Renormalization-guided inverse blocking for lattice field generation:

critical upscaling with conditional flows

Anna Hasenfratz
University of Colorado Boulder

Bridging Machine Learning and Statistical Field Theory
ECT* Trento, Sep 8, 2026

AH, Ethan Neil, Letizia Parato, Noah Schwartz
arXiv: 2608.28580,2608.28581

ECT* “Machine Learning for Lattice Field Theory and Beyond” June 2023

Machine learning for lattice field theory and beyond

Jun 26, 2023, 9:00 AM → Jun 30, 2023, 6:00 PM Europe/Rome

Aula Renzo Leonardi (ECT*)

Biagio Lucini (Swansea University) , Daniel Hackett (Massachusetts Institute of Technology, United States) , Dimitrios Bachtis (ENS, Paris) , Gert Aarts (Swansea University, UK) , Phiala Shanahan (Massachusetts Institute of Technology, United States)

Description

The past few years have seen rapid exploration of how machine learning (ML) techniques can be applied in theoretical particle and nuclear physics, particularly in numerical lattice quantum field theory (LQFT). Promising early works have already demonstrated potential for ML to accelerate computationally demanding LQFT calculations and add new capabilities to the LQFT toolkit.

This workshop aims to provide a forum for the community working on this topic to cross-pollinate methods, generate ideas for new applications, and assess the state of the field to guide further exploration. Highlighted topics include generative models for configuration generation, ML-accelerated algorithms, ML approaches to inverse problems, physics from novel machine-learned observables, and new calculational techniques enabled by ML methods.



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with the help of AI coding assistants

OpenAI Codex

Anthropic Claude Code

Critical slowing down is a scale problem

Near criticality

- Correlation length $\xi \to \infty$
  - All modes contribute to dynamics
- Most updating algorithms are local
  - Autocorrelation grows as $\sim L^z$, $z \approx 2 - 3$
  - Topology freezes
- Cluster algorithms can eliminate CSD in some spin/scalar systems, but have no general analogue for gauge theories

Machine learning and Normalizing flow

Albergo, Kanwar & Shanahan (2019)

Upscaling flow transforms Gaussian noise to dynamical configurations

$\{z\} \in N(0,1)$ random fields $\longrightarrow$ $\{\phi = F(z)\} \in G$ field configurations

$\xi = 0$ $\longrightarrow$ $\xi \sim \infty$

- Autocorrelation is set by $\{z\}$
- But
  - ✦ The flow has to learn the dynamics from short to large scales - hard
  - ✦ The corrective reweighting or accept/reject step is $p \sim e^{-c\,V}$
    - Maintaining global acceptance requires increasingly accurate modeling as V grows

✦

Renormalization group guided inverse blocking

Renormalization group guided inverse blocking

The machine does not know physics - we do

- Supply it with the (complicated) long distance IR physics
- Let it learn the (simple) short distance UV details

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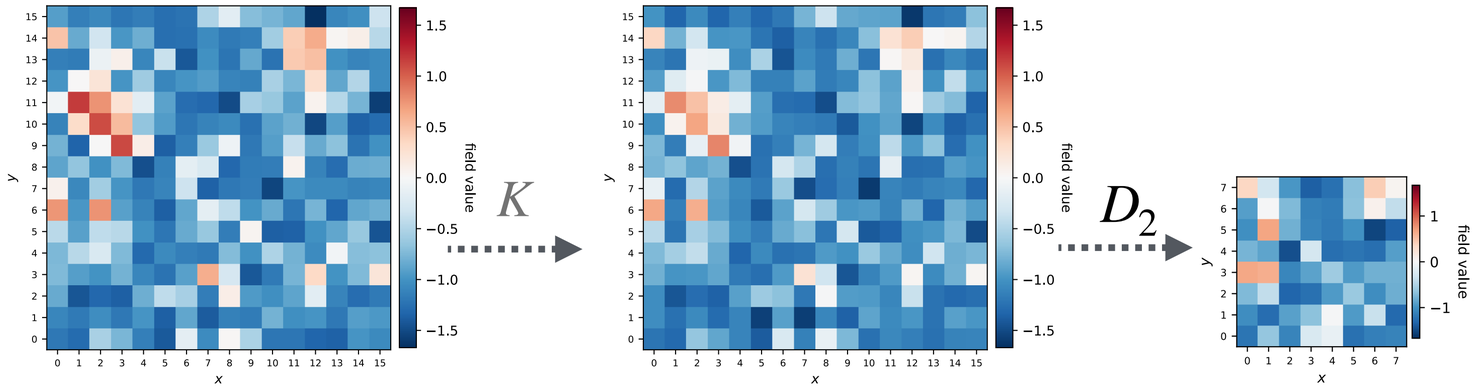
RG blocking :

Fine field L^d | Smoothed field L^d | Blocked field $(L/2)^d$

$\{\phi_f\} \sim e^{-S_f(\kappa)}$ $\xrightarrow{K}$ $\psi = K\phi_f$ $\xrightarrow{D_2}$ $\phi_b = D_2\psi \sim e^{-S_b}$



RG blocking:

- Smooth fields $\psi = K\phi$;
  K : linear convolution
- Coarse grain or decimate:
  $\phi_b = D_2\psi = D_2K\phi$

$\{\phi_f\}$, $\{\psi\}$ and $\{\phi_b\}$ have the same mid-range and IR physics

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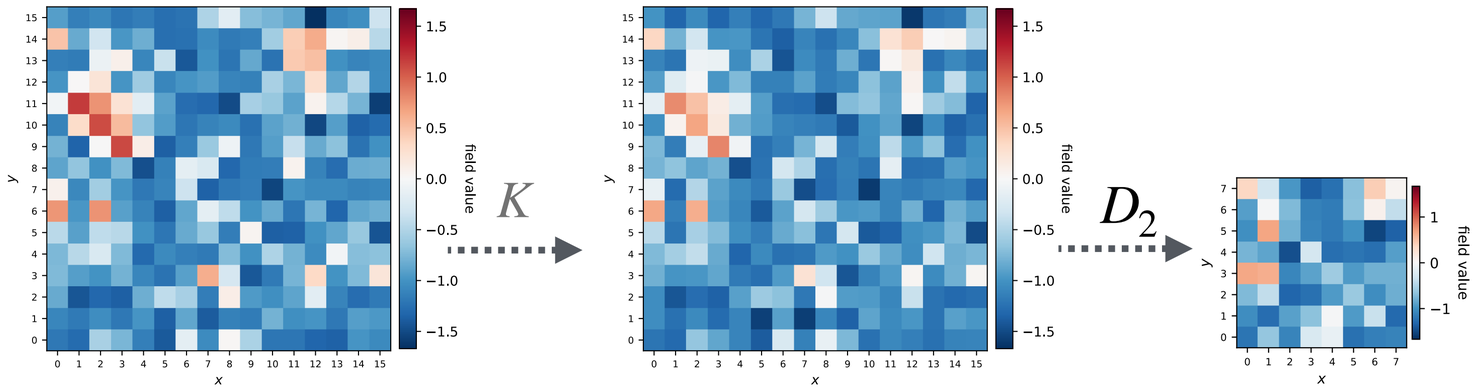

K^{-1} Upscale

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Endres et al. (2015); Detmold, Endres (2018); Bachtis et al. (2022)

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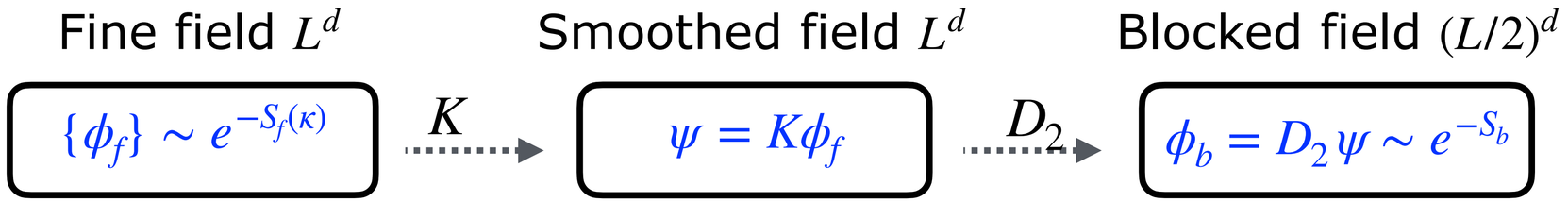
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But: we do not know S_b !

RG blocking:

- $S_f(\kappa) \to S_b(\tilde{\kappa})$, $S_b(\tilde{\kappa})$ is generally complicated, contains many induced terms

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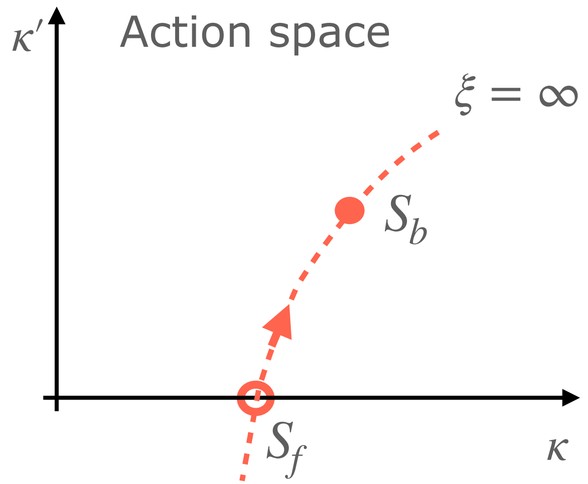

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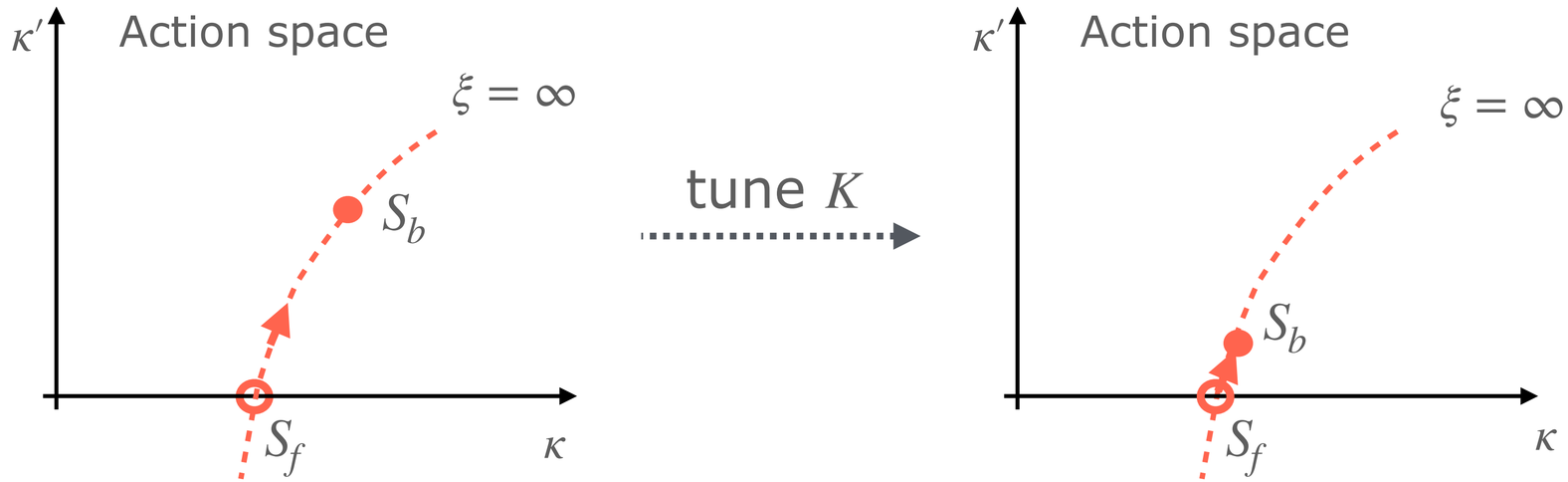

