Generative Sampling

with physics-informed kernels

Friederike Ihssen, Kenji Fukushima, **Renzo Kapust**, Jan M. Pawlowski

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$$\langle \mathcal{O} \rangle = \int D\varphi \, \mathcal{O}(\varphi) \, p(\varphi)$$

Physics prescribes action S for a continuous field $\varphi(x)$



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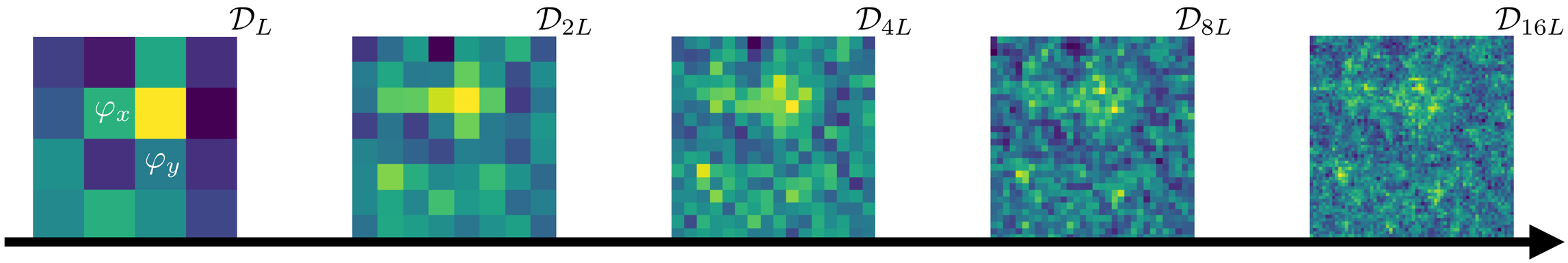

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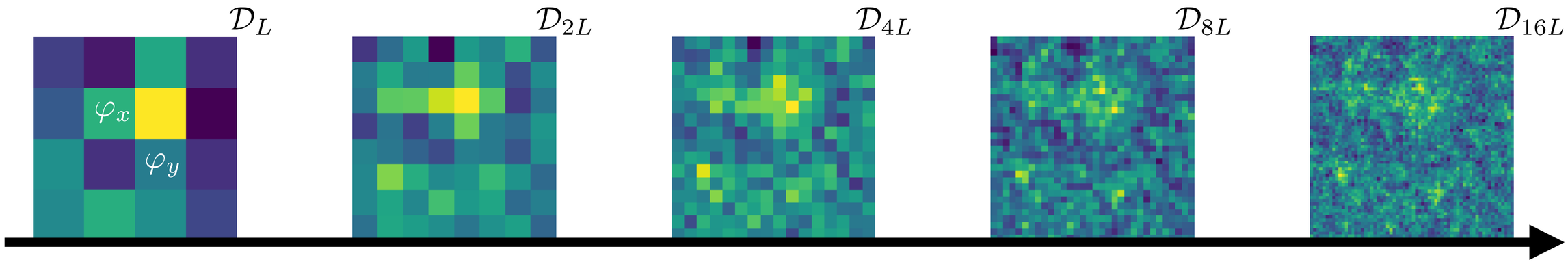

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Markov chain Monte Carlo

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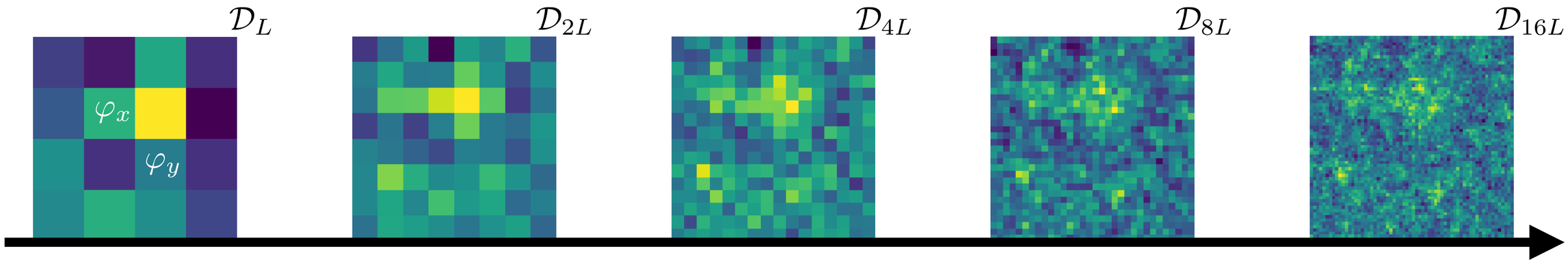

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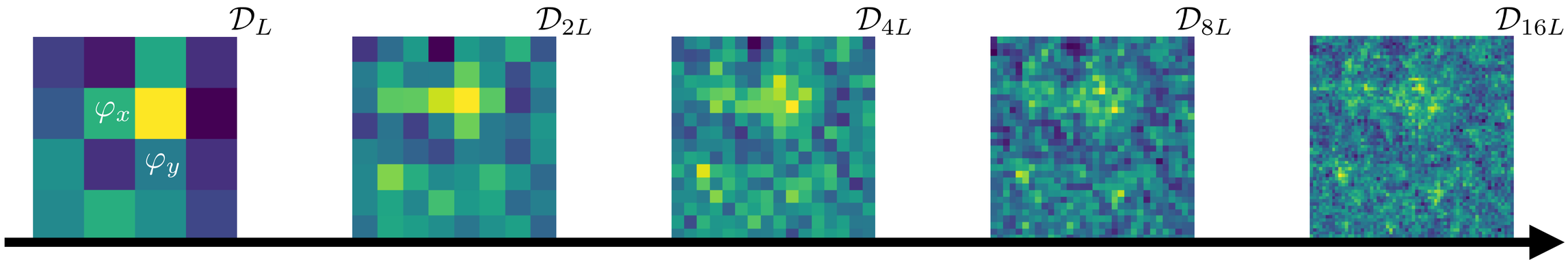

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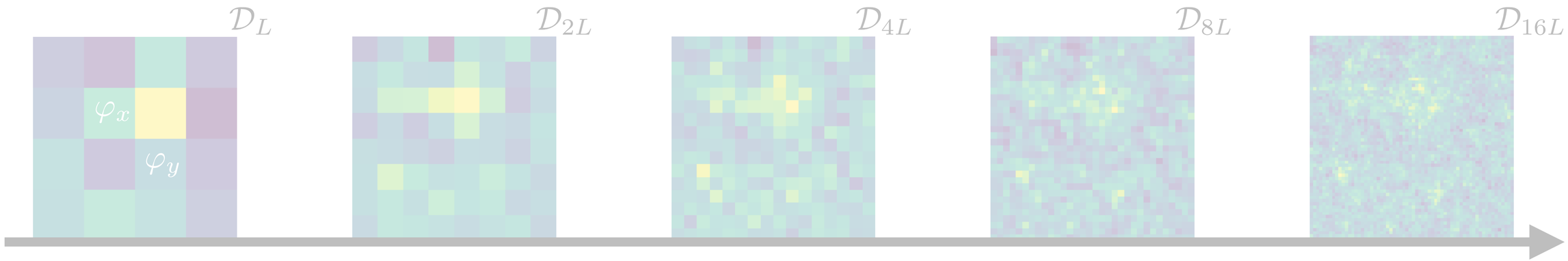

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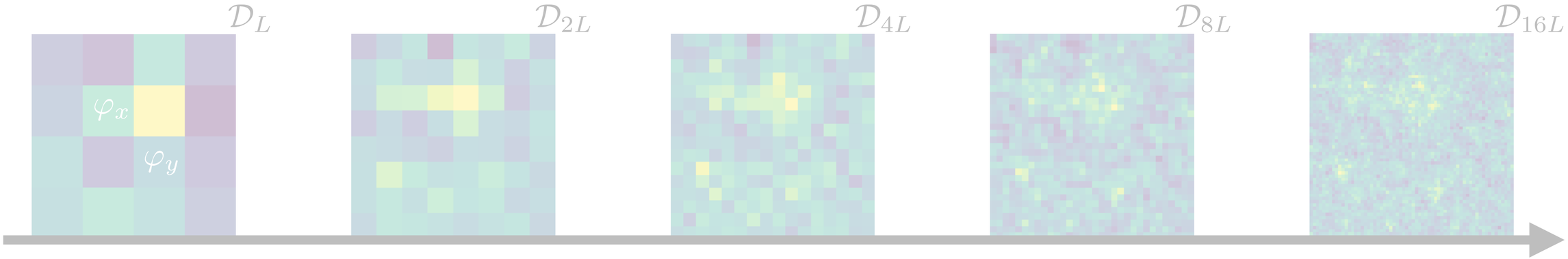

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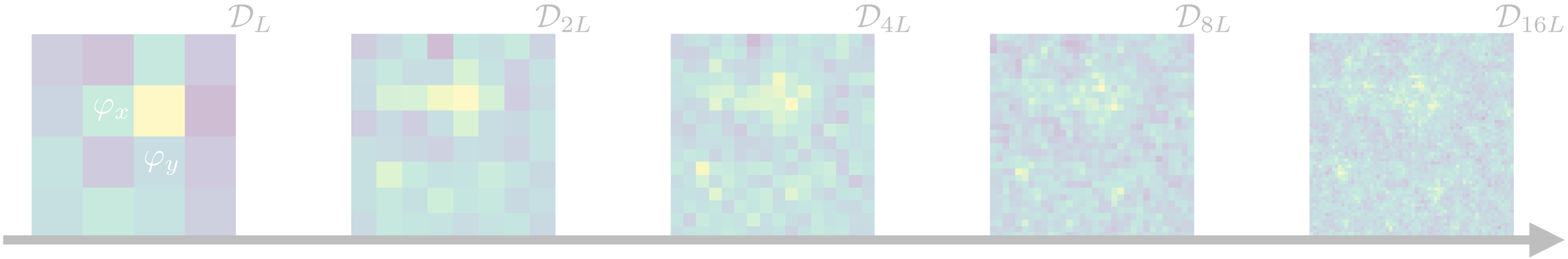
$\uparrow$ $\downarrow$	φ	U_μ	$\bar{\psi}$ ψ	...
Discrete	Scalar	Gauge	Fermions	



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$p : X \to \mathbb{R}^+$

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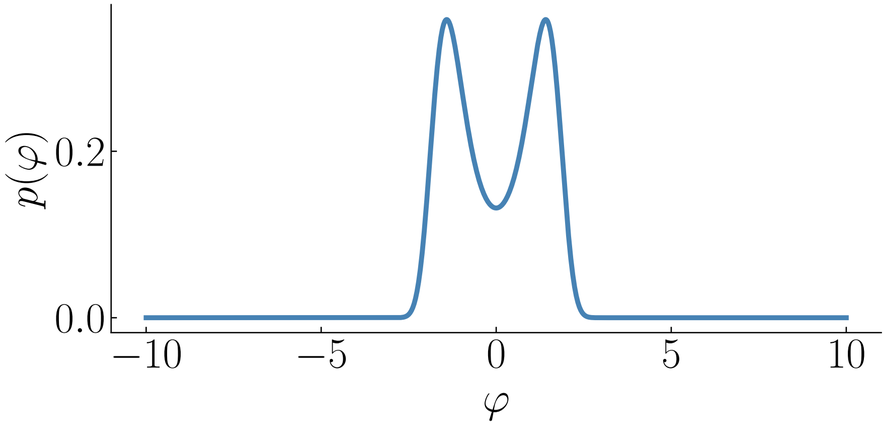

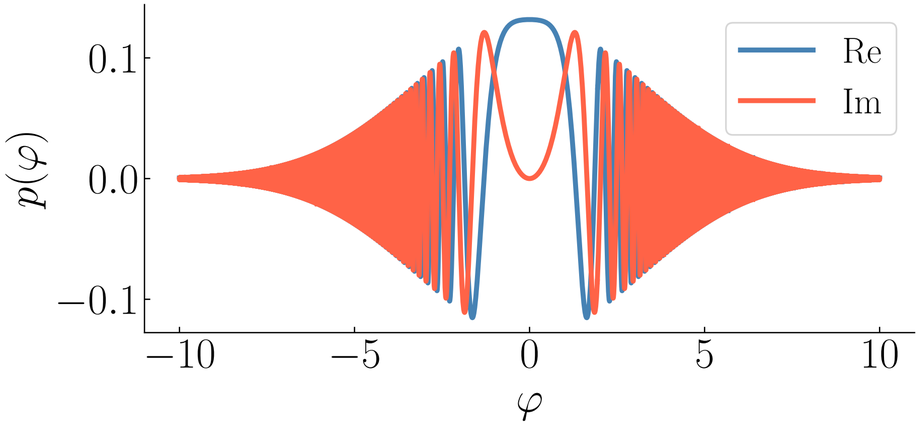

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Wegner, J.Phys.C 7 (1974)
Ihssen, **RK**, Pawlowski, arXiv: 2510.26678
Ihssen, **RK**, Pawlowski: arXiv:2603.03159

