Towards Precision Double-Beta-Decay Predictions

Lukáš Gráf

(Charles University, Prague)

Trento, August 2026

Towards Precision Double-Beta-Decay Predictions

And what is it good for?

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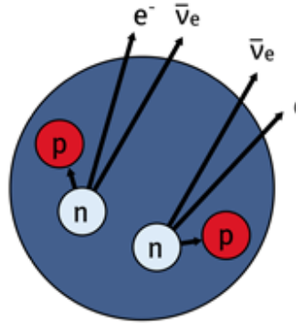
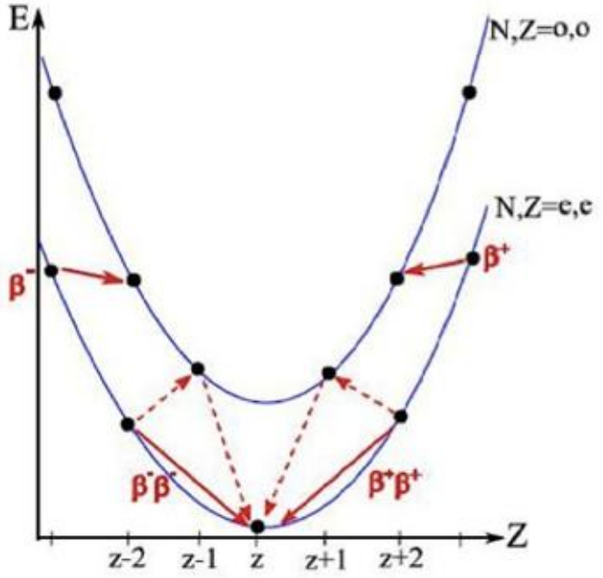
Double Beta Decay

- double beta decay $2\nu\beta\beta: \ (A,Z) \rightarrow (A,Z+2) + 2e^- + 2\bar{\nu}_e$





- one of the rarest processes ever observed in Nature (age of the Universe ~ 10^{10} y) $T_{1/2}^{2\nu\beta\beta} \sim 10^{18}$ - 10^{21} y
- first proposed by Maria Goeppert Mayer in 1935
  - born in Katowice (Prussia) in 1906
  - studied in Goettingen, PhD in 1931 (Max Born)
  - Johns Hopkins, Columbia, Sarah Lawrence College (NY, teaching science, part-time), Chicago (voluntary (!) associate professor), Argonne, UCSD (professor 1960)
- NP for physics in 1963 (shell model)

Double Beta Decay

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WORLD RENOWNED—Dr. Maria Goeppert Mayer, 57, holds the slide rule she uses in the study of nuclear physics that won her a Nobel Prize today. She is a University of California professor here.

S.D. Mother Wins Nobel Physics Prize

Dr. Mayer 1st Woman in U.S., 2nd in History So Honored

By FRANK BOGAN

Dr. Maria Goeppert Mayer, 57, a research physicist at the University of California here, today was named 1963 Nobel Prize winner in physics.

The red-haired college professor, mother of two, is the first woman residing in America to win a Nobel

Double Beta Decay

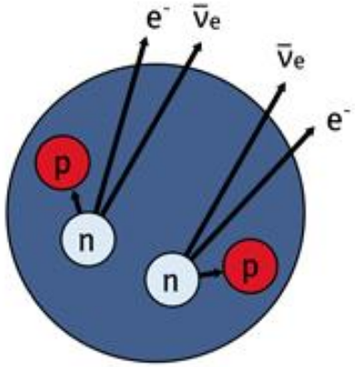
- Majorana's idea (1937): neutral fermion = antifermion
- Furry (1939): $0\nu\beta\beta: \ (A,Z) \rightarrow (A,Z+2) + 2e^-$
- neutrinoless double beta decay
  → mediated by Majorana neutrinos
  → lepton number violation
- experiments: $T^{2\nu\beta\beta}_{1/2} \sim 10^{18} - 10^{21}$ y

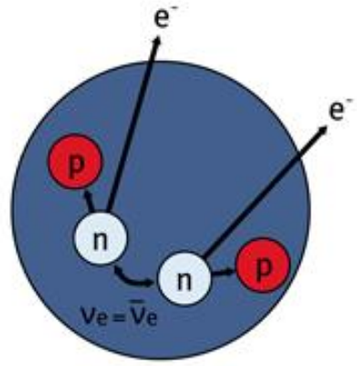
  $T^{0\nu\beta\beta}_{1/2} \sim (0.1 \text{ eV}/m_\nu)^2 \times 10^{26}$ y

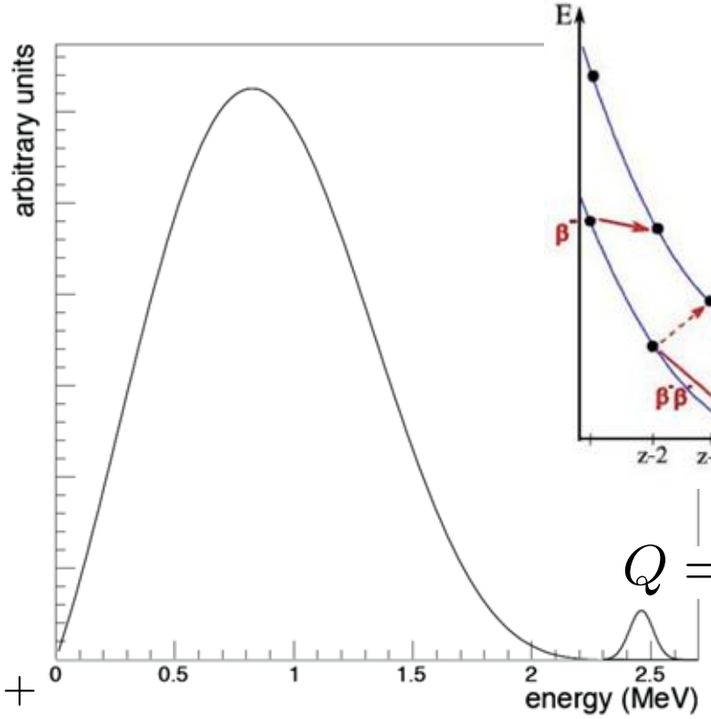
  KamLAND-Zen, LEGEND, CUORE, NEMO-3, CUPID, (n)EXO, …
- a variety of isotopes: ^{76}Ge, ^{136}Xe, …
- variants: $0\nu\beta^+\beta^+: \ (A,Z) \rightarrow (A,Z-2) + 2e^+$

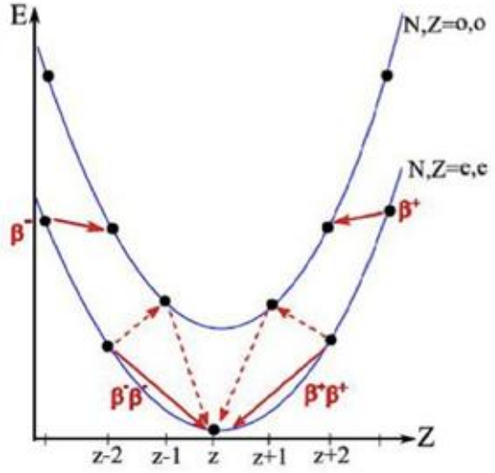
  $0\nu\beta^+EC: \ (A,Z) + e^- \rightarrow (A,Z-2) + e^+$

  $0\nu ECEC: \ (A,Z) + 2e^- \rightarrow (A,Z-2)$









$$Q = E_i - E_f - 2m_e$$

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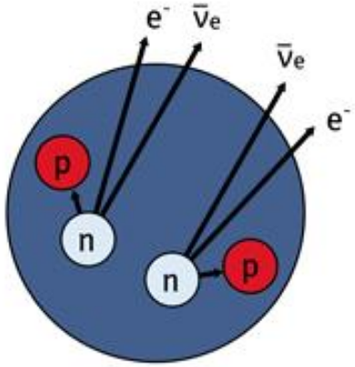

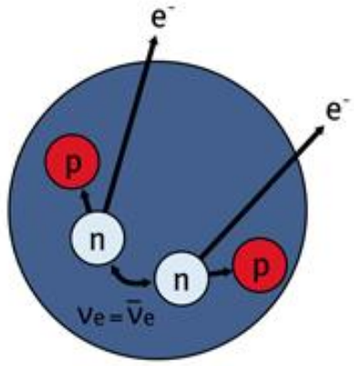

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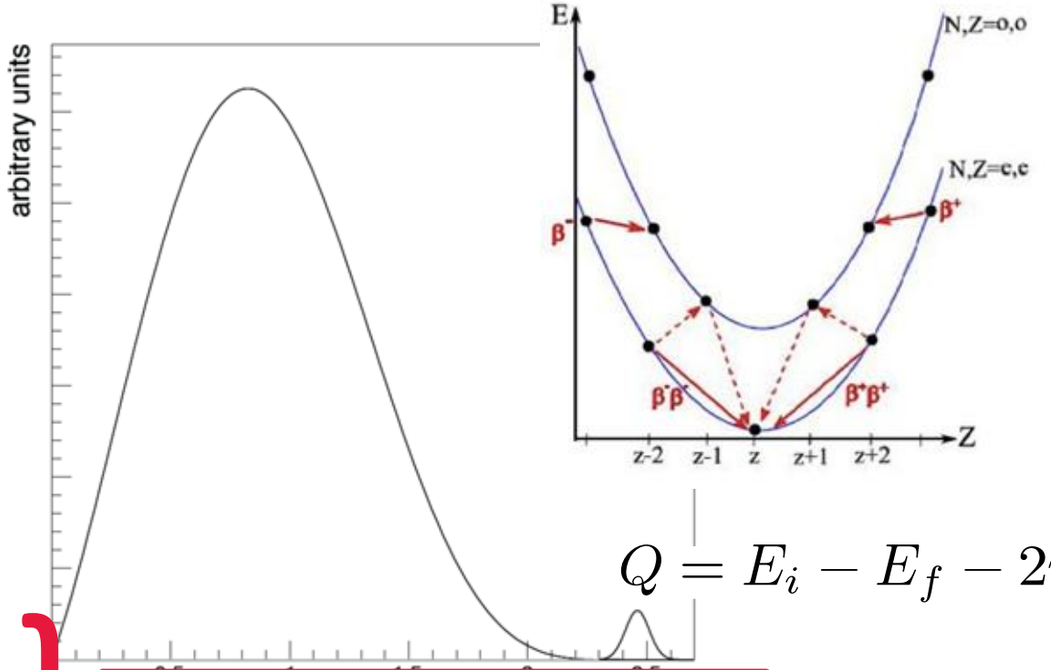


$$Q = E_i - E_f - 2m_e$$

LG, J. Kotila, O. Scholer: 2606.26097

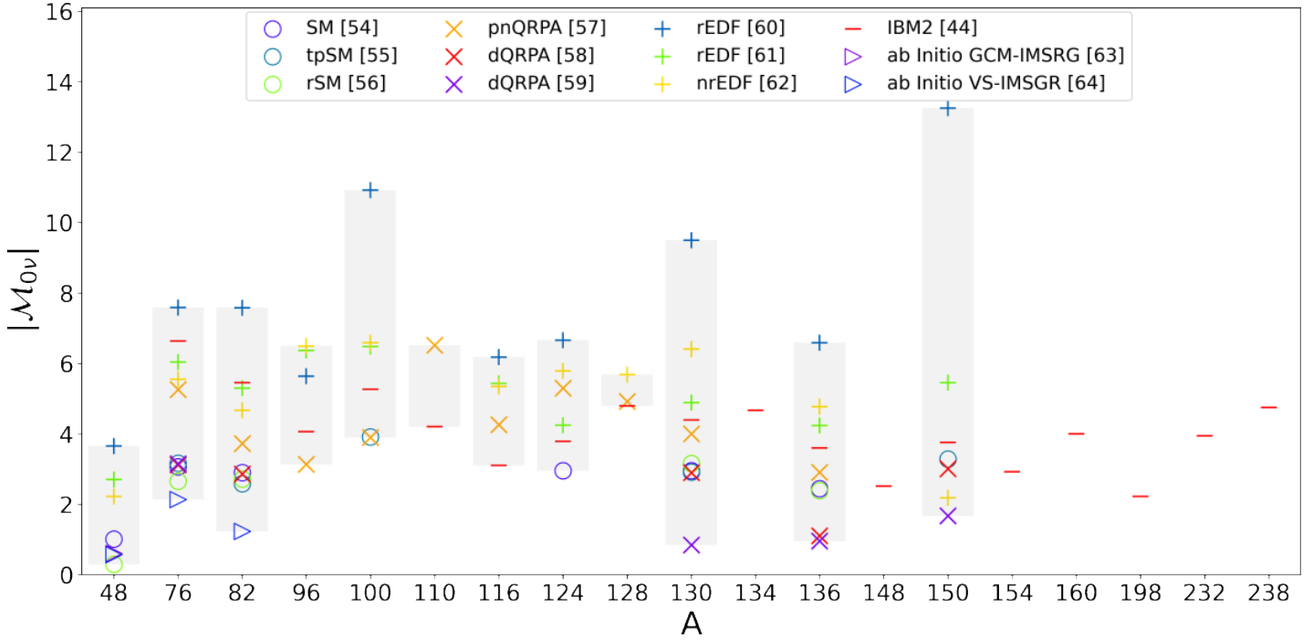
Nuclear Uncertainties

- hadronic currents $J^\mu(q) = g_V\gamma^\mu - g_A\gamma^\mu\gamma^5 + \frac{ig_W}{2m_N}\sigma^{\mu\nu}q_\nu - g_P\gamma^5 q^\mu$
- non-relativistic limit → nuclear matrix elements

$$\mathcal{M}_{0\nu} = g_A^2\mathcal{M}_{GT} - g_V^2\mathcal{M}_F + g_A^2\mathcal{M}_T$$
$$\mathcal{M}_F = \langle h^F(q^2)\rangle$$
$$\mathcal{M}_{GT} = \langle h^{GT}(q^2)(\sigma_a\cdot\sigma_b)\rangle$$
$$\mathcal{M}_T = \langle h^T(q^2)3(\sigma_a\cdot r_{ab})(\sigma_b\cdot r_{ab}) - (\sigma_a\cdot\sigma_b)\rangle$$

- dependence on isotope and operator
- different nuclear structure models, factor of 2-3 difference
- + unknown LECs!