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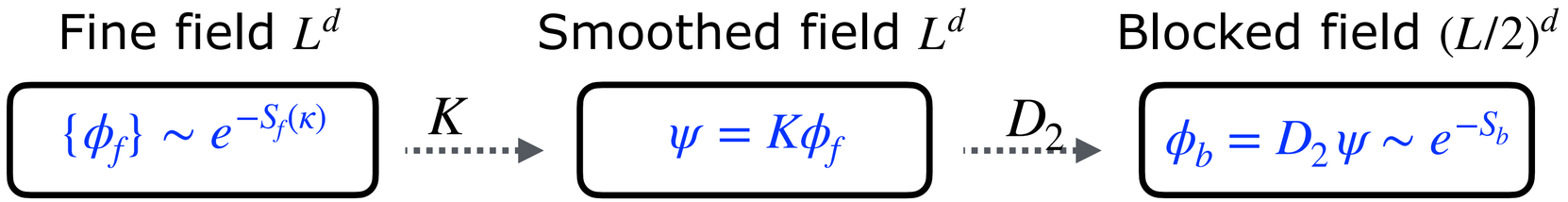

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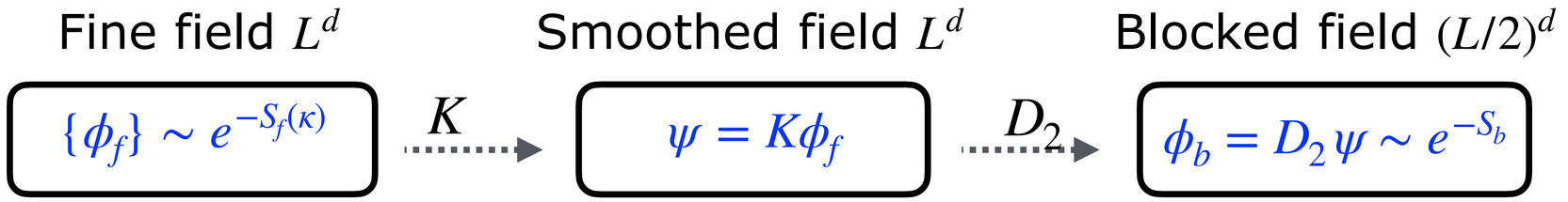

Perfect blocking: choose K such that the blocked ensemble is well described by a simple coarse action S_c

$$\{\phi_b\} \simeq \{\phi_c\} \ , \ \ \phi_c \sim e^{-S_c} \ ;$$

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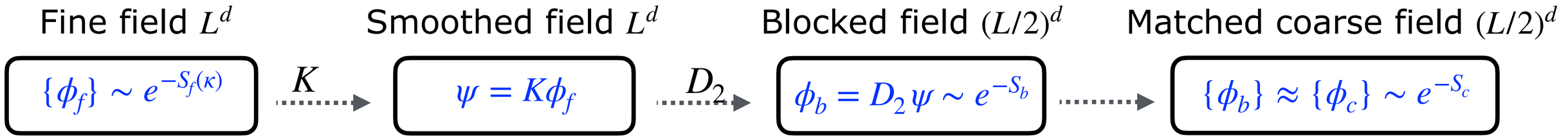

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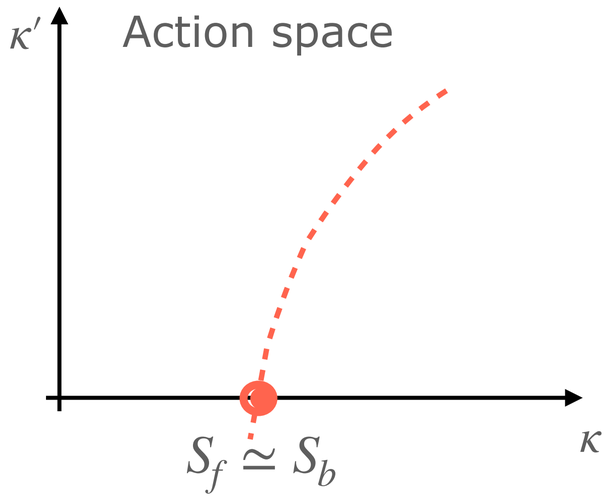
At criticality: $S_c \simeq S_b \simeq S_f$



Upscaling: conditional flow

Albergo, Kanwar & Shanahan (2019)
Bachtis et al. (2022)

Keep coarse fields fixed, generate the missing UV details

Matched coarse field $(L/2)^d$

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Contains

IR physics

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conditional flow fill in the details

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$$d_{01} \sim q_\theta(d_{01} \mid \phi_c)$$
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The network **does not** learn the critical distribution
— it learns the **missing UV conditional distribution**.

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K^{-1}

fine proposal

$$\phi_{\text{prop}} = K^{-1}\psi_{\text{prop}}$$

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Endres et al. (2015)

Correction: rethermalization

Repair the residual mismatch

fine proposal

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IR physics : inherited from coarse ensemble

UV structure : from the flow might need repair

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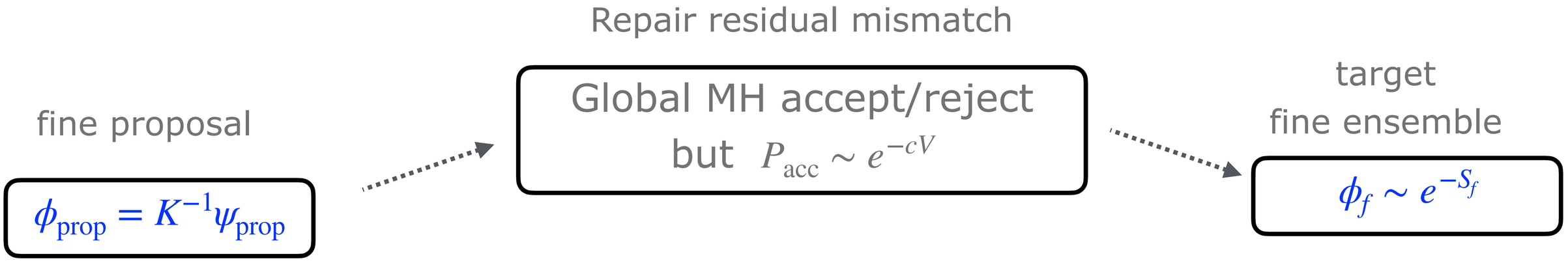

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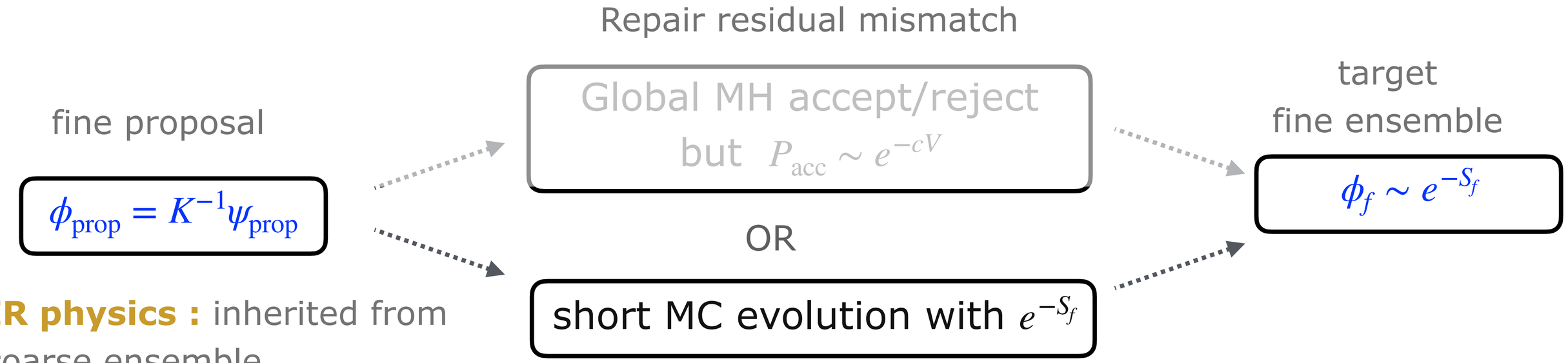

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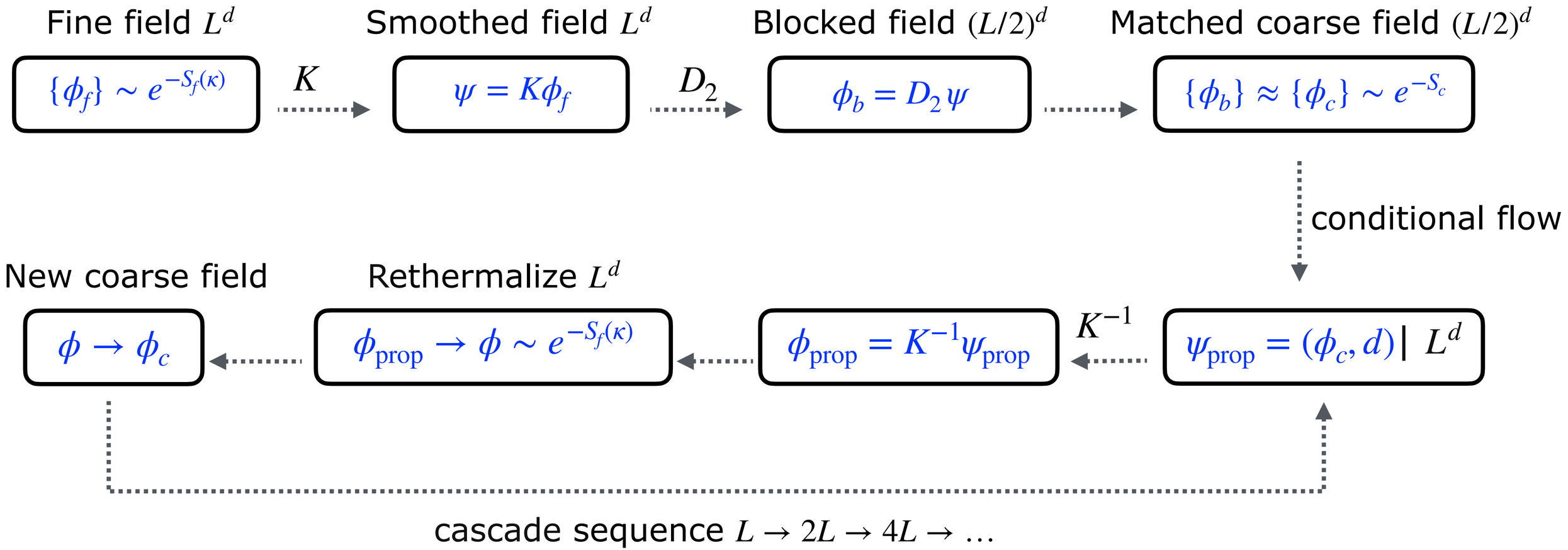
UV structure : from the flow might need repair

Rethermalization does not need to regenerate the long distance modes.

RG-guided inverse blocking:
Keep coarse fields fixed, generate the missing UV details



