Precision calculations with NLEFT

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by ERC, EXOTIC

by NRW-FAIR

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Short introduction

Introductory remarks

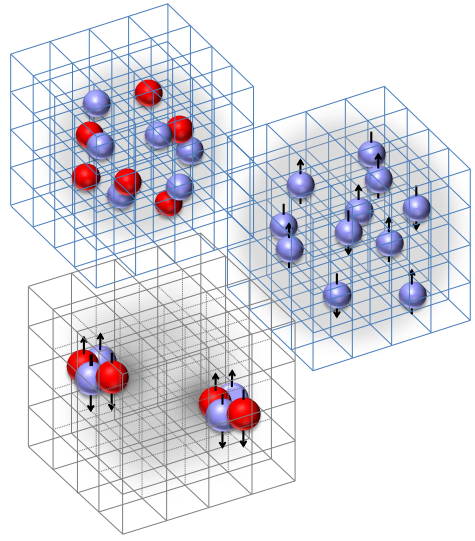
- To hunt BSM physics, we need to have high-precision experiments & theory
- High intensities rather than highest energies are the way to go
- At low energies, we need to cope with the nonperturbative strong interactions
- Nuclei are long known to be a great test bed for fundamental symmetries
- But: Lattice QCD fails in providing accurate nuclear physics, are we doomed?

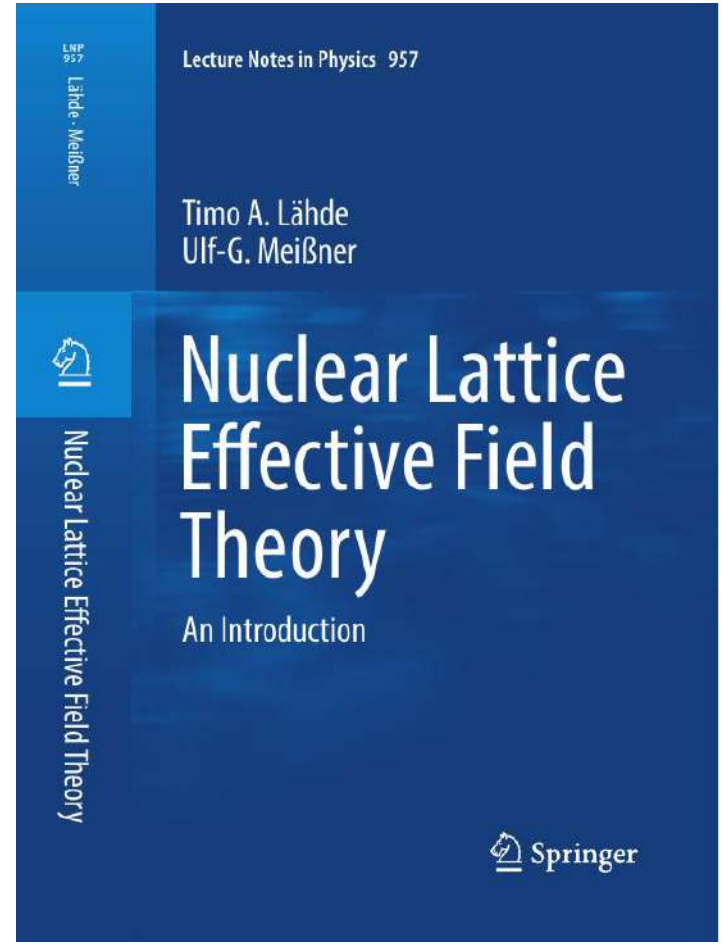
$\hookrightarrow$ NO - scale separation comes in handy (effective field theories)

$\hookrightarrow$ NLEFT has positioned itself as a main player in this field

$\hookrightarrow$ So let us discuss in present status and its perspectives

Nuclear physics on a lattice





T. Lähde & UGM

Nuclear Lattice Effective Field Theory - An Introduction

Springer Lecture Notes in Physics **957** (2019) 1 - 396 [2nd edition in the works]

Foundations

Nuclear lattice effective field theory (NLEFT)

- *new method* to tackle the nuclear many-body problem

- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

  → see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- EFT on the lattice, maximal momentum:

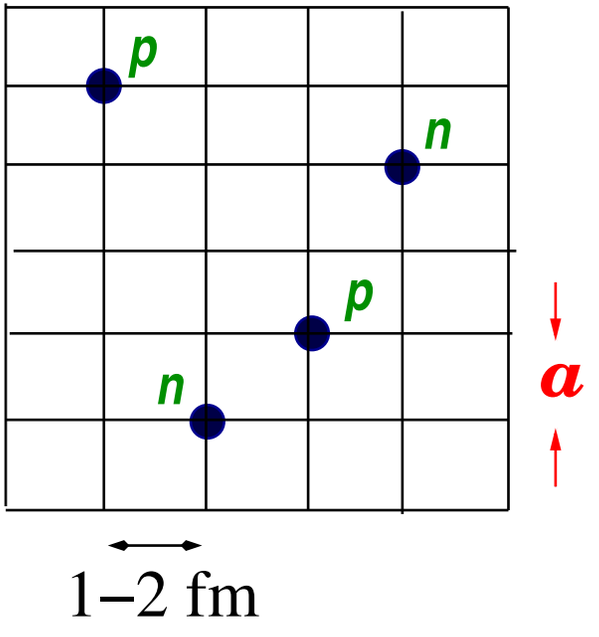
$$p_{\max} = \frac{\pi}{a} \simeq 315 - 630\,\text{MeV [UV cutoff]}$$

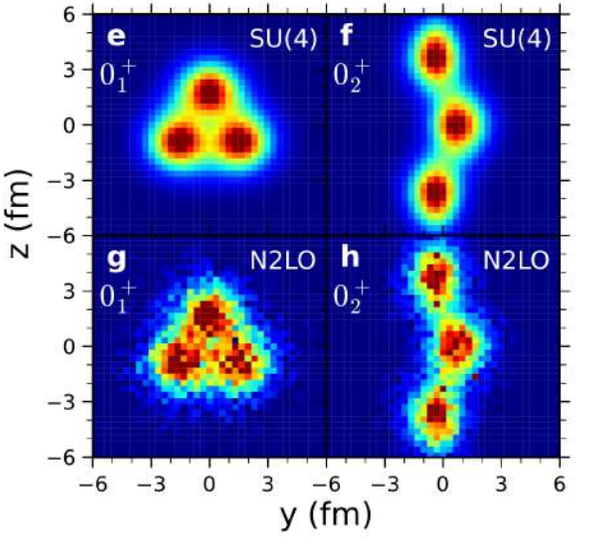
- strong suppression of sign oscillations SU(4)
  due to approximate Wigner (spin-isospin) symmetry

  Wigner, Phys. Rev. **51** (1937) 106; Chen et al., Phys. Rev. Lett. **93** (2004) 242302

↪ works well for even-even nuclei

↪ we still need another method



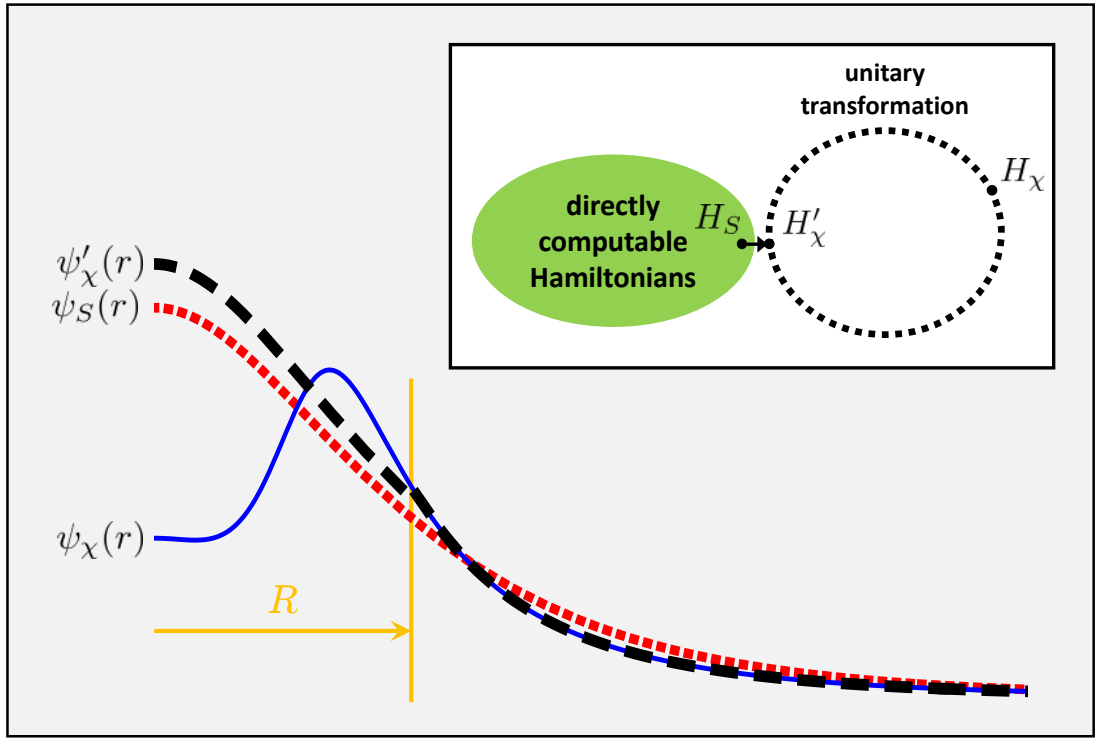


Shen et al., Nature Commun. **14** (2023) 2777

Wave function matching

Elhatisari et al., Nature **630** (2024) 59

- A new quantum many-body method: Bring a complex Hamiltonian H_χ close to a simple one H_S (with H_S essentially free of sign oscillations)
  $\hookrightarrow$ treat H_S non-perturbatively & $H'_\chi - H_S$ in perturbation theory
- Graphical representation of w.f. matching



$\Rightarrow$ Efficient suppression of sign oscillations, applicable in many fields!

Pinhole algorithm

Elhatisari et al., PRL **119** (2017) 222505

- Solution to the CM-problem:

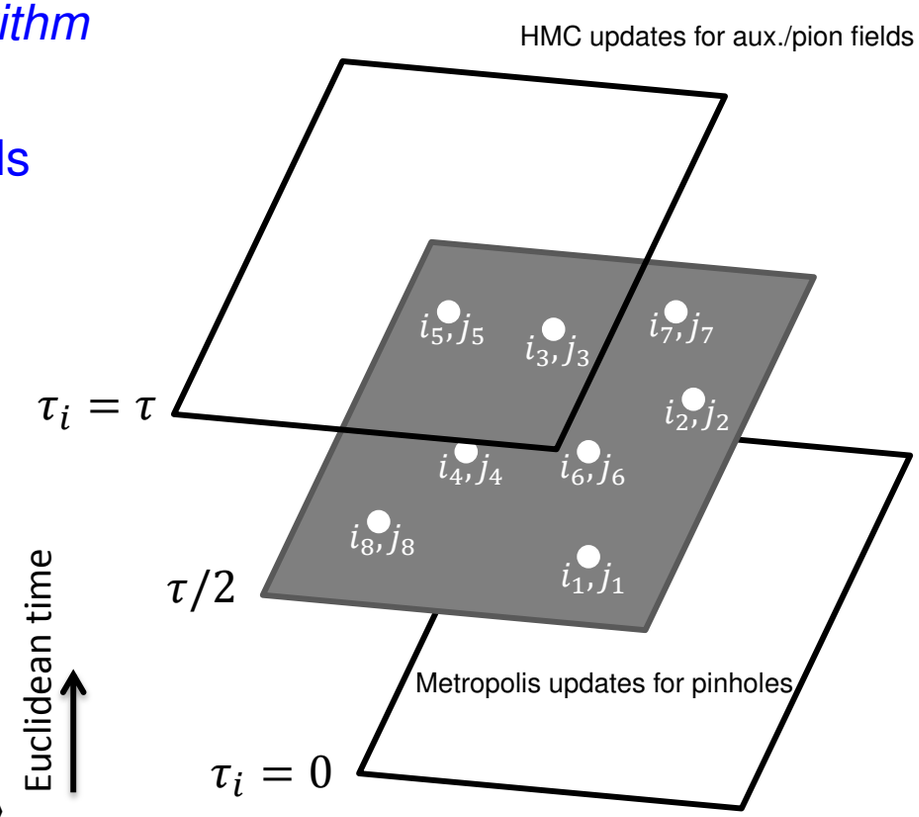
  track the individual nucleons using the *pinhole algorithm*

- Insert a screen with pinholes with spin & isospin labels that allows nucleons with corresponding spin & isospin to pass = insertion of the A-body density op.:

$$\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots \mathbf{n}_A) =: \rho_{i_1,j_1}(\mathbf{n}_1)\cdots\rho_{i_A,j_A}(\mathbf{n}_A):$$

- MC sampling of the amplitude:

$$A_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\ldots,\mathbf{n}_A,L_t) = \langle\Psi_A(\tau/2)|\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\ldots,\mathbf{n}_A)|\Psi_A(\tau/2)\rangle$$



- Allows to measure proton and neutron distributions

- Resolution scale $\sim a/A$ as cm position $\mathbf{r}_{\mathbf{cm}}$ is an integer $\mathbf{n}_{\mathbf{cm}}$ times a/A

Partial pinhole algorithm

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Nuclear radii can be expressed in terms of two-particle correlations G_{pp}, G_{pn}, G_{nn}

$$\langle r^2 \rangle = \sum_{c_1,c_2} \langle : \rho_{i_1 j_1}(\mathrm{n}_1)\rho_{i_2 j_2}(\mathrm{n}_2) : \rangle (\mathrm{n}_1 - \mathrm{n}_2)^2 \,, \quad c_k = (i_k, j_k, \mathrm{n}_k)$$

↪ the number of pinholes needed in such a calculation can be reduced:

$$\rho_M(c_1, \ldots, c_M) = \frac{(A-M)!}{A!} : \rho_{i_1 j_1}(\mathrm{n}_1) \cdots \rho_{i_M j_M}(\mathrm{n}_M) : ,$$

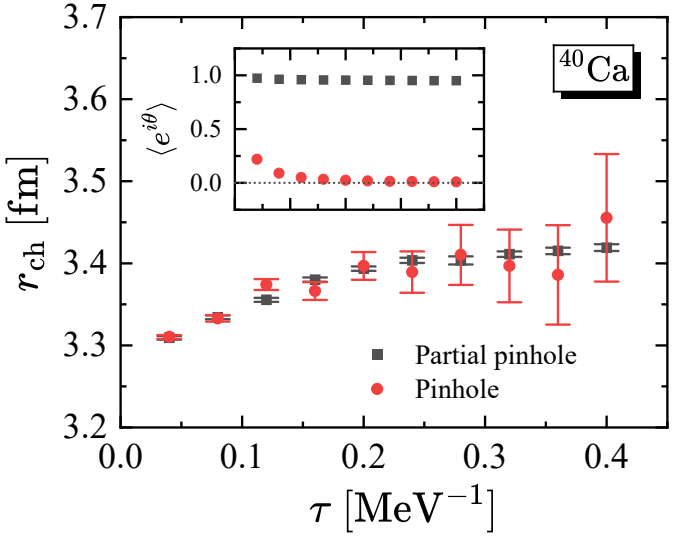
with $\sum_{c_1,\ldots,c_M} \rho_M = 1$

- However, $\rho_M |\Psi_A\rangle$ is not a Slater determinant, even if $|\Psi_A\rangle$ is

↪ use the rank-one operator method for ρ_M Ma et al. (2024)

$$: \rho_{i_1 j_1}(\mathrm{n}_1) \cdots \rho_{i_M j_M}(\mathrm{n}_M) :$$
$$= \lim_{\varepsilon \to \infty} \frac{1}{\varepsilon^M} : e^{\varepsilon [\rho_{i_1 j_1}(\mathrm{n}_1) + \cdots]} :$$

↪ much improved signal allows for a more precise extraction



The chiral Hamiltonian at N3LO

The chiral Hamiltonian

- Chiral Hamiltonian with **smeared** N3LO (2N) and N2LO (3N) interactions

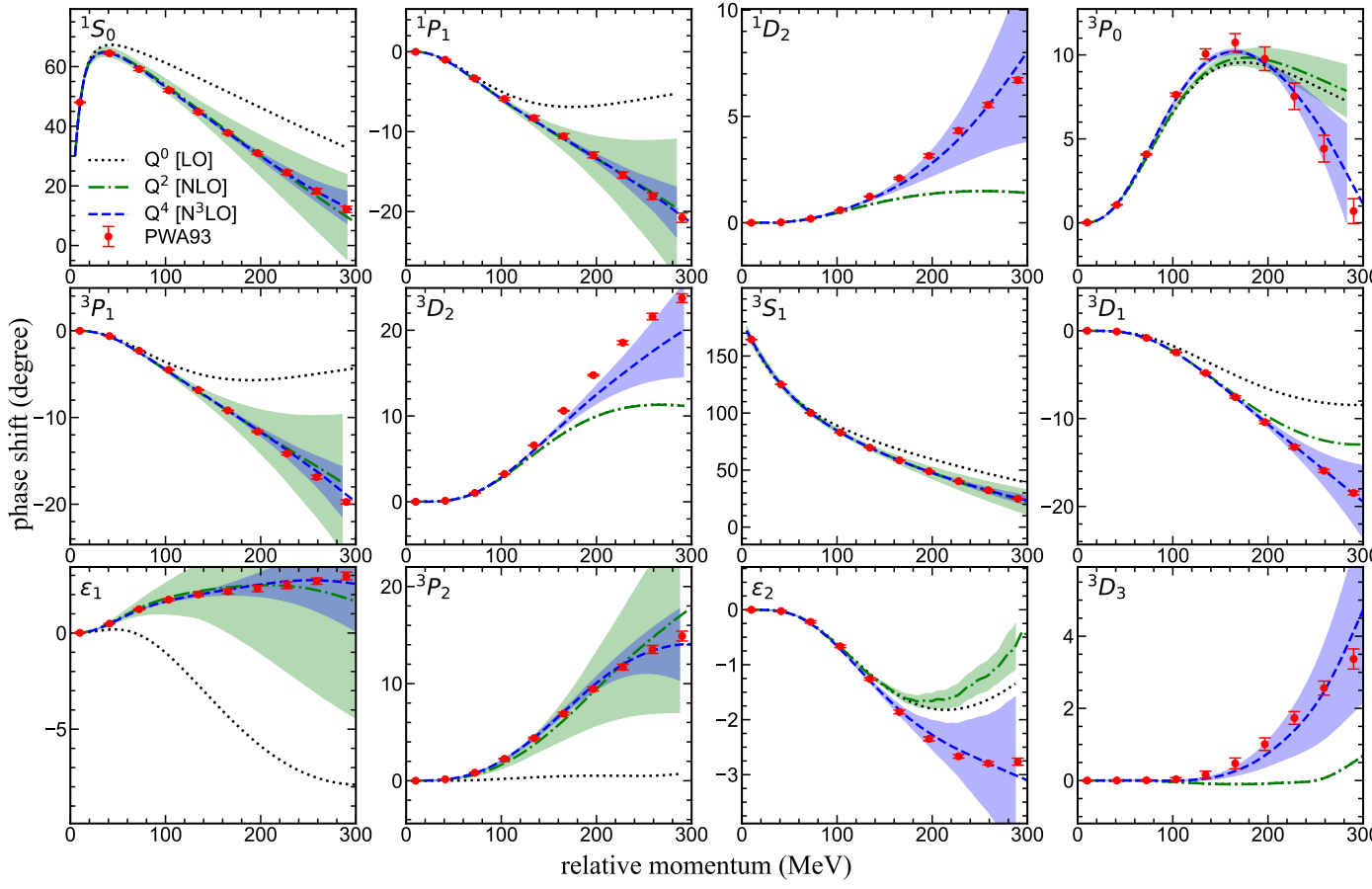
$$H = K + V_{\rm OPE} + V_{\rm C} + V_{\rm 3N}^{\rm Q^3} + V_{\rm 2N}^{\rm Q^4} + W_{\rm 2N}^{\rm Q^4}$$

