

PIRGs & PIKs

The eclectic renormalisation groug

From the reparametrisation of quantum effective actions

to the sampling of theories with sign problems

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

Non-perturbative renormalization in fundamental and effective theories

Trento, 10th of September 2026

isoQuant SFB1225

EMMI

STRUCTURES CLUSTER OF EXCELLENCE

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

Outline

- **Physics-Informed Renormalisation Group**
- **Generative architectures & their problems**
- **Generative RGitectures with Physics-Informed Kernels in lattice field theories**
- **PIKs step by step aka if PIKs can fly** X
- **Solving sign problems with complex PIKs**

Jump forward to: ML-spinning renormalisation group

Physics-Informed Renormalisation Group

Quantum field theory & the functional RG in a nutshell

Classical action $S(\phi)$

Simple example: ϕ **real scalar field with** $S(\phi) = \int d^d x \left\{ \frac{1}{2}\phi \left(-\partial_\mu^2 + m_\phi^2 \right) \phi + \frac{\lambda_\phi}{8}\phi^4 \right\} + \log \mathcal{N}_\phi$

Expectation values $\langle \mathcal{O}(\hat{\phi}) \rangle = \int \prod_x d\hat{\phi}(x)\, p(\hat{\phi})\, \mathcal{O}(\hat{\phi})$

Euclidean measure $p(\hat{\phi}) = e^{-S(\hat{\phi})}$

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Linear block-spinning renormalisation group



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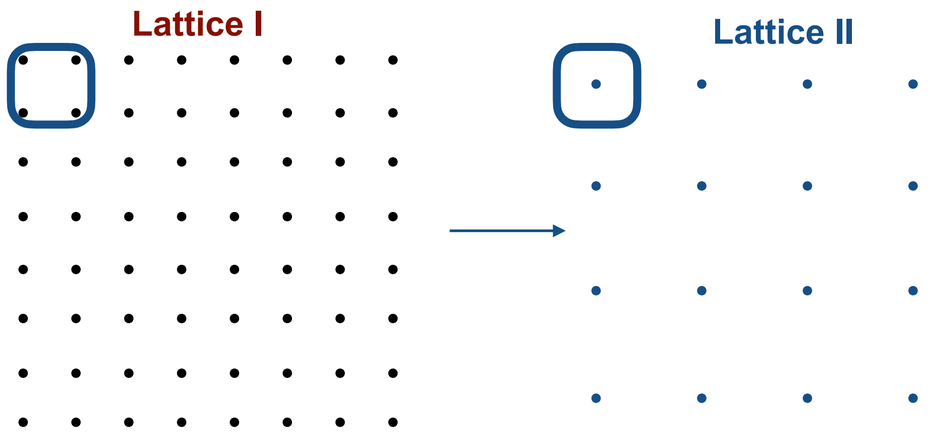
Linear block-spinning renormalisation group



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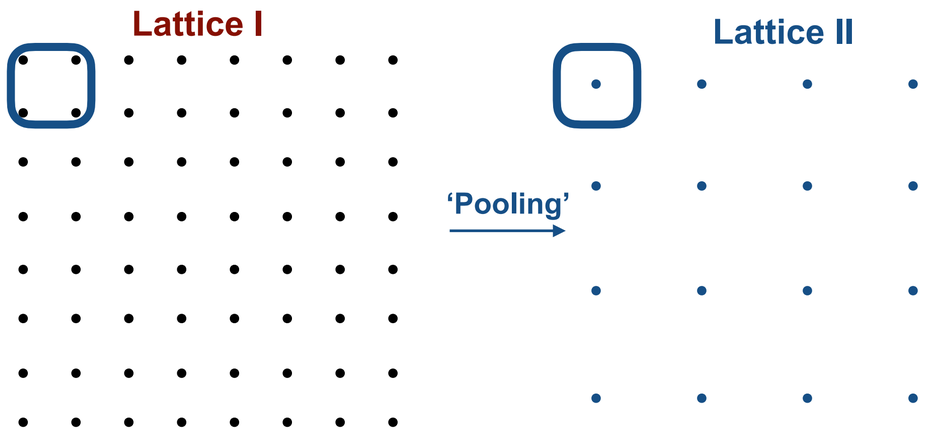

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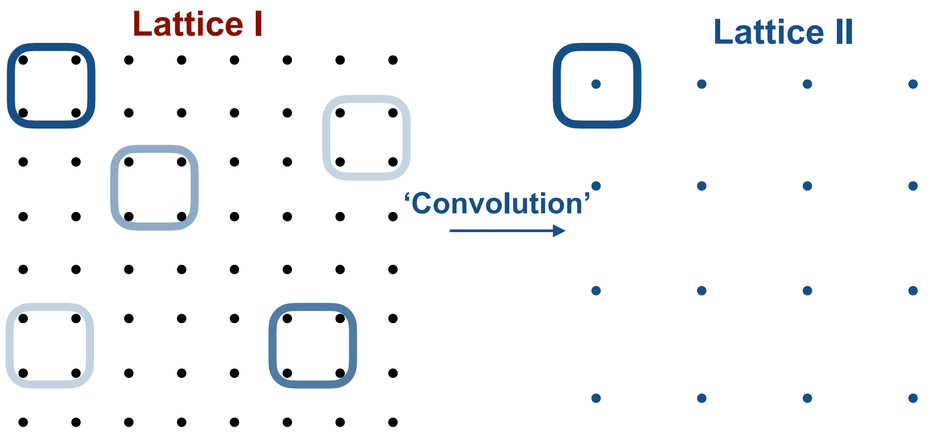

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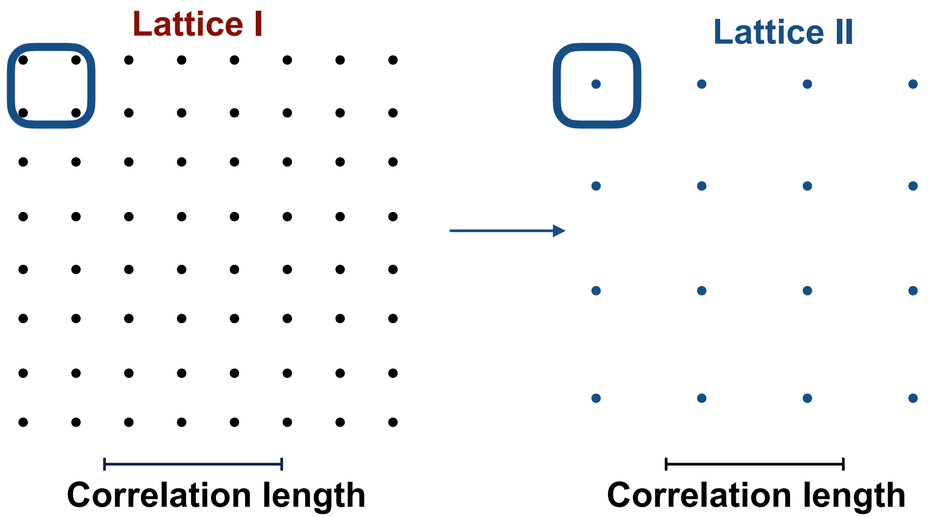

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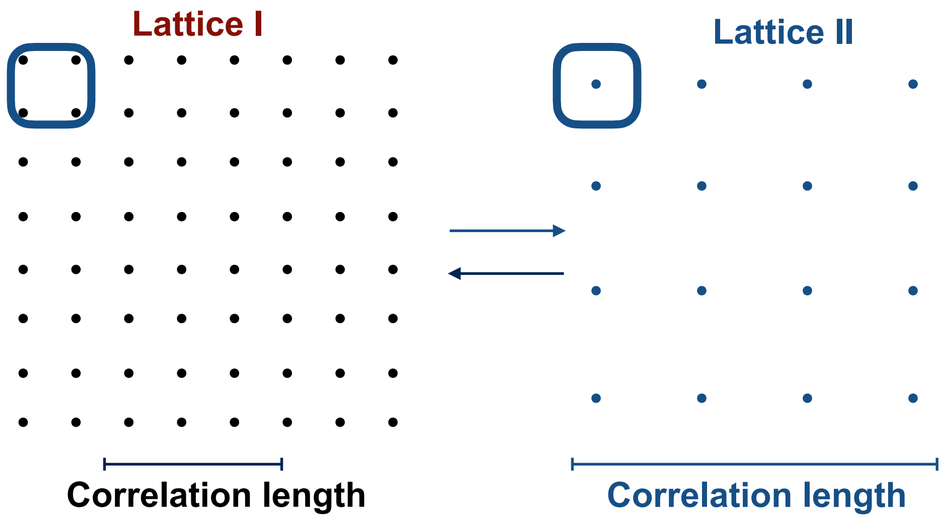

Possible by re-adjusting all parameters

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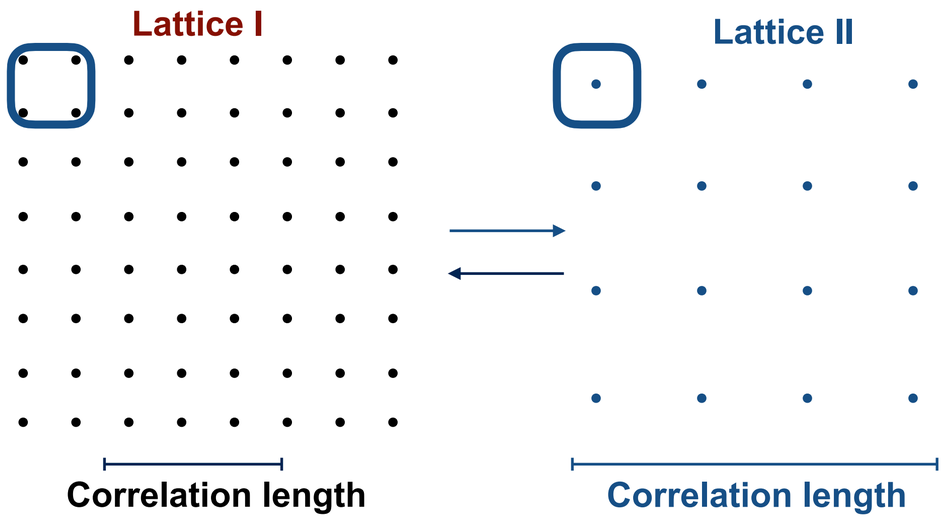

Possible by re-adjusting all parameters

⟶ Finite lattices: RG step involves loss of information (hypothesis of RG entropy that increases)

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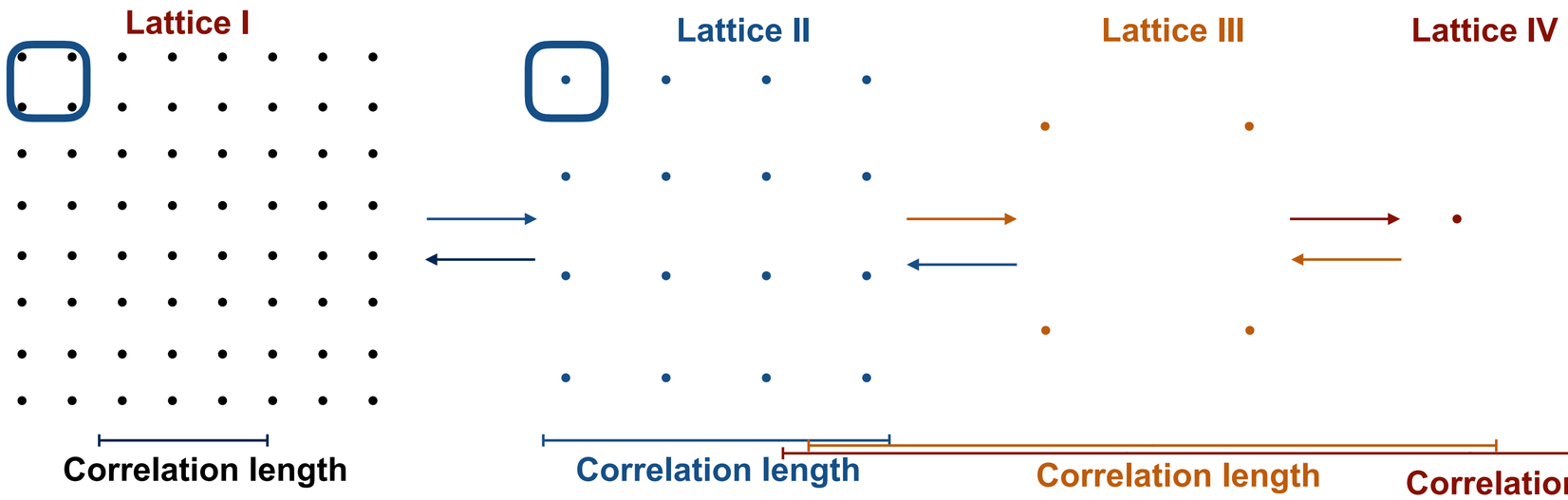

Possible by re-adjusting all parameters

→ Finite lattices: RG step involves loss of information (hypothesis of RG entropy that increases)

Quantum field theory & the RG in a nutshell

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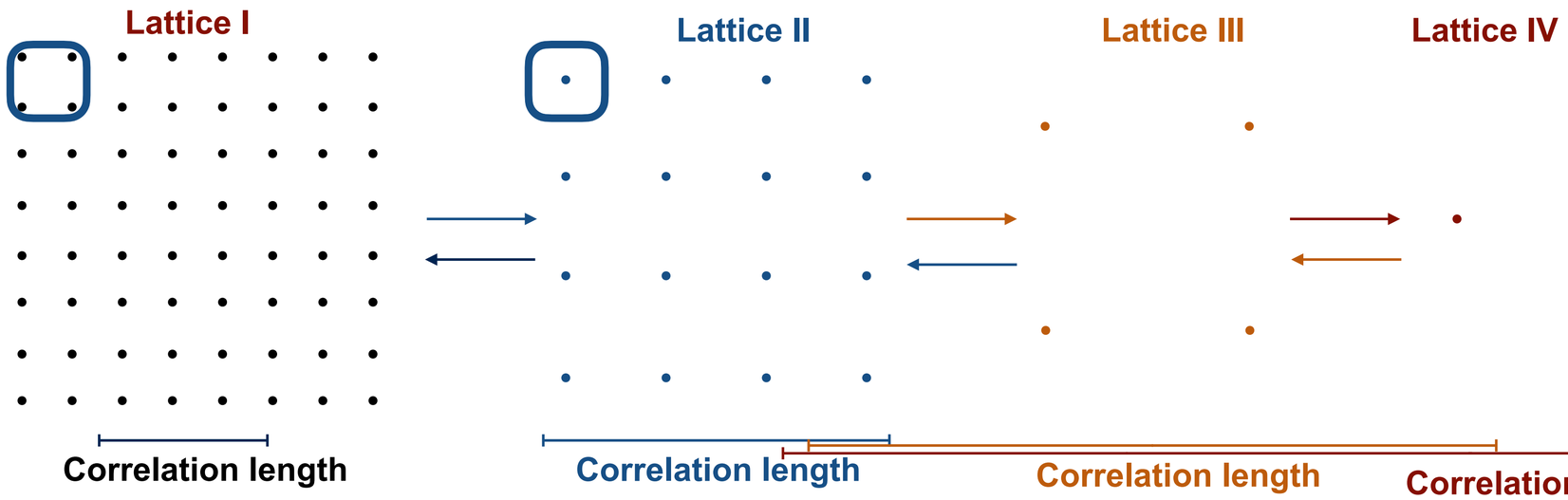

Possible by re-adjusting all parameters

← Full inversion only possible for lattice (system) with infinite extend/infinite # of degrees of freedom

Inverse RG

Quantum field theory & the RG in a nutshell

Jump back to: Physics-Informed Renormalisation Group

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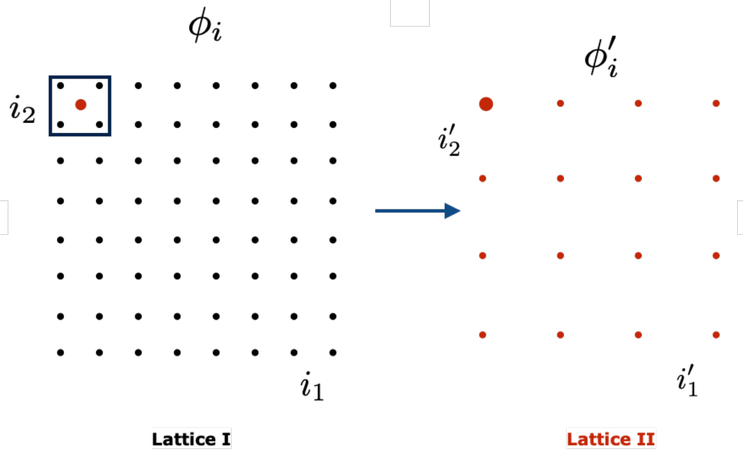
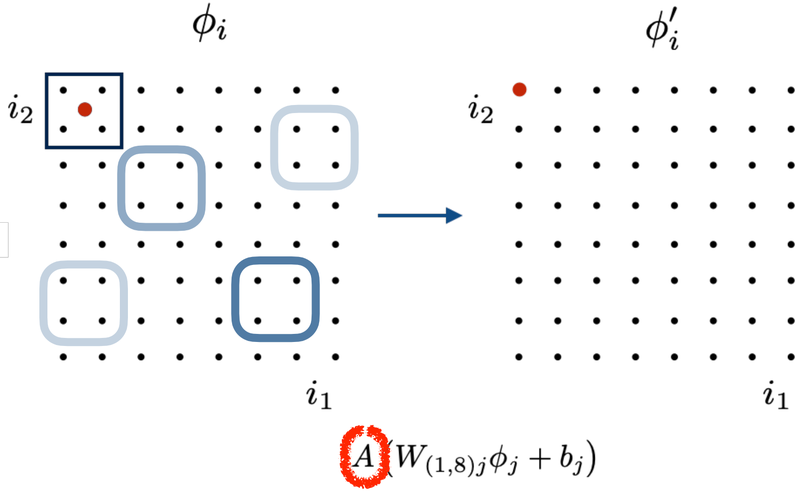
ML-spinning renormalisation group

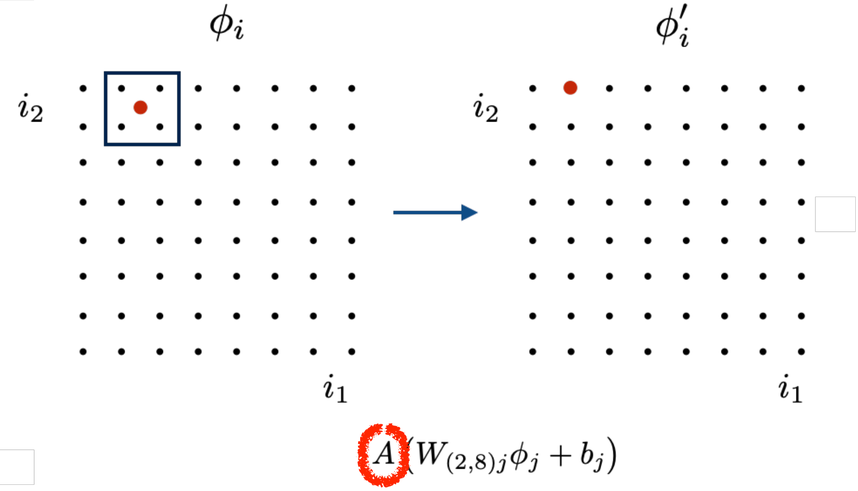
Pooling = scale transformation



$$\phi'_{i'} = \frac{1}{4}\sum_{i \in \mathcal{C}_{(1,4)}} \phi_i$$

Convolution = linear reparametrisation





Activation function: linear → non-linear

Quantum field theory & the functional RG in a nutshell

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RG: General rescalings & reparametrisations via general differential map $\dot{\phi}(\phi)$

Wegner equation

$$\partial_t p_t(\phi) = -\int_x \frac{\delta}{\delta\phi(x)} \left(\dot{\phi}(\phi)\, p_t(\phi) \right)$$

Wegner, J. Phys. C 7 (1974) 2098

flow of distribution aka optimal transport

RG-time: t

Legendre transform

Generalised flow equation

$$\left(\partial_t + \int_x \dot{\phi} \frac{\delta}{\delta\phi} \right) \Gamma_\phi = \frac{1}{2} \operatorname{Tr} \left[G_\phi \left(\partial_t + 2 \frac{\delta \dot{\phi}}{\delta \phi} \right) R_\phi \right]$$

JMP, *Annals Phys.* 322 (2007) 2831

flow of target action aka rate function

Locality

Quantum field theory & the functional RG in a nutshell

Jump forward to: Physics-informed RG flows

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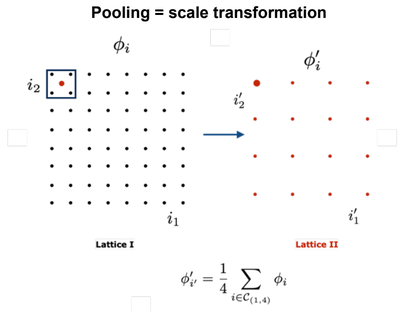
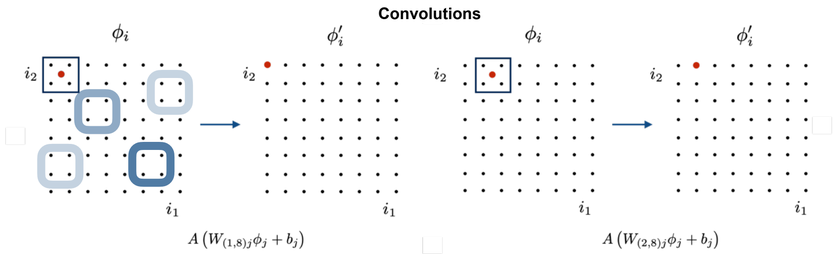


Continuous ML-spinning



Generating functionals $F[\phi]$

perfect for sampling

Probability measure

$$F[\phi] = p[\phi]$$

Wilson effective action = log likelihood

$$F[\phi] \simeq \log p[\phi] = S_{\text{eff}}[\phi]$$

Special case: Polchinski flow ‘83

$$\dot{\phi} \propto \mathcal{C}_t \frac{\delta S_{\text{eff}}}{\delta \phi}$$

Wegner flow ’74

Least redundant **perfect for optimisation**

1PI effective action = rate function

$$F[\phi] \simeq \mathcal{L}\left(\log p[\phi]\right) = \Gamma[\phi]$$

Special case: Wetterich flow ’93

Legendre transform

Generalised flow equation ‘05

Sampling

A brief history of generalised flows

Direct computation & optimisation

$$\partial_t p_t[\phi] = -\int_x \frac{\delta}{\delta\phi(x)}\left(\dot{\phi}[\phi]\, p_t[\phi]\right)$$

Wegner flow

Wegner, J. Phys. C 7 (1974) 2098

Legendre transform

$$\left(\partial_t + \int_x \dot{\phi}\frac{\delta}{\delta\phi}\right)\Gamma_t[\phi] = \frac{1}{2}\mathrm{Tr}\, G_t[\phi]\,\partial_t R_t + \mathrm{Tr}\, G_t[\phi]\,\frac{\delta\dot{\phi}}{\delta\phi}R_t$$