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Perfect blocking

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$\{\phi_f\} \sim e^{-S_f(\kappa)}$ —K→ $\psi = K\phi_f$ —D_2→ $\phi_b = D_2\psi$ ⋯→ $\{\phi_b\} \approx \{\phi_c\} \sim e^{-S_c}$

conditional flow

New coarse field | Rethermalize L^d

$\phi \to \phi_c$ ⋯← $\phi_{\text{prop}} \to \phi \sim e^{-S_f(\kappa)}$ ⋯← $\phi_{\text{prop}} = K^{-1}\psi_{\text{prop}}$ —K^{-1}→ $\psi_{\text{prop}} = (\phi_c, d)\,|\,L^d$

cascade sequence $L \to 2L \to 4L \to \ldots$

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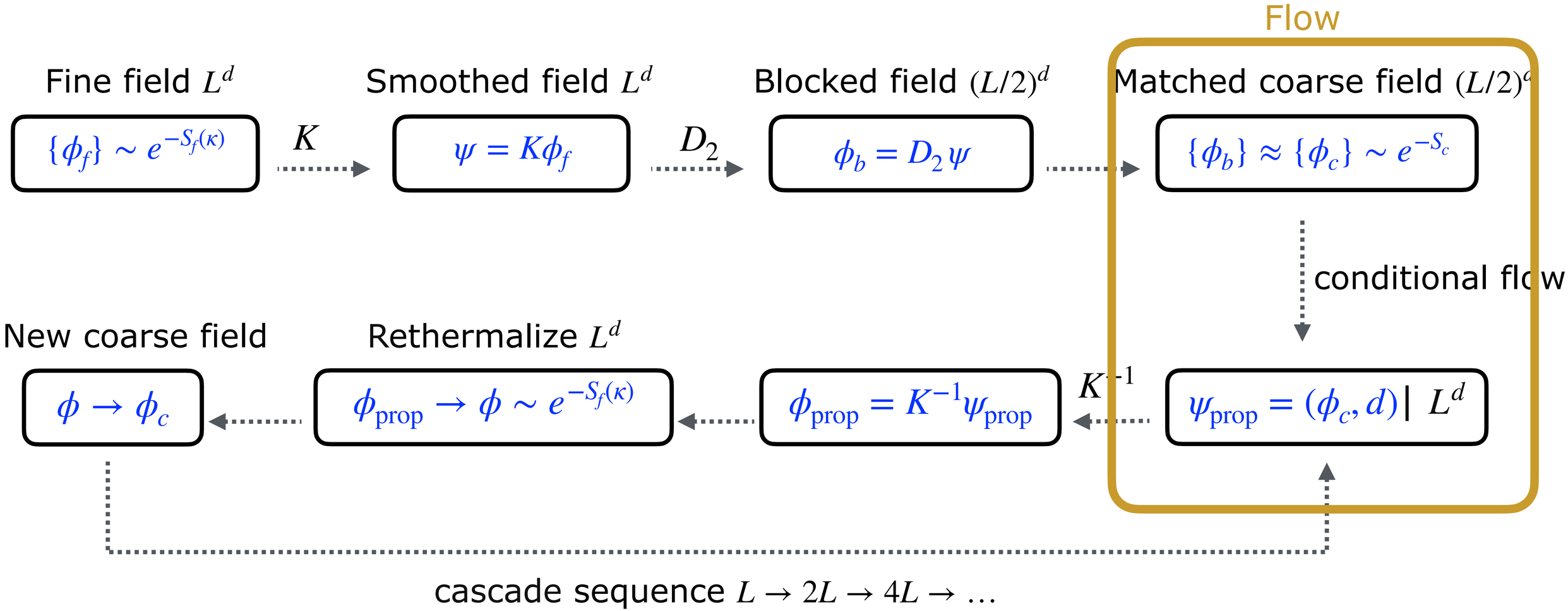

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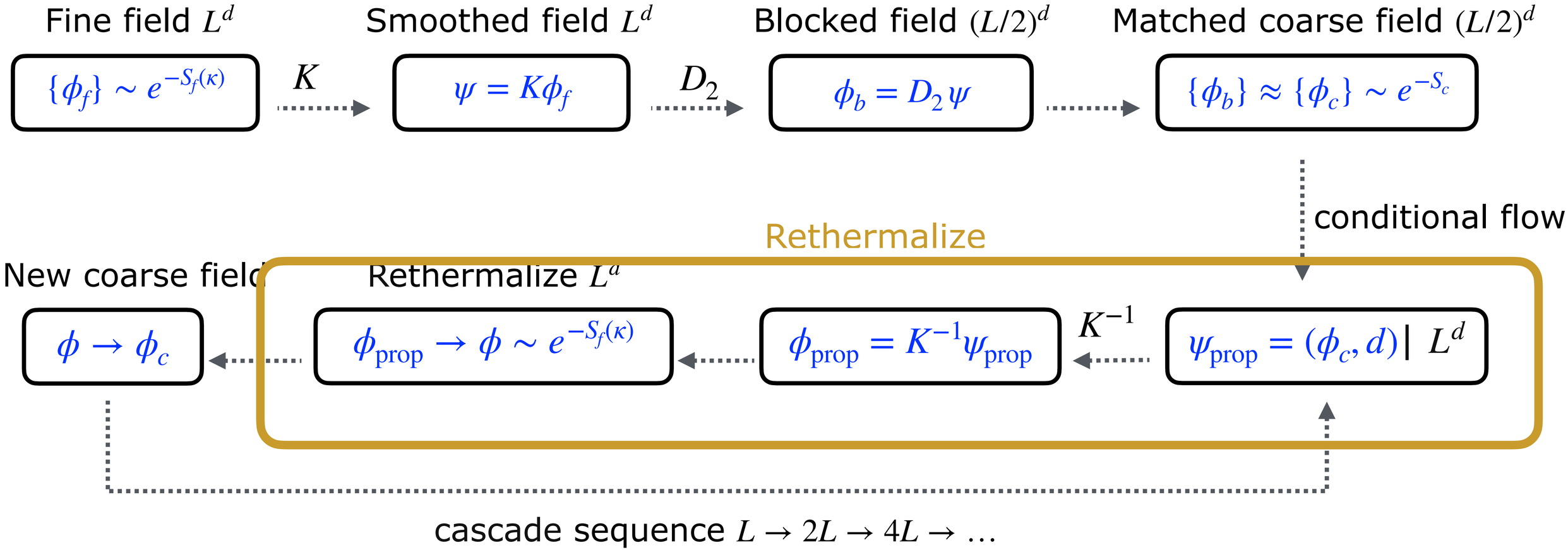

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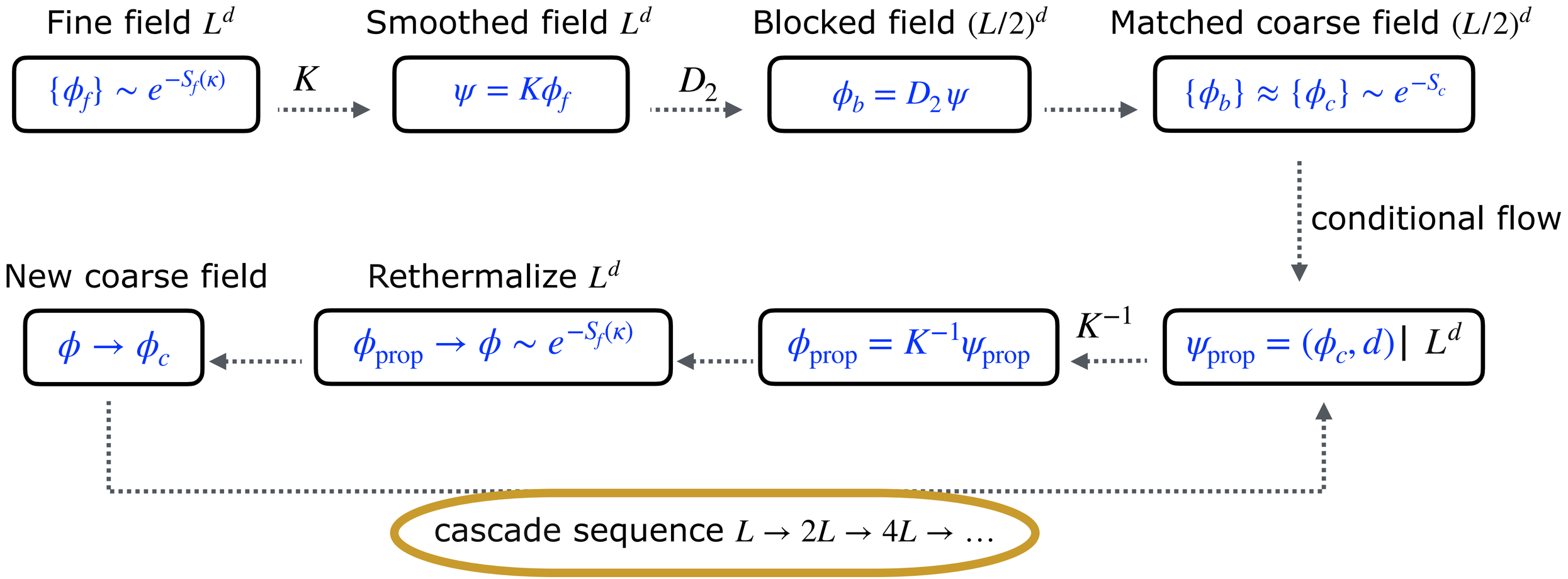


Testbed: 2-d ϕ^4 model at criticality

Model:

$$S[\phi] = -2\kappa \sum_{x,\mu} \phi_x \phi_{x+\hat{\mu}} + \sum_x \left[\phi_x^2 + \lambda(\phi_x^2 - 1)^2\right]$$

$$\lambda = 1.0 \ , \quad \kappa_{\rm cr} = 0.340301(5)$$

2-d Ising universality class: $\eta = 1/4$, $\nu = 1$

Simulations:

- Training: $32^2 \to 16^2 \to 32^2$
- Direct generation up to 128^2 with Wolff cluster update for testing
- Upscaled ensembles up to 2048^2 volumes

(RG guided upscaling is faster that cluster update for $L \gtrsim 64$)

1) The perfect blocking

K is a linear convolution: parametrize it as

$$K\,\phi(\vec{x}) = \sum_{\vec{r}} K(\vec{r})\,\phi(\vec{x}+\vec{r}) \;;\quad \sum_{r} K(r) = 2^{\eta/2} \text{ , fixed}$$

Perfect blocking: choose K such that $S_c \approxeq S_b$

Require that expectation values of

- blocked fields $\phi_b = D_s K \phi$
- direct generated ϕ_c

on the same volume agree. Use both short- and long-distance operators

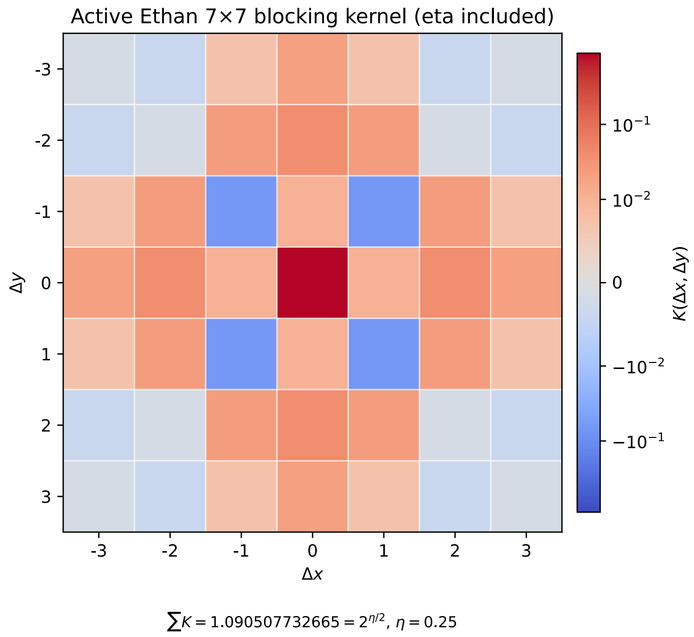
Minimize

$$\chi^2_K = \sum_{i,j} \Delta\mathcal{O}_i C^{-1}_{ij} \Delta\mathcal{O}_j \quad , \quad \Delta\mathcal{O}_i = \langle O_i \rangle_{S_b} - \langle O_i \rangle_{S_c}$$

The perfect blocking kernel

Kernels of size 5x5 - 5 free parameters or
size 7x7 - 9 free parameters (η normalization fixed) both work well

Offset orbit $(\lvert\Delta x\rvert, \lvert\Delta y\rvert)$	Multiplicity	$K(\Delta x, \Delta y)$
$(0,0)$	1	0.888641822
$(1,0)$	4	0.010508374
$(1,1)$	4	−0.066254410
$(2,0)$	4	0.037116968
$(2,1)$	8	0.022657096
$(2,2)$	4	−0.001649881
$(3,0)$	4	0.020222068
$(3,1)$	8	0.007299892
$(3,2)$	8	−0.003848342
$(3,3)$	4	−0.001693935