Known LECs		Unknown LECs	
g_A	1.271	$\lvert g_T'\rvert$	$\mathcal{O}(1)$
g_S	0.97 [51]	$\lvert g_T^{\pi\pi}\rvert$	$\mathcal{O}(1)$
g_M	4.7	$\lvert g_{1,6,7,8,9}^{\pi N}\rvert$	$\mathcal{O}(1)$
g_T	0.99 [51]	$\lvert g_{VL}^{\pi N}\rvert$	$\mathcal{O}(1)$
B	2.7 GeV	$\lvert g_T^{\pi N}\rvert$	$\mathcal{O}(1)$
$g_1^{\pi\pi}$	0.36 [52]	$\lvert g_{1,6,7}^{NN}\rvert$	$\mathcal{O}(1)$
$g_2^{\pi\pi}$	2.0 [52]	$\lvert g_{2,3,4,5}^{NN}\rvert$	$\mathcal{O}(16\pi^2)$
$g_3^{\pi\pi}$	−0.62 [52]	$\lvert g_{VL}^{NN}\rvert$	$\mathcal{O}(1)$
$g_4^{\pi\pi}$	−1.9 [52]	$\lvert g_T^{NN}\rvert$	$\mathcal{O}(1)$
$g_5^{\pi\pi}$	−8.0 [52]	$\lvert g_{VL,VR}^{E,m_e}\rvert$	$\mathcal{O}(1)$
g_ν^{NN}	$-92.9\,\mathrm{GeV}^{-2} \pm 50\%$ [53–55]		



LG, M. Lindner, O. Scholer: PRD 106 (2022)

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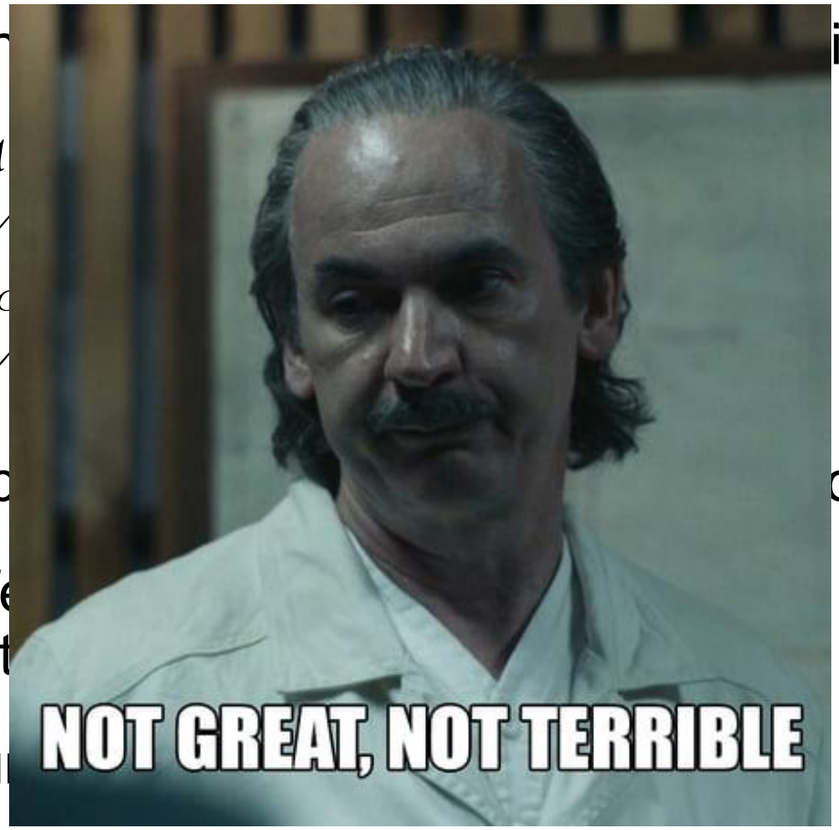

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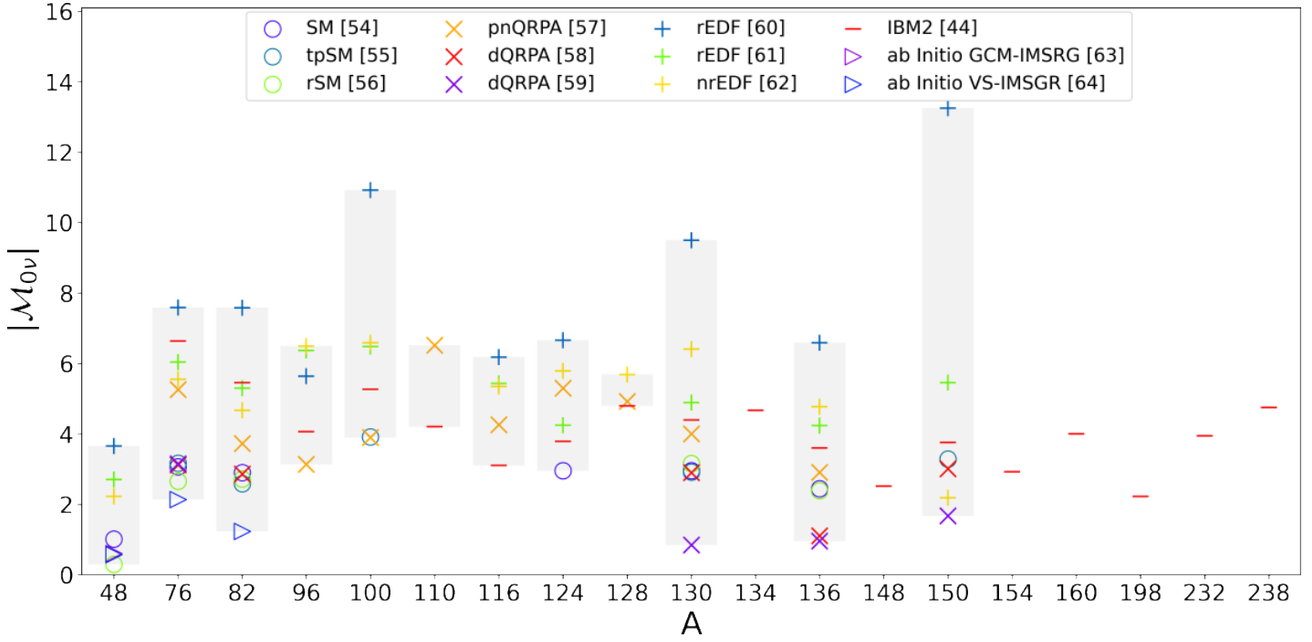


LG, M. Lindner, O. Scholer: PRD 106 (2022)

But let’s focus on two-neutrino double beta decay.

(2ν)ββ Decay – why shall we care?

- searches for 0νββ decay growing – tonne-scale experiments planned
- → background: large amount of the 2νββ decay data collected ~ 10^5 – 10^6 events
- decay rate: $[\mathrm{T}^{2\nu\beta\beta}_{1/2}]^{-1} = G_{2\nu\beta\beta}|M_{2\nu\beta\beta}|^2$

  phase space factor (lepton part)

  nuclear matrix element (nucleon part)

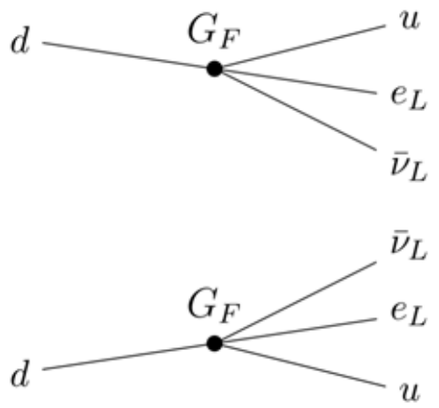
$$\frac{\mathrm{d}\Gamma^{2\nu}}{\mathrm{d}E_{e_1}\mathrm{d}E_{e_2}\mathrm{d}\cos\theta} = \frac{\Gamma^{2\nu}}{2}\frac{\mathrm{d}\Gamma^{2\nu}_{\mathrm{norm}}}{\mathrm{d}E_{e_1}\mathrm{d}E_{e_2}}\left(1+\kappa^{2\nu}(E_{e_1},E_{e_2})\cos\theta\right)$$

- new physics in 2νββ decay? what would it look like?
  - right-handed currents? F. F. Deppisch, LG, F. Simkovic: PRL 125 [2003.11836]
  - sterile neutrinos or light fermions?

    P. D. Bolton, F. F. Deppisch, LG, F. Simkovic: PRD 103 [2011.13387]

    M. Agostini, E. Bossio, A. Ibarra, X. Marcano: PLB 815 [2012.09281]
  - neutrino self-interactions?

    F. F. Deppisch, LG, W. Rodejohann, X. Xu: PRD RC 102 [2004.11919]



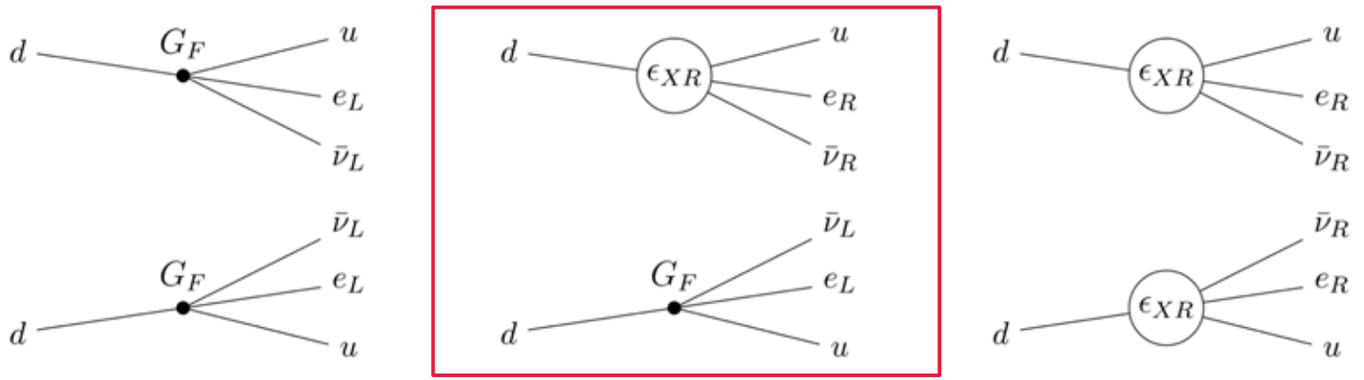
Exotic 2νββ Decay – RHCs

- 2νββ decay in presence of right-handed currents? → Lagrangian:

$$\mathcal{L} = \frac{G_F \cos\theta_C}{\sqrt{2}} \left((1 + \delta_{\mathrm{SM}} + \epsilon_{LL}) j_L^\mu J_{L\mu} + \epsilon_{RL} j_L^\mu J_{R\mu} + \epsilon_{LR} j_R^\mu J_{L\mu} + \epsilon_{RR} j_R^\mu J_{R\mu} \right) + \mathrm{h.c.}$$

with $j^\mu_{L,R} = \bar{e}\gamma^\mu(1 \mp \gamma_5)\nu$, $J^\mu_{L,R} = \bar{u}\gamma^\mu(1 \mp \gamma_5)d$

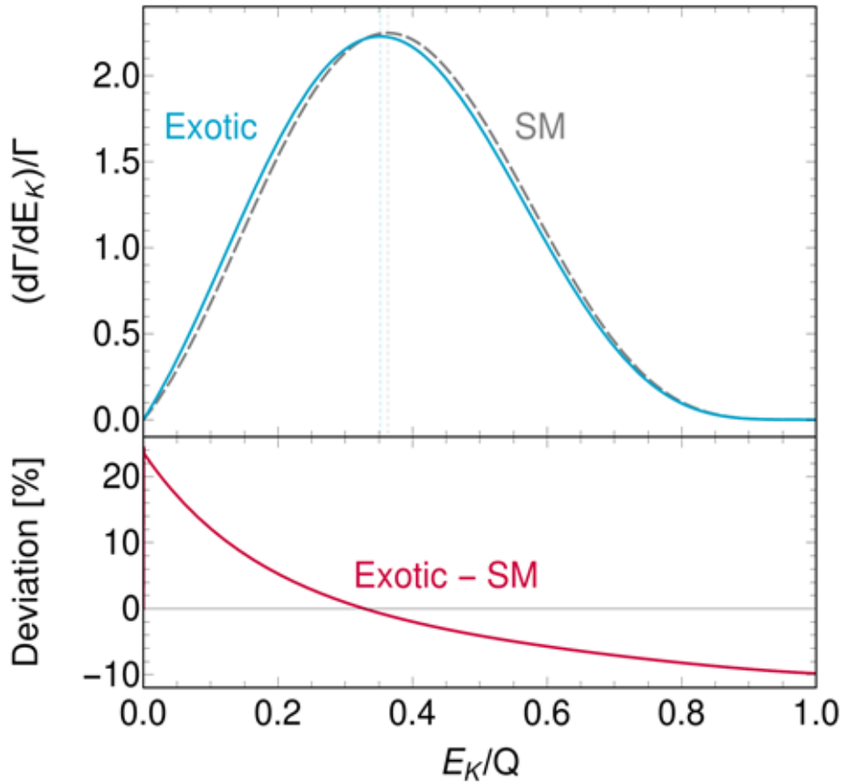
- contributions:

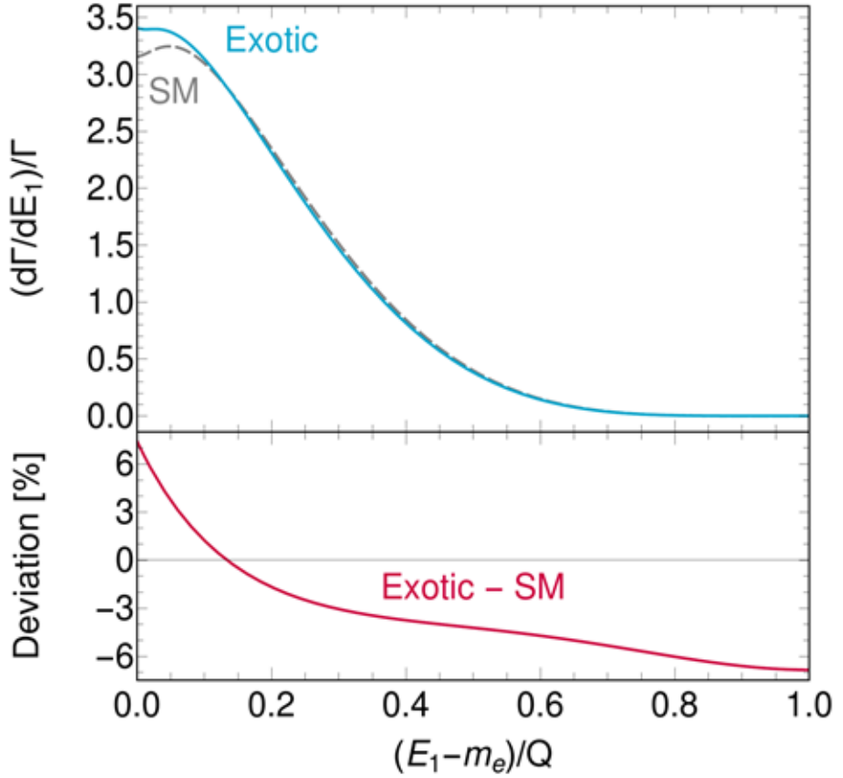


- take the one linear in exotic effective coupling ϵ_{XR}, calculate the observables and get the bound imposed by the experimental data
- rate: $[T^{2\nu\beta\beta}_{1/2}]^{-1} = \epsilon^2_{XR} G_{2\nu\beta\beta} |M_{2\nu\beta\beta}|^2$
- here, results for ^{100}Mo, but other isotopes similar

Electron Energy Distribution

- 2vββ decay (both the SM and the exotic RHC-induced contributions) distribution in total kinetic (left) and single electron kinetic (right) energy



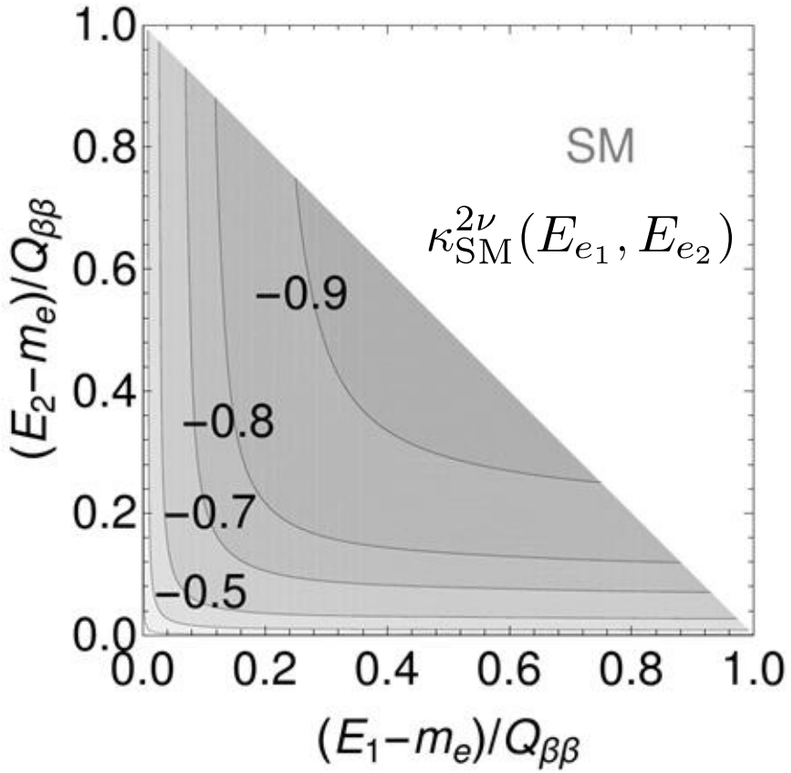


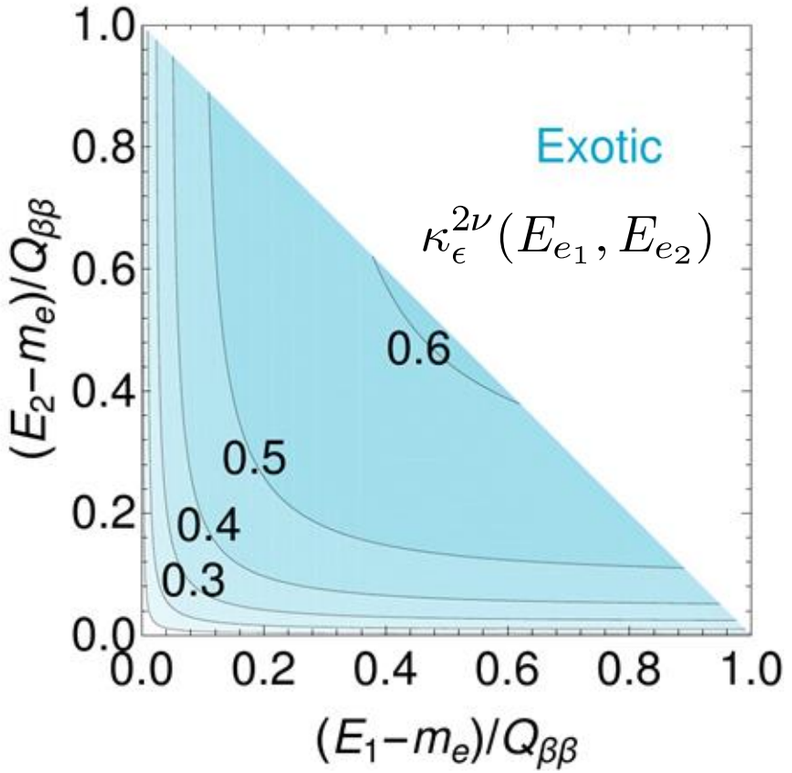
F. F. Deppisch, LG, F. Simkovic: PRL 125 [2003.11836]

