Nuclear size effects in isotope shift

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MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

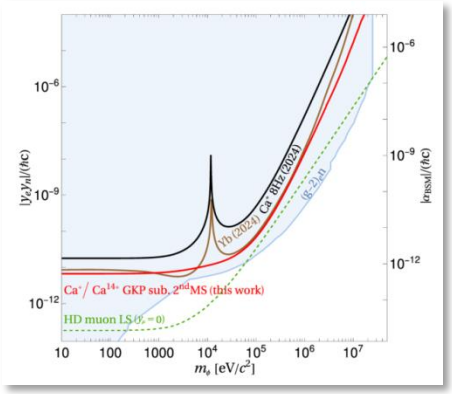
MAX-PLANCK-GESELLSCHAFT

From nuclear structure to new physics, 3-7 August 2026, Trento

Introduction

Isotope shifts: can be measured very accurately. What can we do with them?

- Test theory
- Determine nuclear square radii difference $\delta\langle r^2 \rangle$
- Constrain new physics (with King plots)



From: Wilzewski et al. Phys. Rev. Lett. 134, 233002 (2025)

King plot

Isotope shift for transitions *a* and *b*:

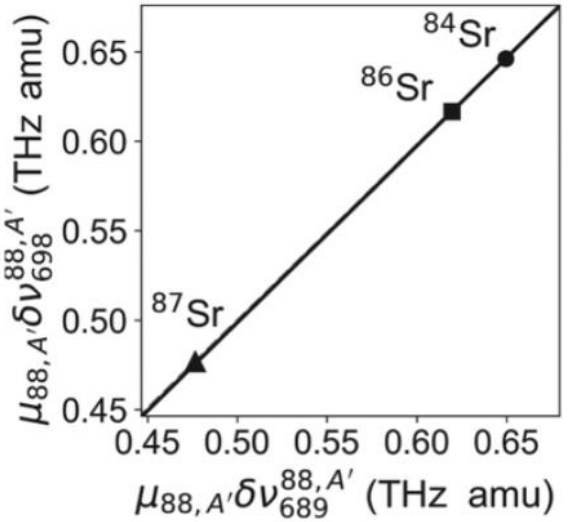
Mass shift + Field shift

$$\nu_a = K_a^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_a^{(1)} \, \delta\langle r^2 \rangle$$

$$\nu_b = K_b^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_b^{(1)} \, \delta\langle r^2 \rangle$$

$\delta\langle r^2 \rangle$ can be eliminated => linear dependence between ν_a and ν_b

Linear King plot



From: Miyake et al. Phys. Rev. Research 1, 033113 (2019)

(Nonlinearities of) King plot

$$\nu = K^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F^{(1)} \, \delta \langle r^2 \rangle$$

New-physics

nonlinearity

$$\ldots + \alpha_{\mathrm{NP}} \left\langle \frac{e^{-m_\phi r}}{r} \right\rangle$$

Delaunay et al. Phys. Rev. D 96, 093001 (2017)
Berengut et al. Phys. Rev. Lett. 120, 091801 (2018)

(Nonlinearities of) King plot

PHYSICAL REVIEW LETTERS **125,** 123002 (2020)

Editors' Suggestion | Featured in Physics

Evidence for Nonlinear Isotope Shift in Yb^+ Search for New Boson

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3σ

PHYSICAL REVIEW LETTERS **128,** 163201 (2022)

Featured in Physics

Evidence of Two-Source King Plot Nonlinearity in Spectroscopic Search for New Boson

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240 σ

PHYSICAL REVIEW LETTERS **134,** 233002 (2025)

Featured in Physics

Nonlinear Calcium King Plot Constrains New Bosons and Nuclear Properties

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1000 σ

PHYSICAL REVIEW LETTERS **134,** 063002 (2025)