Generalised flow equation

JMP, *Annals Phys.* 322 (2007) 2831

$$G_t[\phi](p,q) = \langle\hat{\phi}(p)\,\hat{\phi}(q)\rangle_c = \frac{1}{\frac{\delta^2\Gamma_t[\phi]}{\delta\phi^2} + R_t}(p,q)$$

Sampling

A brief history of generalised flows

Direct computation & optimisation

Wetterich equation $\dot{\phi} = 0$

Wetterich, PLB 301 (1993) 90

$$\partial_t p_t[\phi] = -\int_x \frac{\delta}{\delta\phi(x)}\left(\dot{\phi}[\phi]\, p_t[\phi]\right) \quad \longleftrightarrow \quad \left(\partial_t \right) \Gamma_t[\phi] = \frac{1}{2}\mathrm{Tr}\, G_t[\phi]\, \partial_t R_t$$

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Wegner, J. Phys. C 7 (1974) 2098

Generalised flow equation

JMP, *Annals Phys.* 322 (2007) 2831

Wegner flow or Wegner-Morris flow

Wegner, J. Phys. C 7 (1974) 2098

Morris, hep-th/9810104 (gauge theories, lecture ERG 1998)

Latorre, Morris, JHEP 11 (2000) 004

⋮

Rosten, Phys.Rept. 511 (2012) 177

Generalised 1PI flows

Gies, Wetterich, PRD 65 (2002) 065001 (bilinear composites)

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⋮

Novel conceptual developments based on general flows

Essential RG: **Baldazzi, Falls, *SciPost Phys.* 13 (2022) 085, … (generalised flow eq. with essential/inessential operators)** **fixed theory**

Physics-Informed RG: **Ihssen, JMP, *Annals Phys.* 481 (2025) 170177, … (introducing Physics-informed pairs with generalised flow eq. & Wegner flows)**

flowing theories

Sampling

Physics-informed RG flows

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Key observation: solve for PIRG pair

‘Seemingly trivial’ but extremely powerful

$$\left(F_t[\phi]\,,\dot{\phi}_t[\phi]\right)$$

Jump back to: Continuous ML-spinning

Ihssen, JMP, *Annals Phys.* 481 (2025) 170177

Sampling

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$$\left(F_t[\phi]\,,\dot{\phi}_t[\phi]\right)$$

$$\Gamma_t[\phi] = S[\phi]$$

Extreme example

Jump back to: Continuous ML-spinning

Ihssen, JMP, *Annals Phys.* 481 (2025) 170177

Physics-informed RG flows

PIRG pair

$$\Big(F_t[\phi]\,,\dot{\phi}_t[\phi]\Big)$$

$$\dot{\phi}_t[\phi]$$

Flowing field or Physics-Informed Kernel

PIRG idea

$$F_0(\phi) \xrightarrow{\Big(F_t(\phi)\,,\dot{\phi}_t(\phi)\Big)} F_1(\phi)$$

$$\partial_t F_t + \dot{\phi}_t \frac{\partial F_t}{\partial \phi} = \mathrm{RG}_1(F_t) + \frac{\partial \dot{\phi}}{\partial \phi}\mathrm{RG}_2(F_t)$$

Ihssen, JMP, *Annals Phys.* 481 (2025) 170177

Physics-informed RG flows

PIRG pair

$$\left(F_t[\phi]\,,\dot{\phi}_t[\phi]\right)$$

Direct computation & optimisation

How to: direct computation

Different strategies

Example

$$\left(\partial_t + \int_x \dot{\phi}\frac{\delta}{\delta\phi}\right)\Gamma_t[\phi] = \frac{1}{2}\mathrm{Tr}\,G_t[\phi]\,\partial_t R_t + \mathrm{Tr}\,G_t[\phi]\,\frac{\delta\dot{\phi}}{\delta\phi}R_t$$

Step 1: Solve approximate flow for $\Gamma_{t,\mathrm{app}}$

Step 2: Use $\Gamma_t = \Gamma_{t,\mathrm{app}}$ **as target action in (1)**

Solve generalised flow for $\dot{\phi}_t$

Advantages

(i) Trade intricate diffusion eq. for a simple linear diff. eq.

(ii) Optimised and more versatile expansion schemes

- **(a) Ground state expansion**
- **(b) Feed-down flows**

⋮

Reconstruction & observables

Simplified operator flow

$$\left(\partial_t + \int_x \dot{\phi}\frac{\delta}{\delta\phi}\right)\mathcal{O}_t[\phi] = -\frac{1}{2}\left[G_t\left(\mathcal{D}_t\,R_t\right)G_t\right]\frac{\delta\mathcal{O}_t[\phi]}{\delta^2\phi^2}$$

observable **propagator insertion**

Ihssen, JMP, arxiv:2507.13011

Ihssen, JMP, *Annals Phys.* 481 (2025) 170177

Physics-informed RG flows

PIRG pair

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Direct computation & Optimisation

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Generalised flow equation

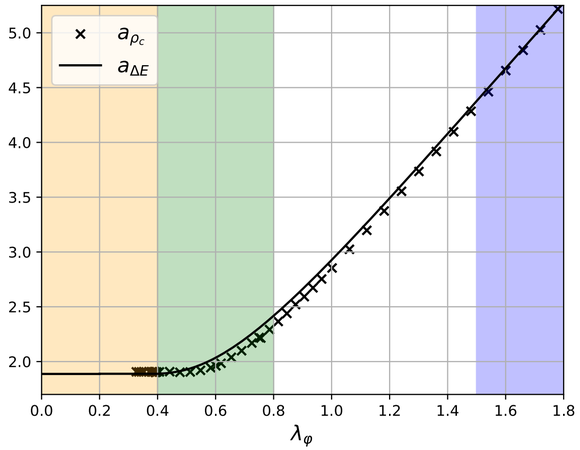
Solved in lowest order ground state expansion

$$Z_\phi(\phi) \equiv 1$$

Inessential coupling: Baldazzi, Falls, *SciPost Phys.* 13

Results

Instanton tunneling in quantum mechanics



$$\Delta E(\lambda_\varphi \to 0) \propto \frac{1}{\sqrt{\lambda_\varphi}}\, e^{-a_{\mathrm{inst}}/\lambda_\varphi}$$

$$a_{\Delta E}(0) = a_{\mathrm{inst}} \approx 1.886$$

$$a_{\mathrm{inst},\rho_\phi} = 1.910(2)$$

Bonanno, Ihssen, JMP, SciPost Phys.Core 9 (2026)

Ihssen, JMP, *Annals Phys.* 481 (2025) 170177

Physics-informed RG flows

PIRG pair

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Direct computation & Optimisation

Solved in lowest order ground state expansion

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Inessential coupling: Baldazzi, Falls, *SciPost Phys.* 13

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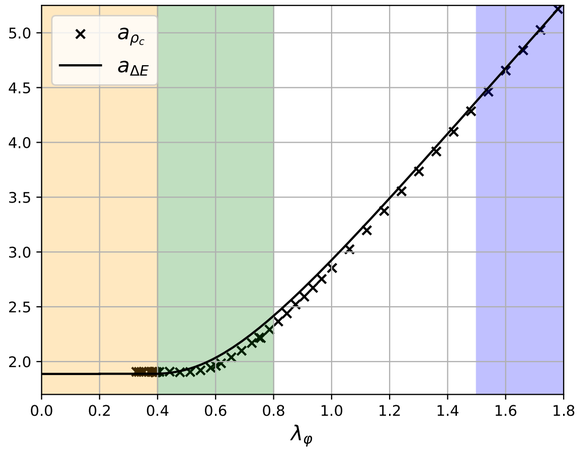
Generalised flow equation

Results

Instanton tunneling in quantum mechanics



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Bonanno, Ihssen, JMP, SciPost Phys.Core 9 (2026)

Gauge invariant/gauge consistent flows

Ihssen, JMP, PRD 112 (2025) 105005

Diffeomorphism invariant gravity flows 猪

Ihssen, Knorr, Mezger, JMP, Sprenger, arXiv:2609.07829

Ihssen, JMP, *Annals Phys.* 481 (2025) 170177

Generative architectures & their problems



'… and now for something completely different…'

OOD problem 1: ***Curse of limited information*** **in training data (extrapolation problem)**

OOD problem 2: ***Curse of dimensionality***

Renormalisation Group

General set-up and some ideas/problems

Data sets

$\{\varphi_i\}$

Experimental data

Sampling data

General data sets

⋮

Probability distributions

$p(\varphi)$

Known distributions ← **in physics**

Unknown distributions

action

$e^{-S[\varphi]}$

log likelihood

Tasks

Known distributions: How to sample efficiently from the distribution?

Unknown distributions: How to learn the distribution from the data set?

and all in between

General set-up and some ideas/problems

Data sets

$\{\varphi_i\}$

Probability distributions

$p(\varphi)$

What can possibly go wrong?

Out Of Domain problems!

General set-up and some ideas/problems

Data sets

$\{\varphi_i\}$

Probability distributions

$p(\varphi)$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

$$|\{\varphi_i\}| \longrightarrow |\{\mathcal{C}_n\}| \lesssim N\left(|\{\varphi_i\}|\right)$$

$$\mathcal{C}_n = \langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_{\text{irr}}$$

Irreducible part of nth moment

$p(\varphi) \neq p_{\mathcal{C}_n}(\varphi)$ **Architecture *has to* encode structural information about *all* $\mathcal{C}_n$**

General set-up and some ideas/problems

Data sets $\{\varphi_i\}$

Probability distributions $p(\varphi)$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

Why does it not always go wrong?

aka why are generative networks *without structural information* successful!

General set-up and some ideas/problems

Data sets

$\{\varphi_i\}$

Probability distributions

$p(\varphi)$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 1: *Curse of limited information* in training data (extrapolation problem)

Why does it not always go wrong?

Probability distribution approximately ‘finite’

$$p(\varphi) \approx p_{\mathcal{C}_n}(\varphi)$$

General set-up and some ideas/problems

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Probability distributions

$p(\varphi)$

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No use of higher $\mathcal{C}_n$

General set-up and some ideas/problems

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Afterburner

Final sampling step with $p(\varphi)$

Typically done in lattice architectures (and MC codes for LHC)

critical slowing down

General set-up and some ideas/problems

Data sets $\{\varphi_i\}$

Probability distributions $p(\varphi)$

What can possibly go wrong?

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No use of higher $\mathcal{C}_n$ $p(\varphi) \approx p_{\mathcal{C}_n}(\varphi)$

Afterburner … Mircacles

Final sampling step with $p(\varphi)$

Typically done in lattice architectures (and MC codes for LHC)

critical slowing down

General set-up and some ideas/problems

Data sets $\{\varphi_i\}$

Probability distributions $p(\varphi)$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 2: *Curse of dimensionality*

Generative sampling in large dimensional data spaces requires interpolations of high dimensional functions

interpolations in high dimensions are in the same problem class as extrapolations

General set-up and some ideas/problems

Data sets $\{\varphi_i\}$

Probability distributions $p(\varphi)$

What can possibly go wrong?

Out Of Domain problems!

OOD problem 2: *Curse of dimensionality*

Generative sampling in large dimensional data spaces requires interpolations of high dimensional functions

interpolations in high dimensions are in the same problem class as extrapolations

Architecture *has to* encode structural information about the interpolations

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

Architectures (Arc) with analytic access to local and global structures

Analytic access to reparametrisations

$$p(\varphi) \to \tilde{p}(\phi(\varphi)) \qquad \text{with} \qquad \langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_p = \langle \varphi_{i_1} \cdots \varphi_{i_n} \rangle_{\tilde{p}}$$

Analytic access to extrapolations

$$\mathcal{C}_{N+m} = \mathcal{C}_{N+m}\left(\{\mathcal{C}_{n<N}\}, \mathrm{Arc}\right)$$

Scale transformations

Symmetries

⋮

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

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⋮

Renormalisation Group

General set-up and some ideas/problems

Data sets

$$\{\varphi_i\}$$

Probability distributions

$$p(\varphi)$$

Architectures (Arc) with analytic access to local and global structures

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Scale transformations

Symmetries

⋮

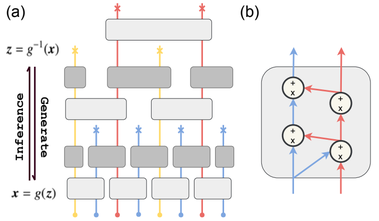
Renormalisation Group

Generative RGitectures

Jump forward to: Generative RGitectures with PIKs in lattice field theories

Example: the inverse renormalisation group

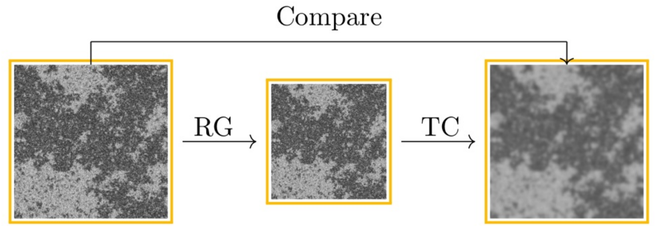
Neural Network Renormalization Group



Li, Wang, PRL 121 (2018) 26, 260601

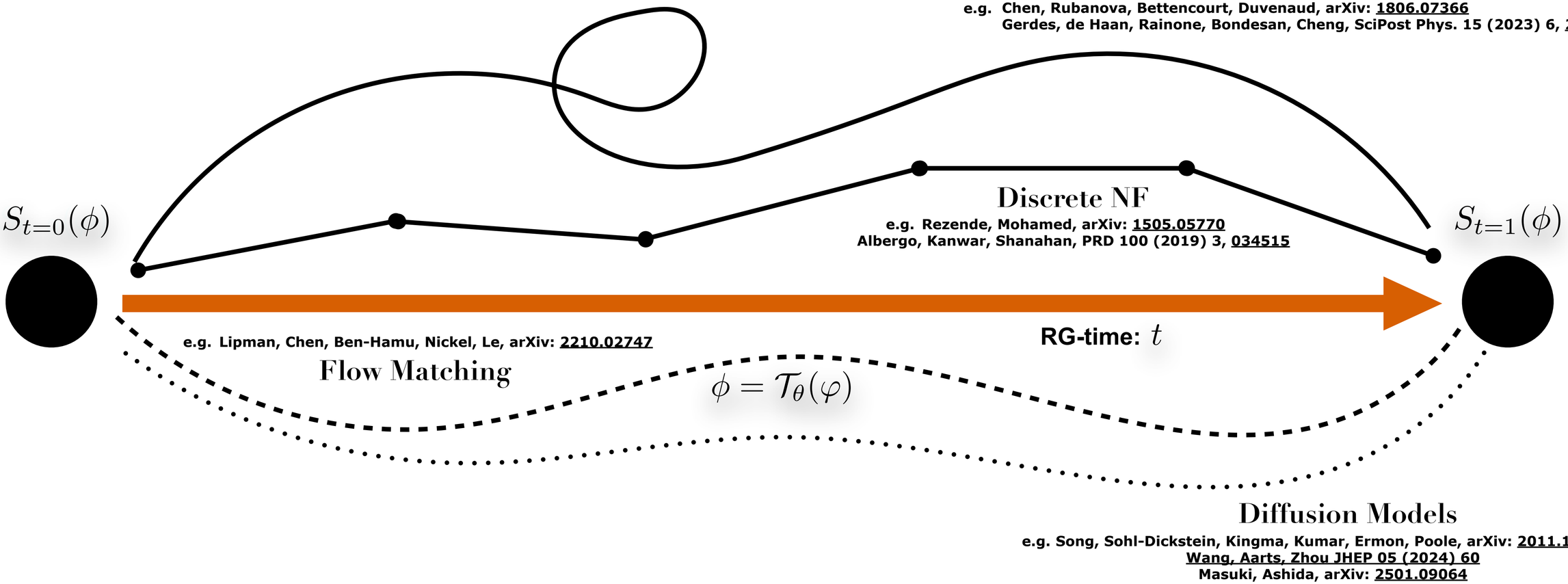
Inverse Renormalization Group in Quantum Field Theory

Bachtis, Aarts, Di Renzo, Lucini, PRL 128 (2022) 8, 081603



Sampling with RGitectures in lattice field theories

Many Possible Generative Models



Learn the transformation between the two distributions from data

Sampling with RGitectures in lattice field theories

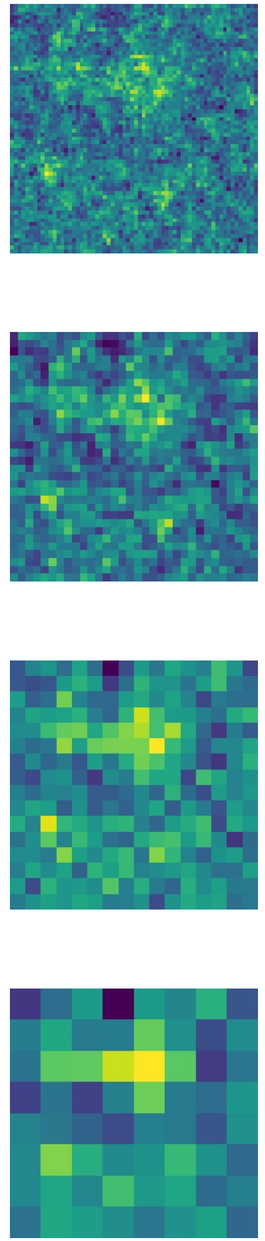
Continuum

$$S[\phi] = \int d^d x \left[\frac{1}{2} \phi(x) \left(-\partial_\mu^2 + M^2 \right) \phi(x) + \frac{\lambda}{4} \phi^4(x) \right]$$

Observables $\langle \mathcal{O} \rangle = \int D\phi \, p(\phi) \, \mathcal{O}(\phi)$ $\qquad D\phi = \prod_{x \in \Lambda} d\phi_x$

Coarse Graining — RG steps

Fine Graining — Inverse RG steps



Lattice

$$S(\phi) = \sum_{x \in \Lambda} \left[(1 - 2\lambda') \, \phi_x^2 + \lambda' \phi_x^4 - 2\kappa \sum_\mu \phi_x \phi_{x+\mu} \right] + \log \mathcal{N}$$

Bauer, Kapust, JMP, Temmen, *SciPost Phys.* 19 (2025) 3, 077

Sampling with RGitectures in lattice field theories

Continuum

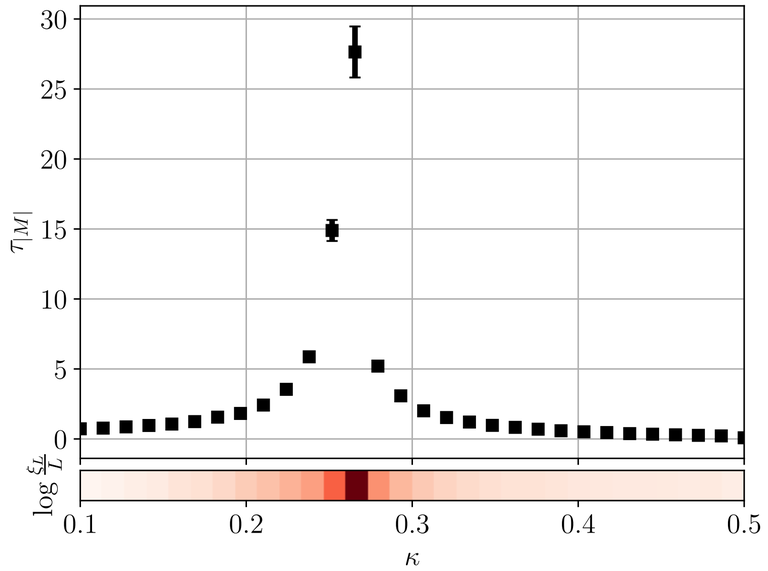
$S[\phi]$

Coarse Graining — RG steps

Observables $\langle \mathcal{O} \rangle = \int D\phi \, p(\phi) \, \mathcal{O}(\phi)$

Monte Carlo Methods

$\phi^{(0)} \longrightarrow \phi^{(1)} \longrightarrow \cdots \longrightarrow \phi^{(N-1)} \longrightarrow \phi^{(N)}$



Correlation length

$\xi = \xi_L a_L$

Autocorrelation time

$\tau_{\mathcal{O}}$

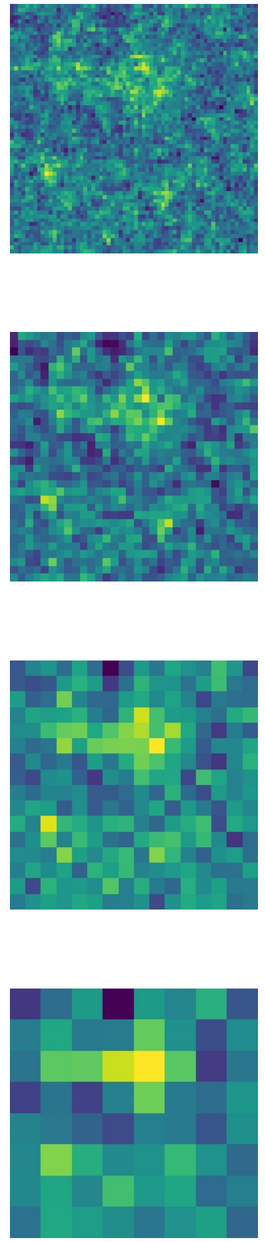
Critical slowing down

$\tau_{\mathcal{O}} \sim \xi_L^{z_{\mathcal{O}}}$

Wolff Nucl.Phys.B. 17 (1990)

similar for topological freezing

Fine Graining — Inverse RG steps

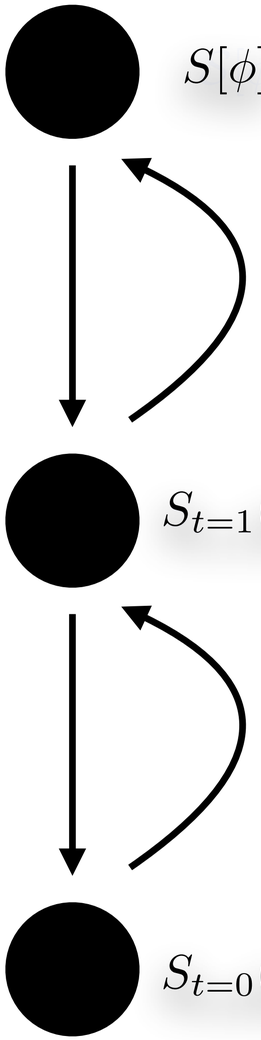


$S(\phi)$

Lattice

Bauer, Kapust, JMP, Temmen, *SciPost Phys.* 19 (2025) 3, 077

Sampling with RGitectures in lattice field theories (no PIKs)



