Color decomposition	Amplitudes
$\mathrm{P}\bar{\mathrm{P}} \to \mathrm{P}\bar{\mathrm{P}}$	$N \otimes \bar{N}$	singlet, adjoint
$\mathrm{PP} \to \mathrm{PP}$	$N \otimes N$	symmetric, antisymmetric

$$\mathrm{P} = \psi \text{ or } \phi \qquad G = \mathrm{SU}(N)_k \text{ or } \mathrm{U}(N)_{k,k},$$

Four independent channel amplitudes $T_M(s,t)$ for each matter theory and gauge group

$$S_{\mathrm{P}\bar{\mathrm{P}}} = \left(\delta_i^k \delta_\ell^j - \frac{\delta_i^j \delta_\ell^k}{N}\right) \boldsymbol{S_{\mathbf{adj}}} + \frac{\delta_i^j \delta_\ell^k}{N} \boldsymbol{S_{\mathbf{sing}}} \qquad S_{\mathrm{PP}} = \frac{\delta_i^k \delta_j^\ell + \delta_i^\ell \delta_j^k}{2} \boldsymbol{S_{\mathbf{sym}}} + \frac{\delta_i^k \delta_j^\ell - \delta_i^\ell \delta_j^k}{2} \boldsymbol{S_{\mathbf{asym}}}$$

$$\{\boldsymbol{S_{\mathbf{adj}}}, \boldsymbol{S_{\mathbf{sing}}}\} \overset{\text{crossing?}}{\leftrightarrow} \{\boldsymbol{S_{\mathbf{sym}}}, \boldsymbol{S_{\mathbf{asym}}}\}$$

The large-N crossing singlet puzzle

[Jain et al 15]

- In 2015 **Jain et al** compute S_{adj}, S_{sym}, S_{asym} at large-N with $\lambda = N/k$ fixed

- Use naive crossing to extract S_{sing}

  ➜ **unitarity violation $S_{\text{sing}}^{\dagger} S_{\text{sing}} \neq 1$ and Aharonov–Bohm limit fails**

  [Jain et al 15, confirmation in 2022 by Gabai et al directly in a light-cone Hamiltonian calculation]

- **Jain et al** propose a **modification** of the S matrix at $\mathcal{O}(1/N)$

$$S_{\text{sing}}^{\text{phys}} = \cos(\pi\lambda)\mathbf{1} + \frac{\sin(\pi\lambda)}{\pi\lambda} i\, T_{\text{sing}}^{\text{naive}} + \mathcal{O}(1/N)$$

- Modification **restores unitarity** and AB!

The conjecture for modification at finite-N

[Mehta et al 22]

- $G = \{\mathrm{SU}(N)_k, \mathrm{U}(N)_{k,k}\}$ and $M = \{\text{sing}, \text{adj}, \text{sym}, \text{asym}\}$

$$S_M^{\text{phys}} = \underbrace{\cos(\pi\nu_M)\mathbf{1}}_{\text{modified forward identity}} + \underbrace{i\,T_M^{\text{mod}}}_{\text{modified crossing}}$$

- Channel-dependent Aharonov–Bohm phase: $\boldsymbol{\nu_M}$ $\quad [\eta_{\mathrm{U}(N)_{k,k}} = 1 \text{ and } \eta_{\mathrm{SU}(N)_k} = 0]$

$$\nu^G_{\text{sing}} = \frac{N^2 - 1 + \eta_G}{Nk} + \frac{\eta_G}{k(|k| - N)}, \qquad \nu^G_{\text{adj}} = -\frac{1 - \eta_G}{Nk} + \frac{\eta_G}{k(|k| - N)},$$

$$\nu^G_{\text{sym}} = -\frac{N - 1 + \eta_G}{Nk}, \qquad \nu^G_{\text{asym}} = \frac{N + 1 - \eta_G}{Nk},$$

- Crossing coefficients: ordinary integers $\longrightarrow$ q-numbers

  Example:

$$T^{\text{mod}}_{\text{adj}} = \mathcal{AC}_{\text{PP}\to\text{P}\bar{\text{P}}}\left[\frac{T^{\text{mod}}_{\text{sym}} - T^{\text{mod}}_{\text{asym}}}{[2]_q}\right] \qquad [n]_q = \frac{\sin(\pi n/k)}{\sin(\pi/k)} \qquad q = e^{2\pi i/k}$$