Probing New Bosons and Nuclear Structure with Ytterbium Isotope Shifts

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Julian C. Berengut,[7] Jacek Bieroń,[8] Klaus Blaum,[1] Laura S. Dreissen,[3,
Melina Filzinger,[3] Elina Fuchs,[3,6] Henning A. Fürst,[3,10] Gediminas Gaigalas
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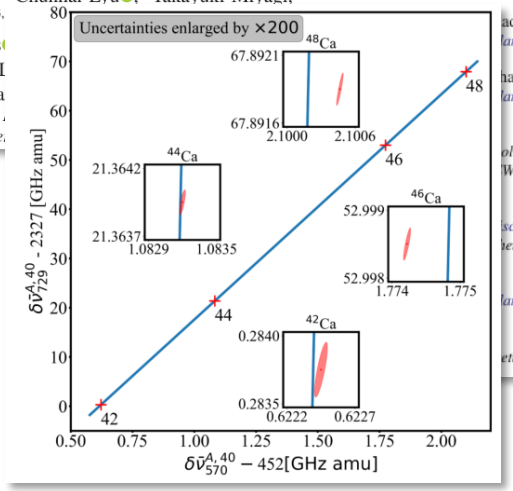
1000 σ



(Nonlinearities of) King plot

$$\nu = K^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F^{(1)} \, \delta\langle r^2 \rangle$$

New-physics nonlinearity

$$\ldots + \alpha_{\mathrm{NP}} \left\langle \frac{e^{-m_\phi r}}{r} \right\rangle$$

Delaunay et al. Phys. Rev. D 96, 093001 (2017)
Berengut et al. Phys. Rev. Lett. 120, 091801 (2018)

"Old"-physics nonlinearities

$$\ldots + G \, \delta\langle r^4 \rangle + F^{(2)} \left(\delta\langle r^2 \rangle \right)^2 + ????$$

$$\ldots + K^{(2)} \left(\frac{m^2}{M^2} - \frac{m^2}{M_0^2} \right)$$

$$\ldots + \text{Nuclear polarization}$$

$$\ldots + ????$$

Higher-order field shift

Field shift

Field shift == shift of atomic energy levels caused by the difference in nuclear charge distributions between two isotopes.

Field shift is largely proportional to $\delta\langle r^2\rangle$ -- but there are higher-order corrections. What are they?

Seltzer, 1969 (first order in perturbation theory!):

$$\delta E_{\rm FS} = C_1\,\delta\langle r^2\rangle + C_2\,\delta\langle r^4\rangle + C_3\,\delta\langle r^6\rangle + \ldots$$

Counts et al. PRL 125, 123002 (2020):

$$\delta E_{\rm FS} = F\,\delta\langle r^2\rangle + G\,\left(\delta\langle r^2\rangle\right)^2$$

Door et al. PRL 134, 063002 (2025):

$$\delta E_{\rm FS} = F\,\delta\langle r^2\rangle + G^{(2)}\,\left(\delta\langle r^2\rangle\right)^2 + G^{(4)}\,\delta\langle r^4\rangle$$

Flambaum et al. PRA 97, 032510 (2018):

$$\delta E_{\rm FS} = F\,\delta\langle r^{2\gamma_1}\rangle + G\,\delta\langle r^{2\gamma_2}\rangle$$

What is the systematic expansion of field-shift energies?

Agenda

- Systematic expansion of field-shift energies – checking against direct numerical evaluation for H-like ions.
- Alkali-like ions.
- General atoms. Conclusions for King plots.

VAY, Harman, Keitel,
Higher-order corrections to field shift in atomic systems,
arXiv: 2607.26902

Field shift, fixed nuclear shape

Fixed nuclear shape == one-parameter nuclear model.

$$r_C \equiv \langle r^2 \rangle^{1/2}$$

Nuclear potential depends only on nuclear radius.

Field shift is induced by $\delta V = V(r_C) - V(r_{C0})$

$$\delta E_{\rm FS} = \langle \psi_0 | \delta V | \psi_0 \rangle + \langle \psi_0 | \delta V \frac{1}{(E_0 - H_0)'} \delta V | \psi_0 \rangle$$

$$\delta E_{\rm FS} = \left\langle \frac{\partial V(r_C)}{\partial (r_C^2)} \right\rangle \delta r_C^2 + \left\langle \frac{1}{2} \frac{\partial^2 V(r_C)}{\partial^2 (r_C^2)} \right\rangle (\delta r_C^2)^2 + \left\langle \frac{\partial V(r_C)}{\partial (r_C^2)} \frac{1}{(E_0 - H_0)'} \frac{\partial V(r_C)}{\partial (r_C^2)} \right\rangle (\delta r_C^2)^2$$

$$\delta E_{\rm FS} = F^{(1)}\, \delta r_C^2 + \left[G_{r_C} + F^{(2)} \right] (\delta r_C^2)^2$$

Accuracy: $7 \cdot 10^{-6}$ for Z=26,
$8 \cdot 10^{-5}$ for Z=62.