Governed by the Renormalisation Group

Bauer, Kapust, JMP, Temmen, *SciPost Phys.* 19 (2025) 3, 077

Sampling with RGitectures in lattice field theories

Normalising Flow

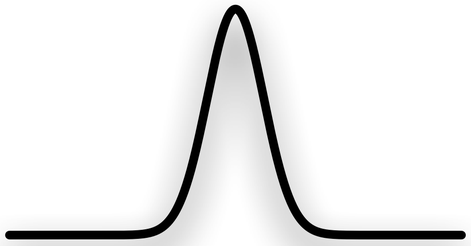


$$\phi = \mathcal{T}_\theta(\varphi)$$

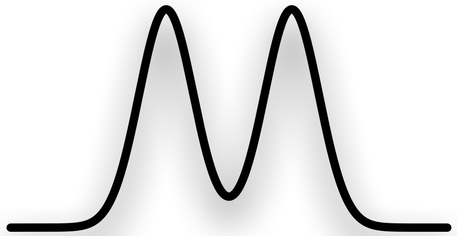
θ : (Neural Network) Parameters

$$\tilde{p}_{t=1}(\phi) = p_{t=0}(\varphi)\,|\det J_{\mathcal{T}}(\varphi)|^{-1}$$

$$\text{Minimize } D_{KL}(\tilde{p}\|p) = \int D\phi\,\tilde{p}(\phi)\log\frac{\tilde{p}(\phi)}{p(\phi)}$$



Free Gaussian Theory



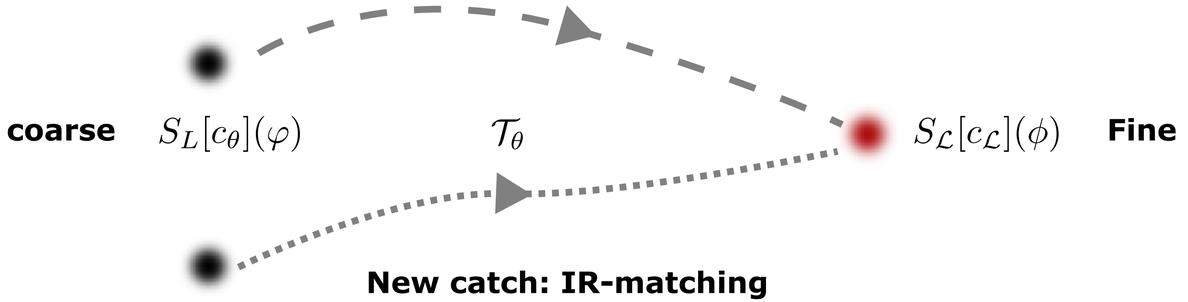
Interacting Theory

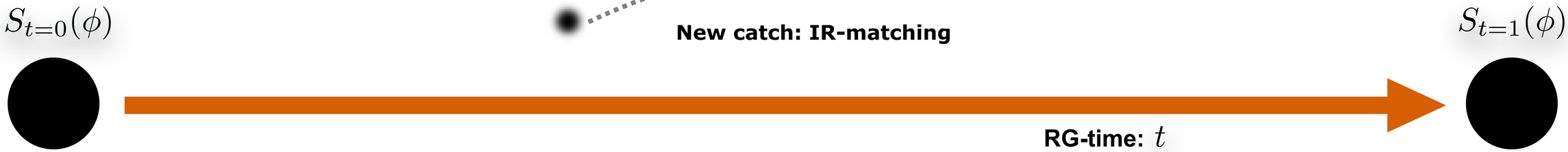
e.g. Lüscher Commun.Math.Phys. 293 (2010) Rezende, Mohamed, arXiv: 1505.05770

Bauer, Kapust, JMP, Temmen, *SciPost Phys.* 19 (2025) 3, 077

Sampling with RGitectures in lattice field theories

Super-resolving Normalising Flow with IR-matching





$$\phi = \mathcal{T}_\theta(\varphi)$$

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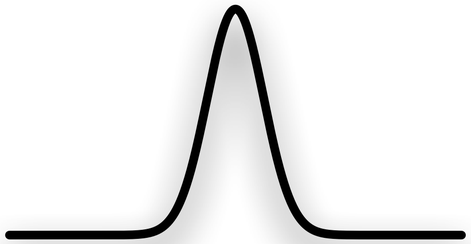
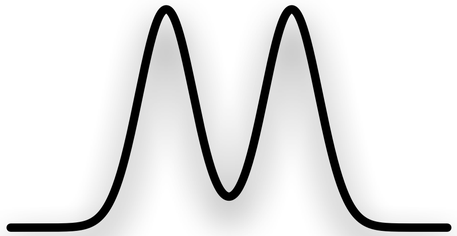
$$\text{Minimize } D_{KL}(\tilde{p}\|p) = \int D\phi\, \tilde{p}(\phi) \log \frac{\tilde{p}(\phi)}{p(\phi)}$$



Free Gaussian Theory



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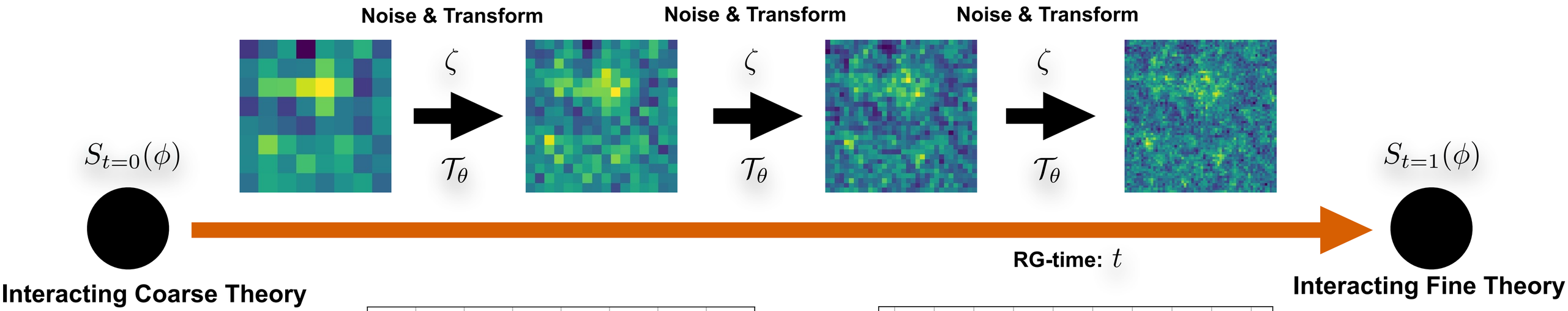
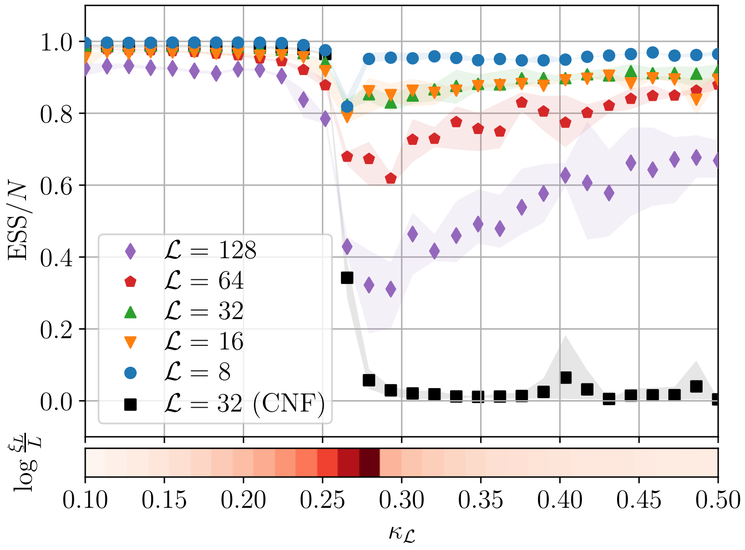
e.g. Lüscher Commun.Math.Phys. 293 (2010) Rezende, Mohamed, arXiv: 1505.05770

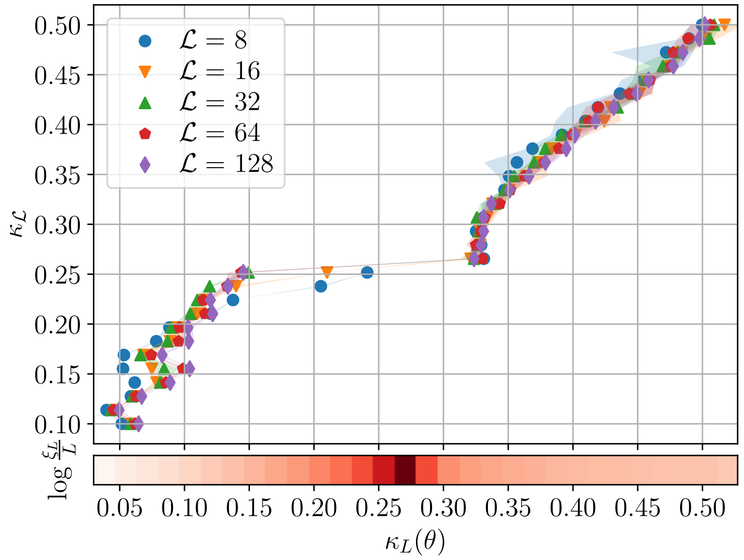
Bauer, Kapust, JMP, Temmen, *SciPost Phys.* 19 (2025) 3, 077

Sampling with RGitectures in lattice field theories

Super-resolving Normalising Flow with IR-matching

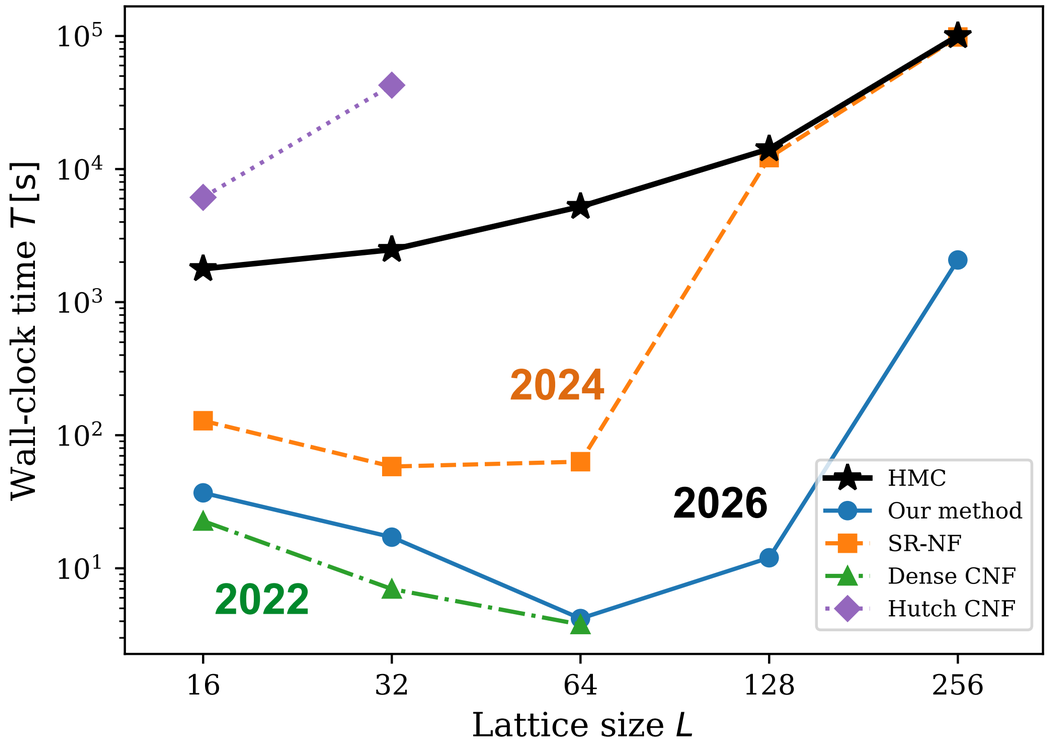






Bauer, Kapust, JMP, Temmen, *SciPost Phys.* 19 (2025) 3, 077

Scalable Generative Sampling and Multilevel Estimation for Lattice Field Theories Near Criticality



- *Our Multilevel method*: The multilevel coarse-to-fine sampler introduced in Sec. III, evaluated at $L \in \{16, 32, 64, 128, 256\}$ (see Appendix B 1).
- *Super Resolving CNF* [64]: the same hierarchical structure but with full-lattice CNF upsampling and without coarse-site preservation, evaluated at $L \in \{16, 32, 64, 128, 256\}$ (see Appendix B 2).
- *Dense CNF* [30]: A single CNF on the target distribution with a Gaussian prior, evaluated at $L \in \{16, 32, 64\}$; training becomes prohibitive beyond $L = 64$ (see Appendix B 3).
- *Hutch CNF*: The Hutch CNF uses multiple CNN layers with the Hutchinson stochastic trace estimator [72] estimating the divergence stochastically, evaluated at $L \in \{16, 32\}$ (see Appendix B 4).

TABLE I. Integrated autocorrelation time $\tau_{\text{int}}(|m|)$ at $\kappa_c = 0.2705$, $\lambda = 0.022$. HMC: Wolff method on 2×10^6-step chains ($L \leq 128$) and 3×10^6 steps ($L = 256$). Generative methods: IMH chains of 5×10^4 steps, 2×10^3 burn-in. The corresponding importance-sampling efficiency ESS/N for all methods is reported in Fig. 7 (Appendix B 7).

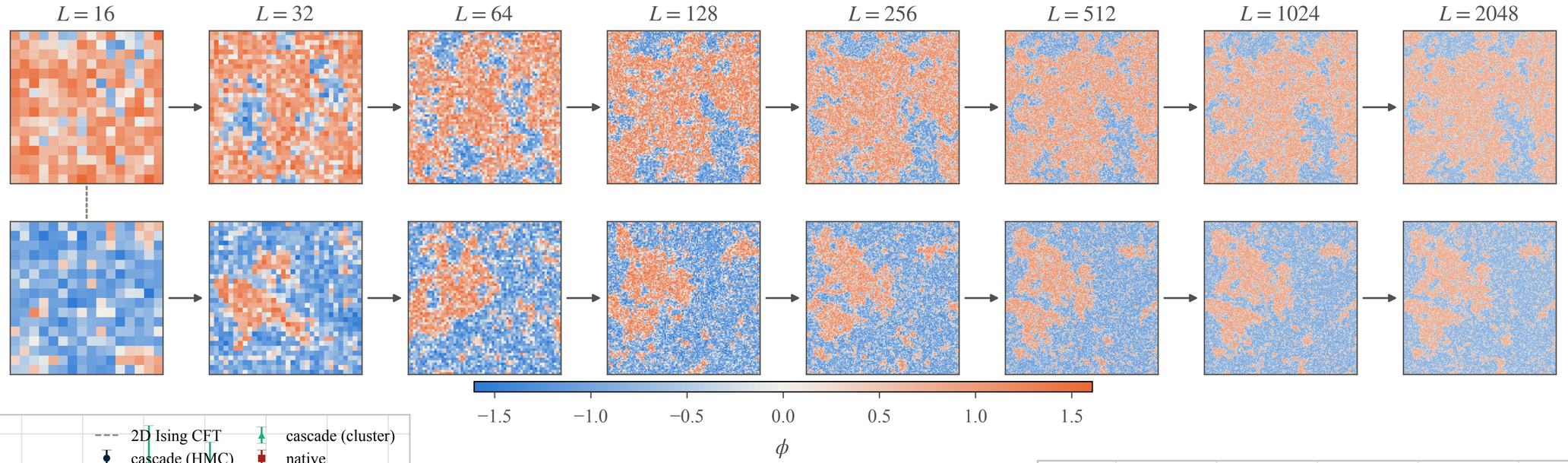
Method	$L = 16$	$L = 32$	$L = 64$	$L = 128$	$L = 256$
HMC	64.6 ± 1.3	228 ± 9	1370 ± 120	4930 ± 850	13265 ± 4321
Our method	0.72 ± 0.02	0.80 ± 0.02	1.16 ± 0.03	3.73 ± 0.15	302 ± 105
SR-NF	0.86 ± 0.01	1.46 ± 0.01	6.96 ± 0.12	161.4 ± 13.0	—[a]
Dense CNF	0.70 ± 0.02	0.76 ± 0.02	0.95 ± 0.03	—	—
Hutch CNF	25.7 ± 2.6	55.4 ± 8.3	—	—	—

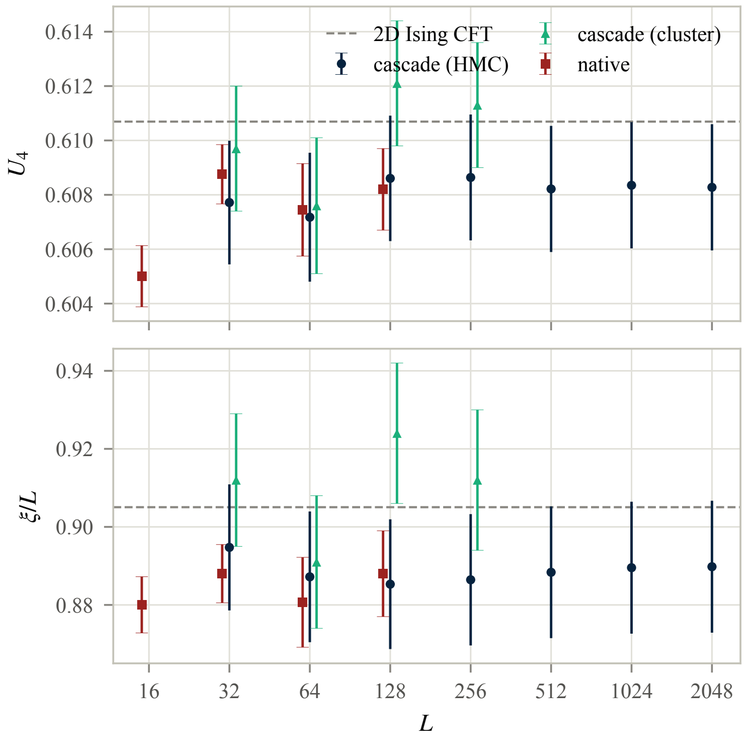
[a] The independence Metropolis–Hastings chain for SR-NF at $L = 256$ has an acceptance rate below 0.2% and sometimes gets stuck in a single configuration, so the τ_{int} estimate unreliable; we therefore omit it.

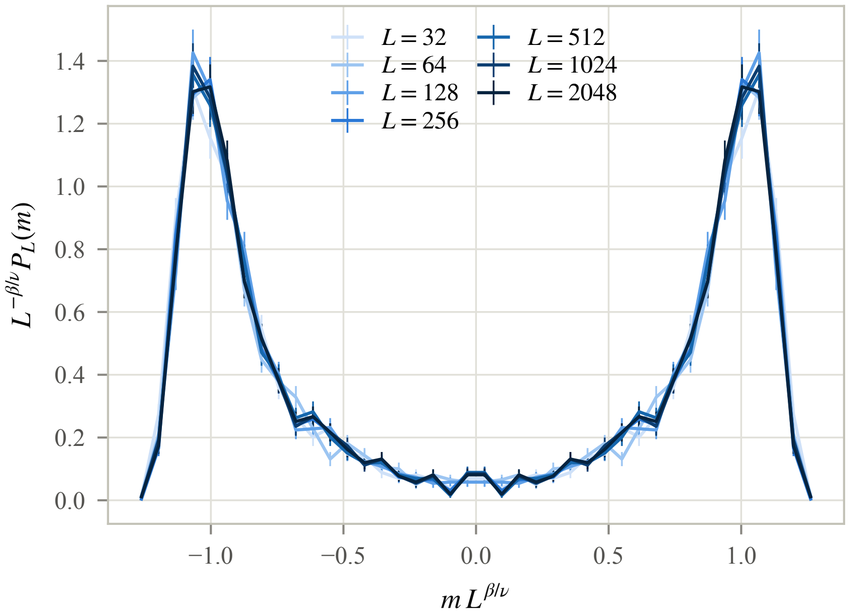
Singha, Kauffmann, Cellini, Jansen, Nakajima, arXiv:2604.10209

Recent developments

Renormalization-guided cascade upscaling for lattice field generation





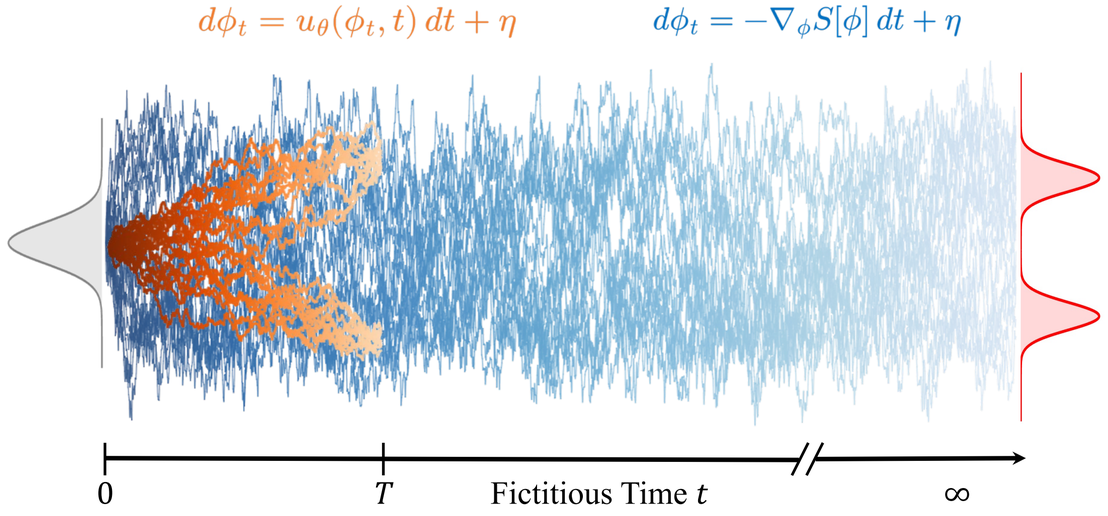


Hasenfratz, Neil, Parato, Schwartz, arxiv:2608.28581

Recent developments

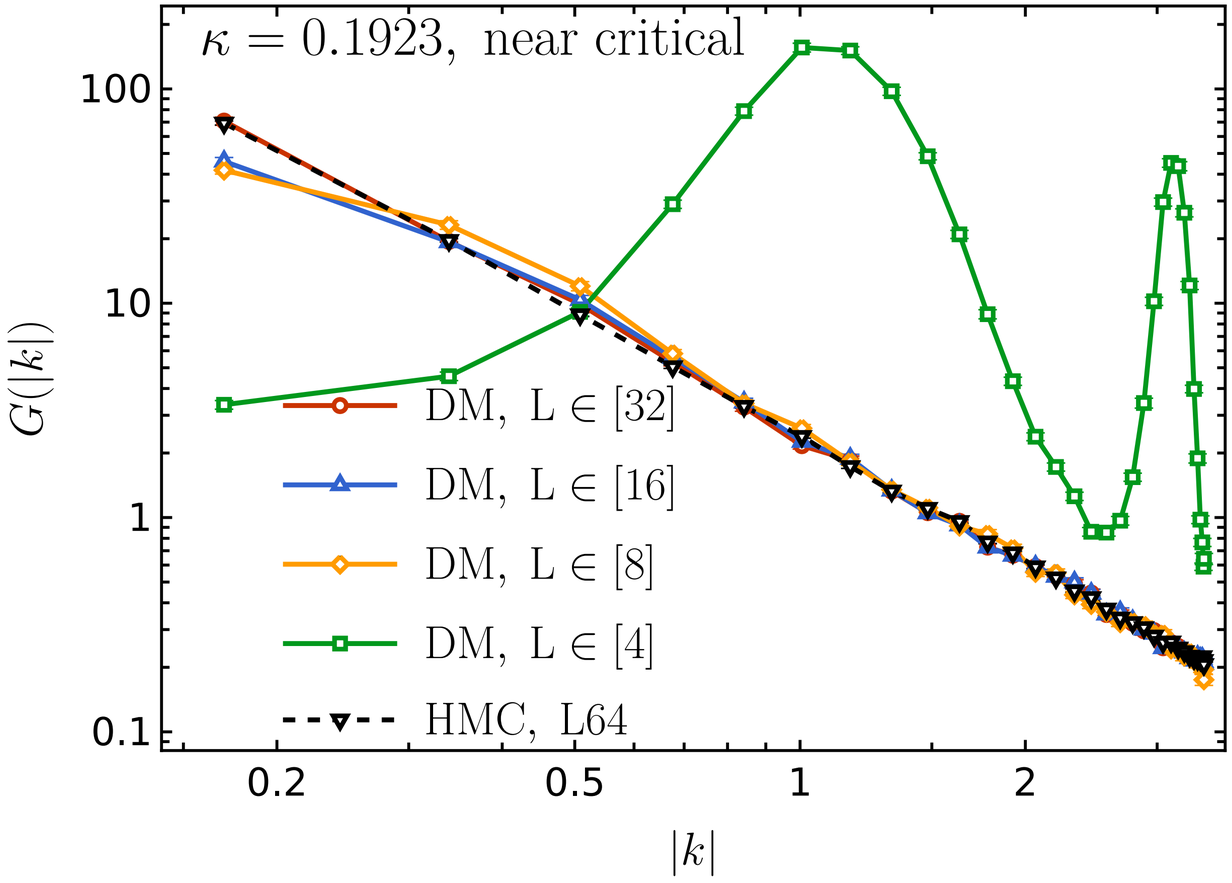
Diffusion Models in Lattice Field Theories