[Mehta et al 22]

Test the conjecture at large-k and finite N

[with Di Pietro, Di Salvo, Uetrecht]

- CS amenable to large-k perturbation theory

$$1/k \ : \ \text{tree-level} \qquad\qquad 1/k^2 \ : \ \text{1-loop}$$

- Forward identity changes at $\mathbf{1/k^2}$

$$\cos(\pi\nu) = 1 - \frac{\pi^2\nu^2}{2} + \cdots, \qquad \nu = \mathcal{O}(1/k)$$

- Modifications of crossing coefficients affect amplitudes at $\mathcal{O}(1/k^3)$

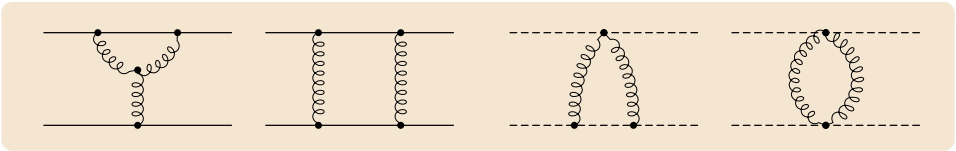
$$[n]_q = n + \mathcal{O}(1/k^2) \qquad T^{\text{tree}} = \mathcal{O}(1/k) \qquad T^{\text{1-loop}} = \mathcal{O}(1/k^2)$$

$$\boldsymbol{S^{\text{phys}} = \cos(\pi\nu)1 + i\,T^{\text{mod}} \simeq 1 + i\,T + i\frac{\pi^2\nu^2}{2}1 + \mathcal{O}(1/k^3)}$$

Goal: test the conjecture for all channels at order $1/k^2$ and finite N

The S matrix at $\mathcal{O}(1/k^2)$

Performed full S matrix computation of all $2 \to 2$ channels



$$\mathcal{L} = \mathcal{L}_{\text{CS}} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge fixing}} + \mathcal{L}_{\text{ghost}}$$

- Fixed N, large k: $T = \frac{1}{k}T^{(1)} + \frac{1}{k^2}T^{(2)} + \cdots$ [with Di Pietro, Di Salvo, Uetrecht]
- Fermions and scalars; $\mathrm{SU}(N)_k$ and $\mathrm{U}(N)_{k,k}$
- General R_ξ gauge; on-shell external matter

Checks and details on the amplitudes

- Gauge parameters and $1/\epsilon$ poles cancel in final amplitude ✓
  [consistent BMHV treatment of $\epsilon_{\mu\nu\rho}$ essential / otherwise ξ dependence]
- IBP: `LiteRed` / `HypExp` reductions, numerical integral checks with `pySecDec` ✓
- Physical-sheet continuation: checked ordinary crossing for $T^{(1,2)}$ ✓
- No massless branch cuts in the final amplitude ✓
- Agreement with the large-N small-λ expansion ✓

One-loop amplitudes: ugly but physical

Example: scalar, symmetric channel

$$i\boldsymbol{T}^{(n),G}_{\boldsymbol{\phi\phi},\mathbf{sym}} = \boldsymbol{\mathcal{A}}^{(n),G}_{\boldsymbol{\phi},+} + \text{(reflected version)}$$

$$\mathcal{A}^{(1),G}_{\phi,+} = \frac{8\pi[N-(1-\eta_G)]}{Nt}\,\mathcal{E}$$

$$\begin{aligned}\mathcal{A}^{(2),G}_{\phi,+} = \frac{\pi}{4N^2}\Bigg(&8im[N-1+\eta_G]\left[\frac{N(2m^2-s)}{t}+\frac{1}{2}\big(2N^2+(3-2\eta_G)N-4(1-\eta_G)\big)\right]\\&+8\sqrt{s}\,[N-1+\eta_G]^2(\pi+i\ell_s)+[N-1+\eta_G]\frac{\sqrt{-t}\,\ell_t}{t^2}\Big[16Nm^4-8Nm^2s\\&-8\big([1+4\eta_G]N-4(1-\eta_G)\big)m^2t+2\big([1+8\eta_G]N-8(1-\eta_G)\big)st\\&+\Big(8N^2+(1-\eta_G)\big(9N-16\big)+9\eta_GN\Big)t^2\Big]\Bigg)\end{aligned}$$

$$\mathcal{E}=\epsilon_{\mu\nu\rho}q_1^\mu q_2^\nu q_3^\rho \qquad \ell_s=\log\frac{\sqrt{s}-2m}{\sqrt{s}+2m} \qquad \ell_t=\log\left(-\frac{\sqrt{-t}-2im}{\sqrt{-t}+2im}\right)$$