Term	$^{154,144}\mathrm{Sm}^{61+}$ [eV]
V'_{r_C}	0.432 549
$V'_{r_C} \frac{1}{(E-H)'} V'_{r_C}$	−0.001 078
$\frac{1}{2} V''_{r_C}$	−0.000 511
Sum	0.430 959
Direct	0.430 994

Field shift, varied nuclear shape

Varied nuclear shape == two (or more) parameter nuclear model.

$$r_C \equiv \langle r^2 \rangle^{1/2}$$

$$r_{C4} \equiv \langle r^4 \rangle^{1/4}$$

$$\eta \equiv V_{24}^{-1} = r_{C4}/r_C$$

What is the second nuclear parameter (apart from r_C)? – Some freedom.

Leading-order field shift $\left\langle \frac{\partial V(r_C, \xi)}{\partial (r_C^2)} \right\rangle \delta r_C^2$ depends on the choice of ξ !

The usual choice of r_{C4} as the second nuclear parameter is not good. Large correlation between r_{C4} and r_C => strong cancellations.

Our choice of the second parameter: the diffuseness parameter *a* of the Fermi model.

$$\delta E_{\rm FS} = F^{(1)}\, \delta r_C^2 + G_a\, \delta a^2 + \left[F^{(2)} + G_{r_C} \right] (\delta r_C^2)^2$$

$$G_a = \left\langle \frac{\partial V(r_C, a)}{\partial (a^2)} \right\rangle_0$$

Effect of change in nuclear shape between isotopes

δa^2 can be expressed in terms of model-independent parameters:

$$\delta a^2 = \frac{\partial a^2}{\partial (r_C^2)} \delta r_C^2 + \frac{\partial a^2}{\partial (\eta^4)} \delta \eta^4$$

~~$$\delta a^2 = \frac{\partial a^2}{\partial (r_C^2)} \delta r_C^2 + \frac{\partial a^2}{\partial (r_{C4}^4)} \delta r_{C4}^4$$~~

Fermi nuclear charge model

$$\rho_{\rm 2pF}(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-c}{a}\right)}$$

$$a^2 = \frac{r_C^2}{8\pi^2} \left[\sqrt{84\eta^4 - 75} - 5 \right]$$

Field shift, varied nuclear shape

$$\delta E_{\rm FS} = F^{(1)}\,\delta r_C^2 + G_a\,\delta a^2 + \left[F^{(2)} + G_{r_C}\right]\,(\delta r_C^2)^2$$

Effect of change in nuclear shape between isotopes

$$F^{(1)} = \left\langle \frac{\partial V(r_C,a)}{\partial(r_C^2)} \right\rangle$$

$$G_a = \left\langle \frac{\partial V(r_C,a)}{\partial(a^2)} \right\rangle$$

$$G_{r_C} = \left\langle \frac{1}{2}\frac{\partial^2 V(r_C,a)}{\partial^2(r_C^2)} \right\rangle$$

$$F^{(2)} = \left\langle \frac{\partial V(r_C,a)}{\partial(r_C^2)}\,\frac{1}{(E-H)'}\,\frac{\partial V(r_C,a)}{\partial(r_C^2)} \right\rangle$$

Term	^{154,144}Sm^{61+} [eV]
V'_{r_C}	0.432 525
V'_a	−0.004 961
$V'_{r_C}\frac{1}{(E-H)'}V'_{r_C}$	−0.001 078
$\frac{1}{2}V''_{r_C}$	−0.000 510
Sum	0.425 976
Direct	0.425 953

Accuracy $3*10^{-6}$ for Z=26 ,
$5*10^{-5}$ for Z=62.