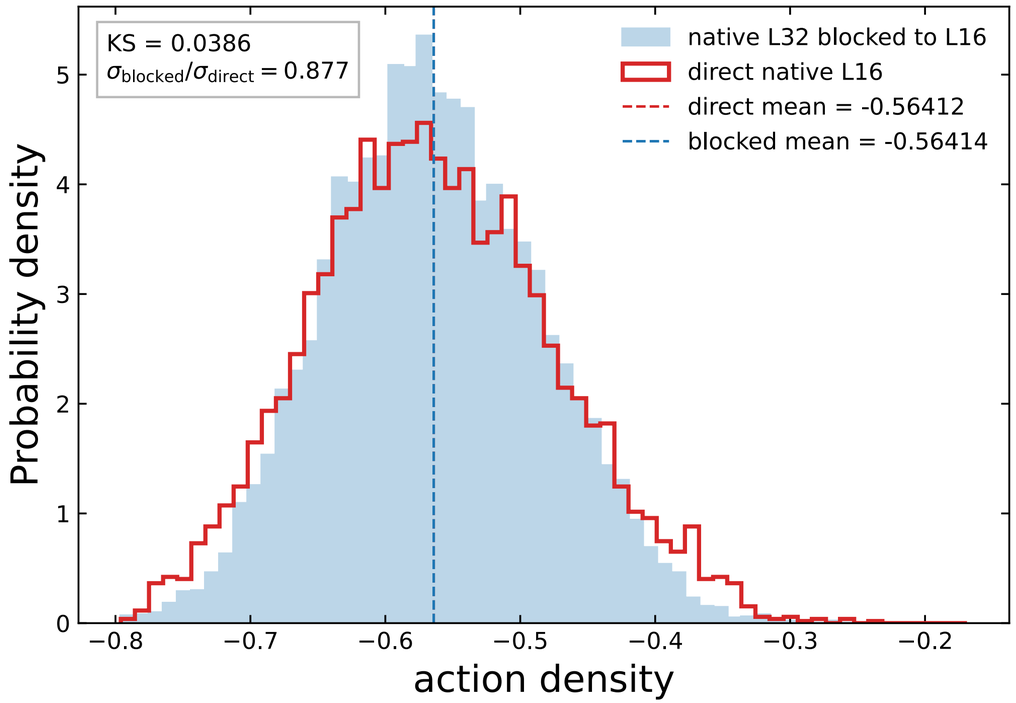


Train once on $16 \to 32$ **;** reuse at larger L

Validation of the perfect blocking kernel - action density

Verify approximate perfect blocking $S_b(\kappa') \approxeq S_c(\kappa_c)$
by comparing blocked and native distributions

Direct $L = 16$ and blocked $L = 32 \rightarrow 16$



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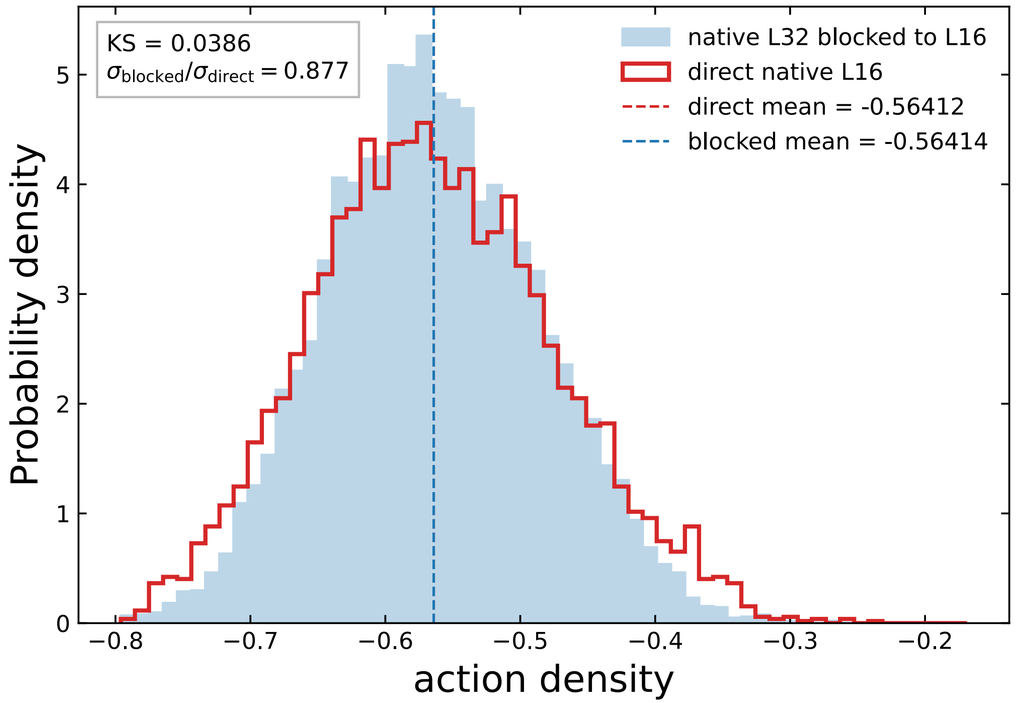
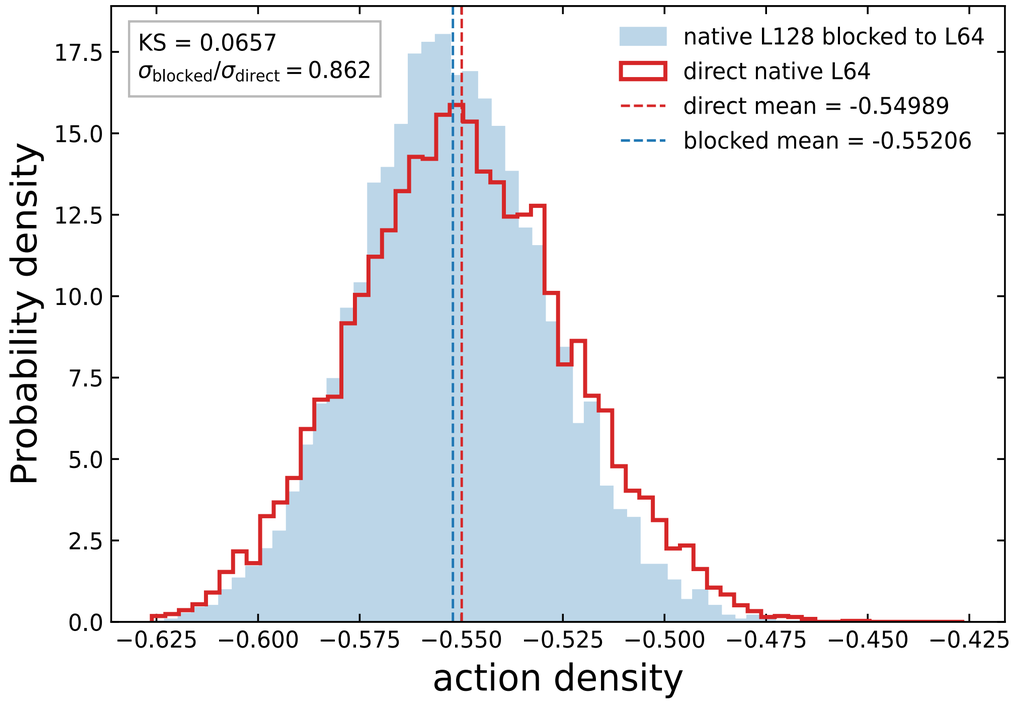


Direct $L = 64$ and blocked $L = 128 \to 64$
Same kernel, no retuning



Validation of the perfect blocking kernel - ϕ^2

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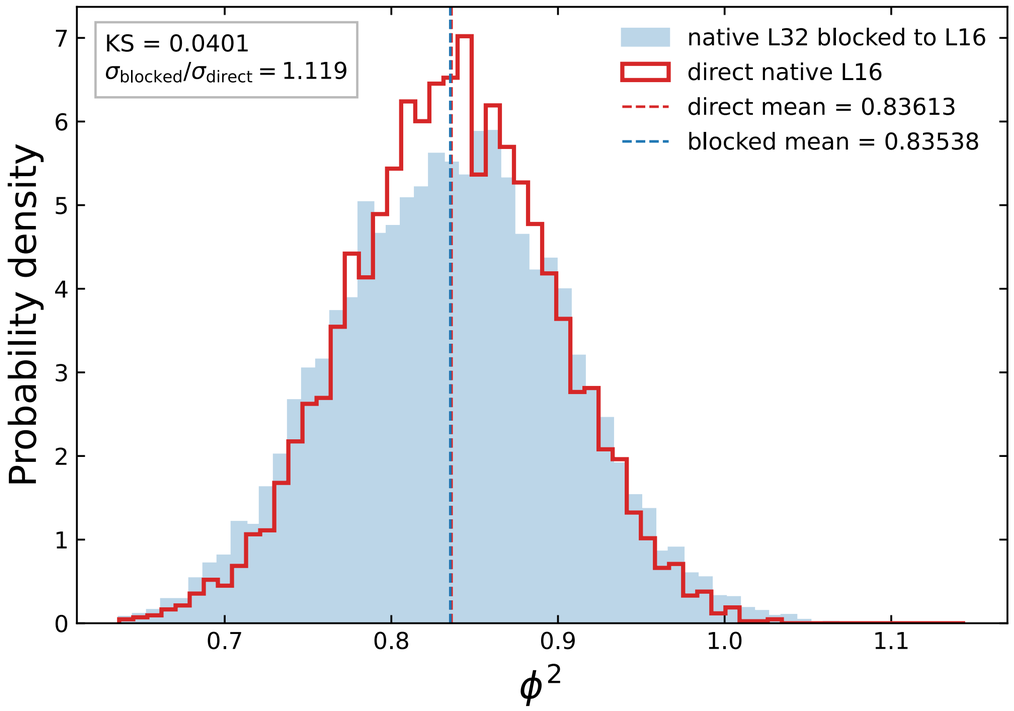

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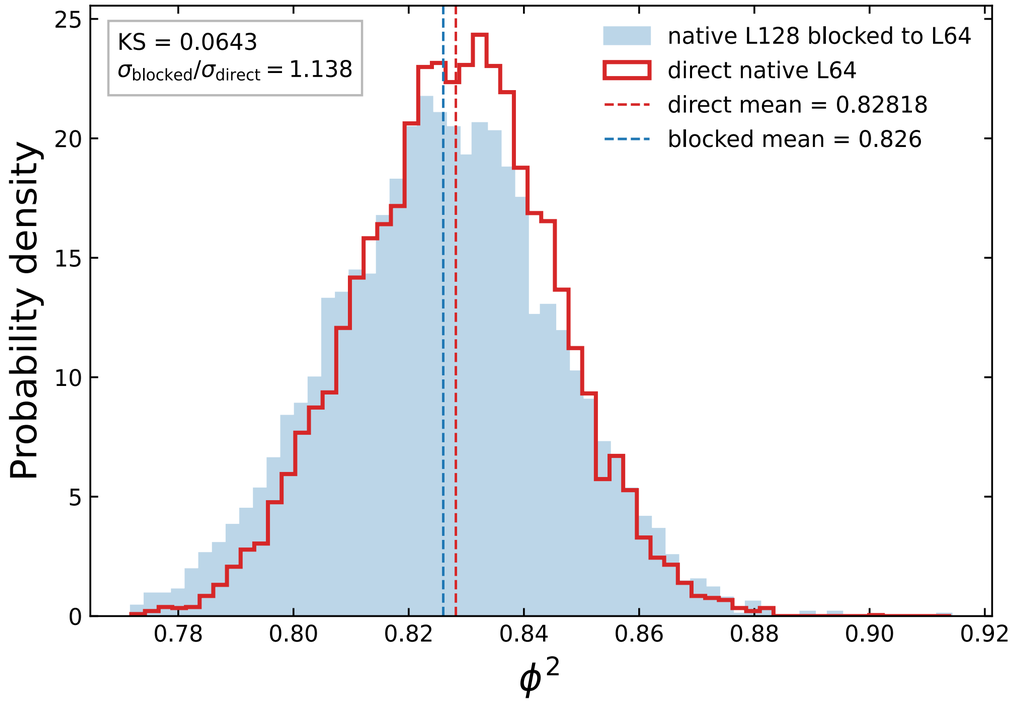

Validation of the perfect blocking kernel - m^2

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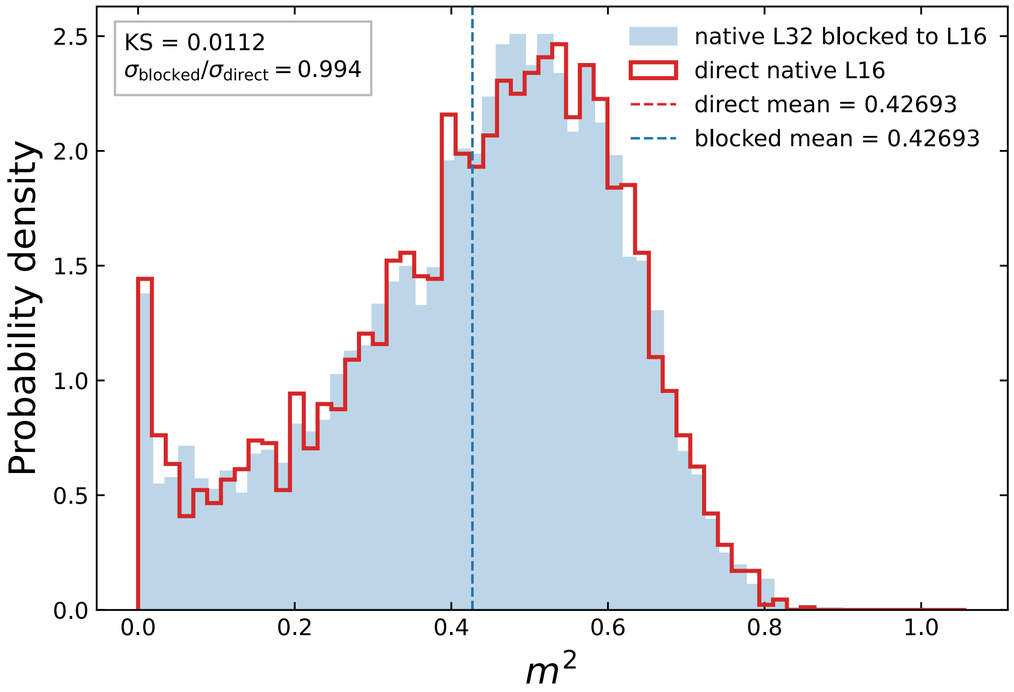