  - K – kinetic energy term with correct nucleon dispersion relation (FFT)
  - $V_{\rm OPE}$ – regulated OPE (similar to continuum case) Reinert et al. (2018)
  - $V_{\rm C}$ – Coulomb interaction
  - $V_{\rm 3N}^{\rm Q^3}$ – 3N interaction at N2LO $\to$ to be discussed later
  - $V_{\rm 2N}^{\rm Q^4}$ – 2N short-range interaction at N3LO (2π exchange absorbed) $\to$ next slide
  - $W_{\rm 2N}^{\rm Q^4}$ – 2N Galilean invariance restoration (GIR) interaction at N3LO Li et al. (2019)

- Smearing explained in what follows
- Throughout, we use $a = 1.32$ fm, corresponding to the magic momentum cutoff $\Lambda \simeq 470$ MeV Lee et al. (2021)

Nucleon-nucleon interaction: 2N LECs

- Here, we absorb the TPEP in the short distance LECs → works, see also Li et al. (2018)

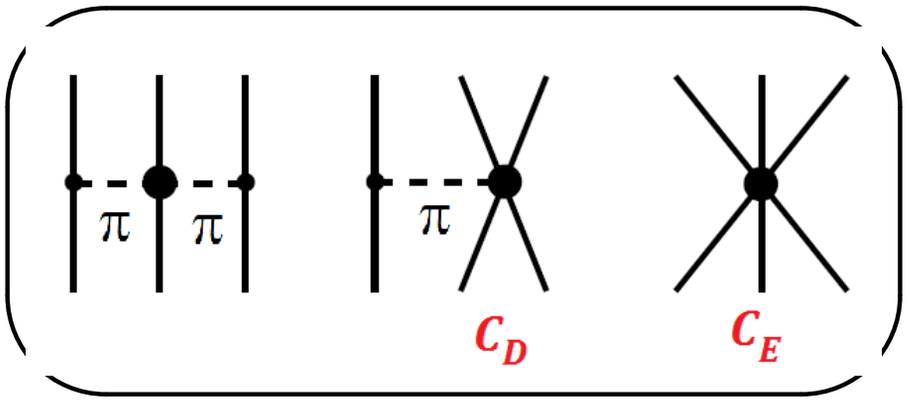


↪ needs to be improved (work underway)

↪ especially important for getting consistent currents beyond LO

Three-nucleon forces

- Basic topologies at N2LO:

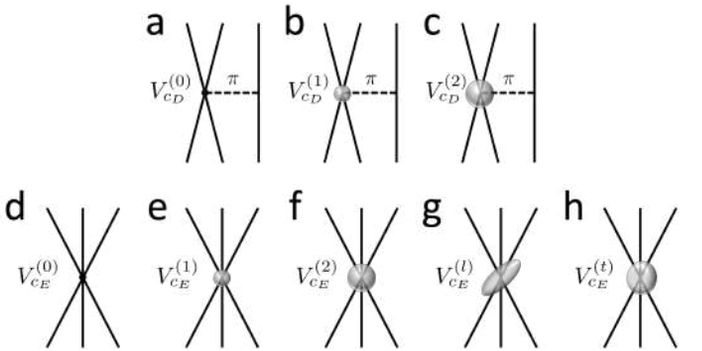


  - 2-pion topology entirely fixed from πN scattering Hoferichter, de Elvira, Kubis, UGM, Phys. Rept. **625** (2016) 1

  - Smearing of the one-pion and contact term topologies

    ↪ more LECs

    ↪ effectively higher-order op's



  - On the market so far (more in the works!)

    ↪ $\mathcal{F}_p = 8$, only local smearings Elhatisari et al. (2024)

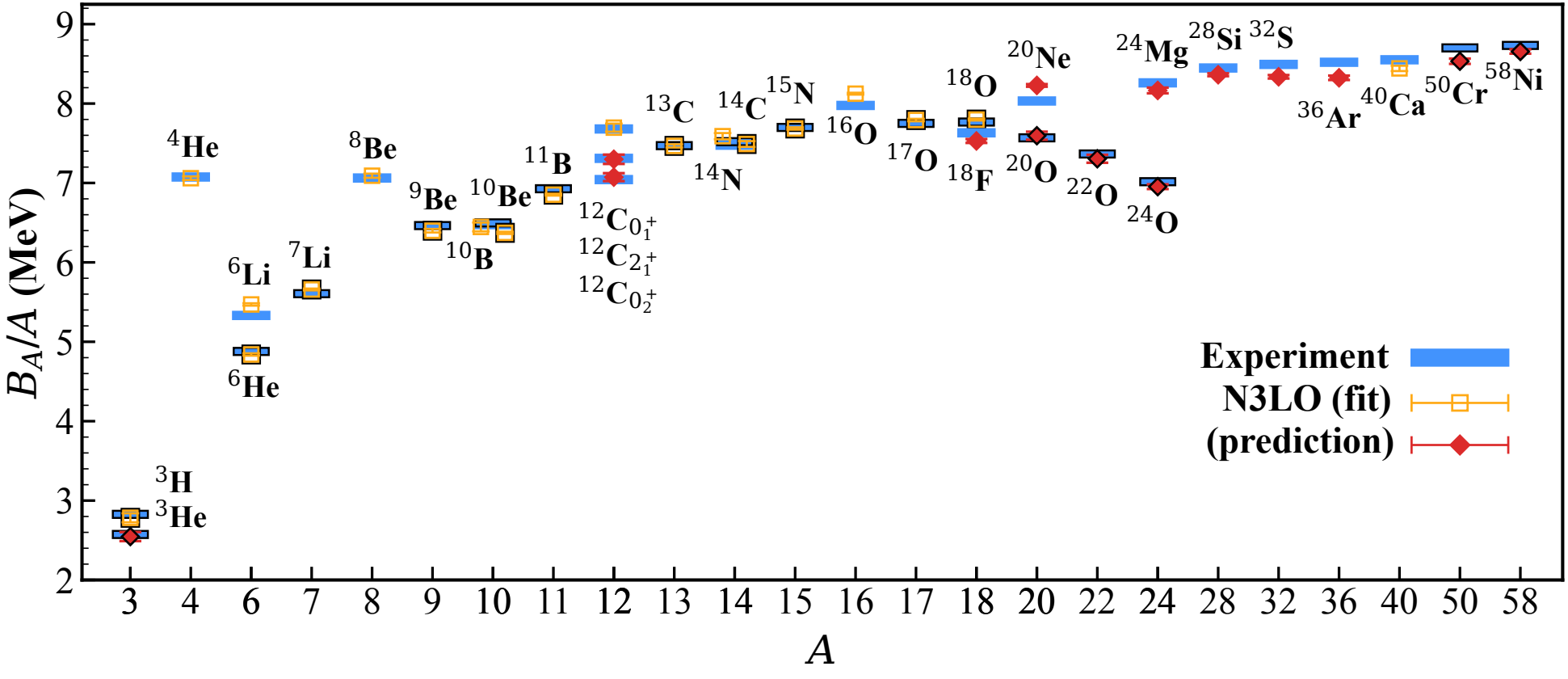
    ↪ $\mathcal{F}_p = 6$, a few non-locally smeared ops Ren et al. (2025)

Fixing the 3NF LECs: Binding Energies at N3LO

Elhatisari et al., Nature **630** (2024) 59

- Quality of the fit: $\mathrm{RMSD}(\mathcal{S}) = \sqrt{\frac{1}{M_{\mathcal{S}}} \sum_{i \in \mathcal{S}} \left(\frac{E_i - E_i^{\mathrm{exp}}}{A_i} \right)^2}$ with size $M_{\mathcal{S}}$

↪ here, $\mathrm{RMSD} = 0.079\ \mathrm{MeV}$ = 1.11% of the average BE per nucleon



A minimal set of 3NFs

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Consider sets of 6 3NF op's (called minimal set)

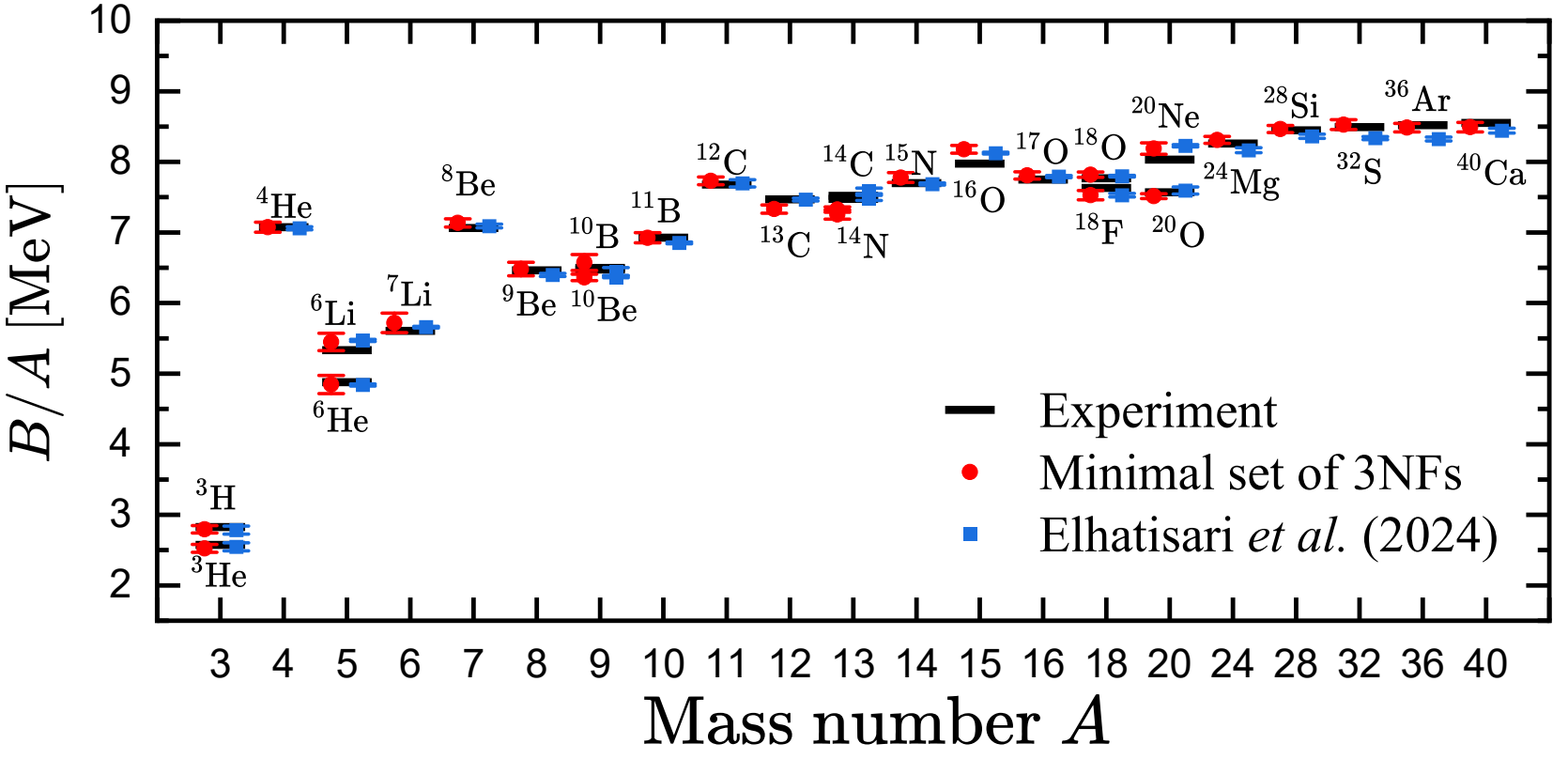
No.	V_{3N}'s
1	$V^{(0)}_{c_E,s_{NL1}}, V^{(3)}_{c_D}, [V^{(0)}_{c_E}]_{044}, [V^{(0)}_{c_E}]_{224}, [V^{(0)}_{c_E}]_{125}, [V^{(0)}_{c_E}]_{145}$
2	$[V^{(1)}_{c_E}]_{000}, V^{(3)}_{c_D}, [V^{(0)}_{c_E}]_{044}, [V^{(0)}_{c_E}]_{222}, [V^{(0)}_{c_E}]_{114}, [V_{c_E,s_L}]_{033}$
3	$V^{(0)}_{c_E,s_{NL1}}, V^{(3)}_{c_D}, [V^{(0)}_{c_E}]_{044}, [V^{(0)}_{c_E}]_{224}, [V^{(0)}_{c_E}]_{125}, [V^{(0)}_{c_E}]_{235}$
4	$V^{(0)}_{c_D}, V^{(2)}_{c_D}, [V^{(0)}_{c_E}]_{044}, [V^{(0)}_{c_E}]_{222}, [V^{(0)}_{c_E}]_{224}, [V^{(0)}_{c_E}]_{125}$
5	$V^{(1)}_{c_D}, V^{(3)}_{c_D}, [V^{(0)}_{c_E}]_{033}, [V^{(0)}_{c_E}]_{044}, [V^{(0)}_{c_E}]_{222}, [V^{(0)}_{c_E}]_{125}$
6	$V^{(0)}_{c_E,s_{NL1}}, V^{(0)}_{c_D}, V^{(3)}_{c_D}, [V^{(0)}_{c_E}]_{044}, [V^{(0)}_{c_E}]_{224}, [V^{(0)}_{c_E}]_{125}$
7	$V^{(1)}_{c_D}, V^{(3)}_{c_D}, V^{(0)}_{c_D,s_{NL2}}, [V^{(0)}_{c_E}]_{044}, [V^{(0)}_{c_E}]_{224}, [V^{(0)}_{c_E}]_{125}$
8	$V^{(0)}_{c_D}, V^{(2)}_{c_D}, [V^{(0)}_{c_E}]_{044}, [V^{(0)}_{c_E}]_{224}, [V^{(0)}_{c_E}]_{125}, [V^{(0)}_{c_E}]_{145}$
9	$[V^{(0)}_{c_E}]_{000}, V^{(3)}_{c_D}, [V^{(0)}_{c_E}]_{224}, [V^{(0)}_{c_E}]_{125}, [V_{c_E,s_L}]_{044}, [V^{(0)}_{c_E}]_{334}$
10	$[V^{(0)}_{c_E}]_{000}, V^{(0)}_{c_E,s_{NL3}}, V^{(1)}_{c_D}, V^{(0)}_{c_D,s_{NL3}}, [V^{(0)}_{c_E}]_{224}, [V^{(0)}_{c_E}]_{125}$

- contains also non-local operators (marked in blue)
- only some sets contain the SU(4) symmetric $V^{(l,t)}_{c_E}$ [here: $[V^{(0)}_{c_E}]_{144}, [V^{(0)}_{c_E}]_{222}$]

Determination of the 3NF LECs

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Fit to the BE of a selected number of nuclei and also include the ^{4}He charge radius



↪ Quality similar to the WFM 3NFs: RMSD = 0.101 MeV (1.4%) [0.093 MeV (1.1%)]

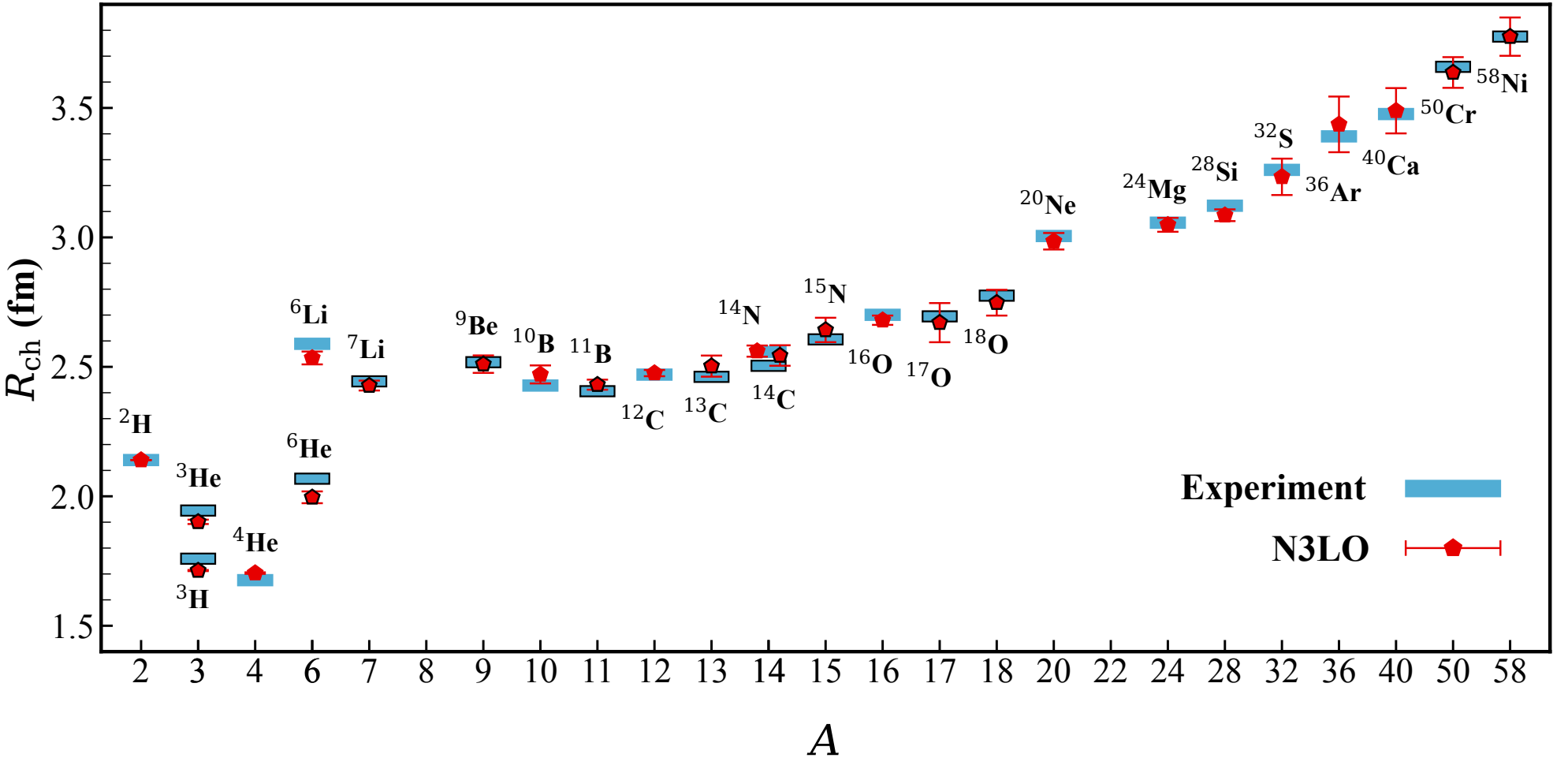
↪ use these later for the calculation of selected oxygen radii