Physics-informed kernels (PIKs)

$$p_t(\phi) = \frac{1}{\mathcal{N}_t} e^{-\hat{S}_t(\phi)}$$

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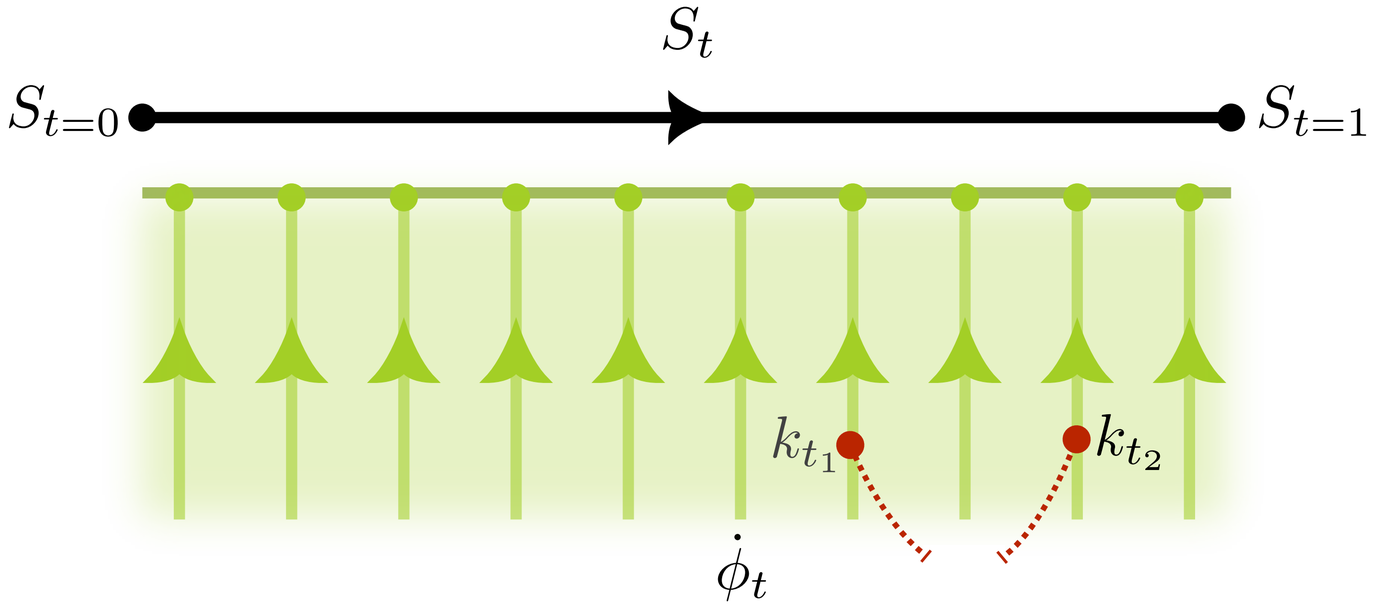

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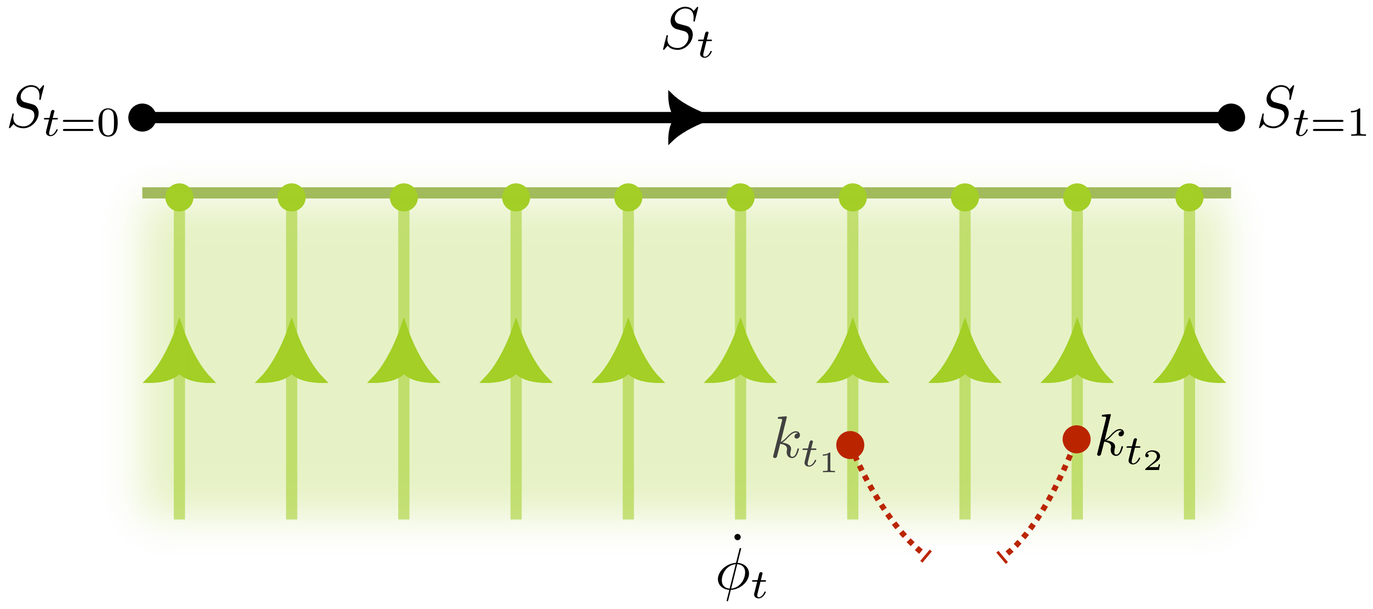

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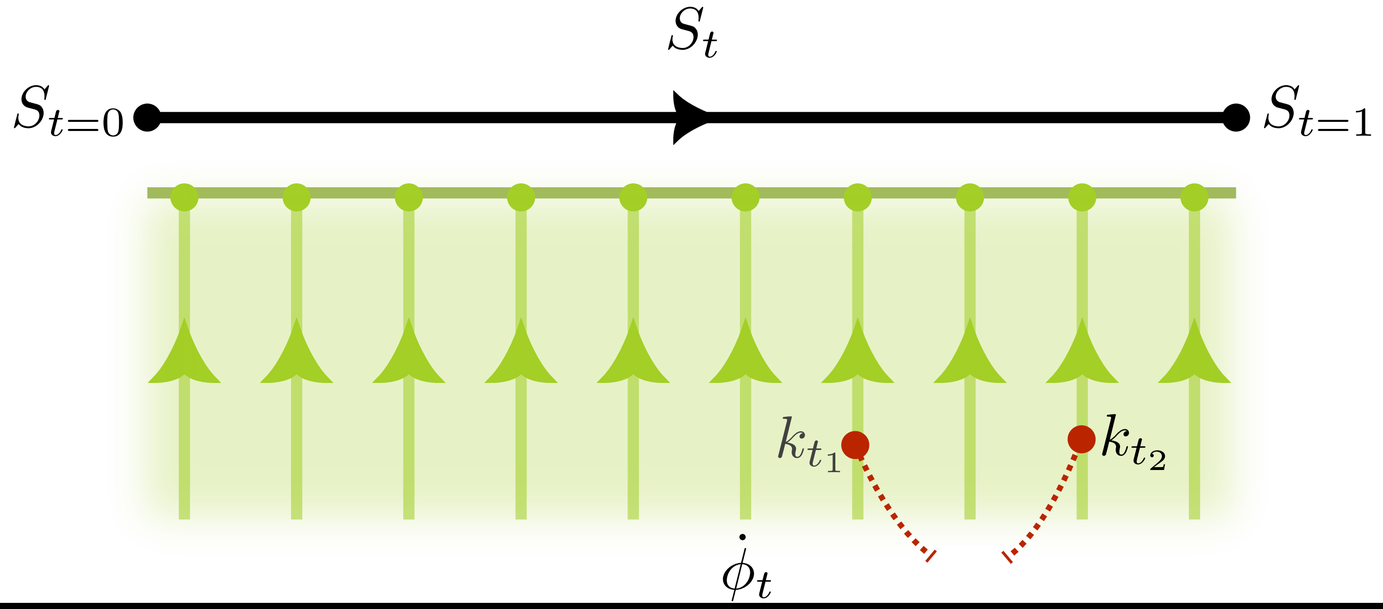

Analytically set action path, e.g. $\hat{S}_t(\phi) = \frac{1}{2}\phi^T\Sigma^{-1}\phi + t\frac{\lambda}{4}\sum_{i\in\mathcal{D}}\phi_i^4$

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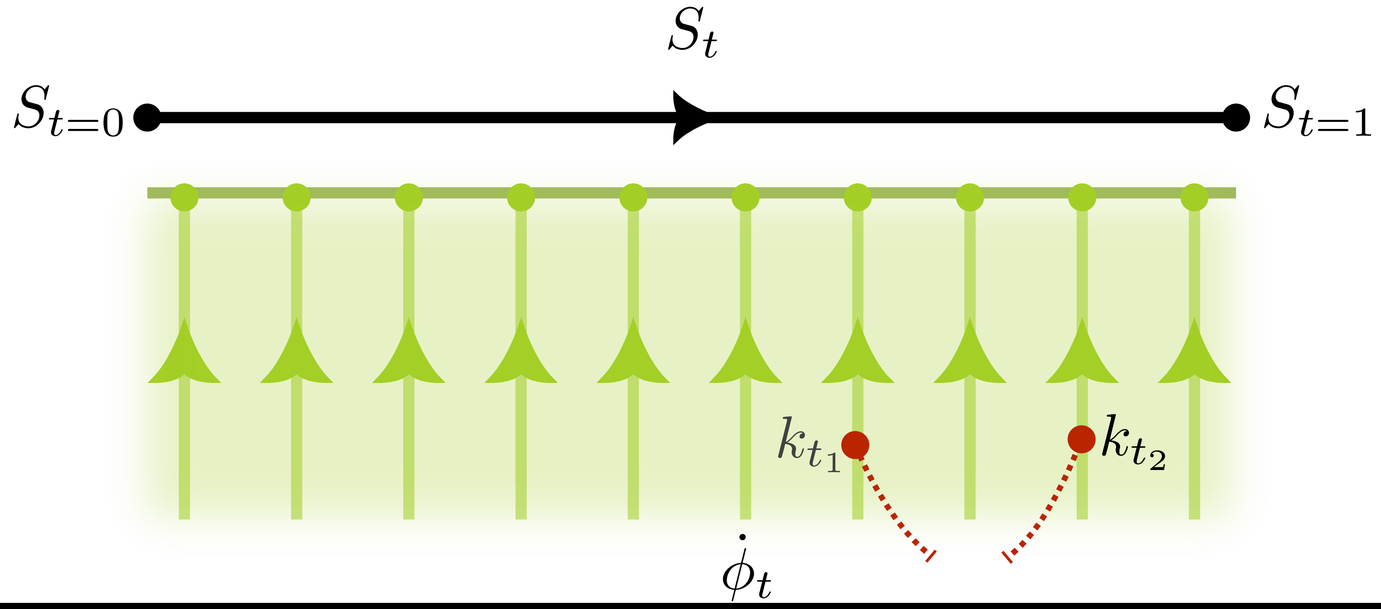

field-independent $\dfrac{d\log\mathcal{N}_t}{dt} = \dfrac{d\hat{S}_t(\phi)}{dt} - \left[\dfrac{\partial}{\partial\phi_i} - \dfrac{\partial\hat{S}_t(\phi)}{\partial\phi_i}\right]\dot{\phi}_{t,i}(\phi)$ field-dependent $\dot{\phi}_{t,i} \neq \dot{\phi}_{t,i}$