Electron Angular Correlation

- angular correlation of the electrons in the SM 2νββ decay (left) and the exotic 2νββ decay induced by right-handed lepton currents (right)

$$\frac{\mathrm{d}\Gamma^{2\nu}}{\mathrm{d}E_{e_1}\mathrm{d}E_{e_2}\mathrm{d}\cos\theta} = \frac{\Gamma^{2\nu}}{2}\frac{\mathrm{d}\Gamma^{2\nu}_{\mathrm{norm}}}{\mathrm{d}E_{e_1}\mathrm{d}E_{e_2}}\left(1+\kappa^{2\nu}(E_{e_1},E_{e_2})\cos\theta\right)$$





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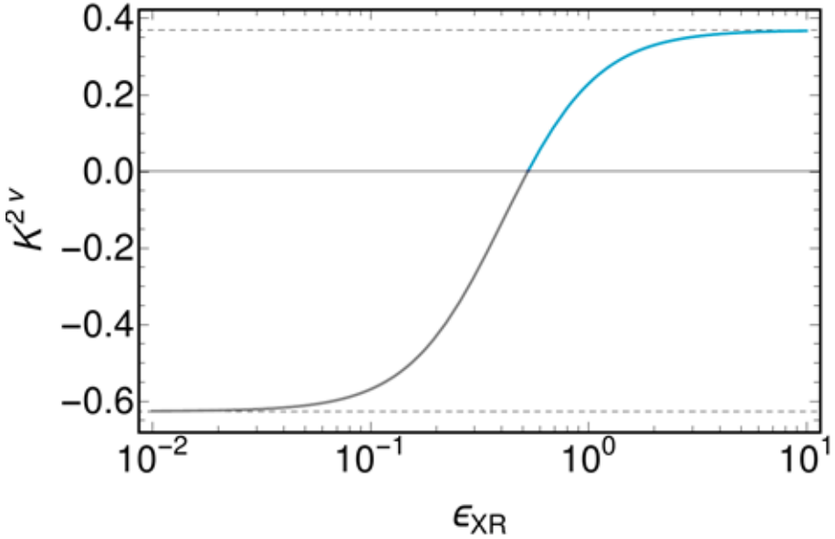
Bound on ϵ_{XR} Coupling

- total rate – nuclear matrix elements not accurate enough
- electron energy distribution – small deviations, could be probed, but certain degeneracy with nuclear structure effects
- best option: measure the angular correlation of the emitted electrons and look for the forward-backward asymmetry
- possible at NEMO-3 experiment using ^{100}Mo (or in future at SuperNEMO?)
- insensitive to the overall rate, largely insensitive to the NMEs

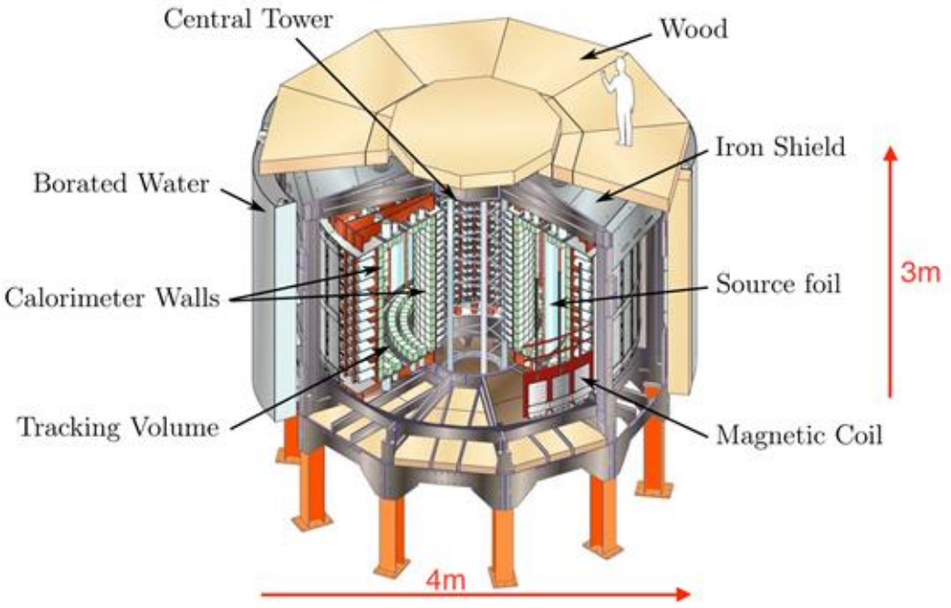


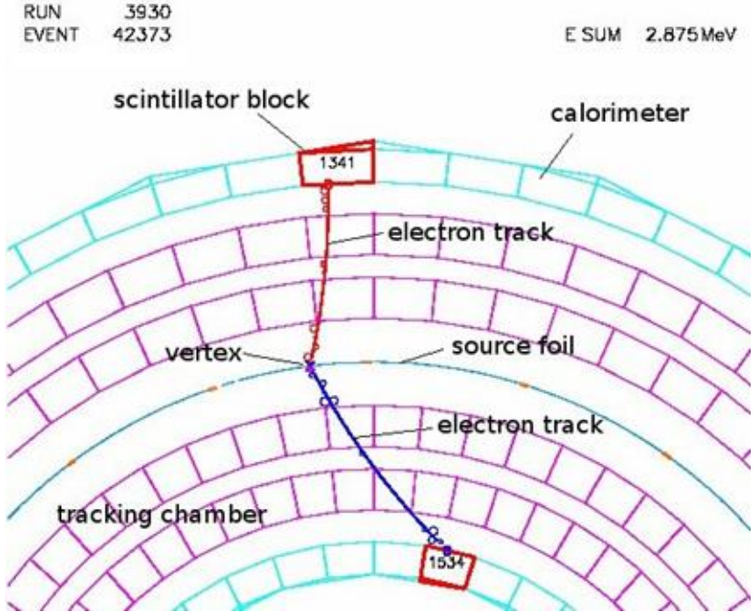
- NMEs: QRPA used in our case, but very similar results expected to be obtained with other nuclear structure models

F. F. Deppisch, LG, F. Simkovic: PRL 125 [2003.11836]

NEMO-3 Experiment

- thin foils of source material (mainly Mo) surrounded by a separate tracking calorimeter
- → better accuracy – reliable detection of two electrons coming from the same place
- allows for measurement of electron energies and angular correlation
- next generation → SuperNEMO





Bound on ϵ_{XR} Coupling

- angular distribution: $\frac{\mathrm{d}\Gamma^{2\nu}}{\mathrm{d}\cos\theta} = \frac{\Gamma^{2\nu}}{2}\left(1 + K^{2\nu}\cos\theta\right)$
- with correlation factor: $K^{2\nu} \approx K^{2\nu}_{\mathrm{SM}} + \alpha\,\epsilon^2_{XR}$, for ^{100}Mo: $\alpha \approx 6.1$
- forward-backward asymmetry: $A^{2\nu}_{\theta} = \frac{N_{\theta>\pi/2} - N_{\theta<\pi/2}}{N_{\theta>\pi/2} + N_{\theta<\pi/2}} = \frac{1}{2}K^{2\nu}$
- estimated accuracy of NEMO-3: $K^{2\nu}_{\mathrm{SM}} = -0.63 \pm 0.0027$
- → bound on the effective coupling at 90% CL: $\boldsymbol{\epsilon_{XR} \lesssim 2.7 \times 10^{-2}}$
- more stringent limit than the one obtained from the standard beta decay measurements, SuperNEMO would further improve this bound to $\epsilon_{XR} \lesssim 4.8 \times 10^{-3}$
- estimates only, a dedicated experimental analysis necessary
- but constraint could be also derived from the shape of the energy distribution – more data, possibly better accuracy, could be also stringent

Exotic 2νββ – Sterile Neutrino

- idea: one of the neutrinos emitted in the decay is sterile

- consider either mixing with the active neutrinos, or right-handed current

$$\mathcal{L} = \frac{G_F \cos\theta_C}{\sqrt{2}} \left[(1+\delta_{\mathrm{SM}}) j_L^{\mu} J_{L\mu} + V_{eN} j_L^{N\mu} J_{L\mu} + \epsilon_{LR} j_R^{N\mu} J_{L\mu} + \epsilon_{RR} j_R^{N\mu} J_{R\mu} \right] + \mathrm{h.c.}$$

- total differential 2νββ decay rate – incoherent sum of the sterile neutrino and SM rates for a given sterile mass m_N and total electron kinetic energy E_K

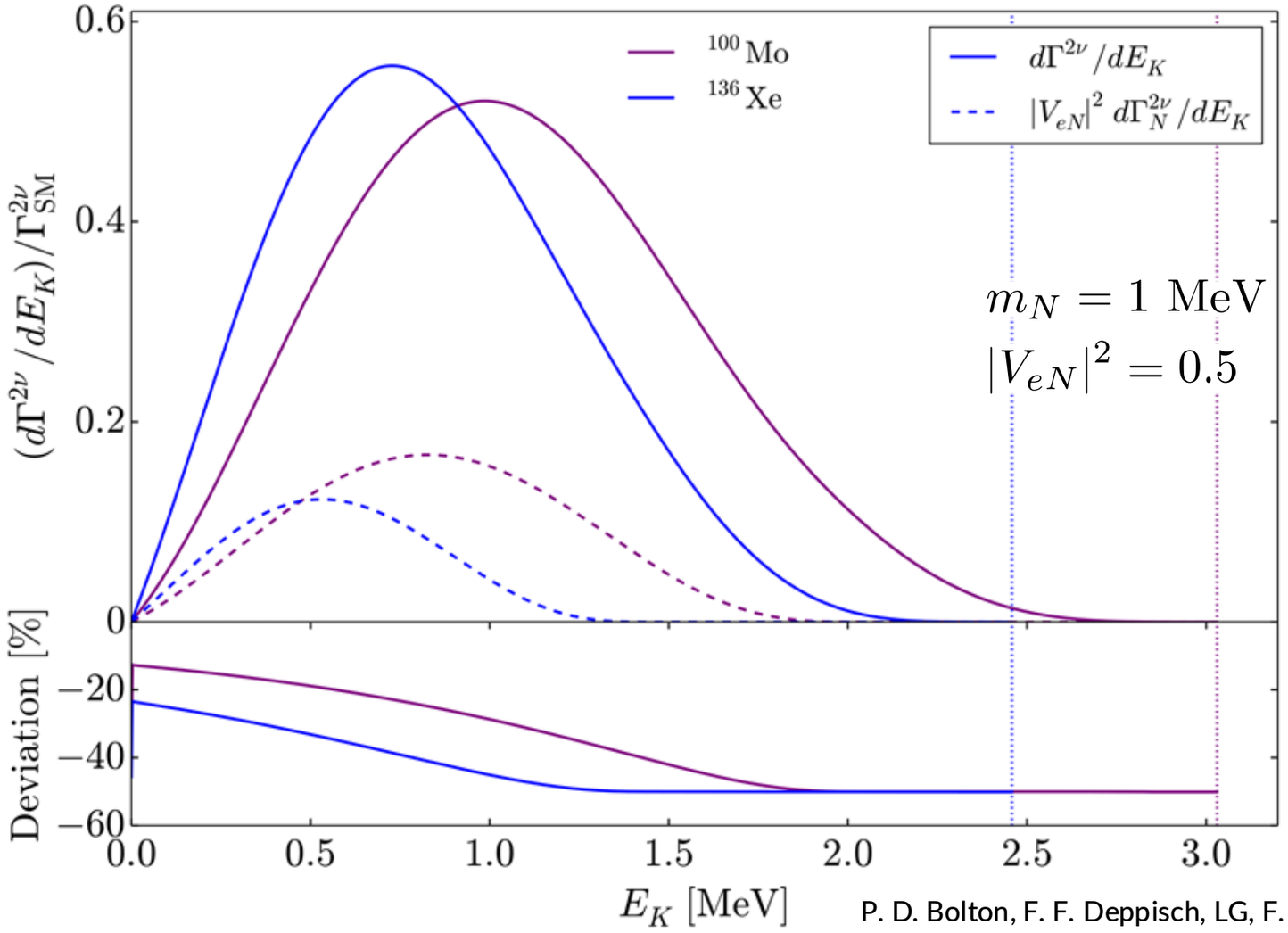
$$\frac{d\Gamma^{2\nu}(\boldsymbol{\xi})}{dE_K} = (1-|V_{eN}|^2)^2 \frac{d\Gamma^{2\nu}_{\mathrm{SM}}}{dE_K} + (1-|V_{eN}|^2)|V_{eN}|^2 \frac{d\Gamma^{2\nu}_{N}(m_N)}{dE_K}$$

- similar expression for the case with the right-handed coupling

$$\frac{d\Gamma^{2\nu}(\boldsymbol{\xi})}{dE_K} = \frac{d\Gamma^{2\nu}_{\mathrm{SM}}}{dE_K} + |\epsilon_{XR}|^2 \frac{d\Gamma^{2\nu}_{N}(m_N)}{dE_K}$$

- again: right-handed current modifies also the angular correlation

Exotic 2νββ – Sterile Neutrino



P. D. Bolton, F. F. Deppisch, LG, F. Simkovic: PRD 103 [2011.13387]

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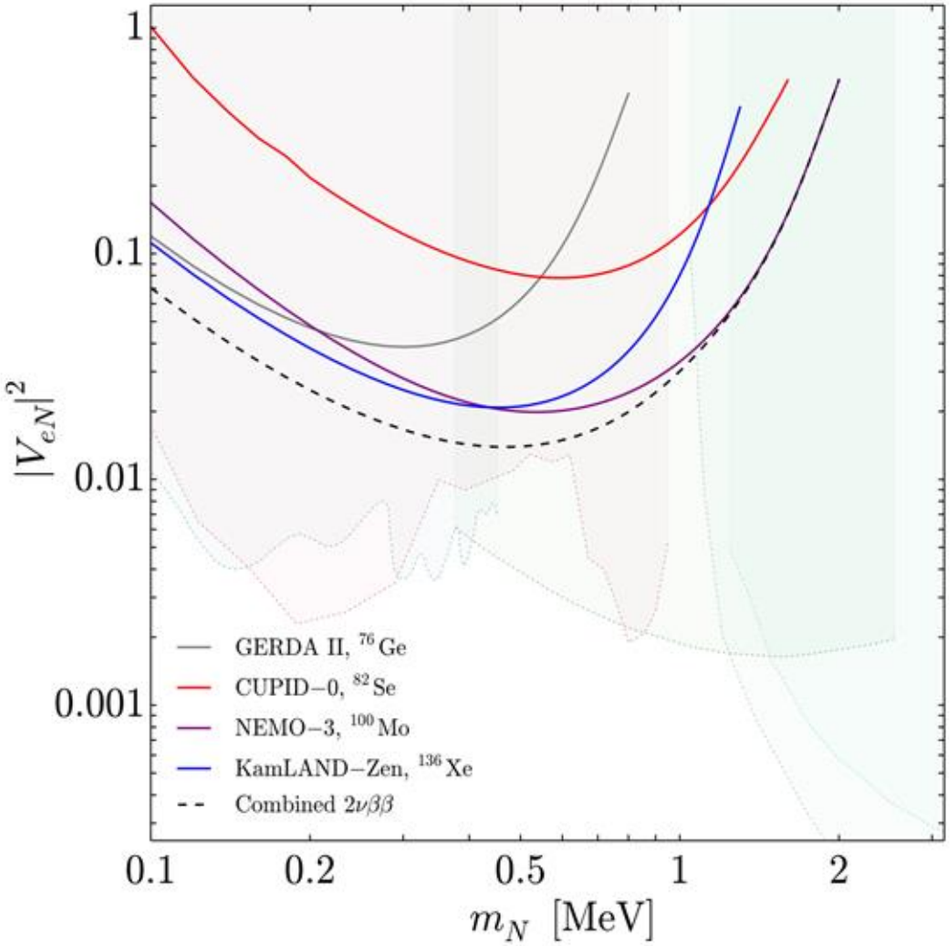