Stochastic Quantization as Optimal Control



Wang, arXiv:2607.21436

Diffusion Models for Sampling Near Criticality in Lattice Field Theories



Tan, Aarts, Habibi, Lucini, Wang, arxiv:2607.08505

Jump back to: Generative architectures & their problems

Generative RGitectures with PIKs in lattice field theories

Physics-informed kernels

PIRG pair

$$\Big(F_t(\phi)\,,\dot\phi_t(\phi)\Big)$$

Sampling

$$\partial_t p_t(\phi) = -\frac{\partial}{\partial\phi}\Big(\dot\phi(\phi)\,p_t(\phi)\Big)$$

$$F_t(\phi) = S_t(\phi)$$

$$\frac{dS_t(\phi)}{dt} + \dot\phi_t(\phi)\,\frac{\partial}{\partial\phi}S_t(\phi) = \frac{\partial}{\partial\phi}\,\dot\phi_t(\phi)$$

$$\partial_t\left[\int \mathcal{D}\phi\, e^{-S_t(\phi)}\right] = 0$$

Ihssen, Kapust, JMP, arxiv:2510.26678

Physics-informed kernels

PIRG pair

$$\left(F_t(\phi)\,,\dot{\phi}_t(\phi)\right)$$

Sampling

$$\partial_t p_t(\phi) = -\frac{\partial}{\partial\phi}\left(\dot{\phi}(\phi)\,p_t(\phi)\right)$$

$$F_t(\phi) = S_t(\phi)$$

Physics-informed kernel

$$\frac{dS_t(\phi)}{dt} + \dot{\phi}_t(\phi)\frac{\partial}{\partial\phi}S_t(\phi) = \frac{\partial}{\partial\phi}\dot{\phi}_t(\phi)$$

PIK-architecture

$$S_{t=0}[\phi] = S_{\text{free/simple}}[\phi]$$

Diffusive RG flow

'Generalised' normalising (RG) flow

$$S_{t=1}[\phi] = S[\phi]$$

Ihssen, Kapust, JMP, arxiv:2510.26678

Physics-informed kernels

PIRG pair

$$\left(F_t(\phi)\,,\dot{\phi}_t(\phi)\right)$$

Sampling

$$\partial_t p_t(\phi) = -\frac{\partial}{\partial\phi}\left(\dot{\phi}(\phi)\,p_t(\phi)\right)$$

$$F_t(\phi) = S_t(\phi)$$

Physics-informed kernel

$$\frac{dS_t(\phi)}{dt} + \dot{\phi}_t(\phi)\frac{\partial}{\partial\phi}S_t(\phi) = \frac{\partial}{\partial\phi}\dot{\phi}_t(\phi)$$

PIK-architecture

$$S_{t=0}[\phi] = S_{\text{free/simple}}[\phi]$$

Diffusive RG flow

'Generalised' normalising (RG) flow

$$S_{t=1}[\phi] = S[\phi]$$

Normalisation problem solved via $\dfrac{d\,\log\mathcal{N}_t}{dt} = \dfrac{d\,\hat{S}_t(\phi)}{dt} - \left[\dfrac{\partial}{\partial\phi_i} - \dfrac{\partial\hat{S}_t(\phi)}{\partial\phi_i}\right]\dot{\phi}_{t,i}(\phi)$

Ihssen, Kapust, JMP, arxiv:2510.26678

See also talk of Kenji

PIRGs & PIKs

Generative PIRGs (general PIKs)

encode Normalising Flows

Diffusion Models

Stochastic Quantisation (real and complex (kernel) Langevin)

Lefschetz thimbles (‘you may seek it with thimbles—and seek it with care’)

⋮

Physics-informed kernels

PIRG pair

$$\Big(F_t(\phi)\,,\dot\phi_t(\phi)\Big)$$

Sampling

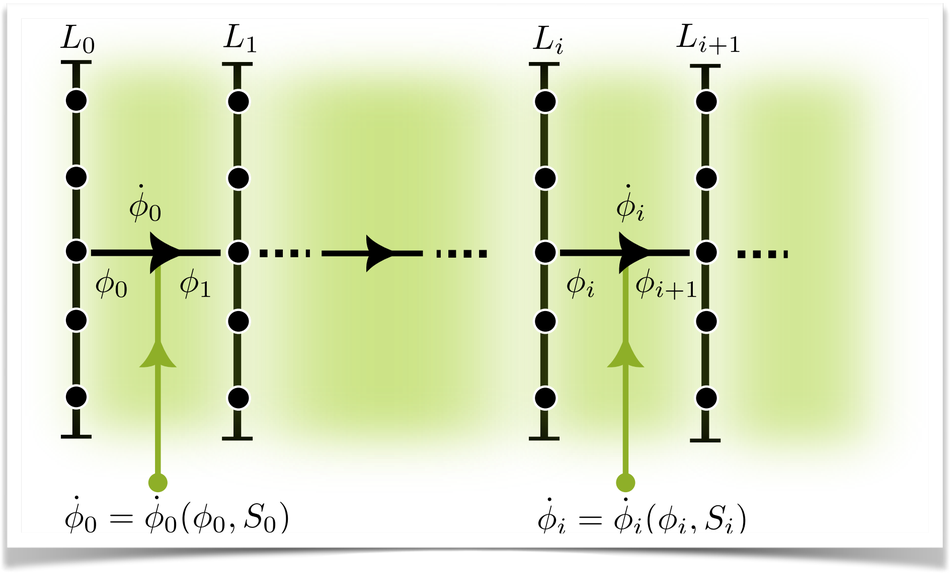
$$\partial_t p_t(\phi) = -\frac{\partial}{\partial\phi}\Big(\dot\phi(\phi)\,p_t(\phi)\Big)$$

Wegner flow

$$F_t(\phi) = S_t(\phi)$$

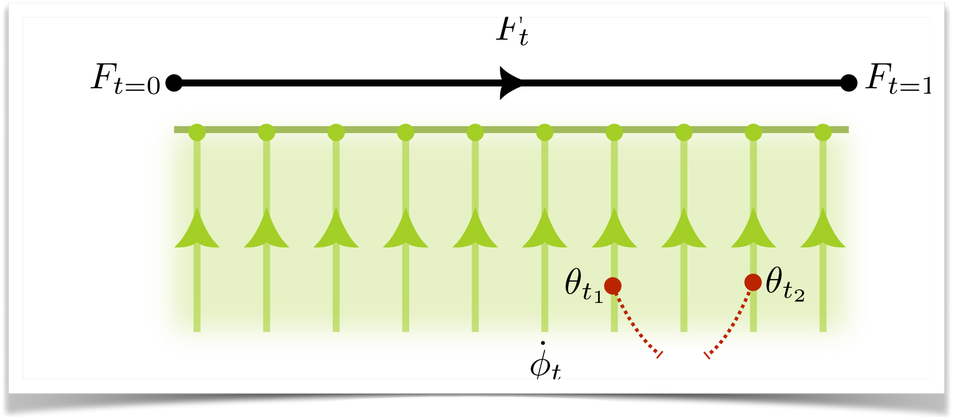
$$\frac{dS_t(\phi)}{dt} + \dot\phi_t(\phi)\,\frac{\partial}{\partial\phi}S_t(\phi) = \frac{\partial}{\partial\phi}\,\dot\phi_t(\phi)$$

PIK-architecture



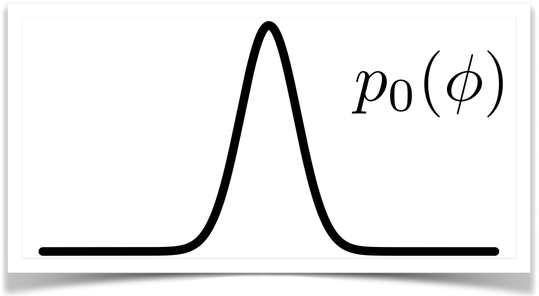
tasks are separable

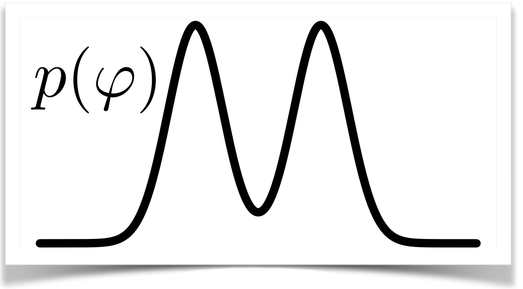
completely parallelisable



Ihssen, Kapust, JMP, arxiv:2510.26678

Sampling with physics-informed kernels







Efficient sampling



$\{\phi_i\}$

Sample drawn from $p_0(\phi)$

Efficient sampling

$$\langle \mathcal{O}(\varphi) \rangle_{p(\varphi)} = \sum_i \mathcal{O}\big(\varphi(\phi_i)\big)$$

$$\varphi(\phi) = \phi + \int_0^1 ds\, \dot{\phi}_s(\phi_s)$$

Sampling problem

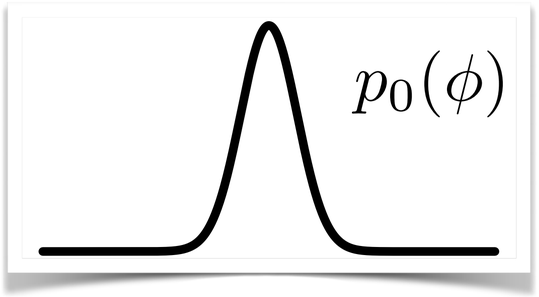
$$\langle \mathcal{O}(\varphi) \rangle_{p(\varphi)}$$

All information is in the PIK $\dot{\phi}_t$

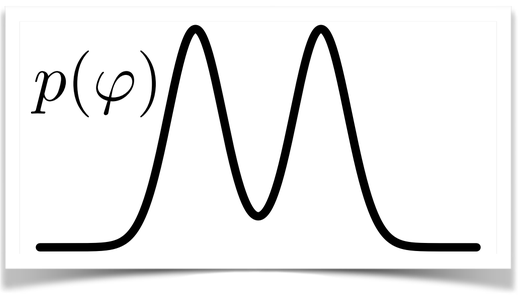
Ihssen, Kapust, JMP, arxiv:2510.26678

Sampling with physics-informed kernels

Example: zero-dimension ϕ^4- theory

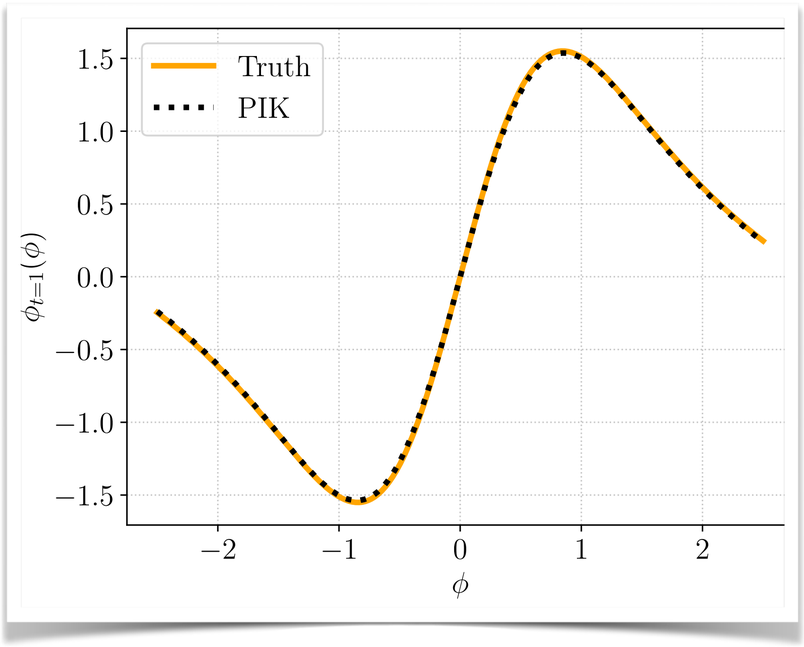


$$\hat{S}_t(\phi) = \frac{1}{2}m^2(t)\phi^2 + \frac{\lambda(t)}{4}\phi^4$$



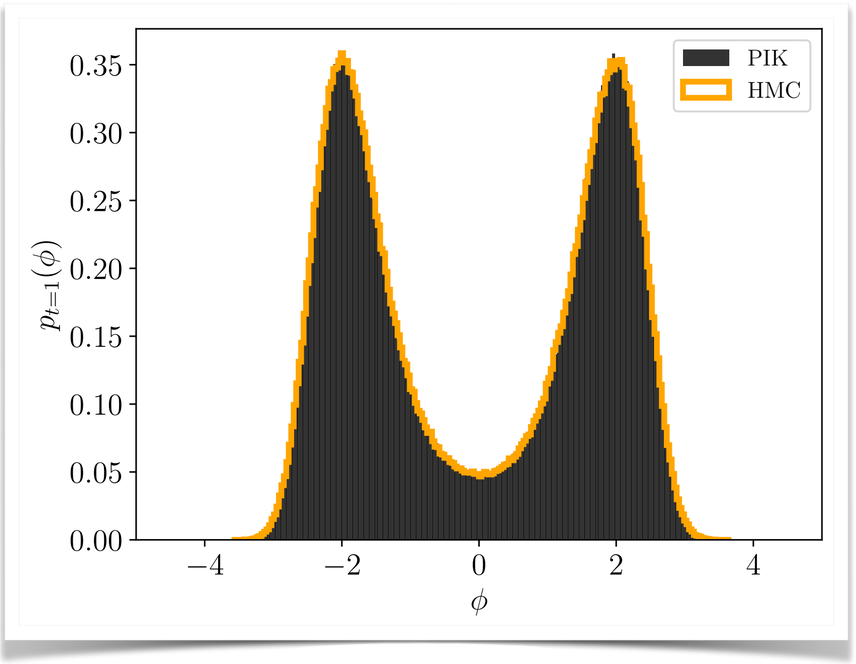


$$\left(m_0^2\,,\,\lambda_0\right) = (1\,,\,0) \longrightarrow \left(m_t^2\,,\,\lambda_t\right) = \left(m_0^2 + t\,(m_1^2 - m_0^2)\,,\,\lambda_0 + t(\lambda_1 - \lambda_0)\right) \longrightarrow \left(m_1^2\,,\,\lambda_1\right) = (-2\,,\,1/2)$$



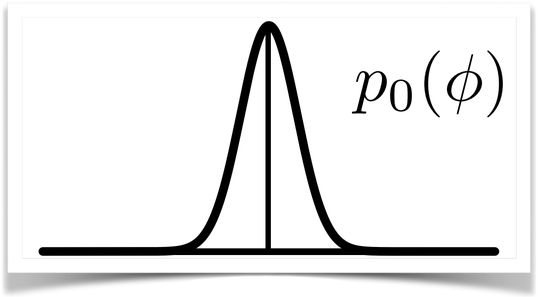
no overlap problem

PIK-architectures are weight-preserving



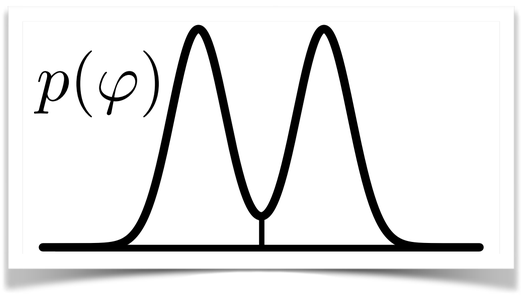
Ihssen, Kapust, JMP, arxiv:2510.26678

Sampling with physics-informed kernels



$$\int_{\mathcal{M}_0} p_0 \, d\mu_0 = \int_{\mathcal{M}_t} p_t \, d\mu_t$$

$$\mathcal{M}_0 \xrightarrow{\dot{\phi}} \mathcal{M}_t$$



Simple: easy to sample

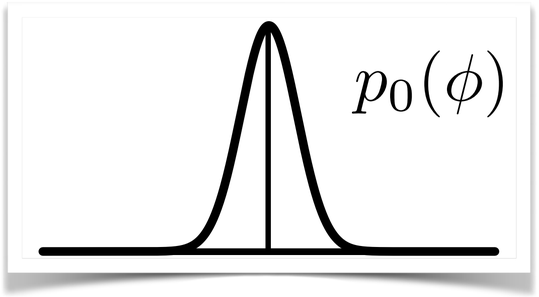
Full distribution

Sampling observables

$$\langle \mathcal{O}(\hat{\phi}) \rangle_p = \sum_{\{\phi_0\}_{p_0}} \mathcal{O}\left(\varphi(\phi_0)\right)$$

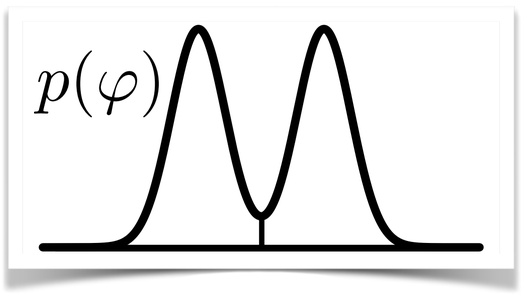
$$\varphi(\phi_0) = \phi_0 + \int_0^1 ds \, \dot{\phi}_s(\phi_s)$$

Sampling with physics-informed kernels



$$\int_{\mathcal{M}_0} p_0 \, d\mu_0 = \int_{\mathcal{M}_t} p_t \, d\mu_t$$

$$\mathcal{M}_0 \xrightarrow{\dot{\phi}} \mathcal{M}_t$$



Simple: easy to sample

Full distribution

Weight-preserving property

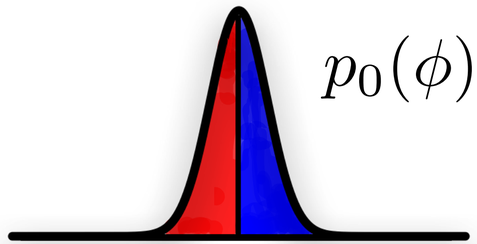
$$\int_{\mathcal{I}_0} p_0 \, d\mu_0 = \int_{\mathcal{I}_t} p_t \, d\mu_t$$

$\mathcal{I}_0 \subset \mathcal{M}_0$

$\mathcal{I}_1 \subset \mathcal{M}_1$

$$\mathcal{I}_0 \xrightarrow{\dot{\phi}} \mathcal{I}_t$$

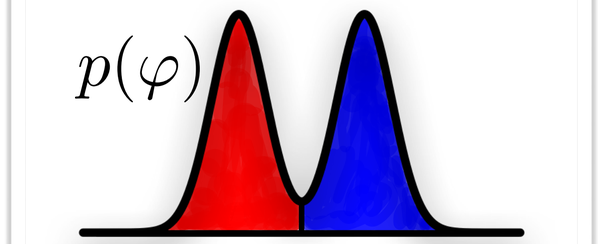
Sampling with physics-informed kernels



Simple: easy to sample

$$\int_{\mathcal{M}_0} p_0 \, d\mu_0 = \int_{\mathcal{M}_t} p_t \, d\mu_t$$

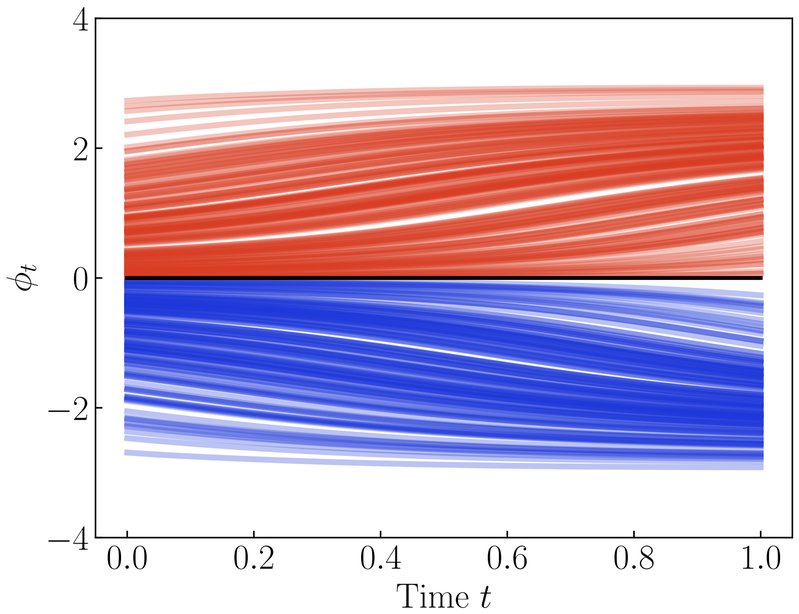
$$\mathcal{M}_0 \xrightarrow{\dot{\phi}} \mathcal{M}_t$$