Applications to structure, reactions and matter

Prediction: Charge radii at N3LO

Elhatisari et al., Nature **630** (2024) 59

- Charge radii ($a = 1.32$ fm, statistical errors can be reduced)

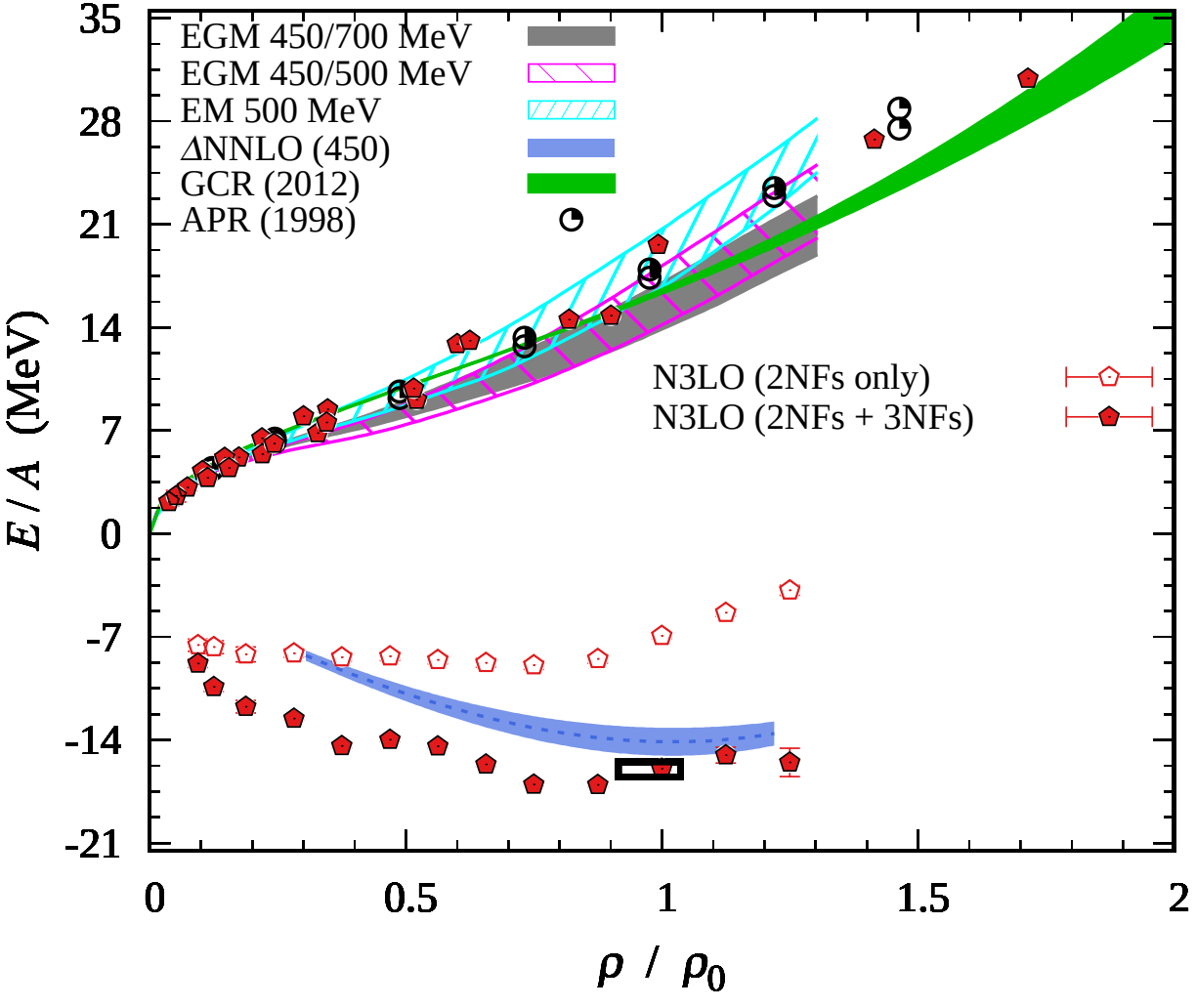


↪ no radius problem! but radii to be improved, see later

Prediction: Neutron & nuclear matter at N3LO

Elhatisari et al., Nature **630** (2024) 59

- Equation of State (EoS) of pure neutron matter & nuclear matter ($a = 1.32$ fm)



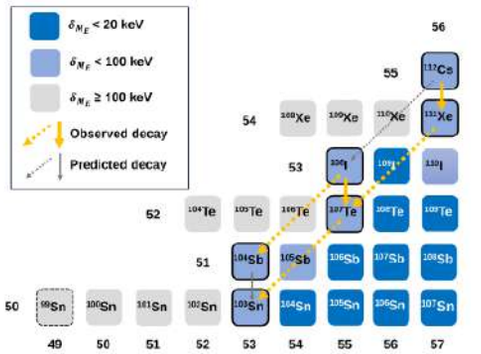
↪ can be improved using twisted b.c.'s

Is it possible to go to heavier nuclei?

- Consider the Sn isotopes close to the proton dripline:

↪ Standard candle of nuclear structure theory

↪ Even $N = Z$ nucleus close to the dripline



Ireland et al., PRC **111** (2025) 014314

↪ Inconsistencies in 2020 AME → new experiments at ISOLTRAP and LEBIT

Nies al. PRC **111** (2025) 014315; Ireland et al., PRC **113** (2026) L021302

↪ Dripline so far based on extrapolations

- Premier playground for the newest generation of supercomputers (Exascale) here: JUPITER @ FZJ

Prediction: Tin isotopes close to the proton dripline

Hildenbrand et al., PRL **136** (2026) 062501

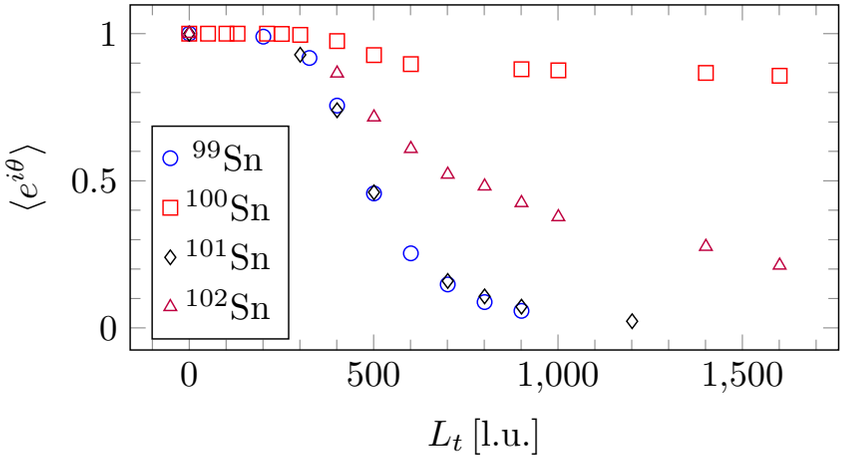
- Signal still strong enough for even-odd isotopes

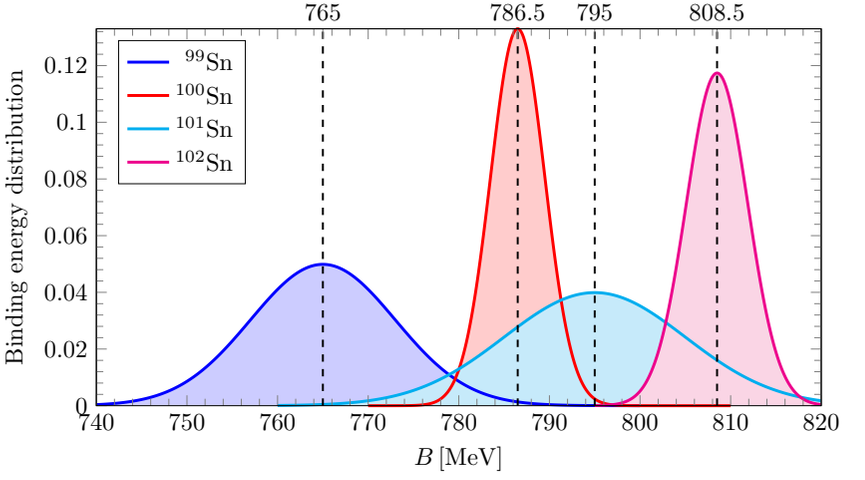
↪ can extract the BEs of ^{99}Sn to ^{102}Sn

↪ small mod's of 3NFs to describe g.s. data [$V_{c_E}^{(l,t)}$]

Nucleus	N3LO	N3LO*	Exp.
^{99}Sn	765±8	804±8	807.9± 0.6
^{100}Sn	786.5±3.0	825.2±3.0	825.16±0.24
^{101}Sn	795±10	834±10	836.39±0.30
^{102}Sn	808.5±3.4	848.1±3.4	849.09±0.10

Isotopes	N3LO	N3LO*	Exp.
102-101	13.5±10.6	14.1±10.6	12.7±0.3
102-100*	22.5±4.6	22.9±4.6	23.9±0.3
102-99	43.5±8.7	44.1±8.7	41.2±0.6
101-100	8.5±10.4	8.9±10.4	11.2±0.4
101-99*	30.0±12.8	30.1±12.8	28.5±0.7
100-99	21.5±8.6	21.2±8.6	17.3±0.7





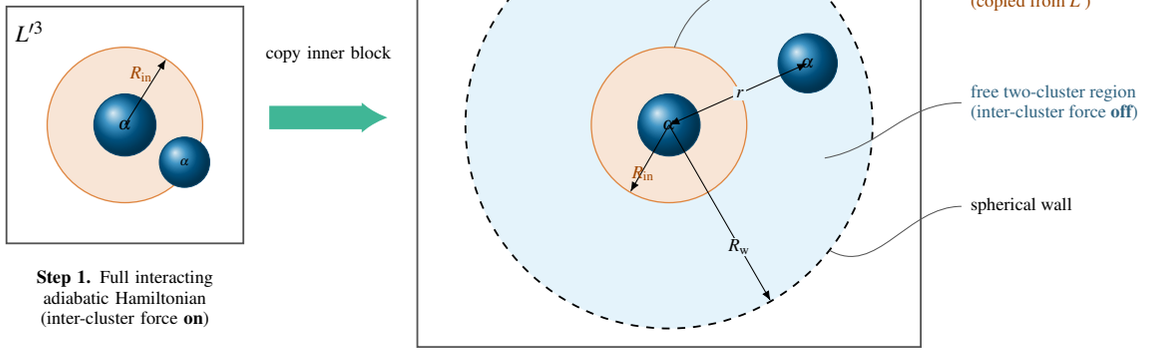
Prediction: Alpha-alpha scattering at N3LO

- Alpha-alpha scattering (at N2LO) one of the breakthrough calc's of NLEFT

Elhatisari et al., Nature **528** (2015) 111

↪ extend this calculation using the APM with high-fidelity forces

- for a fine lattice with $a = 1.32$ fm, the small eigenvalues of $H^{\rm APM}$ are extremely noisy



Step 1. Full interacting adiabatic Hamiltonian (inter-cluster force **on**)

Step 2. Free clusters out to the wall, with the interacting block inserted at short range

↪ overcome this with regularization

↪ we use two different methods: Tikhonov & truncated singular value deomposition

$$N_{\rm Tik} = U \begin{pmatrix} \Lambda_i + \lambda I_i & 0 \\ 0 & \Lambda_r \end{pmatrix} U^T \,, \qquad N_{\rm TVSD} = U \begin{pmatrix} 0_i & 0 \\ 0 & \Lambda_r \end{pmatrix} U^T$$

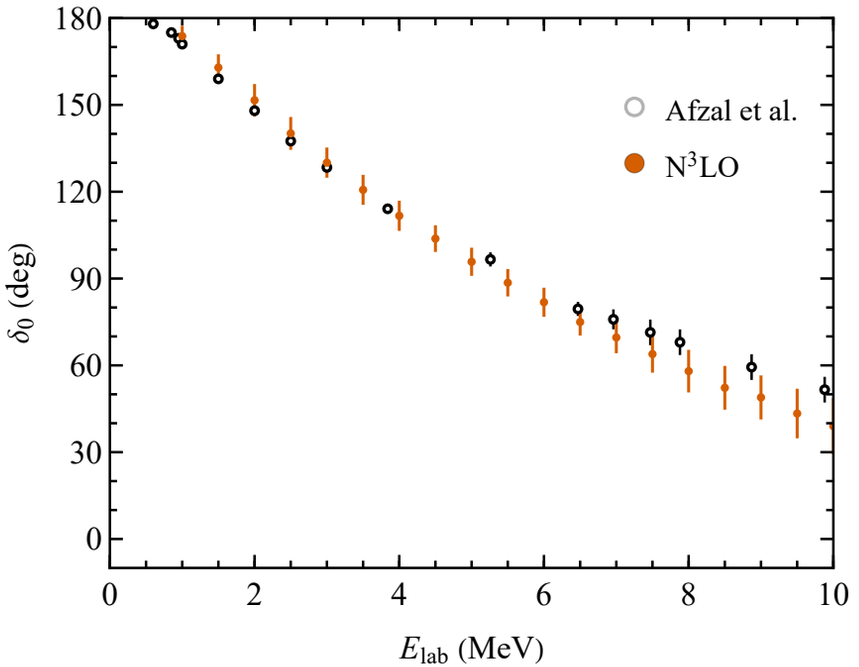
⇒ both give consistent results, nice check

Prediction: Alpha-alpha scattering at N3LO cont'd

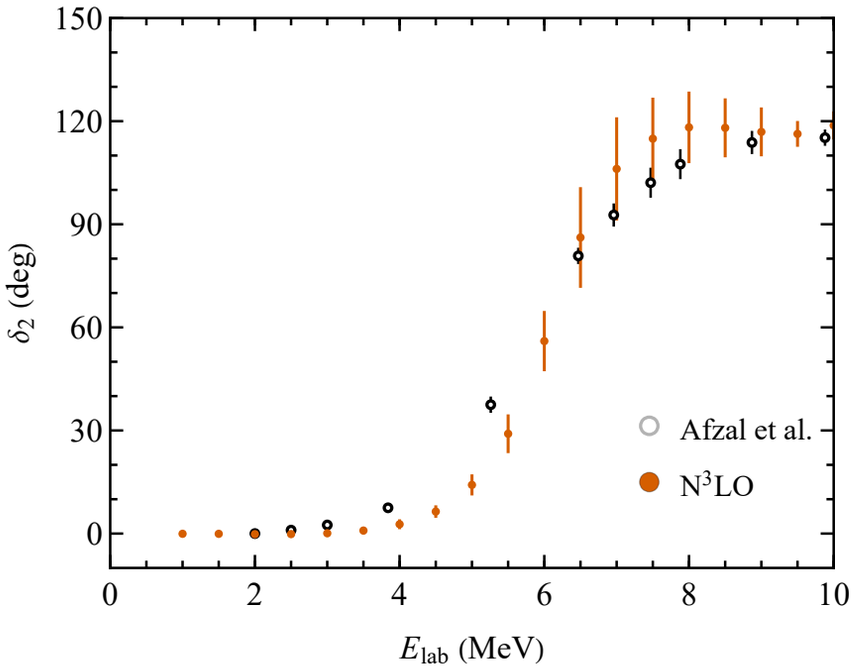
- Alpha-alpha scattering at N3LO

Sarkar, Elhatisari, Lähde, UGM, 2606.28987 [nucl-th]

- S-wave



- D-wave



- Improved over N2LO!

$$E_R = 3.25(25) \text{ MeV} \quad [2.92(18) \text{ MeV}]$$

$$\Gamma_R = 0.75(25) \text{ MeV} \quad [1.34(50) \text{ MeV}]$$

Precision calculations of nuclear radii

Basic formalism

- Nuclear charge radii are computed via

$$r_{\rm ch}^2 = \langle r_p^2 \rangle + R_p^2 + \frac{N}{Z} R_n^2 + \frac{3}{4m_p^2}$$

$\langle r_p^2 \rangle = \frac{1}{Z} \sum_{\vec{n},\sigma,\tau=p} \langle \rho_{\sigma\tau} \left(\vec{n} - \hat{r}_{\rm cm}\right)^2 \rangle$ = mean-square point proton radius

$R_p^2 = \quad 0.7056\,\text{fm}^2$ Lin, Hammer, UGM, PRL **128** (2022) 052002

$R_n^2 = -0.105\,\text{fm}^2$ Filin et al., PRC **103** (2021) 024313

- Alternative representation of $\langle r_p^2 \rangle$ in terms of two-body correlations functions $G_{\alpha\beta}$:

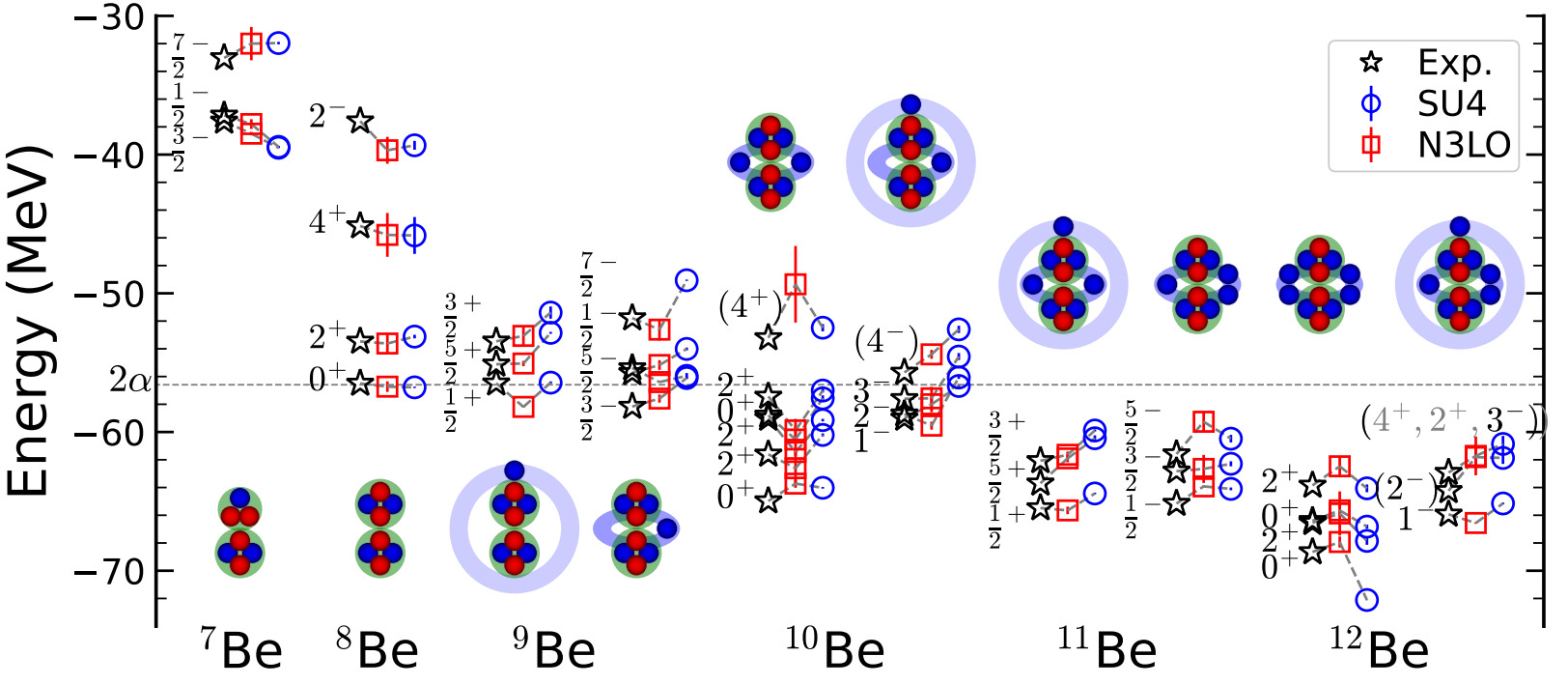
$$\langle r_p^2 \rangle = \frac{N}{ZA^2} G_{pp} + \frac{N-Z}{ZA^2} G_{pn} - \frac{1}{A^2} G_{nn}$$

- Can be improved by including relativisitic and two-nucleon current contributions

Prediction: Be isotopes

Shen et al., Phys. Rev. Lett. **134** (2025) 162503

- Systematic study of the Be isotopes & their radii & their em transitions:



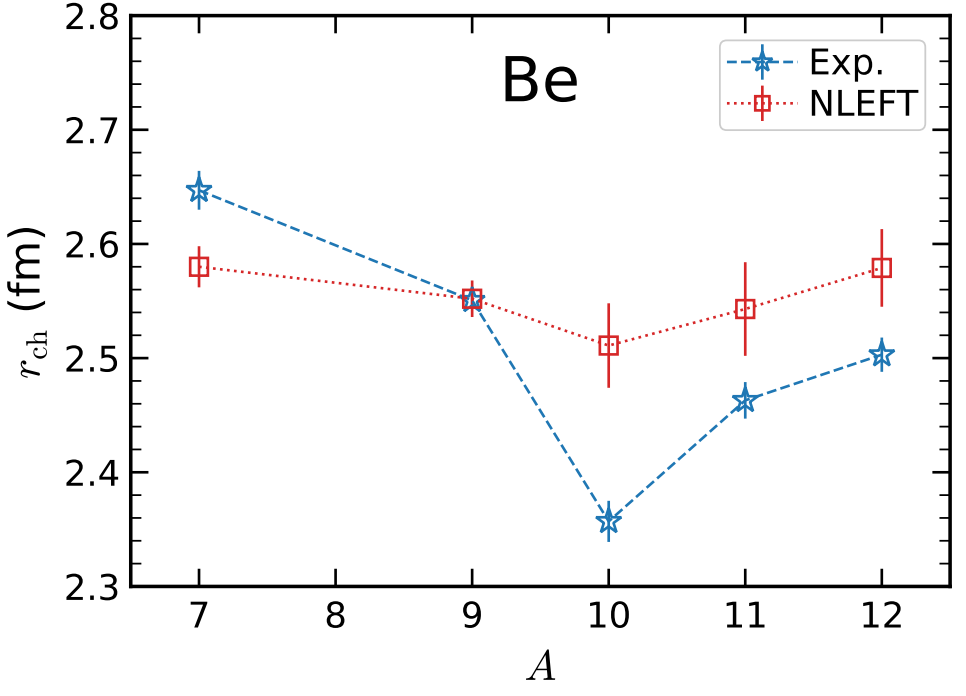
↪ new method to quantify nuclear shapes

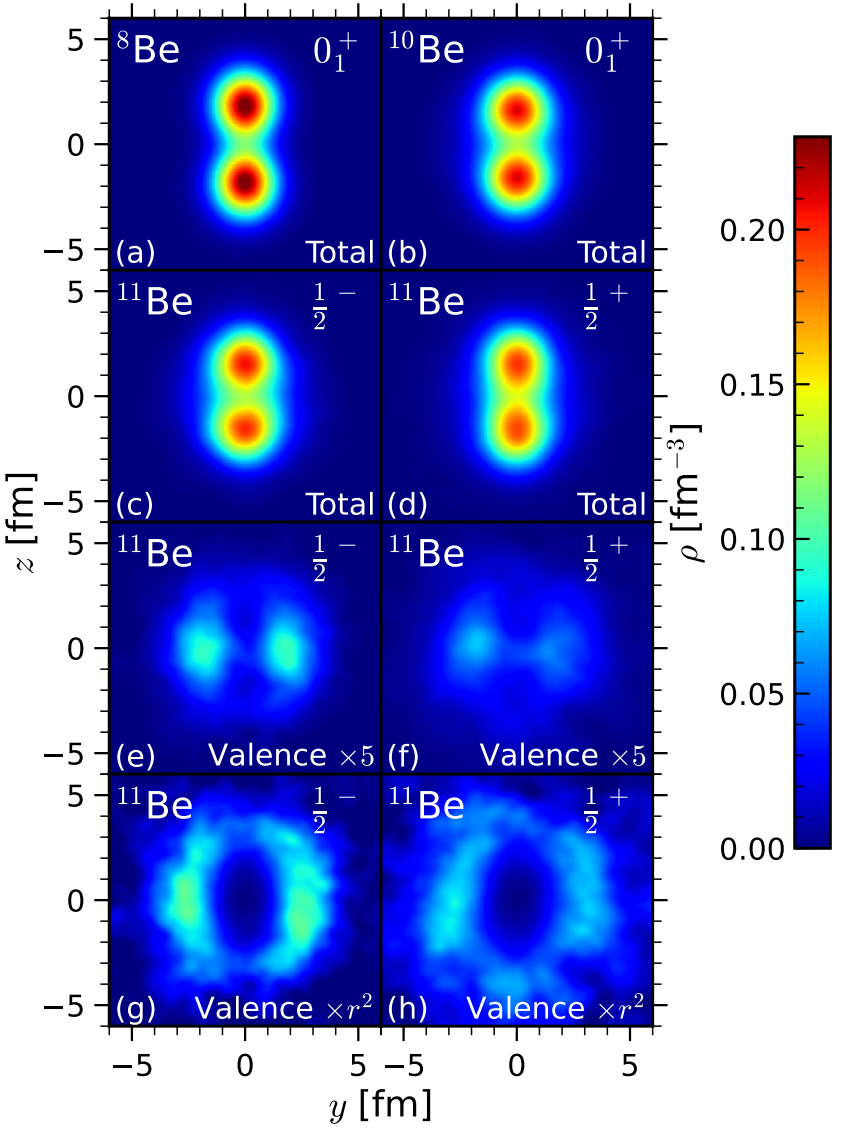
↪ clusters, halos, molecular orbitals in **one shot**

Radii of the Be isotopes

Shen et al., Phys. Rev. Lett. **134** (2025) 162503

- Trend of the data reproduced





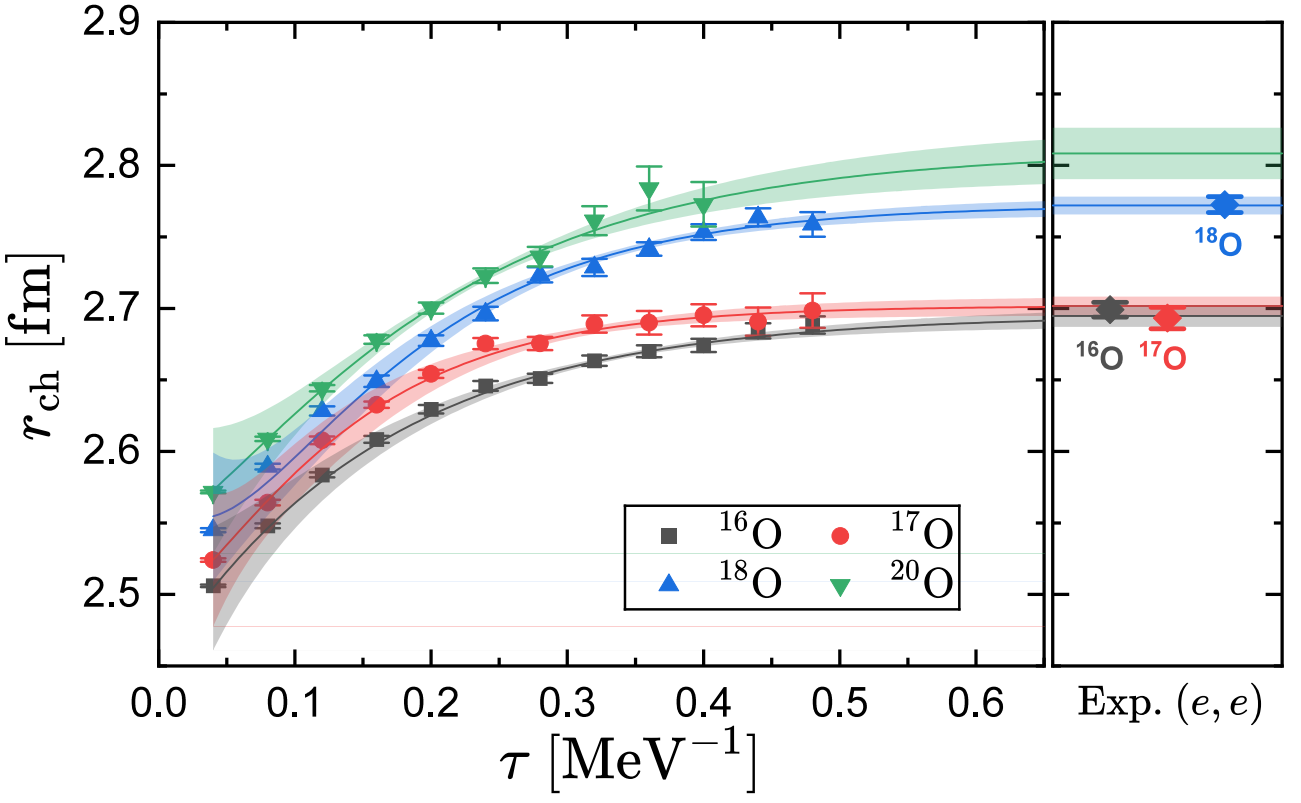
- $r_{\rm ch}(^9{\rm Be})$ agrees with the recent high-precision atom spectroscopy result!

  Eizenberg et al., 2607.13690 [nucl-ex]

Radii of selected oxygen isotopes

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Use the partial pinhole algorithm to study the radii of ^{16,17,18,20}O with $\mathcal{F}_p = 6$ & $M = 4$



Isotope	NLEFT	Exp.
^{16}O	2.704(17)	2.699(5)
^{17}O	2.709(15)	2.693(8)
^{18}O	2.768(17)	2.776(2)
^{20}O	2.810(32)	

- Trend of the data reproduced: $r_{\rm ch}(^{16}{\rm O}) \simeq r_{\rm ch}(^{17}{\rm O}) < r_{\rm ch}(^{18}{\rm O})$
- Prediction for $r_{\rm ch}(^{20}{\rm O})$ → nice test

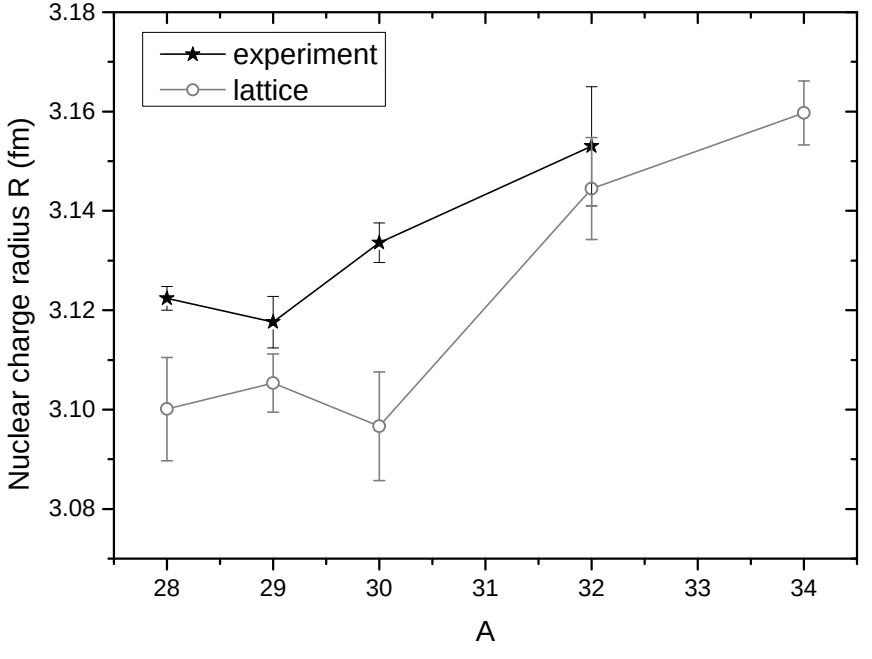
Nuclear charge radii of silicon isotopes

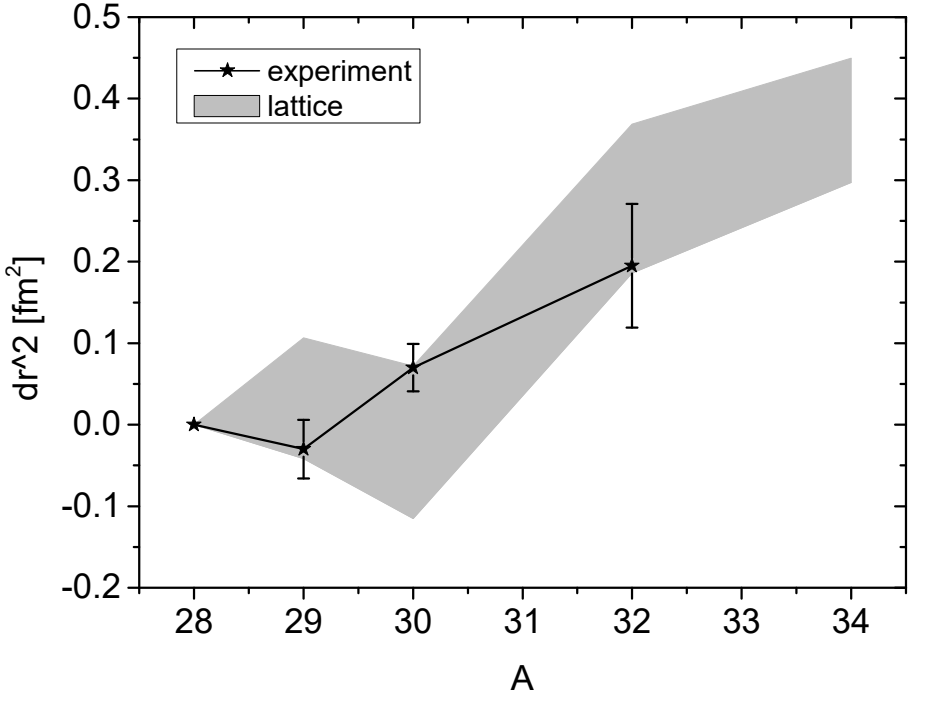
König et al., PRL **132** (2024) 162502

- Laser spectroscopy with BECOLA at FRIB to measure the charge radius of ^{32}Si

$\hookrightarrow \delta\langle r^2\rangle^{A,A'}$ is directly related to the measured atomic factors

- Use the $\mathcal{F}_p = 8$ basis, modify two LECs [$V_{c_E}^{(l,t)}$] to get the exact BE's of ^{28}Si to ^{32}Si



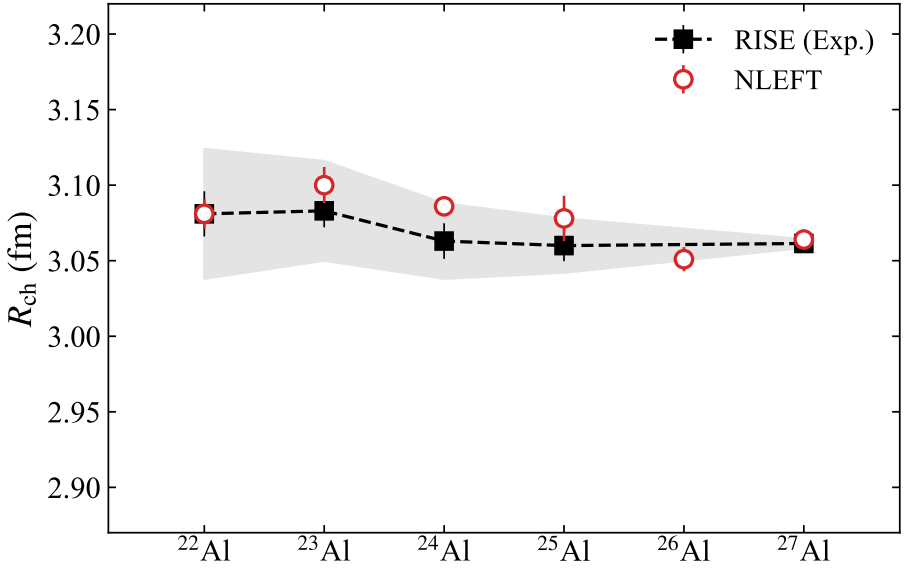


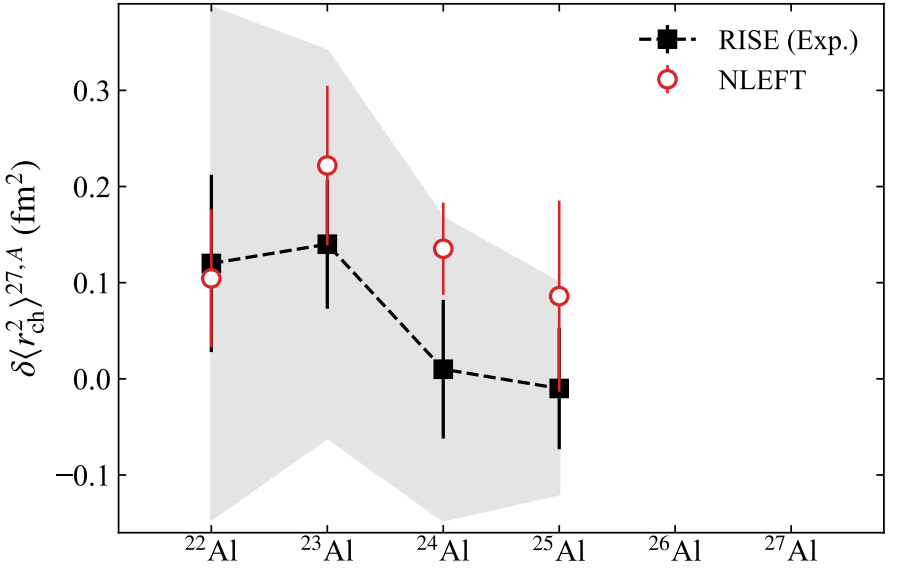
$\hookrightarrow$ NLEFT works quite well