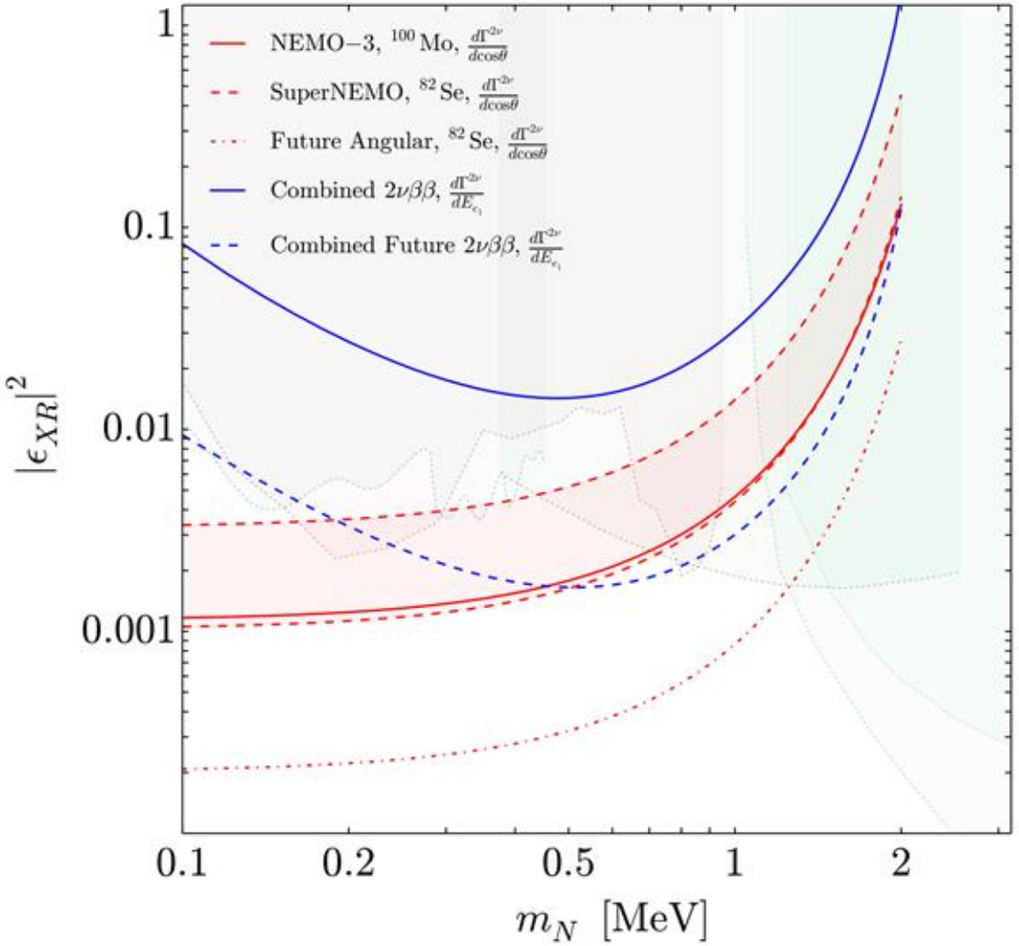

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Neutrino Self-Interactions

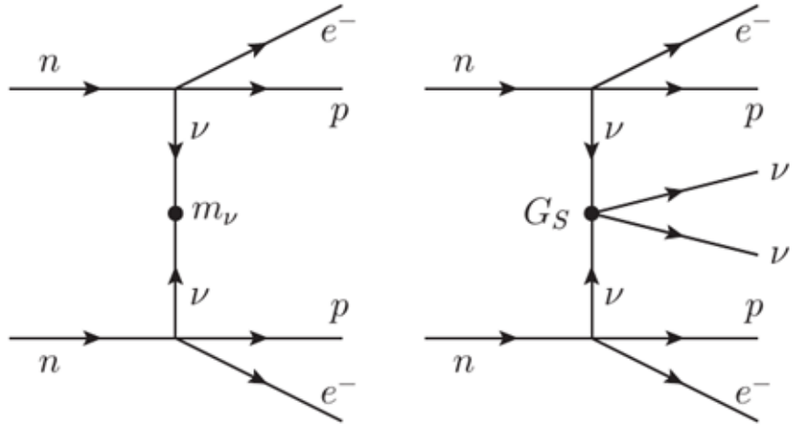
- observations: discrepancy between Cosmic Microwave Background (CMB) and local measurements (redshifts of Type Ia supernovae) of the Hubble constant
- if correct, new cosmo or particle physics; possible solution: neutrino self-interaction (vSI)
- vSI would inhibit the free streaming of neutrinos in the early Universe
- effectively: $G_S\,(\nu\nu)\,(\nu\nu)$, where G_S should be much larger than G_F
- strong interaction, but only neutrinos involved → not easy to test in a lab, light scalar mediator probed by meson decays, tau decay, Z invisible decay, supernova neutrinos … what about double beta decay?

N. Blinov, K. J. Kelly, G. Z. Krnjaic, S. D. McDermott: PRL 123 [1905.02727]

K.-F. Lyu, E. Stamou, Emmanuel, L.-T. Wang: PRD 103 [2004.10868]

ββ Decay Induced by vSI

- idea: effective vSI $G_S\,(\nu\nu)\,(\nu\nu)$ would induce a new, exotic contribution to double beta decay – see the figure:

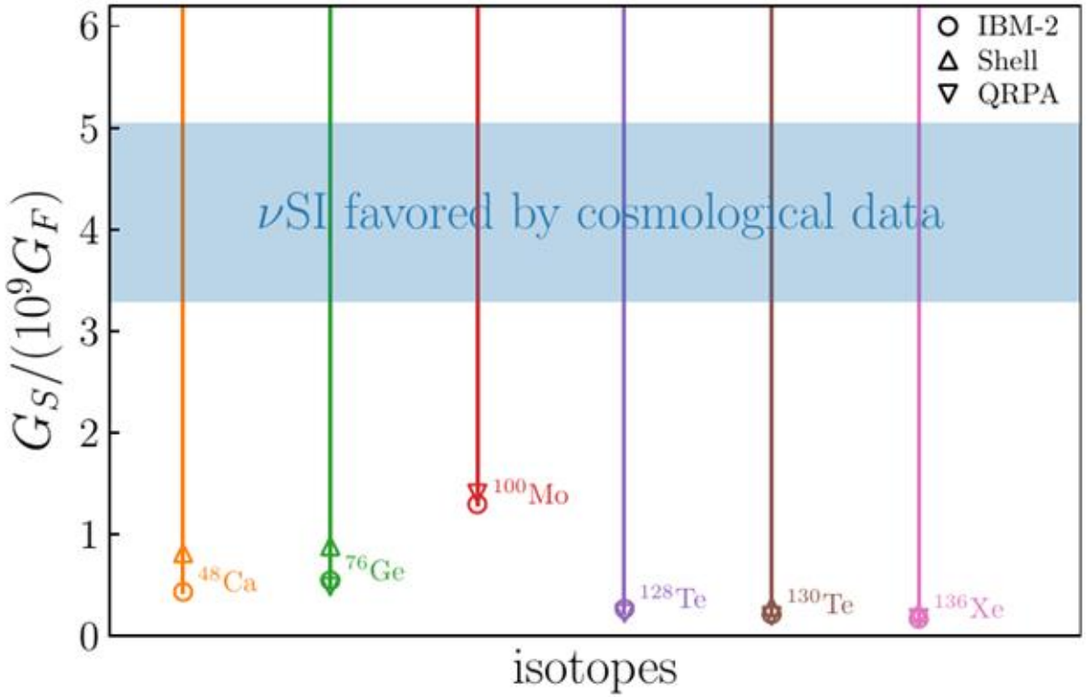


- rate: $\Gamma_{\nu\mathrm{SI}} = \left|\frac{G_S m_e}{2R}\right|^2 \mathcal{G}_{\nu\mathrm{SI}} |\mathcal{M}_{0\nu}|^2$
- nuclear matrix element approximately same as for 0νββ decay
- phase space ~ 2νββ decay $\mathcal{G}_{\nu\mathrm{SI}} = \frac{2c_{\nu\mathrm{SI}}}{15} \int dp_1 dp_2 p_1^2 p_2^2 (Q - T_{12})^5 F^2(p_1, p_2)$
  → vSI could affect 2νββ decay – total rate, but also spectra
- compare to 2νββ rate: $\Gamma_{2\nu} = G_{2\nu} |M_{2\nu}|^2$

F. F. Deppisch, LG, W. Rodejohann, X. Xu: PRD RC 102 [2004.11919]

ββ Decay Induced by vSI

- using condition $\Gamma_{\nu\mathrm{SI}}/\Gamma^{\mathrm{ex}}_{2\nu} < 1$ effective self-interaction G_S can be constrained using the predicted total rate
- result: vSI preferred by cosmology is disfavoured by 2vββ decay experimental data

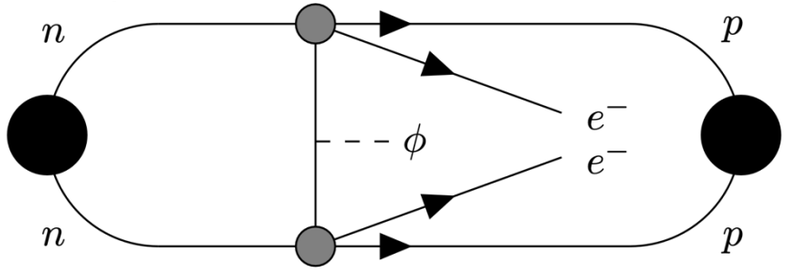


	^{48}Ca	^{76}Ge	^{136}Xe	^{100}Mo	^{128}Te	^{130}Te
Q/MeV	4.263 [46]	2.039 [47]	2.458 [48]	3.034 [47]	0.8659 [49]	2.527 [50]
$T^{2\nu}_{1/2}$/year	5.30×10^{19}	1.88×10^{21}	2.17×10^{21}	6.88×10^{18}	2.25×10^{24}	7.91×10^{20}
$(\Gamma_{\nu\mathrm{SI}})^{-1}$/year	2.52×10^{18}	1.42×10^{20}	1.55×10^{19}	2.94×10^{18}	4.04×10^{22}	9.08×10^{18}
$\Gamma_{\nu\mathrm{SI}}/\Gamma^{\mathrm{ex}}_{2\nu}$	78.6	49.5	528	8.76	209	326
$G_S/G_F <$	4.32×10^{8}	5.44×10^{8}	1.67×10^{8}	1.29×10^{9}	2.65×10^{8}	2.12×10^{8}

F. F. Deppisch, LG, W. Rodejohann, X. Xu: PRD RC 102 [2004.11919]

Scalarful Double Beta Decay

- SSB of global $U(1)_{B-L}$ (arising in seesaw-inspired models) → a massless Goldstone boson, called Majoron $\mathcal{L} \supset \sum_{i,j} \phi \bar{\nu}_i (g_{ij} + \gamma_5 \bar{g}_{ij}) \nu_j + h.c.$



- Majoron can also be a dark matter candidate
- or axion/ALP: induced CP-violating non-derivative axion interactions with the SM
- allows for $0^+(E_i) \to 0^+(E_f) + e^-(E_{e1}) + e^-(E_{e2}) + \phi(E_\phi)$, with amplitude

$$\mathcal{A} = \left\langle 0_f^+ \middle| \sum_{m,n} \int \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot(\mathbf{r}_n - \mathbf{r}_m)} V(\mathbf{q}^2) \middle| 0_i^+ \right\rangle = \frac{(g_A^{\text{eff}})^2 G_F^2 V_{ud}^2 m_e}{\pi R} \mathcal{A}^\phi \, \bar{u}(p_1) P_R u^c(p_2)$$

- A_ϕ depends only on the nuclear and hadronic matrix elements
- hard (short-distance) region with $q_v^0 \sim |\mathbf{q}_v| \sim \Lambda_\chi$ and the chiral scale $\Lambda_\chi \sim m_N \sim O(1)$ GeV; soft region, $q_v^0 \sim |\mathbf{q}_v| \sim m_\pi \sim k_F$; potential (long-distance) region, $q_v^0 \sim (k_F)^2 / m_N$, $|\mathbf{q}_v| \sim k_F$; ultrasoft region with neutrino momenta scaling as $q_v^0 \sim |\mathbf{q}_v| \ll k_F$

J. de Vries, LG, V. Plakkot, D. Starý: JHEP 03 [2511.19173]

Scalarful Double Beta Decay

- for active ν, $m_i \ll |\mathbf{q}|$, $V(\mathbf{q}^2)$ similar to the regular 0νββ potential, usual NMEs

$$\left(T^{0\nu}_{1/2}\right)^{-1} = g_A^4\, G^{0\nu} \left| V_{ud}^2 \sum_i U_{ei}^2 \frac{m_i}{m_e} A_\nu(m_i) \right|^2 , \quad m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

just take amplitude for m = 0, and with $g_{ee} = \sum_{i,j} U_{ei}\, U_{ej} \left(g_{ij} + \bar{g}_{ij}\right)$

$$\mathcal{A}_\nu(m_i) = \begin{cases} \mathcal{A}_\nu{}^{(\mathrm{ld})}(m_i) + \mathcal{A}_\nu{}^{(\mathrm{sd})}(m_i)\,, & m_i < 2\ \mathrm{GeV} \\ \mathcal{A}_\nu{}^{(9)}(m_i)\,, & 2\ \mathrm{GeV} \leq m_i \end{cases}$$

Dekens et al., JHEP [2402.07993]

- for large m_i, ν is integrated out at the quark level, and the amplitude stemming from the dim-9 operator, with $\mu_0 \simeq 2$ GeV, is given by

$$\mathcal{A}_\nu{}^{(9)}(m_i) = -2\eta(\mu_0, m_i) \frac{m_\pi^2}{m_i^2} \left[\frac{5}{6} g_1^{\pi\pi} \left(\mathcal{M}^{PP}_{GT,sd} + \mathcal{M}^{PP}_{T,sd} \right) + \frac{g_1^{\pi N}}{2} \left(\mathcal{M}^{AP}_{GT,sd} + \mathcal{M}^{AP}_{T,sd} \right) - \frac{2}{g_A^2} g_1^{NN} \mathcal{M}_{F,sd} \right]$$

- with LECs $g_1^{\pi\pi} = 0.36$, $g_1^{\pi N} = 0$, and $g_1^{NN} = (1 + 3g_A^2)/4$, Fermi, Gamow-Teller, and tensor NMEs expressed in terms of the axial (A) and pseudoscalar (P) form factors, and with the QCD evolution given by $\eta(\mu_0, m_i)$

J. de Vries, LG, V. Plakkot, D. Starý: JHEP 03 [2511.19173]

Scalarful Double Beta Decay

- for small m_i: $\mathcal{A}_\nu^{(\mathrm{ld})}(m_i) = -\mathcal{M}(0)\dfrac{1}{1+m_i/m_a+(m_i/m_b)^2}\,, \quad \mathcal{A}_\nu^{(\mathrm{sd})}(m_i) = -2g_\nu^{NN}(m_i)\,m_\pi^2\dfrac{\mathcal{M}_{F,sd}}{g_A^2}$

  where $g_\nu^{NN}(m_i) = g_\nu^{NN}(0)\dfrac{1+a_2\,m_i^2+a_4\,m_i^4}{1+|b_4|m_i^4+|b_6|m_i^6}$ Cirigliano, Dekens and Urrutia Quiroga, JHEP [2505.09679]

	$\mathcal{M}_{F,sd}$	$\mathcal{M}^{AP}_{GT,sd}$	$\mathcal{M}^{PP}_{GT,sd}$	$\mathcal{M}^{AP}_{T,sd}$	$\mathcal{M}^{PP}_{T,sd}$	m_a [MeV]	m_b [MeV]	$\mathcal{M}(0)$
^{76}Ge	-2.21	-2.26	0.82	-0.05	0.02	117	218	3.4
^{136}Xe	-1.94	-1.99	0.74	0.05	-0.02	157	221	2.7