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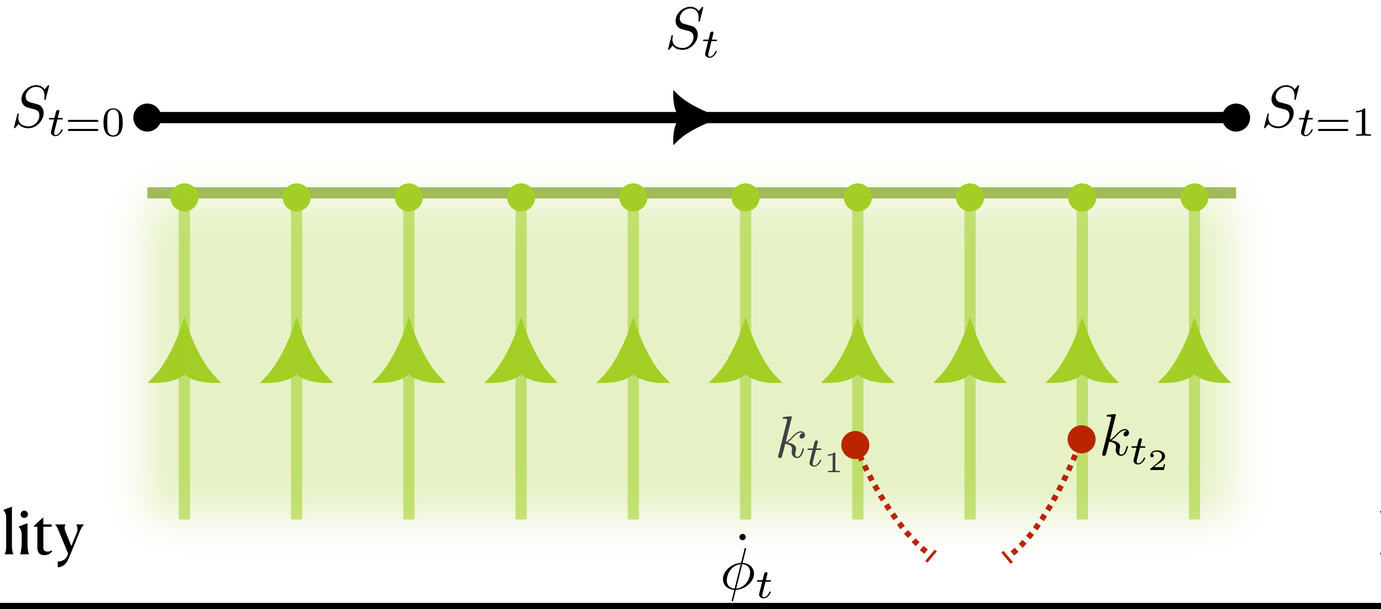

(Iterative) Correctability

Informed Expansions

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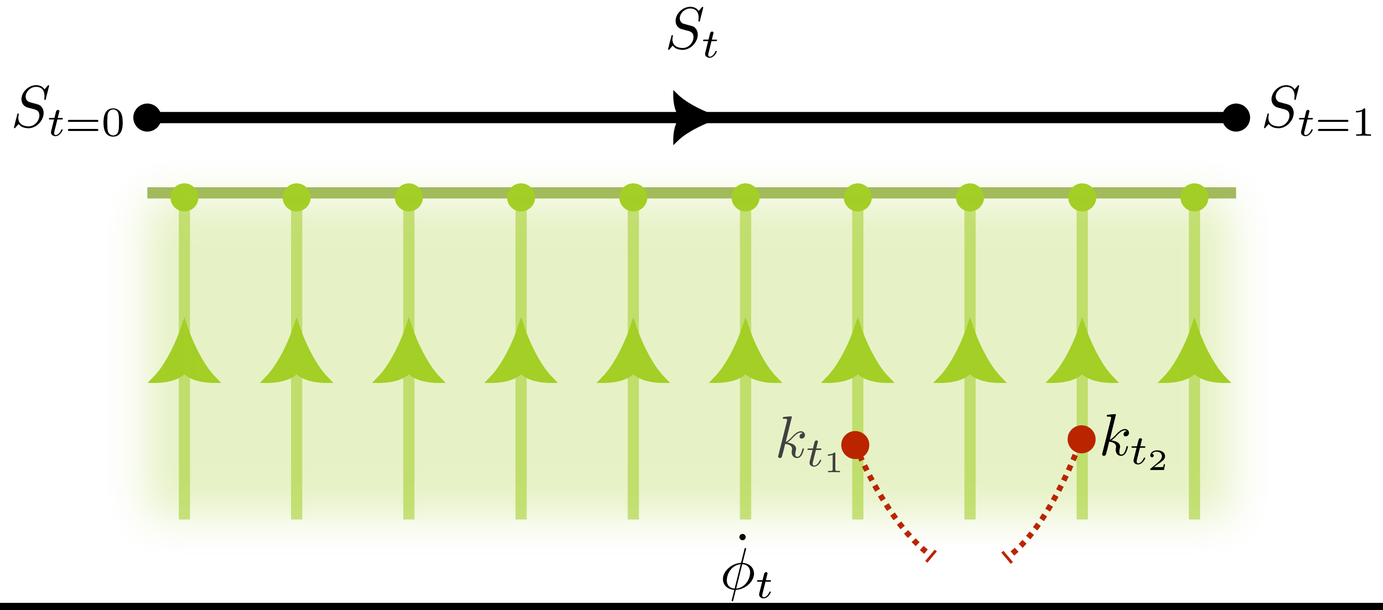

Dissects measure transport into independent sub-problems

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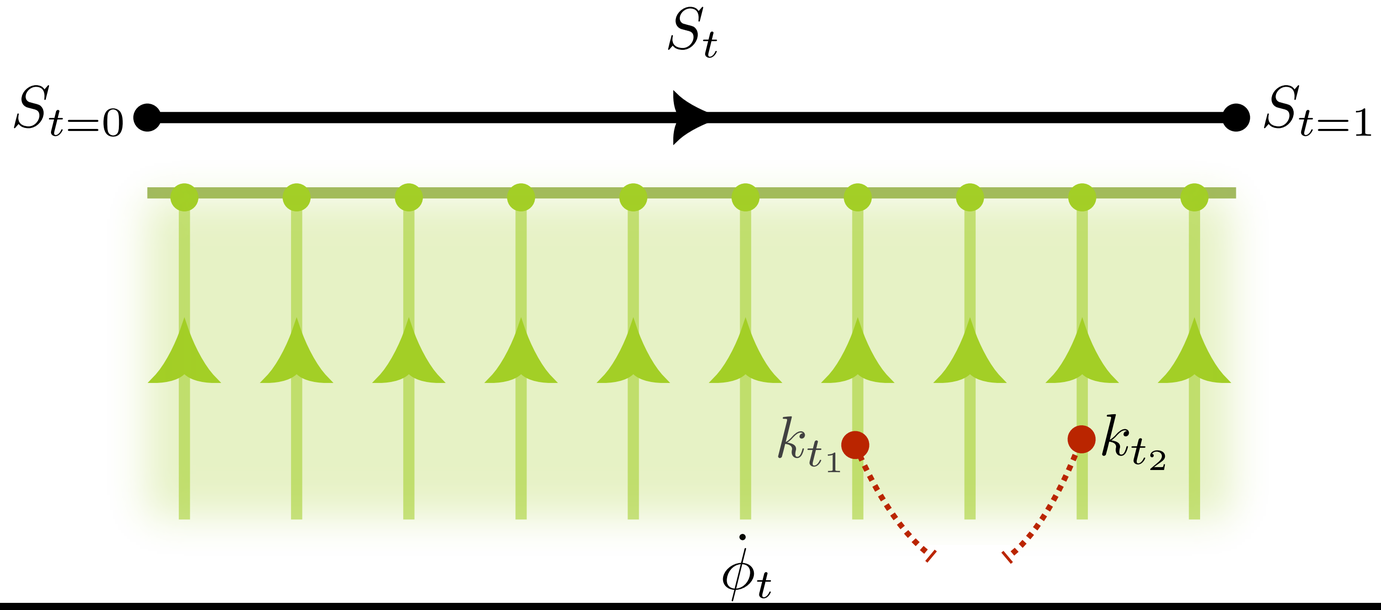

Each sub-problem does not require the notion of a statistical divergence

We want to transform easy theories into hard ones. → complex

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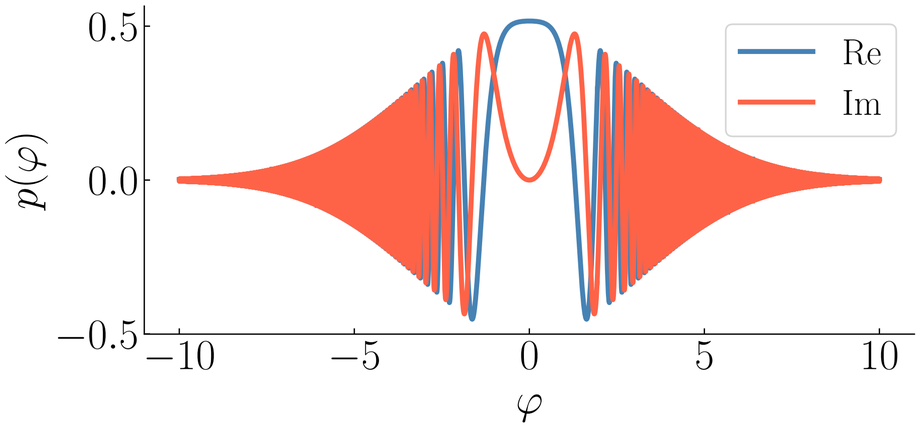

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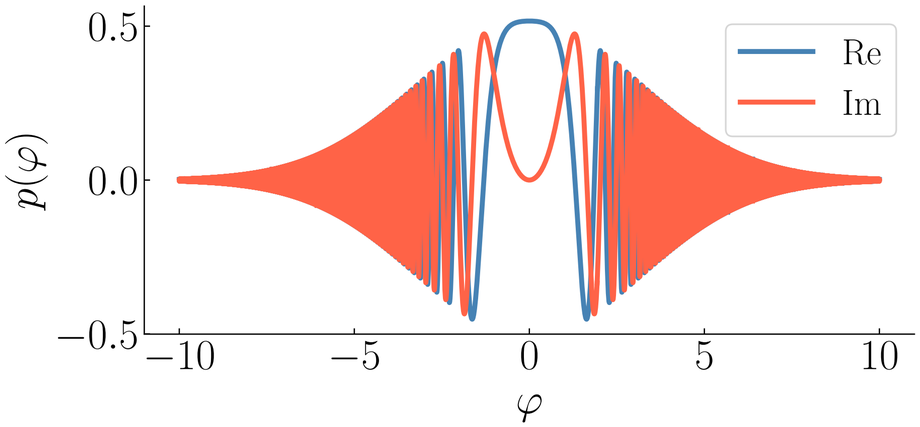

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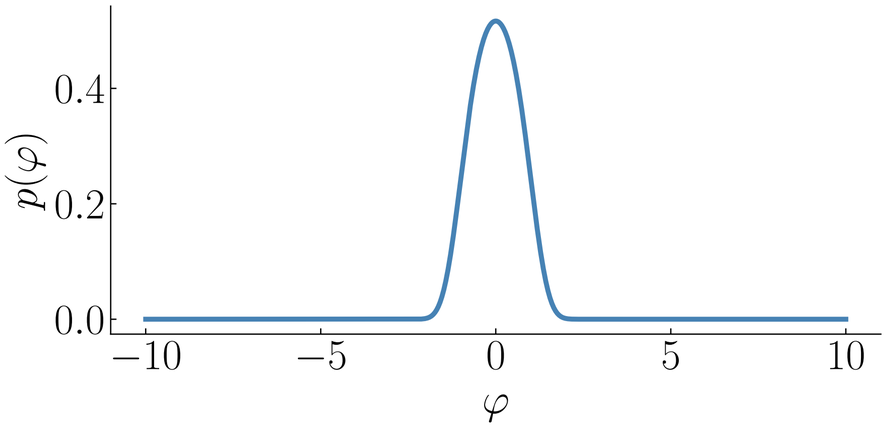