Ready to test finite-N conjectures

Unitarity

$$S^{\text{phys}} \simeq \mathbf{1} + \underbrace{i\,T + i\frac{\pi^2\nu^2}{2}\mathbf{1}}_{\equiv i\widehat{T}} + \mathcal{O}(1/k^3)$$

- **Unitarity:** $-i(\widehat{T} - \widehat{T}^\dagger) = \widehat{T}^\dagger\widehat{T}$
- **Unitarity relation in c.o.m.:** (α: outgoing scattering angle)

$$-i\Big(\widehat{T}(s,\alpha) - \widehat{T}^*(s,-\alpha)\Big) = \frac{1}{8\pi\sqrt{s}}\int_0^{2\pi} d\theta\, \widehat{T}(s,\theta)\widehat{T}^*(s,\theta-\alpha)$$

- **Relations in large-k perturbation theory**

$$\mathcal{O}(1/k): \quad -\frac{i}{k}\Big(\widehat{T}^{(1)}(s,\alpha) - \widehat{T}^{(1)*}(s,-\alpha)\Big) = 0 \quad \checkmark$$

$$\mathcal{O}(1/k^2): \quad -\frac{i}{k^2}\Big(\widehat{T}^{(2)}(s,\alpha) - \widehat{T}^{(2)*}(s,-\alpha)\Big) = \frac{1}{8\pi\sqrt{s}\,k^2}\int_0^{2\pi} d\theta\, \widehat{T}^{(1)}(s,\theta)\widehat{T}^{(1)*}(s,\theta-\alpha)\,.$$

Unitarity in forward scattering

- Most important check: unitarity in forward scattering ($\alpha = 0$) because modified
- **Problem:** $T^{(1)}$ singular from t-channel pole: $\sim \frac{\mathcal{E}}{t+i0} \sim \mathrm{PV}\cot\frac{\alpha}{2}$

$$\widehat{T}_M^{(1)}(s,\alpha) = \underbrace{W_M(s)\,\mathrm{PV}\cot\frac{\alpha}{2}}_{\text{singular at } \alpha=0} + \underbrace{R_M(s,\alpha)}_{\text{regular at } \alpha=0}$$

- **Unitarity:** $-i\big(\underbrace{\widehat{T}^{(2)}(s,\alpha) - \widehat{T}^{(2)*}(s,-\alpha)}_{\text{contains: } \mathbf{1}\propto\delta(\alpha) \text{ from modification}}\big) = \frac{1}{8\pi\sqrt{s}}\int_0^{2\pi} d\theta\, \widehat{T}^{(1)}(s,\theta)\widehat{T}^{(1)*}(s,\theta-\alpha)$
- **Question:** what gives a $\delta(\alpha)$ on r.h.s.? ➜product of two PV induces $\delta(\alpha)$ [Mehta et al 22]

$$\mathrm{PV}\int_0^{2\pi} d\theta\,\cot\frac{\theta}{2}\cot\frac{\alpha-\theta}{2} = 2\pi - 4\pi^2\delta(\alpha)\,.$$

➜ Two checks per channel: $\delta(\alpha)$ terms and regular terms

Verification of unitarity with modified crossing

$$-i\big[\widehat{T}_M^{(2)}(s,\alpha) - \widehat{T}_M^{(2)*}(s,-\alpha)\big] = \widehat{T}_M^{(1)} \star \widehat{T}_M^{(1)*}$$

- Verified unitarity for **fermionic** and **scalar** CS theory in all channels

➜ No additional forward delta from the connected one-loop diagrams
[a priori not clear whether one-loop diagrams could induce the required $\delta(\alpha)$ terms]