Fermi nuclear charge model

$$\rho_{\rm 2pF}(r) = \frac{\rho_0}{1+\exp\left(\frac{r-c}{a}\right)}$$

Alkali-like atoms: theory

Energies are calculated by many-body perturbation theory (MBPT):

$$E = E^{(0)} + E^{(1)} + E^{(2)} ,$$

$$E^{(0)} = \varepsilon_v + \sum_a \varepsilon_a ,$$

$$E^{(1)} = \left(V_{\rm HF} - U\right)_{vv},$$

$$E^{(2)} = -\sum_{amn} \frac{I_{vamn}\, I_{mn;va}}{\epsilon_{mn} - \epsilon_{va}} + \sum_{abm} \frac{I_{abmv}\, I_{mv;ab}}{\epsilon_{vm} - \epsilon_{ab}} + 2\sum_{am} \frac{(V_{\rm HF} - U)_{am}\, I_{mv;va}}{\varepsilon_m - \varepsilon_a} - \sum_{i\neq v} \frac{(V_{\rm HF} - U)_{vi}\, (V_{\rm HF} - U)_{iv}}{\varepsilon_i - \varepsilon_v} .$$

Field-shift constants are obtained by finite-field method:

$$V_{\rm nucl} \to V_{\rm nucl} + h\, \frac{\partial V(r_C)}{\partial r_C^2}$$

$$F^{(1)} = \left.\frac{\partial E(h)}{\partial h}\right|_{h=0}, \quad F^{(2)} = \frac{1}{2}\left.\frac{\partial^2 E(h)}{\partial h^2}\right|_{h=0}$$

Alkali-like atoms: results

Let us look at the ratio $G_a/F^{(1)}$

N = 1

Z	1s, H-like
10	−0.0186
20	−0.0242
30	−0.0452
40	−0.0754
50	−0.1128
60	−0.1579
70	−0.2089
80	−0.2681
90	−0.3325

Alkali-like atoms: results

Let us look at the ratio $G_a/F^{(1)}$

N = 1 N = 3

Z	$1s$, H-like	$2p_{1/2}$ - $2s$, Li-like	
			Diff.
10	−0.0186	−0.0186	0.06%
20	−0.0242	−0.0242	0.10%
30	−0.0452	−0.0453	0.21%
40	−0.0754	−0.0757	0.32%
50	−0.1128	−0.1134	0.45%
60	−0.1579	−0.1588	0.60%
70	−0.2089	−0.2106	0.78%
80	−0.2681	−0.2707	0.97%
90	−0.3325	−0.3366	1.23%

Alkali-like atoms: results

Let us look at the ratio $G_a/F^{(1)}$

N = 1 N = 3 N = 11

Z	$1s$, H-like	$2p_{1/2}$ - $2s$, Li-like	Diff.	$3p_{1/2}$ - $3s$, Na-like	Diff.
10	−0.0186	−0.0186	0.06%		
20	−0.0242	−0.0242	0.10%	−0.0242	0.13%
30	−0.0452	−0.0453	0.21%	−0.0454	0.23%
40	−0.0754	−0.0757	0.32%	−0.0757	0.35%
50	−0.1128	−0.1134	0.45%	−0.1134	0.50%
60	−0.1579	−0.1588	0.60%	−0.1589	0.67%
70	−0.2089	−0.2106	0.78%	−0.2108	0.89%
80	−0.2681	−0.2707	0.97%	−0.2711	1.11%
90	−0.3325	−0.3366	1.23%	−0.3373	1.42%

Alkali-like atoms: results

Let us look at the ratio $G_a/F^{(1)}$

N = 1 N = 3 N = 11 N = 19

Z	$1s$, H-like	$2p_{1/2}$ - $2s$, Li-like		$3p_{1/2}$ - $3s$, Na-like		$4p_{1/2}$ - $4s$, K-like	
			Diff.		Diff.		Diff.
10	−0.0186	−0.0186	0.06%				
20	−0.0242	−0.0242	0.10%	−0.0242	0.13%		
30	−0.0452	−0.0453	0.21%	−0.0454	0.23%	−0.0453	0.18%
40	−0.0754	−0.0757	0.32%	−0.0757	0.35%	−0.0757	0.35%
50	−0.1128	−0.1134	0.45%	−0.1134	0.50%	−0.1134	0.51%
60	−0.1579	−0.1588	0.60%	−0.1589	0.67%	−0.1590	0.69%
70	−0.2089	−0.2106	0.78%	−0.2108	0.89%	−0.2108	0.92%
80	−0.2681	−0.2707	0.97%	−0.2711	1.11%	−0.2712	1.15%
90	−0.3325	−0.3366	1.23%	−0.3373	1.42%	−0.3374	1.47%