Radii of aluminum isotopes at the proton drip line

Brinson et al., 2605.09139 [nucl-ex]

- Laser spectroscopy with RISE at FRIB → charge radius of Al isotopes towards the dripline
- Use the $\mathcal{F}_p = 8$ basis, modify two LECs [$V_{c_E}^{(l,t)}$] to get the exact BE's of ^{22}Al to ^{27}Al





↪ step-wise increase of the radii towards the dripline

↪ these trends are recovered by NLEFT

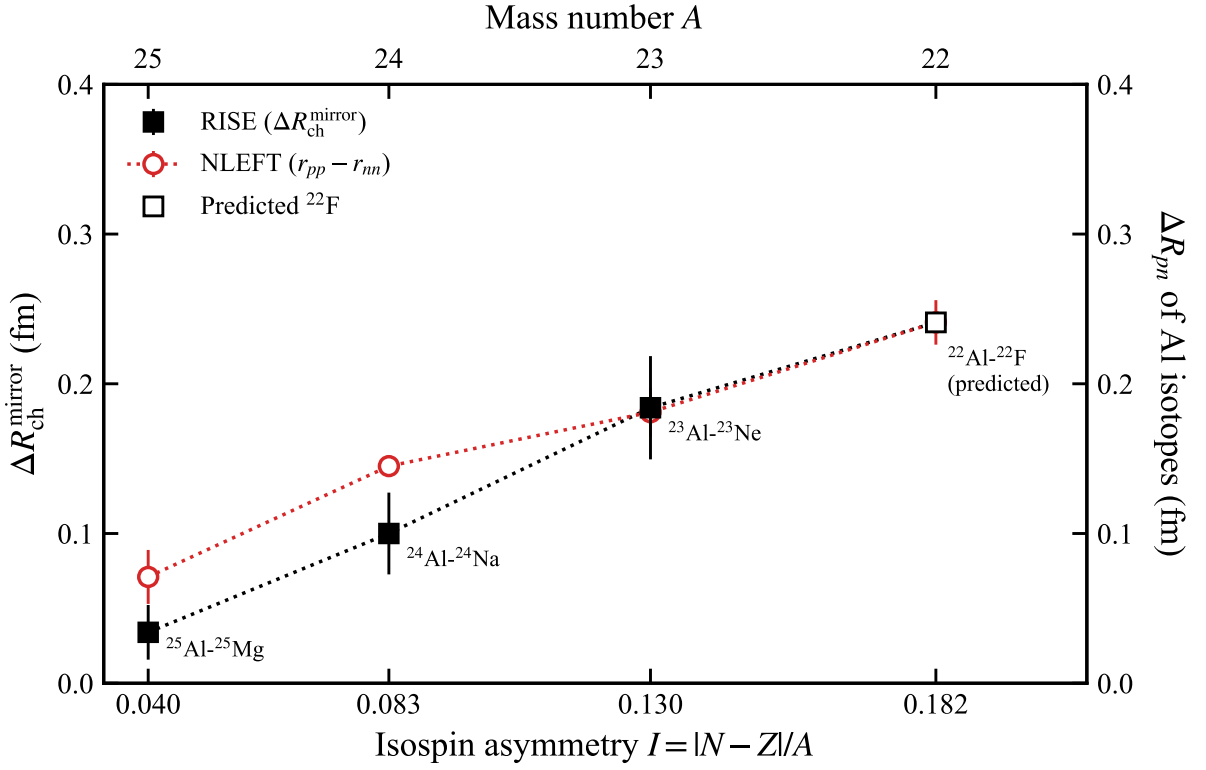
↪ excludes a proton halo in ^{22,23}Al as suggested by earlier studies (next slide)

Radii of aluminum isotopes at the proton drip line II

Brinson et al., 2605.09139 [nucl-ex]

- Can we understand these trends? More insights from mirror nuclei!
- Use the measured value for ^{23}Ne, ^{24}Na, ^{25}Mg

Ohayon, At. Data Nucl. Data Tables **165** (2025) 101732



$\hookrightarrow$ $\boldsymbol{\Delta R_{pn} = r_{pp} - r_{nn}}$ follows the trend from well bound nuclei $\rightarrow$ no halo for ^{22,23}Al

Towards precision calculations of nuclear β-decays

Prediction: Triton β-decay at N3LO

Elhatisari, Hildenbrand, UGM, Phys. Lett. B **859** (2024) 139086

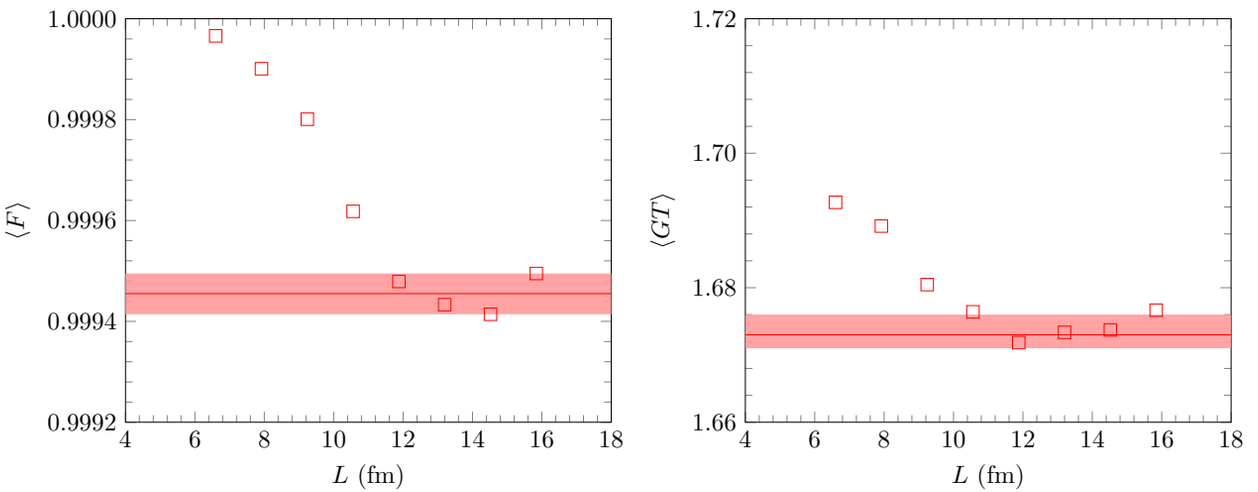
- Use the N3LO interaction and leading order currents
- Master formula: $(1+\delta_R)\, t_{1/2}\, f_V = \dfrac{K/G_V^2}{\langle \mathbf{F} \rangle^2 + \frac{f_A}{f_V}\, g_A^2\, \langle \mathbf{GT} \rangle^2}$
- Experiment: $\langle \mathbf{F} \rangle = \sum_{n=1}^{3} \langle {}^3\mathrm{He} \| \tau_{n,+} \| {}^3\mathrm{H} \rangle = 0.9998$ [theory!]

$$\langle \mathbf{GT} \rangle = \sum_{n=1}^{3} \langle {}^3\mathrm{He} \| \sigma_n \tau_{n,+} \| {}^3\mathrm{H} \rangle = 1.6474(23)$$

- NLEFT:

$$\langle \mathbf{F} \rangle_{\mathrm{N3LO}} = 0.99949(11)$$

$$\langle \mathbf{GT} \rangle_{\mathrm{N3LO}} = 1.6743(58)$$



↪ Important first step

↪ PKU group: less precise wfs but improved currents Wang, Feng, Lu, PRC **112** (2025) 025502

The anomalously long lifetime of ^{14}C

Wang, Elhatisari, Feng, Lu, UGM, 2607.15984 [nucl-th]

- Basis of all radiocarbon dating: $\boxed{\tau(^{14}\mathrm{C}) \simeq 5730\,\mathrm{yrs}}$

- The β-decay $^{14}\mathrm{C}(0_1^+) \to {}^{14}\mathrm{N}(1_1^+)$ is a challenge to nuclear theory

for a review, see Talmi, Physics **4** (2022) 940

$\hookrightarrow$ an allowd GT transition, but $M_{\mathrm{GT}}^{\mathrm{exp}} = 2 \times 10^{-3}$ not $\mathcal{O}(1)$

- Early attempts in the shell-model show the need of tensor forces (D wave in the g.s. $^{14}\mathrm{N}(1_1^+)$)

Jancovici, Talmi, PR **95** (1954) 289; Zamick et al., PRC **51** (1995) 1253, ...

- Modern *ab initio* calculations focus more on the 3NFs & higher order currents

Holt, Kaiser, Weise, PRC **79** (2009) 054331; Vary et al., PRL **106** (2011) 202502; Ekström et al. PRL **113** 262504

- Study all these effects consistently in NLEFT at N3LO

  $\hookrightarrow$ adopt the 3NFs to the region of interest

  $\hookrightarrow$ use multi-channel variational wf's as WFM converges too slowly here

  $\hookrightarrow$ use consistent currents

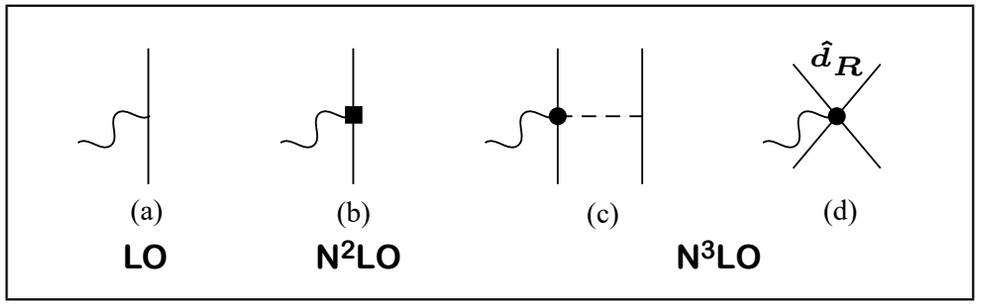
Currents and wave functions

Wang, Elhatisari, Feng, Lu, UGM, 2607.15984 [nucl-th]

- Transition operators and axial currents:

$$O^{\rm LO}_{{\rm GT},\lambda} = A^{\rm LO}_\lambda = -g_A \sum_{n=1}^{A} \sigma_\lambda(n)\tau_+(n)$$

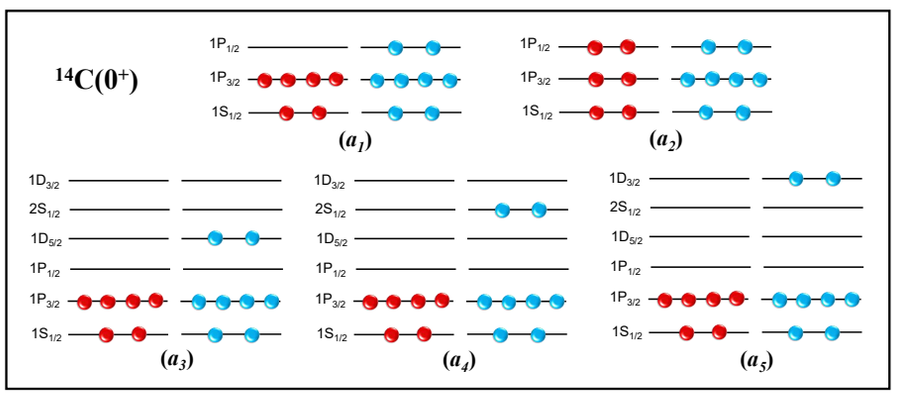
$$O^{>\rm LO}_{{\rm GT},\lambda} = A^{\rm N^2LO}_\lambda + A^{\rm N^3LO}_\lambda({\rm OPE}) + A^{\rm N^3LO}_\lambda({\rm CT})$$

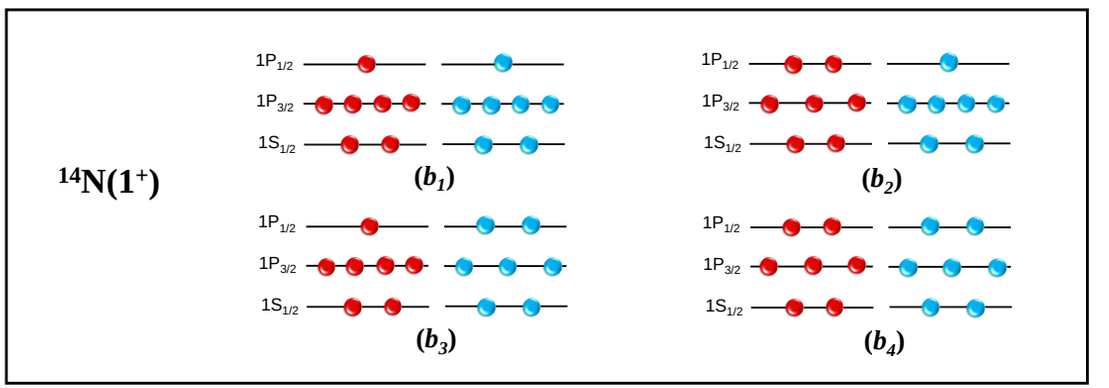


- LECs from RS analysis c_3, c_4 (OPE) and the relation $\hat{d}_R \sim c_D$ (CT) Park et al. (2003), Gazit et al. (2009), ...

- Wavefunctions: Simple Hamiltonian H_S too far from the true solution (H_s leads to $O(1)$)

  ↪ Superposition of many initial states to construct the g.s. wf's

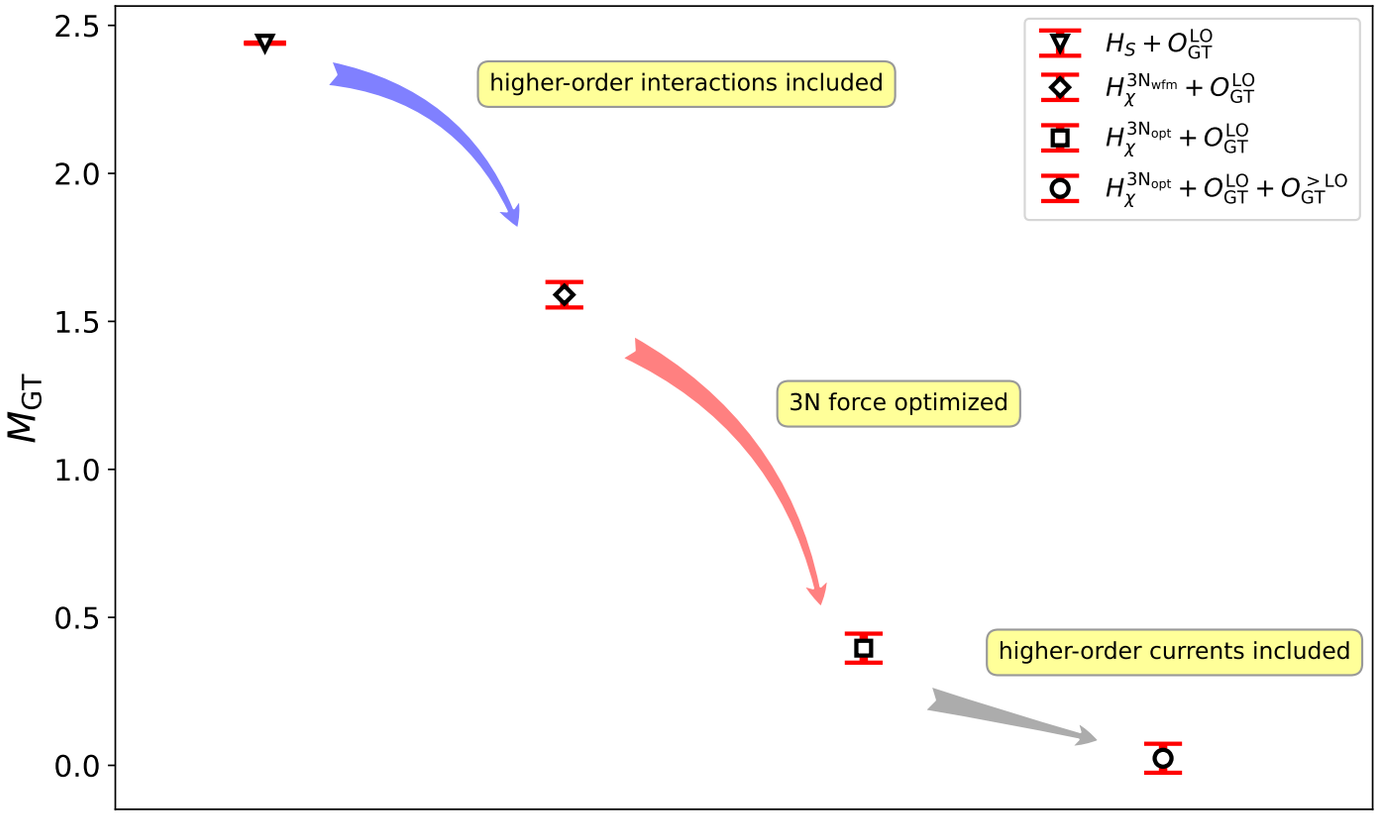




The long lifetime of ^{14}C explained

Wang, Elhatisari, Feng, Lu, UGM, 2607.15984 [nucl-th]

- Putting pieces together leads to the following picture $M_{\rm GT} = 0.02(5)$:



$\hookrightarrow$ statistically consistent w/ the exp result $M_{\rm GT}^{\rm exp} \approx 2 \times 10^{-3}$

$\hookrightarrow$ intricate interplay of nuclear structure, 3NFs and h.o. currents

Some calculational details

Wang, Elhatisari, Feng, Lu, UGM, 2607.15984 [nucl-th]

- Determine 3NFs from some masses up to ^{16}O

  plus the magnetic moment $\mu(^{14}\mathrm{N}(1^+_1))$

  and $\Delta E = E(^{14}\mathrm{N}(1^+_1)) - E(^{14}\mathrm{N}(1^+_2))$

  $\hookrightarrow$ still fine up to ^{40}Ca

- Only with $H_\chi^{3\mathrm{N}_{\mathrm{opt}}}$ there is a sizeable

  D-wave admixture in the ^{14}N g.s.