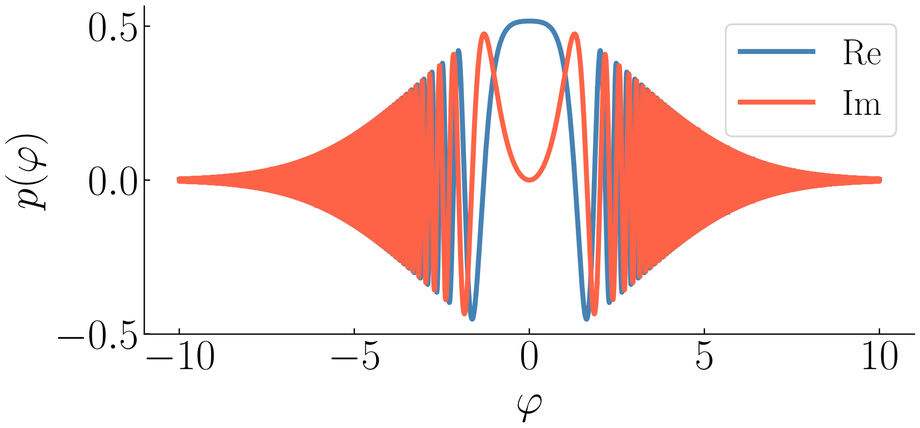

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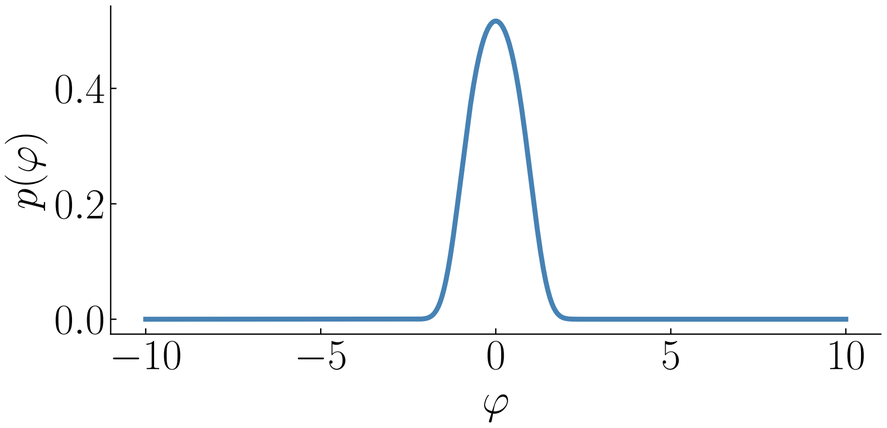
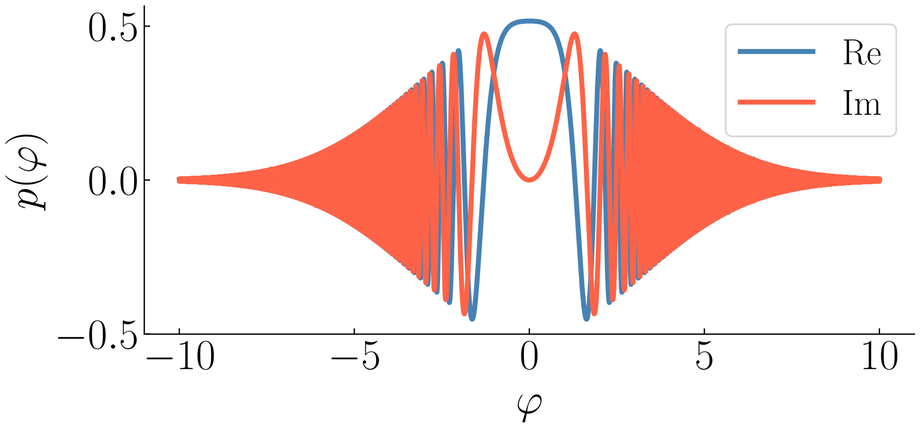


Friederike's talk



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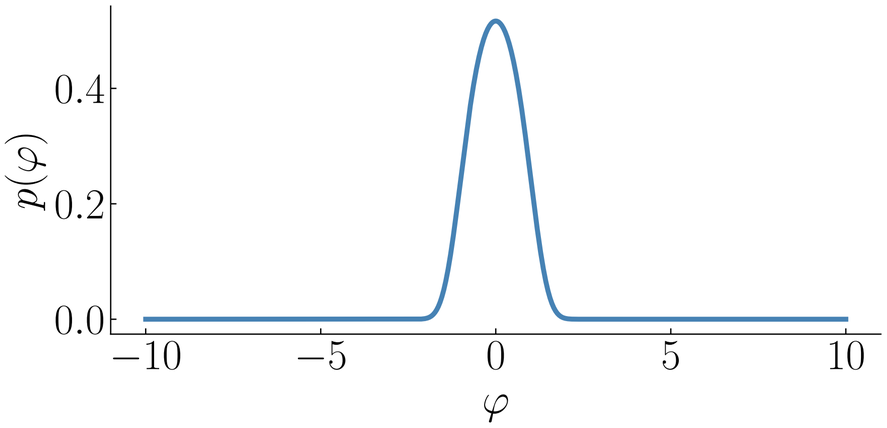

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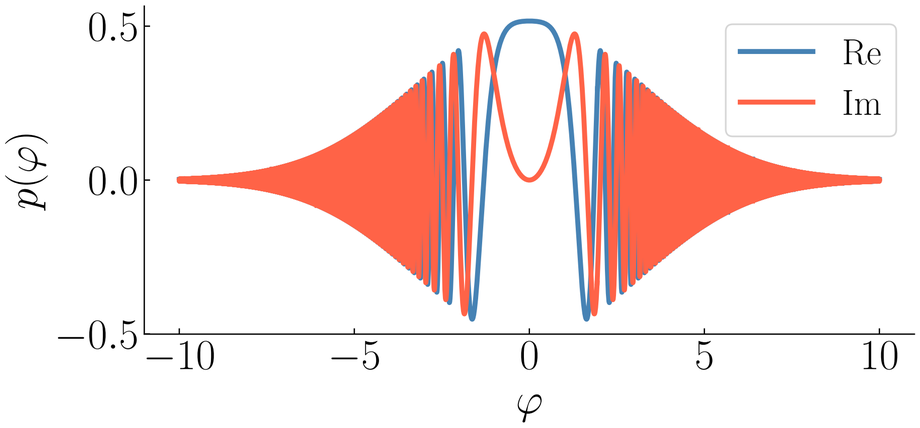

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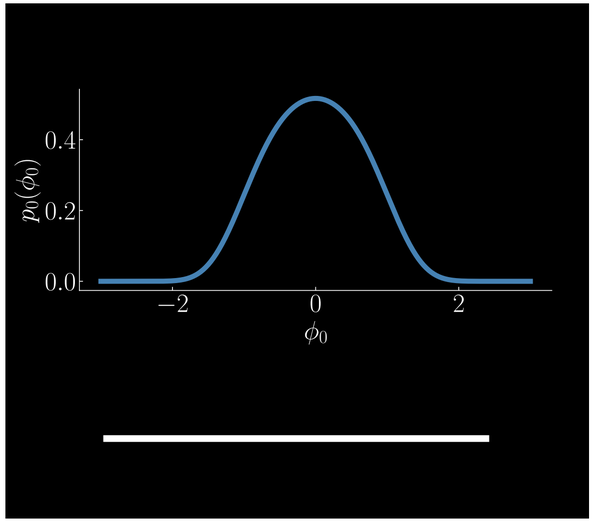
e^{-S} $S_0(\phi_0)$ $\hat{S}_t(\phi) = \frac{1}{2}m_t^2\phi^2 + \frac{1}{4}\lambda_t\phi^4$ $S_1(\phi_1)$ e^{iS}

Each sub-problem does not require the notion of a statistical divergence

Ihssen, **RK**, Pawlowski: arXiv:2603.03159

PIKfolds: Taming a sign problem

$$\frac{d\phi_t}{dt} = \dot{\phi}_t(\phi) \qquad \frac{dS_t(\phi)}{dt} = \left[\frac{\partial}{\partial \phi_i} - \frac{\partial S_t(\phi)}{\partial \phi_i} \right] \dot{\phi}_{t,i}(\phi) \qquad \Rightarrow \quad (p_t, \mathcal{M}_t)$$



$(p_0, \mathcal{M}_0)$

Typically

$p_0(\phi_0) \in \mathbb{R}$

$\mathcal{M}_0 = \mathbb{R}^{\mathcal{D}}$

Ihssen, **RK**, Pawlowski: arXiv:2603.03159

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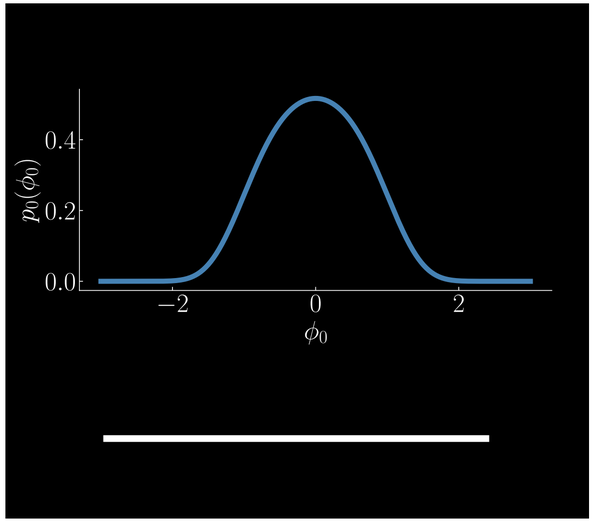
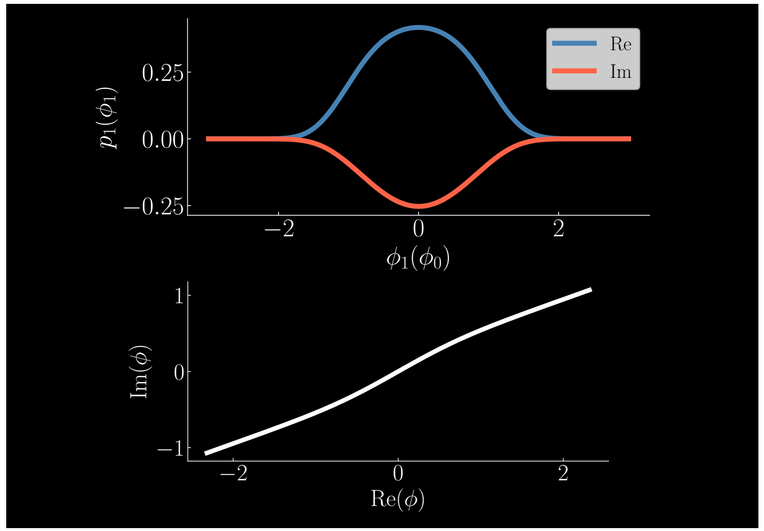
$$\frac{d\phi_t}{dt} = \dot{\phi}_t(\phi) \qquad \frac{dS_t(\phi)}{dt} = \left[\frac{\partial}{\partial \phi_i} - \frac{\partial S_t(\phi)}{\partial \phi_i} \right] \dot{\phi}_{t,i}(\phi) \qquad \Rightarrow \quad (p_t, \mathcal{M}_t)$$



$(p_0, \mathcal{M}_0)$



$(p_1, \mathcal{M}_1)$

Typically

$$p_0(\phi_0) \in \mathbb{R}$$
$$\mathcal{M}_0 = \mathbb{R}^{\mathcal{D}}$$

$$p_1 = p$$
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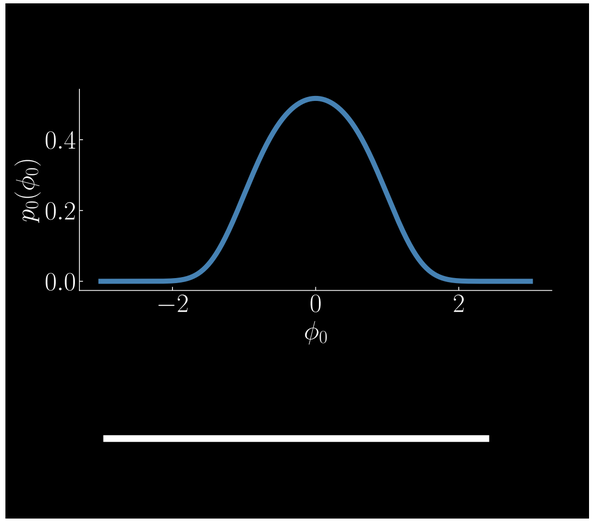

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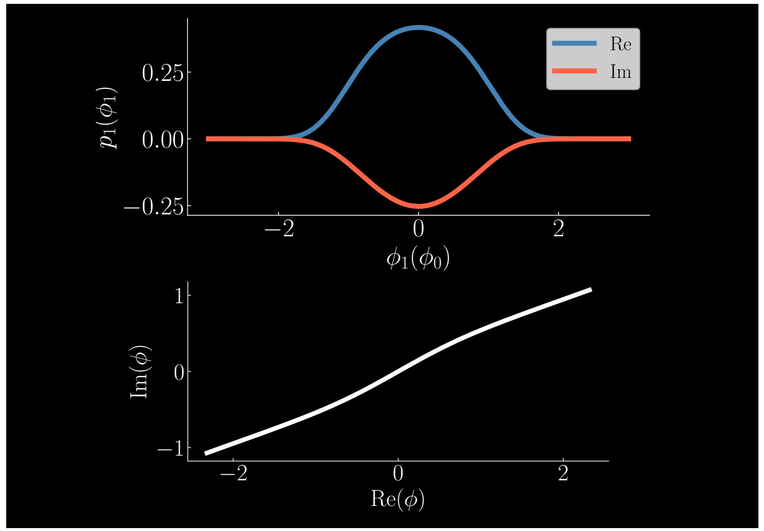