Table: NSM NMEs and fit parameters used in $0\nu\beta\beta$ computations. Menéndez, J. Phys. G **45** (2018), L. Jokiniemi, P. Soriano and J. Menéndez, Phys. Lett. B **823** (2021)

- the inverse half-life reads: $\left(T^\phi_{1/2}\right)^{-1} \equiv \dfrac{\Gamma^\phi}{\ln 2} = |g_{ee}|^2|\mathcal{A}^\phi|^2 G^\phi\,,$

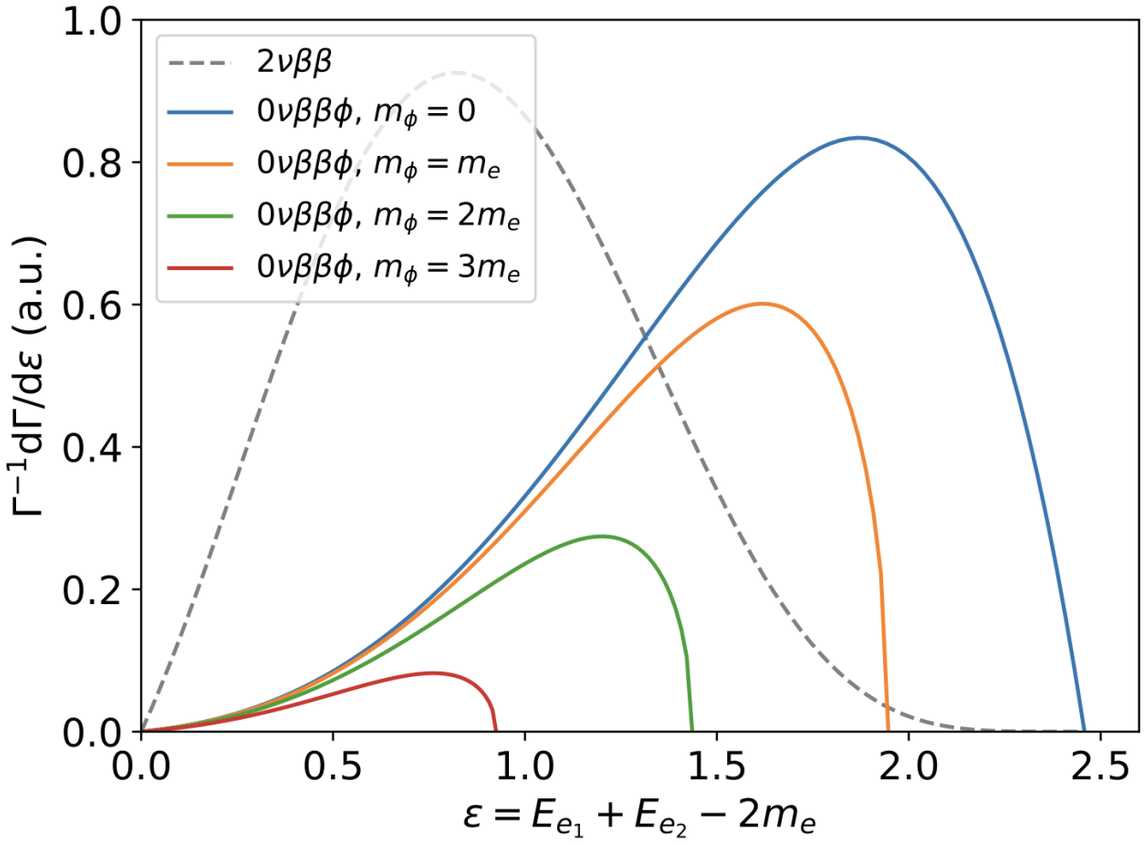
  where $G^\phi = \dfrac{1}{\ln 2}\dfrac{(G_F V_{ud} g_A^{\mathrm{eff}})^4}{128\pi^7 R^2}\displaystyle\int_{m_e}^{E_i-E_f-m_e-m_\phi} dE_{e_1} F(Z_f,E_{e_1})|\mathbf{p}_{e_1}|E_{e_1}$

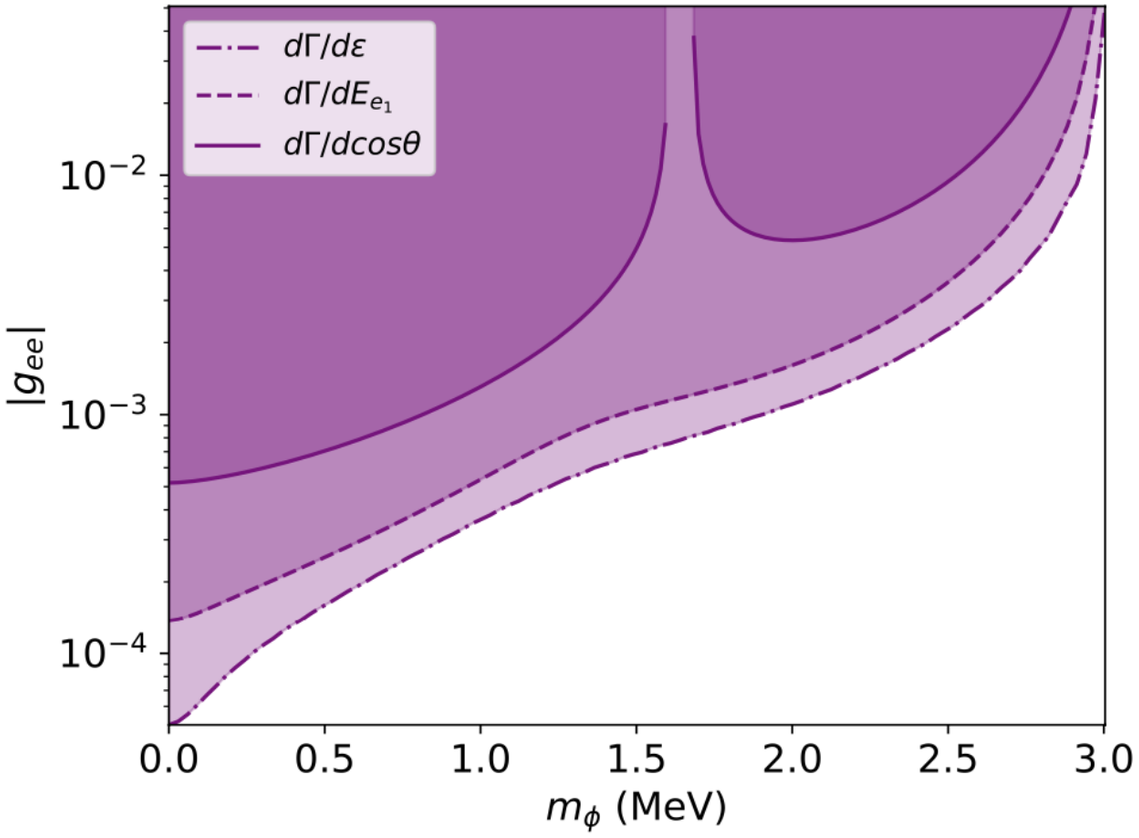
  $\times \displaystyle\int_{m_e}^{E_i-E_f-E_{e_1}-m_\phi} dE_{e_2} F(Z_f,E_{e_2})|\mathbf{p}_{e_2}|E_{e_2}\int dE_\phi|\mathbf{p}_\phi|\,\delta(E_i-E_f-E_\phi-E_{e_1}-E_{e_2})$

J. de Vries, LG, V. Plakkot, D. Starý: JHEP 03 [2511.19173]

Scalarful Double Beta Decay

- constraint from summed/single-electron spectrum and from angular correlation





J. de Vries, LG, V. Plakkot, D. Starý: JHEP 03 [2511.19173]

Towards Precision Double Beta Decay Spectrum

- double beta decay spectrum: upcoming experiments aim to reach (sub-)percent level precision
- theoretical description not as accurate → efforts to match the precision necessary
- expansion in the leptonic energies Šimkovic, Dvornicky, Stefanik, Faessler: PRC [1804.04227]
- corrections at the level of heavy baryon chiral effective field theory – pion exchanges, weak magnetism
  El Morabit, Bouabid, Cirigliano, de Vries, LG, Mereghetti: JHEP [2412.14160]
- electron wave functions – Fermi function, realistic wave functions in central field
  Kotila, Iachello: PRC [1209.5722]
  Neascu, Horoi: PRC [1510.00882]
- atomic effects: screening, exchange corrections, etc. Kotila, Iachello: PRC [1209.5722]
  Niţescu, Šimkovic: PRC [2411.05405]
- radiative corrections (Sirlin function and beyond) Niţescu, Šimkovic: PRC [2411.05405]
  de Vries, El Morabit, Mereghetti, Sandner: [2603.23592]

Taylor Expansion in Leptonic Energies

- Šimkovic et al.: improved description of 2vββ — Šimkovic, Dvornicky, Stefanik, Faessler: PRC [1804.04227]
- expansion in small leptonic energies allows for separation of nuclear and leptonic parts, works very well

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$$

$$\varepsilon_K = (E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1})/2,$$
$$\varepsilon_L = (E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1})/2.$$

$$\mathcal{A}_0^{2\nu} = 1, \quad \mathcal{A}_2^{2\nu} = \frac{\varepsilon_K^2 + \varepsilon_L^2}{(2m_e)^2},$$
$$\mathcal{A}_{22}^{2\nu} = \frac{\varepsilon_K^2 \varepsilon_L^2}{(2m_e)^4}, \quad \mathcal{A}_4^{2\nu} = \frac{\varepsilon_K^4 + \varepsilon_L^4}{(2m_e)^4}$$

$$M_{GT-1}^{2\nu} \equiv M_{GT}^{2\nu}$$
$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4\, m_e^3}{(E_n - (E_i + E_f)/2)^3}$$
$$M_{GT-5}^{2\nu} = \sum_n M_n \frac{16\, m_e^5}{(E_n - (E_i + E_f)/2)^5}$$

$$M_n = \langle 0_f^+ \parallel \sum_m \tau_m^- \sigma_m \parallel 1_n^+ \rangle \langle 1_n^+ \parallel \sum_m \tau_m^- \sigma_m \parallel 0_i^+ \rangle$$

$$G_N^{2\nu} = \frac{c_{2\nu}}{m_e^{11}} \int_{m_e}^{E_i - E_f - m_e} F_0(Z_f, E_{e_1}) p_{e_1} E_{e_1} dE_{e_1}$$
$$\times \int_{m_e}^{E_i - E_f - E_{e_1}} F_0(Z_f, E_{e_2}) p_{e_2} E_{e_2} dE_{e_2} \qquad (14)$$
$$\times \int_0^{E_i - E_f - E_{e_1} - E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathcal{A}_N^{2\nu} dE_{\nu_1}, \quad (\mathrm{N}{=}0,\ 2,\ 4,\ 22)$$

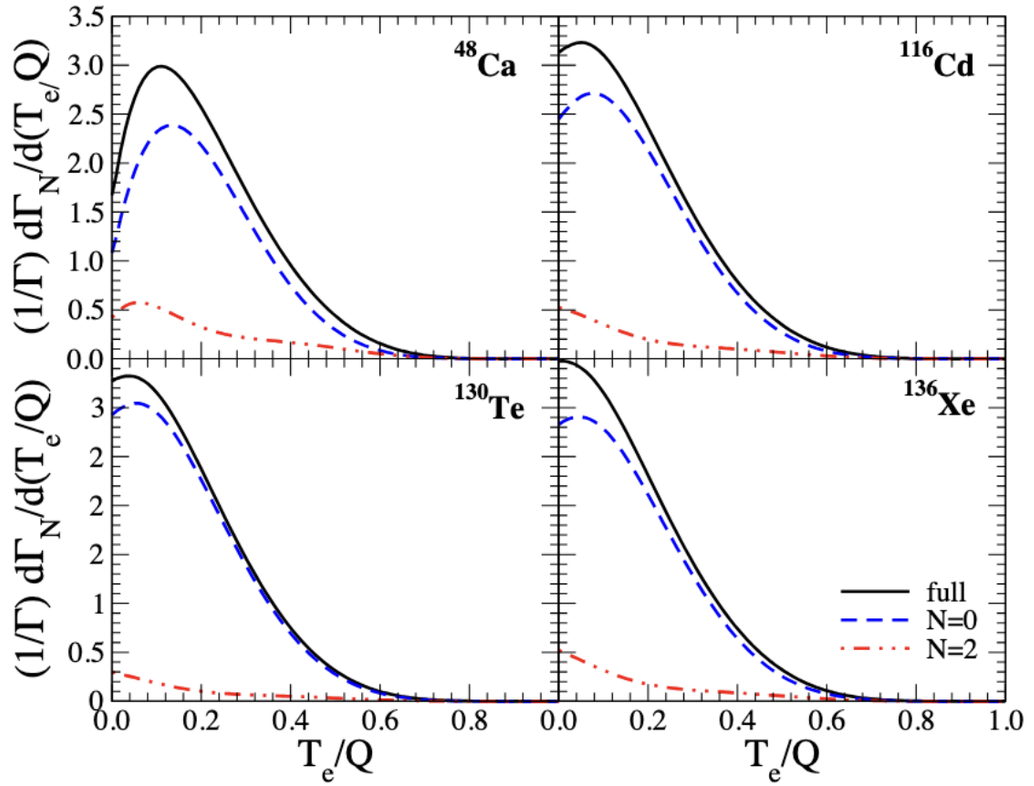
$$\xi_{31}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}, \quad \xi_{51}^{2\nu} = \frac{M_{GT-5}^{2\nu}}{M_{GT-1}^{2\nu}}$$

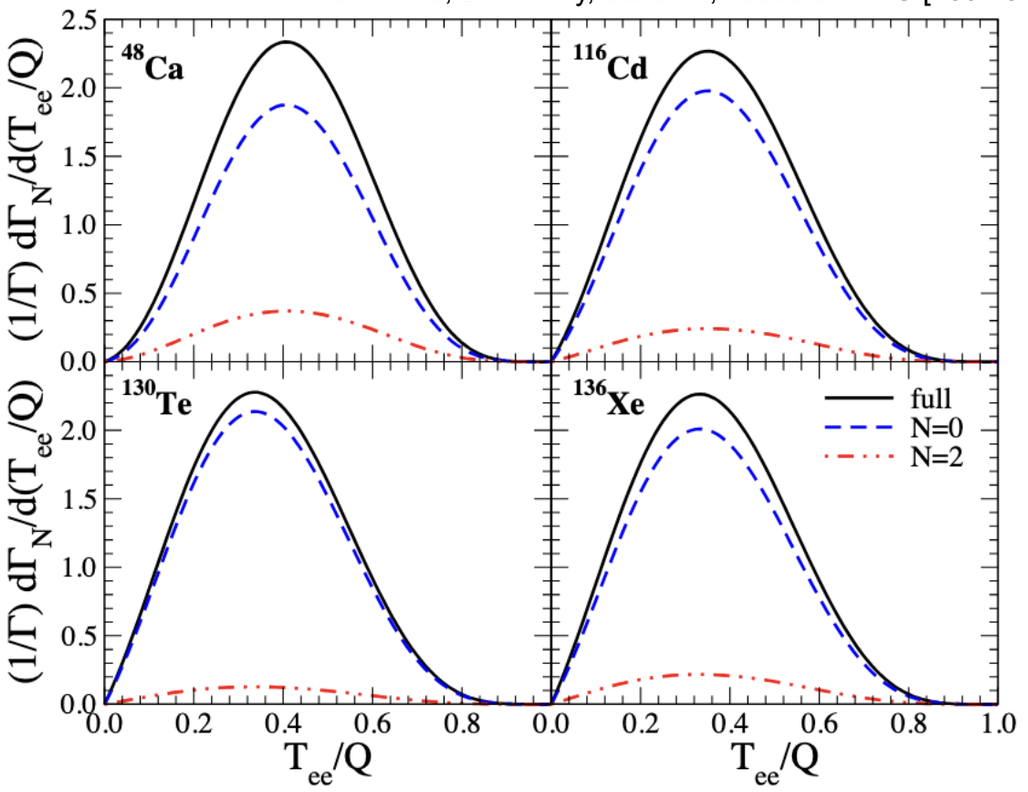
$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} = \left(g_A^{\mathrm{eff}}\right)^4 \left|M_{GT-1}^{2\nu}\right|^2 \left(G_0^{2\nu} + \xi_{31}^{2\nu} G_2^{2\nu} + \frac{1}{3}\left(\xi_{31}^{2\nu}\right)^2 G_{22}^{2\nu} + \left(\frac{1}{3}\left(\xi_{31}^{2\nu}\right)^2 + \xi_{51}^{2\nu}\right) G_4^{2\nu}\right),$$