Alkali-like atoms: results

Let us look at the ratio $G_a/F^{(1)}$

	N = 1	N = 3		N = 11		N = 19		N = 37	
Z	$1s$, H-like	$2p_{1/2}$ - $2s$, Li-like	Diff.	$3p_{1/2}$ - $3s$, Na-like	Diff.	$4p_{1/2}$ - $4s$, K-like	Diff.	$5p_{1/2}$ - $5s$, Rb-like	Diff.
10	−0.0186	−0.0186	0.06%						
20	−0.0242	−0.0242	0.10%	−0.0242	0.13%				
30	−0.0452	−0.0453	0.21%	−0.0454	0.23%	−0.0453	0.18%		
40	−0.0754	−0.0757	0.32%	−0.0757	0.35%	−0.0757	0.35%	−0.0757	0.37%
50	−0.1128	−0.1134	0.45%	−0.1134	0.50%	−0.1134	0.51%	−0.1134	0.49%
60	−0.1579	−0.1588	0.60%	−0.1589	0.67%	−0.1590	0.69%	−0.1589	0.68%
70	−0.2089	−0.2106	0.78%	−0.2108	0.89%	−0.2108	0.92%	−0.2108	0.91%
80	−0.2681	−0.2707	0.97%	−0.2711	1.11%	−0.2712	1.15%	−0.2711	1.15%
90	−0.3325	−0.3366	1.23%	−0.3373	1.42%	−0.3374	1.47%	−0.3374	1.47%

Conclusion:
Ratios of higher-order to leading-order field shift are (almost) independent on electronic configuration

Arbitrary atoms

"Global approximation": ratios of higher-order to lowest-order field-shift constant do not depend on electronic structure.

$$E_{\rm FS} = F^{(1)}\,\lambda_* = F^{(1)}\left\{\delta r_C^2 + \frac{G_a}{F^{(1)}}\,\delta a^2 + \left[\frac{F^{(2)}}{F^{(1)}} + \frac{G_{r_C}}{F^{(1)}}\right]\left(\delta r_C^2\right)^2\right\}$$

Depends only on nuclear parameters of two isotopes

Accuracy:

$\leq 1\%$ for most of atomic transitions

$\sim 10\%$ for fine-structure transitions (between states differing only by total moment *J*)

Consequences for King plot

Standard King plot (leading-order mass and field shift):

$$\nu_a = K_a^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_a^{(1)} \, \delta\langle r^2 \rangle$$

$$\nu_b = K_b^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_b^{(1)} \, \delta\langle r^2 \rangle$$

Consequences for King plot

Standard King plot (leading-order mass and field shift):

$$\nu_a = K_a^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_a^{(1)} \, \delta\langle r^2 \rangle$$

$$\nu_b = K_b^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_b^{(1)} \, \delta\langle r^2 \rangle$$

… + higher-order field shift in the global approximation:

$$\nu_a = K_a^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_a^{(1)} \, \lambda_*$$

$$\nu_b = K_b^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_b^{(1)} \, \lambda_*$$

Consequences for King plot

Standard King plot (leading-order mass and field shift):

$$\nu_a = K_a^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_a^{(1)} \, \delta\langle r^2 \rangle$$
$$\nu_b = K_b^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_b^{(1)} \, \delta\langle r^2 \rangle$$

… + higher-order field shift in the global approximation:

$$\nu_a = K_a^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_a^{(1)} \, \lambda_*$$
$$\nu_b = K_b^{(1)} \left(\frac{m}{M} - \frac{m}{M_0} \right) + F_b^{(1)} \, \lambda_*$$

Conclusion:
In the global approximation, higher-order field shift does not contribute to the King-plot nonlinearity