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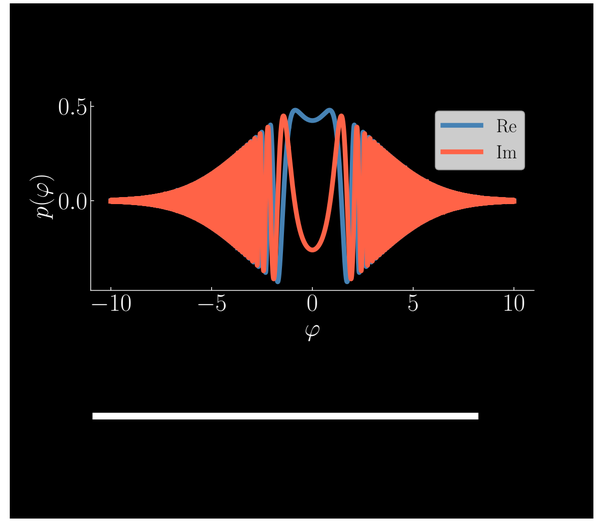

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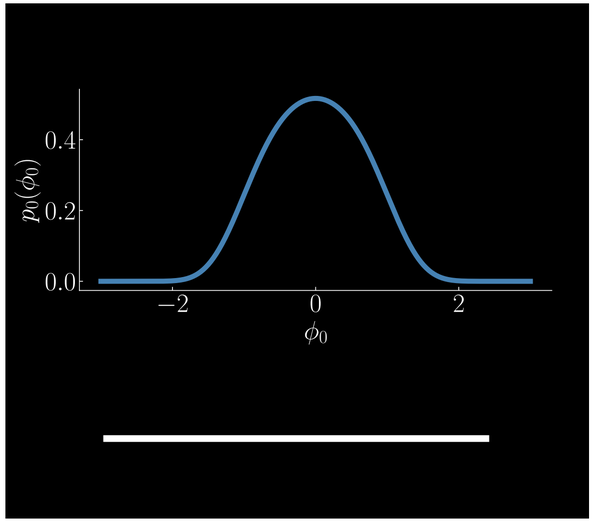

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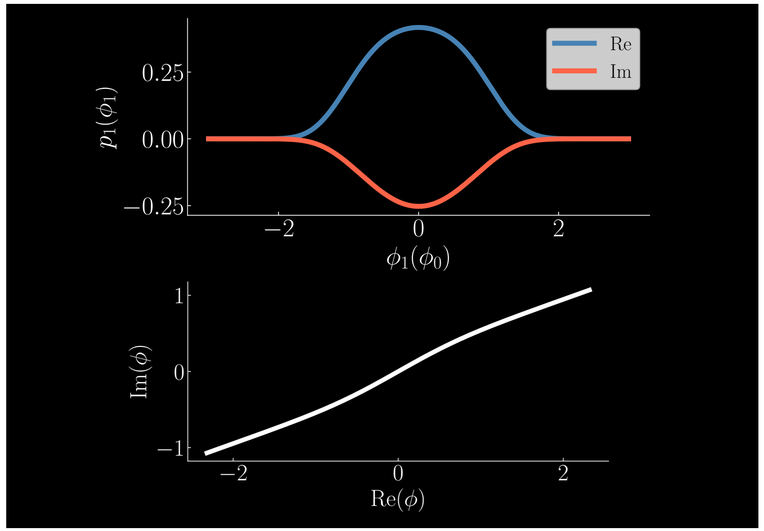

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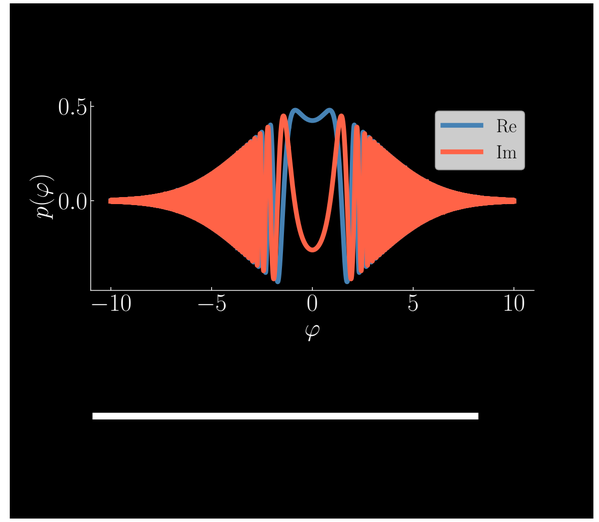

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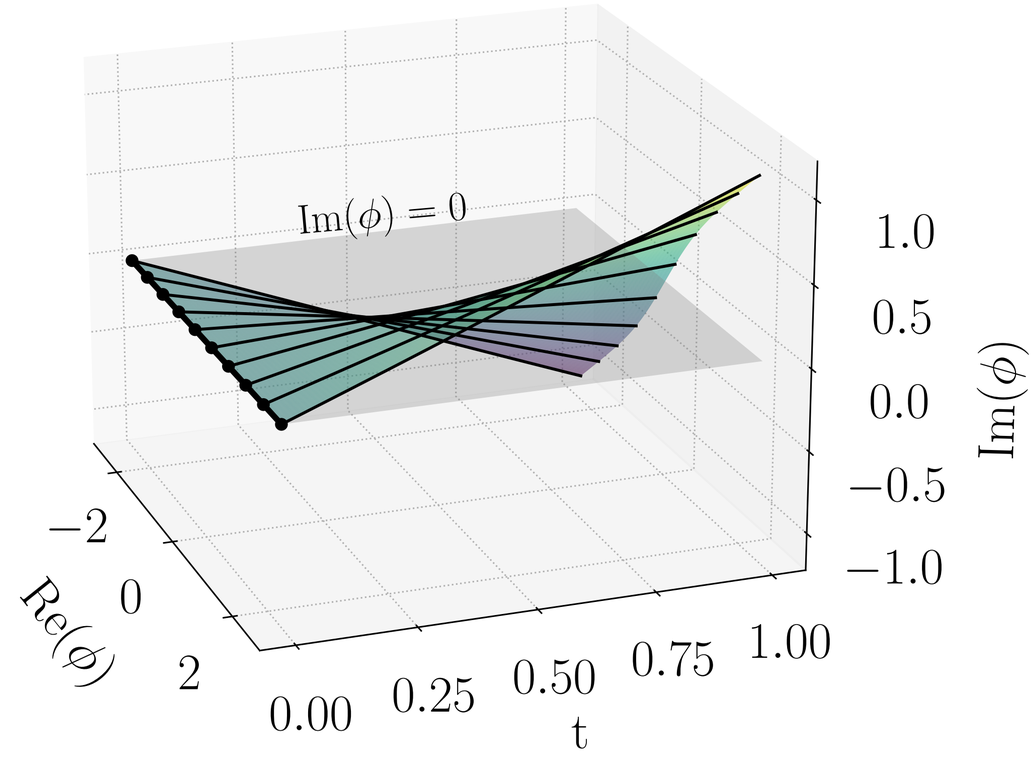

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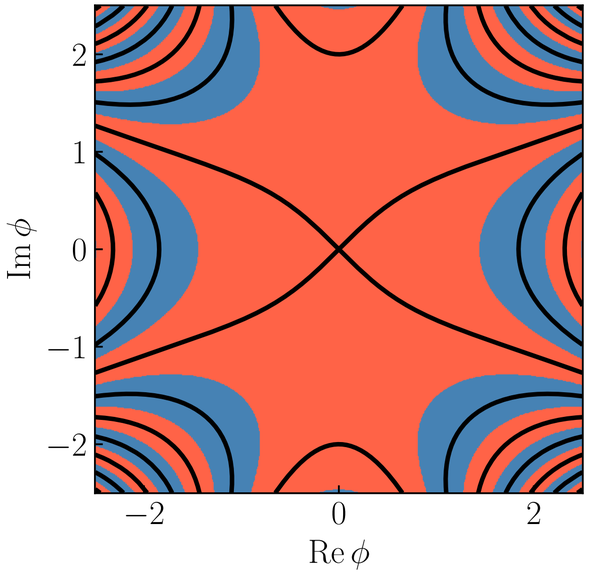

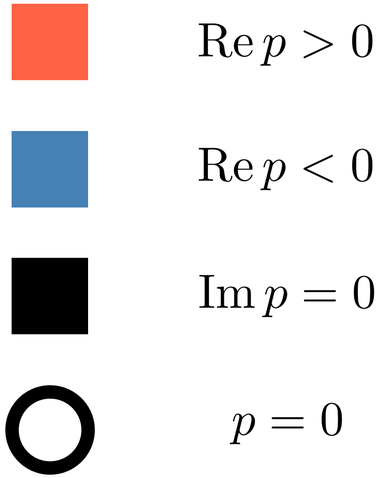

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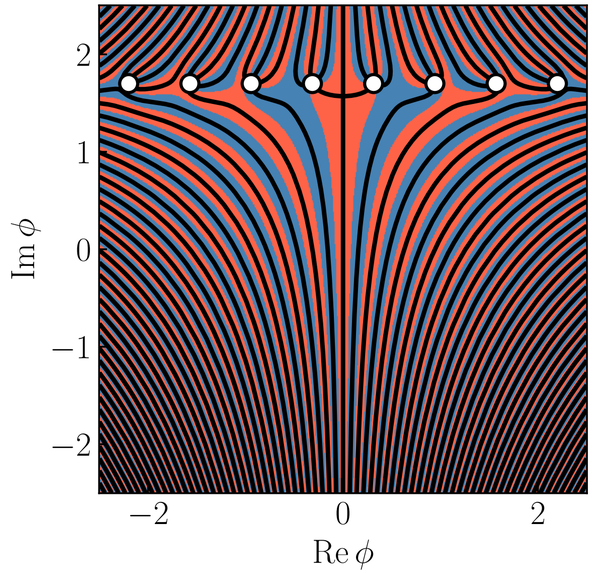

What do zeros change for the measure transport?

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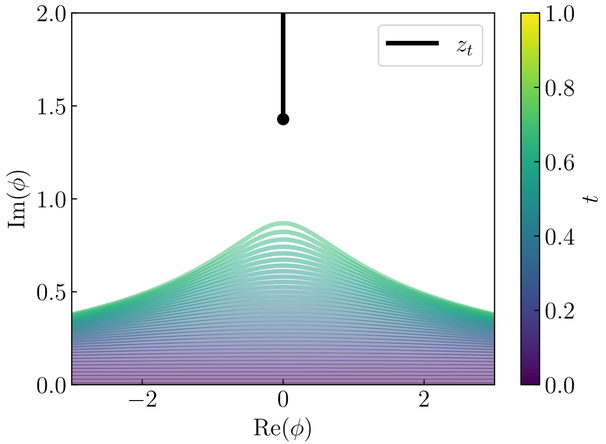

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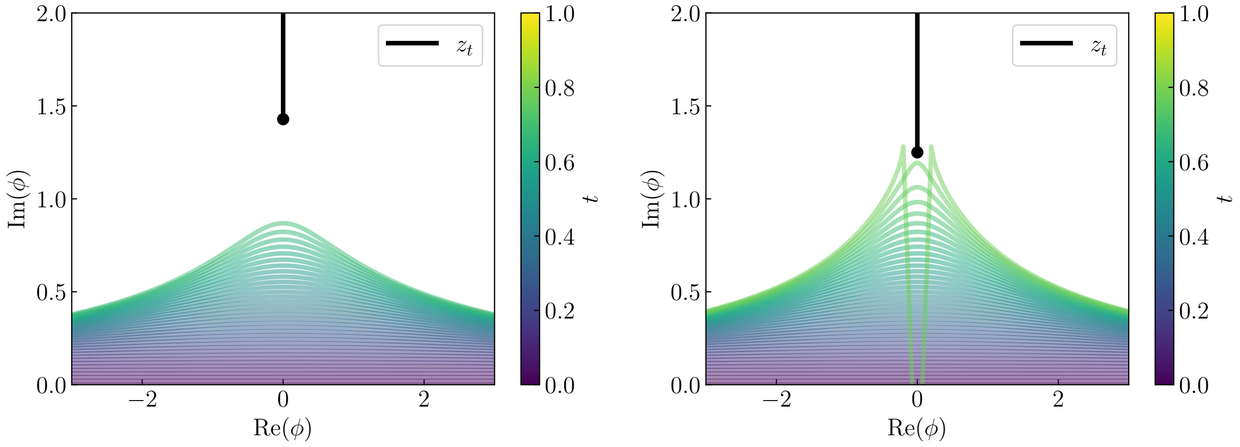