- Sensitivity analysis based on Sobol's

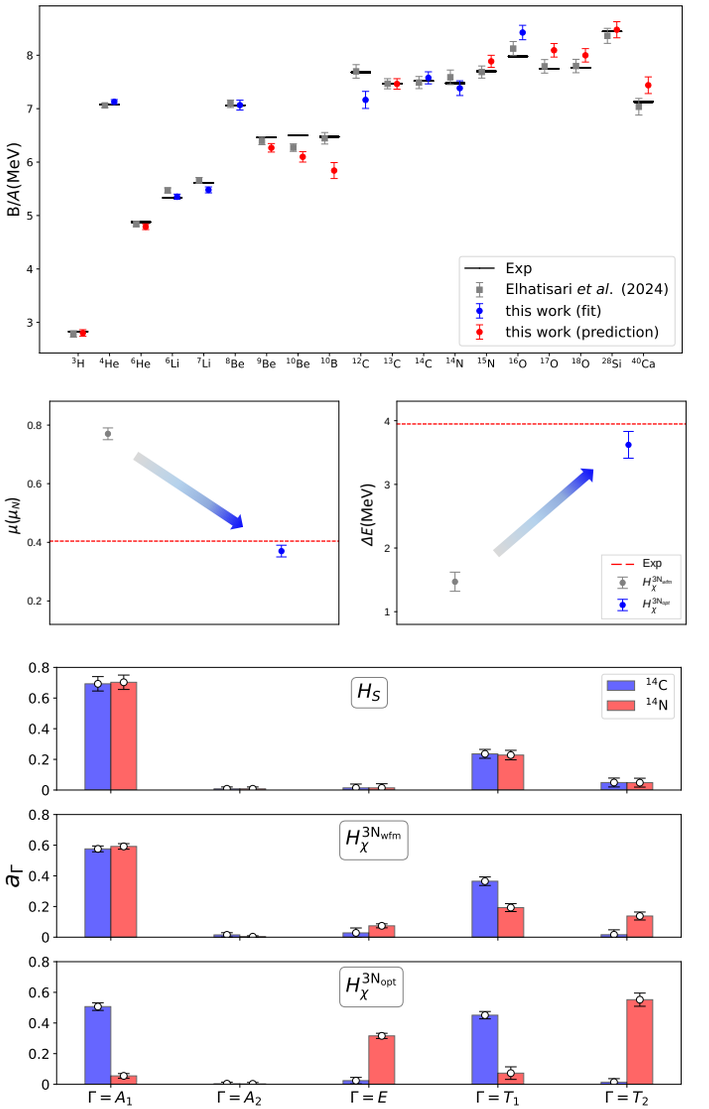
  global sensitivity analysis (GSA) reveals

  strongest sensitivity to the LEC c_D

  Sobol, Math. Comput. Simul. **55** (2001) 271

  Saltelli, Comput. Phys. Commun. **145** (2002) 280

$\Rightarrow$ towards high-precision β-decay studies



Summary & outlook

- NLEFT can provide precision calculations over a wide range of masses
- To-do-list:
  - ☐ improving the 3NFs (work under way)
  - ☐ fully include the TPE in the 2NFs (in the works)
  - ☐ systematic investigations of β-decays (started)
  - ☐ $\mathbf{0\nu2\beta}$ decays into the Ge region (started)
  - ☐ precision calculation of $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ (started)
  - ☐ strangeness and neutron star physics to higher precision

Hildenbrand et al., EPJA **60** (2024) 215; Tong et al., Sci.Bull. **70** (2025) 825; 2509.26148 [nucl-th]

- A short comment on Rothmann et al., 2606.12166 [nucl-th]: “It’s not even wrong”

It's not even wrong

Niu, Lu, Zhang, Ma, Elhatisari, Lee, UGM, 2607.06044 [nucl-th]

- The authors of 2606.12166 performed HF calculations with 2016/17 NLEFT interactions

  ↪ they claim that there are problems with published AFQMC calculations, variational bounds, correlation energies, and saturation mechanisms

- The central issue: In that paper they do not keep the regulated lattice problem fixed

  ↪ A regulated lattice interaction is not specified by its LECs or spatial Hamiltonian density alone.

  It is defined together with the regulator scheme used to calibrate it, including the spatial lattice spacing, temporal regulator or Hamiltonian limit, kinetic-energy discretization, smearing prescription, and operator content

- A separate issue arises in finite-volume benchmarks:

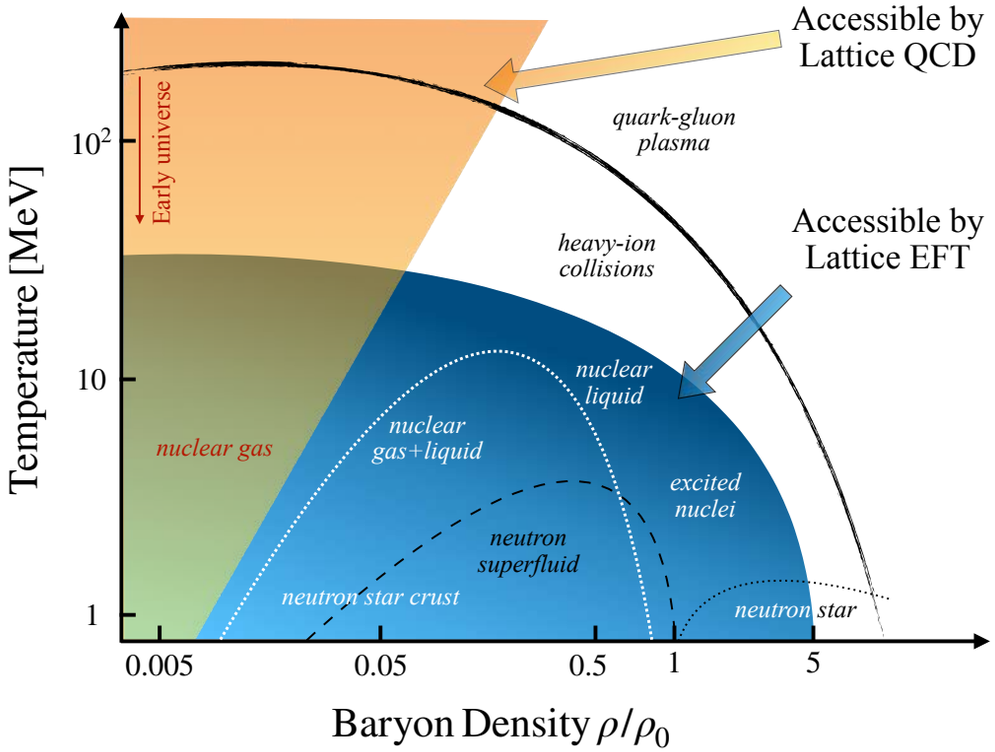
  ↪ Even for a fixed interaction, one must compare calculations at the same volume, particle number, and boundary conditions, or else account explicitly for the extrapolation to the infinite-volume or thermodynamic limit

- For details, please consult 2607.06044, where things are done correctly! [NLEFT standard]

SPARES

Comparison to lattice QCD

LQCD (quarks & gluons)	NLEFT (nucleons & pions)
relativistic fermions	non-relativistic fermions
renormalizable th'y	EFT
continuum limit	no continuum limit
(un)physical masses	physical masses
Coulomb - difficult	Coulomb - easy
high T/small ρ	small T/nuclear densities
sign problem severe	sign problem moderate



- For nuclear physics, NLEFT is the far better methodology!

Transfer matrix method

- Correlation–function for A nucleons: $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

  with Ψ_A a Slater determinant for A free nucleons

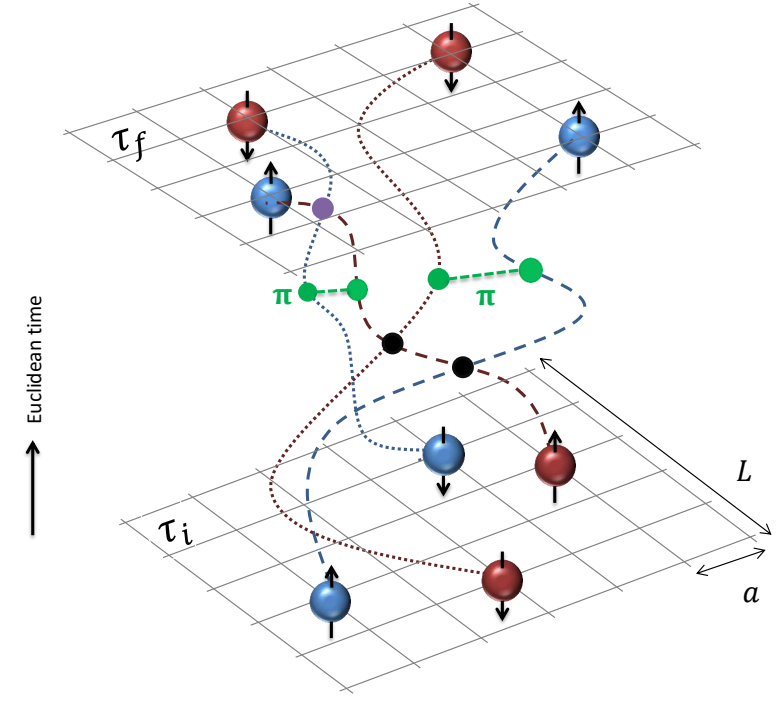
  [or a more sophisticated (correlated) initial/final state]

Euclidean time

- Transient energy

$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

$\rightarrow$ ground state: $E_A^0 = \lim_{\tau\to\infty} E_A(\tau)$



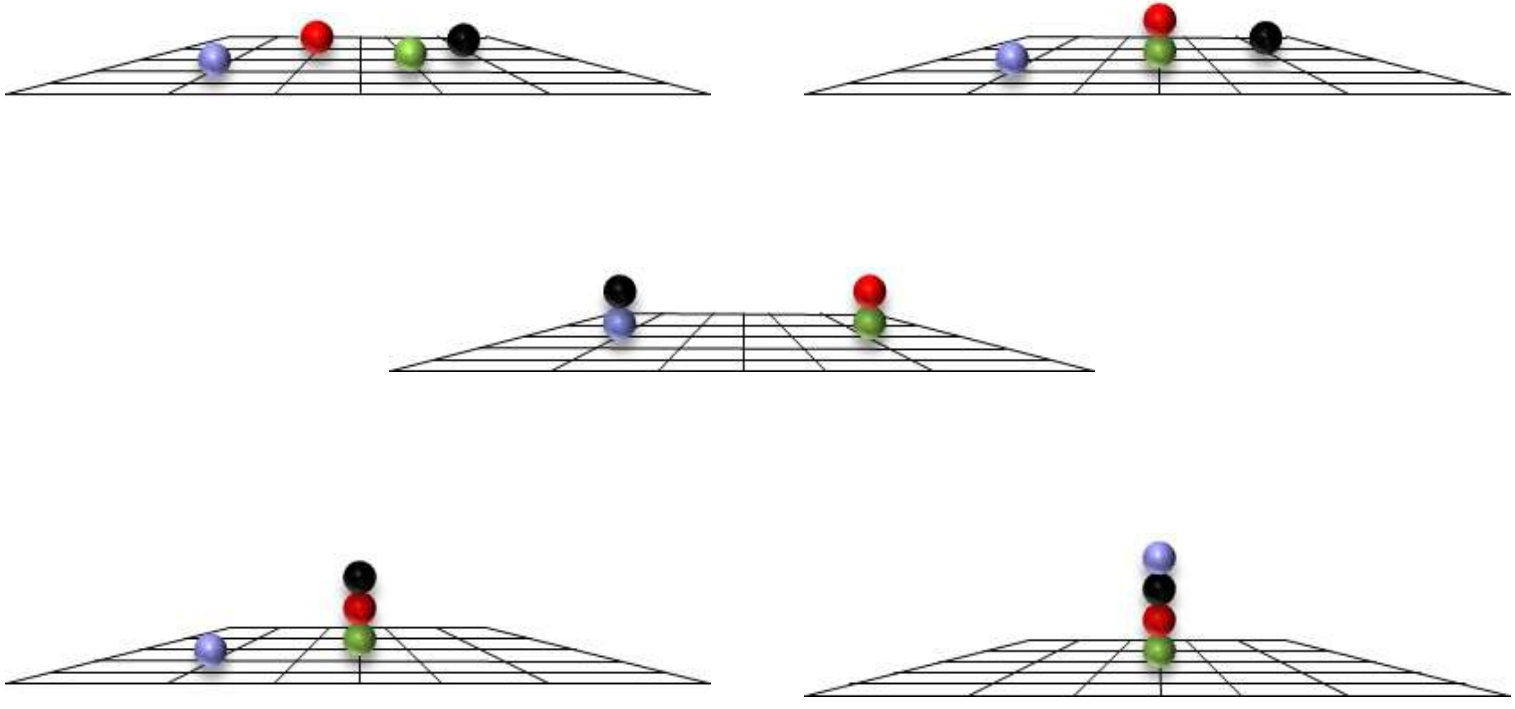
- Exp. value of any normal–ordered operator $\mathcal{O}$

$$Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-\tau H/2)\, \mathcal{O}\, \exp(-\tau H/2)\, | \Psi_A \rangle$$

$$\lim_{\tau\to\infty} \frac{Z_A^{\mathcal{O}}(\tau)}{Z_A(\tau)} = \langle \Psi_A | \mathcal{O} | \Psi_A \rangle$$

- Excited states: $Z_A(\tau) \to Z_A^{ij}(\tau)$, diagonalize, e.g. $0_1^+, 0_2^+, 0_3^+, \ldots$ in ^{12}C

Configurations



$\Rightarrow$ all *possible* configurations are sampled

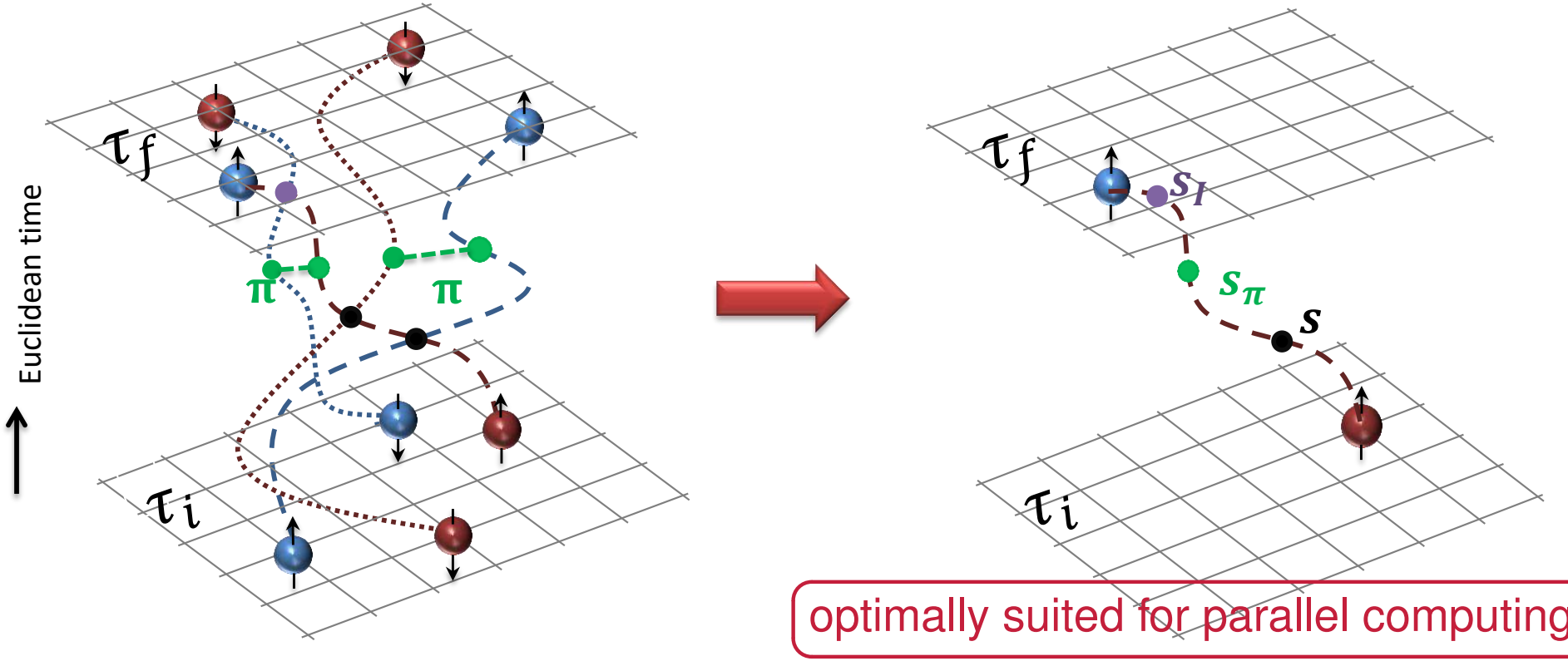
$\Rightarrow$ preparation of *all possible* initial/final states

$\Rightarrow$ *clustering* emerges *naturally*

Auxiliary field method

- Represent interactions by auxiliary fields (Gaussian completion):

$$\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}}\int ds \exp\left[-\frac{s^2}{2} + \sqrt{C}\; s\left(N^\dagger N\right)\right]$$



optimally suited for parallel computing!