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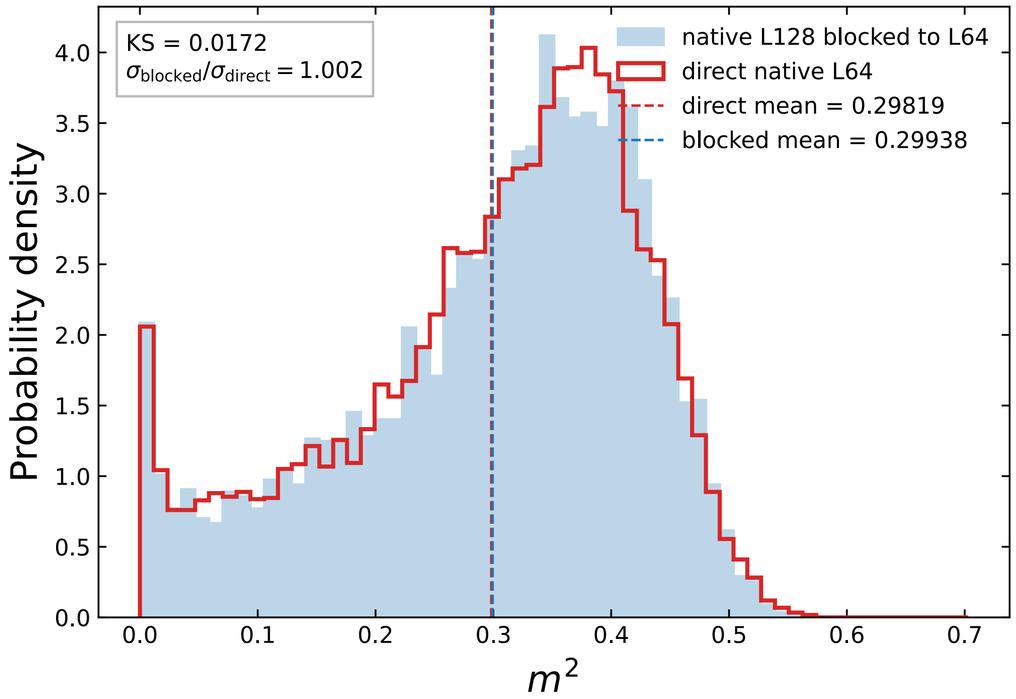
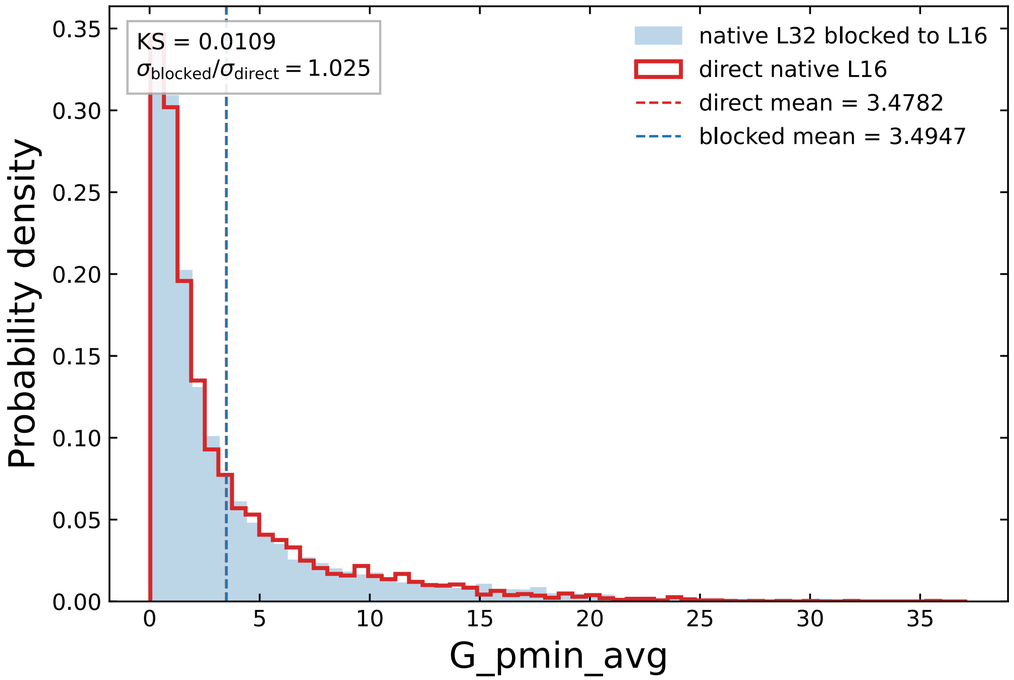


Long distance observables are nearly perfectly reproduced

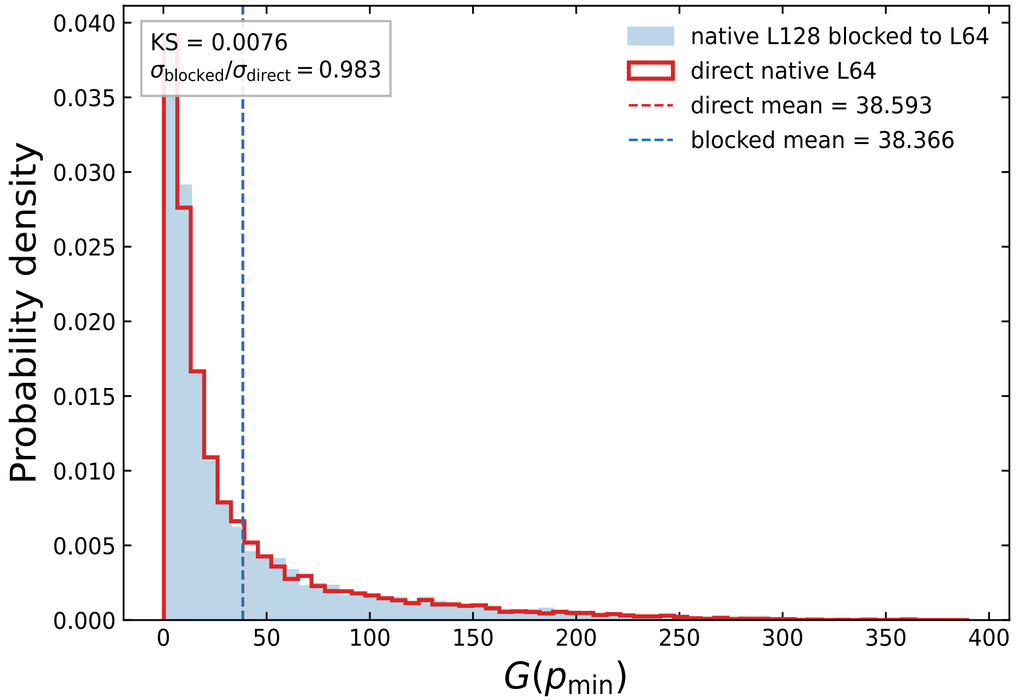
Validation of the perfect blocking kernel - $G(p = 2\pi/L)$

Verify approximate perfect blocking $S_b(\kappa') \approx S_c(\kappa_c)$
by comparing blocked and native and distributions

Direct $L = 16$ and blocked $L = 32 \rightarrow 16$



Direct $L = 64$ and blocked $L = 128 \rightarrow 64$
Same kernel, no retuning



Long distance observables are nearly perfectly reproduced

2) Upscaling with conditional flow

$$\phi_c \quad \longrightarrow \quad (\phi_c, d_{01}, d_{10}, d_{11}) = \psi_{\text{prop}} \quad \longrightarrow \quad \phi_{\text{prop}} = K^{-1}\psi_{\text{prop}}$$

Upscaling with normalizing flow :

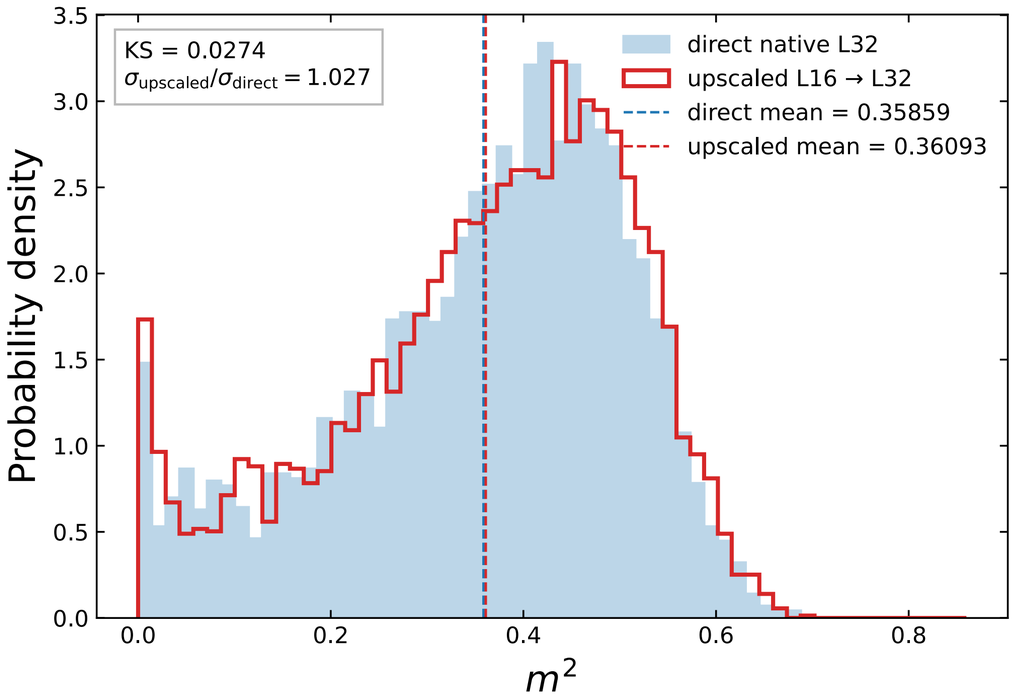
- train on $\psi = K\phi$, conditioned only ϕ_c
- no action enters the network
  - $d_{01} \sim q_\theta(d_{01} \mid \phi_c)$
  - $d_{10} \sim q_\theta(d_{10} \mid \phi_c, d_{01})$
  - $d_{11} \sim q_\theta(d_{11} \mid \phi_c, d_{01}, d_{10})$