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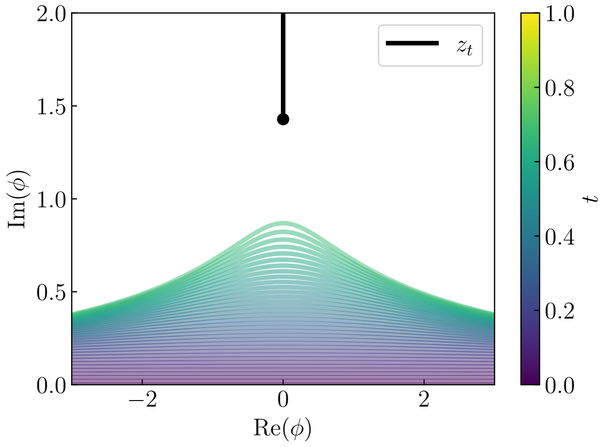

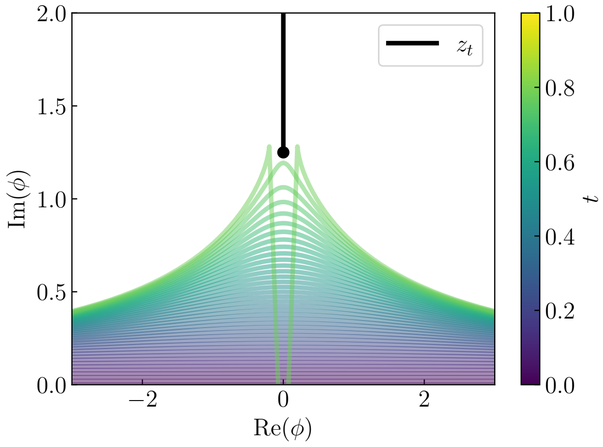

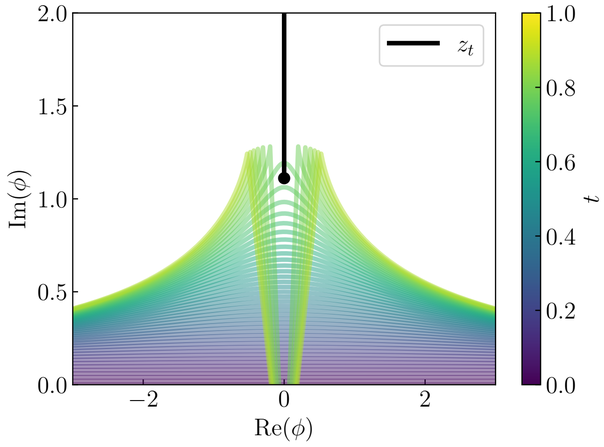

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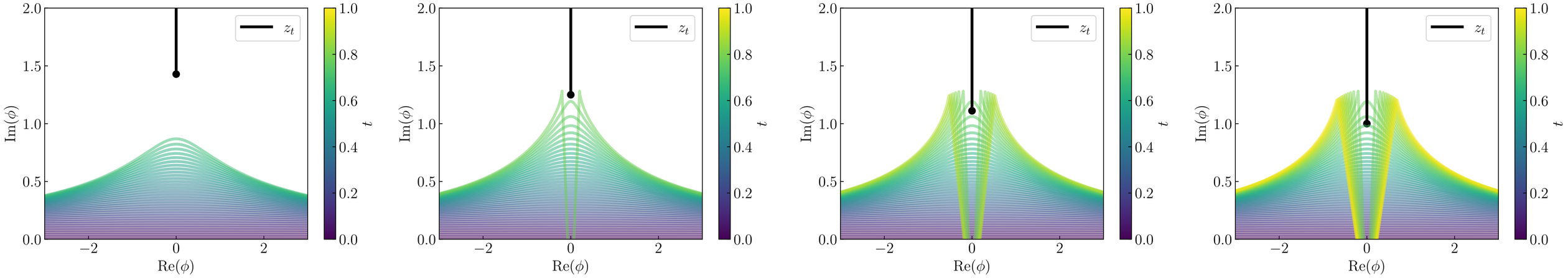

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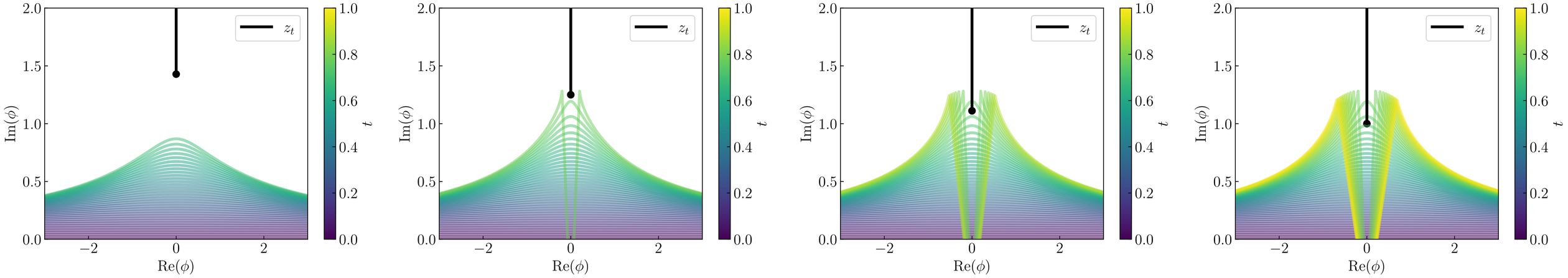

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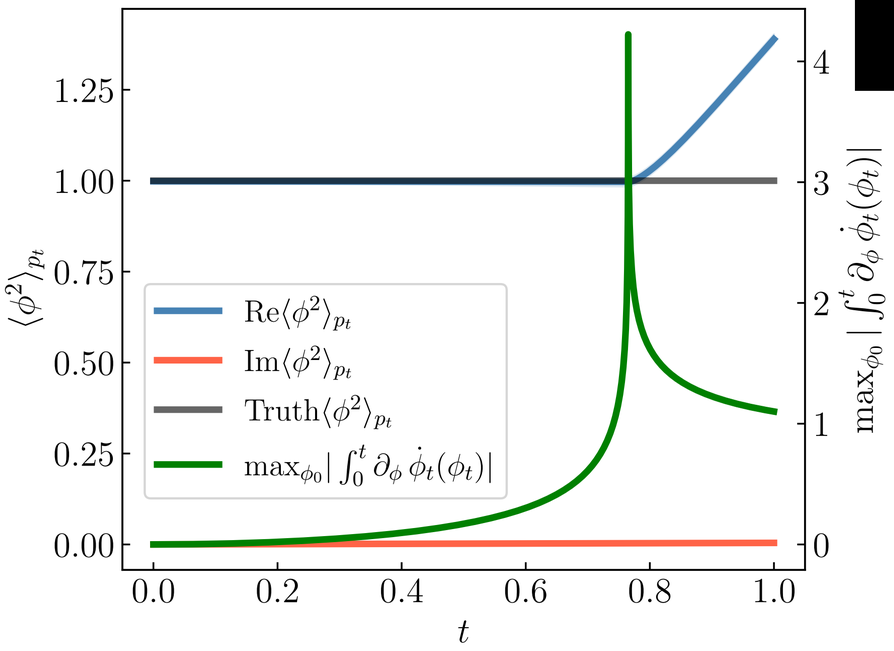

see also eg.
Lawrence, Yamauchi, Phys.Rev.D 107 (2023) 11, 114505

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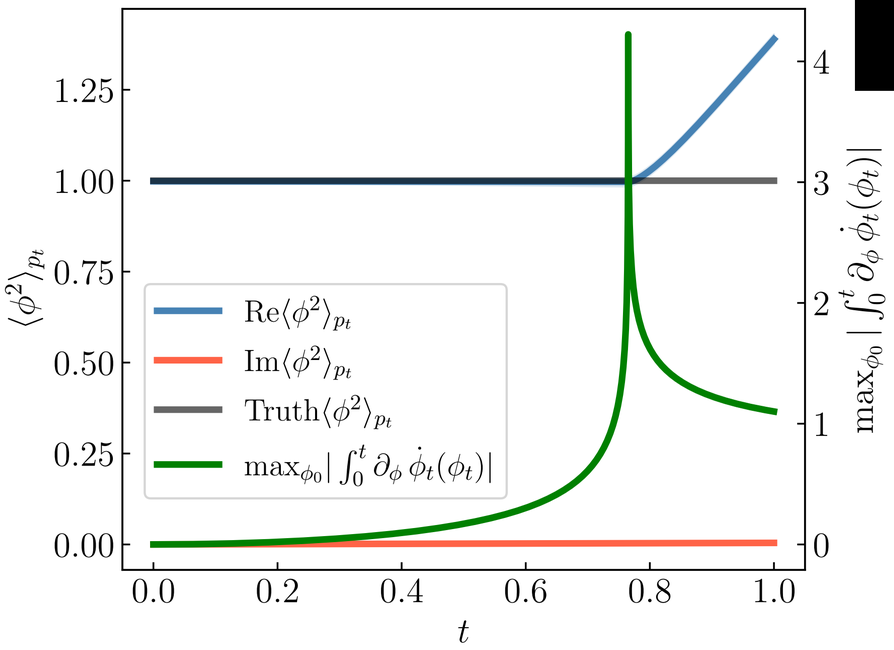
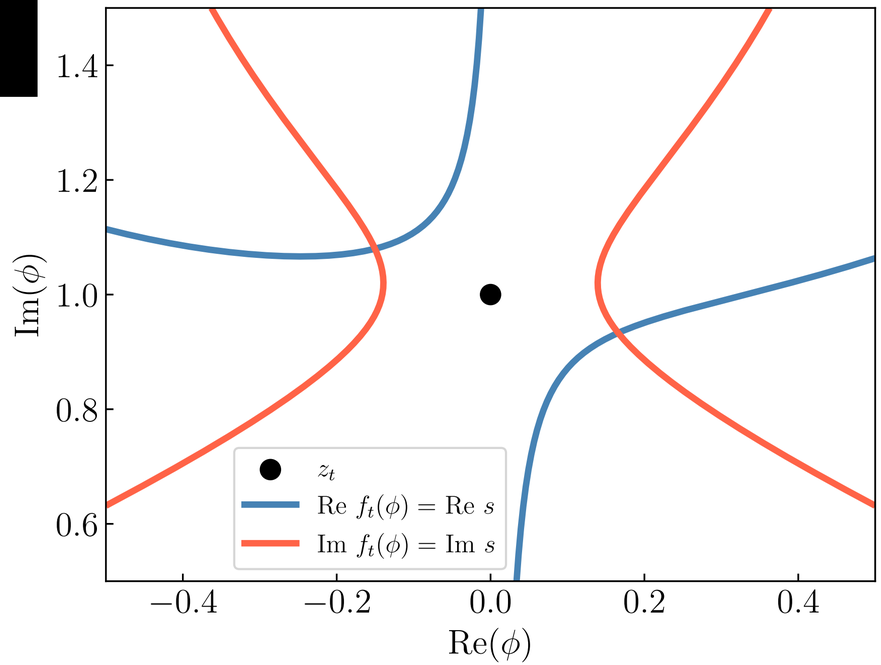


$$s = f_t(z_t) + \epsilon$$



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$(p_0, \mathcal{M}_0)$ —PIK→ $(p, \mathcal{M})$