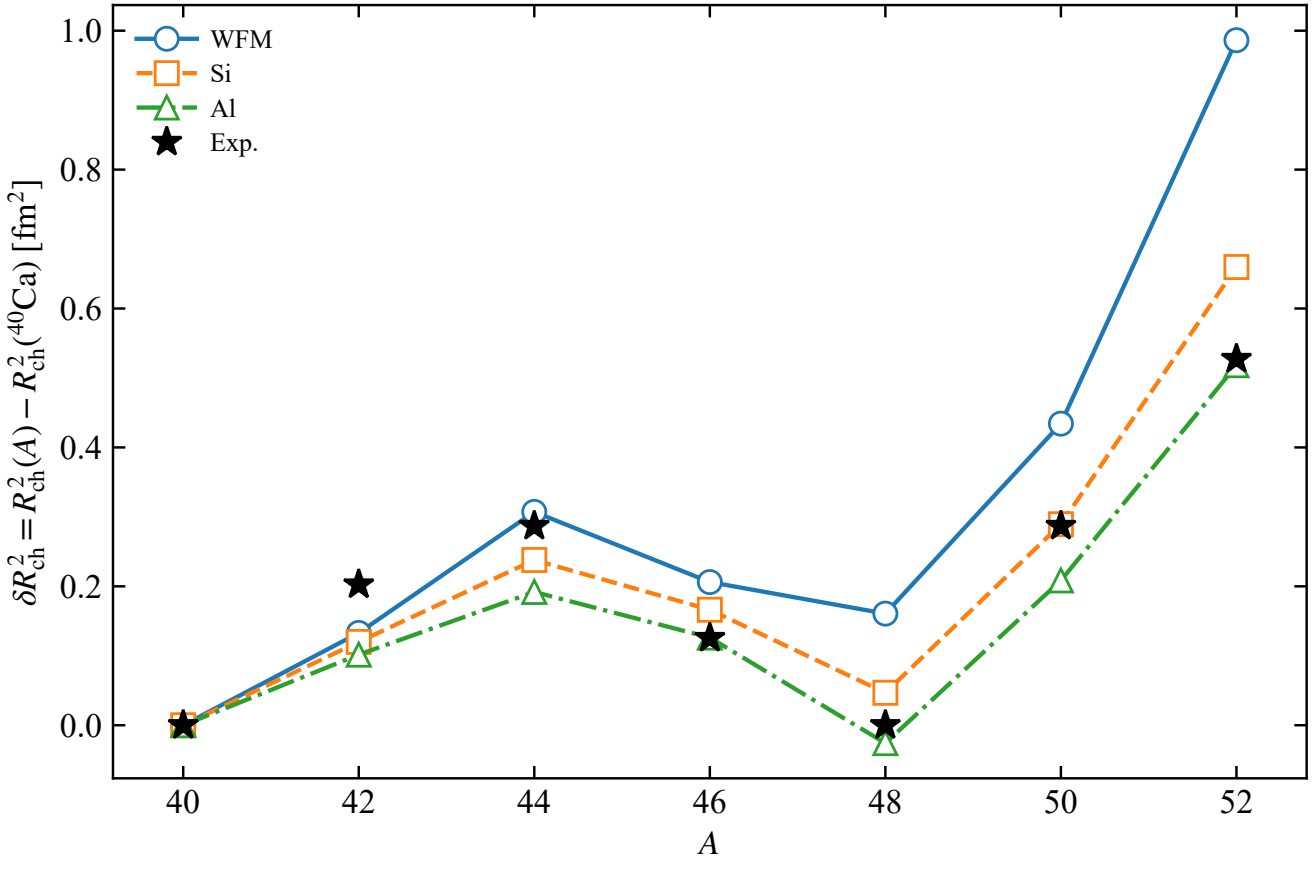
Computational equipment

- Present = JUPITER + FRONTIER + ...

Radii of calcium isotopes

Zhang et al., in preparation

- Study the radii of the Ca isotopes from $A = 40$ to $A = 52$:

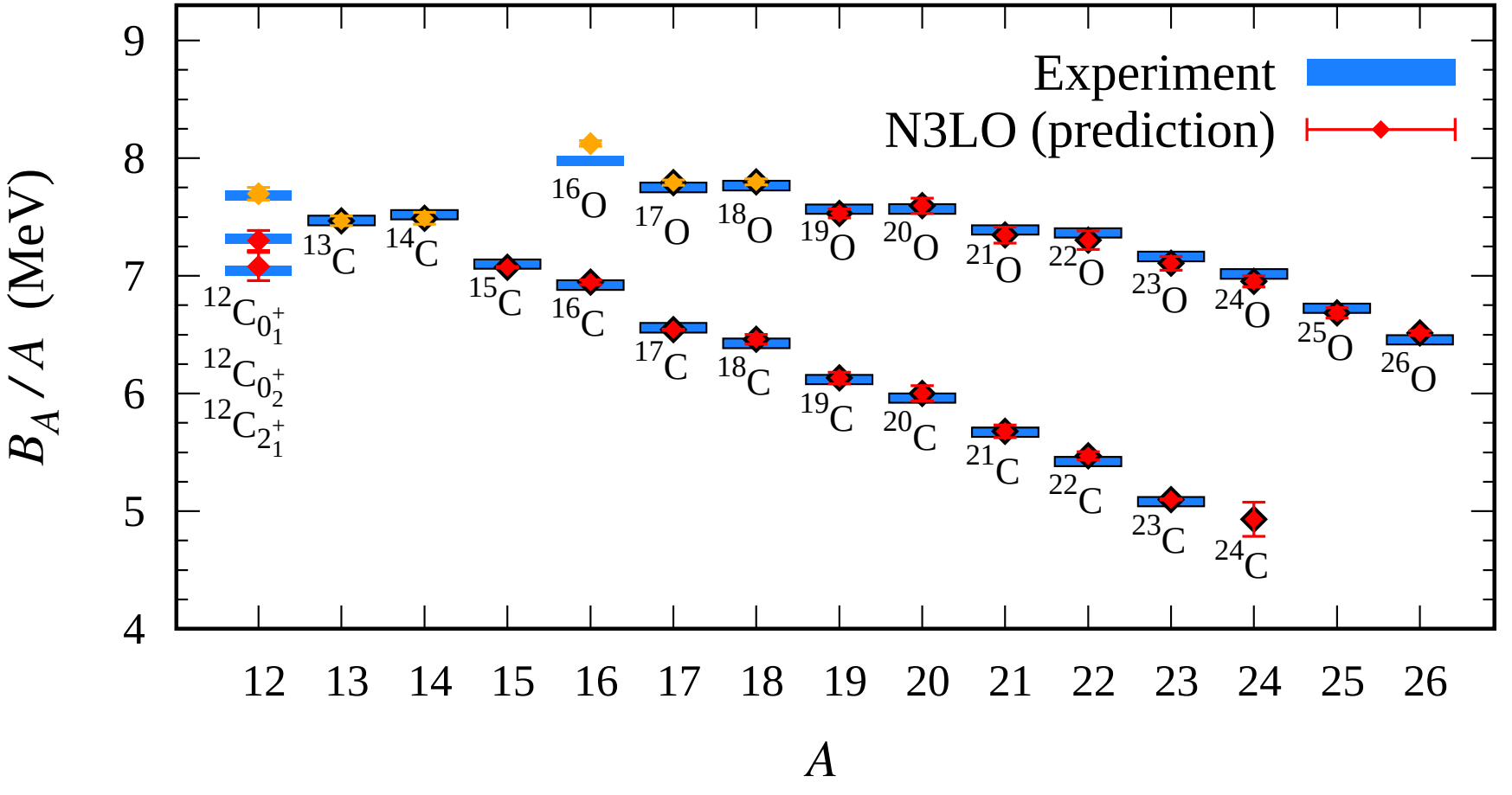


$\hookrightarrow$ identical trends, some overall shifts $\rightarrow$ improve 3NFs

Prediction: Isotope chains of carbon & oxyen

Song et al., Phys. Lett. **B 872** (2026) 140086

- Towards the neutron drip-line in carbon and oxygen:



↪ 3NFs of utmost importance for the n-rich isotopes!

↪ universal features of neutron correlations

Prediction: Neutron-alpha scattering

Elhatisari, Hildenbrand, UGM, J. Phys. **G 52** (2025) 125102

- n-α scattering know to be a good testbed of 3NFs

  Pieper, Pandharipande (1993), Quaglioni, Navratil (2008), Kravvaris et al. (2020), ...

- Use the time-honoured Lüscher approach

- Compare to R-matrix results

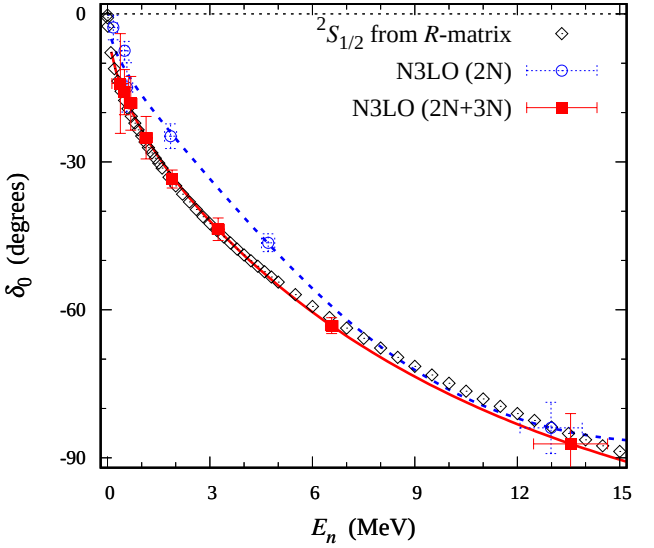
  G. M. Hale, private communication (2023)

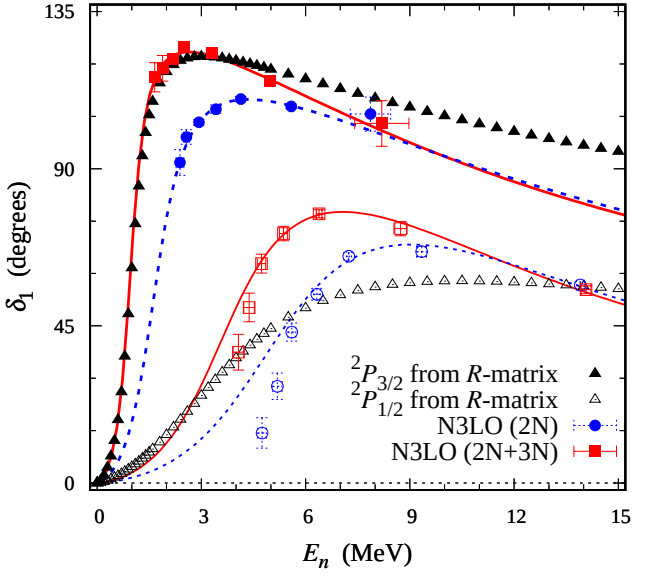
$\hookrightarrow$ S-wave ($^2S_{1/2}$) well described when 3NFs are included

$\hookrightarrow$ Large $^2P_{3/2}$ well described when 3NFs are included

$\hookrightarrow$ Visible deviation in the $^2P_{1/2}$ wave for $E_n \geq 3$ MeV

- What is the reason for the deviations in the $^2P_{3/2}$-$^2P_{1/2}$ splitting?



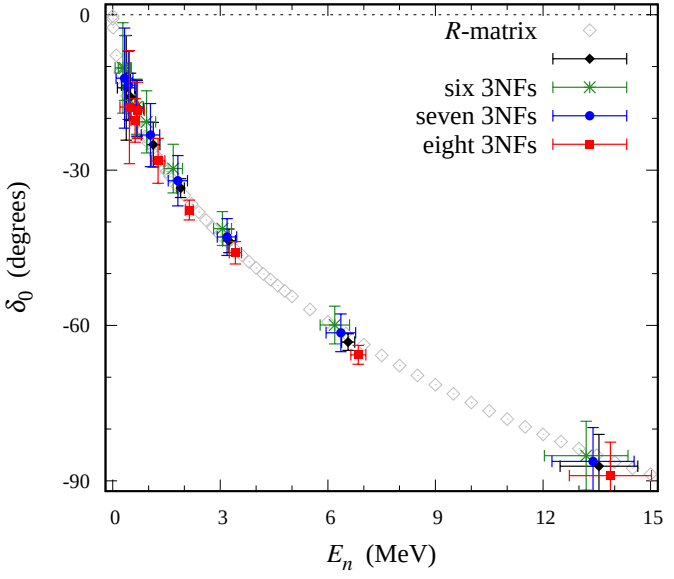


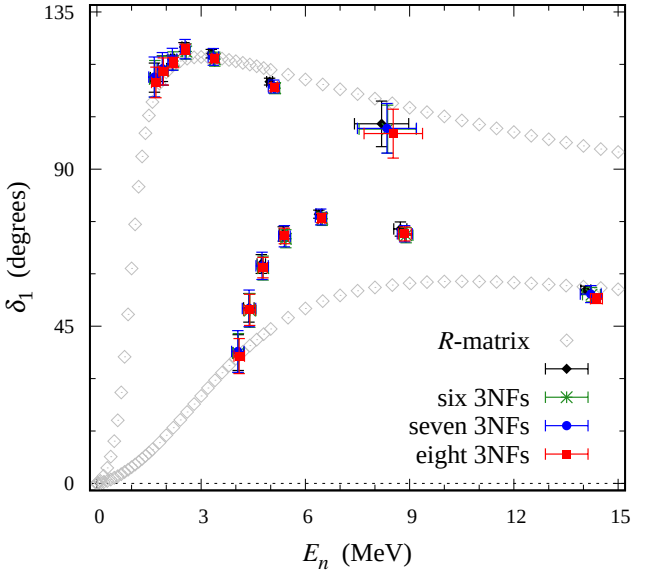
Neutron-alpha scattering revisited

Elhatisari, Hildenbrand, UGM, J. Phys. **G 52** (2025) 125102

- Investigate one possible reason: Choice of nuclei to fix the LECs?

- Perform MCMC searches to explore systematically the set of 8 3NF ops
  and also reduce to sets with 6 and 7 ops and quantify the uncertainty using RSMD





$\hookrightarrow$ clearly, this is not the source of the problem

$\hookrightarrow$ must improve the 3NFs!

Prediction: Multi-neutron correlations in light nuclei I

Zhang, Elhatisari, UGM, 2512.18849 [nucl-th]

- $3n$ and $4n$ clusters much thought about

↪ breakup, knockout & double charge exchange reactions

↪ investigate multi-n correlations in light nuclei

- Use 282 chiral interactions to predict:

$$E_{\rm gs}(^6{\rm H}) = -4.94 \pm 0.14\,{\rm MeV}$$

$$E_{\rm gs}(^7{\rm H}) = -5.29 \pm 0.16\,{\rm MeV}$$

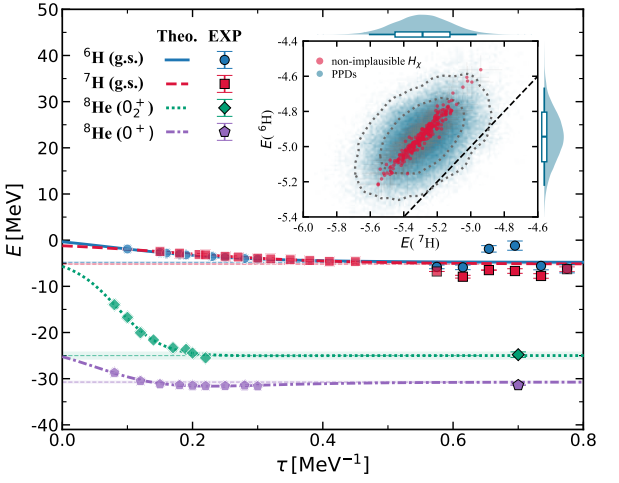
- Marginal posteriors suggest:

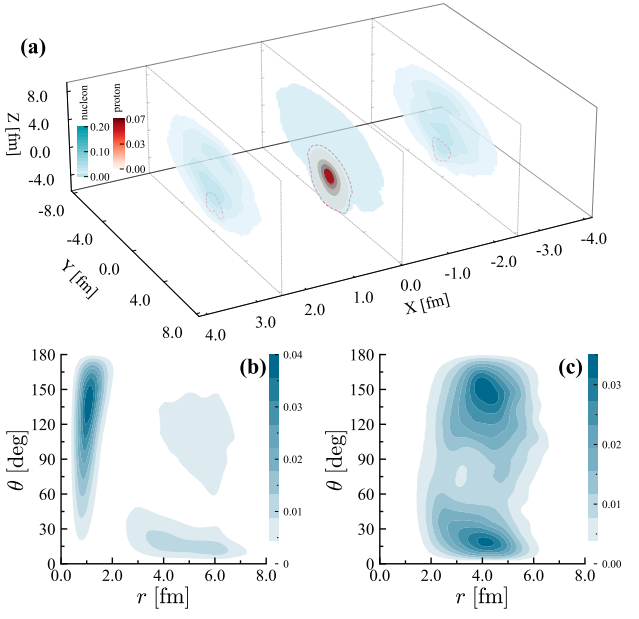
$$S_n(^7{\rm H}) = 0.35 \pm 0.32\,{\rm MeV}$$

↪ sequential ^{6}H+n decay disfavored

↪ enhances the relevance of multi-n correlations

↪ two-neutron correlations clearly visible



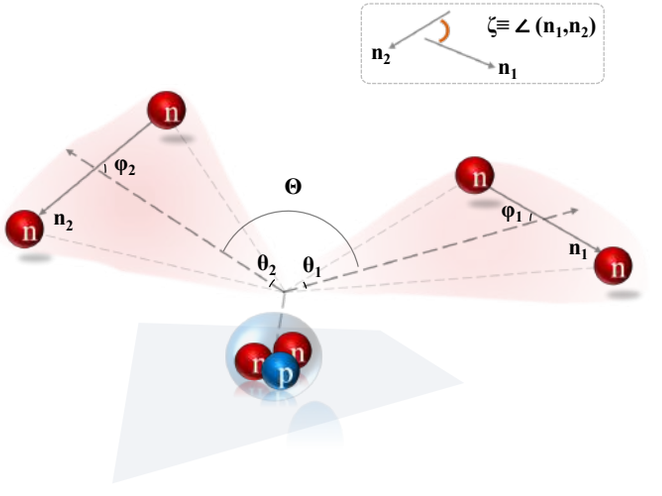


Prediction: Multi-neutron correlations in light nuclei II

Zhang, Elhatisari, UGM, 2512.18849 [nucl-th]

- Define four-neutron correlations via:

$$\rho_4(\theta_1,\varphi_1,\theta_2,\varphi_2;\Theta,\zeta) = \sum_{i<j,\,k<l}^{(ij),(kl)} \Big\langle \delta(\theta_{ij}-\theta_1)\,\delta(\varphi_{ij}-\varphi_1) \times \delta(\theta_{kl}-\theta_2)\,\delta(\varphi_{kl}-\varphi_2) \times \delta(\Theta_{ij,kl}-\Theta)\,\delta(\zeta_{ij,kl}-\zeta)\Big\rangle,$$

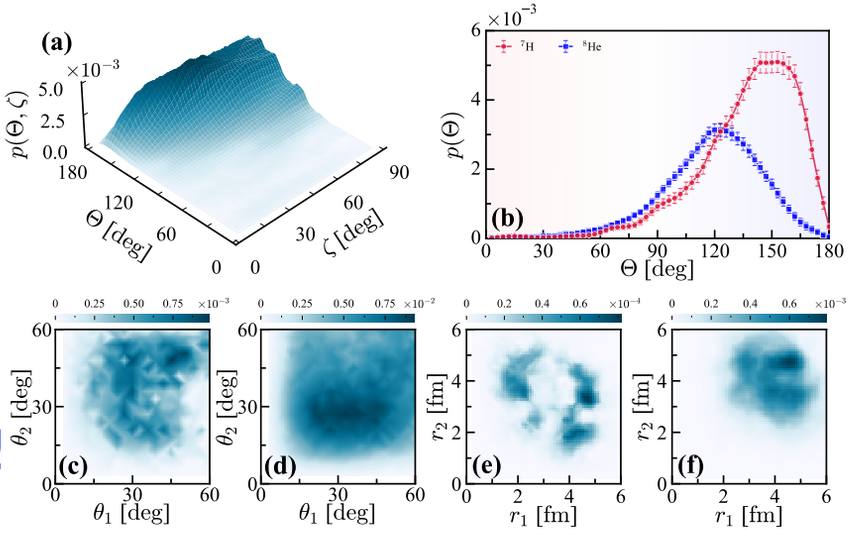


- The four-neutron correlations show:

↪ symmetric $2n$-$2n$ configuration is dominant
$(\sim 95\%,\ \ \Theta \simeq 140^\circ - 160^\circ)$

↪ compact tetraneutron configuration small
$(\sim 5\%,\ \ \Theta \simeq 60^\circ - 90^\circ)$

↪ must consider the $2n$-$2n$ config. in exp analysis!