Train once on $16 \to 32$; reuse at larger L

Validation of Upscaling - m^2

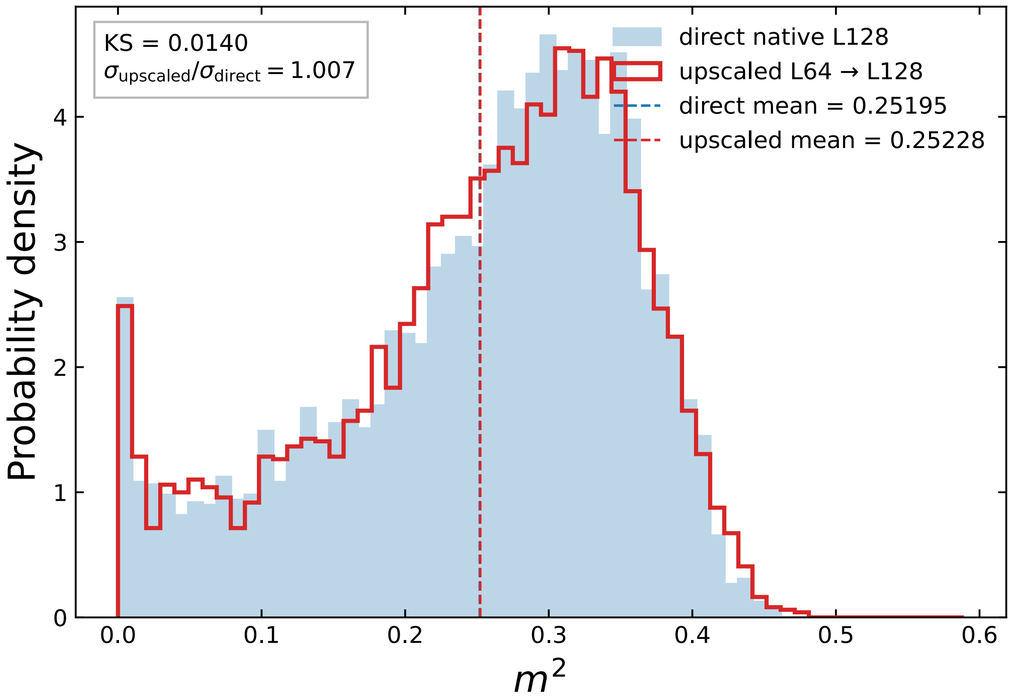
$$\phi_c \longrightarrow (\phi_c, d_{01}, d_{10}, d_{11}) = \psi_{\text{prop}} \longrightarrow \phi_{\text{prop}} = K^{-1}\psi_{\text{prop}}$$

Direct $L = 32$ and upscaled $L = 16 \to 32$



Direct $L = 128$ and upscaled $L = 64 \to 128$

Same flow, no retraining

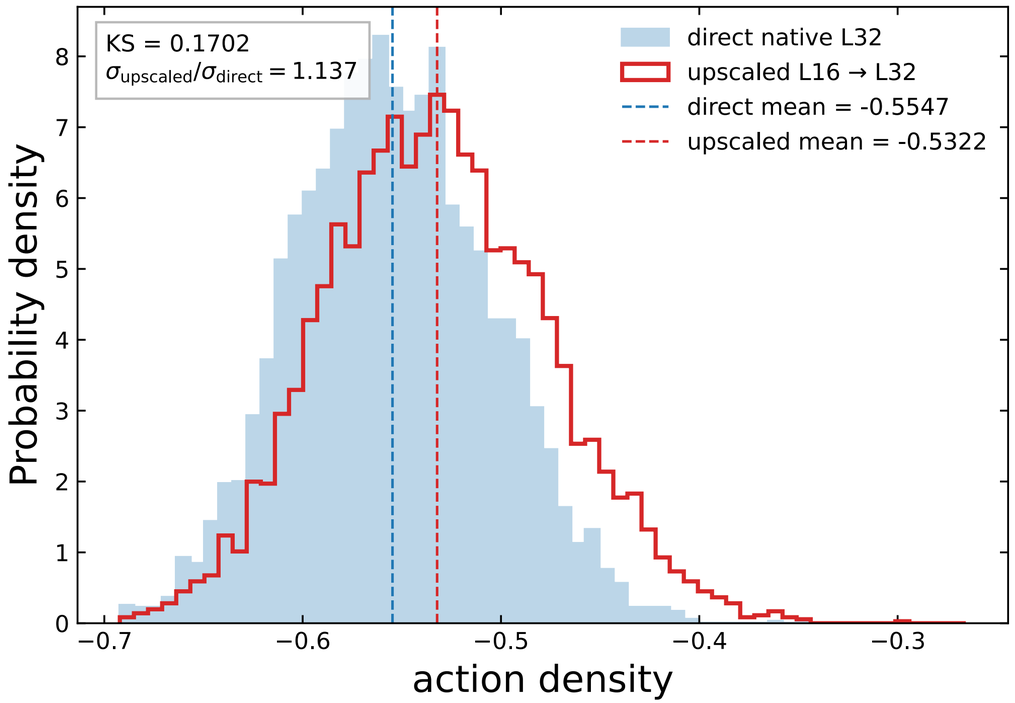


Long-distance observables are nearly perfectly reproduced

Validation of Upscaling - action density

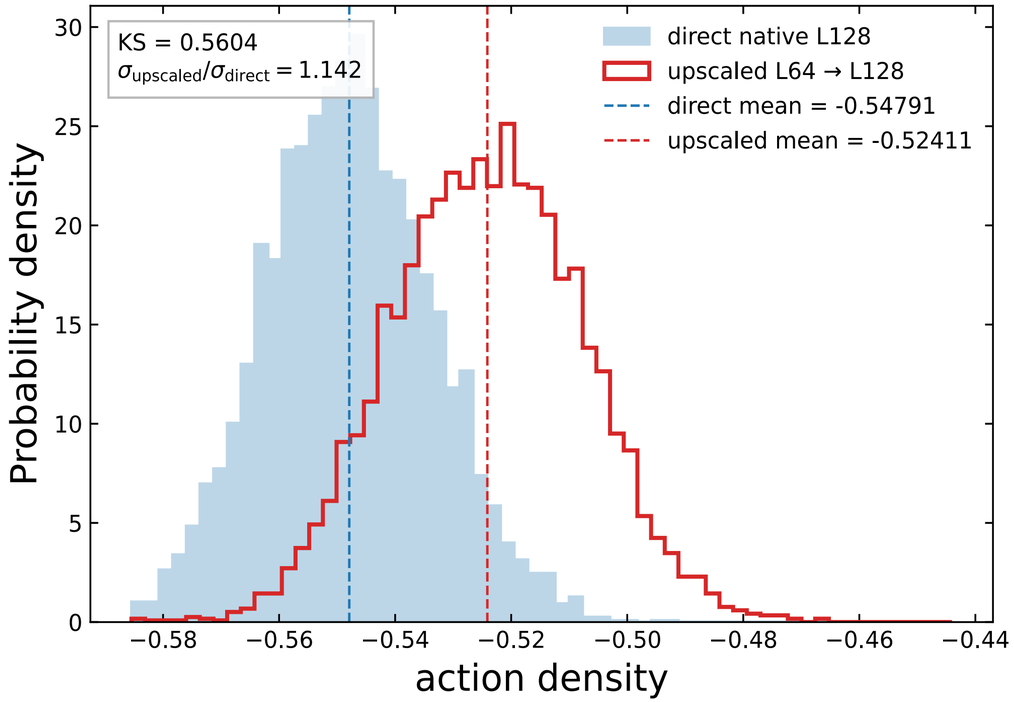
$$\phi_c \longrightarrow (\phi_c, d_{01}, d_{10}, d_{11}) = \psi_{\text{prop}} \longrightarrow \phi_{\text{prop}} = K^{-1}\psi_{\text{prop}}$$

Direct $L = 32$ and upscaled $L = 16 \to 32$



Direct $L = 128$ and upscaled $L = 64 \to 128$

Same flow, no retraining

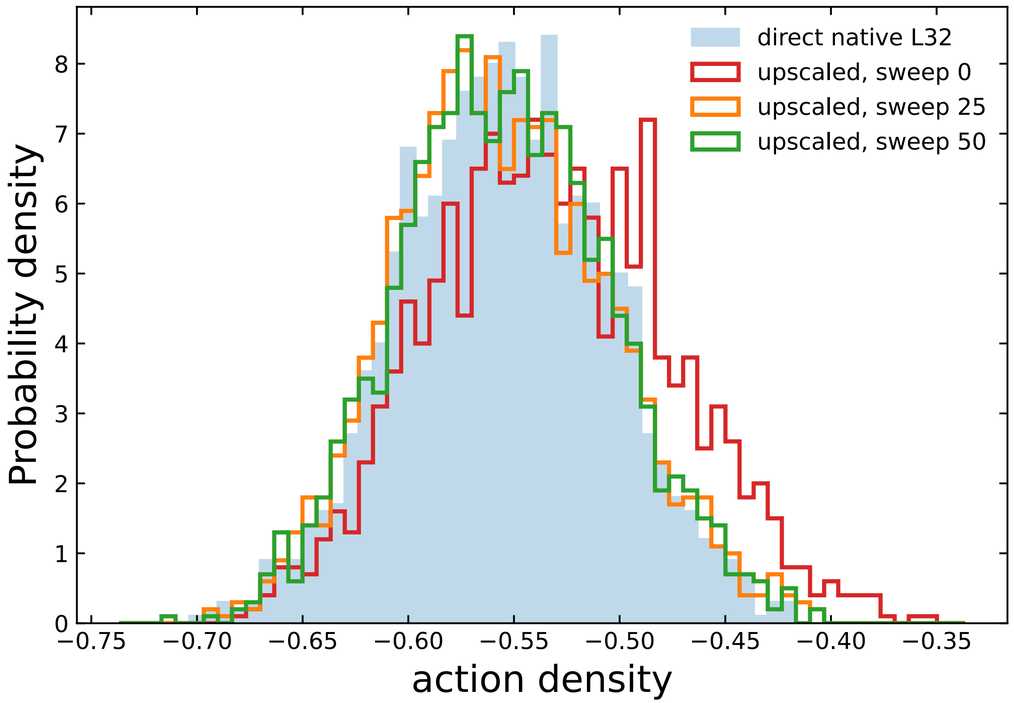


Short-distance mismatch grows with volume

3) Rethermalization:
Can short exact evolution repair UV mismatch?

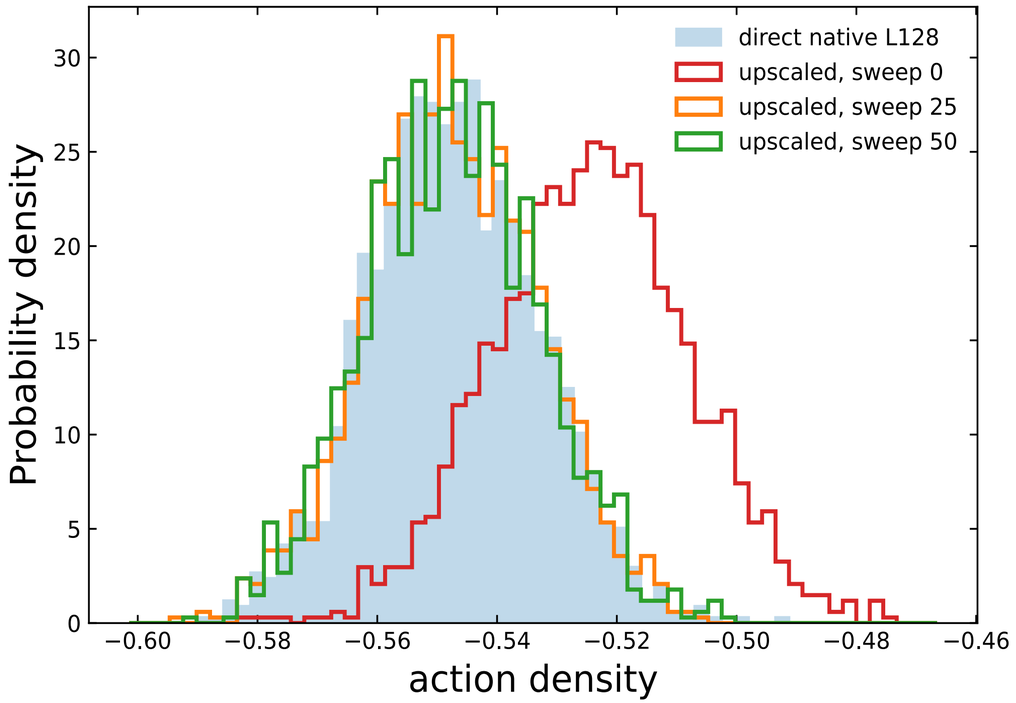
Rethermalize with 25 or 50 full-volume HMC sweeps (or 5-10 cluster update)