➜ Forward $\delta(\alpha)$ from tree-convolution cancels $\delta(\alpha)$ from modification
[one-loop amplitude required in $\delta(\alpha)$ at $\mathcal{O}(1/k^3)$]

➜ First perturbative verification of the conjecture at finite N

Crossing modification beyond the planar singlet

Forward contribution $\propto \boldsymbol{\nu_M^2}$
confirmed for fermions and scalars in both gauge theories

$k\nu_M$	$\mathbf{SU}(N)_k$	$\mathbf{U}(N)_{k,k}$	visible at LO large-N
singlet	$N - 1/N$	N	✓
adjoint	$-1/N$	0	✘
symmetric	$-1 + 1/N$	-1	✘
antisymmetric	$1 + 1/N$	$+1$	✘

Entries show the leading $1/k$ phase; the $\mathrm{U}(N)$ adjoint starts at higher order

[with Di Pietro, Di Salvo, Uetrecht]

Crossing and Unitarity in CS: take-away messages

ETC*: relativistic matter in flatland and CS gauge fields

- In CS matter theory unitarity incompatible with naive crossing and AB
- So far evidence only in singlet channel at large-N [Jain et al 15]

➜ **Our work:** [with Di Pietro, Di Salvo, Uetrecht]

- Full one-loop $2 \to 2$ S matrix through $1/k^2$ at fixed finite N
  [all channels, scalar/fermion matter, $\mathbf{SU}(N)_k$, $\mathbf{U}(N)_{k,k}$]
- Channel-dependent forward identity: one-loop unitarity restored
  [First verification for finite N]

Questions for the future: modified crossing at $1/k^3$, same origin of $\delta(\alpha)$?, ABJM theory?

Part II – Perturbative Fixed Points

Perturbative Fixed Point

» Perturbation theory: challenging $\beta = 0$

$$\beta = -\epsilon\,\alpha^2 + C\,\alpha^3 + \dots$$

$$\alpha^* = \epsilon/C + \mathcal{O}(\epsilon^2)$$

→ both LO and NLO
→ sign $C\epsilon > 0$
→ perturbative $|\epsilon| \ll |C|$

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» NOT $\epsilon = d - d_0$, no change of dimension

$$\epsilon \sim \frac{1}{N} \qquad \epsilon \sim \left(\frac{N_1}{N_2} - \text{const}\right)$$

» $|\epsilon| \neq 0$ but small and tunable → systematic expansion

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» NLO orders in perturbation theory !

» unicorns !

Perturbative Critical Phenomena

d = 4: Non-Abelian Gauge sector

[Coleman & Gross 1973]

» Asymptotic Freedom

[Gross & Wilczek, Politzer 1973]

» IR gauge fixed-points

[Caswell 1974, Banks & Zaks 1982]

» IR or UV gauge-Yukawa

[Litim, Sannino 2014]
[Bond, Litim 2016 & 2018]

d = 3

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d = 3

[1] » UV in pure scalar theories

[Heinz & Appelquist 1981]
[Pisarski 1982]
[Kvedaraitė & TS & Uetrecht 2025]

[2] » IR in scalar & fermionic

[Schröder, Stamou, TS, Uetrecht 2025]

Facts of Life in d=3

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{i}{2}\overline{\psi}\gamma^\mu\partial_\mu\psi + \mathcal{L}_{\text{gauge}}$$
$$-\frac{M}{2}\overline{\psi}\psi - \frac{y}{2}\phi\overline{\psi}\psi - \frac{Y}{4}\phi^2\overline{\psi}\psi$$
$$-\frac{m^2}{2}\phi^2 - \frac{h}{6}\phi^3 - \frac{\lambda}{4!}\phi^4 - \frac{\tau}{5!}\phi^5 - \frac{\eta}{6!}\phi^6$$

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» fermions in d=4: Dirac 4 complex → Weyl 2 complex

d=3: Dirac 2 complex → Majorana 2 real

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» fermions in d=4: Dirac 4 complex → Weyl 2 complex
d=3: Dirac 2 complex → Majorana 2 real

» RGEs only at even loops: 2-loop, 4-loop, 6-loop, ...

critical phenomena in d=3

Part II.1 – Scalar Theories

Tricritical O(N) model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 - \frac{\eta}{6!}\phi^6$$