Conclusions

I. Systematic expansion of field-shift energies:

$$\delta E_{\rm FS} = F^{(1)}\,\delta r_C^2 + G_a\,\delta a^2 + \left[F^{(2)} + G_{r_C}\right]\,(\delta r_C^2)^2$$

$$\delta a^2 = \frac{\partial a^2}{\partial (r_C^2)}\delta r_C^2 + \frac{\partial a^2}{\partial (\eta^4)}\delta\eta^4$$

Shape effects are important!

$$r_C \equiv \langle r^2\rangle^{1/2}$$

$$r_{C4} \equiv \langle r^4\rangle^{1/4}$$

$$\eta \equiv V_{24}^{-1} = r_{C4}/r_C$$

$$a^2 = \frac{r_C^2}{8\pi^2}\left[\sqrt{84\eta^4 - 75} - 5\right]$$

II. Global approximation, valid for most atomic transitions:

$$\delta E_{\rm FS} = F^{(1)}\,\delta r_C^2 \to F^{(1)}\,\lambda_* \equiv F^{(1)}\left\{\delta r_C^2 + \frac{G_a}{F^{(1)}}\,\delta a^2 + \left[\frac{F^{(2)}}{F^{(1)}} + \frac{G_{r_C}}{F^{(1)}}\right]\left(\delta r_C^2\right)^2\right\}$$

where ratios $\dfrac{G_a}{F^{(1)}}$ $\dfrac{G_{r_C}}{F^{(1)}}$ $\dfrac{F^{(2)}}{F^{(1)}}$ are tabulated (hydrogenic 1s values).

III. Accuracy: within 1% in most cases.
exception – fine-structure transitions, about 10%.

IV. Within global approximation, no King-plot nonlinearities.

Message to nuclear physics community:

1. Nuclear shape (= model) effects are encoded (for electronic atoms) mostly by a single parameter:

$$\eta \equiv V_{24}^{-1} = \frac{\langle r^4 \rangle^{1/4}}{\langle r^2 \rangle^{1/2}}$$

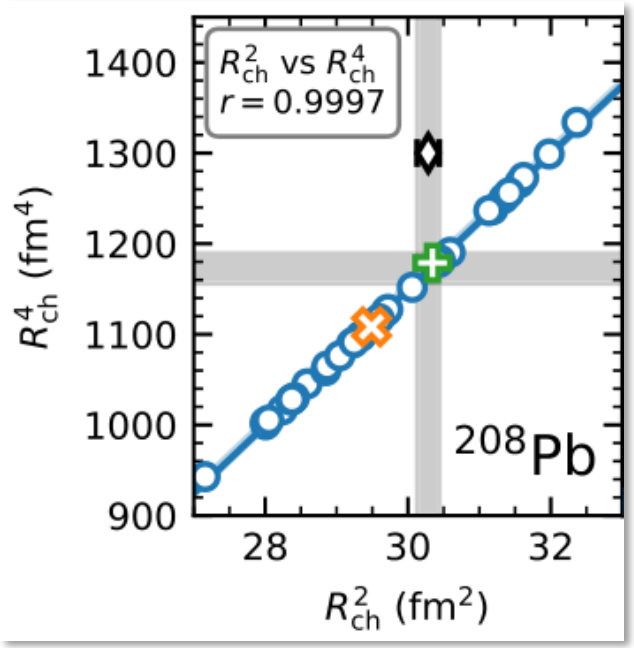
2. It is calculable!

V_{24} (^{208}Pb)

Nucl. theory: 0.9409(5)

Electr. scattering: 0.9409(2)

3. Calculations are very welcome!



Miyagi , Heinz, Schwenk
Phys. Lett. B 872, 140032 (2026)

Thanks for your attention
and
let's discuss!