Direct $L = 32$ and upscaled $L = 16 \to 32$



Direct $L = 128$ and upscaled $L = 64 \to 128$

Same flow, no retraining

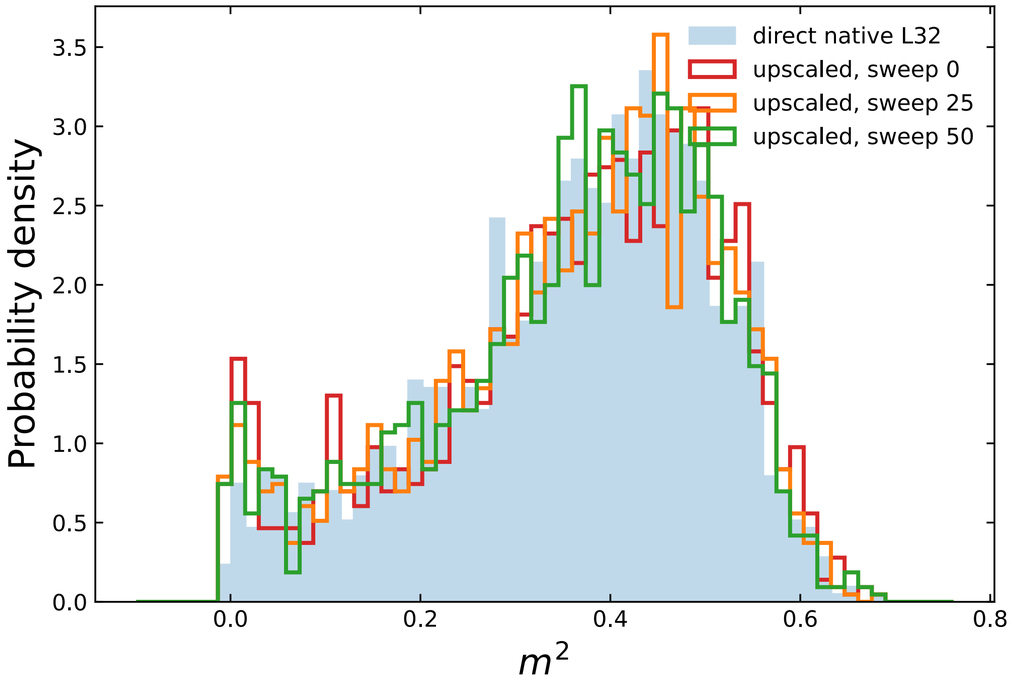


A few rethermalization steps repair the initial UV mismatch

Rethermalization :
Can short exact evolution repair UV mismatch?

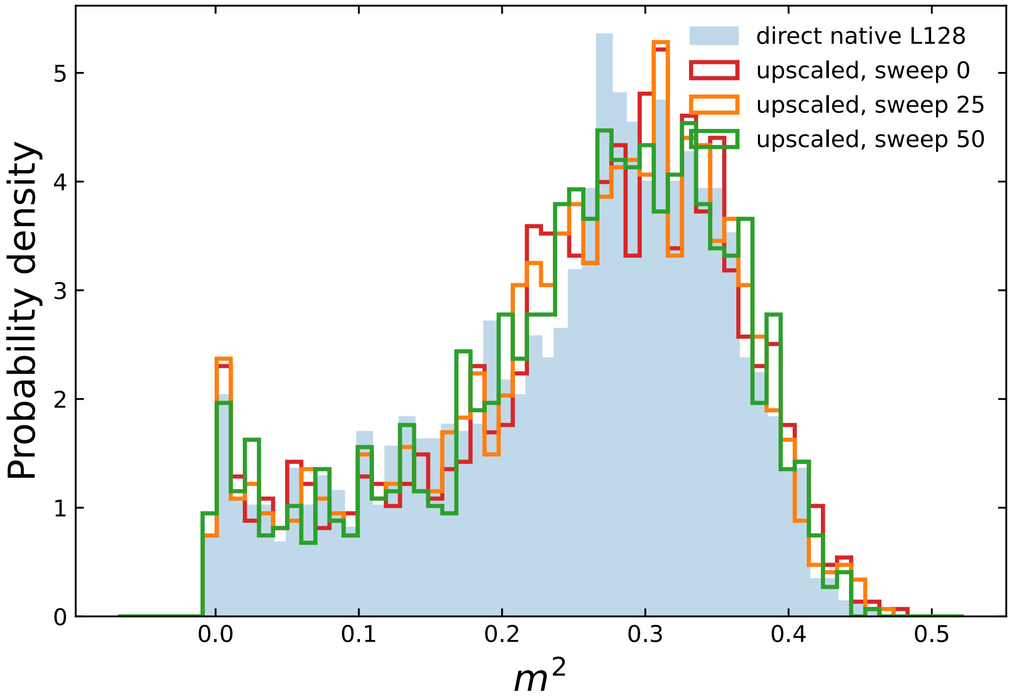
Rethermalize with 25 or 50 full-volume HMC sweeps

Direct $L = 32$ and upscaled $L = 16 \rightarrow 32$



Direct $L = 128$ and upscaled $L = 64 \rightarrow 128$

Same flow, no retraining

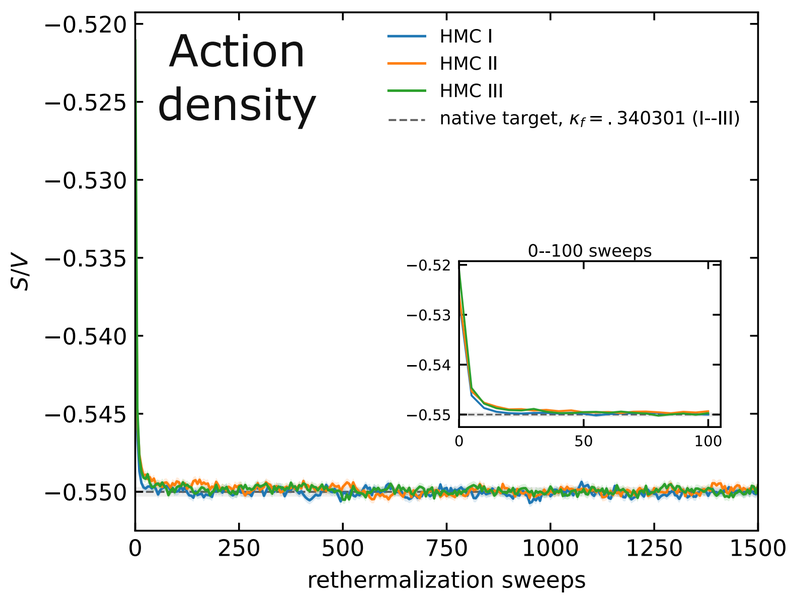


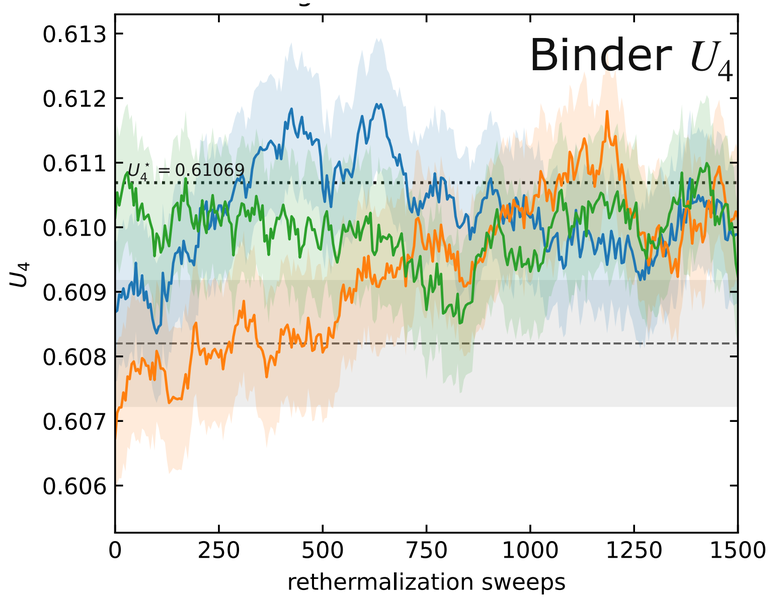
Long-distance modes are preserved by rethermalization

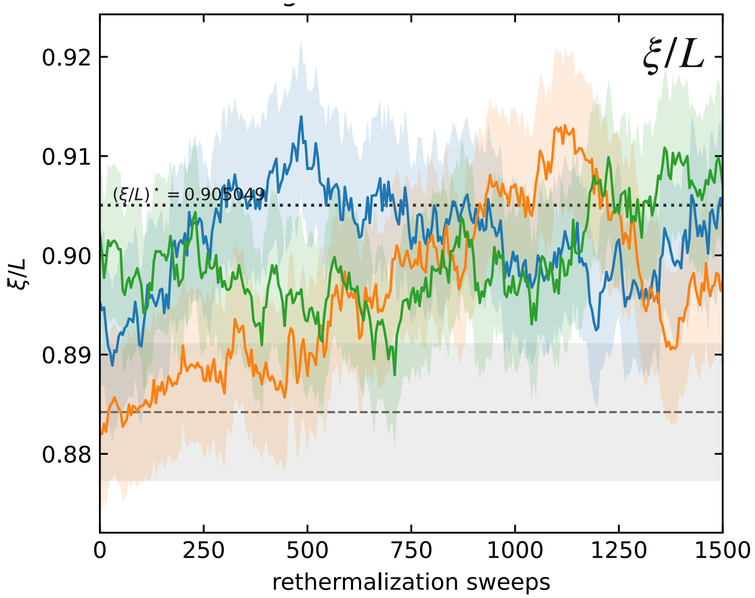
Rethermalization :

Repairing mistuned couplings

Why is rethermalization so effective? It needs to repair short distance mismatch only







HMC I: $\kappa_c = \kappa_f$

HMC II : κ_c, λ mistuned but critical

HMC III : κ_c mistuned, not critical

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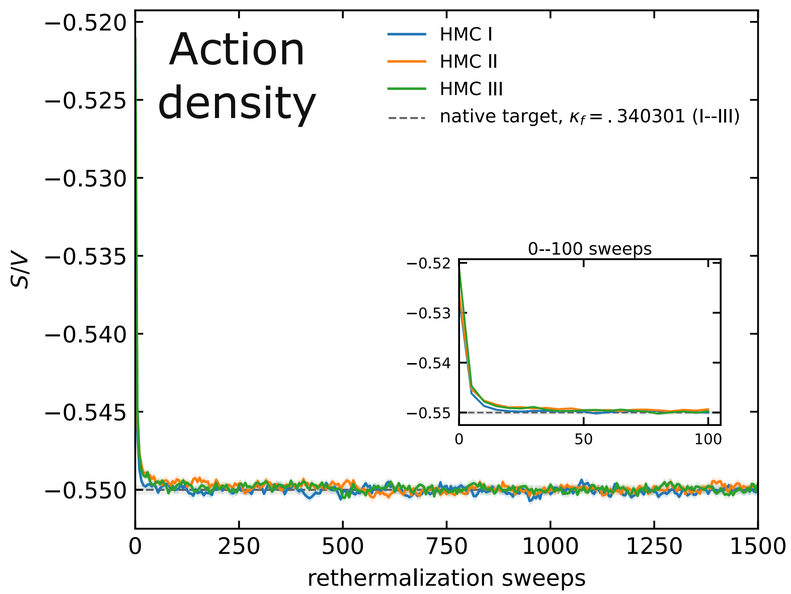