$\dot{\phi}_t^{(w)}$

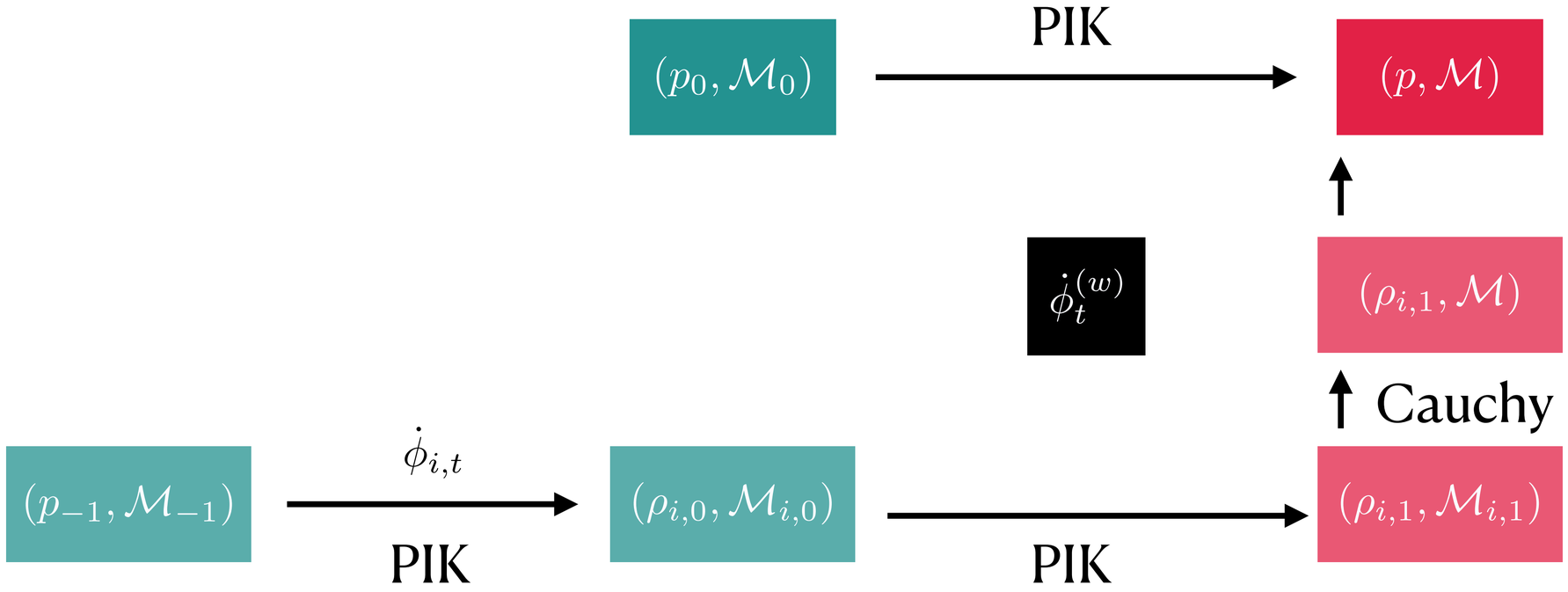
$(p_{-1}, \mathcal{M}_{-1})$ —$\dot{\phi}_{i,t}$, PIK→ $(\rho_{i,0}, \mathcal{M}_{i,0})$ —PIK→ $(\rho_{i,1}, \mathcal{M}_{i,1})$

Uniquely match

$$p_0(\phi) = \sum_i w_i \rho_{i,0}(\phi) \qquad p_0(\phi) = 0 \longleftrightarrow p_t(\,\Phi_t(\phi)\,) = 0 \qquad p(\phi) = \sum_i w_i \rho_{i,1}(\phi)$$

We want to transform easy theories into hard ones.

complex ← **easy theories** **into** **hard ones.** → **complex**



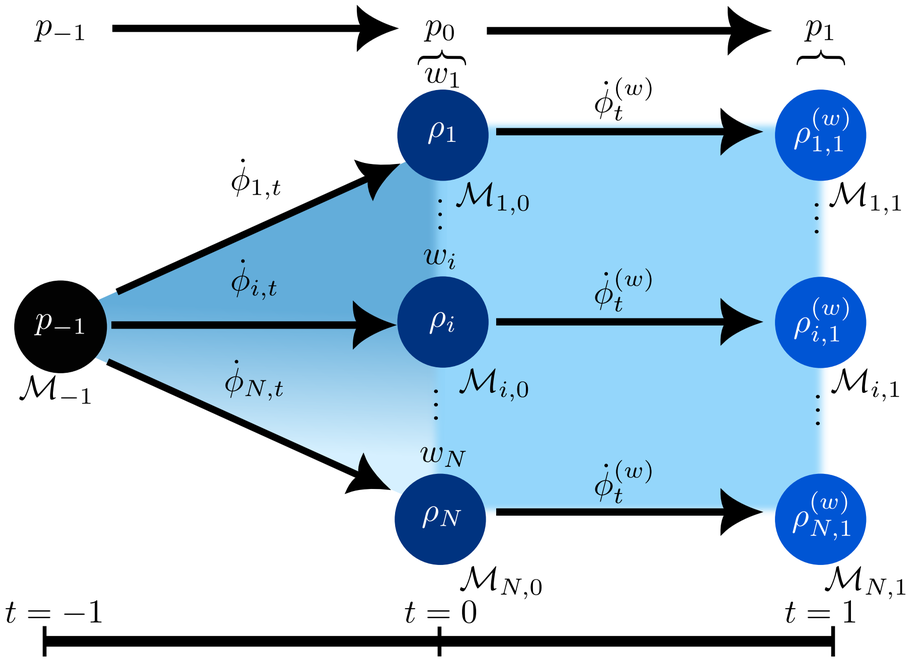
The PIKfan

We want to transform

complex ← **easy theories** into **hard ones.** → **complex**



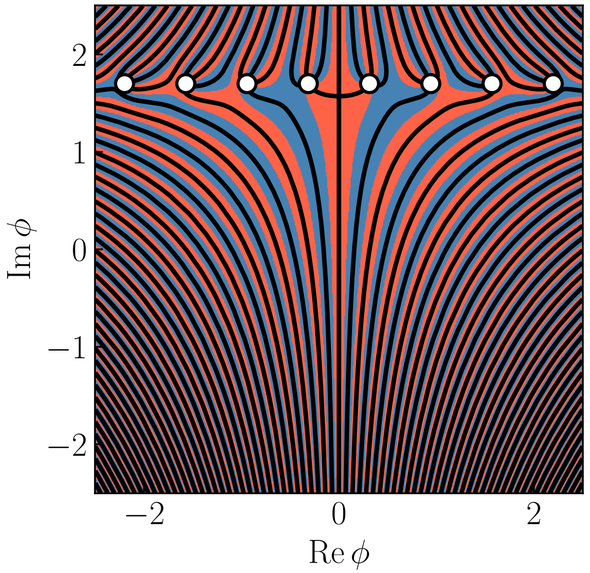
Kind regards to Kai Zhou



The PIKfan

Hubbard model

$$p(\varphi) = \frac{1}{\mathcal{N}} e^{\frac{\beta}{2U}\varphi^2} \left[1 + e^{\beta(i\phi+\mu+\frac{1}{2}U)}\right]^2$$



$$\mu = 1.2$$

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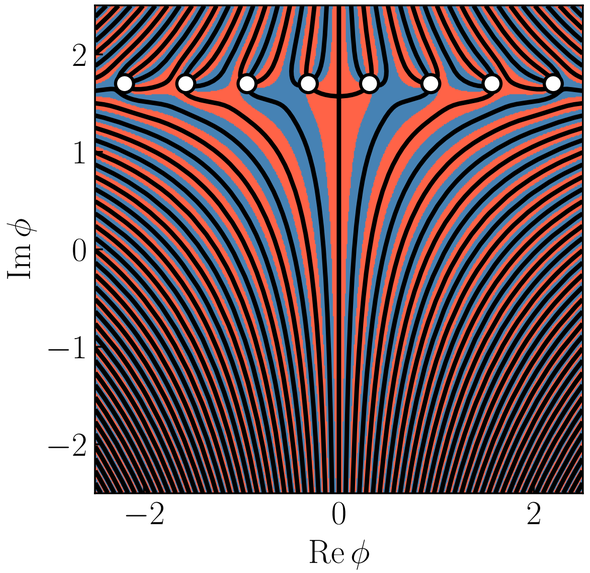
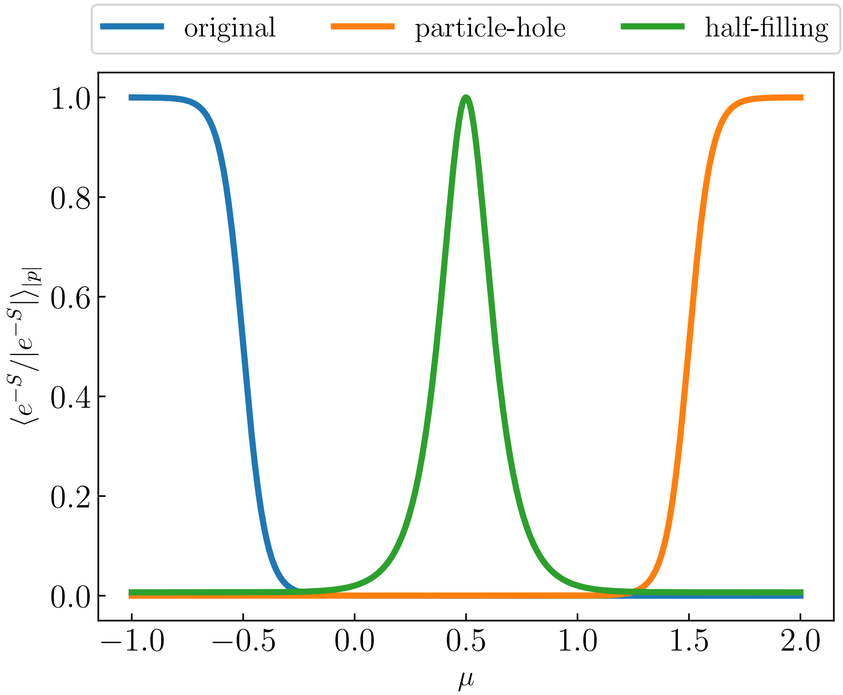


$$\mu = 1.2$$

Average sign



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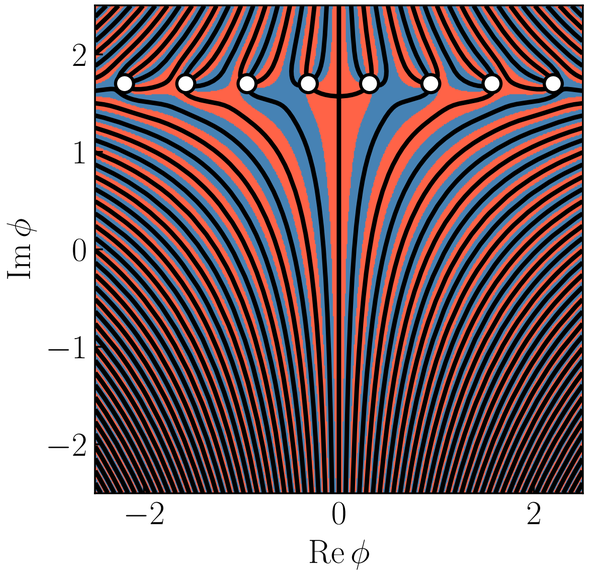

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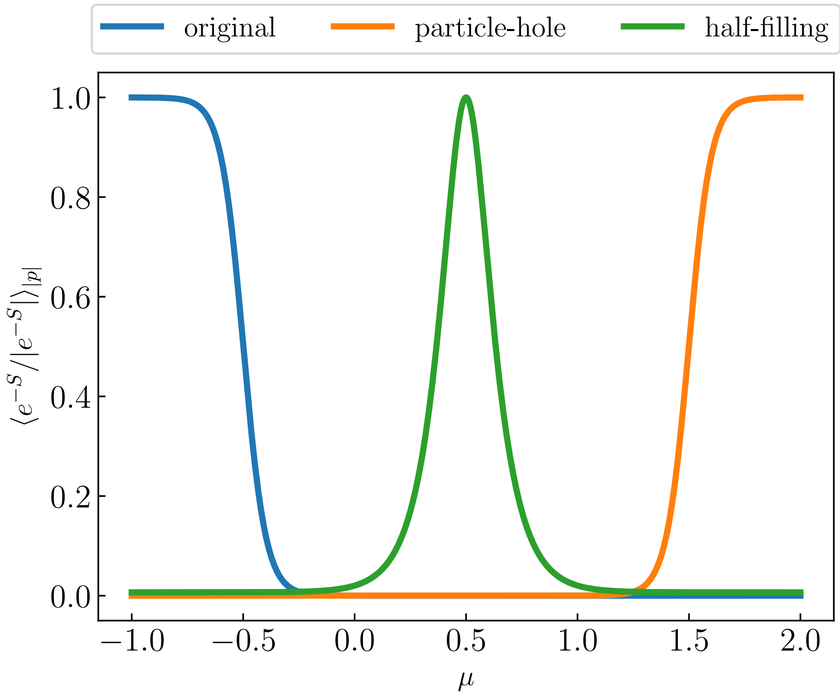
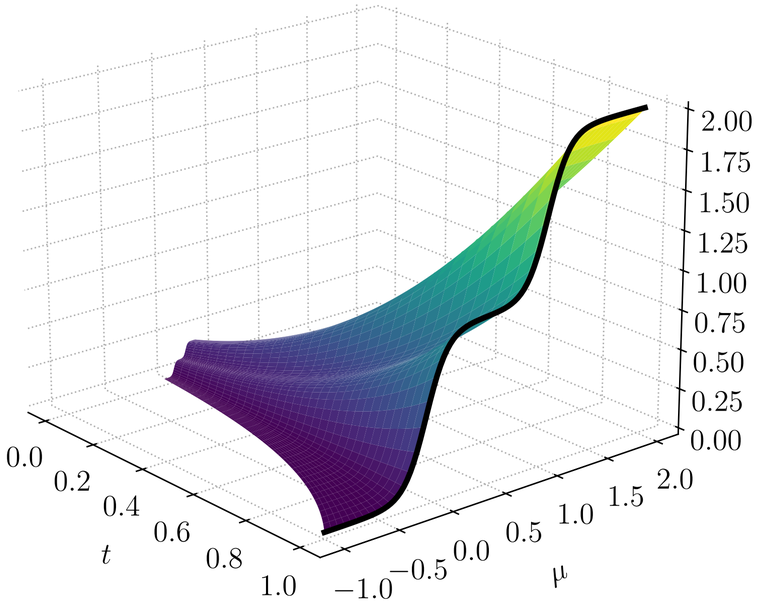