Tricritical O(N) model

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& large N

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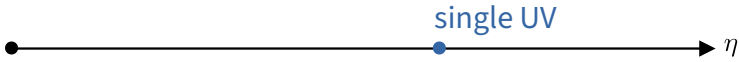

perturbation theory

[Applequist, Heinz (1981-82)]
[Pisarski (1982)]

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& large N

saddle point methods

[Bardeen, Moshe, Bander (1984)]
[David et al. (1984-85)]

FRG [Litim et al. (2017-20)]
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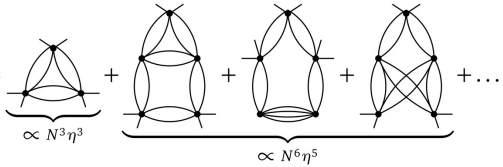
Only “N = ∞” (???)

line or single FP?
breaking of scale invariance?

Line of FPs?

$\beta_\eta =$  $+\dots$

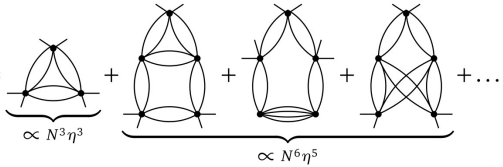
$$\hat{\eta} \propto N^{3/2}\eta$$

$$\epsilon = N^{-1/2}$$

$$\beta_{\hat{\eta}} = \frac{\epsilon}{5}\hat{\eta}^2 - \frac{\pi^2}{7200}\hat{\eta}^3 + \dots$$

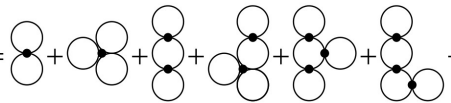
$$\hat{\eta}^* = \frac{1440}{\pi^2}\epsilon + \mathcal{O}(\epsilon^3)$$

Line of FPs?

$\beta_\eta =$ 

$$\hat{\eta} \propto N^{3/2}\eta \qquad \epsilon = N^{-1/2}$$

$$\beta_{\hat{\eta}} = \frac{\epsilon}{5}\hat{\eta}^2 - \frac{\pi^2}{7200}\hat{\eta}^3 + \dots \qquad \hat{\eta}^* = \frac{1440}{\pi^2}\epsilon + \mathcal{O}(\epsilon^3)$$

$V_{\text{eff}} =$ 

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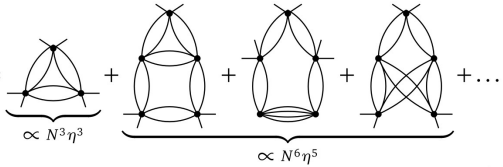
$\beta_\eta =$ 

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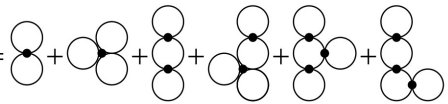
$V_{\text{eff}} =$ 

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no smooth limit for finite N

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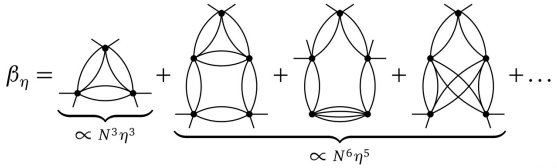

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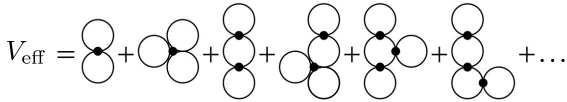

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FRG: large-N too early!

LPA truncation!

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Effective Potential

» large-N: $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{6!}\eta\,\phi^6$ $\qquad\qquad$ $\phi(x) = (\varphi + H(x),\, G_1(x), \ldots G_{N-1}(x))^\mathsf{T}$

integrate out H(x), G(x) – only G(x) at LO

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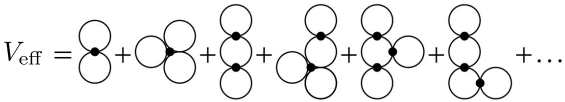

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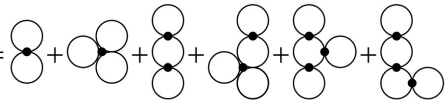
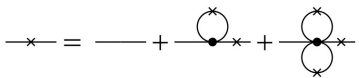
$V_{\text{eff}} =$  $+\ldots$