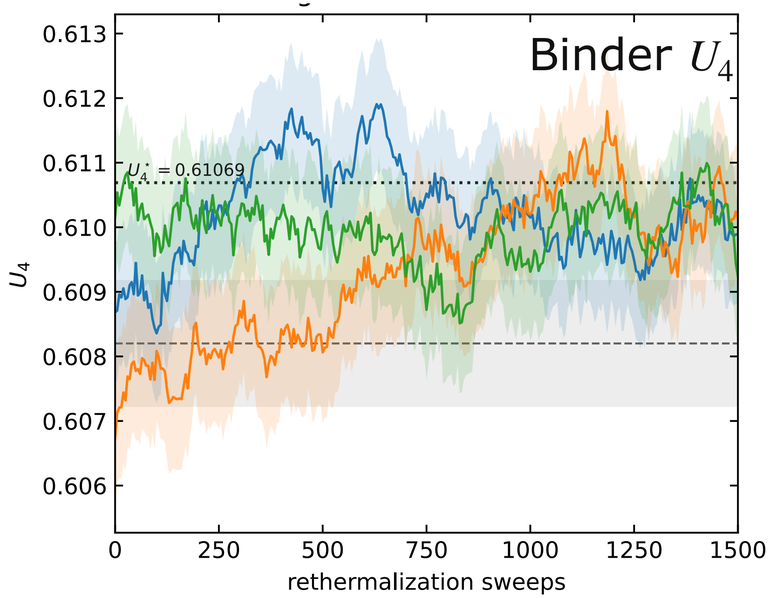

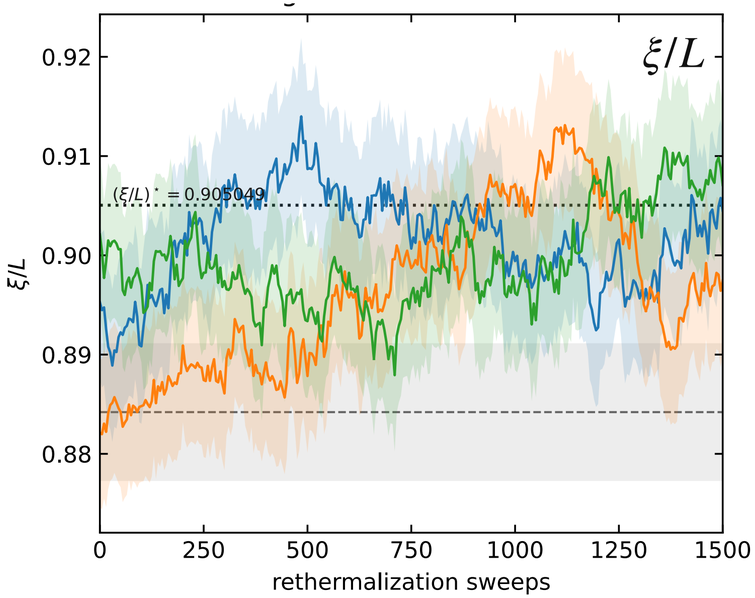

HMC I: $\kappa_c = \kappa_f$

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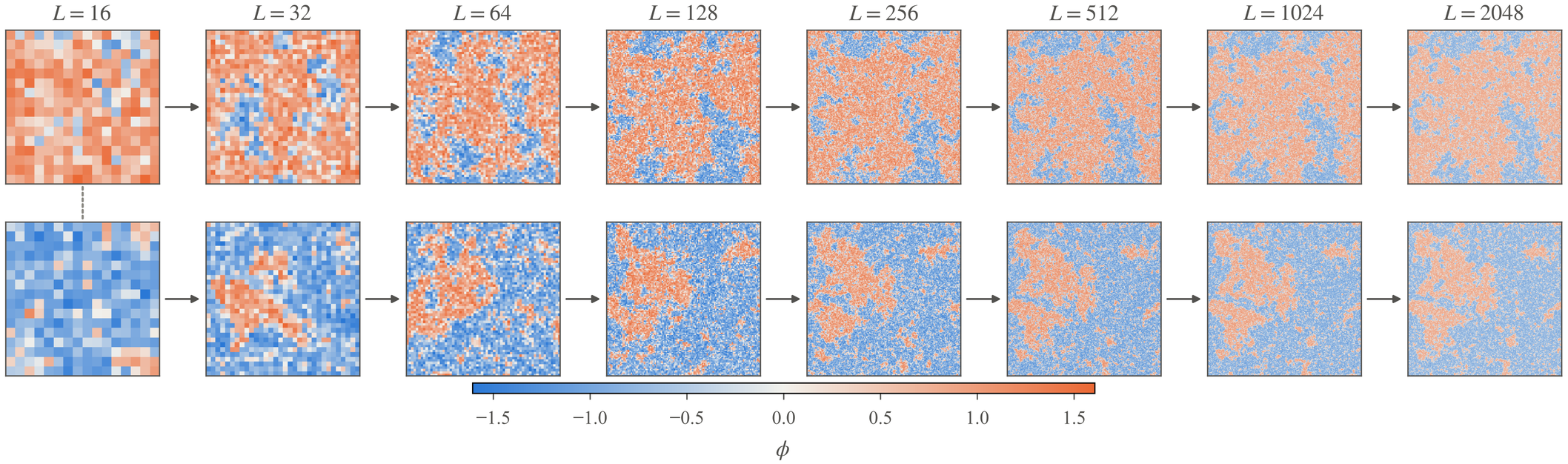
HMC III : κ_c mistuned, not critical

Irrelevant directions are repaired fast
Relevant directions need more rethermalization steps

4) Cascade : $16 \to 32 \to 64 \to \ldots \to 2048$

Rethermalization repairs mismatch from upscaling

$\Longrightarrow$ use the rethermalized field as coarse input for the next step (only at κ_{cr})



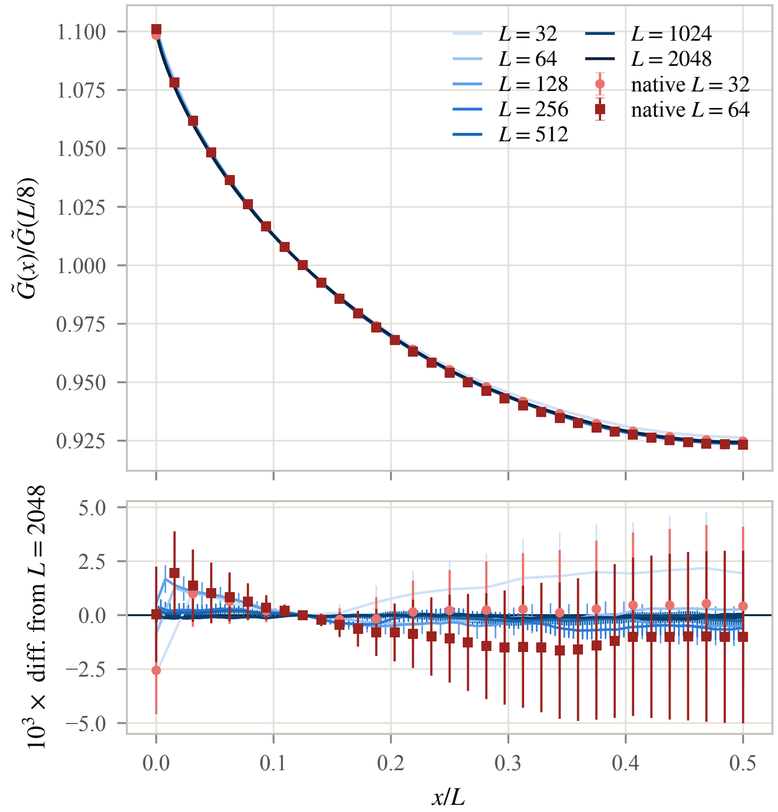
Volumes within a cascade are strongly correlated;

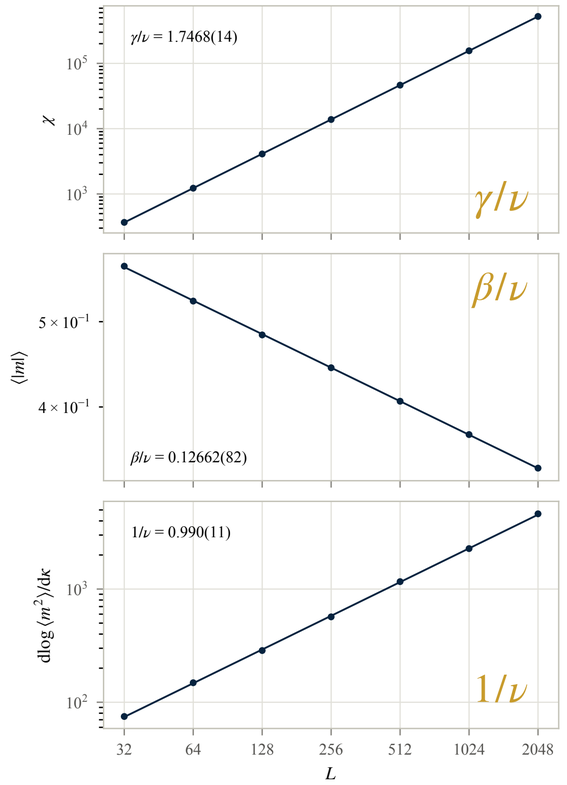
Different cascades are independent if their root configurations are independent.

Critical properties

Volumes within a cascade are strongly correlated; **different cascades are independent**

Finite size scaling of observables





~2000 independent roots reproduce the known critical scaling up to L=2048

Conclusion

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Train small, upscale large, repair locally

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Train small, upscale large, repair locally

RG-guided inverse blocking separates the problem by scale
- coarse ensembles supply the **IR**
- conditional flow generates only the missing **UV details**

Approximate perfect blocking makes the coarse theory simple
- compact kernel
- transfers across volumes

Short exact rethermalization repairs residual UV mismatch
- preserves long-distance modes
- avoids a global acceptance penalty that worsens with volume

Cascade upscaling reaches $L = 2048$ from $L = 16$
- critical observables and scaling are consistent with 2-d Ising behavior

Generalization away from criticality
- requires identifying coupling pairs with ξ and $\xi/2$ correlation length
- include as a tunable parameter

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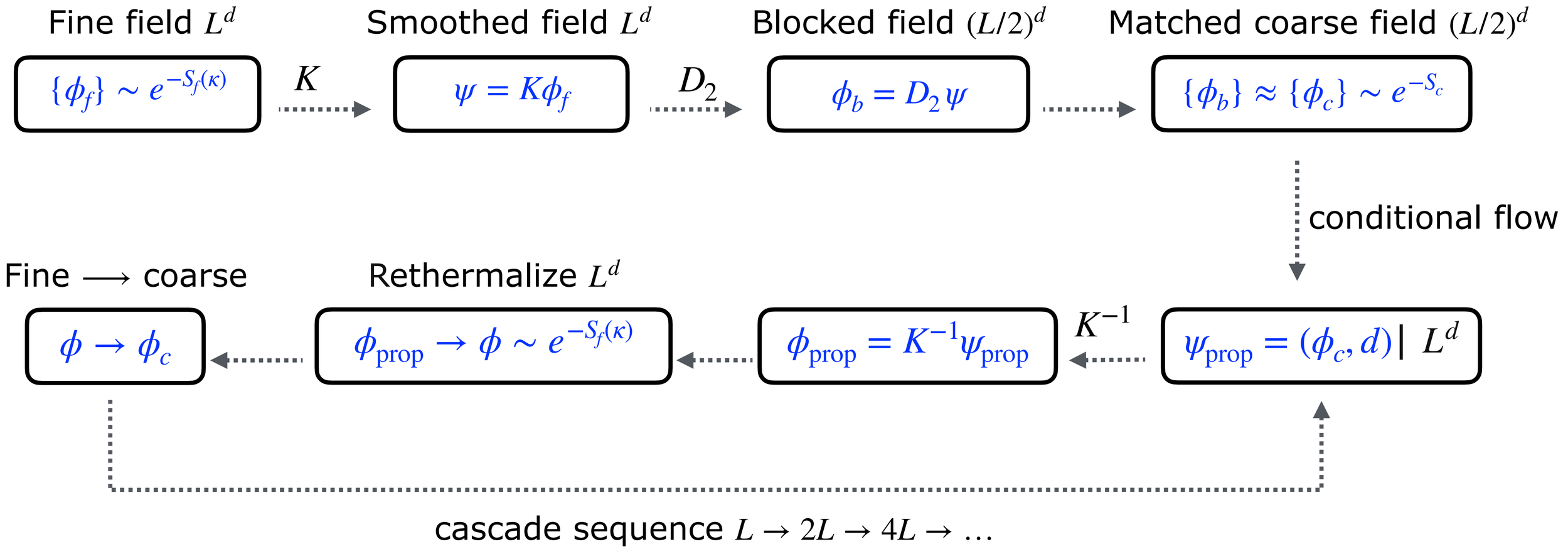
- requires identifying coupling pairs with ξ and $\xi/2$ correlation length
- include as a tunable parameter

Outlook: compact groups $\longrightarrow$ gauge systems $\longrightarrow$ fermions

RG blocking is known; equivariant flow technology exists — combine them.

EXTRA SLIDES

RG-guided inverse blocking:
Supply the IR; generate only the missing UV details



Observables

IR observables considered:

$$U_4 = 1 - \frac{\langle m^4 \rangle}{3\,\langle m^2 \rangle^2}\,, \qquad \frac{\xi}{L} = \frac{1}{2\,L\,\sin(\pi/L)}\sqrt{\frac{\chi}{G_p} - 1}\,, \qquad \chi = V\left(\langle m^2 \rangle - \langle m \rangle^2\right)$$