Full distribution

Weight-preserving property

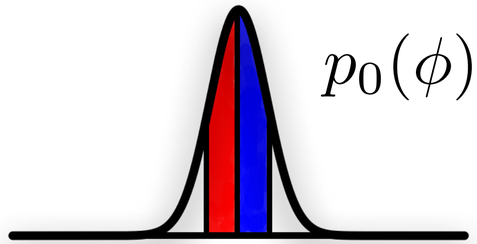
$\mathcal{I}_0 \subset \mathcal{M}_0$



$\mathcal{I}_1 \subset \mathcal{M}_1$

Ihssen, Kapust, JMP, arxiv:2603.03159, arxiv:2510.26678

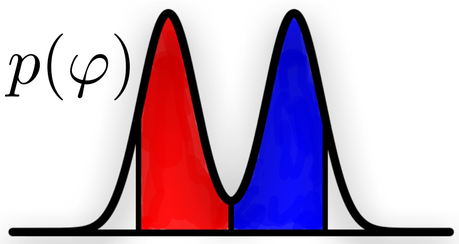
Sampling with physics-informed kernels



Simple: easy to sample

$$\int_{\mathcal{M}_0} p_0 \, d\mu_0 = \int_{\mathcal{M}_t} p_t \, d\mu_t$$

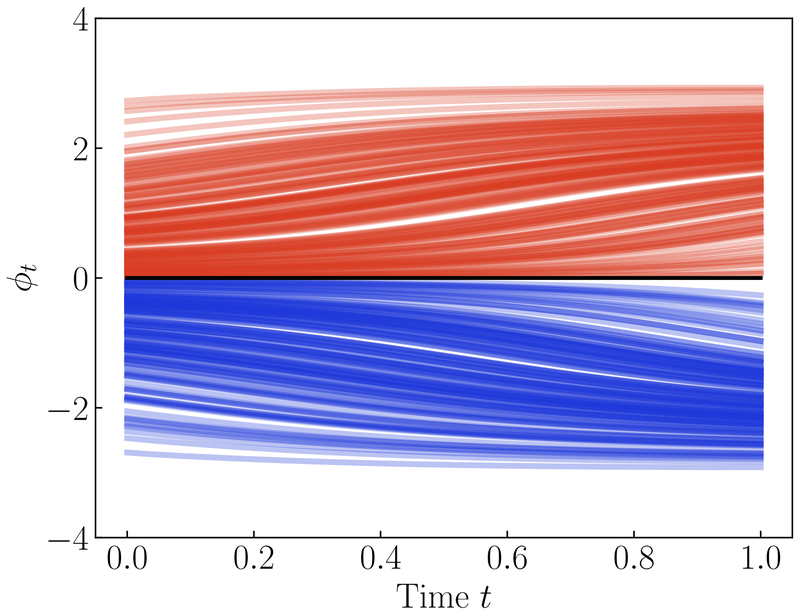
$$\mathcal{M}_0 \xrightarrow{\dot{\phi}} \mathcal{M}_t$$



Full distribution

Weight-preserving property

$\mathcal{I}_0 \subset \mathcal{M}_0$



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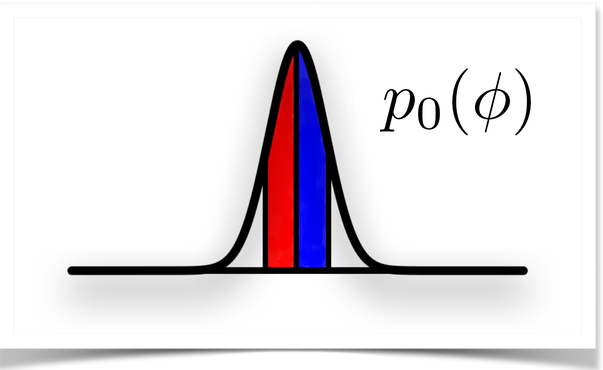
Ihssen, Kapust, JMP, arxiv:2603.03159, arxiv:2510.26678

Sampling with physics-informed kernels



Simple: easy to sample

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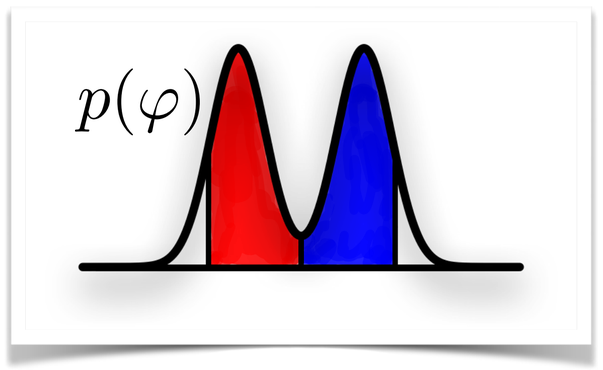
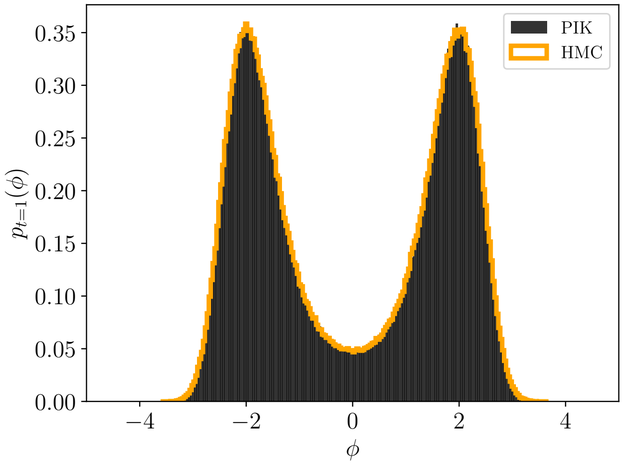
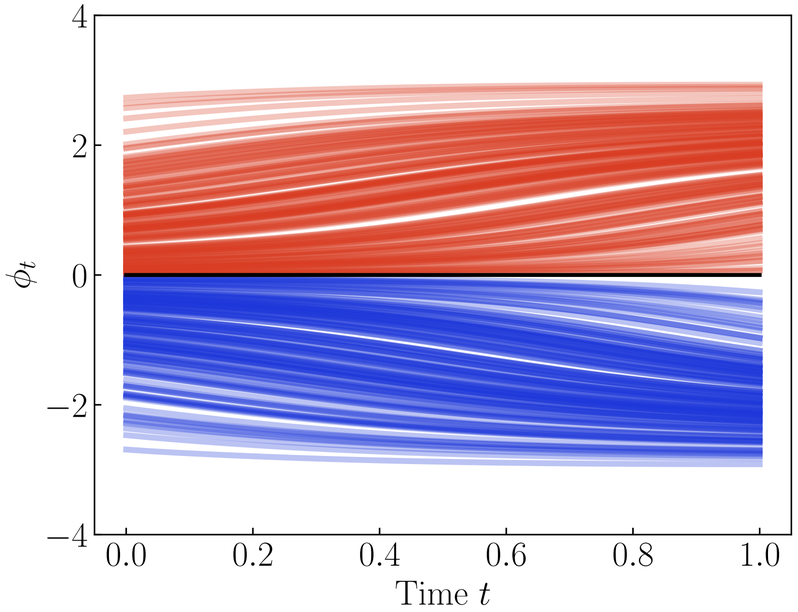


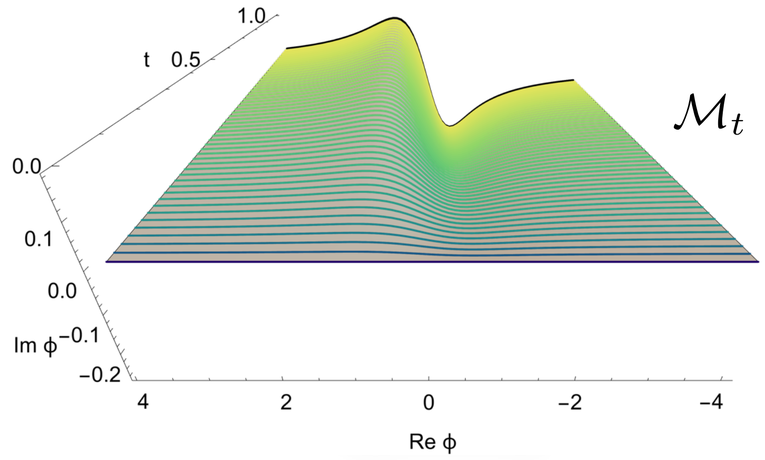
Full distribution



No overlap problems
&
no mode collaps

Weight-preserving property





$$p_0 \, d\mu_0 \in \mathbb{R} \longrightarrow p_t \, d\mu_t \in \mathbb{R}$$

Crucial for complex sampling

Jump forward to: Solving sign problems with complex PIKs

PIKs step by step

aka

If PIKs can fly

Sampling with physics-informed kernels

$$\frac{dS_t(\phi)}{dt} = \left[\frac{\partial}{\partial\phi} - \frac{\partial S_t(\phi)}{\partial\phi}\right]\dot{\phi}_t(\phi) \qquad \frac{d\phi_t}{dt} = \dot{\phi}_t(\phi_t)$$

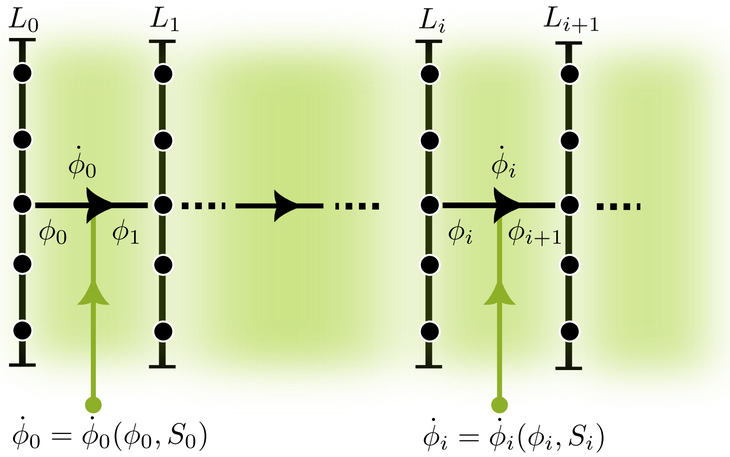
Linear partial differential equation for the kernel $\dot{\phi}_t(\phi)$ for a freely chosen path $S_t(\phi)$

Parametrisation of the action

$$S_t(\phi) = \hat{S}_t(\phi) + \log\mathcal{N}_t$$

$$\hat{S}_t(\phi) = \sum_i c_{i,t}\mathcal{O}_{i,t}(\phi)$$

Change in normalisation is expensive



Parametrisation of the kernel

$$\dot{\phi}_t(\phi) = \sum_j k_{j,t}K_{j,t}(\phi)$$

complete basis $K_{j,t}(\phi)$

expansion coefficients $k_{j,t}$

(i) Independent kernels

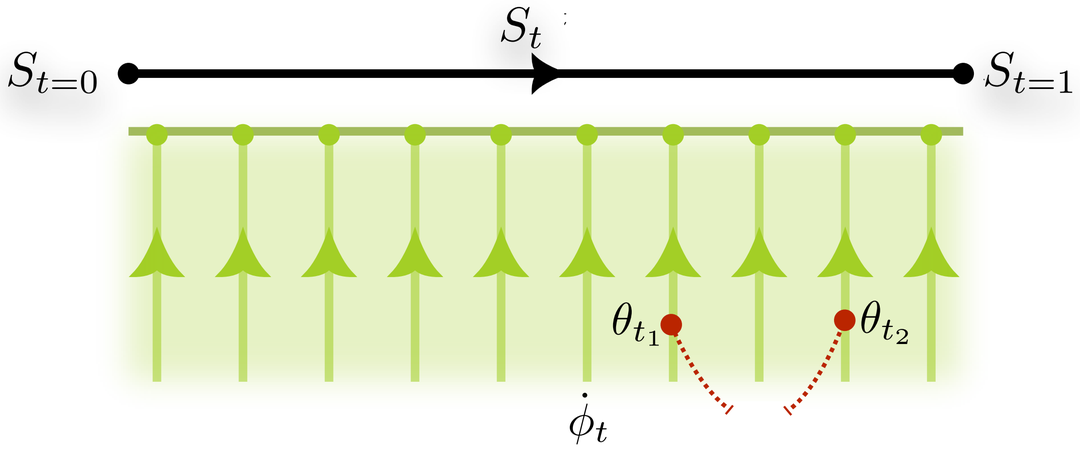
(ii) OOD resolution

(iii) Optimisation

Ihssen, Kapust, JMP, arxiv:2510.26678

Sampling with physics-informed kernels

(ii) OOD resolution 1 (curse of limited information)



$$\mathcal{L}_\chi \, \dot{\phi}_t(\phi) = \frac{d\hat{S}_t(\phi)}{dt} - \frac{d\hat{S}(\chi)}{dt}$$

Rough approximation $\dot{\phi}_t^{(0)}(\phi)$

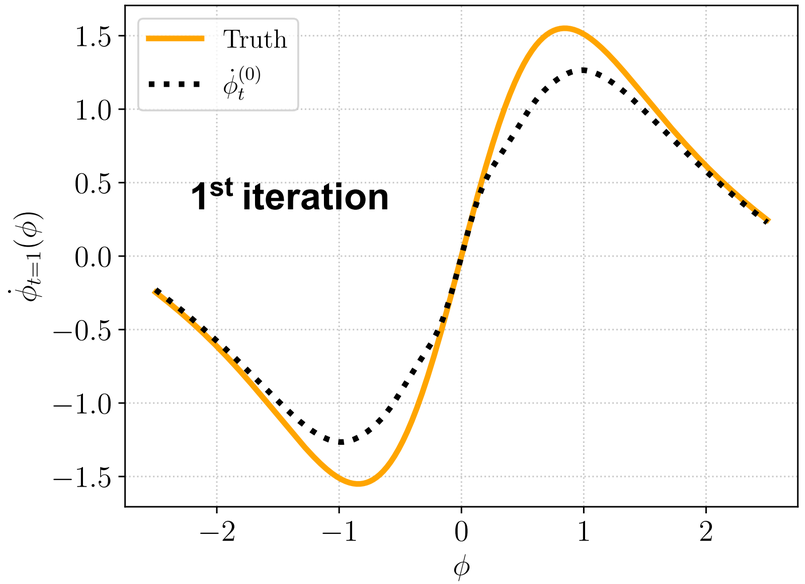
Error $r_\chi^{(0)}(\phi)$

$$\mathcal{L}_\chi \Delta\dot{\phi}_t^{(0)}(\phi) = r_\chi^{(0)}(\phi)$$

Radial basis functions with compact support

Kansa 1990 Wendland 1995 DeVore, Temlyakov 1996

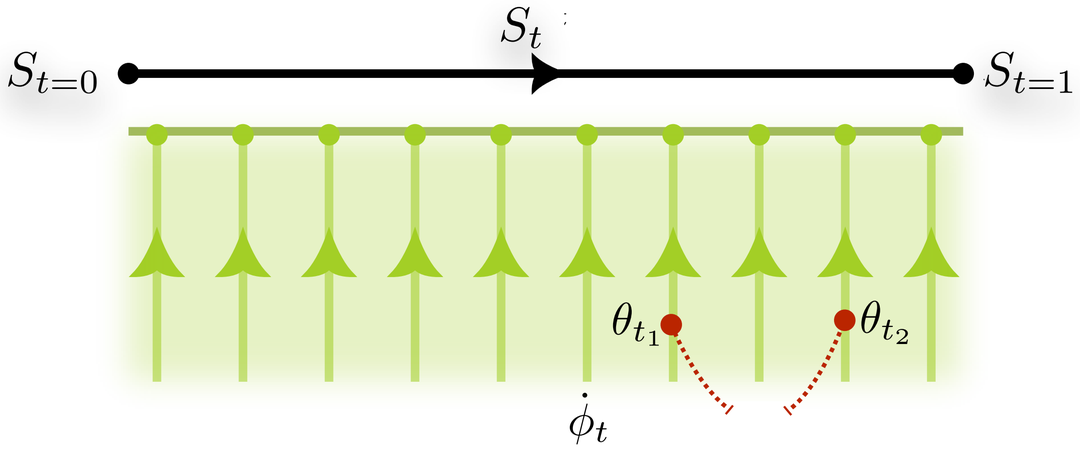
$$\dot{\phi}_t^{(1)}(\phi) = \dot{\phi}_t^{(0)}(\phi) + \Delta\dot{\phi}_t^{(0)}(\phi)$$



Ihssen, Kapust, JMP, arxiv:2510.26678

Sampling with physics-informed kernels

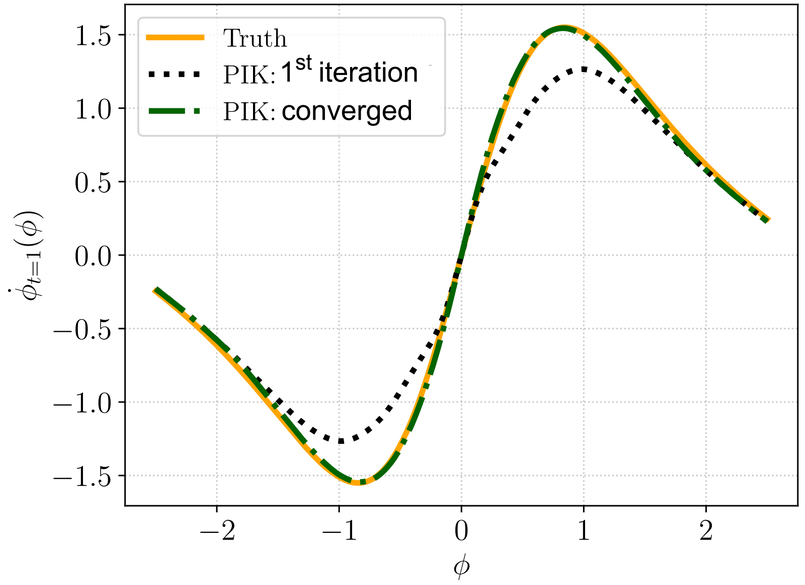
(ii) OOD resolution 1 (curse of limited information)



$$\mathcal{L}_\chi \, \dot{\phi}_t(\phi) = \frac{d\hat{S}_t(\phi)}{dt} - \frac{d\hat{S}(\chi)}{dt}$$

Rough approximation $\dot{\phi}_t^{(0)}(\phi)$

Error $r_\chi^{(0)}(\phi)$



$$\mathcal{L}_\chi \Delta\dot{\phi}_t^{(0)}(\phi) = r_\chi^{(0)}(\phi)$$

Radial basis functions with compact support

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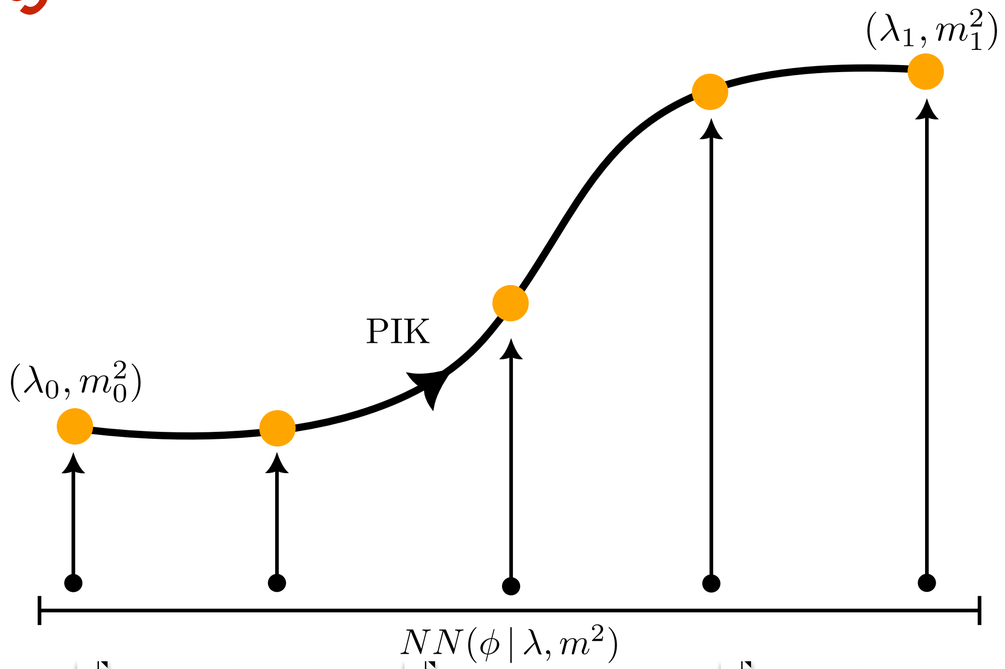
Ihssen, Kapust, JMP, arxiv:2510.26678

Some thoughts

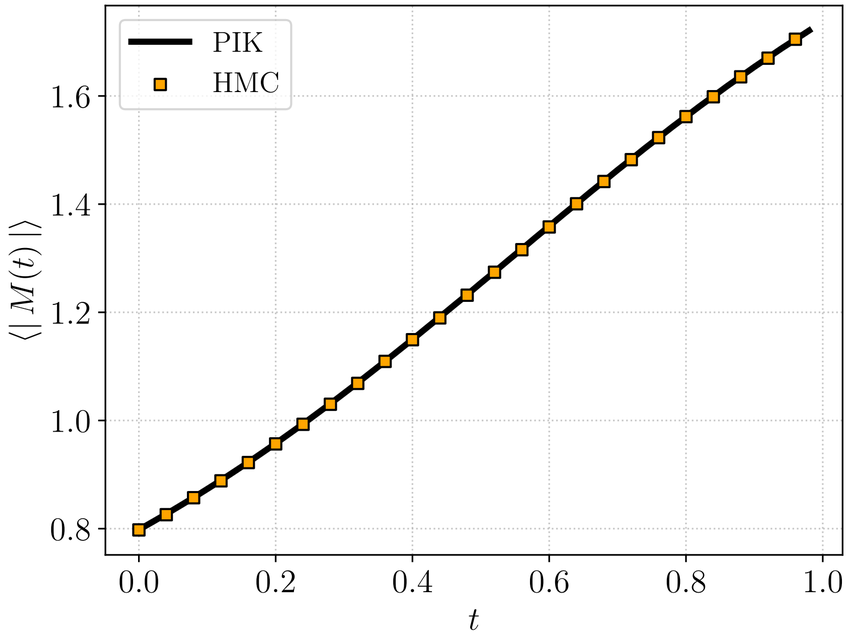
Sampling with physics-informed kernels

(iii) Optimisation

Parameter-conditional flows



e.g. Singha et al. 2022 Gerdes et al. 2023 Wang et al. 2024



Choose the path for the action that covers the required parameters efficiently

Ihssen, Kapust, JMP, arxiv:2510.26678

Some thoughts

Sampling with physics-informed kernels

(iii) Optimisation

Functional optimisation of irreducible cumulants

Physics-informed RG flows

Sampling | **Optimisation**

$$\partial_t p_t[\phi] = -\int_x \frac{\delta}{\delta\phi(x)}\Big(\dot\phi[\phi]\, p_t[\phi]\Big) \quad \longleftrightarrow \quad \Big(\partial_t + \int_x \dot\phi \frac{\delta}{\delta\phi}\Big)\,\Gamma_t[\phi] = \frac{1}{2}\mathrm{Tr}\, G_t[\phi]\,\partial_t R_t + \mathrm{Tr}\, G_t[\phi]\,\frac{\delta\dot\phi}{\delta\phi} R_t$$