Prediction: Searching for the tetraneutron

Wu, Elhatisari, UGM, Shen, Geng, Kim, 2601.01801 [nucl-th]

- Latest tetraneutron sighting in ^{4}He(^{8}He,^{8}Be)$4n$

  Duer et al., Nature **606** (2022) 678

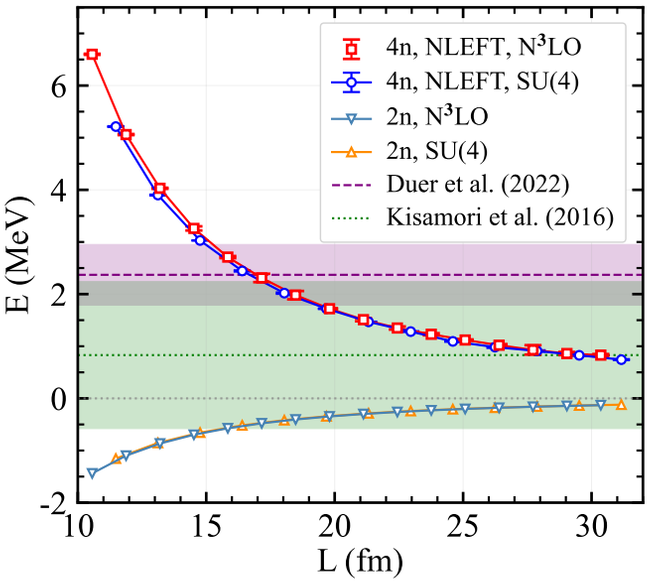
↪ analysis presumably too simplified,
various re-analyses show no resonance

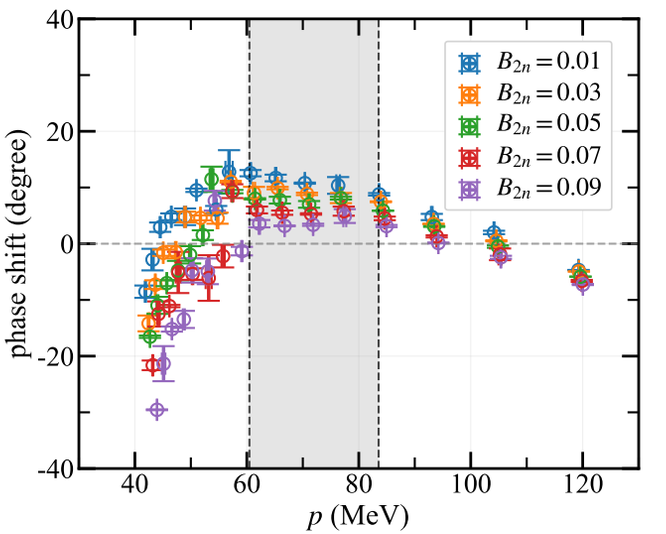
↪ most theory analyses do not give a $4n$ resonance

- Search on the lattice (SU(4) and N3LO interactions)

↪ volume-dependence shows no sign of a resonance

↪ $2n$-$2n$ scattering shows a shallow attraction
in the momentum range $p \simeq 60 - 85$ MeV
corresponds to confined $4n$ energies of $1.7 - 3.3$ MeV
similar to the $4n$ peak energies in experiment





Multi-strangeness matter

Multi-strangeness matter: Formalism

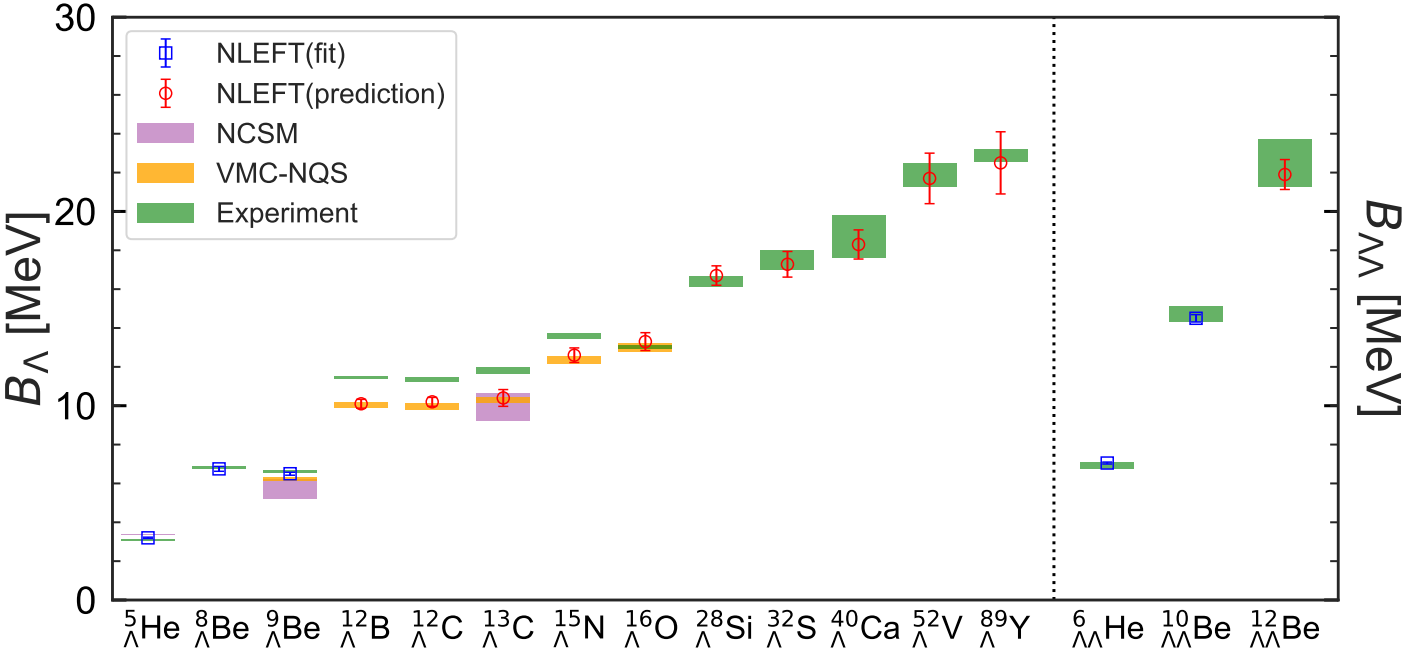
Tong, Elhatisari, UGM, Sci.Bull. **70** (2025) 825; Astrophys.J. **982** (2025) 164; +Ren, 2509.26148

- Neutron stars gained prominence in the multi-messenger area (e.g. the hyperon puzzle)

↪ construct a minimal model that captures the essential features of hypernuclei and neutron stars

- Hamiltonian: $H = H_{\rm free} + V_{NN} + V_{N\Lambda} + V_{\Lambda\Lambda} + V_{NNN} + V_{NN\Lambda} + V_{N\Lambda\Lambda}$

↪ fix LECs from scattering data, symmetric nuclear matter and hypernuclei



NCSM: Knöll, Roth, PLB **846** (2023) 138258; VMC-NQS: DiDonna et al., PRR **8** (2026) 013160

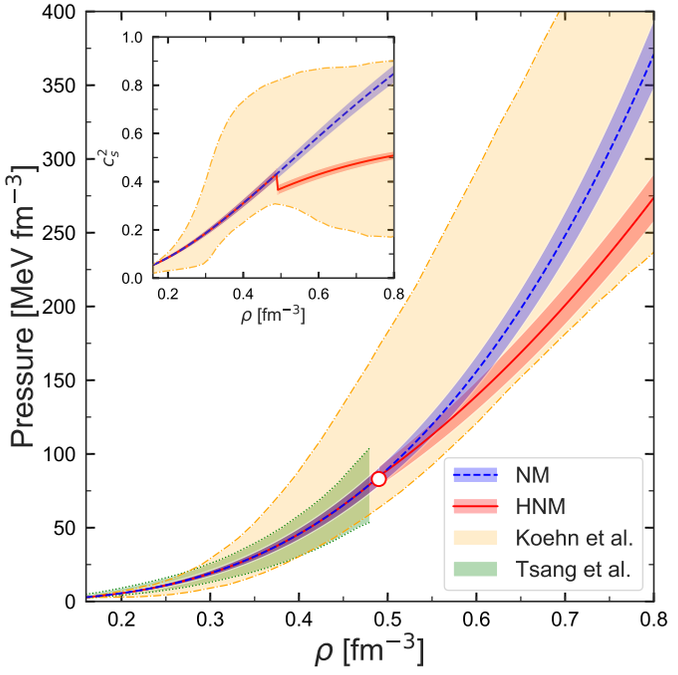
Neutron star properties

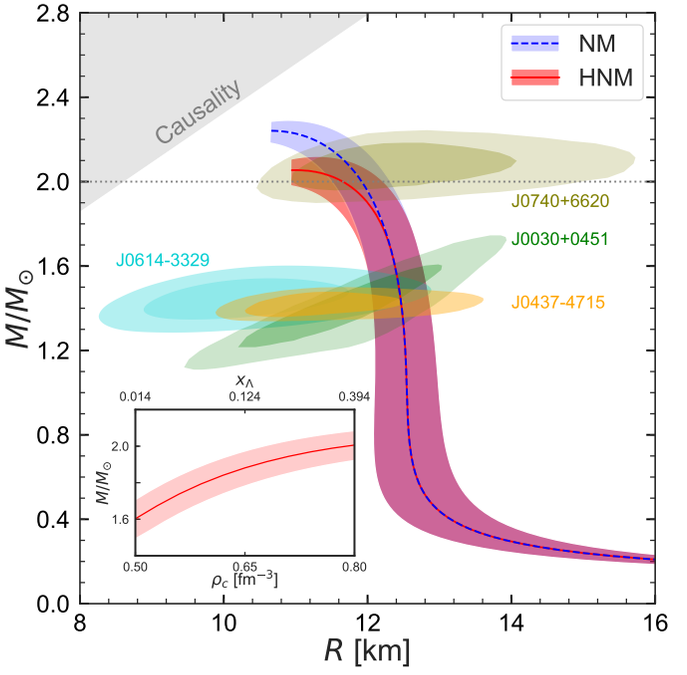
Tong, Elhatisari, UGM, Sci.Bull. **70** (2025) 825; Astrophys.J. **982** (2025) 164; + Ren, 2509.26148

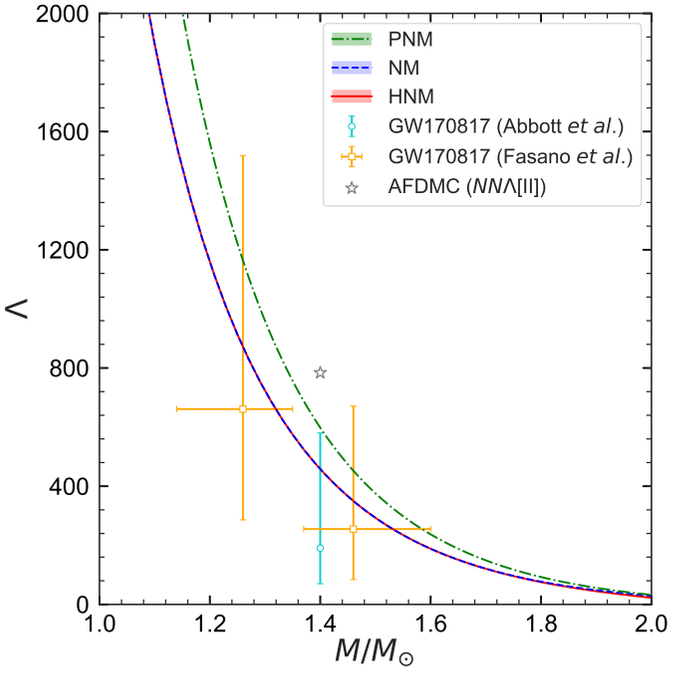
- New AFQMC algorithm allows to consider up to 232 neutrons, 25 protons and 50 Λ's

↪ parameter-free prediction of β-stable matter

↪ consistent with all experimental and deduced constraints







Koehn et al., PRX **15** (2025) 021014;
Tsang et al., Nature Astron. **8** (2024) 328;

Vinciguerra et al., Astrophys. J. **961** (2024) 62;
Salmi et al., Astrophys. J. **974** (2024) 294;
Choudhury et al., Astrophys. J. Lett. **971** (2024) L20
Mauviard et al., Astrophys. J. **995** (2025) 60

Abbott et al., PRL **121** (2018) 161101
Fasano et al., PRL **123** (2019) 141101

Neutron star properties II

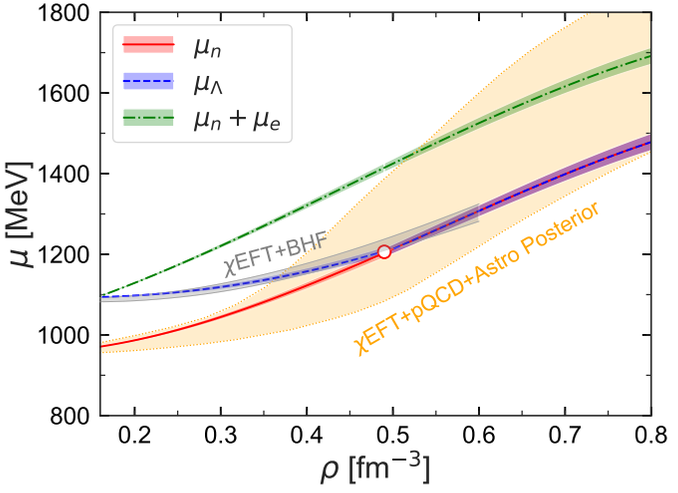
Tong, Elhatisari, UGM, Sci.Bull. **70** (2025) 825; Astrophys.J. **982** (2025) 164; + Ren, 2509.26148

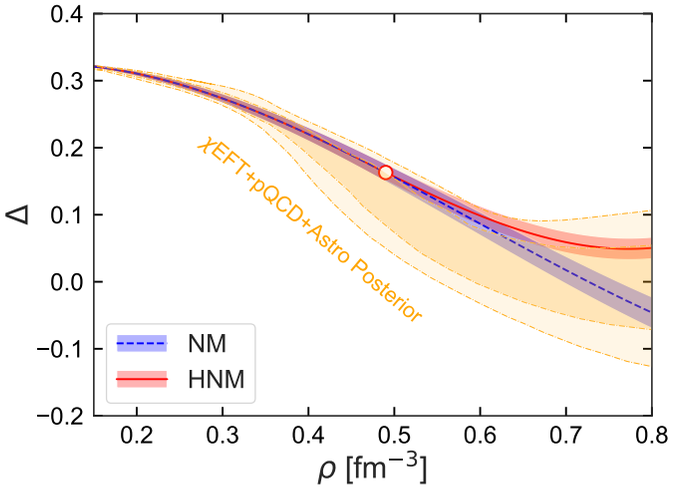
- Λs appear at $\rho_\Lambda^{\rm th} \simeq 0.490(0.002)(0.005)$ fm^{-3}

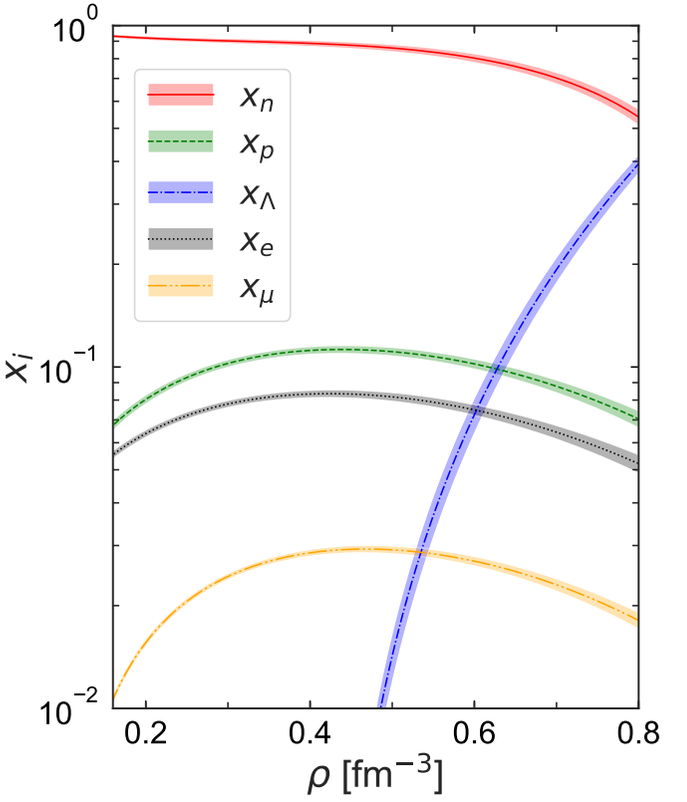
↪ so hyperons can exist peacefully inside neutron stars

↪ proton fraction barely exceeds 10%

- Chemical potential μ_n and trace anomaly Δ also consistent







Brandes, Weise, PRD **111** (2025) 034005 (2025);

Annala et al. Nature Commun. **14** (2023) 8451

Annala et al. Nature Commun. **14** (2023) 8451XXX

⇒ Very promisng results, now extend interactions to high-fidelity forces