Application



Transport between different Hubbard models

$$\beta_t \mu_t = \text{const.}$$

e.g.

$$\beta_t U_t = \text{const.}$$

Adding a quartic term

$$p(\varphi) = \sum_i w_i f_i(\varphi)$$

$$f_i(\varphi) \propto \exp\left[-\frac{1}{2\sigma}(\varphi - im_i)^2 - \frac{1}{4}\lambda\varphi^4\right]$$

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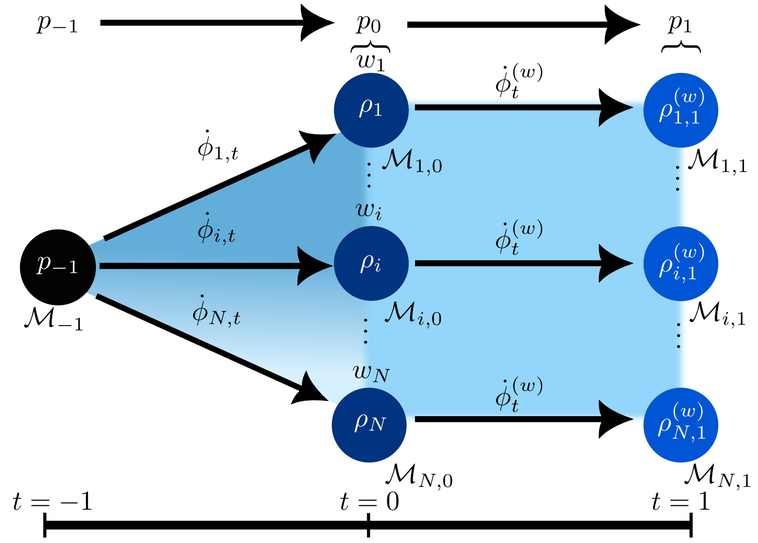

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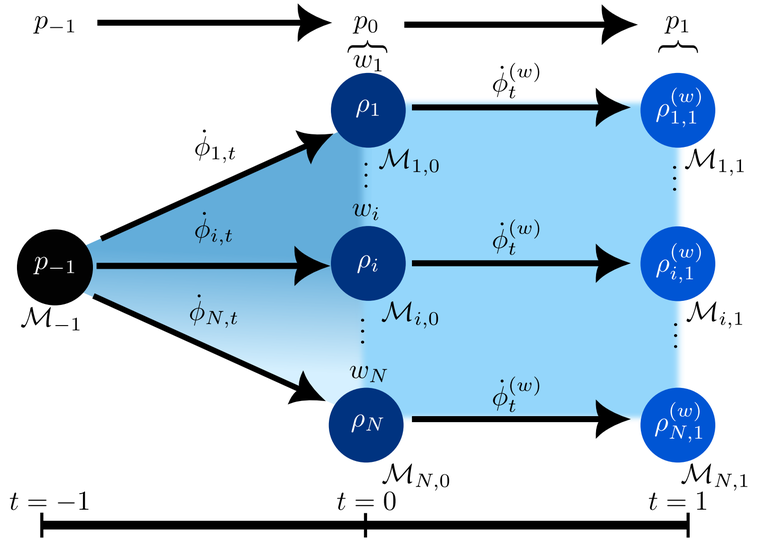

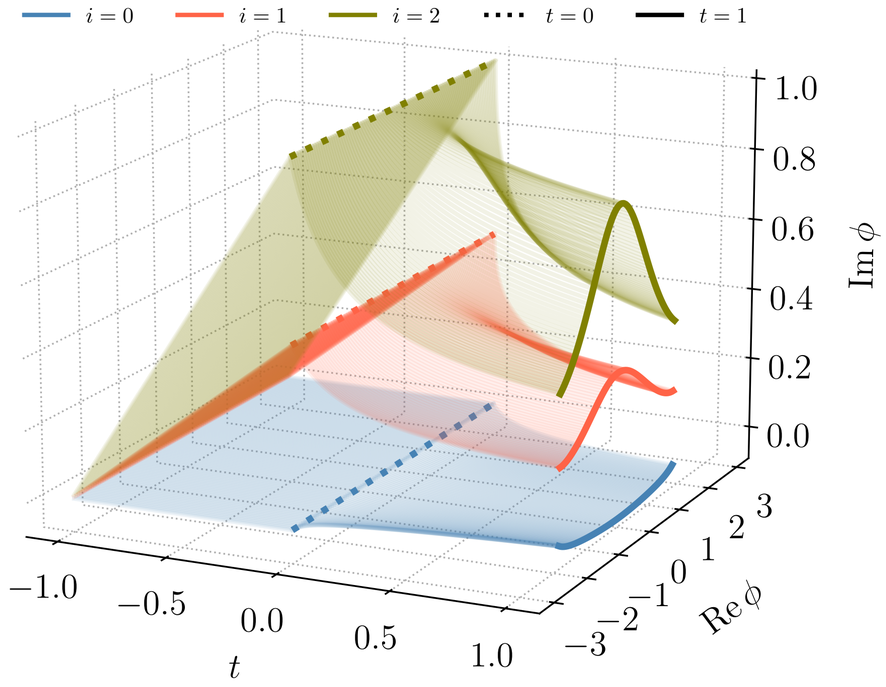

The PIKfan

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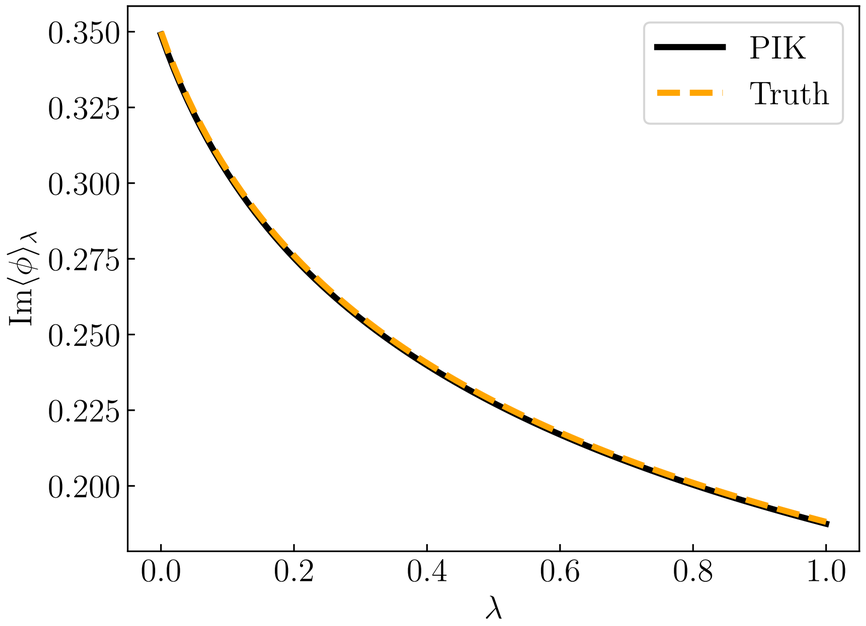
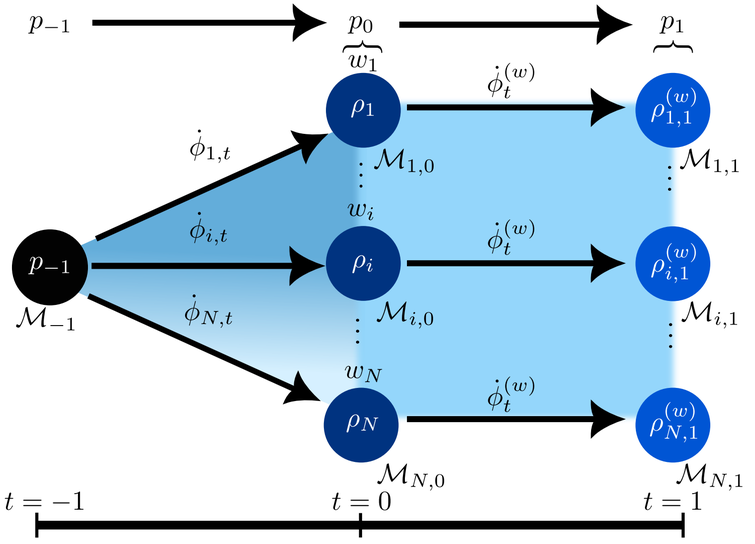
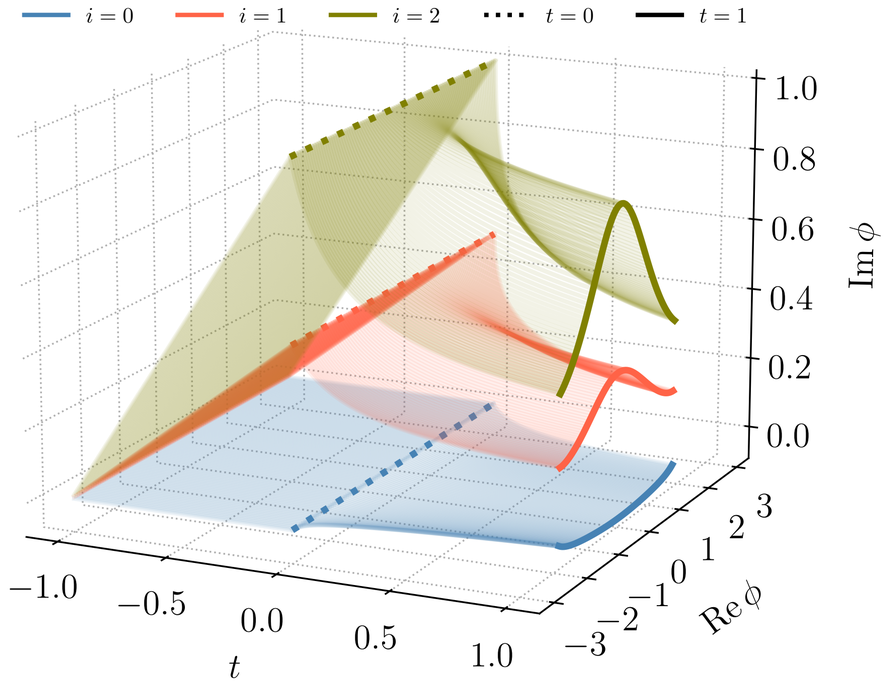






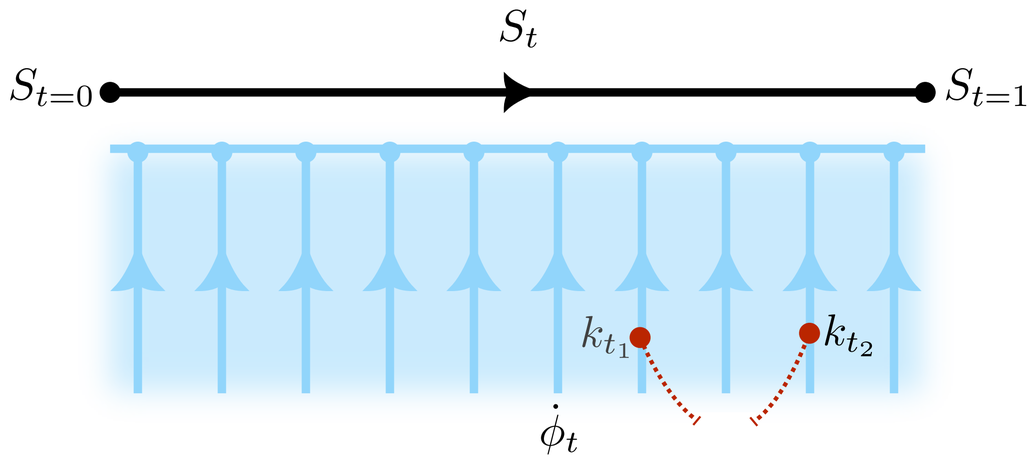
Generative Sampling

with physics-informed kernels

Dissects measure transport into independent, correctable sub-problems

Applicability to real-time problems: weight-preserving transport

Applicability to fermionic sign problems: PIKfan



Thank you
for your attention !