Legendre transform

Wegner flow

Wegner, J. Phys. C 7 (1974) 2098

Generalised flow equation

JMP, *Annals Phys.* 322 (2007) 2831

$$G_t[\phi](p,q) = \langle \hat\phi(p)\,\hat\phi(q)\rangle_c = \frac{1}{\frac{\delta^2\Gamma_t[\phi]}{\delta\phi^2} + R_t}(p,q)$$

Key observation: solve for PIRG pair

$$\Big(F_t[\phi]\,,\dot\phi_t[\phi]\Big)$$

'Seemingly trivial' but extremely powerful

$$F_t[\phi] = S[\phi]$$

Extreme example

'Generalised' normalising (RG) flows

Ihssen, JMP, *Annals Phys.* 481 (2025) 170177

Ihssen, Kapust, JMP, arxiv:2510.26678

Some thoughts

Sampling with physics-informed kernels

(ii) OOD resolution 2 (curse of dimensionality)

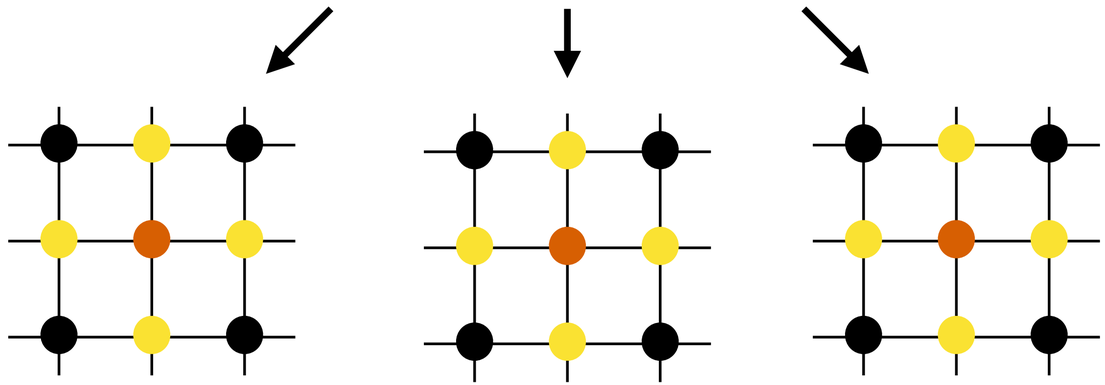


Higher-dimensional ϕ^4- theories

Truncation

Reduce the volume scaling of the basis → Reduce the curse of dimensionality

$$\left[\frac{\partial}{\partial\phi} - \frac{\partial S_t(\phi)}{\partial\phi}\right]\dot{\phi}_t(\phi) = \frac{dS_t(\phi)}{dt}$$



Local action ensures that the number of basis functions grows with the dimension and not with the volume

Ihssen, Kapust, JMP, arxiv:2510.26678

Some thoughts

Sampling with physics-informed kernels

(ii) OOD resolution 2 (curse of dimensionality)



Higher-dimensional ϕ^4- theories

Physics-Informed Basis: Hermite Polynomials

$$\left[\frac{\partial}{\partial \phi} - \frac{\partial S_t(\phi)}{\partial \phi} \right] \dot{\phi}_t(\phi) = \frac{dS_t(\phi)}{dt}$$

$$\left[\frac{\partial^2}{\partial \phi^2} - \frac{\partial S_t(\phi)}{\partial \phi} \frac{\partial}{\partial \phi} \right] \Omega_t(\phi) = -\frac{dS_t(\phi)}{dt}$$

Helmholtz decomposition

$$\dot{\phi}_t(\phi) = -\frac{\partial}{\partial \phi} \Omega_t(\phi) + H_t(\phi)$$

$$0 = \frac{\partial}{\partial \phi} \left[p_t(\phi) H_t(\phi) \right]$$

Köhler et al. 2020

Albergo et al. 2025

Ihssen, Kapust, JMP, arxiv:2510.26678

Some thoughts

Sampling with physics-informed kernels

(ii) OOD resolution 2 (curse of dimensionality)



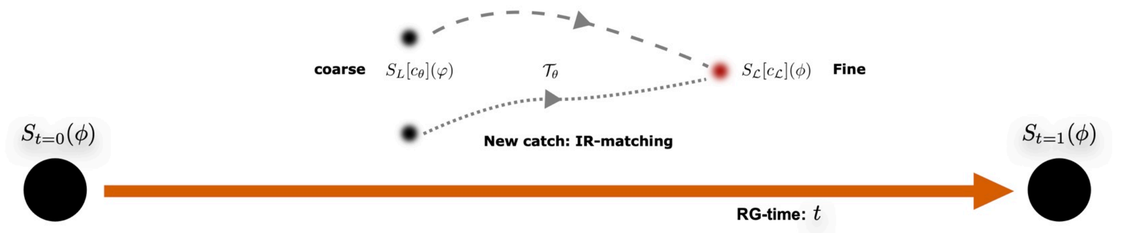
Higher-dimensional ϕ^4- theories

Use RG-nature of the map

Renzo Kapust

Sampling with RGitectures in lattice field theories

Normalising Flow



$$\phi = \mathcal{T}_\theta(\varphi)$$

θ : (Neural Network) Parameters

$$\tilde{p}_{t=1}(\phi) = p_{t=0}(\varphi)\,|\det J_{\mathcal{T}}(\varphi)|^{-1}$$

$$\text{Minimize } D_{KL}(\tilde{p}\|p) = \int D\phi\,\tilde{p}(\phi)\log\frac{\tilde{p}(\phi)}{p(\phi)}$$



Free Gaussian Theory



Interacting Theory

Lüscher 2009 Rezende, Mohamed 2015

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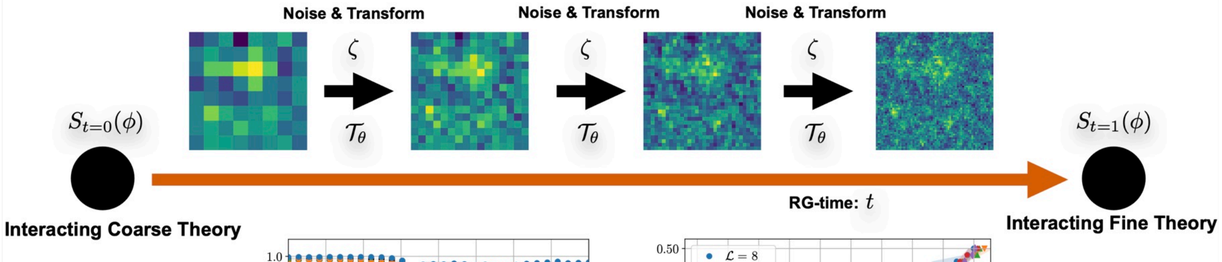
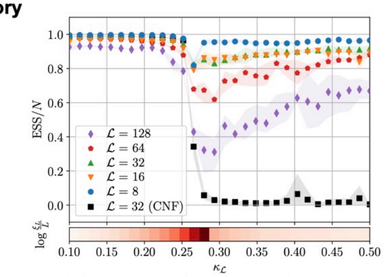
Bauer, Kapust, JMP, Temmen, *SciPost Phys.* 19 (2025) 3, 077

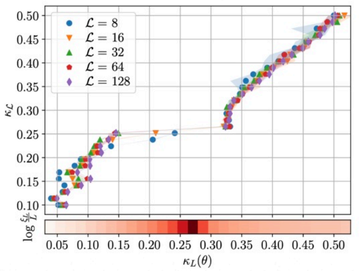
Renzo Kapust

Sampling with RGitectures in lattice field theories

Super-resolving Normalising Flow with IR-matching







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Bauer, Kapust, JMP, Temmen, *SciPost Phys.* 19 (2025) 3, 077

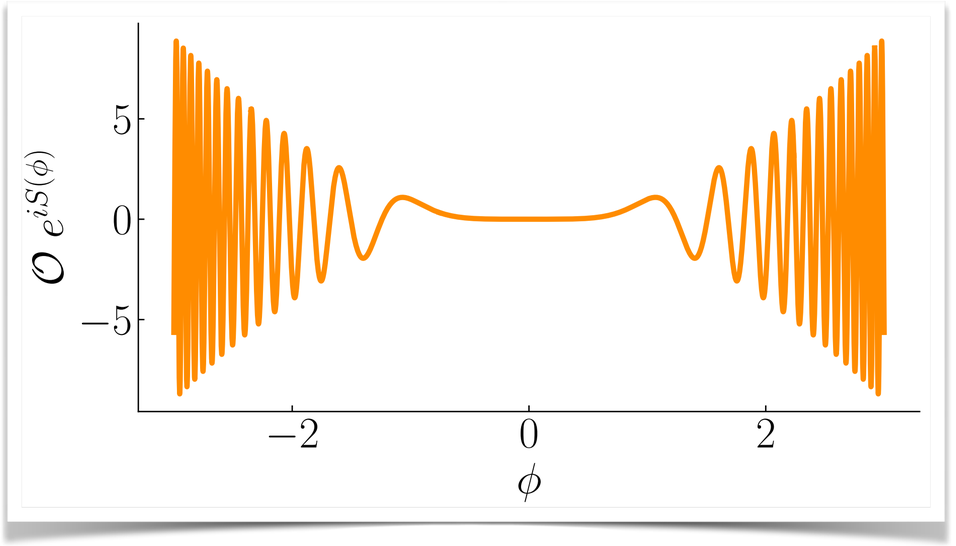
Ihssen, Kapust, JMP, arxiv:2510.26678

Jump back to: PIKs step by step aka If PIKs can fly

Real sign problems: Ihssen, Kapust, JMP, arXiv:2603.03159

Complex sign problems: Fukushima, Ihssen, Kapust, JMP, arXiv:2609.XXXXX

Solving sign problems with complex PIKs



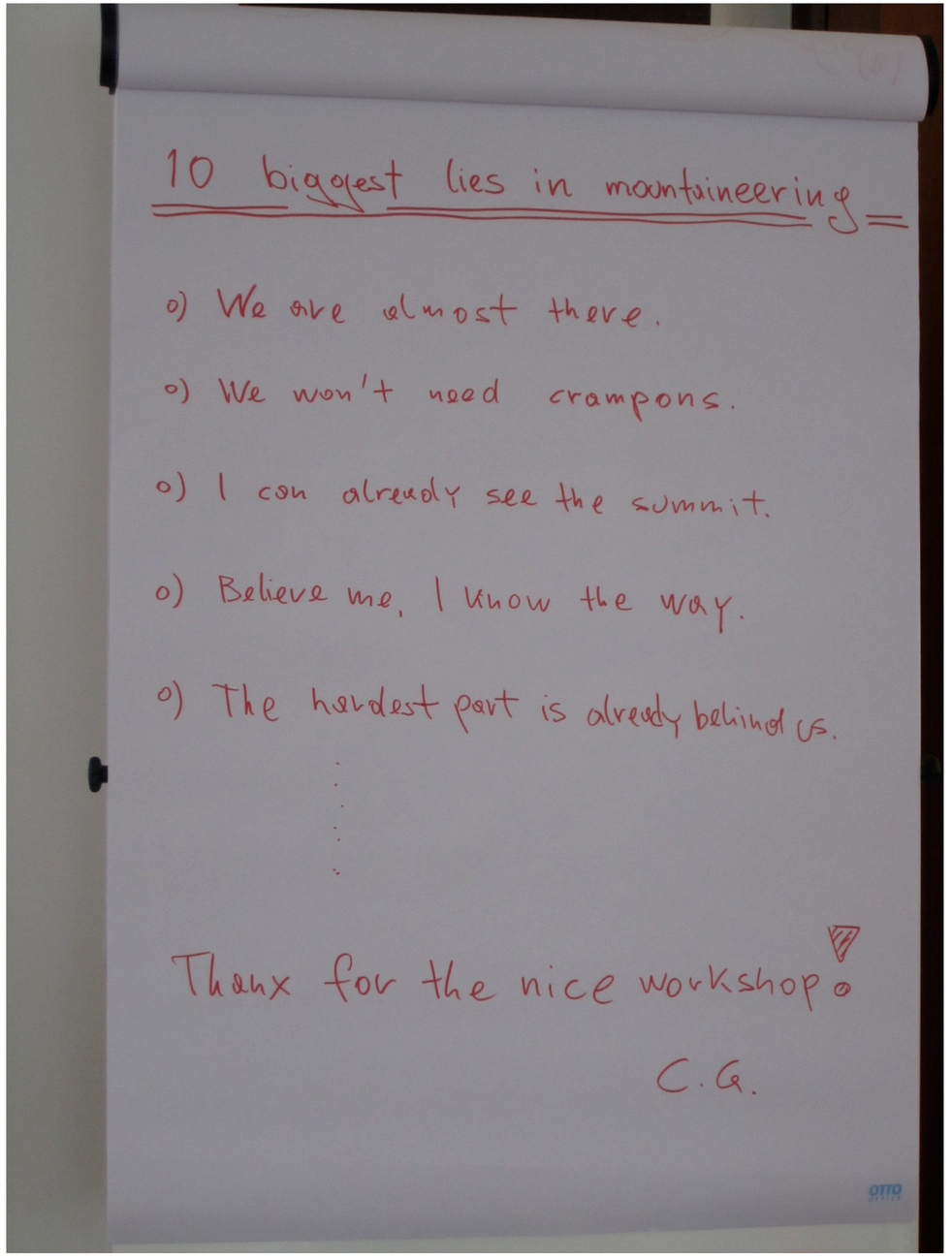
Real sampling

$$\langle \mathcal{O}(\varphi) \rangle_{p(\varphi)} = \sum_i \mathcal{O}(\varphi(\phi_i)) \quad \longleftarrow \quad \{\phi_i\}$$ **Sample drawn from** $p_0(\phi)$

Trying to solve the sign problem is like trying to climb Mt Everest

…and ending up on another peak

Philippe de Forcrand
DELTA 2013 Heidelberg

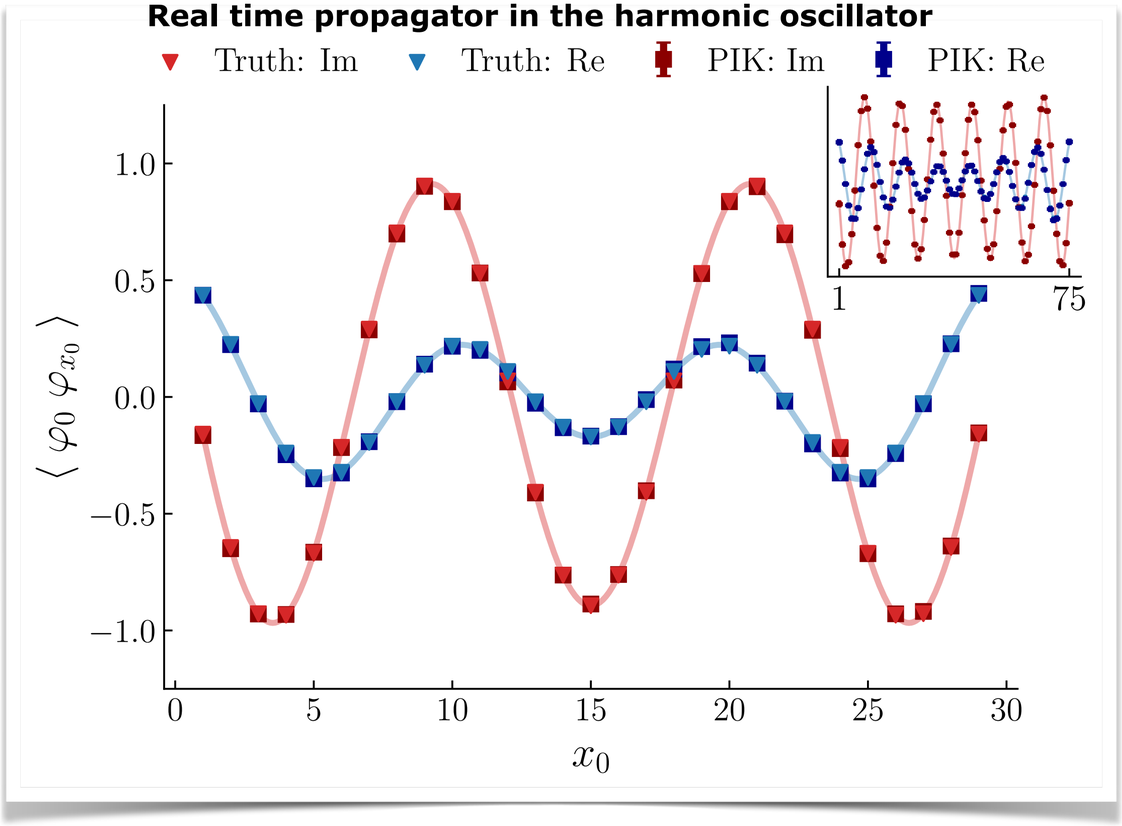


Christof Gattringer
DELTA 2013 Heidelberg

Solving real sign problems

Real PIKfolds

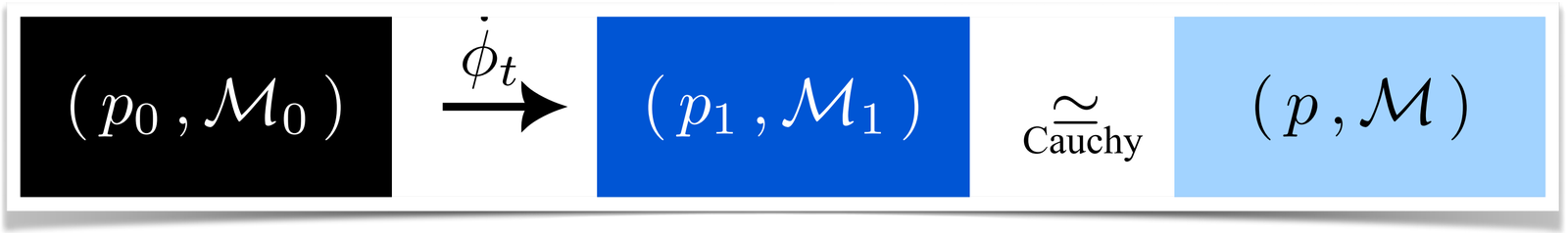
See talk of Friederike



See talk of Friederike

Solving real sign problems

Real sampling



$$\sum_i \mathcal{O}\big(\varphi(\phi_i)\big) = \langle \mathcal{O}(\varphi)\rangle_{p(\varphi),\mathcal{M}_1} = \langle \mathcal{O}(\varphi)\rangle_{p(\varphi),\mathcal{M}}$$

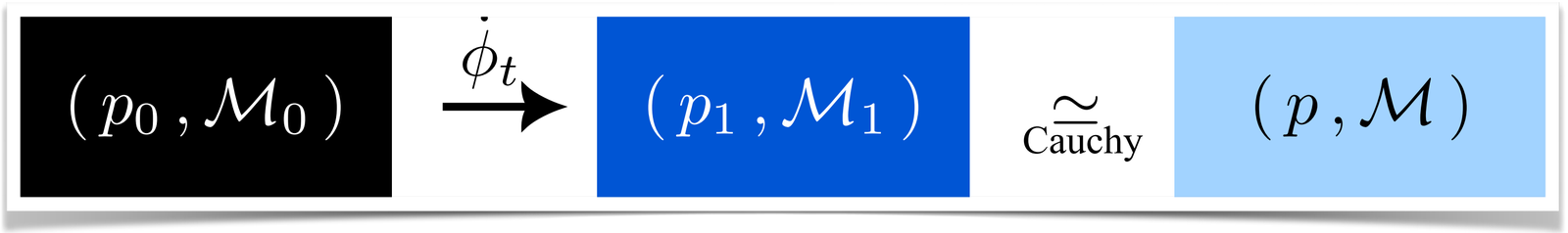
$$\varphi(\phi) = \phi + \int_0^1 ds\, \dot{\phi}_s(\phi_s)$$