Finite nuclear size effect to energy

$$E_{\rm fns} = \frac{2\pi Z\alpha}{3}\,\phi^2(0)\,R_{\rm rms}^2 + \frac{\pi(Z\alpha)^2}{3}\,\phi^2(0)\,r_F^3 - \frac{2\pi(Z\alpha)^3}{3}\,\phi^2(0)\,R_{\rm rms}^2\,\ln(Z\alpha R_{\rm rms}) + \ldots$$

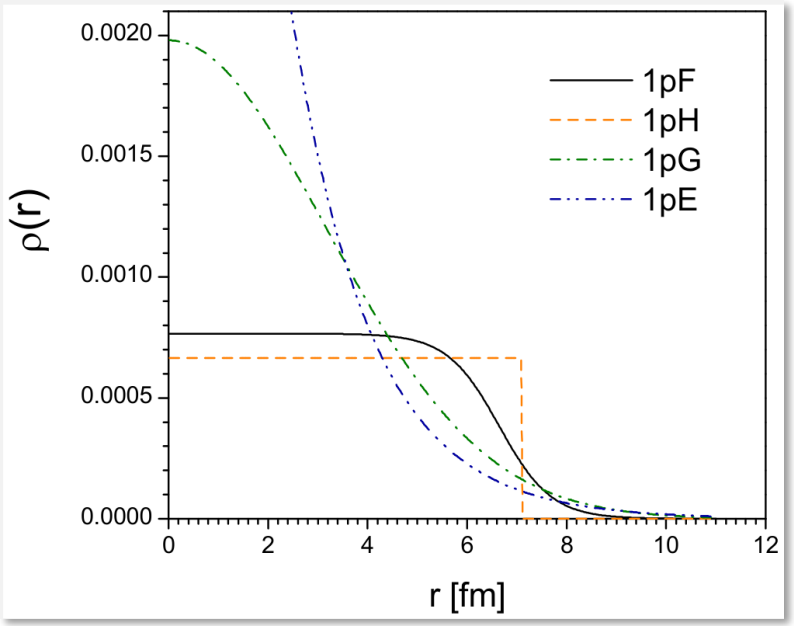
Leading fns

Higher-order fns -> model dependence

$$r_F^3 = \int d^3r_1 \int d^3r_2\,\rho(r_1)\,\rho(r_2)\,\left|\vec{r}_1 - \vec{r}_2\right|^3$$

Finite nuclear size correction, Z = 82, 1s state, one-parameter models (rms radius 5.5030 fm):

Model	$E_{\rm fns}$ [eV]	Diff.
Fermi (1pF)	67.21373	
Sphere (1pH)	67.34240	0.19%
Gaussian (1pG)	66.66235	−0.82%
Exponential (1pE)	65.63333	−2.35%



RMS radius alone is not sufficient !

Finite nuclear size effect to energy

Let's fix more nuclear parameters and compare different models…

$$V_{24} = \frac{\langle r^2\rangle^{1/2}}{\langle r^4\rangle^{1/4}}\,, \quad V_{26} = \frac{\langle r^2\rangle^{1/2}}{\langle r^6\rangle^{1/6}}\,,$$

$$\langle r^2\rangle^{1/2} = 5.5030\ \text{fm}\,, \quad V_{24} = 0.9406\,, \quad V_{26} = 0.8983\,.$$

Finite nuclear size correction, Z = 82, 1s state:

n	Model	r_C	V_{24}	V_{26}	V_{28}	r_F/r_C	$E_{\rm fns}$	Diff.
1	1pF	5.5030	0.9399	0.8959	0.8600	1.4912	67.21373	
	1pH	5.5030	0.9573	0.9302	0.9112	1.4856	67.34240	0.19%
	1pG	5.5030	0.8801	0.7974	0.7356	1.5146	66.66235	−0.82%
	1pE	5.5030	0.7953	0.6640	0.5718	1.5590	65.63333	−2.35%
2	2pF	5.5030	0.9406	0.8974	0.8621	1.4910	67.21939	
	2pG	5.5030	0.9406	0.9041	0.8772	1.4912	67.20764	−0.018%
	2pE	5.5030	0.9406	0.8763	0.7884	1.4905	67.24369	0.036%
3	3pF	5.5030	0.9406	0.8983	0.8645	1.4910	67.21806	
	3pG	5.5030	0.9406	0.8983	0.8640	1.4910	67.21788	−0.0003%
	3pE	5.5030	0.9406	0.8983	0.8618	1.4910	67.21716	−0.0013%

The model dependence can be eliminated !!

VAY and Ohayon, Phys. Rev. A 113, 012804 (2026)