Taylor Expansion in Leptonic Energies

- Šimkovic et al.: improved description of 2νββ – spectra:

Šimkovic, Dvornicky, Stefanik, Faessler: PRC [1804.04227]



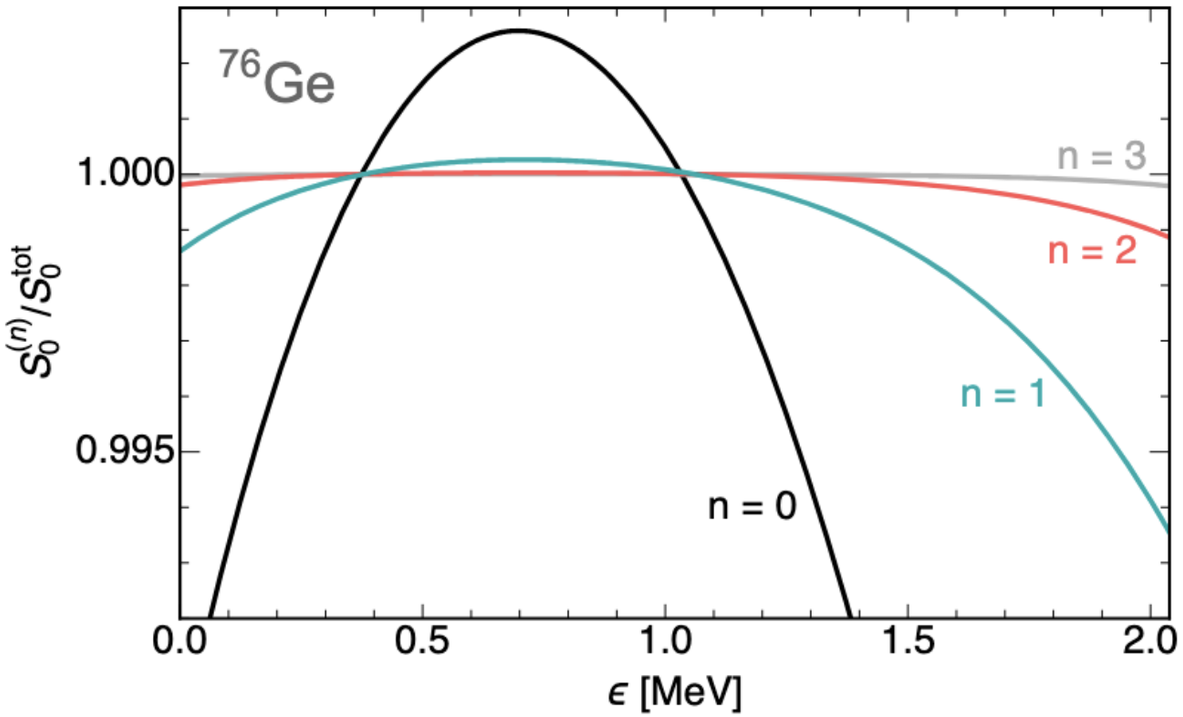


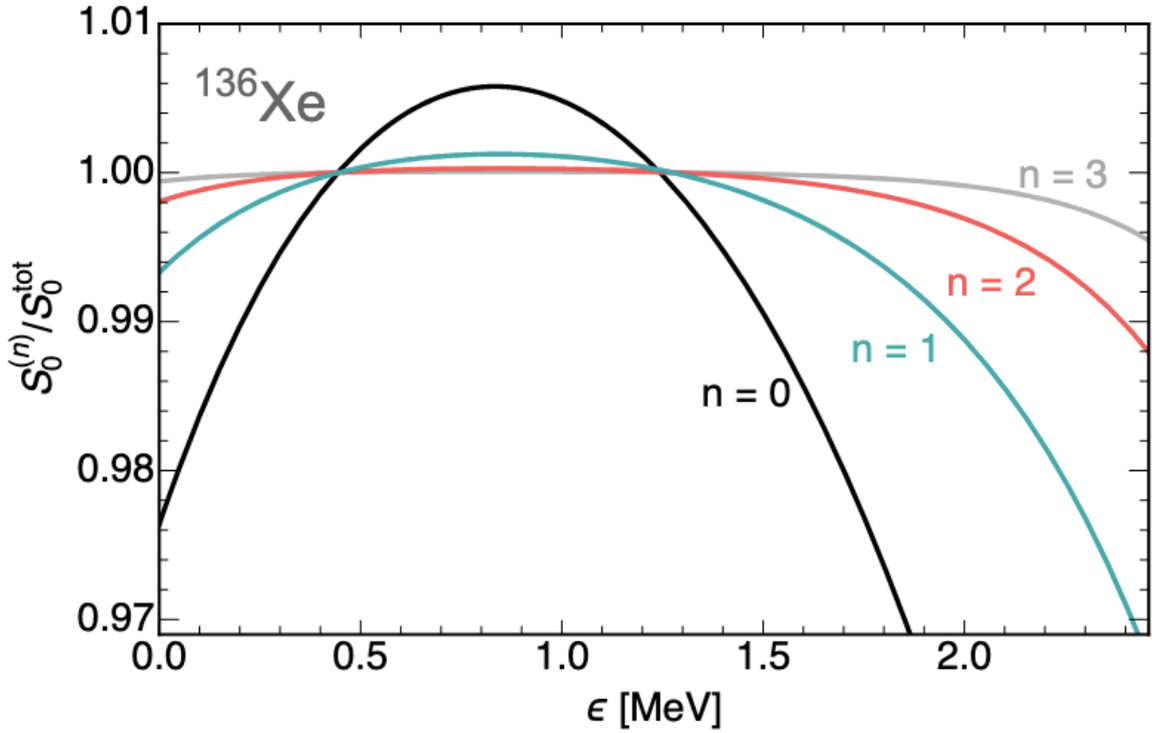
- fitted by experiments:

Experiment	Isotope	ξ_{31}	Source
KamLAND-ZEN	^{136}Xe	-0.26 ± 0.28	Phys. Rev. Lett. 122, 192501
CUPID-Mo	^{100}Mo	0.45 ± 0.06	Phys. Rev. Lett. 131, 162501
CUORE	^{130}Te	0.15 ± 0.15	Phys. Rev. Lett. 135, 082501

Taylor Expansion in Leptonic Energies

- the Taylor-expanded spectrum converges fairly quickly



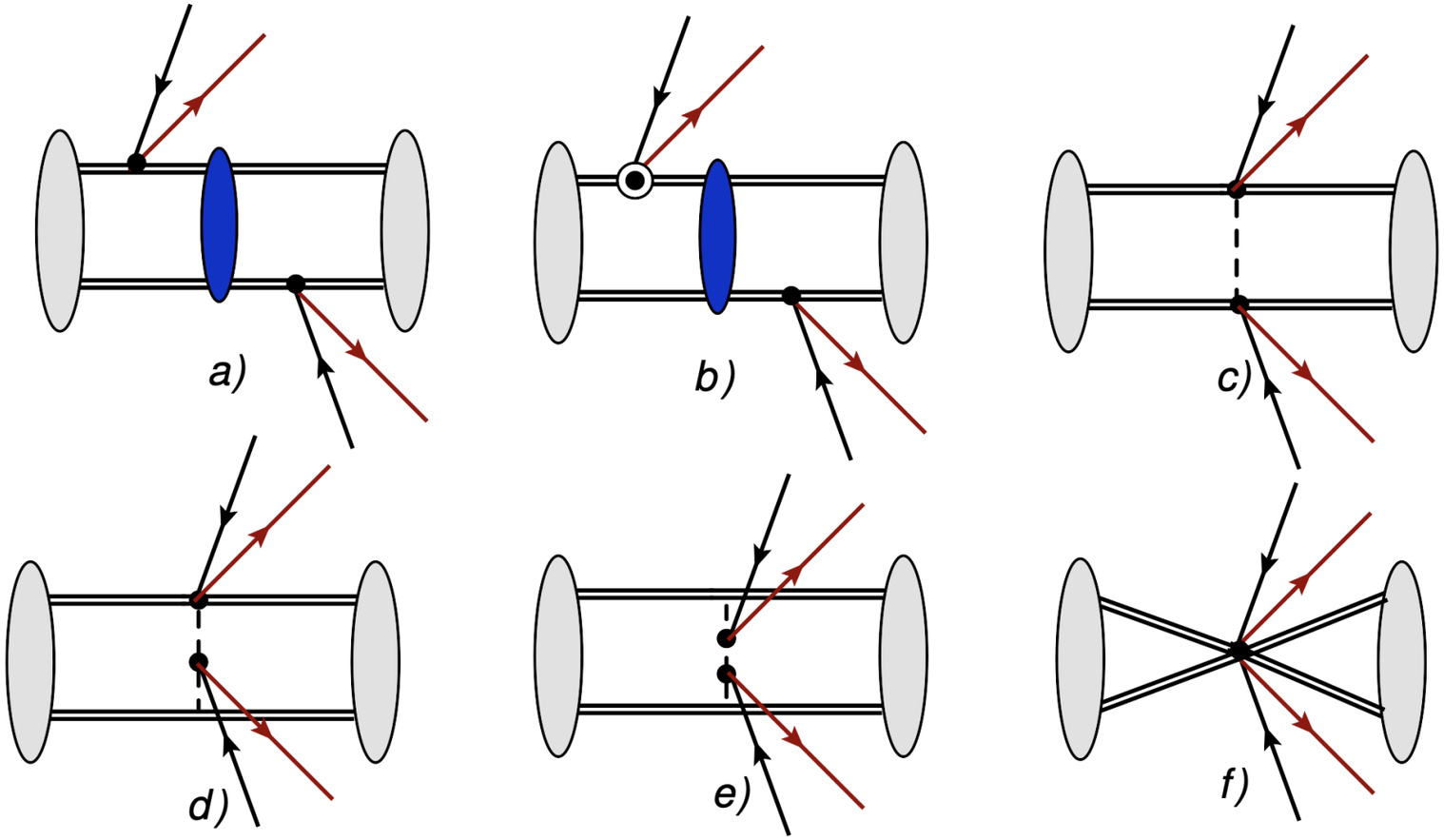


$$S(\epsilon) \equiv \frac{1}{\Gamma}\frac{d\Gamma}{d\epsilon} \qquad \delta S_i^{(m)}(\epsilon) = S_i^{(m)}(\epsilon) - S_0^{(m)}(\epsilon)$$

El Morabit, Bouabid, Cirigliano, de Vries, LG, Mereghetti: JHEP 03 [2412.14160]

Chiral Corrections to the Spectrum

- corrections determined at the level of heavy baryon chiral effective field theory: pion exchanges, weak magnetic term (~ g_M)



El Morabit, Bouabid, Cirigliano, de Vries, LG, Mereghetti: JHEP 03 [2412.14160]

Pion-Exchange Contributions

- corrections at the level of heavy baryon chiral EFT: pion exchanges

$$\frac{\Gamma^{2\nu}}{\ln 2} \simeq (g_A^{\text{eff}})^4 |\mathcal{M}^{2\nu}_{GT-1}|^2 \Delta_0 \left[G_0^{2\nu} + \xi_{31} \frac{\Delta_2}{\Delta_0} G_2^{2\nu} + \frac{1}{3}\xi_{31}^2 G_{22}^{2\nu} + \left(\frac{1}{3}\xi_{31}^2 + \xi_{51} \frac{\Delta_2}{\Delta_0} \right) G_4^{2\nu} + \frac{G_M^{2\nu}}{\Delta_0} \right]$$

$$V_{2\nu}(\vec{q}) = -\frac{(8G_F^2 V_{ud}^2)}{2F_\pi^2} (\tau_1^+ \tau_2^+) \frac{1}{\vec{q}^2 + m_\pi^2} (L_{11}^\mu L_{22}^\nu - L_{12}^\mu L_{21}^\nu) \left\{ g_V^2 \delta^{\mu 0}\delta^{\nu 0} + \frac{g_A^2}{3}\delta^{\mu i}\delta^{\nu i} \right.$$

$$\left. \times \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 \left(1 + \frac{4}{3}\frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} + \frac{4}{3}\left(\frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} \right)^2 \right) - \frac{4}{3} S_{12} \frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} \left(1 + \frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} \right) \right] \right\}$$

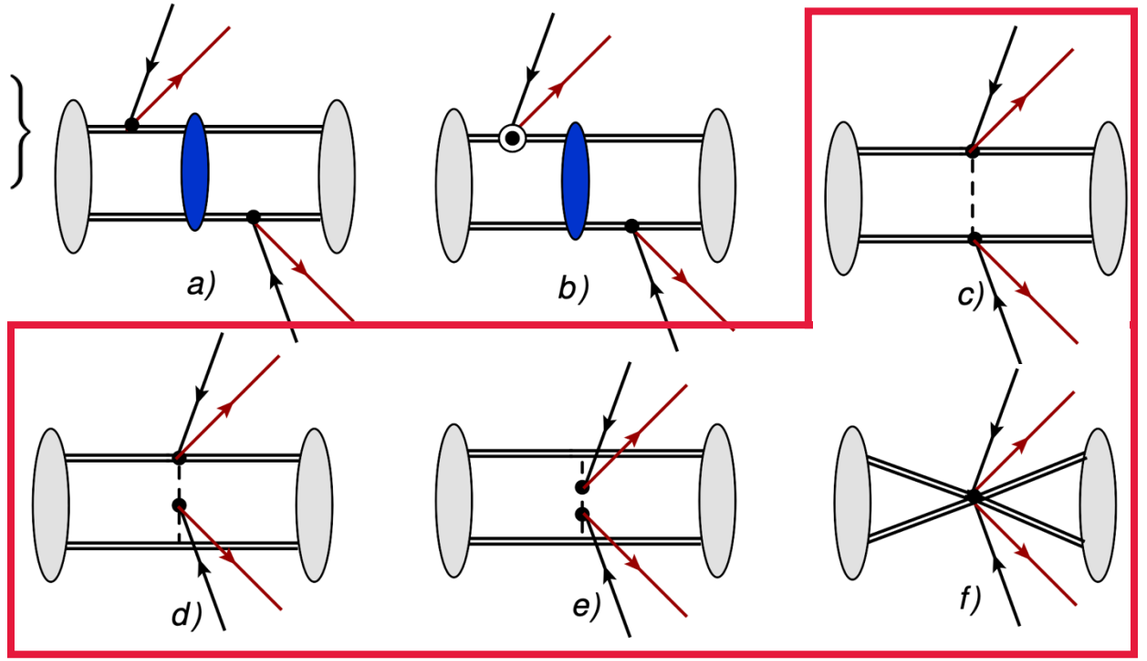
$$\Delta_0 = 1 + \frac{4}{g_A^2} \frac{(3\epsilon_{GT} - \epsilon_F)}{M_{GT}^{(-1)}} - \frac{2 g_M}{3 m_N g_A} (\mathcal{Q} + 2m_e),$$

$$\Delta_2 = 1 + \frac{2}{g_A^2} \frac{(3\epsilon_{GT} - \epsilon_F)}{M_{GT}^{(-1)}}$$

$$\epsilon_F = \frac{m_e g_V^2}{2F_\pi^2} \frac{1}{4\pi R_A} \left(M_F(m_\pi) + \frac{m_\pi^2}{F_\pi^2} g_{2\nu,\text{F}}^{\text{NN}} M_{F,sd} \right)$$

$$\epsilon_{GT} = \frac{m_e g_A^2}{6F_\pi^2} \frac{1}{4\pi R_A} \left(M_{GT}^{AA}(m_\pi) - 2M_{GT}^{AP}(m_\pi) + 4M_{GT}^{PP}(m_\pi) - \frac{3m_\pi^2}{g_A^2 F_\pi^2} g_{2\nu,\text{GT}}^{\text{NN}} M_{F,sd} \right)$$

El Morabit, Bouabid, Cirigliano, de Vries, LG, Mereghetti: JHEP 03 [2412.14160]



- just like 0νββ NMEs for sterile-neutrino exchange with $M_N = m_\pi$!