Real sign problems: Ihssen, Kapust, JMP, arXiv:2603.03159

See talk of Friederike

Solving real sign problems

Real sampling



$$\sum_i \mathcal{O}\big(\varphi(\phi_i)\big) = \langle \mathcal{O}(\varphi)\rangle_{p(\varphi),\mathcal{M}_1} = \langle \mathcal{O}(\varphi)\rangle_{p(\varphi),\mathcal{M}}$$

$$\varphi(\phi) = \phi + \int_0^1 ds\, \dot{\phi}_s(\phi_s)$$

No poles on the PIKfold

Real sign problems: Ihssen, Kapust, JMP, arXiv:2603.03159

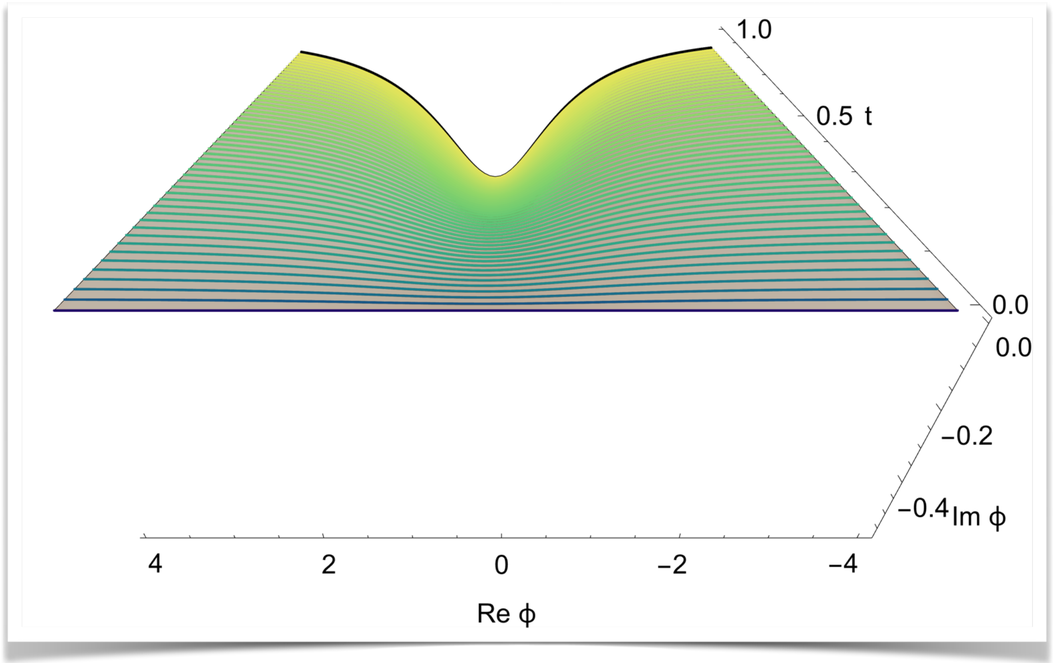
See talk of Friederike

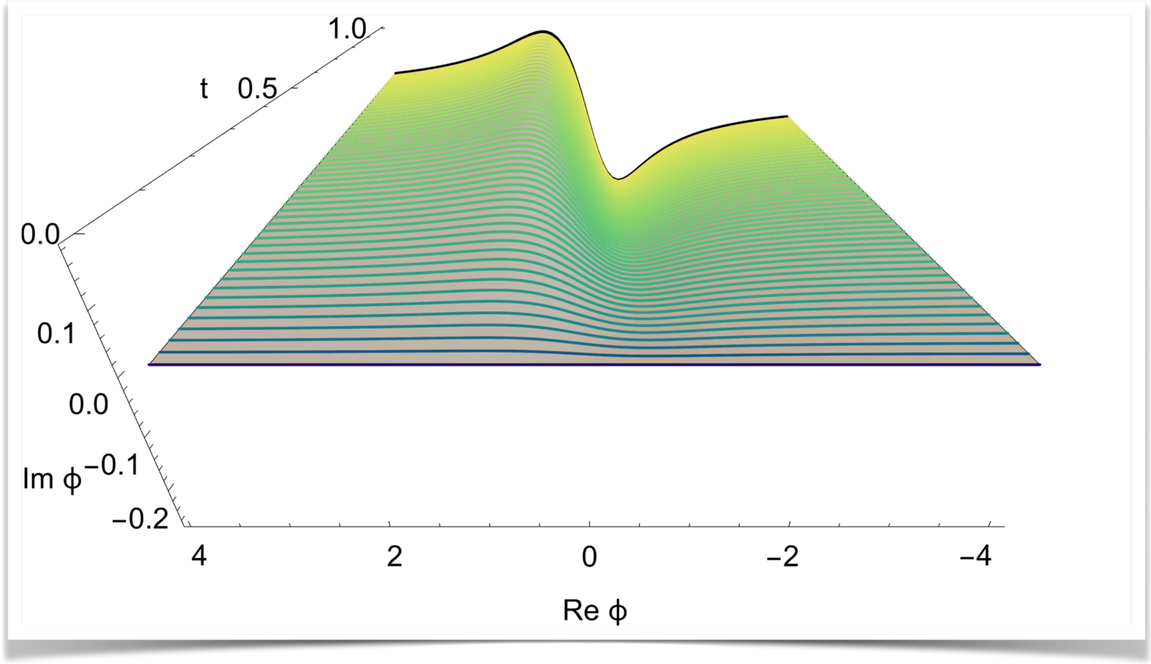
Solving real sign problems

Jump forward to: Solving sign problems with complex PIKs

Real PIKfolds $\{M_t\}$

$$p_t \, d\mu_t \in \mathbb{R}$$





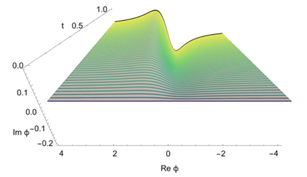
$$\hat{S}_t(\phi) = \frac{1}{2}\phi^2 + \frac{1}{4}\phi^4 + (1+it)\phi$$

$$\hat{S}_t(\phi) = -(1+it)\phi^2 + \phi^4 + \frac{1}{2}\phi$$

Ihssen, Kapust, JMP, arxiv:2603.03159

See talk of Friederike

Solving real sign problems



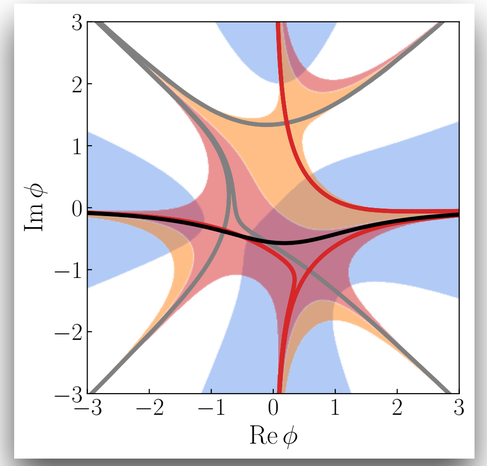
Real PIKfolds

PIK architectures and Lefschetz thimbles

$$\hat{S}_t(\phi) = \frac{1}{2}\phi^2 + \frac{1}{4}\phi^4 + (1+it)\phi$$

$$\hat{S}_t(\phi) = \frac{1}{2}m^2\phi^2 + it\phi$$

Manifold $\mathcal{M}_1$



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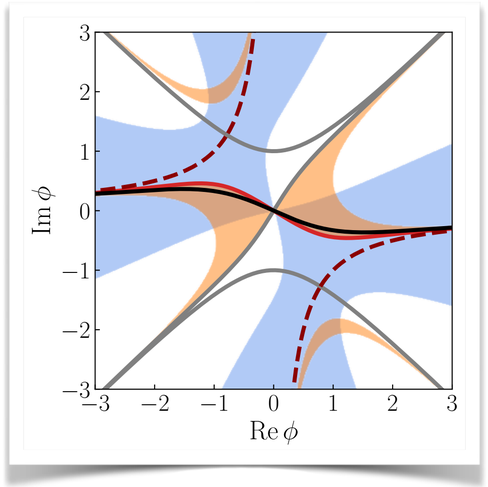
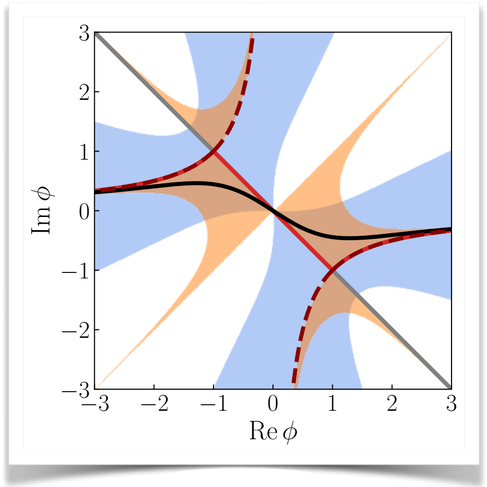


$m^2 = 1 + 2i$



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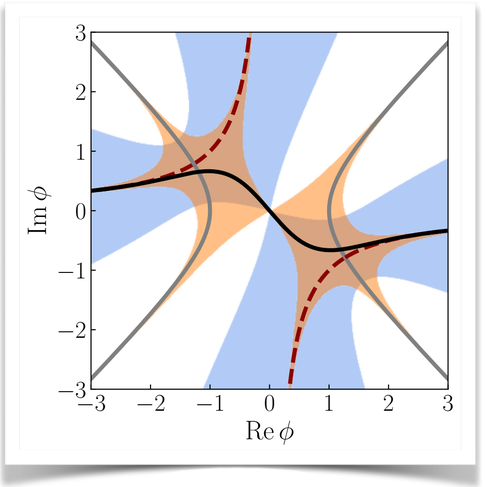


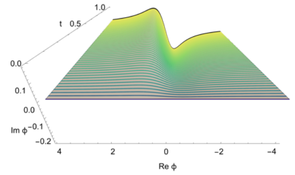
$m^2 = -1 + 2i$

Ihssen, Kapust, JMP, arxiv:2603.03159

See talk of Friederike

Solving real sign problems

Sneak preview of the real time harmonic oscillator



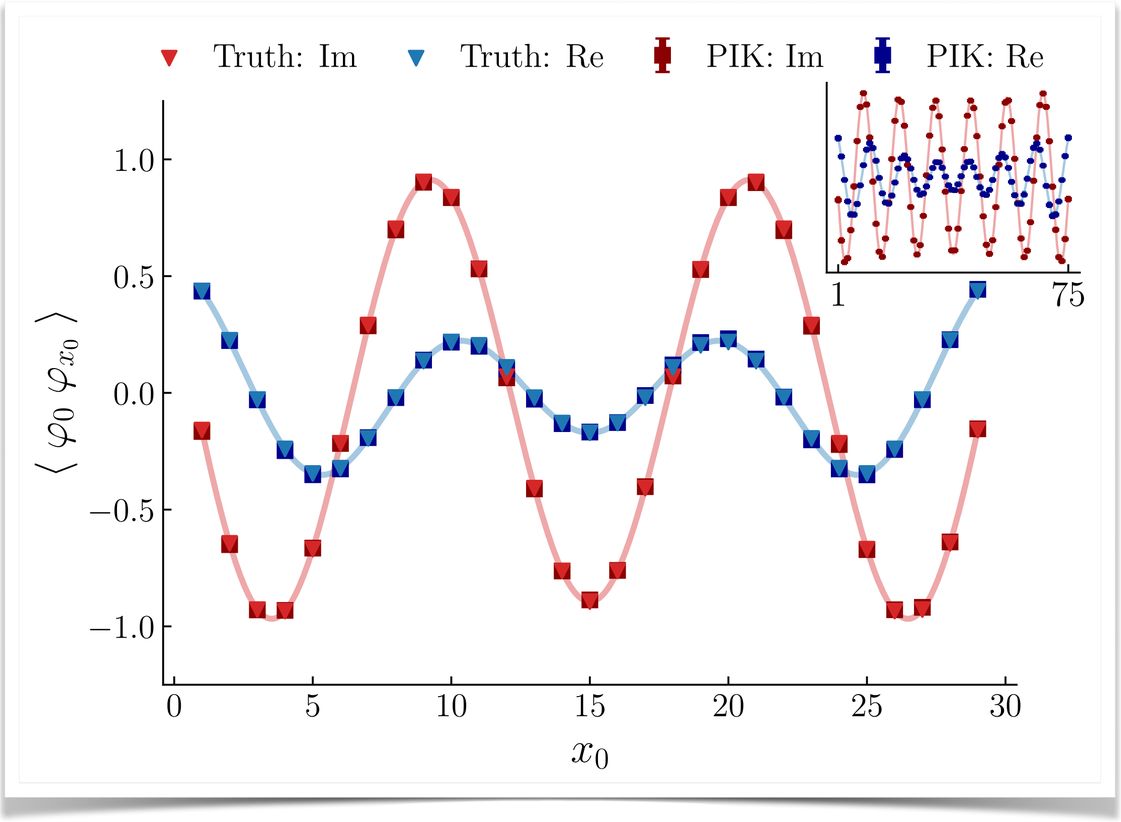
Real PIKfolds

Also for interacting real-time quantum systems

Ihssen, Kapust, JMP, in preparation

$$\hat{S}_t(\phi) = \sum_{i \in \mathcal{D}} \frac{\kappa_t}{2} (\phi_{i+1} - \phi_i)^2 - \left(m_t^2 + \epsilon\right) \frac{\phi_i^2}{2}$$

$$m_t^2 = m^2 e^{+i\frac{\pi}{2}t}$$



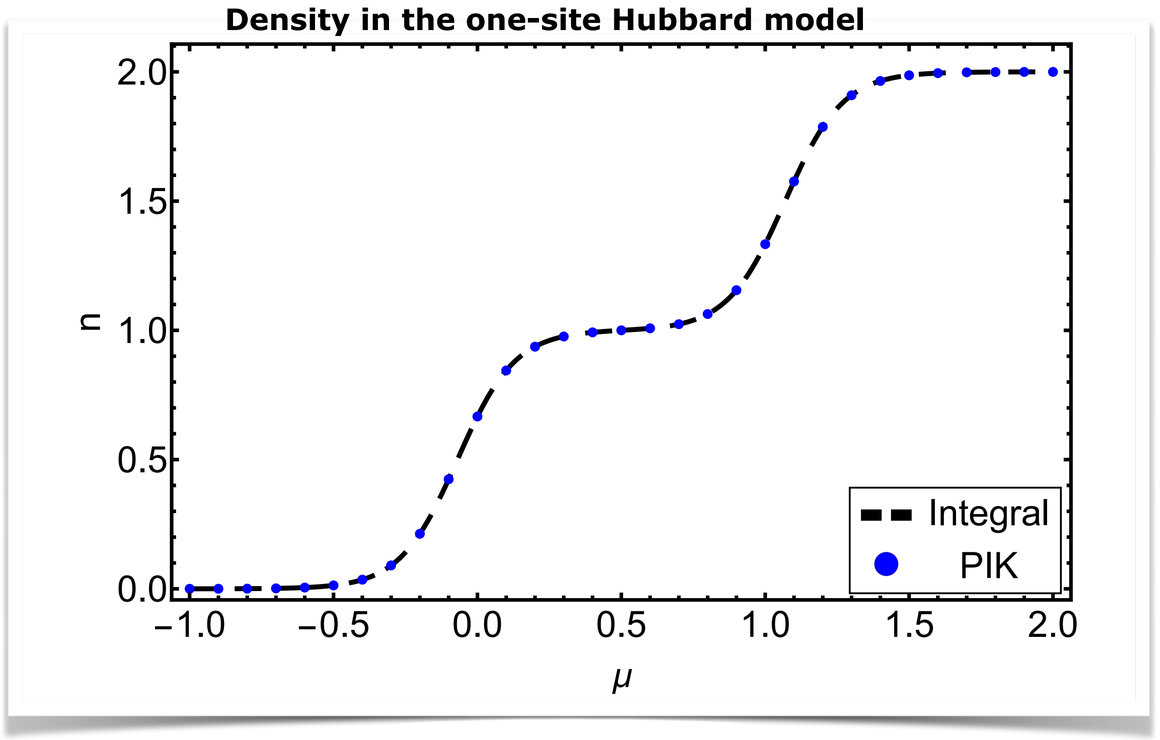
$$\kappa_t = e^{-i\frac{\pi}{2}t}$$

Ihssen, Kapust, JMP, arxiv:2603.03159

Solving sign problems with complex PIKs

PIKfan: Beyond the PIKfold

See talk of Renzo



Jump back to: Solving real sign problems

Fukushima, Ihssen, Kapust, JMP, arXiv:2609.XXXXX

See talk of Renzo

Classification of sign problems

$$\{q\,,\,\mathcal{M}_q\} \simeq \{p\,,\,\mathcal{M}\}$$

if there is a PIK $\dot{\phi}_t$

$$\{p\,,\,\mathcal{M}\} \xrightarrow{\dot{\phi}_t} \{q\,,\,\mathcal{M}_1\}$$

$$\int_{\mathcal{M}_q} d\mu\, q(\phi)\, \mathcal{O}(\phi) = \int_{\mathcal{M}_1} d\mu\, q(\phi)\, \mathcal{O}(\phi)$$

Fukushima, Ihssen, Kapust, JMP, arXiv:2609.XXXXX

See talk of Renzo

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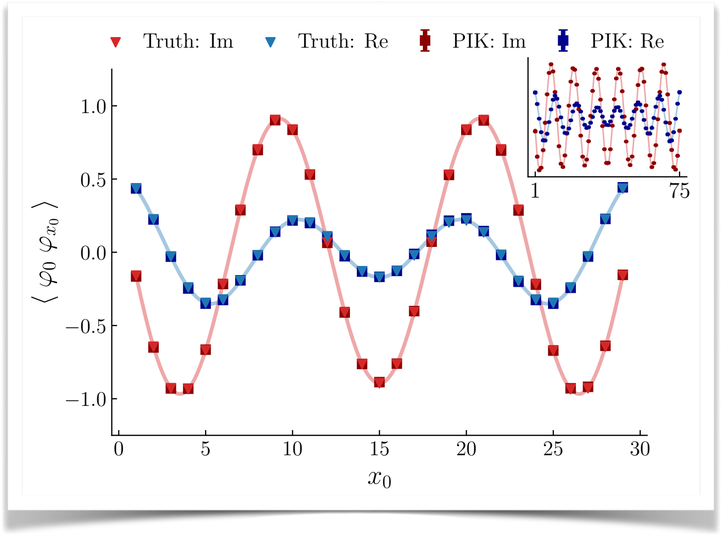
No singularity of the PIK $\dot{\phi}_t$ **on the PIKfold and 'between'** $\mathcal{M}_q$ **and** $\mathcal{M}_1$

Fukushima, Ihssen, Kapust, JMP, arXiv:2609.XXXXX

See talk of Friederike

Trivial example: *Real* sign problems



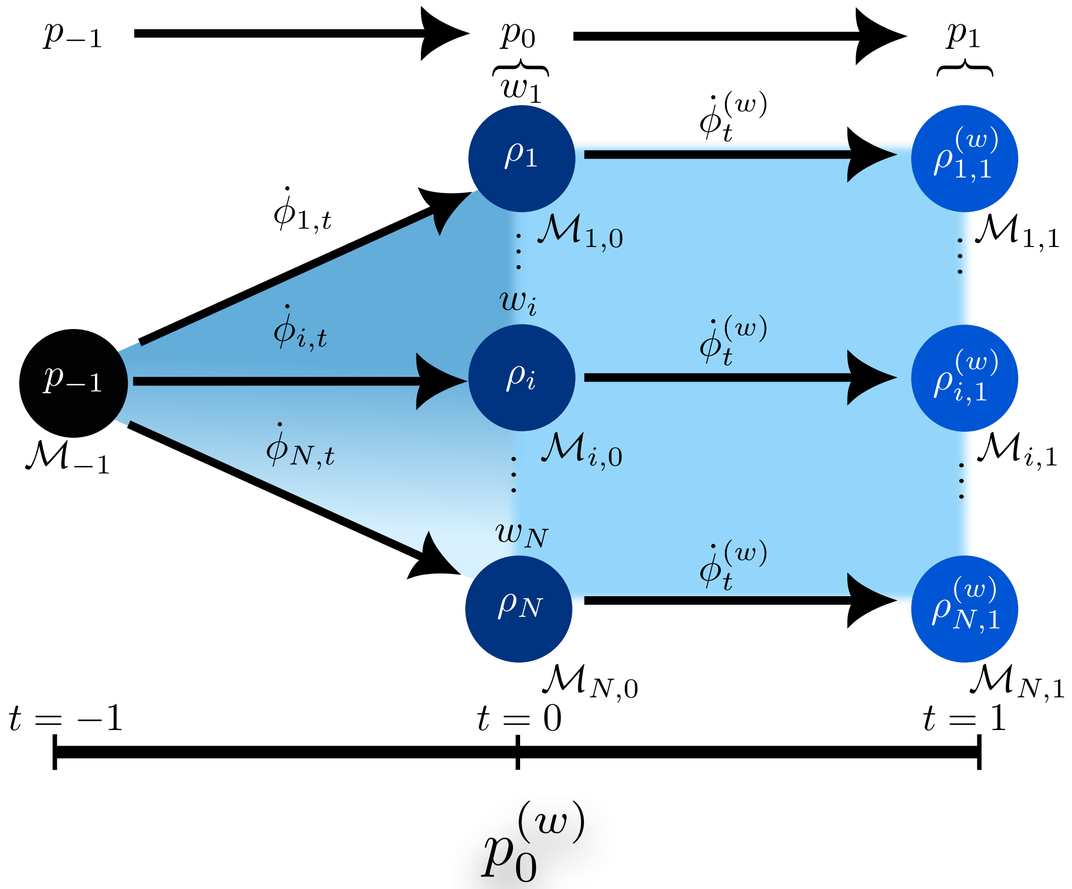
$$\{p_0\,,\,\mathbb{R}^{\mathcal{D}}\} \xrightarrow{\dot{\phi}_t} \{p\,,\,\mathcal{M}\}$$

***real* PIKfold** $p_t\, d\mu_t \in \mathbb{R}$

Fukushima, Ihssen, Kapust, JMP, arXiv:2609.XXXXX

See talk of Renzo

Complex sign problems: the PIKfan



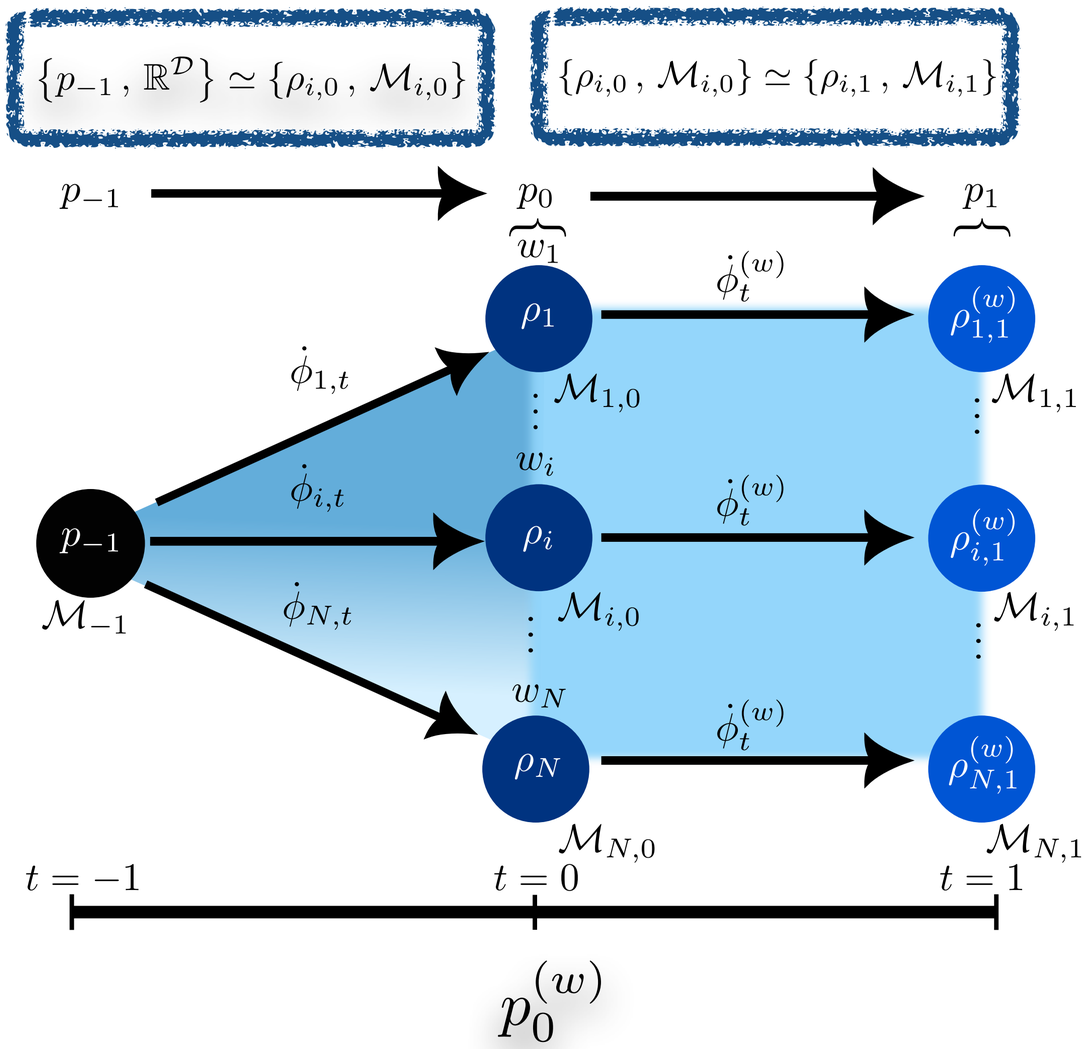
Weighted sum of e.g. Gaußians

$$p_t^{(w)}(\phi) = \sum_{i=1}^{N} w_{i,t}\, \rho_{i,t}(\phi) \qquad \sum_{i=1}^{N} w_{i,t} = 1$$