» resummed propagator



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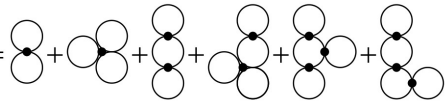
$V_{\text{eff}} =$  $+\ldots$

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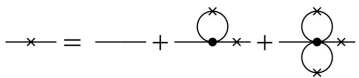

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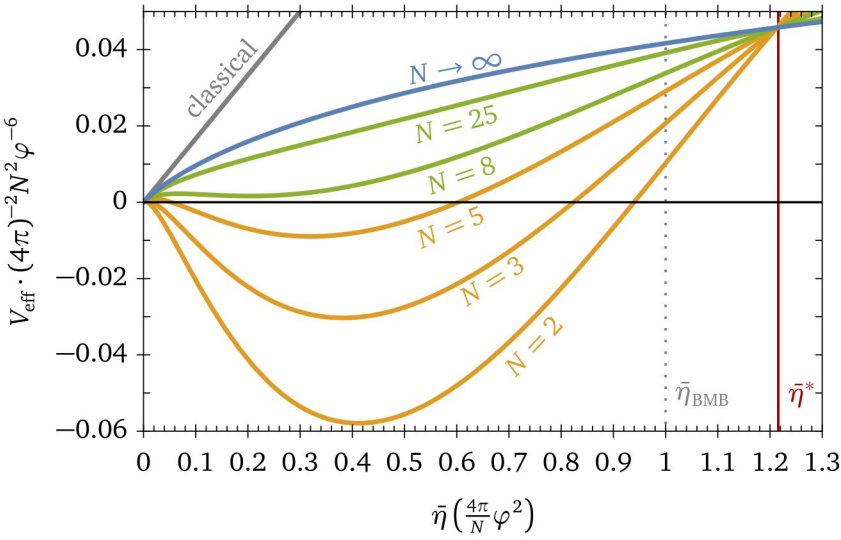
→ 2 PI effective potential: constant field φ and resummed mass M [Cornwall, Jackiw 1974]

$$V_{\text{2PI}}^{\text{LO}} = \frac{NM^3}{24\pi^2} + \frac{(4\pi)^2}{6N^2}\bar{\eta}\left(\varphi^2 - \frac{NM}{4\pi}\right)^3$$

» NLO: resummation of diagrams – stability for large-N

Effective Potential (NLO)



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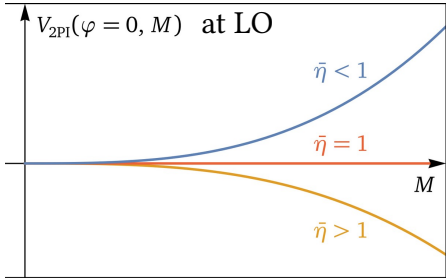

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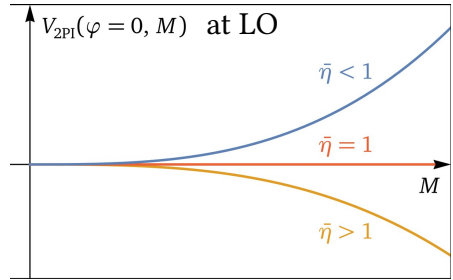
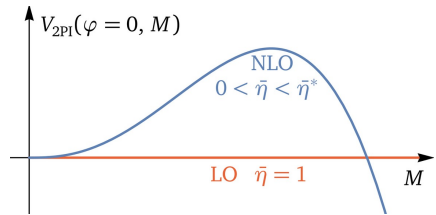


NLO:

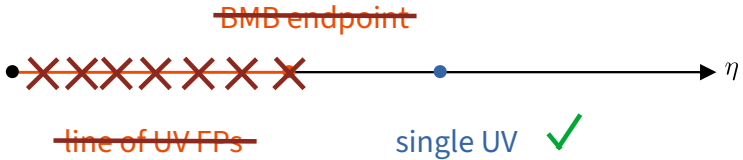
$$V_{\rm 2PI}^{\rm NLO}(0, M) = M^3\left[c(\bar{\eta}) - \beta_{\bar{\eta}}\log\frac{M}{\mu}\right]$$