Pion-Exchange Contributions

- corrections at the level of heavy baryon chiral EFT: pion exchanges

$$\frac{\Gamma^{2\nu}}{\ln 2} \simeq (g_A^{\text{eff}})^4 |\mathcal{M}^{2\nu}_{GT-1}|^2 \Delta_0 \left[G_0^{2\nu} + \xi_{31} \frac{\Delta_2}{\Delta_0} G_2^{2\nu} + \frac{1}{3}\xi_{31}^2 G_{22}^{2\nu} + \left(\frac{1}{3}\xi_{31}^2 + \xi_{51} \frac{\Delta_2}{\Delta_0} \right) G_4^{2\nu} + \frac{G_M^{2\nu}}{\Delta_0} \right]$$

$$\Delta_0 = 1 + \frac{4}{g_A^2} \frac{(3\epsilon_{GT} - \epsilon_F)}{M_{GT}^{(-1)}} - \frac{2 g_M}{3 m_N g_A} (\mathcal{Q} + 2m_e) \,, \qquad \epsilon_F = \frac{m_e g_V^2}{2F_\pi^2} \frac{1}{4\pi R_A} \left(M_F(m_\pi) + \frac{m_\pi^2}{F_\pi^2} g_{2\nu,\mathrm{F}}^{\mathrm{NN}} M_{F,\,sd} \right)$$

$$\Delta_2 = 1 + \frac{2}{g_A^2} \frac{(3\epsilon_{GT} - \epsilon_F)}{M_{GT}^{(-1)}} \,. \qquad \epsilon_{GT} = \frac{m_e g_A^2}{6F_\pi^2} \frac{1}{4\pi R_A} \left(M_{GT}^{AA}(m_\pi) - 2M_{GT}^{AP}(m_\pi) + 4M_{GT}^{PP}(m_\pi) - \frac{3m_\pi^2}{g_A^2 F_\pi^2} g_{2\nu,\mathrm{GT}}^{\mathrm{NN}} M_{F,\,sd} \right)$$

- hence, 2vββ in principle does contain info on 0vββ NMEs

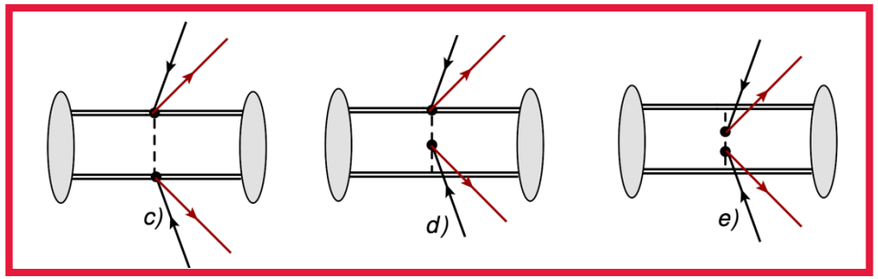
$$M_{F,(sd)}(m_\pi) = \langle 0^+ | \sum_{m,n} h_{F,(sd)}(r, m_\pi) \tau_m^+ \tau_n^+ | 0^+ \rangle \,,$$

$$M_{GT}^{ab}(m_\pi) = \langle 0^+ | \sum h_{GT}^{ab}(r, m_\pi) \vec{\sigma}_m \cdot \vec{\sigma}_n \tau_m^+ \tau_n^+ | 0^+ \rangle$$

$$h_F(r, m_\pi) = \frac{2R_A}{\pi} \int_0^\infty dq \frac{q^2}{q^2 + m_\pi^2} h_F(q^2) \frac{\sin qr}{qr} \,,$$

$$h_{F,sd}(r) = \frac{2R_A}{\pi m_\pi^2} \int_0^\infty dq q^2 h_F(q^2) \frac{\sin qr}{qr} \,,$$

$$h_{GT}^{ab}(r, m_\pi) = \frac{2R_A}{\pi} \int_0^\infty dq \frac{q^2}{q^2 + m_\pi^2} h_{GT}^{ab}(q^2) \frac{\sin qr}{qr}$$



$$h_F(q^2) = h_{GT}^{AA}(q^2) = 1 \,, \qquad h_{GT}^{AP}(q^2) = -\frac{2}{3} \frac{q^2}{q^2 + m_\pi^2} \,, \qquad h_{GT}^{PP}(q^2) = \frac{1}{3} \frac{q^4}{(q^2 + m_\pi^2)^2}$$

Hyvärinen and Suhonen: Phys. Rev. C 91, 024613 (2015)

Menéndez: J. Phys. G [1804.02105]

Weak Magnetism

- corrections at the level of heavy baryon chiral EFT: weak magnetic term (~ g_M)

$$\frac{\Gamma^{2\nu}}{\ln 2} \simeq (g_A^{\text{eff}})^4 |\mathcal{M}^{2\nu}_{GT-1}|^2 \Delta_0 \left[G_0^{2\nu} + \xi_{31} \frac{\Delta_2}{\Delta_0} G_2^{2\nu} + \frac{1}{3}\xi_{31}^2 G_{22}^{2\nu} + \left(\frac{1}{3}\xi_{31}^2 + \xi_{51} \frac{\Delta_2}{\Delta_0} \right) G_4^{2\nu} + \frac{G_M^{2\nu}}{\Delta_0} \right]$$

- arising from term: $(ig_M/m_N)\epsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma q_\nu$ with g_M = 4.7

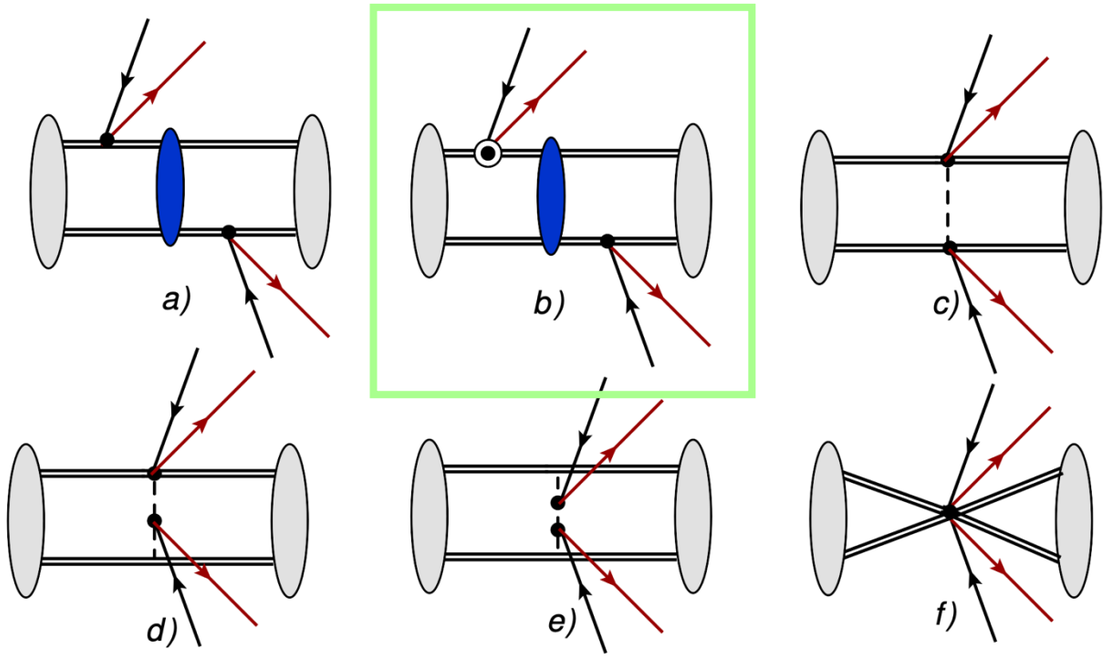
- resembles QED magnetic coupling, with W instead of photon

$$\Delta_0 = 1 + \frac{4}{g_A^2} \frac{(3\epsilon_{GT} - \epsilon_F)}{M_{GT}^{(-1)}} - \frac{2g_M}{3m_N g_A}(\mathcal{Q} + 2m_e),$$

$$\Delta_2 = 1 + \frac{2}{g_A^2} \frac{(3\epsilon_{GT} - \epsilon_F)}{M_{GT}^{(-1)}}.$$

- new phase-space term distortion

$$\mathcal{A}_M^{2\nu} = \frac{2g_M}{3m_N g_A^{\text{eff}}} \frac{(E_{e1} + E_{e2})(2E_{e1}E_{e2} - m_e^2)}{E_{e1}E_{e2}}$$



El Morabit, Bouabid, Cirigliano, de Vries, LG, Mereghetti: JHEP 03 [2412.14160]

Differential Decay Rate & Spectral Shape

- differential decay rate – additional fit parameter

$$\frac{d\Gamma}{d\epsilon} = g_A^4 \left(M_{GT}^{(-1)}\right)^2 \Delta_0 \left[\frac{dG_0^{2\nu}}{d\epsilon} + \frac{dG_2^{2\nu}}{d\epsilon}\xi_{31}\frac{\Delta_2}{\Delta_0} + \frac{dG_4^{2\nu}}{d\epsilon}\left(\xi_{51}\frac{\Delta_2}{\Delta_0} + \frac{1}{3}\xi_{31}^2\right) + \frac{dG_{22}^{2\nu}}{d\epsilon}\frac{1}{3}\xi_{31}^2 + \frac{dG_M^{2\nu}}{d\epsilon}\right]$$

- differential phase-space factors

$$\frac{dG_i^{2\nu}}{d\epsilon} = \frac{1}{\ln 2}\frac{(G_F V_{ud})^4}{8\pi^7 m_e^2}\int_{-\epsilon/2}^{\epsilon/2} d\Delta \int_0^{\mathcal{Q}-\epsilon} dE_{\nu 1}\, E_{e1}p_{e1}E_{e2}p_{e2}E_{\nu 1}^2 E_{\nu 2}^2 \times F(E_{e1}, Z_f) \times F(E_{e2}, Z_f) \times \mathcal{A}_i$$

where: $\mathcal{A}_0 = 1\,, \qquad \mathcal{A}_2 = \frac{\epsilon_K^2 + \epsilon_L^2}{(2m_e)^2}\,, \qquad \mathcal{A}_4 = \frac{\epsilon_K^4 + \epsilon_L^4}{(2m_e)^4}\,, \qquad \mathcal{A}_{22} = \frac{\epsilon_K^2 \epsilon_L^2}{(2m_e)^4}$

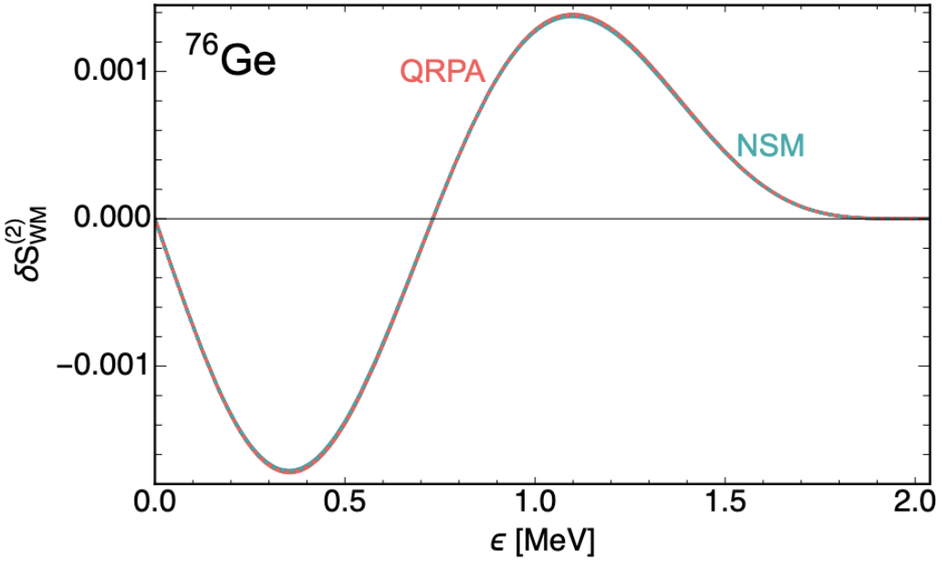
$$\mathcal{A}_M = \frac{2g_M}{3m_N g_A}\frac{(E_{e1}+E_{e2})(2E_{e1}E_{e2} - m_e^2)}{E_{e1}E_{e2}}\,,$$

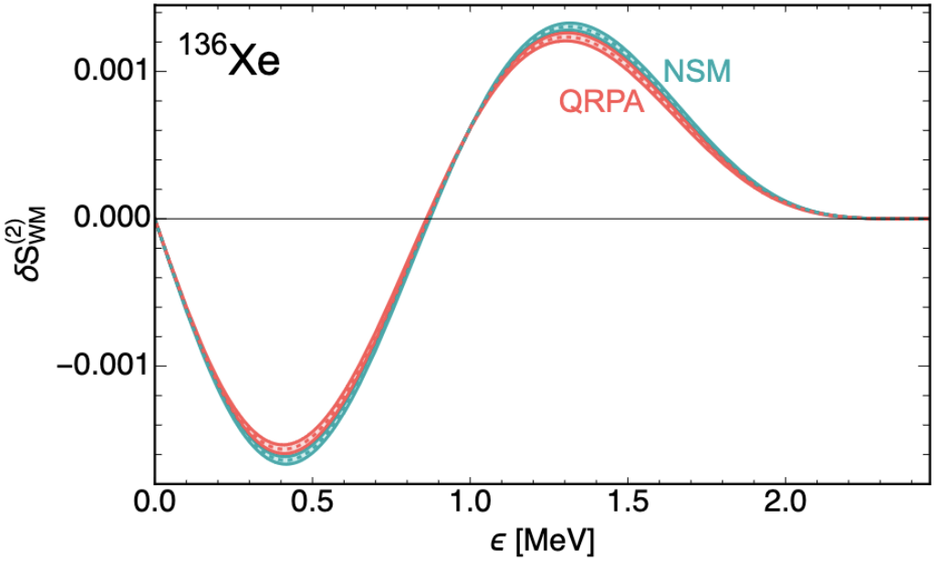
- we study the effects in terms of the shape factors S and their deviations δS

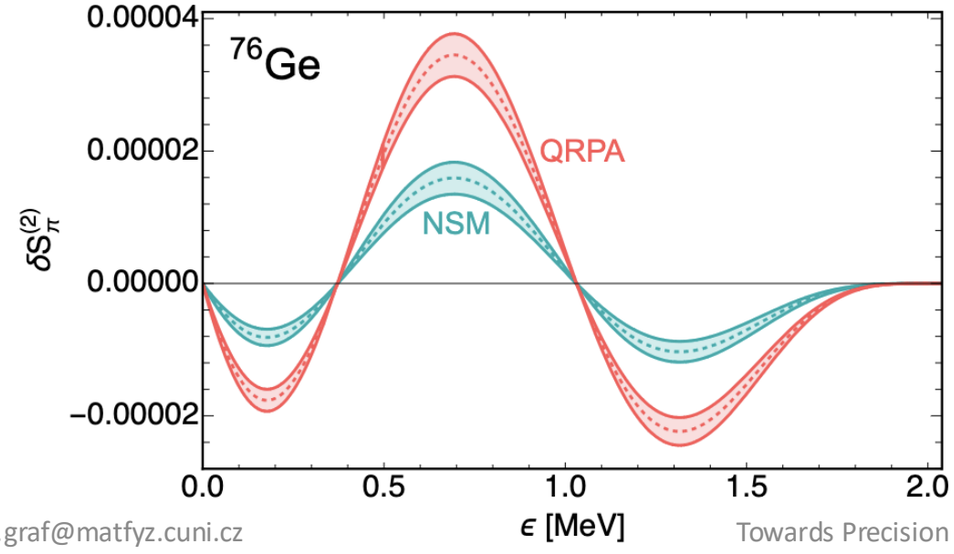
$$S(\epsilon) \equiv \frac{1}{\Gamma}\frac{d\Gamma}{d\epsilon} \qquad \delta S_i^{(m)}(\epsilon) = S_i^{(m)}(\epsilon) - S_0^{(m)}(\epsilon)\,, \quad \delta \bar{S}_i^{(m)}(\epsilon) = \delta S_i^{(m)}(\epsilon)/S_0^{(m)}(\epsilon)$$