Fukushima, Ihssen, Kapust, JMP, arXiv:2609.XXXXX

See talk of Renzo

Complex sign problems: the PIKfan



Real PIKfolds

$$\{p_{-1}\,,\,\mathbb{R}^{\mathcal{D}}\} \simeq \{\rho_{i,0}\,,\,\mathcal{M}_{i,0}\} \simeq \{\rho_{i,1}\,,\,\mathcal{M}_{i,1}\}$$

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See talk of Renzo

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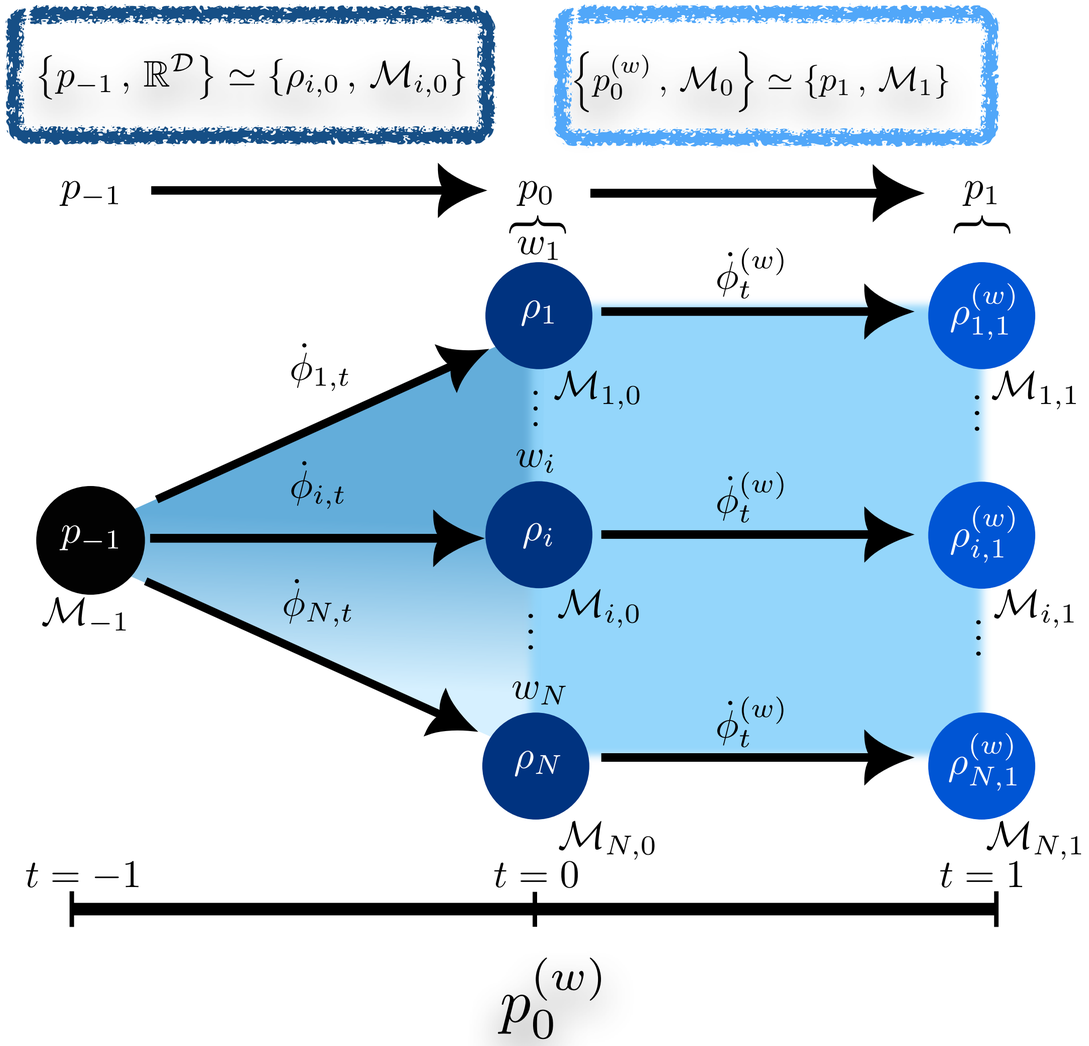

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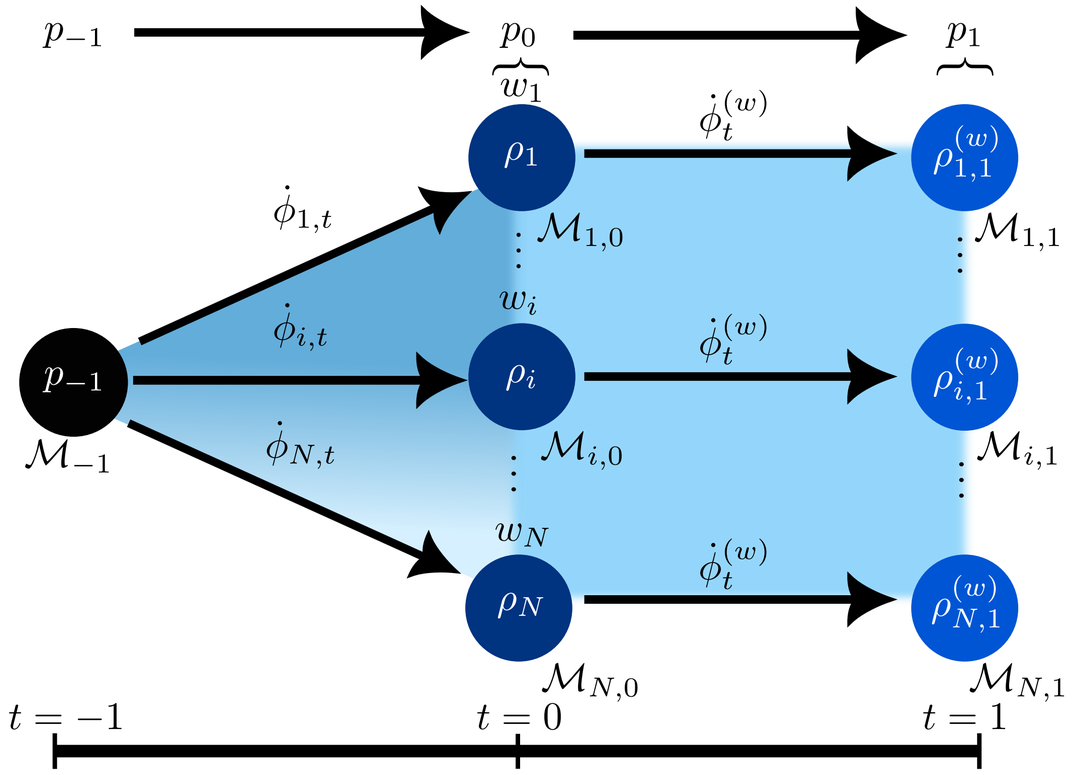
Equivalence class

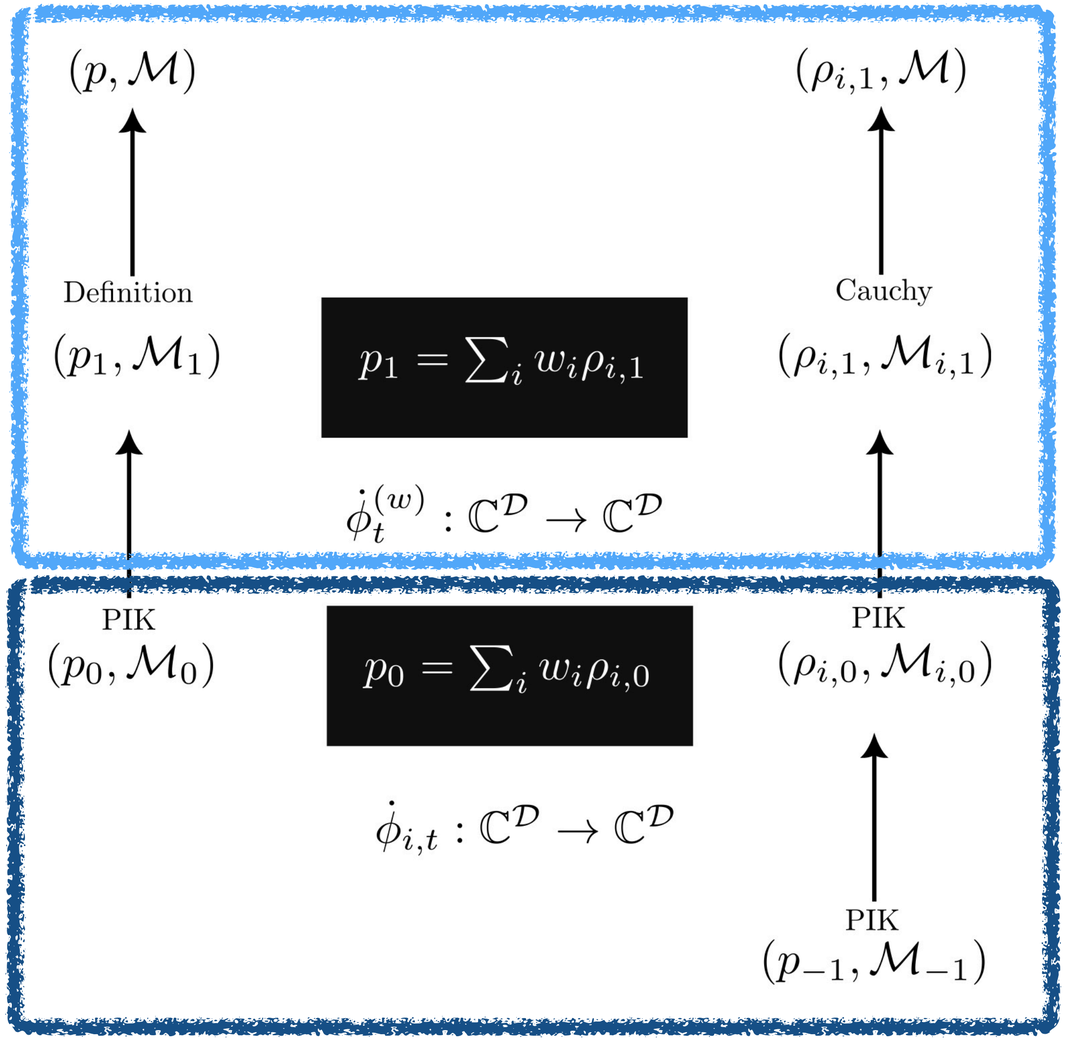
$$\left\{p_0^{(w)}\,,\,\mathcal{M}_0\right\} \simeq \{p_1\,,\,\mathcal{M}_1\}$$

Fukushima, Ihssen, Kapust, JMP, arXiv:2609.XXXXX

See talk of Renzo

Complex sign problems: the PIKfan



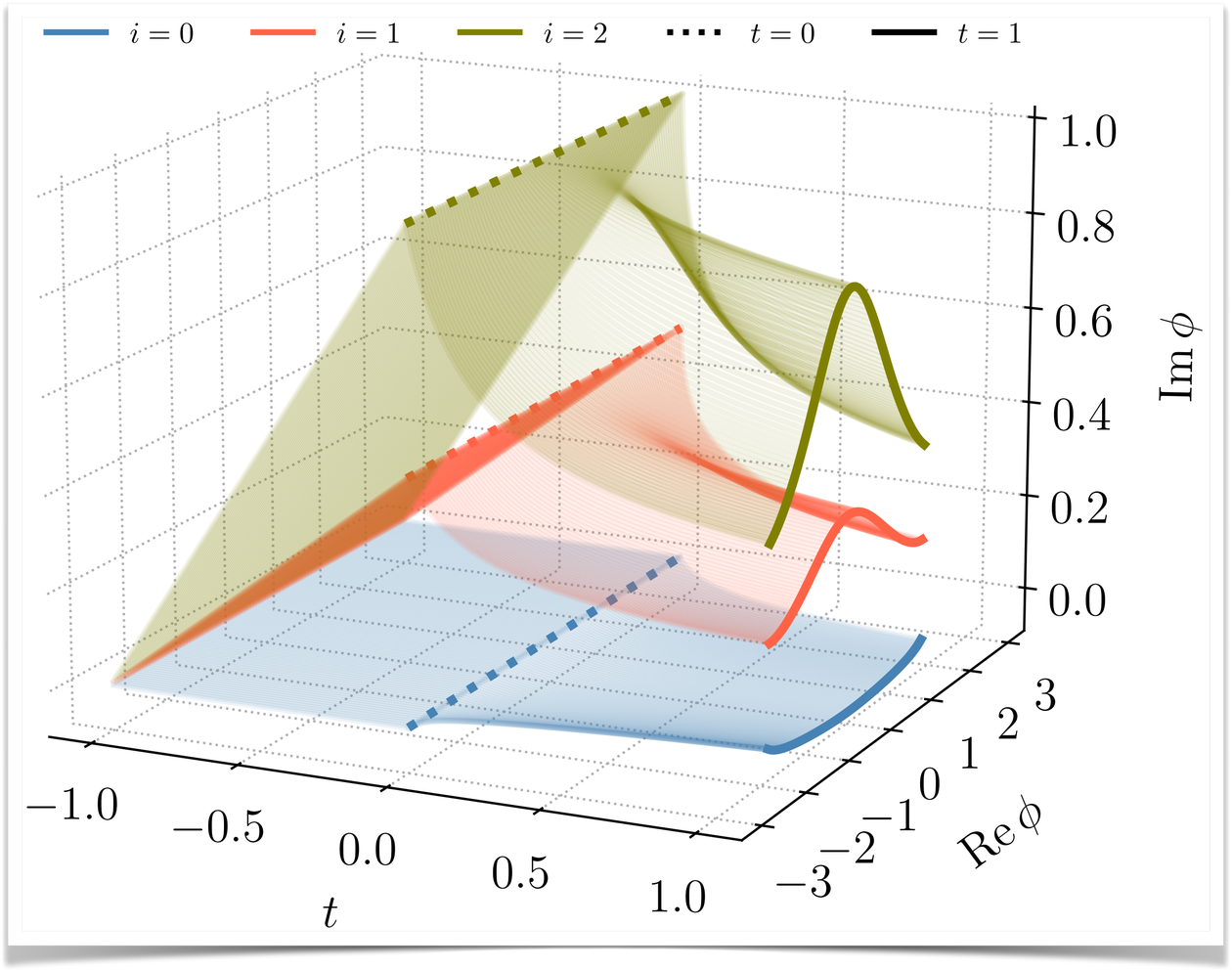


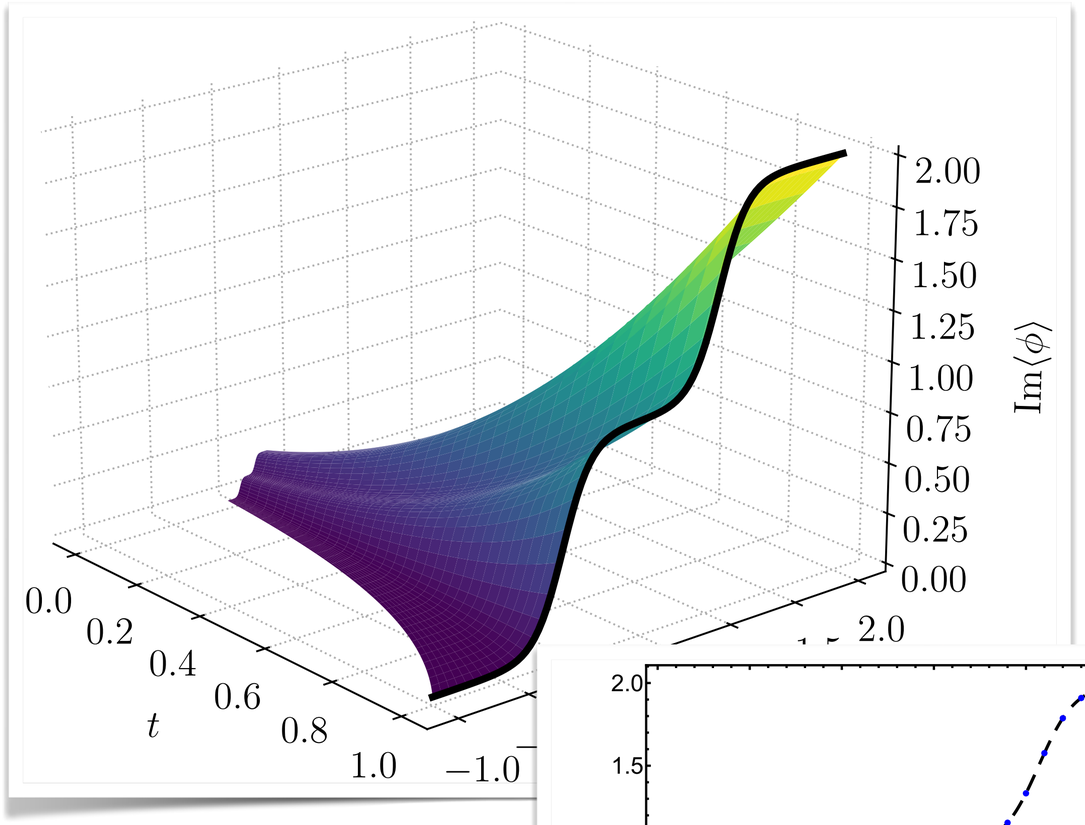
Fukushima, Ihssen, Kapust, JMP, arXiv:2609.XXXXX

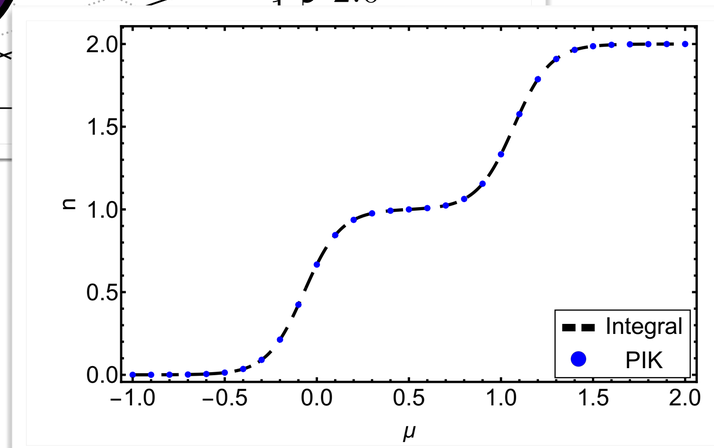
See talk of Renzo

Complex sign problems: the PIKfan

Sneak preview Hubbard model







Fukushima, Ihssen, Kapust, JMP, arXiv:2609.XXXXX

Summary

PIRG pair

$$\Big(F_t(\phi)\,,\dot{\phi}_t(\phi)\Big)$$

$F_t = \Gamma_t$

Generalised flow

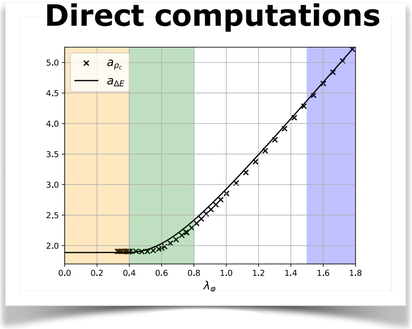
$F_t = p_t$

Wegner flow

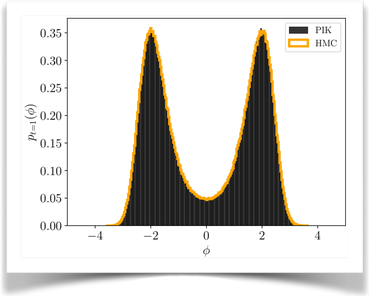
What is it good for?

Direct computations & generative sampling

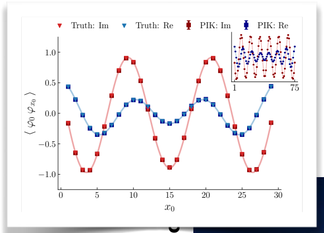
Direct computations

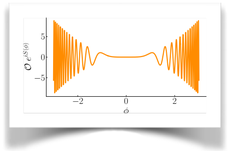


Generative sampling

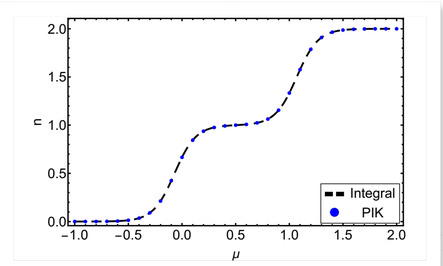


Real-time quantum theories





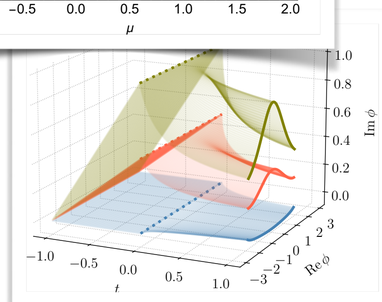
(Fermionic) sign problems



Trying to solve the sign pr



https://commons.wikimedia.org/wiki/File:Chogori.jpg



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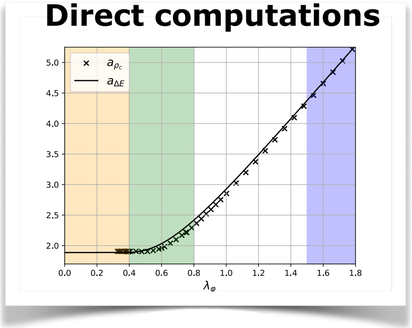
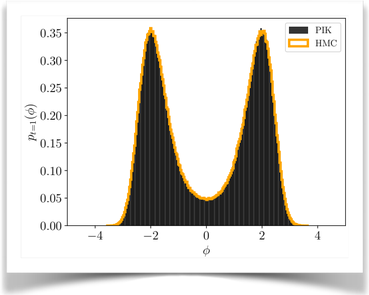


Generative sampling



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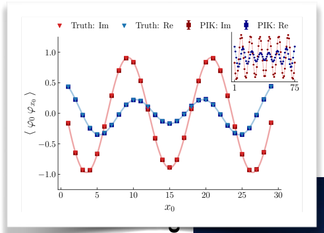
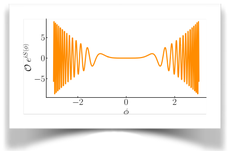
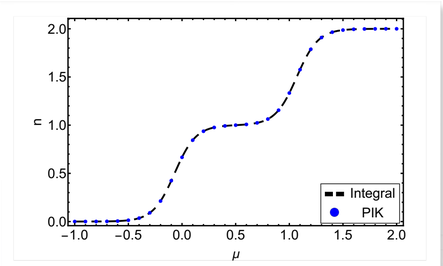




(Fermionic) sign problems



Trying to solve the sign problem



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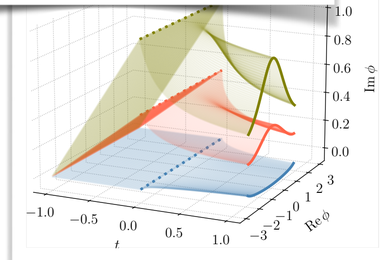



Boiling coffee?

Stay tuned


The end