→ quantum corrections break scale symmetry, not BMB

→ no sign of BMB, line of fixed-points

Summary Part II.1



line and BMB are classical artefacts

perturbation theory can compete with FRG here

Part II.2 – Scalar & Fermionic Theories

Tom Steudtner

Template Formalism

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_a \partial^\mu \phi_a + \frac{i}{2}\overline{\psi}_i \gamma^\mu \partial_\mu \psi_i - \frac{1}{4} Y^{abij} \phi_a \phi_b \overline{\psi}_i \psi_j - \frac{1}{6!} \eta^{abcdef} \phi_a \phi_b \phi_c \phi_d \phi_e \phi_f \,,$$

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field anomalous dimensions

$$\gamma_\phi^{ab} = \gamma_{\phi,1}\,\underbrace{Y^{acij}Y^{cbji}}_{\text{tensor structures}} + \ldots \quad = [1\text{ term}]_{\text{2-loop}} + [9\text{ terms}]_{\text{4-loop}}$$

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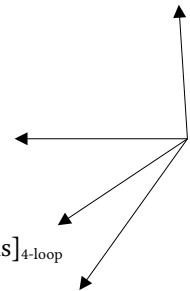
γ_ψ^{ij} = [1 term]$_{\text{2-loop}}$ + [8 terms]$_{\text{4-loop}}$



+ permutations

vertex corrections

$\hat{\beta}_Y^{abij}$ = [5 terms]$_{\text{2-loop}}$ + [132 terms]$_{\text{4-loop}}$

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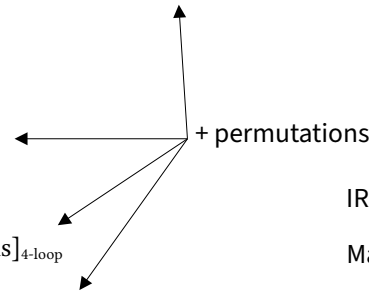

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IRA mass [Chetyrkin, Misiak, Munz, 1997]

MaRTIn [Brod, Hüdepohl, Stamou, TS 2401.04033]

+ FMFT [Pikelner, 1707.01710]

+ PSLQ & York Schröder

Crosscheck: SUSY

Generalised WZ theory $W(\Phi) = \frac{1}{4!}\lambda^{ABCD}\,\Phi_A\Phi_B\Phi_C\Phi_D$

$\mathcal{N}=1$: real superfields Φ → real scalar ϕ, fermion ψ

$$\mathcal{L}_{\mathcal{N}=1} = -\tfrac{1}{4}\lambda^{ABCD}\,\phi_A\phi_B(\psi_C\psi_D) - \tfrac{1}{36}\lambda^{ABCG}\lambda^{GDEF}\,\phi_A\phi_B\phi_C\phi_D\phi_E\phi_F + \dots$$

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$\mathcal{N}=2$: complex superfields Φ → complex scalar ϕ, fermion ψ

$$\mathcal{L}_{\mathcal{N}=2} = \left[-\tfrac{1}{4}\lambda^{ABCD}\,\phi_A\phi_B(\psi_C\psi_D) + \text{h.c.}\right] - \tfrac{1}{36}\lambda^{ABCG}(\lambda^{GDEF})^*\,\phi_A\phi_B\phi_C\phi_D^*\phi_E^*\phi_F^*$$

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SUSY across Dimensions

$d = 3$ at 4-loop

[Schröder, Stamou, TS, Uetrecht, 2512.02123]

$\mathcal{N} = 2$ ✅ ⟶ $\mathcal{N} = 1$ ✅

$\mathcal{N} = 1$ ✅ ⟶ $\mathcal{N} = 1/2$ ❌

$\mathrm{Tr}\left[\gamma^{\mathrm{odd}}\right] \neq 0$ $\quad$ $\mathrm{Tr}\left[\gamma^{\mathrm{odd}}\right] = 0$

[Zerf, Mihaila, Marquard, Herbut, Scherer, 1709.05057]

$d = 4$ at 4-loop [TS 2507.18710]

Perturbative Fixed Points ?

$$\beta_\eta = \#\,\eta^2 + \#\,Y^2\eta - \#\,Y^4$$

$$\beta_Y = \boxed{\#\,Y^3} + \#\,Y^5 + \#\,\eta^2 Y - \#\,\eta Y^3$$

small? negative?