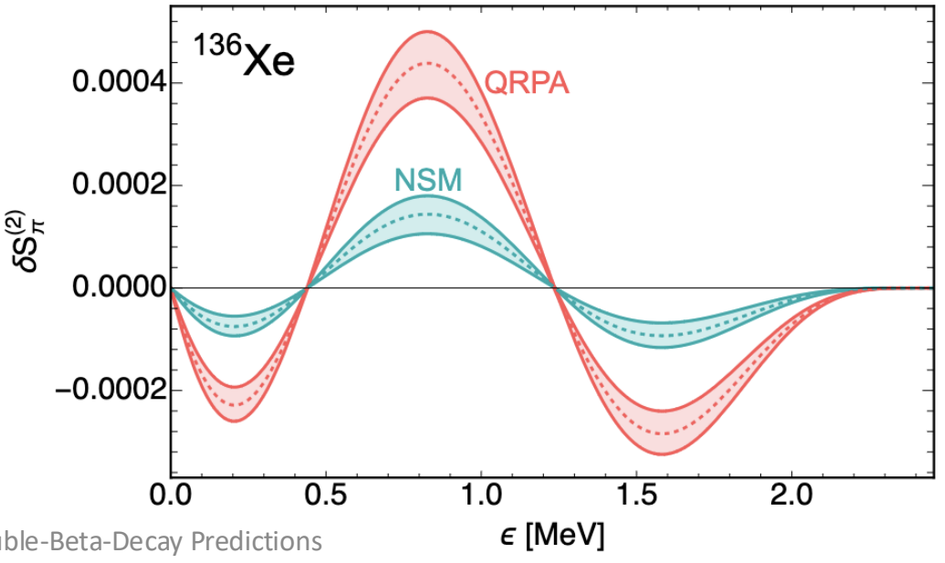
Effect on Spectral Shape

El Morabit, Bouabid, Cirigliano, de Vries, LG, Mereghetti: JHEP 03 [2412.14160]



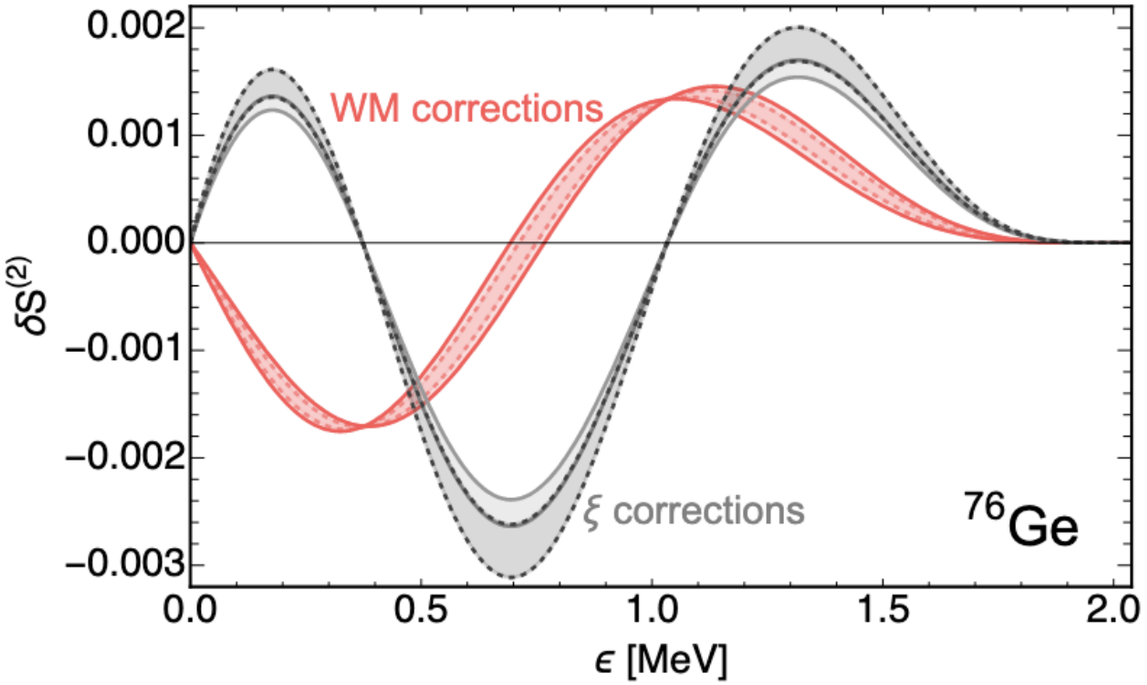


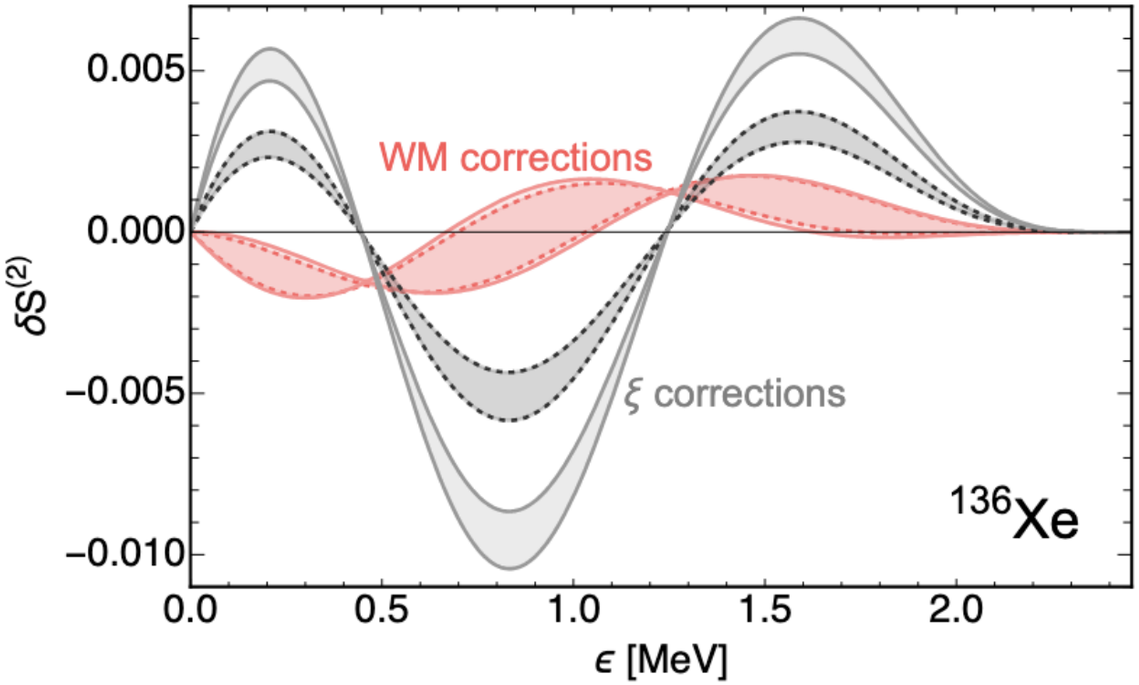




Spectral Shape – WM vs ξ

- compare the effect of the ξ corrections (leptonic energies) with the WM effect
- using both QRPA (lighter, solid) and NSM (darker, dashed); “nodes” for $\frac{dG_2^{2\nu}}{d\epsilon} \simeq \frac{G_2^{2\nu}}{G_0^{2\nu}} \frac{dG_0^{2\nu}}{d\epsilon}$

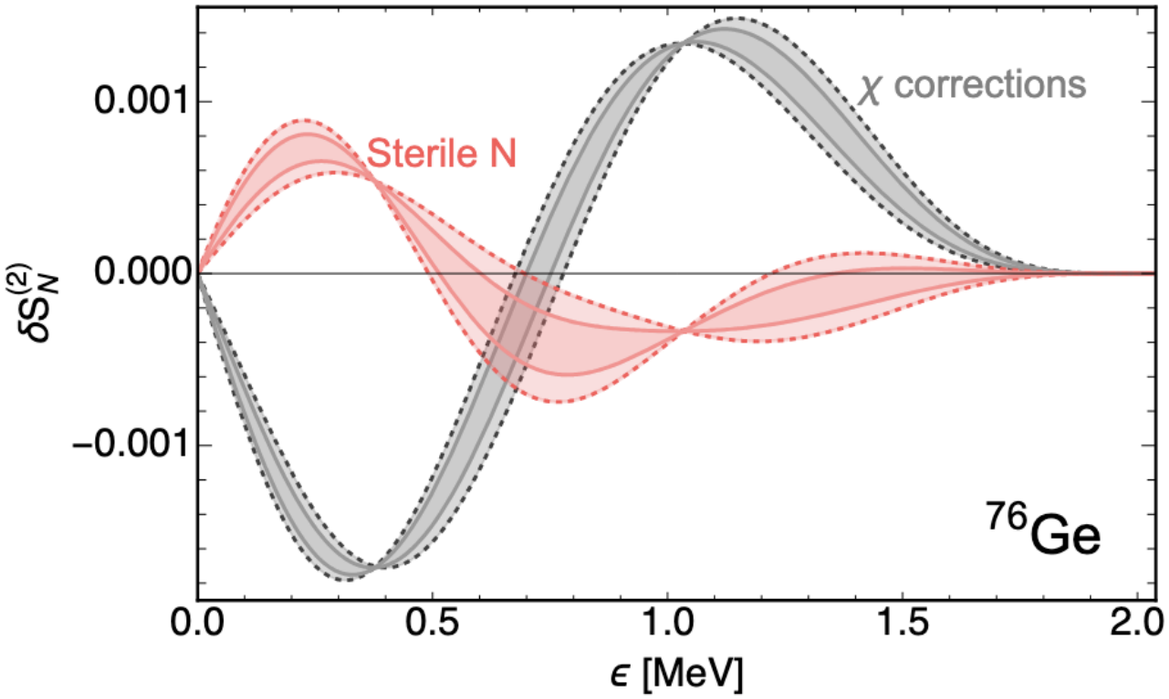


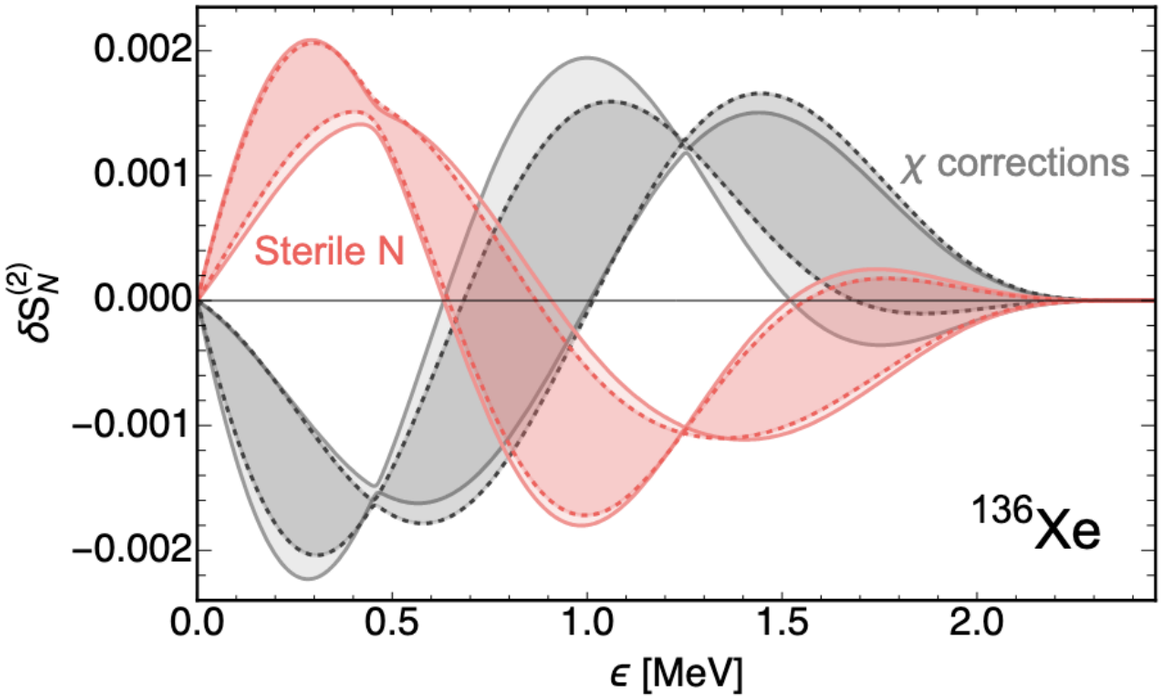


$$S(\epsilon) \equiv \frac{1}{\Gamma}\frac{d\Gamma}{d\epsilon} \qquad \delta S_i^{(m)}(\epsilon) = S_i^{(m)}(\epsilon) - S_0^{(m)}(\epsilon)$$

Spectral Shape – Sterile Neutrino Contribution

- contribution from sterile neutrino with m_N = 1 MeV and $|V_{eN}|^2$ = 0.1





$$S_N(\epsilon) = \frac{1}{\Gamma^{2\nu}(m_N, |V_{eN}|)} \frac{d\Gamma^{2\nu}(m_N, |V_{eN}|)}{d\epsilon} \qquad \delta S_N(\epsilon) = S_N(\epsilon) - S_0(\epsilon)$$

El Morabit, Bouabid, Cirigliano, de Vries, LG, Mereghetti: JHEP 03 [2412.14160]

Spectral Shape – Tensor Current

- contribution from exotic tensor current $\epsilon_T \bar{e}_R \sigma^{\mu\nu} \nu_L \, \bar{u}_R \sigma_{\mu\nu} d_L$

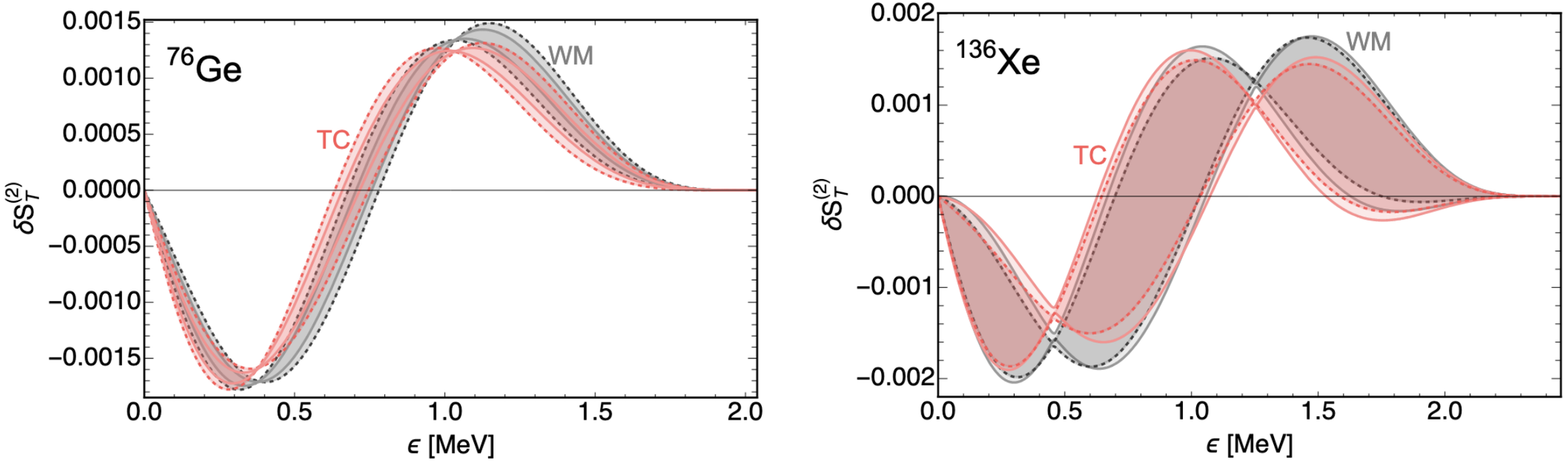


Figure 10: Shape-factor deviations δS induced by the BSM tensor current (TC) with $\epsilon_T = -0.0014$ in comparison with the deviations caused by the weak magnetism (WM) correction term.

$$C_{2\nu}^{\mathrm{BSM}} = \frac{8 g_A^4}{3} \left\{ \epsilon_T \frac{g_T}{g_A} \left[(M_{GT}^K)^2 + (M_{GT}^L)^2 + M_{GT}^K M_{GT}^L \right] \frac{m_e (E_{e1} + E_{e2})}{E_{e1} E_{e2}} \right\}$$

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Conclusion & Outlook

- 2νββ not merely a background process; can help to unravel BSM physics and/or provide additional input for nuclear structure from its spectrum
- understanding 2νββ is crucial for understanding upcoming experiments, theory should match the (planned) experimental precision
- first chiral-EFT framework for 2νββ electron spectra: identifies the leading SM spectrum and organizes subleading corrections systematically in the chiral expansion
- NLO SM corrections from weak magnetism and pion exchange modify the electron spectra and should be included in precision/BSM spectral analyses
- the pion-exchange terms link 2νββ spectra to 0νββ nuclear matrix elements
- relevant for precision and BSM searches, since SM effects can mimic spectral distortions

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Thank you!