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» naive: all positive, no cancellations → no fixed point?

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» naive: all positive, no cancellations → no fixed point?

» remove tensor structure: $\mathrm{Tr}\left[Y^{ab}\,\beta_Y^{ab}\right]$

→ almost always positive

→ subtle opportunity when Y^{ab} furnish clifford algebra

Example

$$\mathcal{L} = \partial_\mu \phi_i^* \partial^\mu \phi_i + i\overline{\psi}_{ia} \not{\partial} \psi_{ia} - Y\,(\phi_i^* \sigma_{ij}^A \phi_j)(\overline{\psi}_{ak} \sigma_{kl}^A \psi_{al}) - \tfrac{1}{36}\eta(\phi_i^* \phi_i)^3$$

$SO(3) \times U(N)$: $\quad \mathbf{2} \times \mathbf{1} \qquad \mathbf{2} \times \mathbf{N}$

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$SO(3) \times U(N)$: $\quad \mathbf{2} \times \mathbf{1} \qquad \mathbf{2} \times \mathbf{N}$

» large-N: $\quad \epsilon = \dfrac{1}{N} \qquad \alpha_Y = \dfrac{NY^2}{(4\pi)^2} \qquad \alpha_\eta = \dfrac{\eta}{(4\pi)^2}$

$$\beta_{\alpha_Y} = -4\epsilon\,\alpha_Y^2 + \mathcal{O}(\alpha^3)$$

spinor rep SO(3)

Example

$$\mathcal{L} = \partial_\mu \phi_i^* \partial^\mu \phi_i + i\overline{\psi}_{ia} \not{\partial} \psi_{ia} - Y\,(\phi_i^* \sigma_{ij}^A \phi_j)(\overline{\psi}_{ak} \sigma_{kl}^A \psi_{al}) - \tfrac{1}{36}\eta(\phi_i^* \phi_i)^3$$

$$SO(3) \times U(N): \qquad \mathbf{2} \times \mathbf{1} \qquad \mathbf{2} \times \mathbf{N}$$

» large-N: $\epsilon = \dfrac{1}{N}$ $\alpha_Y = \dfrac{NY^2}{(4\pi)^2}$ $\alpha_\eta = \dfrac{\eta}{(4\pi)^2}$

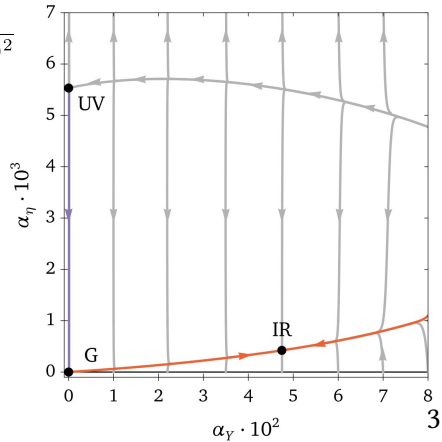
$$\beta_{\alpha_Y} = -4\epsilon\,\alpha_Y^2 + \mathcal{O}(\alpha^3)$$

spinor rep SO(3)

» perturbative control

$$\alpha_Y^* = \frac{\epsilon}{6+\pi^2} + \mathcal{O}(\epsilon^2)$$

$$\alpha_\eta^* = \frac{12(3+\pi^2)}{(6+\pi^2)^2}\epsilon^2 + \mathcal{O}(\epsilon^3)$$



Conclusion Part II

» towards overview of perturbative critical phenomena in d=3

» purely scalar theory: NLO effective potential, BMB argument weakened

» scalar-fermionic theory: 4L templates
new kind of IR fixed points

» up next: gauge sector

Conclusions

I. Crossing and unitarity in Chern–Simons matter theory

- What changes in an anyonic S matrix?
- Naive crossing violates unitarity $\rightarrow$ modify naive S matrix
- Test conjecture with full S-matrix computation at $1/k^2$ and finite N

II. Fixed-point structure of 3d systems

- general 4-loop RGEs
- scalar UV fixed point
- new scalar–fermionic IR fixed point.